GUIDO LORENZONI
University of Chicago

IVÁN WERNING
Massachusetts Institute of Technology

Wage-Price Spirals

ABSTRACT We interpret recent inflation experience through the lens of a New Keynesian model with price and wage rigidities and nonlabor inputs in inelastic supply. The model provides a natural interpretation of some features of the recent episode: an initial surge of noncore inflation, followed by a lagged response of core inflation and a further lagged, persistent response of wage inflation. The model also provides a natural way of discussing the role and the strength of wage-price spiral dynamics in price-setting models. The model interprets recent developments as symptoms of underlying supply constraints, which can be triggered by both demand and supply shocks. The immediate manifestation of these constraints is in the relative price of scarce, inelastic nonlabor inputs (including energy). The secondary effects arise because they produce a gap between lowered real wage aspirations of firms—that try to make up for higher nonlabor costs—and increased real wage aspirations of workers—caused by increased labor demand. The gap produces a wage-price spiral, which continues as long as the initial relative scarcity of nonlabor inputs persists, even though input prices are falling. In this view, the fact that nominal wage growth is currently exceeding price inflation can be given a benign interpretation, as a sign of real wages going back to trend and not necessarily as a concern of an ongoing spiral.

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The recent inflation surge in the United States and in the rest of the world has reignited debates about inflation's origins and propagation mechanisms. In particular, it has brought to the forefront the separate roles and interaction of prices, wages, and profits, and indeed it has done so at two key junctures.

Early on, at the first juncture, many worried that inflation would emanate from a tight labor market, stimulated by expansionary fiscal and monetary policies, causing wage inflation that would then produce price inflation.[1] This is not how inflation played out though. Instead, price inflation and profit margins soared, while wage growth picked up later and more gradually, implying an initial fall in real wages as shown in figure 1.[2]

More recently, as price inflation started falling, wage growth rose, surpassing inflation and leading to a rise in real wages. At this second juncture, the concern is that higher wage growth would prevent inflation from going back to target, or even set off an out-of-control wage-price spiral.

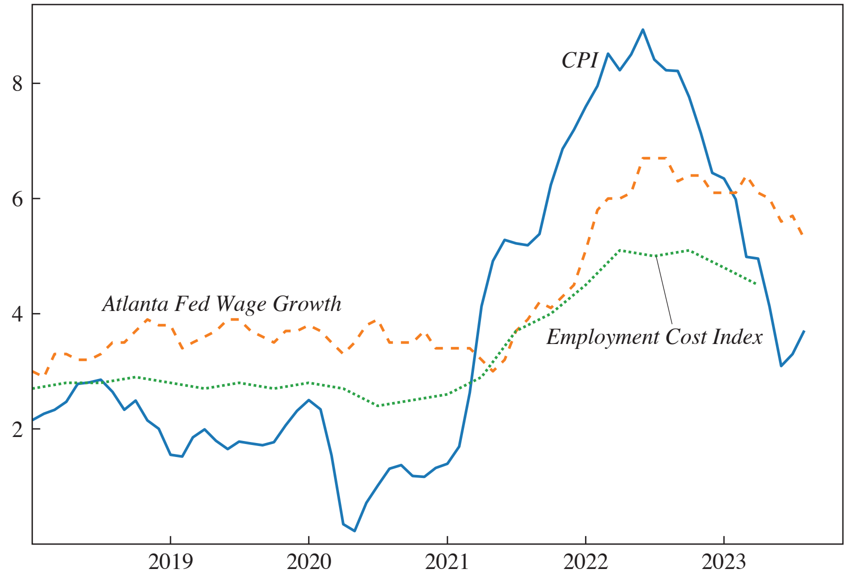
This paper aspires to simultaneously improve our understanding of these recent events while sharpening underlying economic concepts and intuitions surrounding inflation. To this end, we lay out a simple macroeconomic model. We show that this simple model is capable of capturing some key features of the recent episode. Our conceptual analysis dissects the role of prices and wages, isolating their interaction to provide a working definition of wage-price spiral and to understand the dynamics of the real wage.

