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## What Next for $r^*$ ? A Capital Market Equilibrium Perspective on the Natural Rate of Interest

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# What next for $r^*$ ?

## A capital market equilibrium perspective on the natural rate of interest

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### Abstract

The natural rate of interest ( $r^*$ ) is a central benchmark for monetary and fiscal policy, shaping decisions by firms and households across the economy. In advanced economies,  $r^*$  declined steadily for half a century, but the recent sharp increases in market-based real rates have raised the prospect of a turnaround. This paper revisits the long-run decline in real rates through a structural model in which  $r^*$  equilibrates the supply and demand in the capital market. I show that advanced economies have undergone profound shifts in both capital demand but – especially – capital supply, driving a fall in  $r^*$  of about 5 percentage points and nearly doubling the wealth-to-GDP ratio since 1970, despite weak investment and low saving. To trace the dynamics, I construct a limited-foresight transition in which agents notice and understand the shocks hitting the economy only as they arrive. I argue this is a more plausible alternative to the usual perfect foresight assumption. The model’s business-as-usual scenario computed this way implies  $r^*$  of around 0%-1.2% today, and points to a small gentle decline over the years to come. But  $r^*$  could instead rise, due to new forces such as AI, deglobalization, or a persistent fall in the safety premium following the recent inflation episode. The model is capable of generating a sharp turnaround in the path for the natural rate, but for  $r^*$  to return to pre-2008 levels, several upside risks to  $r^*$  likely need to crystallize all at once.

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# 1 Introduction

How high will the costs of financing government debt be over the long term? What will mortgages cost in the future, and how will this affect house prices? What is the opportunity cost of capital investment? How will asset valuations evolve?

None of these questions – critical to the decision-making of central bankers, finance ministers, households, firms, and financial market participants – can be answered without forming a view on the natural rate of interest, or  $r^*$ . Up until recently, interest rates have been trending down for three decades, and a large literature has studied the causes and consequences of this decline, ascribing much of it to the fall in the natural rate. Over the same period, the value of accumulated wealth in relation to income has nearly doubled, despite subdued saving and investment flows.

Recently, this long-running trend came to a sharp end: since the post-Covid inflation, long-term real interest rates in the financial markets have risen sharply, often by as much as the short-term policy rates (Figure 1), suggesting that the markets price in the possibility that higher interest rates will be the new normal.

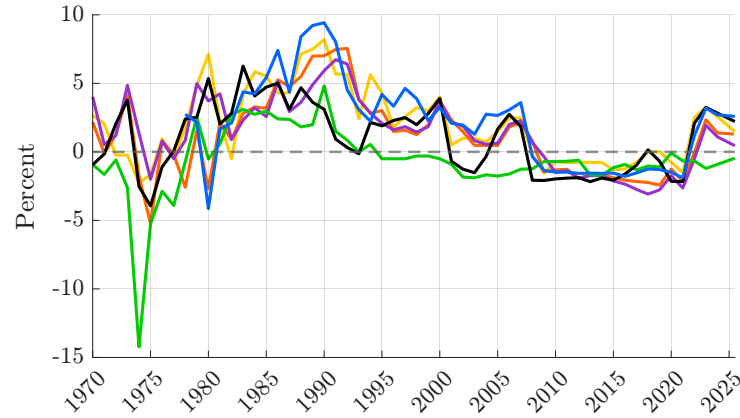
This paper constructs a framework for understanding the joint dynamics of  $r^*$  and the wealth-to-GDP ratio in the advanced economy bloc centered on capital market equilibrium and assesses the possibility that the era of low and falling  $r^*$  – one that Summers (2015) described as secular stagnation – has come to an end.

At the core of the analysis is a general equilibrium model of a macro-economy that is able to capture many of the most important trends relevant for saving and investment decisions discussed in the literature. These *forces* include: the changing outlook for future living standards, population and demographic dynamics, shifts in market power and risk premia, technological shifts in capital depreciation rates and capital intensity of production, changes in government fiscal and social policies, and fluctuations in foreign demand for advanced economy assets. The goal of studying these phenomena jointly is to establish a unified and quantitatively plausible narrative as to the absolute and relative influences over the past, and the likely decisive drivers over the future. The advantage of doing so through the lens of the capital market equilibrium is that this imposes additional discipline: the narrative must reconcile not only the decline in  $r^{*1}$  but also the doubling of the wealth-to-GDP ratio over

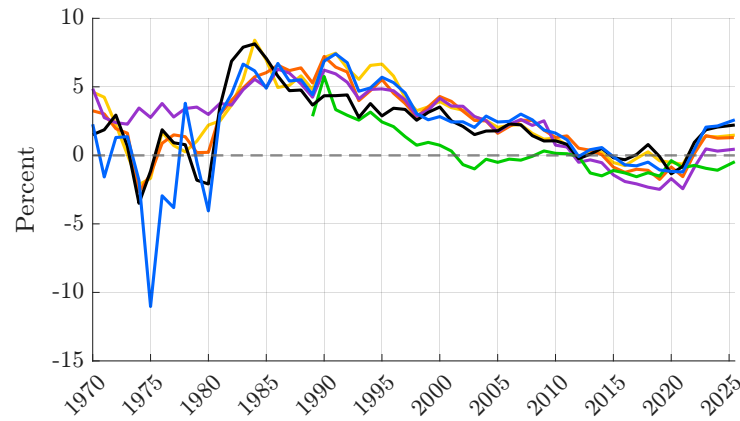
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<sup>1</sup>There are several concepts for the natural rate and it is useful to be clear upfront about the ones that are studied here. The various  $r$ -stars differ in the horizon in question (short-term / business cycle vs. medium run vs. long run / steady state), risk characteristics (safe vs. risky vs. effective, or weighted), or geography (global, regional, or country-level). In this paper,  $r^*$  stands for the *medium-to-long-term safe real rate of interest*, which is quantified for the advanced economy bloc. This is the equilibrium interest rate that, on the

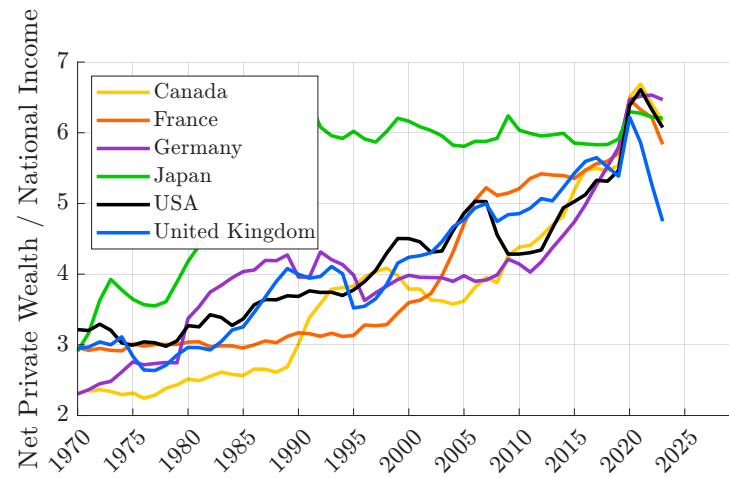
Figure 1: Real interest rates and wealth-to-income ratios in selected advanced economies



(a) Policy rates



(b) 10-year government yields



(c) Wealth-to-income ratio in advanced economies

*Notes:* Real interest rates in Panels (a) and (b) are constructed as follows: from 1991 onwards, real rates are the corresponding nominal rates minus the 5-year ahead IMF World Economic Outlook inflation forecast. Prior to 1991, actual inflation outturns are used. Panel (c) shows the ratio of net private wealth to national income from the World Inequality Database.

*Sources:* IMF WEO, FRED, World Inequality Database.

the same period.<sup>2</sup>

I proceed in four steps: first, within the GE model I write down, I characterize the steady state equilibrium in the capital market as the intersection of the *long run capital demand and supply curves*. Second, I show how these curves looked in around 1970 in the industrialized economies, how they have changed, and why. Having described the corresponding steady states, I compute the full transition path between them, pinning down the level of  $r^*$  today as well as the future path in the business-as-usual scenario, and decomposing the dynamics into the various exogenous drivers. Finally, motivated by the recent market moves, I quantify the impact of six common upside risk narratives popular among the commentators (de-globalization, re-militarization, AI, intangibles, debt-fueled rise in social security, and persistent inflation risk) to see whether any or all of them jointly can plausibly generate a sharp turnaround in the natural rate.

The capital demand – capital supply framework<sup>3</sup> provides a very useful way of understanding the economics, and quantitative effects, of the structural shifts that took place in the industrialized world over the past 50 years and as such lets us go beyond the purely numerical results in the literature. Specifically, I explain why some of the forces move only one of the schedules, while others shift both, and why some of the drivers shift the schedules only horizontally, while others move them vertically too. Conceptually, the framework is useful for reconciling the flow-based (saving/investment) intuitions with the stocks-based (wealth/capital) approach (as elegantly shown by [Auclert et al. \(2024\)](#)). Moreover, because I use quantified, model-based schedules and not just a stylized representation, I am able to take account of the nonlinearities (which are behind the sometimes mysterious but quantitatively significant interaction effects between the various forces). Furthermore, I show that the elasticities of the capital demand and supply schedules are time-varying and respond to the structural shifts, providing a cautionary tale for approaches that rely on fixed slope estimates to calculate the equilibrium impact of exogenous shifts.

I find that the structural changes have led to large shifts in the two schedules, and one hand, abstracts from nominal rigidities (and so is applicable at frequencies lower than a business cycle), but, on the other hand, moves in response to real, or structural, changes to the economic environment. Once all structural changes work their way through the economy,  $r^*$  settles at its steady state value. Below I also characterize the risky natural rate of interest and the weighted combination of the two returns.

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<sup>2</sup>As well as subdued net saving rates and net investment-to-GDP ratios.

<sup>3</sup>The capital demand schedule  $\mathcal{D}(r)$  tells us the aggregate capital absorption by firms and government at any interest rate  $r$  in the long run. The capital supply schedule  $\mathcal{S}(r)$  instead answers the following question: what would the value of aggregate wealth accumulated by households (relative to income) in the long run be if the interest rate was equal to  $r$ ?

especially in the capital supply  $\mathcal{S}(r)$ .<sup>4</sup> As a result, the model-implied steady state safe  $r^*$  has fallen from circa 5% in the 1970 steady state to essentially 0% in the new long-run steady state. The model generates a large increase in the steady state wealth, from 3 to over  $6 \times$  annual GDP, while at the same time predicting lower steady state net investment and saving rates.

In order to give a steer on  $r^*$  at any given point in time (not least answer the question: what is  $r^*$  right now?), research on  $r^*$  must go beyond steady state analysis and consider the transition between steady states. This is because the forces shifting the equilibrium arrive gradually and unexpectedly, and the adjustment of the economy is not instantaneous (and may be non-monotonic). The literature that computes transitions usually relies on perfect foresight. But this assumes that agents perfectly foresee the entire path of future shocks – for example, agents anticipate and react to a growth slowdown long before it occurs. A common result is a smooth and monotonic path for  $r^*$  that might not be entirely plausible. A contribution of this paper is to compute transitional dynamics under the more realistic assumption that agents have limited foresight and are surprised by new shocks that arrive over time.<sup>5</sup> I show that computing the transition this way makes a (quantitatively significant) difference to the estimate of  $r^*$ : specifically,  $r^*$  declines faster than under perfect foresight.

In what I call the business-as-usual scenario (i.e. without taking into account any of the recently debated forces that could drive a turnaround), safe real  $r^*$  in 2025 in advanced economies stands at around 0.6%. Caution is warranted: the point estimate is highly uncertain and sensitive to the precise calibration. Reflecting these sensitivities, it might be more helpful and robust to think of the model as predicting a range for current  $r^*$ , between 0% and 1.2% (and even around this range there is a lot of uncertainty).

A decomposition of the dynamics of  $r^*$  along the transition shows, unsurprisingly, the important role played by the growth slowdown and demographics. More novel, the framework illustrates the importance of the rise in mark-ups, which temporarily depress  $r^*$  before pushing it up, and of the rising risk premia, which push down on safe  $r^*$  close to 1-to-1. I also provide a decomposition of the rise in the wealth-to-GDP ratio, where the longer expected retirement and slower growth emerge as dominant (while the rise in the risk premium plays essentially no role).

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<sup>4</sup>I also find that the capital demand schedule has steepened, and the capital supply schedule has flattened. However, measured at the respective state states, both schedules are more elastic (flatter) at the long-run (2050) steady state as compared to the 1970 steady state.

<sup>5</sup>Technically speaking, along the limited foresight transition path the agents in the economy experience a sequence of MIT shocks. In other words, they make expectational errors.

A reasonably robust prediction of the model is that  $r^*$  has fallen by less than the government bond yields in the period immediately before the Covid pandemic. Viewed through that lens, some (perhaps a half) of the recent spike in yields can be seen as a correction. Nonetheless, the current long-term real yields stand above the baseline  $r^*$  estimate, suggesting the possibility that  $r^*$  may be higher than the business-as-usual scenario considered thus far.

In the final part of the paper, I take seriously the possibility that seems to be reflected in the bond markets, namely that some new shifts in the environment have acted to raise the natural rate sharply higher. Rather than providing a definite view of the magnitudes of risks and their probabilities (a task beyond the scope of this paper!), the idea behind this exercise is to provide a mapping between quantitative assumptions underlying these scenarios and the impact on  $r^*$ . Such mapping might help policymakers and practitioners form their own view of the central case for  $r^*$  and the risks around it.

I find that AI and inflation risk scenarios are most powerful quantitatively, each raising  $r^*$  by circa 1 percentage point. In addition, the heightened inflation risk raises the safe rate very quickly, while the effects of AI-driven growth boom build up over several years.

Several other scenarios can prevent real rates from the gentle decline predicted in the business-as-usual scenario. At the same time, for real rates to go back to pre-GFC levels, several of the scenarios would need to materialize all at once (or individual shifts would need to be even more powerful than assumed here).

**Literature and contribution.** My paper relates to several strands of the literature on the determinants of the natural rate of interest. One approach, following [Laubach and Williams \(2003\)](#) and [Holston et al. \(2017\)](#), is semi-structural:  $r^*$  is inferred from a representative-agent Euler equation through empirical filtering methods. The logic behind these estimates is that, if the observed interest rate has declined but inflation remained stable, it must be that the natural rate has fallen. These exercises point to long-run shifts in trend growth and the residual component  $z_t$  capturing other forces as the main driver of  $r^*$ . This approach is very distinct from the one pursued in this paper, although one challenge that the two papers share is how to pin down the level of  $r^*$ . [Holston et al. \(2017\)](#) pin down the level by equating it to their prior estimate of trend growth at the start of the sample.<sup>6</sup> I follow a calibration strategy where I target the wealth-to-GDP ratio in the 1970s steady state. Both approaches yield broadly consistent answers, suggesting  $r^*$  around that time of around 4-5%.

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<sup>6</sup>Specifically, [Holston et al. \(2017\)](#) initialize the random walk component  $z_t$  at 0. See footnote 6 in their paper.

A second strand of the literature, much closer to this paper, embeds richer household heterogeneity and demographic structure to study the determinants of the natural rate. Several papers use rich overlapping generation models to study the shifting age composition and its impact on  $r^*$  (Carvalho et al. (2015), Gagnon et al. (2016), Lisack et al. (2017), Auclert et al. (2024)). This paper follows the tradition of using a tractable OLG setup pioneered by Blanchard (1985) and Gertler (1999) and used recently in Rachel and Summers (2019a). It is closest to the papers that consider many of the drivers of  $r^*$  jointly to assess the relative quantitative importance of each factor as well as their ability to jointly account for the decline (Eggertsson et al. (2019), Peruffo and Platzer (2024), Rachel and Summers (2019b)). The present paper incorporates a comprehensive set of factors relevant for  $r^*$ . However, one trend that is not modeled here is the rise in inequality, which may have led to a decline in  $r^*$  in the region of 1 percentage point over the past 30 years (Straub (2017), Peruffo and Platzer (2024), Auclert et al. (2025)).

Several papers have used an explicit capital equilibrium representation to study  $r^*$ , although details vary. Rachel and Smith (2015) used a flow-based desired saving - desired investment setting to consider how the shifts in both curves imply a fall in  $r^*$  and a broadly constant gross investment and saving ratios. Moll et al. (2021) developed a general equilibrium model where capital demand and capital supply have a closed-form representation, and used it to study the implications of equilibrium movements in returns (driven, in this case, by automation) for inequality. Auclert et al. (2024) used the estimates of the elasticities of the steady-state demand and supply schedules and applied a sufficient-statistics approach to infer the impact of the demographic shifts.

A contribution that is closest to mine is the contemporaneous paper of Auclert et al. (2025). Just as I do here, that paper uses a long-run equilibrium framework to study multiple drivers of the capital market equilibrium, including demographics, labor share, productivity growth, social security, and mark-ups, and what they imply for wealth-to-GDP and returns. The precise focus of the two papers is different, however: while Auclert et al. (2025) focus on the relative importance of asset demand and asset supply shifts and on fiscal sustainability questions, my main goal is to study the dynamics of  $r^*$ . The two papers are highly complementary. Auclert et al. (2025) use a sufficient statistics approach, utilizing the estimates of (semi-)elasticities of the asset demand and asset supply schedules to decipher the impact of the structural shifts in both curves on the wealth-to-GDP ratio and the steady-state natural rate. I follow a different route, namely I compute the schedules and their shifts directly using a general equilibrium model. This allows me to present the full set of shifts of these



curves and highlight the importance of interactions between various forces given the inherent non-linearity of the framework. In addition, I show that the curves not only shift but also their slopes change as a result of the shocks I consider. Another area of complementarity is that [Auclert et al. \(2025\)](#) use a model with a more detailed life-cycle structure able to match the age-wealth profiles, while the present paper simplifies the demographic structure and utilizes the resulting tractability to study non-linearities and compute the realistic, limited foresight transition path between steady states. Finally, [Auclert et al. \(2025\)](#) focus on the United States, while I collect data and calibrate my model to the advanced economy bloc.