Our model is relatively close to standard models, but with two essential features not always present in the most basic New Keynesian setups. One important feature of our analysis is the inclusion of a scarce nonlabor input with low substitutability in production (lower than Cobb-Douglas). We do not have in mind general forms of capital but rather inputs like energy, other primary commodities, or intermediate inputs that may be subject to shortages or in relatively fixed supply in the short run, for example, lumber or microchips. These nonlabor inputs provide both a potential supply shock or a supply constraint for demand shocks. This feature of our model is motivated by the 2020–2023 COVID-19 crises and post-COVID-19 recovery.

1. Economists who sent prescient, early warnings on inflation risk, like Blanchard (2021), focused on this transmission mechanism.

2. In the figure, along with Consumer Price Index (CPI) inflation, we show two measures of wage inflation, both of which avoid including compositional effects: the Bureau of Labor Statistics Employment Cost Index (all civilian workers, twelve-month change) and the Federal Reserve Bank of Atlanta's Wage Growth Tracker (overall, twelve-month change).

Figure 1. Post-pandemic Price and Wage Inflation in the United States



Source: Bureau of Labor Statistics and Federal Reserve Bank of Atlanta.

The other important feature of our model is that we include both nominal price and wage rigidities, as in many medium-scale models, but unlike the simplest New Keynesian models with only one form of nominal rigidity.

In a model with these features, supply constraints play a crucial role in inflation dynamics, and when these supply constraints are active, both demand and supply disturbances can set in motion price and wage dynamics that resemble the ones observed.

Namely, the model can produce a three-phase pattern of adjustment in nominal prices. First, there is a bout of very high inflation in the price of the inelastic nonlabor inputs, followed by a prolonged gradual fall in the price of these inputs. Second, there is a more persistent period of high general good price inflation. Third, there is a smaller but even more persistent increase in wage inflation.

The pattern described follows from our assumptions on the role of the inelastic input, which more directly affects price-setting firms, and on the relative degree of price stickiness with the input price being perfectly flexible and goods prices being more flexible than wages. This pattern implies that, at some point, wage inflation crosses price inflation, so a period in which real wages fall is followed by a period in which they recover.

Data are always interpreted with a theoretical lens. At one end of the spectrum, commentators and Federal Reserve governors' speeches often employ standard macroeconomic concepts, such as a Phillips curve, in their simplest incarnations, to fix ideas or make back-of-the-envelope calculations. On the other end of the spectrum, several papers have contributed by calibrating sophisticated multi-sectoral models. Our paper lives in the gap between these extremes—our model is simpler than medium scale calibrated models, allowing us to develop several important concepts, yet it goes beyond textbook tools used in day-to-day policy debates.

Turning to the more conceptual points of our paper, one may ask, what do we mean by a wage-price spiral? While there may not be universal agreement, in this paper we use the expression to describe a feedback mechanism where wages and prices compete adjusting upward: wage earners try to keep up with rising prices; price setters try to keep up with rising wages. This mechanism amplifies and perpetuates the effects of certain inflationary shocks.

Our perspective is that this feedback mechanism is present in virtually all models—including standard New Keynesian varieties. The purpose of this paper is to elucidate and explore this mechanism in detail and focus on the shape of price and wage responses to both supply and demand shocks.

At heart, the economic logic of the wage-price spiral mechanism is that workers and firms disagree on the relative price of goods and labor, that is, on the real wage W/P. When firms adjust nominal prices, they do so with some goal for W/P. But workers may have a different, higher goal for W/P and set nominal wages to reach that goal. If they do, the outcome of this disagreement is nominal escalation, with inflation in both prices and wages.

Our interpretation of the concept of a wage-price spiral, highlighting disagreement or conflict as a proximate cause of inflation, is an idea that we explore more generally in Lorenzoni and Werning (2022). The present paper studies how this conflict plays out in particular variants of the New Keynesian model and places attention on the path of real wages in response to demand and supply shocks.

Beyond providing an interpretation of recent inflation dynamics, we also use our model to derive a number of general positive and normative results.

First, we derive a general condition for the direction of adjustment of the real wage in response to demand shocks. We show that whether the real wage increases or decreases following a demand shock depends on how strong the forces set in motion on the price-setting side of the model and on the wage-setting side are.

A demand shock acts on the price side by producing an endogenous increase in the price of nonlabor inputs. If there is low degree of substitutability between labor and nonlabor inputs, we get both a large price response of nonlabor inputs and a large reduction in the marginal product of labor when nonlabor inputs are relatively scarce. The first force will show up in noncore inflation measures. The second will contribute to a distributional tension between workers and firms that materializes in a wage-price spiral.

A demand shock also acts on the wage side directly. Our model does not feature unemployment and search directly, but the labor supply side of our model captures the basic idea that an overheated labor market will directly affect nominal wage demands by increasing the rate at which workers are willing to exchange labor for consumption goods. Therefore, this piece of the model captures the basic logic of a wage Phillips curve. Through this channel, excess demand will also produce higher real-wage aspirations for workers and contribute to the wage-price spiral.

Excess demand operates and contributes to a wage-price spiral on *both* sides. However, for the movement in the real wage, what matters is the relative strength on the two sides. In our low-elasticity-of-substitution calibration, the effect is stronger on the price side and thus produces overall lower real wages.[3]

An additional observation that comes from our analysis is that both demand and supply shocks create a situation of excess demand. In the demand shock case, natural output is unchanged, but the demand temporally expands. In the supply shock case, the "natural" level of output is lower, but the demand is unchanged. This excess demand leads to a tension between the level of the real wage that firms and workers aspire to, resulting in a wage-price spiral that produces inflation in both wages and prices. However, excess demand is not a sufficient statistic. In the supply shock case, real wages always fall; whereas in the demand shock case, the real wage may fall depending on parameters. Only under some conditions are the effects on wages and prices similar for both shocks.

Excess demand is zero when there is a zero output gap. A result that applies in our model is that, with a zero output gap, there can never be both

3. Incidentally, our analytical result can be taken as a contribution to the classic debate on the cyclicality of the real wage that has spurred a large body of literature, including Christiano and Eichenbaum (1992) and Rotemberg and Woodford (1992). However, our aim here is not to discuss the general cyclical property of real wages but rather to discuss how potentially sizable real wage movements can be set in motion in special circumstances, like the recent post-pandemic recovery.

price and wage inflation, that is, price and wage inflation always have the opposite sign. Furthermore, our definition of conflict inflation (Lorenzoni and Werning 2022), which we use to capture the wage-price spiral force, is closely related to the size of the output gap in the New Keynesian model here. This connects us immediately to the notion of "divine-coincidence inflation" introduced by Rubbo (2020), which in the model here coincides with conflict inflation.[4]

The result just stated can be rephrased as that if the central bank successfully pursues a zero output gap, the central bank can always prevent a wage-price spiral (i.e., achieve zero conflict inflation). But it does not imply that a zero output gap policy is the optimal policy. In section IV, we study optimal policy and ask two questions. First, could it be part of optimal policy to "run the economy hot," that is, to allow for a positive output gap despite high inflation? Second, could it be part of optimal policy to go further and allow for inflation in both prices and wages?

Our answer to the first question is affirmative: if the economy needs a lower real wage, it may be more efficient to reach the adjustment with the help of higher price inflation and moderate wage deflation, rather than through lower price inflation and deeper wage deflation. A positive output gap helps shift the adjustment in the direction of price inflation, so it is socially beneficial in this manner.