While the two papers use the same representation of the long-run equilibrium – the crossing point between two schedules – they follow different conventions in how they label these schedules. [Auclert et al. \(2025\)](#) label the upward sloping schedule “asset demand”, and a downward sloping schedule “asset supply”. This tradition follows the literature that began with [Caballero \(2006\)](#) and includes the important contributions of [Caballero et al. \(2008\)](#) and [Caballero et al. \(2017\)](#). In this tradition, the word *asset* is used from the perspective of households’ balance sheets. These assets are supplied by firms (e.g. in the form of claims to future profits such as stocks), or governments (in the form of government bonds). In this paper I instead refer to the upward sloping curve as “capital supply”, and to the downward sloping curve as “capital demand”. This naming convention goes back to the neoclassical growth model, in which the equation  $MPK = r + \delta$  is the downward sloping *capital demand* schedule, and in which infinite lives result in an infinitely elastic *capital supply*. This convention is also tied to the [Aiyagari \(1994\)](#), heterogeneous agents tradition. Of course, both conventions are equally valid.

An important literature that this paper builds on stressed global developments in  $r^*$  and the convergence of interest rates across countries (e.g. [Del Negro et al. \(2019\)](#)). Another related literature distinguishes between multiple natural rates depending on risk premia ([Del Negro et al. \(2017\)](#), [Farhi and Gourio \(2019\)](#)) or the macro-level financial stability implications ([Akinci et al. \(2023\)](#)). The distinction between safe, risky, and effective rates is important for interpreting the widening spread between government yields and average returns on capital ([Rachel and Smith \(2015\)](#), [Reis \(2022\)](#), [Reis \(2021\)](#), [Moll et al. \(2021\)](#)), and motivates the inclusion of the risk premium as one of the driving forces in this paper.

Finally, by considering the upside risk scenarios beyond the business-as-usual projection, the paper connects to the recent literature that studies the potential game-changers for  $r^*$  which have become a focal point in the last few years since the pandemic. [Obstfeld \(2023\)](#) is a timely survey, and important examples include [Ambrosino et al. \(2024\)](#) and [Mehrotra](#)

and [Waugh \(2025\)](#) who study the impact of trade fragmentation and trade wars on  $r^*$  and [Mehrotra \(2025\)](#) who considers the impact of transition to net zero carbon emissions.

**Roadmap.** The rest of the paper proceeds as follows. Section 2 gives an overview of the model, with further details provided in the appendix. Section 3 develops the capital demand – supply representation of the steady state equilibrium and quantifies this framework for the 1970 steady state. Section 4 describes the calibration of the exogenous shifts, and Section 5 quantifies the impact of these shifts on the long-run schedules and thus characterizes the new long-run steady state in the business-as-usual scenario. Section 6 studies the transition between the two steady states. Section 7 decomposes movements in  $r^*$  and wealth-to-GDP into the exogenous drivers in the business-as-usual scenario. Section 8 presents the six risk scenarios and studies their quantitative impact on  $r^*$ . Section 9 concludes.

## 2 Model

### 2.1 High level overview

The model of this paper builds closely on the important contribution of [Gertler \(1999\)](#), who in turn builds on [Blanchard \(1985\)](#) to construct a tractable model of the life-cycle. This framework assumes only two stages of life, work and retirement, with a constant probability of becoming a retiree. The model remains highly tractable, permitting detailed analysis of the dynamics and non-linearities, while at the same time capturing the essence of the incentives to save and dis-save over a lifetime.

Importantly, Ricardian Equivalence does not hold in this model – finite planning horizons induce households to discount the future at a higher rate than the government, which in turn means that tax-and-transfer policy, including redistribution across generations through the issuance of government debt, affects demand and hence the natural rate of interest. Furthermore, the model offers a general equilibrium framework to study the macroeconomic impact of other important government policies, such as social security.

To account for the increase in the spread between safe and risky returns ([Moll et al. \(2021\)](#), [Reis \(2021\)](#)), I assume that households’ decisions are based on the weighted rate of return that combines the two rates, with an exogenous, time-varying premium between the safe and risky return.<sup>7</sup>

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<sup>7</sup>I do not model the portfolio choice problem or the underlying risk or liquidity explicitly. See [Reis \(2021\)](#) or [Angeletos and Panousi \(2011\)](#) for examples of how to endogenize the risk premium. [Auclert et al. \(2025\)](#)

## 2.2 Demographics and preferences

There are two stages of life, work and retirement, with exogenous transition probabilities. That is, each worker (denoted with superscript  $w$ ) faces a given probability of retirement  $1 - \omega$ , and, once a retiree (superscript  $r$ ), a given probability of death  $1 - \gamma$ . Population grows at a gross rate  $1 + n$ .

Despite this parsimonious structure (which will pay off in terms of the model's tractability), the model can represent the demographic structure of an economy reasonably well. One can readily see that the expected durations of working life and retirement are given by  $\frac{1}{1-\omega}$  and  $\frac{1}{1-\gamma}$ , and thus life expectancy (assuming individuals start working at 20) is  $20 + \frac{1}{1-\omega} + \frac{1}{1-\gamma}$ . In a stationary equilibrium, the old-age dependency ratio – the number of retirees divided by the number of workers – is given by:<sup>8</sup>

$$\psi = \frac{1 - \omega}{1 - \gamma + n}. \quad (1)$$

Thus, the dependency ratio is high when individuals retire relatively early in life, when retirement does not last long, and when population growth is low.

There is no aggregate risk; the only sources of uncertainty facing an individual are the risk of retirement while a worker (associated with a loss of labor income) and the risk of death while a retiree. Left unchecked, these sources of risk would affect agents' behavior. This would make aggregation problematic, and, more importantly, it would be unrealistic: the timing of retirement is, for the most part, known. To neutralize this unrealistic feature, I assume that there are perfect annuity markets for the retirees (removing the influence of the risk of death on their behavior), and that workers' preferences have a certainty equivalence property (such that the *risk* of retirement does not affect workers' behavior in equilibrium).<sup>9</sup>

Specifically, I assume that agents have recursive Epstein-Zin preferences defined as follows:

$$V_t^z = [(C_t)^\rho + \beta^z \mathbb{E}_t\{V_{t+1}^z | z\}^\rho]^{1/\rho} \quad (2)$$

where  $C_t$  denotes consumption,  $V_t^z$  and  $\beta^z$  stands for agent's  $z \in \{w, r\}$  value function and

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provide an elegant microfoundation for the assumptions imposed here.

<sup>8</sup>To see what this demographic structure implies for old-age dependency ratio, let  $N_t^r$  represent the number of retirees at time  $t$ . Under the demographic structure assumed here, we have  $N_{t+1}^r = (1 - \omega)N_t + \gamma N_t^r$  where  $N_t$  is the number of workers. Rearranging gives  $(1 + n)\frac{N_{t+1}^r}{N_{t+1}} = (1 - \omega) + \gamma\frac{N_t^r}{N_t}$ . Letting  $\psi_t := \frac{N_t^r}{N_t}$  denote the old-age dependency ratio and imposing stationarity implies (1).

<sup>9</sup>In particular, workers are assumed to have recursive Epstein and Zin (1991) preferences that generate certainty equivalent decision rules in the presence of income risk.

the discount factor respectively, and  $\sigma = \frac{1}{1-\rho}$  is the intertemporal elasticity of substitution.

Both retirees and workers make consumption-saving decisions to maximize their expected lifetime utility. I now outline the problems of the two types of agents.

## 2.3 Households

### 2.3.1 Retirees

Retirees consume out of savings and social security payments. Each period, some retirees die. We make the assumption – standard in the literature – that those who survive receive the proportional share of the proceeds (Blanchard (1985)). This means that the effective gross return faced by individual retirees is  $R_t/\gamma$ , higher than the ongoing gross interest rate  $R_t$ .<sup>10</sup>

Because the probability of death is independent of age and the government does not discriminate across retirees in its social security transfer policy, each retiree (irrespective of age) solves an identical problem, which is:

$$V_t^r = \max_{C_t^r} [(C_t^r)^\rho + \beta\gamma\mathbb{E}_t\{V_{t+1}^r\}^\rho]^{1/\rho}$$

subject to the flow budget constraint:  $A_{t+1}^r = (R_t/\gamma)A_t^r - C_t^r + E_t^r$ , where  $A_t^r$  stands for retiree's assets,  $C_t^r$  is her consumption expenditure, and  $E_t^r$  is the social security payment.

### 2.3.2 Workers

Individuals are born workers and have no financial assets at the start of life. They consume out of asset wealth and their labor income net of taxes. Because of the demographic structure (in particular the assumption that the probability of retirement is independent of age<sup>11</sup>), the worker's problem is effectively the same no matter the age. Each worker solves:

$$V_t^w = \max_{C_t^w} \{(C_t^w)^\rho + \beta[\omega V_{t+1}^w + (1-\omega)V_{t+1}^r]^\rho\}^{1/\rho}$$

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<sup>10</sup>For retirees *as a group* wealth accumulates at the interest rate  $R_t$ , as the higher individual return cancels out with some retirees dying.

<sup>11</sup>Clearly, this is an unrealistic assumption. But, as explained above, the effect of this assumption on workers' behavior is naturalized through the structure of preferences which exhibit a certainty equivalence property. The role of this assumption is thus only to simplify the model and achieve aggregation.

subject to:  $A_{t+1}^w = R_t A_t^w + (1 - \tau_l)W_t - \tilde{T}_t - C_t^w$ , where  $\tau_l$  is the proportional tax on labor income, and  $\tilde{T}$  represents lump-sum tax/transfer.<sup>12</sup>

### 2.3.3 Consumption of workers and retirees

The key advantage of the tractable Blanchard-Yaari-Gertler setup employed here is that, for every household, individual consumption is a linear function of individual wealth (see Appendix for the derivation). In other words, within each of the two groups, individuals have identical marginal propensities to consume. This feature makes the model highly tractable, since it allows for a straightforward aggregation. For example, adding up individual retirees' consumptions, we obtain the aggregate consumption function of the retirees. With slight abuse of notation, denoting now by  $C_t^r$ ,  $A_t^r$  etc. the aggregate variables, the aggregate consumption of all retirees and all workers in the economy is:

$$C_t^r = \epsilon_t \pi_t (R_t A_t^r + S_t) \quad C_t^w = \pi_t (R_t A_t^w + H_t + S_t^w). \quad (3)$$

The term  $\epsilon_t \pi_t$  is the MPC of a retiree (the Appendix derives the law of motion for this MPC – see equation (19)). Retiree consumption is simply the MPC multiplied by their wealth, which is composed of their financial wealth  $R_t A_t^r$  and their social security wealth  $S_t$ .<sup>13</sup>

Analogously, workers' consumption is a linear function of their financial, human, and social security wealth.<sup>14</sup> Following Gertler (1999), we denote the MPC of workers with  $\pi_t$ ,

<sup>12</sup>There are two key channels through which life-cycle considerations affect workers' behavior. First, a worker takes into account the fact that with probability  $1 - \omega$  she becomes a retiree. This means that, relative to the representative agent case, she discounts the future stream of wages by more: effectively, this is the *saving for retirement* effect. Mechanically, a larger discount rate reduces the value of human wealth in the consumption function, thus leading to lower consumption and higher saving. Second, a worker discounts the future stream of wealth more because she anticipates that inevitably there will come a time when she becomes a retiree, facing the sad truth that her life is finite. With finite life, wealth can be smoothed out across fewer periods, so its marginal utility value is lower. This effect shows up as a higher effective discount rate applied to future wealth.

<sup>13</sup>The latter is given by the discounted sum of social security payments:  $S_t = \sum_{\nu=0}^{\infty} \prod_{z=1}^{\nu} \frac{E_{t+\nu}}{(1+n)R_{t+z}/\gamma}$ . The aggregate pension payments are discounted with a gross interest rate adjusted upwards on account of population growth (the same aggregate flow of social security payments is split more broadly) and on account of the likelihood of death.

<sup>14</sup>The workers' human wealth is given by  $H_t = \sum_{\nu=0}^{\infty} \prod_{z=1}^{\nu} \frac{(1-\tau_l)N_{t+\nu}W_{t+\nu} - \tilde{T}_{t+\nu}}{(1+n)R_{t+z}\Omega_{t+z}/\omega}$ . Human wealth is a discounted sum of the economy-wide net-of-tax wage bill. The discount rate that is applied to the aggregate wage bill is the product of the gross real interest rate, the gross population growth rate (for the same reason as discussed above), the inverse of the probability of retirement and a factor  $\Omega > 1$ , which makes for a heavier discounting of the future as funds received in retirement are less valued than money received earlier. In total, therefore, there are three distinct factors in the life-cycle setting that raise the discount rate on future labor income (relative to the infinite horizon case). They are: (1) finite expected time spent working (reflected by the

meaning that  $\epsilon_t$  is the ratio of retiree-to-worker MPC. It is intuitive that  $\epsilon > 1$  – retirees spend down their wealth faster than workers, who are busy saving for retirement.

### 2.3.4 Aggregate consumption and the share of wealth held by retirees

Denoting by  $\lambda$  the share of financial assets held by retirees, we can add the two aggregate consumptions above to get aggregate consumption:

$$C_t = C_t^w + C_t^r = \pi_t \{(1 - \lambda_t)R_t A_t + H_t + S_t^w + \epsilon_t(\lambda_t R_t A_t + S_t^r)\}$$

Because the MPC of retirees is higher than the MPC of workers ( $\epsilon > 1$ ), higher  $\lambda$  raises aggregate consumption. So transferring resources across the demographic groups changes overall demand.

## 2.4 Production side

Production is undertaken by firms using capital and labor. Aggregate output is given by the Cobb–Douglas production function

$$Y_t = K_t^\alpha (X_t N_t)^{1-\alpha}, \quad (4)$$

where  $N_t$  denotes the number of workers and  $X_t$  is labor-augmenting total factor productivity. This specification can be viewed as a reduced form of task-based production frameworks used in the study of automation ([Acemoglu and Autor, 2011](#); [Acemoglu and Restrepo, 2018](#)), in which  $\alpha$  captures the fraction of tasks performed by capital.<sup>15</sup> There is exogenous technological progress at net rate  $x$  and population growth at net rate  $n$ , so that  $X_{t+1} = (1 + x)X_t$  and  $N_{t+1} = (1 + n)N_t$ .

Firms in this economy have market power, and the aggregate gross mark-up is  $\varphi \geq 1$ .<sup>16</sup> This means that the wage  $W_t$  and the rental rate  $r_t^r$  are equated to mark-down  $1/\varphi$  times the after-tax marginal products of the factors:  $W_t = (1 - \tau_l) \frac{1-\alpha}{\varphi} \frac{Y_t}{N_t}$  and  $r_t^r + (1 - \tau_k)\delta = (1 - \tau_k) \frac{\alpha}{\varphi} \frac{Y_t}{K_t}$ . Note that I assume tax expensing of capital depreciation.

Profits are taxed at the same rate as capital income, and after-tax profits are given by  $(1 - \tau_k)(1 - \frac{1}{\varphi})Y_t$ .

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presence of  $\omega$  in the discount rate); (2) greater discounting of the future owing to expected finiteness of life (reflected by the presence of  $\Omega$ ); and (3) growth of the labor force (reflected by the presence of  $1 + n$ ).

<sup>15</sup>In task-based models, the Cobb–Douglas form typically includes a productivity shifter that depends on parameters such as  $\alpha$ . Here that shifter is absorbed into  $X_t$ .

<sup>16</sup>This can be micro-founded by the standard Dixit–Stiglitz CES demand system.

## 2.5 Government

The government consumes  $G_t + M_t$  each period (where  $G$  is civilian government consumption and  $M$  is military spending)<sup>17</sup>, and pays retirees a total of  $E_t$  in social security and healthcare benefits. To finance its expenditures, the government levies proportional taxes on labor at rate  $\tau_l$  and on capital and profit income at rate  $\tau_k$ , as well engages in a lump sum tax/transfer  $\tilde{T}_t$  on the workers. It can also issue one period government bonds  $B_{t+1}$ , which pay out safe gross return  $R_t^f$ . The government flow budget constraint is:

$$B_{t+1} + \tilde{T}_t + \frac{1}{\varphi} (\alpha\tau_l + (\varphi - \alpha)\tau_k) Y_t = R_t^f B_t + G_t + M_t + E_t.$$

Defining total tax collected  $T_t := \tilde{T}_t + \frac{1}{\varphi} (\alpha\tau_l + (\varphi - \alpha)\tau_k) Y_t$ , rewriting in terms of the households' rate of return  $R_t$ , and iterating forward gives the intertemporal budget constraint of the government:

$$R_t B_t = \sum_{v=0}^{\infty} \frac{T_{t+v}}{\prod_{z=1}^v R_{t+z}} - \sum_{v=0}^{\infty} \frac{G_{t+v} + M_{t+v}}{\prod_{z=1}^v R_{t+z}} - \sum_{v=0}^{\infty} \frac{E_{t+v}}{\prod_{z=1}^v R_{t+z}} + \sum_{v=0}^{\infty} \frac{(R_{t+v} - R_{t+v}^f) B_{t+v}}{\prod_{z=1}^v R_{t+z}}.$$

The sum of the first three terms on the right is the present value of the primary surplus. The final term is the present value of the implicit government revenues that arise from paying  $R_t^f$  on its debt, which is below the marginal return in the private economy  $R_t$ , known as a bubble premium, or a convenience yield (Reis (2021), Brunnermeier et al. (2024)). I discuss the different returns and the relationship between them in detail below.

Government policy is exogenous. In particular, it is characterized by four ratios,  $\bar{g}_t, \bar{m}_t, \bar{b}_t, \bar{e}_t$ , of government consumption (of the two kinds), debt, social security spending, all in relation to GDP, respectively:

$$\begin{aligned} G_t &= \bar{g}_t Y_t \\ M_t &= \bar{m}_t Y_t \\ B_t &= \bar{b}_t Y_t \\ E_t &= \bar{e}_t Y_t \end{aligned}$$

Given the paths of  $G_t, M_t, E_t$  and  $B_t$ , the spread between the risk-free and the market rate, and the proportional tax rates  $\tau_l$  and  $\tau_k$ , lump-sum taxes adjust to satisfy the intertemporal

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<sup>17</sup>I split out the military spending since later in the paper I analyze the re-armament scenario and its impact on the natural rate.

budget constraint.

## 2.6 Resource constraint and international lending / borrowing

The economy-wide resource constraint is

$$K_{t+1} = Y_t - C_t - G_t - M_t + (1 - \delta)K_t - NX_t \quad (5)$$

where net exports satisfy  $NX_t = NFA_{t+1} - R_t^f \cdot NFA_t$  and  $NFA_t$  is the (exogenously given) net foreign asset position of the economy. Treating the net foreign asset position as exogenous allows us to account for the impact of outside forces – such as the increase in the desire to save in safe western assets by the Asian economies and oil producers since the late 1990s – on the natural rate in the advanced economies. To reflect the preference for safety, I use the safe return  $R_t^f$  in (5).

## 2.7 Assets and returns

In the description of the households' problem, I focused on the intertemporal consumption-saving decision in an environment where households have access to a saving vehicle (assets  $A_t$ ) that returns the gross interest rate  $R_t$ . Within their total asset portfolio, households hold both risk-free and risky assets. For simplicity, I do not model the portfolio choice explicitly here. Instead, I assume that households (implicitly) dislike risk (and/or like liquidity) and that this results in a risk premium between the returns on the two assets:<sup>18</sup>

$$R_t^r = R_t^f + \varsigma, \quad (6)$$

where  $\varsigma$  is the exogenous risk premium and  $R_t^r$  is the gross risky rate of return. Furthermore, I assume that the gross effective return faced by households is given by the weighted average of the safe and risky return:

$$R_t = safe \cdot R_t^f + (1 - safe)R_t^r,$$

where the weight  $safe \in [0, 1]$  corresponds to an average share of households' portfolios held in safe assets.

In this economy, the safe assets consist of government bonds, while the risky assets

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<sup>18</sup>See [Auclert et al. \(2025\)](#) for a microfoundation consistent with this setup.



comprise capital  $K_t$  (thus the rental rate is equal to the risky rate) and the aggregate value of the claim to pure profits  $\Pi_t$ , which evolves according to

$$\Pi_t = (1 - \tau_k) \left( 1 - \frac{1}{\varphi} \right) Y_t + \frac{\Pi_{t+1}}{R_t^r}.$$

### 2.7.1 Equilibrium in the capital market

The equilibrium condition that carries much of the economics of the model in this paper is the capital market equilibrium, where the funds accumulated by households are equal to the use of funds by firms and the government:

$$\underbrace{A_t^w + A_t^r - NFA_t}_{\text{Supply of capital: households and foreigners}} = \underbrace{K_t + B_t + \Pi_t}_{\text{Use of capital: firms and govt}}. \quad (7)$$

The supply of funds (or capital) in this economy results from household saving: the stock of savings accumulated by households – workers and retirees – in the domestic economy, netting out the stock of savings that households invest abroad (the negative of the net international investment position,  $NFA_t$ ). This capital is invested in – or absorbed, or used – by firms (in physical capital and equity value) and by the government (in debt).

## 3 Steady state equilibrium

In this section I introduce the steady state capital market equilibrium framework to study movements in  $r^*$  and in the wealth-to-GDP ratio, and then apply it to the developed economies bloc of the 1970s.

On the balanced growth path, output and other aggregate variables grow at the rate  $(1+x)(1+n) \approx 1+x+n$ . To obtain stationarity of the system, we hence normalize all growing variables by output and refer to the balanced growth path as the steady state.

### 3.1 Steady state capital demand

In steady state, the components of capital demand are given by:

$$k = \frac{(1 - \tau_k)^{\frac{\alpha}{\varphi}}}{r + \varsigma + (1 - \tau_k)\delta} \quad (8)$$

$$b = \bar{b} \quad (9)$$

$$\Pi = \frac{(1 - \tau_k)(1 - 1/\varphi)}{1 - (1 + x + n)/(1 + r + \varsigma)} \quad (10)$$

where  $r$  is the net risk-free rate (so that  $r := R_t^f - 1$ ). I define the (steady state) capital demand schedule as a function of the safe real interest rate, and this can be expressed in closed form as follows:

$$\mathcal{D}(r) := k(r) + \bar{b} + \Pi(r) = \frac{(1 - \tau_k)^{\frac{\alpha}{\varphi}}}{r + \varsigma + (1 - \tau_k)\delta} + \bar{b} + \frac{(1 - \tau_k)(1 - 1/\varphi)}{1 - (1 + x + n)/(1 + r + \varsigma)}. \quad (11)$$

The capital demand schedule is downward sloping, and asymptotes to infinity – absorption of saving becomes large and ultimately diverges – when  $r = \max(-(1 - \tau_k)\delta - \varsigma, x + n - \varsigma)$ . In the former case, firms demand an infinite quantity of capital in the long run; in the latter, the present value of the profit flow becomes infinite and so absorbs any quantity of saving in the economy.

### 3.2 Steady state capital supply

Unfortunately, no simple expression for the steady-state capital supply schedule is attainable. Nonetheless, the capital supply schedule can be computed numerically in a few simple steps: first, transform the model economy into a small open economy model, with exogenous returns but an endogenous residual net foreign asset position (over and above the exogenous  $NFA_t$ ). It is straightforward to show that the steady-state residual net foreign asset position, denoted by  $f$ , satisfies:

$$\begin{aligned} (x + n + \delta)k &= 1 - \pi \left\{ (1 - \lambda)(1 + r + (1 - safe)\varsigma) (k + \bar{b} + \Pi + n\bar{f}a + f) + h + s^w \right. \\ &\quad \left. + \epsilon \left[ \lambda(1 + r + (1 - safe)\varsigma) (k + \bar{b} + \Pi + n\bar{f}a + f) + s \right] \right\} - \bar{g} - \bar{m} \\ &\quad - [(1 + x + n) - (1 + r + (1 - safe)\varsigma)] f \\ &\quad - [(1 + x + n) - (1 + r)] n\bar{f}a, \end{aligned} \quad (12)$$

which is simply the stationary resource constraint in a small-open economy version of the model. Clearly, it is straightforward to compute  $f$  for any  $r$ , yielding a schedule  $f(r)$ . Then, capital supply is simply

$$\mathcal{S}(r) = \mathcal{D}(r) + f(r). \quad (13)$$

The steady state safe interest rate,  $r^*$ , is the interest rate at which  $f(r^*) = 0$  and thus

$$\mathcal{S}(r^*) = \mathcal{D}(r^*). \quad (14)$$

### 3.3 The 1970s steady state

I apply the model to the advanced economy (AE) bloc, consisting of the United States, Canada, Western European economies, Japan, and other OECD members. The approach of treating these economies as a bloc reflects a high degree of integration on the one hand, and experience of common driving trends on the other. I leave the important question of cross-country heterogeneity and country-specific  $r^*$  for future work.

The analysis begins in the 1970 steady state.<sup>19</sup> To compute the steady state, the model must be parametrized. I assume that a single time period in the model corresponds to one year. I further assume that the AE bloc is characterized by three time-invariant parameters: the discount factor  $\beta = 0.99$ , the intertemporal elasticity of substitution  $\sigma = 0.5$ , and the share of safe assets  $safe = 0.11$ .<sup>20</sup> I calibrate the remaining parameters of the model (the policy variables, the demographics, and other exogenous forces) using the corresponding data

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<sup>19</sup>To be precise, it is unlikely that an economy is *ever* precisely in steady state – the multitude of shocks that hit the economy move the time- $t$  equilibrium away from the steady state (which in itself is time-varying), and even when the shocks abate, convergence to the steady state can be slow and is only asymptotic. Thus, assuming the economy is in steady state is an approximation. Nonetheless, of the post-war history of advanced economies, early 1970s seem to be a reasonable candidate for the period when the economy was near its steady state: the period after the immediate post-WW2 decades of reconstruction and restructuring, and before the oil shocks of the 1970s and the large revisions, first upward and then sharply downward, of the growth prospects (which we document in detail below).

<sup>20</sup>The value for the elasticity parameter  $\sigma$  aligns with benchmark values in the literature and is consistent with the mean estimate reported in the influential meta-study of [Havranek et al. \(2015\)](#). Parameter *safe* reflects the average share of government bonds in total financial wealth over time, and is the same as the calibration used by [Auclert et al. \(2025\)](#). The calibration of  $\beta$  targets wealth to GDP ratio of 3.2 in the 1970 steady state, which is consistent with the WID data on AE wealth-to-GDP and the US data obtained from FRED. The discount factor of 0.99 is a little higher than commonly used discount factors. But note that the model abstracts from idiosyncratic risk, a key feature of incomplete-markets frameworks a’la Bewley-Huggett-Aiyagari. Standard calibrations of such models suggest that idiosyncratic risk lowers equilibrium real interest rates by 1–2 percentage points. One way to interpret our choice of  $\beta$  is as an upward adjustment relative to the “true” underlying discount factor, to capture the effect of idiosyncratic uncertainty on real rates.

Variable	Symbol	Value
Government debt / GDP	$\bar{b}$	0.28
Civilian government spending / GDP	$\bar{g}$	0.15
Military spending / GDP	$\bar{m}$	0.05
Social Security / GDP	$\bar{e}$	0.04
Capital tax (percent)	$\tau_k$	36
Labor tax (percent)	$\tau_l$	21
Productivity growth (percent per year)	$x$	2.80
Population growth (percent per year)	$n$	1.12
Expected length of working life (years)	$\frac{1}{1-\omega}$	46
Old-age dependency ratio	$(1-\omega)/(1-\gamma+n)$	0.18
Depreciation rate	$\delta$	0.03
Capital intensity of production	$\alpha$	0.32
Gross markup	$\varphi$	1.08
Risk premium (percentage points)	$\varsigma$	3.00
Global savings glut	$nfa$	0

Table 1: 1970s steady state: parameter values

from 1970, reported in Table 1. I present the relevant details in the next section.

Given these parameter values, Figure 2 plots the capital demand schedule  $\mathcal{D}(r)$  and the capital supply schedule  $\mathcal{S}(r)$ , along with their intersection at the steady-state equilibrium. The horizontal axis reports the ratio of financial wealth to GDP, while the vertical axis shows the safe real interest rate.

The steady-state capital demand schedule is downward sloping and convex, consistent with the closed-form expression in (11), and familiar from canonical macro models, not least the neoclassical growth model (NGM).

By contrast, the capital supply schedule is quite different from its perfectly elastic NGM counterpart. In fact, it is C-shaped: it is upward sloping, except when the safe rate is close to the asymptote  $x + n - \varsigma$ . The upward sloping part is the relevant one here: the steady state equilibrium always occurs on the upward sloping section.<sup>21</sup> To make the interpretation

<sup>21</sup>Nonetheless, it is interesting to spell out why the capital supply schedule takes this shape. Two forces account for this pattern. At higher interest rates, the standard mechanism dominates: a higher return to saving encourages greater accumulation, so steady-state capital rises with  $r$ . At lower rates, however, this effect is outweighed by a “valuation effect.” Postponed consumption itself becomes an asset whose steady-state value rises as the interest rate falls. This effect is especially strong when interest rates approach  $x + n - \varsigma$ : here, the net present value of the pure profit flow is very large, so even modest postponement of consumption maps into a large steady-state stock of funds. Importantly, this C-shaped supply curve does not imply multiple steady states.

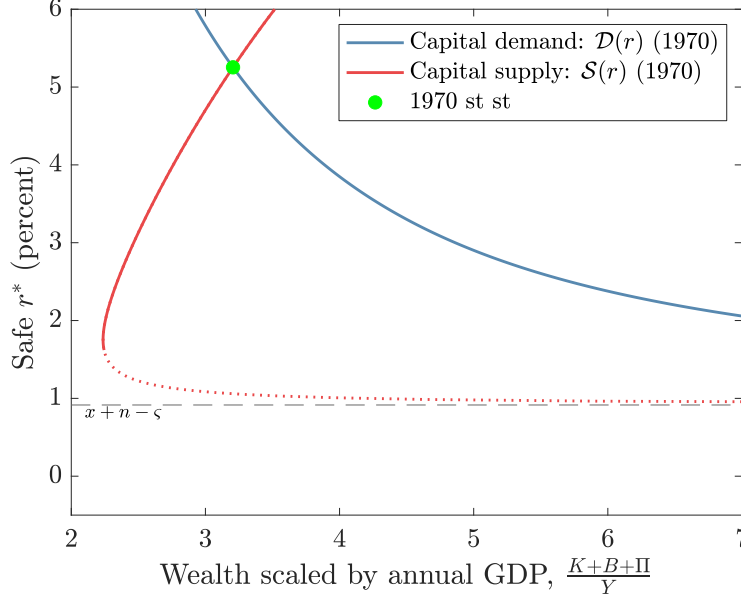


Figure 2: The 1970 steady state: capital demand and capital supply representation

*Notes:* The downward sloping schedule is the capital demand, in equation (11). The C-shaped schedule is capital supply, in equation (13). The dotted part of the  $S(r)$  schedule arises due to the valuation effect, but is not relevant for the analysis. See the discussion in footnote 21. Both schedules asymptote to the vertical dashed line at  $x + n = \varsigma$ .

of the figures cleaner, in what follows I truncate the capital supply schedule's downward sloping part.

The calibrated model implies that in the 1970 steady state the natural real rate was  $5\frac{1}{4}\%$  (and the wealth-to-GDP ratio, which we target in the calibration of  $\beta$ , is 320%). In this steady state, interest rates are high because the future is bright. High growth expectations reduce households' desire to save, and boost firms' desire to invest (trend GDP growth is nearly 4%, as productivity is expected to increase by 2.8% per year and population growth is 1.1%). Moreover, the population is young, with only 18% of the people in retirement. A worker in the 1970s is expected to spend only around 5 to 9 years in retirement, limiting the desire to save for old age. The high interest rates translate into contained asset prices, limiting the value of the wealth-to-GDP ratio. Other endogenous variables are reported in Table 2.

To characterize the slope of the two schedules, we calculate the respective semi-elasticities and evaluate them in the steady state:

$$\varepsilon^S = \frac{1}{A_{ss}} \frac{dS(r)}{dr} \Big|_{r_{ss}} \stackrel{1970}{=} 12.2, \quad \varepsilon^D = \frac{1}{A_{ss}} \frac{dD(r)}{dr} \Big|_{r_{ss}} \stackrel{1970}{=} -13.1.$$

Thus, the model predicts that in the 1970 steady state, the two schedules have nearly identical slopes (in absolute value). Of course, the slopes are useful as they are locally informative of the long-run equilibrium impact of shifts of these curves on interest rates and on wealth (Auclert et al. (2025)). Let  $\kappa^S$  and  $\kappa^D$  denote the proportional horizontal shifts in the respective curves,

$$\kappa^{S \text{ or } D} := \frac{\Delta A}{A_{ss}}.$$

The impact of those shifts is given by:

$$dr = \frac{\kappa^S - \kappa^D}{\varepsilon^D - \varepsilon^S}, \quad (15)$$

$$\frac{dA}{A} = \frac{\varepsilon^D \kappa^S - \varepsilon^S \kappa^D}{\varepsilon^D - \varepsilon^S}. \quad (16)$$

For example, an increase of the government debt-to-GDP ratio of 50 percentage points – roughly the increase we have observed in the data – shifts the capital demand schedule to the right, with  $\kappa^D = \frac{50}{320} \approx 0.16$ . The semi-elasticity approach predicts that such a shift results in  $dr \approx \frac{-0.16}{-13.1-12.2} = 0.63$  of a percentage point rise in the natural interest rate and  $dA/A \approx \frac{-12.2 \cdot 0.16}{-13.1-12.2} = 7.7\%$ , i.e. an increase in wealth-to-GDP of  $7.7\% \cdot 320 = 25$  percentage points. That is, there is about 50% long-run crowding out of private wealth.<sup>22</sup> As I show below, the equilibrium effect of the rise in debt will be different, however (the interest rate increases by more, with greater crowding out), as the capital supply schedule also shifts (leftwards) due to the associated general equilibrium forces.

While this semi-elasticity approach is useful for local changes of a single driving force, it might be insufficient to understand the impact of large shifts in the environment, especially when many of the exogenous drivers change substantially all at the same time. Next, I use the framework provided by the model to study how changes in the economic environment shift the demand for capital and the supply of capital, and thus ultimately how they affect the steady-state interest rate and wealth-to-GDP ratio, taking full account of nonlinearities and interactions.

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<sup>22</sup>Indeed, note that with  $-\varepsilon^D \approx \varepsilon^S$  as is the case here, we have  $\frac{dA}{A} \approx \frac{1}{2} (\kappa^S + \kappa^D)$ .