The answer to the second question is also affirmative. We construct examples in which, at some point along the adjustment path, the output gap is positive, and price and wage inflation are both positive. The economic intuition is that this aspect of policy is a form of "forward guidance": by promising to heat up the economy in the future, we speed up the adjustment of the real wage today. Underlying this result is the assumption of forward-looking price- and wage-setting behavior and the commitment of policy. In contrast, when policy has full discretion, the equilibrium outcome never features both price and wage inflation.

There is a large and growing body of literature analyzing the post-pandemic surge in inflation in the United States and globally. Our paper is part of a group of papers that emphasizes the crucial role of supply disruptions and supply constraints in the recent inflation surge, a group that includes Ball, Leigh, and Mishra (2022), Amiti and others (2023), Bernanke and Blanchard (2023), Comin, Johnson, and Jones (2023), Gagliardone and Gertler (2023), and Kabaca and Tuzcuoglu (2023). We do it here by pointing

4. This is connected to the "divine-coincidence" inflation index of Rubbo (2020), which also only depends on the output gap.

out the explanatory power of this interpretation for the joint dynamics of prices and wages.

The way in which supply constraints play out here is closely related to the approach in Comin, Johnson, and Jones (2023), who develop a quantitative model with an explicit treatment on nonlinearities in the supply of nonlabor inputs and take an explicit open economy approach. We believe the virtue of this way of interpreting the facts is that it shows a state of global excess demand can cause endogenously sharp input price adjustments, which cannot be taken merely as exogenous price shocks.

Our model emphasizes the role of the real wage as a state variable. This plays an important role in our interpretation of recent events. In particular, we see the recent increase in the real wage as fundamentally driven by a desire of wage setters to make up for the accumulated losses in purchasing power during the early stage of the episode. In other words, we interpret the recent high wage inflation as driven by some form of catch-up. The empirical analysis by Bernanke and Blanchard (2023) provides an empirical challenge to this view, as they attempt to measure this catch-up mechanism in the data and fail to find it significant. However, it is not easy to identify structurally this channel of catch-up, and in general, findings of wage inflation responding to past price inflation can be taken as supportive of a lag effect, leading to a lag recovery of real wages.[5]

In terms of the broader idea of wage-price spiral, our paper is connected to a vast amount of literature, and we will make only a few close references here. Blanchard (1986) wrote the seminal paper connecting that idea to New Keynesian models of staggered price setting. The model has nominal prices and wages that are fixed for two periods, with prices reset in even periods and wages in odd periods. The main result in the paper is that the alternating wage and price setting leads to a slow adjustment of the price level in response to a permanent money supply shock and the adjustment features dampening oscillations in the real wage. Our paper instead builds on the canonical New Keynesian setting with sticky price and sticky wages of the Calvo variety as developed by Erceg, Henderson, and Levin (2000). Relative to Blanchard (1986), price and wage setting occur in a staggered fashion without the predictable alternation between wages and prices, so our model is not prone to the same type of oscillations. We also do not focus on a permanent money shock or study monetary policy in terms of money supply. Instead, we focus on supply and demand shocks under different policy responses. Finally, we investigate optimal monetary policy.

5. See, for example, the regressions in Barlevy and Hu (2023) and the literature cited there.

Our analysis of wage-price spirals in section II builds on the idea of inflation as the result of distributional conflict, something we explore in more detail in Lorenzoni and Werning (2022). A seminal contribution on this conflict perspective of inflation is Rowthorn (1977). That paper provides a model where, in each period, wages are first set by workers and then prices are set by firms. Inflation is shown to be increasing in the conflict or "aspirational gap." Because of the assumed sequential timing of price and wage setting, conflict and inflation must not be fully anticipated by workers. Indeed, no rational expectations equilibrium exists with conflict. In contrast, our model features staggered wages and prices, which ensure that there is an equilibrium with finite conflict and inflation, even under rational expectations.