Table 2: 1970s steady state: endogenous outcomes

Variable	Symbol	Value
Safe $r^*$ (percent)	$r_f$	5.3
Risky $r^*$ (percent)	$r_r$	8.3
Capital-output ratio	$k$	1.8
Firm equity value of pure profit / GDP	$\Pi$	1.1
Wealth / GDP	$k + \Pi + \bar{b}$	3.2
Share of wealth held by retirees	$\lambda$	0.17
Social security wealth of workers / GDP	$s^w$	0.48
Social security wealth of retirees / GDP	$s$	0.28
Human wealth / GDP	$h$	4.14
MPC out of wealth – workers (percent)	$\pi$	7.3
MPC out of wealth – retirees (percent)	$\epsilon\pi$	14.7

## 4 The shifts

I now turn to the task of quantifying the changes in the exogenous drivers, listed in Table 1, that characterize the process of structural socio-economic change over the past five decades.

**Expected growth of per-capita income:  $x_t$ .** Growth expectations profoundly shape both the capital demand and capital supply. Since households want to smooth consumption, expectations about the future are an important factor in determining their intertemporal saving decisions. Furthermore, future demand determines the demand for capital investment, as well as the present value of firm profits (since, when the economy grows faster, so do the profits).

On the balanced growth path, the per capita consumption growth is equal to the per capita output growth, which in turn equals  $x$ . Thus we can calibrate the time path  $\{x_t\}$  by directly examining how the expectations for the growth rate of per capita output have evolved. The natural place to start is the forecast produced by the IMF in their World Economic Outlook publications. The vintages of historic forecasts are available starting in the 1990s. Figure 3 shows the real-time 5-year ahead forecast for aggregate GDP growth for the aggregate of advanced economies. The dots denote the final, fifth year of the forecast. The Figure shows that long-term GDP growth expectations for the advanced economy block have trended downwards over time, from about 3% per year in the early 1990s to around 1.6% in the latest forecast. The decline in the per-capita growth expectations is less pronounced, however, since some of this decline is due to the decline in the growth of the labor force.

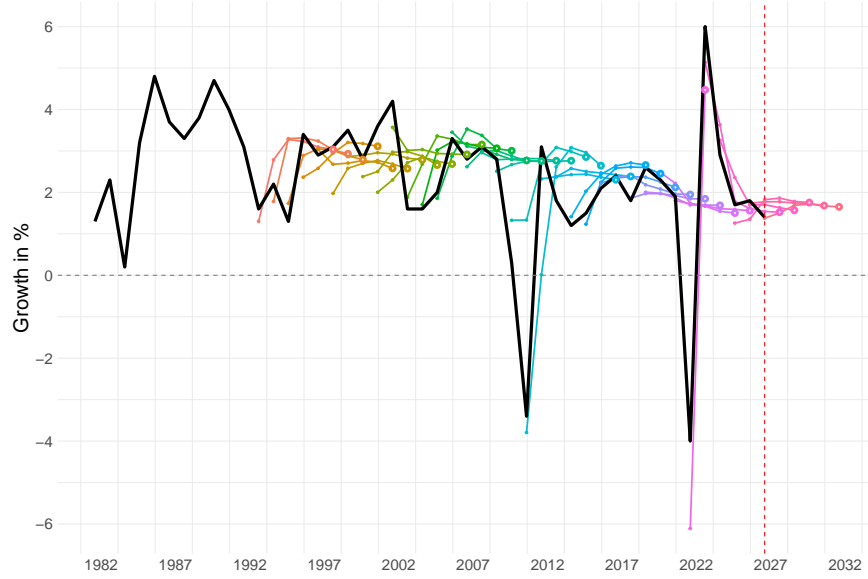


Figure 3: IMF WEO advanced economies' GDP growth projections and outturns

*Notes:* The black solid line is the annual growth of GDP across the advanced economies, as defined by the IMF. The colored lines show the successive World Economic Outlook forecasts, with the dots denoting the final, fifth year of the forecast. In our calibration, we adjust these long-term expectations with population growth expectations and match the resulting series to productivity growth  $x_t$ .

Population growth in high-income economies declined from about 0.8% in the 1970s to around 0.2% per year currently and is expected to decline to around zero by mid-2040s. Thus, per-capita growth expectations have declined by about 1 percentage point since the 1990s, from 2.3% to 1.3%.

Going further back in time, the growth scenario prepared by the OECD in the mid-1970s and reviewed again in 1980 provides a useful perspective. In 1976 the OECD projected labor productivity to increase by 4% over the next five years in advanced economies, representing a strong pick-up in expectations relative to the early 1970s. However, the economies grew significantly slower, and, in 1980, the likely outcome was judged to be around 1.5 percentage points below the original forecast, at around 2.5%. This downgrade in expectations appears to have been persistent, reflecting the broad slowdown in productivity growth around that time.

**Demographics: population growth  $n_t$ , length of working life  $1/\omega_t$ , Length of retirement  $1/\gamma_t$ .** Figure 4 illustrates the scale of the demographic transition underway in advanced economies. The left panel highlights the steady decline in population growth, projected to turn negative around 2050, alongside the sharp rise in the old-age dependency



ratio, which has nearly doubled over the past half-century and is expected to climb further. The right panel shows life expectancy continuing to rise, while the effective retirement age has remained broadly stable. As a result, the expected length of retirement has expanded dramatically – roughly quadrupling over the past century, from 5 to nearly 20 years.

These demographic shifts have far-reaching macroeconomic consequences. Slower population growth reduces the incentives to invest and sets the pace of aggregate output growth given productivity trends. It also shapes expectations about per-capita social security benefits. A rising dependency ratio implies greater political and economic weight of retirees, while longer retirement horizons strengthen incentives for households to accumulate savings.

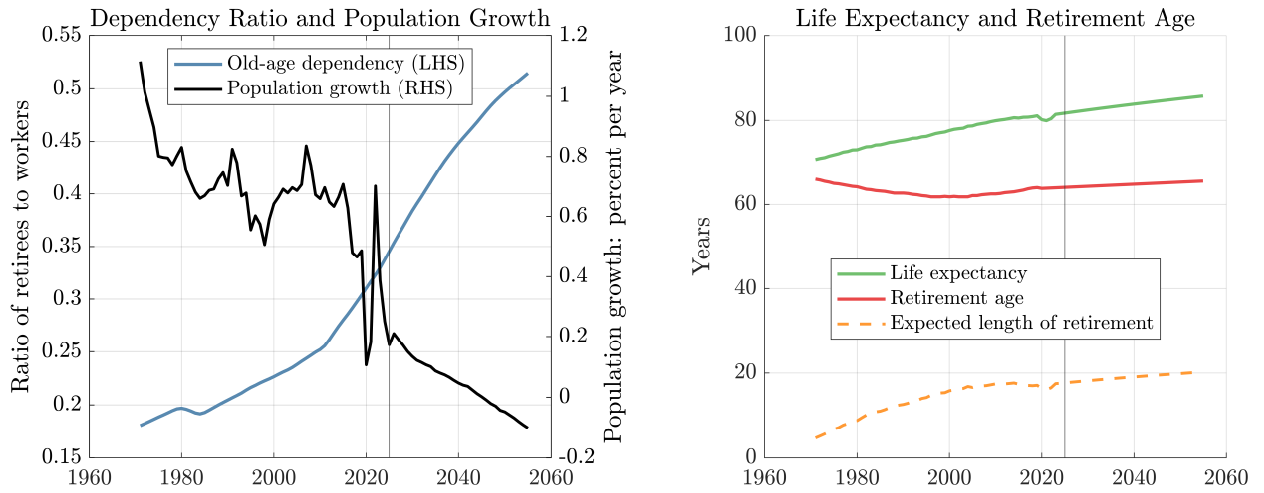


Figure 4: Demographic transition in advanced economies

*Notes:* The data come from the 2024 UN population projections.

To calibrate the time path  $\{n_t\}$ , I use population data and projections from the United Nations World Population Prospects (the medium variant). The calibration of  $\{\omega_t\}$  matches the evolution of the effective retirement data from the OECD.<sup>23</sup> Then, given  $\{n_t\}$  and  $\{\omega_t\}$ , the time path for  $\{\gamma_t\}$  is consistent with the old-age dependency ratio in the UN projections.<sup>24</sup>

**Capital intensity of production  $\alpha_t$ .** The literature on task-based production models showed that the share of tasks carried out by capital is encapsulated in parameter  $\alpha$  (Acemoglu and Autor (2011), Acemoglu and Restrepo (2018)). Automation raises  $\alpha$  and shifts out the demand for capital (Moll et al. (2021)). But it also affects how the economic pie is shared between workers and capital owners, and hence impacts the value of human wealth

<sup>23</sup>We assume that working life start at age 20.

<sup>24</sup>Matching life expectancy delivers similar profile for  $\gamma_t$ .

of workers in the economy, with the associated effects on saving behavior and hence capital supply.

I back out  $\{\alpha_t\}$  from the Penn World Table (10.01) labor share, recognizing that the labor share measured in the data corresponds to the fraction  $\frac{1-\alpha}{\varphi}$  in the model.

**Depreciation  $\delta_t$ .** The depreciation rate of capital affects directly the demand for capital. I calibrate  $\{\delta_t\}$  directly to the depreciation series in PWT. Depreciation increases over time as the composition of capital stock shifts away from heavy machinery and towards ICT and intangible capital.

**Gross markup  $\varphi_t$ .** Changes in gross mark-ups could be driven by shifts in concentration and market power, which in turn could be driven by technological or regulatory changes. Or they could reflect the greater importance of intangibles, R&D, and fixed costs. An increase in gross mark-up raises the ex-post profit share in the economy and increases the equity value of firms, shifting out the capital demand schedule. Saving accumulation by households can also be affected as changes in gross mark-ups alter the profile for wages and for returns.

I calibrate  $\{\varphi_t\}$  directly to the estimates of gross mark-up available in the literature. Specifically, I base the calibration on the mark-up estimates from [Ravn and Cabaco \(2025\)](#). Their estimates are for the US only.<sup>25</sup> On this measure, the gross mark-up falls in the 1970s, from around 1.1 to 1.075, and then increases steadily to 1.25 currently. Since these estimates reflect the US, I adjust this trajectory downwards slightly and impose less of an increase, given the evidence that mark-ups in other advanced economies are lower and have increased by less than in the United States ([De Loecker and Eeckhout \(2021\)](#), [Díez et al. \(2018\)](#)).

**Risk premium  $\varsigma_t$ .** Recall that  $\varsigma_t$  is the spread between the safe and the risky rates of return, where the safe assets are government bonds and the risky assets are firm capital and equity (capitalizing the value of pure profits). Shifts in this spread shift both capital demand and capital supply curves vertically (since we specified these schedules in terms of the risk-free rate).

Following [Moll et al. \(2022\)](#) and [Reis \(2022\)](#), we assume that the spread has increased over time, from 3 percentage points in the early part of the sample to 4.5 percentage points in the recent years.

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<sup>25</sup>It is worth highlighting the two differences between this measure and [De Loecker et al. \(2020\)](#). First, this estimate uses translog production functions; second, the aggregation is done using harmonic means.

**Government debt  $\bar{b}_t$ .** Government debt absorbs household saving, and so is a direct source of capital demand in the economy. Its changing level also has some general equilibrium effects on capital supply through endogenously changing taxes.

Government debt has been on an upward trajectory across most of the advanced economies. For the AE bloc as a whole, government indebtedness increased from 28% of GDP in 1970 to 82% in the most recent data. The baseline forecast (e.g. consistent with the latest IMF projections) is that the government debt-to-GDP ratio will continue to rise, albeit much more steadily than over the past decade, reaching circa 90% of GDP in 2050.

**Government civilian consumption  $\bar{g}_t$  and military spending  $\bar{m}_t$ .** Direct use of resources by the government affects the economy through the resource constraint, and the need to finance such expenditures through taxation. Here we split the civilian government spending and the military spending, to highlight the trends in the latter (and later to analyze the scenario in which explicitly the military spending rises over the forecast horizon).

I use national accounts to measure general-government final consumption expenditure net of defense and social transfers in kind. Defense spending is taken from the SIPRI military expenditure database.

**Social security  $\bar{e}_t$ .** Social security provides a source of income in retirement and, as such, alleviates the need to save for retirement. I measure old-age pensions and related cash benefits from OECD “Social protection - old age” as a share of GDP.

**Tax rates  $\tau_{l,t}$  and  $\tau_{k,t}$ .** Rates at which capital and profits are taxed will affect the value of these assets and the desirability to save.<sup>26</sup>

Effective tax rates on labor and capital come from [Bachas et al. \(2024\)](#) who provide a new comprehensive dataset on effective tax rates across the world.

**Foreign saving  $\bar{nfa}_t$ .** [Bernanke \(2005\)](#) famously pointed out that the increased desire to save by some of the developing economies has acted as an additional source of capital supply in advanced economies, pushing real interest rates down. Indeed, saving by some of the Asian economies and oil-producing countries has increased sharply in the aftermath of the Asian financial crisis. This increase matches in magnitude the net international investment position of the advanced economy bloc (Figure 5).

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<sup>26</sup>With inelastic labor supply in our model, labor taxes do not play much of a role in driving interest rates.

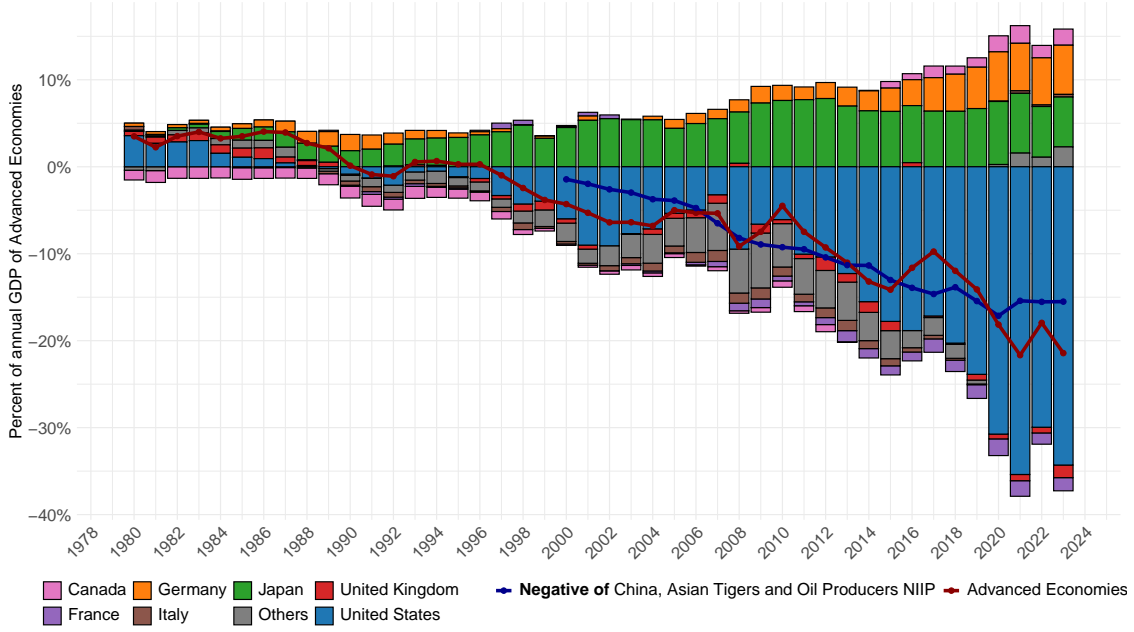


Figure 5: The decomposition of the net international investment position of AEs, and the surplus of the selected economies

*Notes:* All lines and bars are normalized by annual GDP of advanced economies. The data come from the [Milesi-Ferretti \(2024\)](#) database.

Given the external nature of the increase in the desire to save by this group of economies, we calibrate the exogenous increase in the  $n\bar{f}a$  as a share of advanced economies' aggregate GDP, increasing steadily from zero in the late 1990s to around 15% of GDP recently.

## 5 The 2050 steady state

I now feed through all the changes in the environment outlined above, and compute the long-run steady state. For convenience, I refer to this as a 2050 steady state, since the transition towards the long run is largely complete by then.

Figure 6 provides the key result of the paper. It shows that the AE bloc has experienced *large* shifts in both capital demand and (especially) in capital supply, and that these shifts have driven a large decline in the safe natural rate of interest of over 5 percentage points, to essentially 0% in real terms, and a large increase in the wealth-to-output ratio, to more than 6. Table 3 compares the key endogenous variables across the two steady states.

Recomputing the elasticities of the two schedules, we obtain:

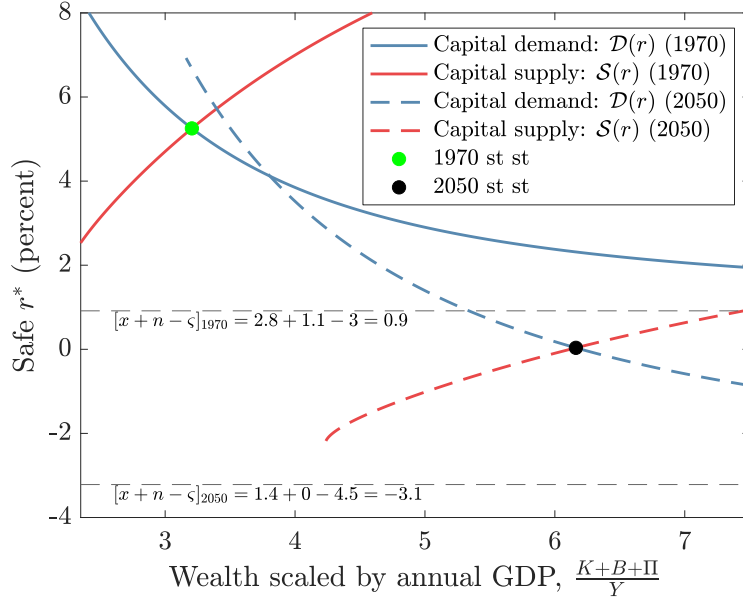


Figure 6: 1970 and 2050 steady states: capital demand and capital supply representation

*Notes:* The figure shows the shifts of the steady state schedules in the capital market equilibrium in the business-as-usual scenario (i.e. between the 1970 and the 2050 steady state). See main text for details.

$$\varepsilon^{\mathcal{S}} = \frac{1}{A_{ss}} \frac{d\mathcal{S}(r)}{dr} \Big|_{r_{ss}} \stackrel{2050}{=} 20.8, \quad \varepsilon^{\mathcal{D}} = \frac{1}{A_{ss}} \frac{d\mathcal{D}(r)}{dr} \Big|_{r_{ss}} \stackrel{2050}{=} -18.6.$$

Thus, both schedules are significantly flatter at the new steady state, compared to the 1970 steady state. The Figure shows that the capital demand schedule has become steeper overall; however, because of its convex shape and the rise in wealth-to-GDP, the schedule around the new steady state equilibrium is flatter than in the 1970 steady state.

The capital supply schedule has shifted down and to the right. In the new steady state, it is now upward sloping at much lower real rates; it is also significantly more elastic around the steady state equilibrium.

What accounts for these large shifts in the steady state capital demand and capital supply schedules? The Figure already gives us a hint, since it shows that the horizontal asymptote given by  $x + n - \varsigma$  shifted downwards as productivity growth has halved, population stopped growing, and the risk premia have increased (and are assumed to remain elevated). Thus, these three factors are likely quantitatively important. I explore all the driving forces next.

Table 3: The 1970 and 2050 steady states: endogenous outcomes

Variable	Symbol	1970	2050
Safe $r^*$ (percent)	$r^*$	5.3	0.0
Risky $r^*$ (percent)	$r^r$	8.3	4.5
Physical capital-output ratio	$k$	1.8	2.6
Value of pure profit flow / GDP	$\Pi$	1.1	2.7
Wealth / GDP	$k + \Pi + \bar{b}$	3.2	6.2
Share of wealth held by retirees	$\lambda$	0.17	0.35
Social security wealth of workers / GDP	$s^w$	0.48	0.92
Social security wealth of retirees / GDP	$s$	0.28	1.13
Human wealth / GDP	$h$	4.1	6.1
MPC out of wealth – workers (percent)	$\pi$	7.3	4.1
MPC out of wealth – retirees (percent)	$\epsilon\pi$	14.7	6.6

## 5.1 Decomposition of capital demand

I first compute an accounting decomposition of the capital demand curve into physical capital, the value of pure profits, and outstanding government debt. Figure 7 shows this decomposition for both steady states.

This decomposition reveals an interesting yet subtle feature of macroeconomic adjustment that has occurred in the developed world over the past 50 years (and that is still underway). While the wealth-to-GDP ratio (the sum of three components,  $k + \Pi + \bar{b}$ ) has increased over time both in the model and in the data, the demand curve for physical capital and the schedule representing the value of firm profits both *shifted in*. The only component of demand for funds that unambiguously expanded is government borrowing. However, even though the capital absorption coming from the firm side of the economy is lower in the long-run steady state compared to the 1970s *for every interest rate*, the steady state capital stock and firm equity have increased (precisely because interest rates declined).

## 5.2 Capital demand and supply shifts, force by force

The shifts depicted in Figure 6 show the joint effect of all the forces simultaneously. To learn more about the economic effects of individual drivers, Figure 8 shows how the schedules would have shifted if only a single force was operational from 1970 onward (keeping the actual size of the shift fixed). The thin solid lines represent the schedules in the initial steady state and are identical across panels. The thick dashed lines show the respective

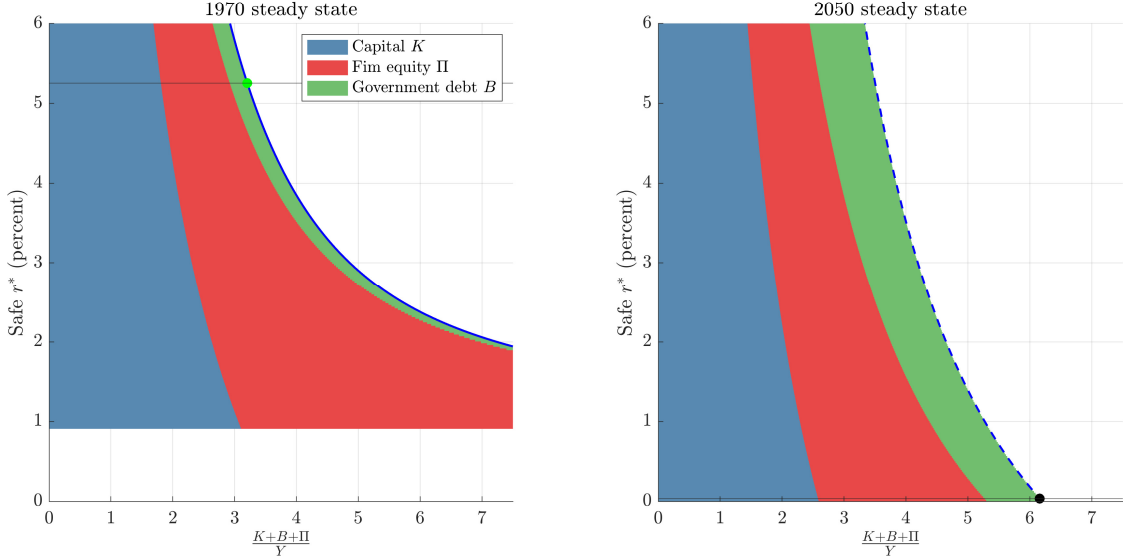


Figure 7: Decomposition of the capital demand curve in the 1970 and 2050 steady states

*Notes:* Both panels decompose the capital demand curve  $\mathcal{D}(r)$  horizontally into components, according to equation (11). The horizontal lines with dots denote the respective steady states.

shifts of capital demand and supply as a result of any single force.

The figure presents a fascinating insight into how the long-run capital market equilibrium is affected by the various drivers we have considered. Noticeably, most of the forces shift both capital demand and capital supply. The four exceptions from this are (1) increasing duration of retirement due to higher life expectancy, (2) rise in social security spending; (3) the bloating of the global saving glut, and (4) the increase in the depreciation rate; the first three shift only capital supply, while the latter one only moves capital demand.

Three of the drivers shift the schedules vertically. Quite intuitively, these are the forces that determine the effective discount rate that is applied to future payoffs: TFP growth  $x$ , population growth  $n$  and risk premium  $\varsigma$ . In effect, these shifters move the horizontal asymptote for both schedules. Clearly, given these changes in the shape of the curves and the inherent nonlinearities, the sum of the impacts of individual shifters is different from the joint effect – in other words, interaction effects between the forces are important. We will see this momentarily in the decomposition of  $r^*$  dynamics.

### 5.3 Desired saving and desired investment

The steady state can also be thought of as an intersection of the desired saving and desired investment schedules – that is, in the space of flows rather than stocks. For example, this is

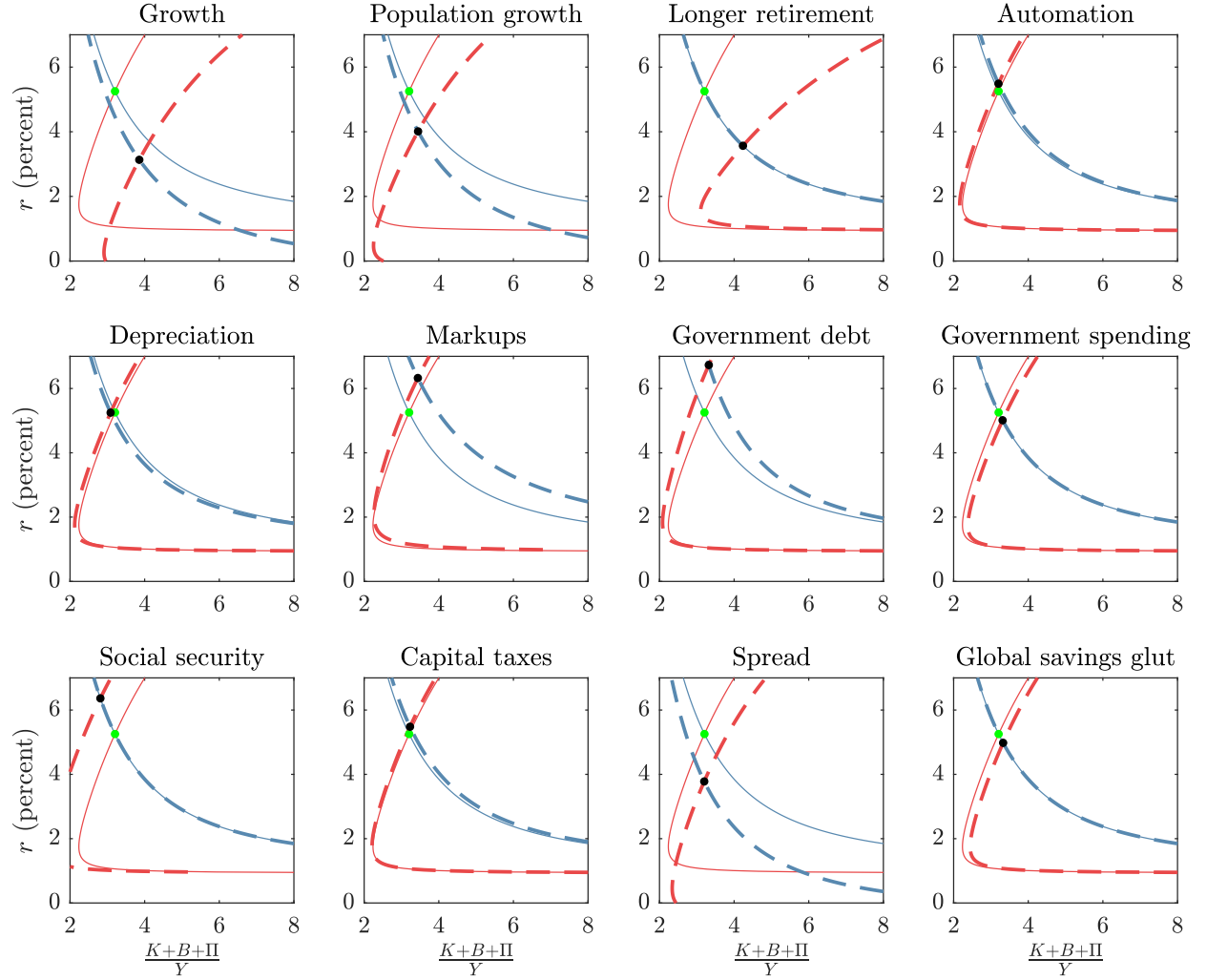


Figure 8: The capital market equilibrium shifts in response to individual exogenous drivers

*Notes:* The solid curves show the 1970 steady state and are identical across panels. The dashed curves show the position of the long-run capital demand and capital supply under the assumption that only the specific force shifts.



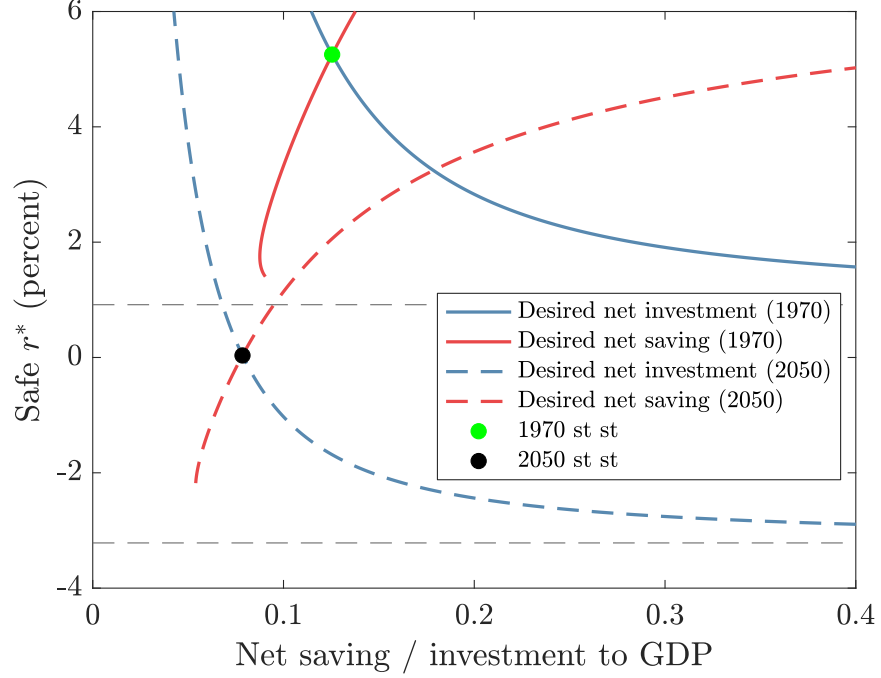


Figure 9: Desired saving – desired investment (flows) representation of the steady state equilibrium.

*Notes:* This figure is the flow equivalent of Figure 6. It depicts the desired saving and the desired investment schedule defined in equation (18).

the approach taken by [Rachel and Smith \(2015\)](#). To go from the capital market equilibrium characterized above to the saving-and-investment equilibrium, note that net investment and saving satisfy

$$\frac{I_t}{Y_t} = \frac{K_{t+1} - K_t + \Pi_{t+1} - \Pi_t + B_{t+1} - B_t}{Y_t} \quad \frac{S_t}{Y_t} = \frac{A_{t+1} - A_t}{Y_t}. \quad (17)$$

Since on the balanced growth path capital and wealth grow at the same rate as output (namely  $g := x + n$ ), we obtain the following formulas for the desired investment and desired saving schedules:

$$\frac{Inv}{Y}(r) = g\mathcal{D}(r) \quad \frac{Sav}{Y}(r) = g\mathcal{S}(r). \quad (18)$$

Figure 9 shows the desired saving – desired investment representation of the equilibrium. It makes the point that even though wealth in the new steady state is much higher than in 1970, the net investment and net saving flows consistent with the long-run steady state are actually lower. The significant decline in growth means that saving and investment flows sufficient to keep asset ratios steady are much lower.

## 6 Transitional dynamics

The steady state analysis can tell us a lot about where the economy started and where it is headed. But it does not tell us what the natural real rate is today, or has / will be at any given point in time. To address this question, I turn to transitional dynamics and ask: what path connects the 1970 steady state to the long-run (2050) steady state, and where along that path are we in 2025?

When analyzing transitional dynamics, researchers must confront the question of how to model expectations. The literature (e.g., [Rachel and Summers \(2019a\)](#)) typically assumes perfect foresight: at the point of departure from the initial steady state, agents anticipate the entire sequence of current and future shocks. This assumption makes more sense in certain contexts, for example when the exogenous drivers that are being studied are predictable. For example, the demographic processes are slow-moving, and dependable demographic projections are widely available. But the perfect foresight assumption is less appealing when forces at play are difficult to predict. For example, it is highly unlikely that those alive in the 1970s anticipated the decline in labor share of income in the 1990s, the rise in mark-ups in the 2000s, or the decline in expected future growth post-GFC. Transition dynamics based on the perfect foresight assumption might provide a misleading steer on the dynamics of the natural rate and the wealth-to-GDP ratio.

### 6.1 Imperfect forecasts and surprises

To tackle this issue, I compute transitional dynamics with a limited degree of foresight. This requires making assumptions about what the agents in the economy expected at each point in time.

To keep the analysis manageable, I first assume that agents can perfectly foresee shifts in some of the drivers, namely: government spending, capital taxes, military spending, and the lengths of working life and retirement. Some of these are genuinely predictable (demographics), while others are quantitatively minor for the results (e.g., government spending; see the previous section).<sup>27</sup>

The other eight exogenous drivers are imperfectly foreseen. To pin down the expectations, for some of them I rely on direct measures: for example, historical IMF forecasts can be informative about expectations about future productivity growth  $x$ , population dynamics

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<sup>27</sup>I also assume there are investment adjustment costs, with the cost parameter calibrated to the value typically used in the literature. This makes very little quantitative difference to the results.

$n$ , and debt-to-GDP ratio  $\bar{b}$ . For others, I make narrative-style assumptions which broadly reflect the limited ability of the agents to anticipate the structural shifts. In that spirit, I assume that agents learned about the rise in the foreign demand for safe assets  $n\bar{f}a$  gradually, only as this demand materialized following the Asian Financial Crisis, or that shifts in the risk premia  $\varsigma$  occurred around the 1990s recession, the burst of the dot-com bubble, and the global financial crisis. Similarly, I assume that the realization of the initially falling then rising markups  $\varphi$  came gradually, with a big shift in the perception of the mark-ups that occurred in the recovery from the GFC when corporate profits recovered more strongly than the rest of the economy. Finally, I assume expectations about capital intensity  $\alpha$  and depreciation rate  $\delta$  are assumed to change only around the same time as the shifts actually happen (i.e., I assume limited anticipation of these shocks). Even though by their nature these assumptions are imprecise, they are likely more realistic than the perfect foresight assumption they replace.

I also assume that expectation revisions occur at discrete, infrequent dates rather than continuously. This “episodic” updating – consistent with limited or costly attention and with market narratives that ebb and flow – induces jumps in expectations and makes it more likely that the limited-foresight transition departs materially from the perfect-foresight benchmark.<sup>28</sup>

The detailed account of all of the assumptions about the historical expectations is in the Appendix.

## 6.2 The natural rate along the transition path

The resulting transition path is shown in Figure 10. The solid lines are the limited foresight paths; for comparison, the dashed lines show the perfect foresight transition. The left panel shows the equilibrium safe and risky rates; the right panel shows the corresponding paths of the wealth-to-GDP ratio and its components.