Our modeling of nonlabor inputs and their connection to price and wage determination connects our analysis to extensive literature on models of energy shocks.[6] An important modeling difference is that we focus on nominal wage rigidities, while they study a form of real-wage rigidity.

On the normative side, our paper is connected to the welfare analysis of alternative policy rules in models where both prices and wages are rigid, going back to the original paper by Erceg, Henderson, and Levin (2000) and to the real rigidity model by Blanchard and Galí (2007b). The starting observation in the literature is that the presence of both price and wage rigidities breaks divine coincidence and introduces potentially interesting trade-offs in the response of monetary policy to supply shocks. We offer a complete characterization of optimal policy and explore conditions for the optimum to have a positive output gap in combination with high inflation, as well as cases where it is optimal to have both wage and price inflation.

I. Model

We build our arguments in a standard New Keynesian model with nominal price and wage rigidities. To capture supply shocks, an important ingredient we include is a scarce nonlabor input X, which is used alongside labor for production. We assume this input has a flexible price, and we allow the production function to have elasticity of substitution different from one.[7] An important example is energy inputs, but we interpret X more broadly to

6. For example, Blanchard and Galí (2007a); in turn, this connects us to the enormous body of literature on the effects of oil shocks, going back to Bruno and Sachs (1985).

7. This is formally equivalent to having labor and capital, with capital rented at a flexible price, although the interpretation is different. Erceg, Henderson, and Levin (2000) have labor and capital. Closer to the interpretation here, Blanchard and Galí (2007a) have an energy input.

also capture shortages, bottlenecks, and capacity constraints in the supply of intermediates like microchips or lumber, which have been in the spotlight during the post-pandemic recovery.

We focus on a closed economy in which the supply of X is given while the price of X adjusts endogenously in equilibrium. The analysis can be easily expanded to the case of an open economy in which the good X is imported, and, in particular, to the limited case of a small open economy that takes the world price of X as given. In that case, a supply shock would take the form of a shock to the world price instead of a shock to the endowment.

I.A. Setup

Time is continuous and infinite. The representative household has preferences

$$\int_0^\infty e^{-\rho t}\left(\frac{1}{1-\sigma}C_t^{1-\sigma}-\frac{\Phi_t}{1+\eta}N_t^{1+\eta}\right)dt,$$

where C_t is an aggregate of a continuum of varieties of goods

$$C_t=\left(\int_0^1 C_{jt}^{1-\frac{1}{\varepsilon_C}}dj\right)^{\frac{1}{1-\frac{1}{\varepsilon_C}}},$$

N_t is labor supply, and Φ_t is a labor supply shock. Each goods variety j is supplied by a monopolistic firm with production function

$$Y_{jt}=F\left(L_{jt},X_{jt}\right)\equiv\left(a_L L_{jt}^{\frac{\epsilon-1}{\epsilon}}+a_X X_{jt}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}},$$

where L_{jt} is the labor input and X_{jt} is the nonlabor input. The labor input L_{jt} of each firm j is an aggregate of a continuum of labor varieties

$$L_{jt}=\left(\int_0^1 L_{jkt}^{1-\frac{1}{\varepsilon_L}}dk\right)^{\frac{1}{1-\frac{1}{\varepsilon_L}}}.$$

Each labor variety k is supplied by a monopolistic union that employs labor from households and turns it, one for one, into specialized labor services of type k. Integrating over firms, total employment of labor variety k is

$N_{kt} = \int_0^1 L_{jkt} dj$. Integrating over unions, total labor supply is $N_t = \int_0^1 N_{kt} dk$. The representative household owns an exogenous endowment X_t of the nonlabor input X and sells it to the monopolistic goods producers on a competitive market, at the price P_{Xt}.