Historically, the model predicts an increase in safe  $r^*$  in the early 1980s, and a large subsequent decline. Safe  $r^*$  is estimated to be 0.6% in 2025 (in the limited forecast transition). Overall, the transition towards the long-run steady state that is currently underway is to a large extent complete, although the natural rate is expected to decline further towards zero (by around half a percentage point) in the decades to come.

The differences between the perfect and imperfect foresight paths for  $r^*$  are substantial.

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<sup>28</sup>Nonetheless, for three of the drivers ( $\alpha, \delta, n\bar{f}a$ ) where the timing of these changes to expectations is particularly uncertain, I assume that expectations themselves evolves continuously.

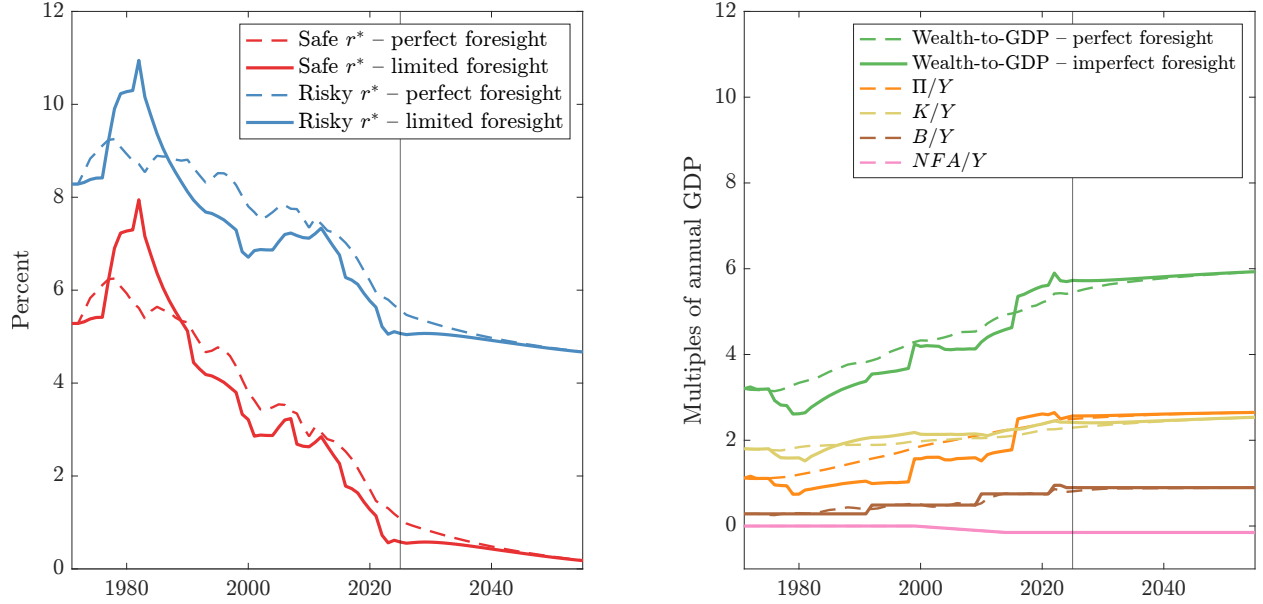


Figure 10: Model-implied estimate of  $r^*$  and wealth along the transition path

*Notes:* The left panel shows the model-implied natural rates in both the baseline limited foresight transition, as well as the perfect foresight transition in the dashed lines. The right panel shows the equivalent paths of the wealth-to-GDP ratio and its components.

Most notably, there is a large divergence in the 1980s, driven by large swings in growth expectations. In the last 25 years, the model predicts that the imperfect foresight  $r^*$  is consistently below its perfect foresight counterpart. This is a surprising prediction, given that the natural rate was on a downward trend over this period, and so one might expect the perfect foresight path to be running ahead of the limited forecast path in predicting the decline. This difference is also present today: the perfect foresight  $r^*$ , which stands at 1.2%, is 0.6 percentage points higher than the limited foresight  $r^*$ . Figure 11 decomposes the difference between the safe  $r^*$  in the limited and perfect foresight transitions respectively, and shows that the gap is largely accounted for by shocks to mark-ups. This is because a surprise shock to mark-ups has a short-run effect that is of opposite sign to the long-run effect (see the Appendix for a more detailed discussion).

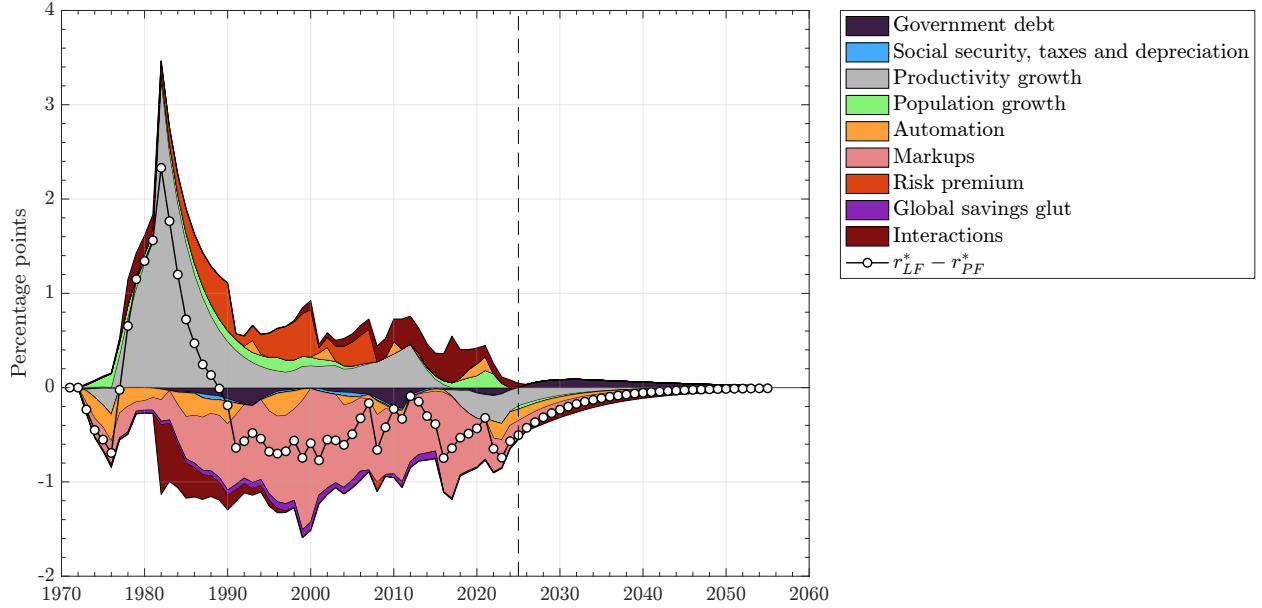


Figure 11: Decomposition of the difference between safe  $r^*$  in the limited and perfect foresight transitions

*Notes:* The figure decomposes the difference between the limited and perfect foresight safe  $r^*$  – the two red lines in the left panel of Figure 10 – into the underlying exogenous forces. Only the forces that are imperfectly foreseen are shown (contributions of all others are zero by construction).

A useful way to visualize the transition path is to return to the capital market equilibrium and plot the transition alongside the steady state shift (Figure 12). The green and orange lines in the figure show the perfect and limited foresight transition paths, respectively, with the stars denoting year 2025. The figure shows that the transition seemed to have followed the capital demand schedule. This is consistent with the capital supply shifts being dominant.

### 6.3 Transition paths vs. sequence of steady states

An alternative method of computing a time path for the natural rate is to ignore the transitional dynamics completely, and instead compute the sequence of steady states consistent with the value of the exogenous drivers at each  $t$ . This approach is followed, for example, in Auclert et al. (2025). A natural question that arises is: how much difference do transitional dynamics make?

To explore this, Figure 13 plots the perfect and limited foresight transition paths for safe  $r^*$ , alongside the sequence of steady state  $r^*$ s. The latter implicitly assumes that the economy transitions to a new steady state instantaneously each period. The differences between the time paths are even larger in this case, with  $r^*$  estimates that differ by over

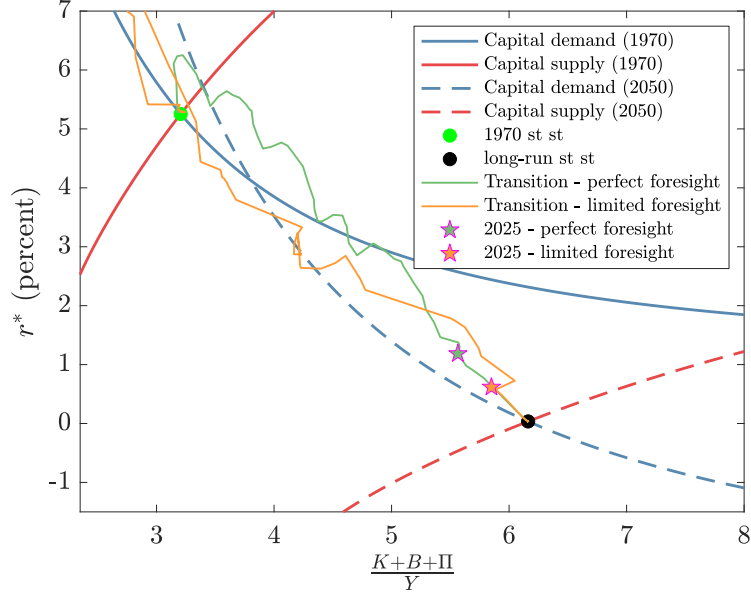


Figure 12:  $r^*$  and wealth along the transition path

*Notes:* This figure overlays the transition paths on the steady state equilibrium representation familiar from previous figures.

a percentage point in most periods. The approaches provide a qualitatively different steer in terms of the levels and the direction of  $r^*$  change in the 1980s, the mid-2000s and much of the 2010s. The steady state sequence also suggests that  $r^*$  is higher today, at 1.5% per annum. These large differences highlight the importance of taking transitional dynamics and paths for expectations into account when estimating the natural rate.

## 6.4 A word of caution, and comparison to data

The preceding analysis illustrates that the estimate of  $r^*$  at any given time is sensitive to specific assumptions about the transitional dynamics. This, of course, is also true for other aspects of the calibration, and so caution is warranted when interpreting any point estimates coming out of the model's simulations. To illustrate, Figure 14 shows the path of  $r^*$  coming out of the (limited foresight) transition under three different calibrations of the pure discount factor  $\beta$ : the baseline  $\beta = .99$ , as well as  $\beta = \{.98, 1\}$ . Either of these alternative calibrations meets the broad targets in terms of the 1970 level of the wealth-to-GDP ratio (of around  $3 \times \text{GDP}$ ). This exercise shows that plausible alternative calibrations can change the level of  $r$ , and the level and sensitivity of wealth to the exogenous drivers. One must exercise caution in order not to over-interpret the point estimates as being precise or definitive. Plenty of

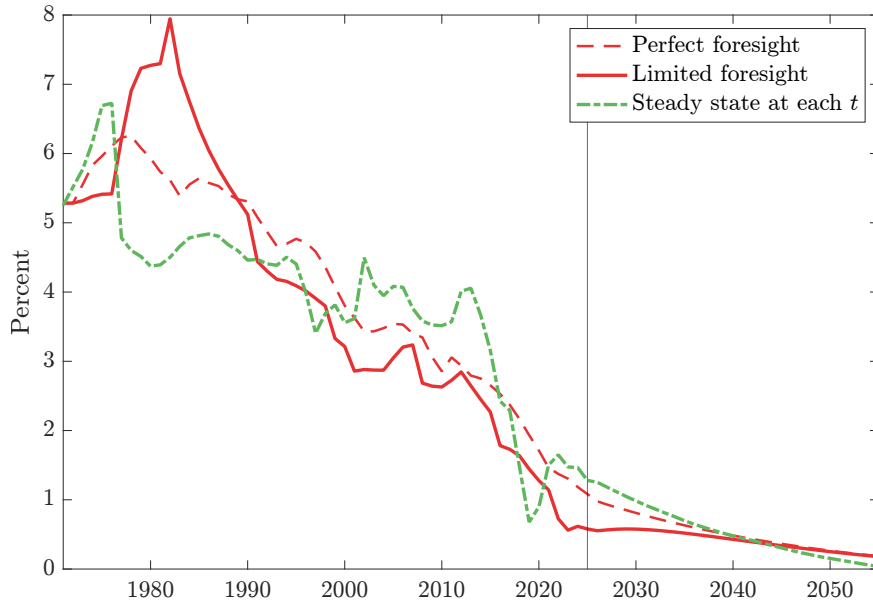


Figure 13: Estimates of safe  $r^*$  along the transition path and the sequence of steady states

*Notes:* The two red curves are the same as in the left panel of Figure 10. The green curve shows the sequence of steady state  $r^*$ .

uncertainty surrounds these model predictions.

This uncertainty notwithstanding, Figure 14 also illustrates how the estimates of  $r^*$  and wealth-to-GDP from the model line up against the data on real 10-year bond yields and the estimate of the wealth-to-GDP ratio. The findings are interesting. The right panel shows that the rise in wealth-to-GDP is broadly in line with the data. Focusing on the left panel, back in the 1970s and 1980s, the realized real rates (which back then are measured using inflation outturns) are well below the estimate of the natural rate. This might help explain why inflation was high and volatile back then. Real rates caught up with the natural rate in the second half of the 1980s and both began climbing down. Market rates fell faster more recently, especially in the run-up to the pandemic. Thus, part of the recent spike in real rates can be interpreted as a correction. Nonetheless, market-based real rates, at around 2%, are somewhat higher than the central estimate from the model.

## 7 What caused the decline in the natural rate?

Figure 15 presents the decomposition of the safe  $r^*$  along the transition path. The pick-up in  $r^*$  in the late 1970s was driven largely by high growth expectations. But these expectations did not last, and by the late 80s, they started to contribute to the decline in  $r^*$ . Other forces

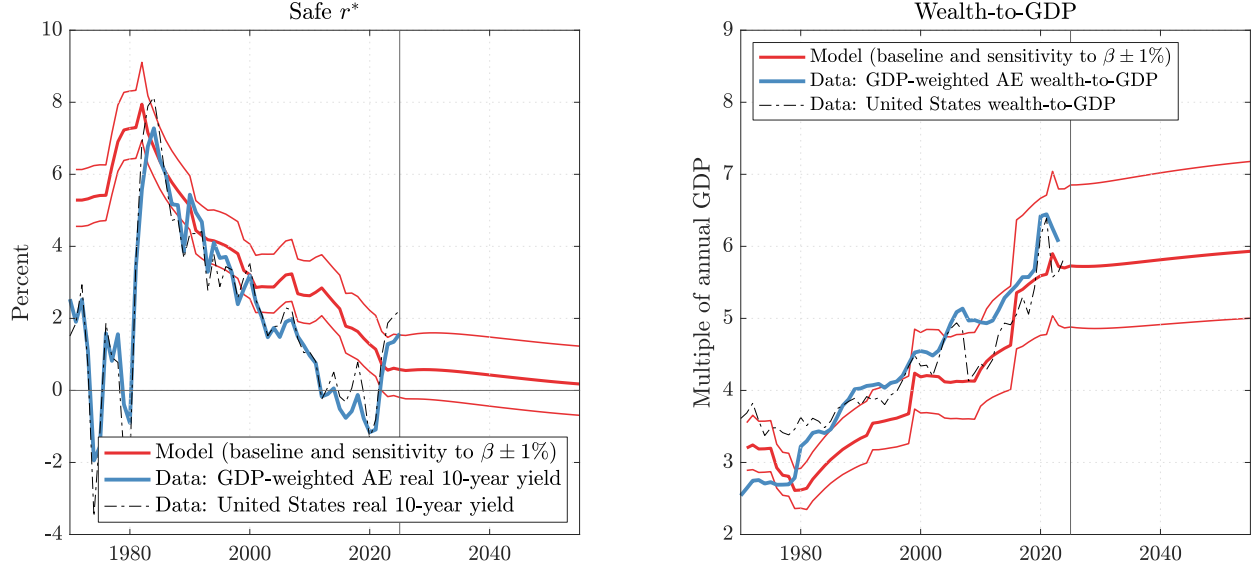


Figure 14: Model predictions vs. the data on long-term real yields and wealth-to-GDP

*Notes:* The thick red lines in both panels show the baseline business-as-usual transition for safe  $r^*$  on the left and wealth-to-GDP ratio on the right. The thin red lines show the equivalent paths when the discount factor  $\beta$  is assumed to be higher or lower, illustrating the sensitivity of the results. The thick blue lines show data on 10-year government bond yields (real rates constructed in the same way as in the introduction) and wealth-to-GDP across advanced economies. The black dotted line shows the data for the United States.

that were important early on included the demographic factors and the transitional effect of mark-ups, which were offset to some extent by public sector forces, not least the climb in social security spending and increased demand for capital due to automation.

More recently, the 2010s saw a slow recovery from the GFC, and long-term growth expectations were eventually revised as well, dragging down  $r^*$ . The continued increases in pension benefits offset some, but not all, of the impact of rising life expectancy and slowing population growth. Rising spread between the risky rate and the safe rate lowered the latter consistently through the episode. On the other hand, government debt ratio continued to increase, pushing  $r^*$  higher, particularly so in the aftermath of the global financial crisis. The long-run effect of the rise in mark-ups started to be visible around 2007, with the contribution turning from negative to positive around that time. It has since become quantitatively large, supporting  $r^*$  by about half a percentage point from about 2020 onwards. Last but not least, the decomposition reveals large and growing interactions between the forces, which can be understood bearing in mind how the various forces – particularly growth and the risk premia – change the shape of the long-run capital demand and supply schedules.

Figure 16 shows the decomposition of the changes in wealth-to-GDP implied by the model. Here, the rise in life expectancy (and the associated rise in the length of retirement)



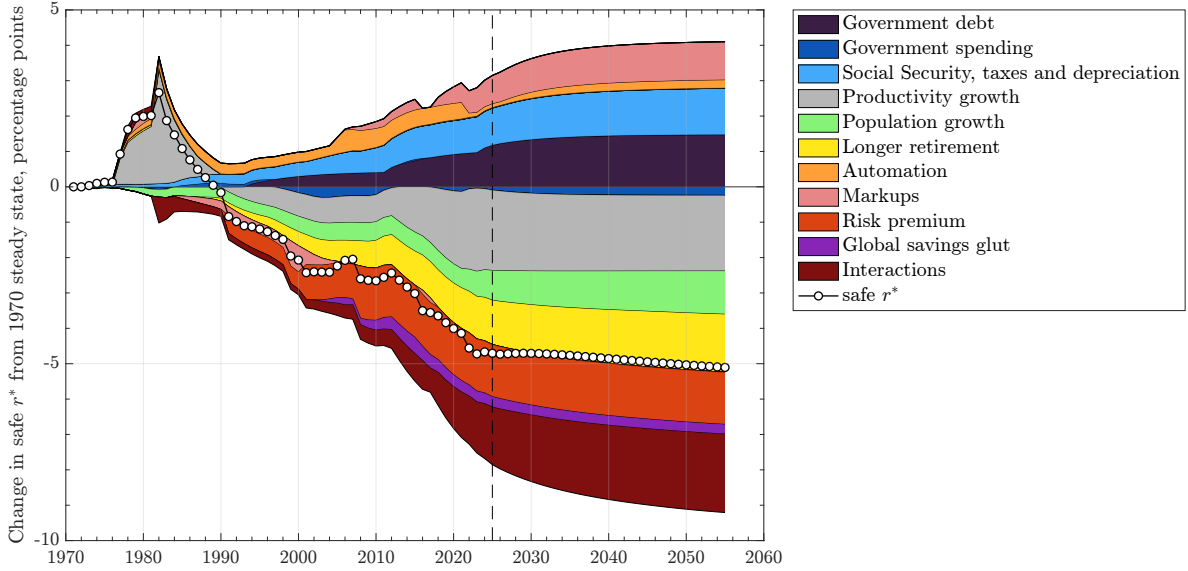


Figure 15: Decomposition of the decline in the safe  $r^*$  in advanced economies

*Notes:* The figure shows the model-based decomposition of the drivers of the decline in the safe real rate of interest along the limited foresight transition. To make the figure easier to read, some of the less quantitatively important forces are summed together (e.g. taxes and depreciation have a relatively minor effect on  $r^*$  and these are shown together in the blue bar).

emerges as the main driver. This is because this force shifts capital supply but does not move capital demand, resulting in large effects on wealth accumulation. In contrast, the decline in population growth shifts both schedules, resulting in large effects on  $r^*$  but small effects on wealth accumulation.

## 8 What next for $r^*$ ? Beyond the business-as-usual

The baseline simulation represents a business-as-usual path for  $r^*$ . It incorporates only the forces emphasized in the pre-pandemic literature – before Covid, the war in Ukraine, and heightened geopolitical tensions, and the emergence of generative AI. In this scenario,  $r^*$  remains broadly stable, with only a gentle downward drift over the coming decades. This stands in stark contrast to recent market developments: long-term government yields in some countries have risen back to levels last seen before the global financial crisis, marking a decisive break from the half-century trend of declining real rates.

The final part of the paper therefore asks: within the framework developed here, what assumptions would one need to make in order to rationalize such a turning point in the natural rate? And are any of the forces discussed by commentators and priced by market partic-

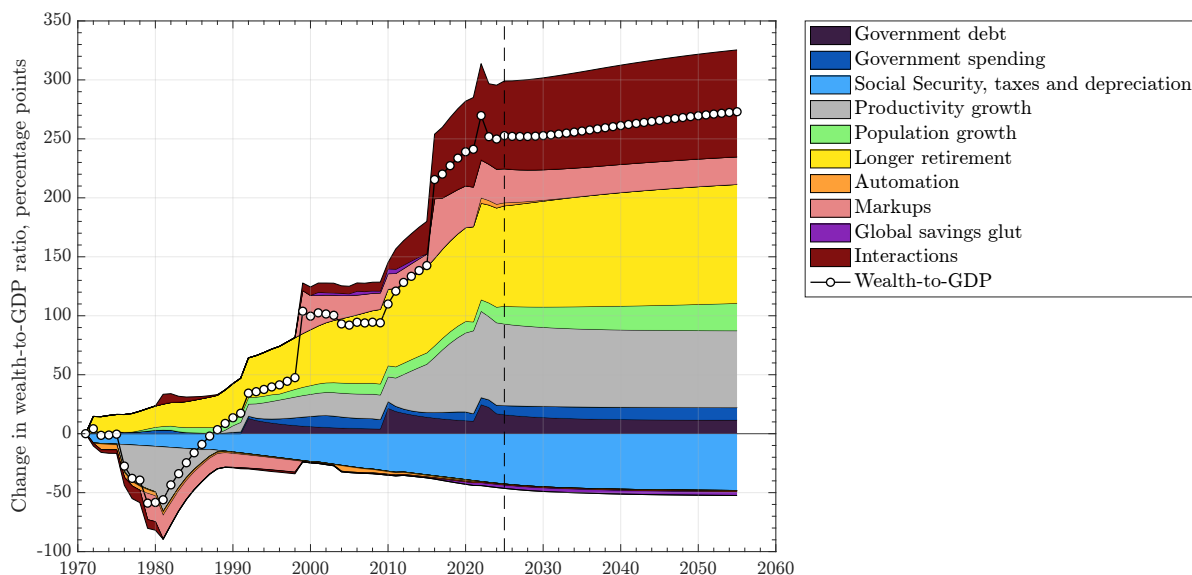


Figure 16: Decomposition of the rise in wealth-to-GDP ratio in advanced economies

*Notes:* The figure shows the model-based decomposition of the drivers of the rise in the wealth to GDP ratio.

ipants quantitatively strong enough to generate the observed upward shift in expectations for  $r^*$ ?

## 8.1 Six scenarios

I consider six scenarios:

1. Scenario 1: De-globalization
2. Scenario 2: Re-militarization
3. Scenario 3: AI
4. Scenario 4: Intangibles
5. Scenario 5: Debt financed social security
6. Scenario 6: Inflation risk

In designing the scenarios, my aim is to highlight material risks to the baseline, or “business-as-usual” projection for the natural rate that I discussed above. I therefore consider quantitatively meaningful departures from the baseline assumptions. While plausible, these

scenarios are best viewed as upside risks for  $r^*$  rather than as predictions. I discuss each in turn.

**Scenario 1: De-globalization** ( $\bar{nfa}, x, \varsigma$ ). With the tariff wave under the Trump administration and rising geopolitical tensions, there is broad concern that globalization may go into reverse.<sup>29</sup> In this scenario, surplus economies (China, other Asian exporters, and oil producers) reduce their holdings of Western assets: their NFA position falls by 10 percentage points, from 15% to 5% of AEs’ annual GDP. Furthermore, I assume re-shoring improves resilience, lowering the risk premium by a quarter of a percentage point (from  $4\frac{1}{2}$  to  $4\frac{1}{4}$  percentage points), but at the cost of loss in efficiency: TFP growth is 0.1 percentage points lower. The net effect on  $r^*$  is in principle ambiguous—slower growth pushes down, while narrower spreads and a partial reverse of the “global savings glut” push the natural rate up.

**Scenario 2: Re-militarization** ( $\bar{m}, \bar{b}$ ). Russia’s invasion of Ukraine ended the “peace dividend” era and raised defense needs in Western democracies. I assume the military-spending share rises from 2.4% (baseline) to 3.8% of GDP, with no offset elsewhere in government consumption. One-third of the increase is debt-financed, lifting the debt-to-GDP ratio by 14 percentage points by 2050.

**Scenario 3: AI** ( $x, \varphi, \alpha$ ). Artificial intelligence is a major technological force with uncertain macro impacts. Beyond efficiency gains, AI may increase market concentration (data-driven scale economies) and, akin to automation, shift the income distribution away from labor. In this scenario I assume TFP growth is boosted by 0.75 percentage points over the next decade and by 0.25 percentage points thereafter—large but in the “middle of the pack” of current guesstimates (Figure 17). We also assume gross markups rise by 2 percentage points and the labor share declines gradually by up to 1.1 percentage points, as AI favors large firms with access to large amounts of data and generates a boom in capital demand as it takes over an increasing share of tasks.

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<sup>29</sup>The processes of globalization and trade integration are complex; the model captures them only at a high level of abstraction. For a more thorough assessment, see [Ambrosino et al. \(2024\)](#) and [Mehrotra and Waugh \(2025\)](#).

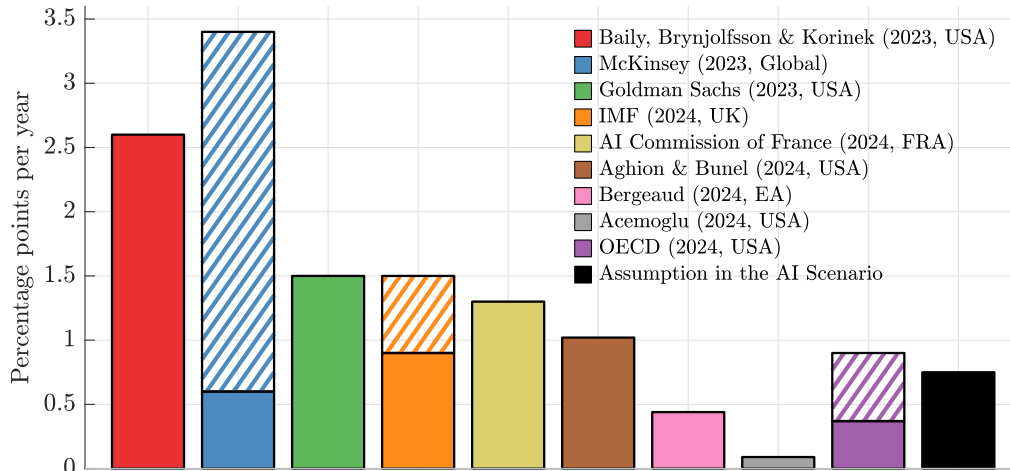


Figure 17: Predicted boost to labor productivity growth over the next decade from AI, by study

*Notes:* As reported in [Filippucci et al. \(2024\)](#). The parts of the bars with stripes represent optimistic scenarios. The regions to which the respective studies refer to are given in the legend.

**Scenario 4: Rise of intangibles** ( $\varphi$ ,  $x$ ,  $\delta$ ). One of the most important and hotly-debated trends of the ones we discussed is the rise of mark-ups. Several interpretations of what underlies this trend have been discussed in the literature: one possibility is that the increase in mark-ups reflects the rise in market power and the associated increase in rents ([Barkai \(2020\)](#), [Gutiérrez and Philippon \(2018\)](#)), perhaps in part driven by less effective anti-trust policy. Another view is that the rise in mark-ups reflects the rise of intangible inputs, which either have an important fixed cost component ([De Ridder \(2024\)](#)), which is covered, ex-post, with a mark-up, or they represent payments to the unmeasured capital stock ([Crouzet and Eberly \(2023\)](#)). The latter interpretation can also explain some of the observed weakness in measured or true productivity ([Crouzet and Eberly \(2021\)](#), [Crouzet et al. \(2024\)](#)). In this scenario, which builds on [Crouzet et al. \(2024\)](#), I assume that a continued rise in intangibles leads to a further rise in concentration and hence in markups (by an additional 1 percentage point), together with a gradual increase in the depreciation rate (by 1.5 percentage points), and a 0.1 percentage point slowdown in productivity growth (which reflects the dominant role for diminished entry incentives that the research has identified).

**Scenario 5: Debt-funded social security** ( $\bar{e}$ ,  $\bar{b}$ ). As the share of (soon-to-be) retirees rises, their political influence likely grows. Here, social-security outlays continue to increase—reaching 13% of GDP by 2050 rather than stabilizing below 8% as in the base-

line—and are financed by debt issuance (a transfer from the young and unborn to the old). The debt-to-GDP ratio climbs to over 145% in this scenario.

**Scenario 6: Inflation risk ( $\varphi$ ).** Post-COVID inflation has eroded the real value of nominal claims—most visibly the “safe” stock of government debt—while the sharp rise in policy rates and the yield curve has generated sizable capital losses on bond portfolios. For instance, in the United States, holders of the nominal 10-year Treasury bonds experienced roughly 25–30% real losses over the past four years, a stark departure from the low but steady real realized returns of the preceding four decades (Figure 18). A natural hypothesis is that a persistent increase in compensation for inflation risk has raised government yields and compressed the safety premium. If this persists, the spread between “safe” and risky assets might narrow persistently. In this scenario we therefore exogenously reduce the safe–risky spread by 1 percentage point—from 4.5 to 3.5 percentage points (recall that in the 1970s–1980s benchmark the spread is 3 percentage points, so the spread remains higher than this starting level, but significantly lower than in the recent decades).

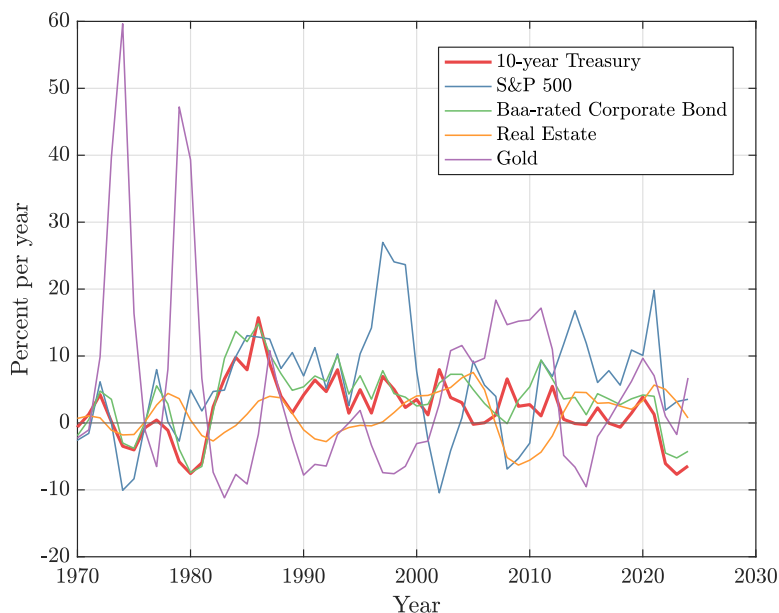


Figure 18: Realized average annual real return on US assets over the preceding 4 year period

*Notes:* Computed based on the returns database of Aswath Damodaran of the NYU Stern. See [Damodaran \(2024\)](#) for details.

## 8.2 Turning point for $r^*$ ?

Figure 19 shows, in the separate lines, the paths for safe  $r^*$  under each of the six scenarios described above, as well as a path that combines all of the scenarios. All scenarios considered result in higher  $r^*$ , except Scenario 4, where the growth slowdown driven by the rise of intangibles dominates the rise in mark-ups and the natural rate is lower than baseline as a result. Quantitatively, the re-militarization scenario is not very important: it builds up slowly and the magnitude of the effect is limited.

Other scenarios matter more: in particular, both AI and the Inflation Risk scenarios raise the safe natural rate by over a percentage point. The decline in the risk premium due to a persistent perception that “safe” assets have become riskier generates a sharp jump in  $r^*$ , not dissimilar to the one we observed in the financial markets in recent years.

Nonetheless, the modeled scenarios can only generate a pick-up in the natural rate to the levels last seen in the pre-GFC period only when they are considered jointly (or if some of the individual scenarios play out more strongly, of course). It is in the combined 1+2+3+4+5+6 scenario that the natural rate returns to the levels last seen prior to the global financial crisis. So while the model with imperfect foresight is capable of generating a sharp turnaround in rates (it does so in the 1980s), and while the narratives that are discussed among economists and market participants *can* generate a large swing in natural rates, one needs to believe a confluence of risk scenarios hitting the economy all at once to generate a return to natural rates of 3% or more. And of course, besides the upside scenarios considered here, there are also risks that lie to the downside.<sup>30</sup>

One important lesson that comes out of the scenario analysis is the quantitative importance of the evolution of the risk premium for safe  $r^*$ . Since the pass-through from shocks to the spread to movements in the safe rate is close to one-to-one in this model, whether the spread remains elevated, shrinks, or increases has a profound impact on the safe natural rate relevant for central banks and finance ministries. The flip side of this is that, in this simple model, changes in the spread have little bearing on the risky  $r^*$ . Given the importance of the spread for the determination of the natural rates, a more in-depth modeling of the drivers of this spread, for example, as in the models of [Angeletos and Panousi \(2011\)](#) and [Reis \(2021\)](#), is warranted.

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<sup>30</sup>For example, the uncertainty associated with the technological revolution powered by AI could translate to greater precautionary saving and lower natural rates.

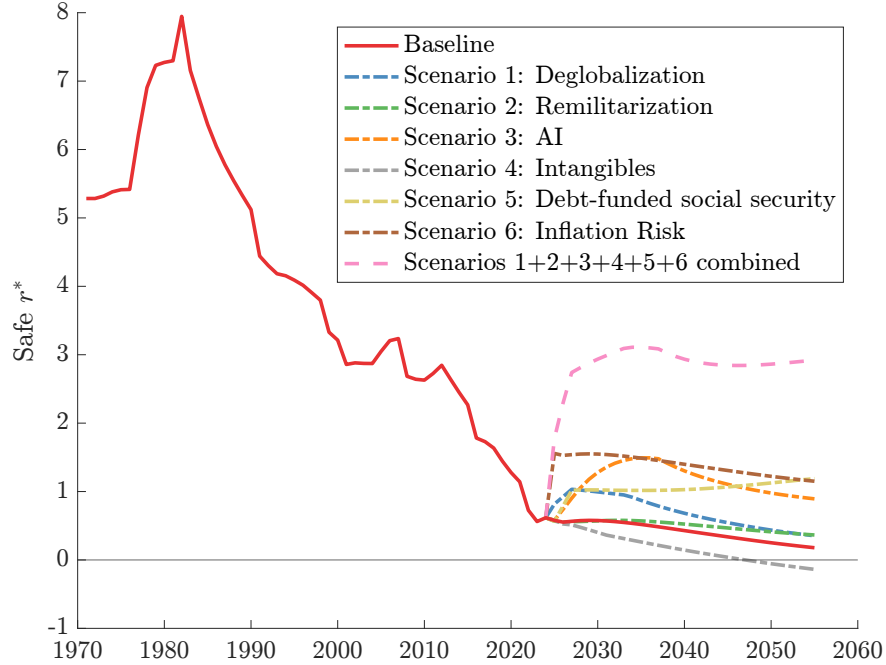


Figure 19: Upside risk scenarios

*Notes:* The figure shows simulations based on surprise increases in exogenous factors described in the main text. The combined scenario is calculated as the sum of the marginal effects of each of the six scenarios considered.

## 9 Conclusions

The natural real interest rate is unobserved but central to economic policy and long-horizon private decisions. The recent departure from the three-decade downward drift in government bond yields calls for a reassessment of the determinants of  $r^*$ .

This paper develops and quantifies a capital-market equilibrium framework to study the joint evolution of  $r^*$  and wealth across advanced economies. The central finding is that the advanced-economy bloc has undergone substantial shifts in both capital demand and—especially—capital supply. These shifts help explain the long-run decline in the natural rate and the sharp increase in the wealth-to-GDP ratio, broadly consistent with empirical trends.

In the new steady state, capital demand and supply are more elastic. Despite the near-doubling of the wealth-to-GDP ratio, equilibrium saving and investment flows remain modest, in line with the data. The business-as-usual scenario—based only on drivers identified prior to the recent economic and geopolitical shocks—implies that  $r^*$  will remain low. It is somewhat higher than market-implied real rates in the pre-pandemic years (0–1.2 percent

rather than negative values), but still far below historical averages.

Scenario analysis points to plausible upside risks to  $r^*$  and highlights two themes. First, the safety premium plays a special role in anchoring long-term rates. Second, understanding how structural forces shape long-run prosperity and growth is essential. Future research should investigate risk premia in greater detail, including idiosyncratic risks to physical and intangible capital, shifts in preferences toward risk, and the evolving supply of safe assets. Quantifying the productivity and welfare gains from artificial intelligence and other technologies will also be crucial for forecasting  $r^*$ .<sup>31</sup>

Finally, this analysis has focused on the advanced-economy bloc. While the resulting  $r^*$  serves as a useful benchmark for an integrated area, country-specific natural rates may diverge from the bloc rate under incomplete financial integration. Extending the framework to incorporate country-level drivers of  $r^*$ , together with measures of cross-border financial linkages, is a promising avenue for future work.

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<sup>31</sup>For example, [Rachel \(2022\)](#) studies leisure-enhancing technologies – technologies that are monetized indirectly through the sale of consumers’ time, attention and data, and considers the implications for growth and for the natural rate.



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# Appendix

## A Solution to retirees' and workers' problems

**Retirees.** The first-order conditions of the retiree's problem yield the Euler Equation:

$$C_{t+1}^{rjk} = (R_{t+1}\beta)^\sigma C_t^{rjk}$$

Denoting by  $\epsilon_t \pi_t$  the retiree's marginal propensity to consume out of wealth<sup>32</sup>, we can write down retiree's consumption function as:

$$C_t^{rjk} = \epsilon_t \pi_t (R_t / \gamma) A_t^{rjk}$$

Plugging this expression into the Euler Equation yields the expression for the evolution of the retiree's MPC:

$$\epsilon_t \pi_t = 1 - (R_{t+1}^{\sigma-1} \beta^\sigma \gamma) \frac{\epsilon_t \pi_t}{\epsilon_{t+1} \pi_{t+1}}. \quad (19)$$

**Workers.** The Euler Equation from the worker's problem is:

$$\omega C_{t+1}^{rjk} + (1 - \omega) \Lambda_{t+1} C_{t+1}^{rj(t+1)} = (R_{t+1} \Omega_{t+1} \beta)^\sigma C_t^{wj}$$

where  $\Lambda$  is the marginal rate of substitution across consumption while being a worker and a retiree, and  $\Omega$  is a weighing factor which captures the fact that workers discount future more:  $\Omega_{t+1} = \omega + (1 - \omega) \epsilon_{t+1}^{\frac{1}{1-\sigma}}$ .<sup>33</sup>

Denoting the MPC of the worker by  $\pi$ , and conjecturing that the consumption function takes the form:

$$C_t^{wj} = \pi_t (R_t A_t^{wj} + H_t^j + S_t^j)$$

(where  $H$  stands for human wealth and  $S$  is social security wealth, given respectively by  $H_t^j = \sum_{\nu=0}^{\infty} \frac{W_{t+\nu} - T_{t+\nu}}{\prod_{z=1}^{\nu} R_{t+z} \Omega_{t+z} / \omega}$  and  $S_t^j = \sum_{\nu=0}^{\infty} \frac{E_{t+\nu}^j}{\prod_{z=1}^{\nu} R_{t+z} \Omega_{t+z} / \omega}$ ), we obtain the time path of worker's MPC :

$$\pi_t = 1 - (R_{t+1} \Omega_{t+1})^{\sigma-1} \beta^\sigma \frac{\pi_t}{\pi_{t+1}}.$$

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<sup>32</sup>Reasons for this notation will become clear momentarily.

<sup>33</sup>For the complete derivation of worker's Euler Equation, see the Appendix of [Gertler \(1999\)](#).

Social security wealth of the workers is:

$$S_t^w = \sum_{\nu=0}^{\infty} \frac{(1-\omega)\omega^\nu N_t \left( \frac{\epsilon_{t+\nu+1} \frac{S_{t+\nu+1}}{\psi N_{t+\nu+1}}}{R_{t+\nu} \Omega_{t+\nu}} \right)}{\prod_{z=1}^{\nu} R_{t+z} \Omega_{t+z}}.$$

The numerator of the sum on the right hand side is a time- $t + \nu$  capitalized value of the social security payments to all the individuals who were in the workforce at  $t$  and retire at  $t + \nu + 1$ . The total social security wealth is just the infinite sum of the discounted value of these capitalized payments.

The evolution of total wealth of retirees is the sum of return on their wealth from last period plus what the newly retired bring in:

$$\lambda_{t+1} A_{t+1} = \lambda_t R_t A_t - C_t^r + (1-\omega)[(1-\lambda_t)R_t A_t + W_t - C_t^w]$$

From this, we get the explicit expression for the evolution of the retiree share:

$$\lambda_{t+1} = \omega(1 - \epsilon_t \pi_t) \lambda_t R_t \frac{A_t}{A_{t+1}} + (1 - \omega).$$

## B Details about the imperfect transition path assumptions

This appendix presents the specifics and the results of the limited foresight exercise. Figures 20 and 21 plot, in solid red, the realized data for each shifter; dotted lines show the expectations formed at different dates. To illustrate, the first panel traces advanced-economy government debt, which rose from about 28% to 82% of GDP and is expected to increase further. The dotted curve labeled “expectations from start” depicts what agents in the 1970 steady state anticipated: a roughly constant debt ratio near 30%. That view held until the early 1980s, when debt began to climb. Initially, this was thought to be temporary (anticipated fiscal consolidation), but after a decade of persistence, expectations for the debt-to-GDP ratio were revised up toward 50%. Subsequent surprises included the jump in debt after the global financial crisis (expectations updated in 2009) and during the COVID period, followed by recent downside surprises as post-pandemic inflation eroded debt ratios. Deficits nevertheless remained elevated, with looser fiscal stances (e.g., suspension of Germany’s debt brake and major U.S. fiscal packages), reflected in the most recent expectations.

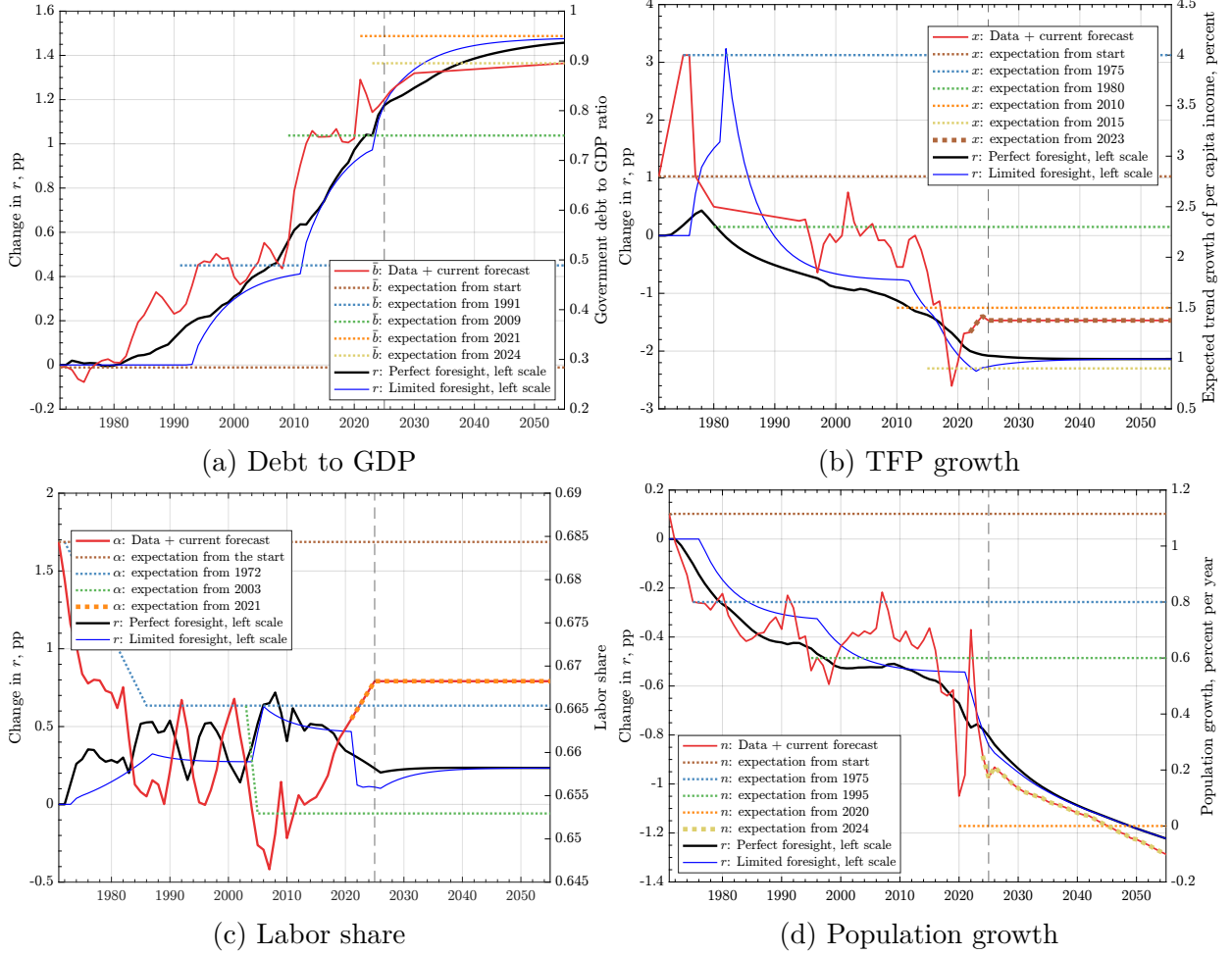


Figure 20: Limited foresight: assumptions on the evolution of expectations and the results in terms of  $r^*$  dynamics

In each panel, the two lines on the left scale show the transition of the safe natural real rate  $r^*$  under perfect foresight (solid black) and under limited foresight using the evolving expectations just described (solid blue). In general, the  $r^*$  profiles under limited forecast are less smooth. This is intuitive: agents are surprised over time; not anticipating the shocks means they are able to smooth through them less. But ultimately both the perfect and limited forecast lines converge to the same endpoint because expectations are assumed to align over the forecast horizon.

Within each panel, comparing the solid black and blue lines—the perfect- and limited-foresight paths for the natural rate—shows how much relaxing perfect foresight matters for each trend. For some drivers the gap is small: changes in population growth, labor share, depreciation, and net foreign assets move  $r^*$  similarly whether anticipated or not.

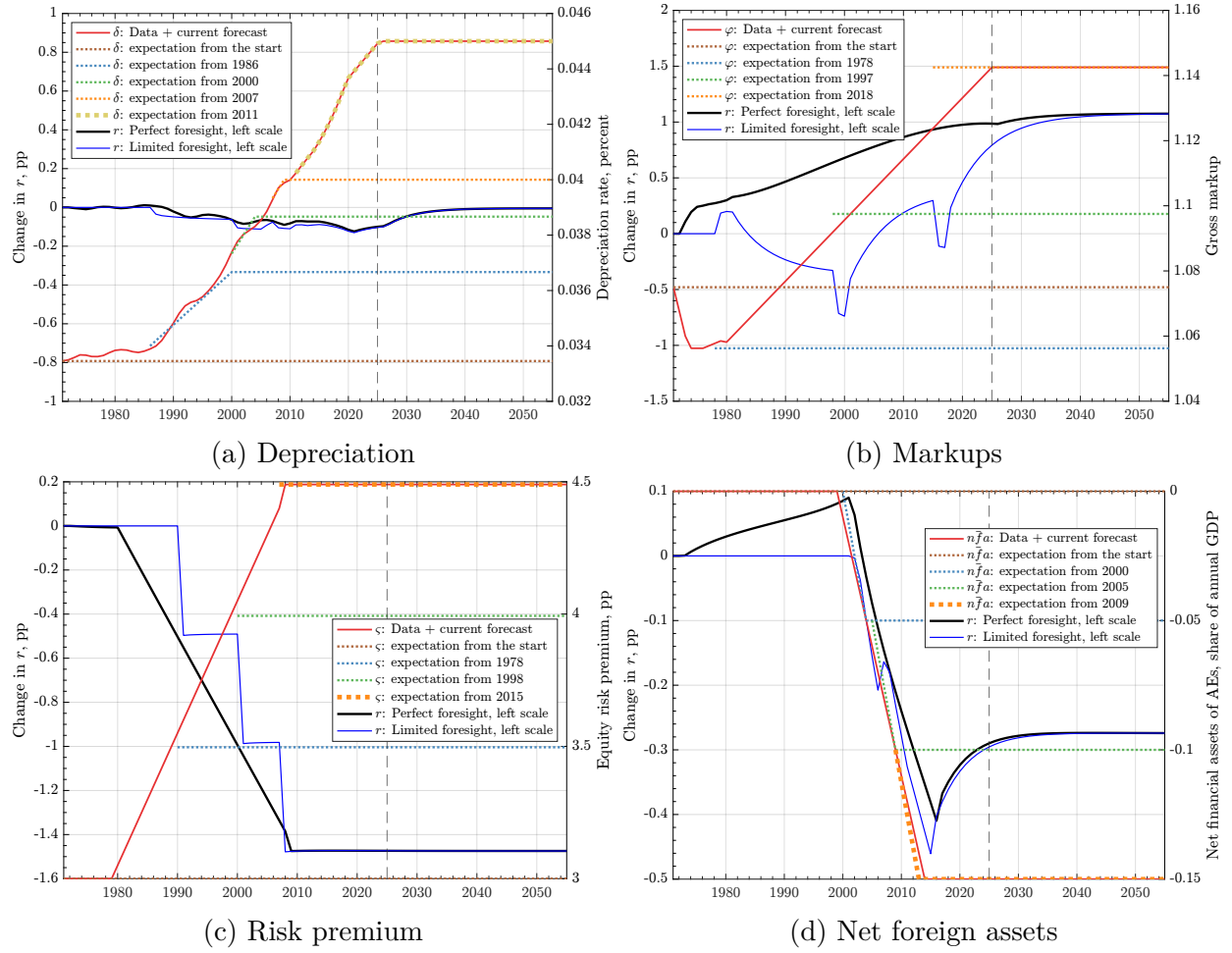


Figure 21: Limited foresight: assumptions on the evolution of expectations and the results in terms of  $r^*$  dynamics

For other drivers, imperfect foresight is pivotal—most notably for markups. An unexpected rise in markups lowers  $r^*$  in the short run before pushing it above baseline in the long run. The short-run decline reflects weaker capital demand with a predetermined capital stock, which forces the rental rate down; over time firms shed capital and  $r^*$  rises. In capital market equilibrium, the valuation effect from higher firm equity offsets (and ultimately dominates) the contraction in capital demand. Because markups fell in the 1970s before rising from the 1980s onward, the perfect-foresight path implies a persistent upward push on  $r^*$ . Under limited foresight, by contrast, markups depressed  $r^*$  for much of the period and only recently become a force raising it as expectations converge to their long-run level.

Imperfect foresight also reshapes the timing of the growth slowdown’s impact. Under perfect foresight, the post-GFC decline in trend growth is anticipated well in advance, dragging on  $r^*$  before the crisis. With limited foresight, elevated mid-1970s expectations lift  $r^*$ , early-1980s revisions pull it down, the Great Moderation stabilizes it, and then the GFC prompts a renewed decline as agents internalize the persistence of weaker growth in living standards.