Monopolistic firms set the nominal price at which they are willing to sell their variety and then supply the amount chosen by consumers. Similarly, monopolistic unions set the nominal wage and supply the amount chosen by firms. Firms and unions are only allowed to reset their price and their wage rate occasionally. Namely, at each point in time, firms are selected randomly to reset their price with Poisson arrival λ_p, and unions are selected with arrival λ_w.

When the exogenous variables X_t and Φ_t are constant, the model has a steady state in which quantities are constant, nominal prices are constant (zero inflation), all goods varieties have the same price, and all labor varieties have the same wage. We will consider an economy in steady state and analyze its response to one-time, unexpected shocks, either due to changes (transitory or permanent) to X_t or Φ_t, or to changes in monetary policy leading to transitory deviations of C_t and N_t from the path consistent with zero inflation.

I.B. Price and Wage Setting

Let P_t^* and W_t^* denote the price and wage set by the firms and unions that can reset at time t, while P_t and W_t denote the price indexes for the goods and labor aggregates.

The nominal marginal cost of producing good j is

$$\frac{W_t}{F_L\left(L_{jt}, X_{jt}\right)} = \frac{W_t}{a_L Y_{jt}^{\frac{1}{\epsilon}} L_{jt}^{-\frac{1}{\epsilon}}}.$$

Using lowercase variables to denote log-linear deviations from steady state and taking a first-order approximation, nominal marginal costs can then be expressed as

$$w_t - mpl_{jt}, \tag{1}$$

where

$$mpl_{jt} = \frac{1}{\epsilon}\left(y_{jt} - l_{jt}\right)$$

is the marginal product of labor. The production function of firm j in log-linear approximation is

(2) $$y_{jt} = s_L l_{jt} + s_X x_{jt},$$

where s_L and s_X are the steady-state shares of the labor and nonlabor inputs, with $s_L + s_X = 1$. All firms being price takers in the input market, they all employ inputs in the same ratio L_{jt}/X_{jt}, so in log-linear approximation

$$l_{jt} - x_{jt} = n_t - x_t,$$

where n_t and x_t are the aggregate supplies of the two inputs. Combining these results, the marginal product of labor is

(3) $$mpl_t = \frac{s_X}{\epsilon}\left(x_t - n_t\right).$$

Following standard steps, optimal price setting requires that firms set their price at time t equal to an average of future nominal marginal costs, conditional on not resetting. This gives the following optimality condition for P_t^* in log-linear approximation:

(4) $$p_t^* = \left(\rho + \lambda_p\right)\int_t^\infty e^{-(\rho+\lambda_p)(\tau-t)}\left(w_\tau - mpl_\tau\right)d\tau.$$

Following similar steps, we can derive the wage-setting equation

(5) $$w_t^* = \left(\rho + \lambda_w\right)\int_t^\infty e^{-(\rho+\lambda_w)(\tau-t)}\left(p_\tau + mrs_\tau\right)d\tau,$$

where

(6) $$mrs_t = \phi_t + \sigma y_t + \eta n_t$$

is the marginal rate of substitution between consumption and leisure of the representative consumer.

The presence of w_τ on the right-hand side of equation (4) and p_τ on the right-hand side of equation (5) captures the logic of a wage-price spiral in our model. Firms aim to get prices to be a constant markup over nominal marginal costs, and since marginal costs depend on nominal wages, they set nominal prices to catch up with current and anticipated future nominal wages. Symmetrically, wage setters aim to achieve a real wage that reflects their willingness to substitute leisure with consumption goods, so they set nominal wages to catch up with current and anticipated future nominal goods prices.

The optimality condition for the input ratio of firms can be written as follows:

$$p_{Xt} = w_t - \frac{1}{\epsilon}\left(x_t - n_t\right). \tag{7}$$

This condition will be used to derive the equilibrium input price p_{Xt}.

I.C. Inflation Equations

To go from equations (4) and (5) to wage and price inflation, combine them with the differential equations for p_t and w_t:

$$\dot{p}_t = \lambda_p\left(p_t^* - p_t\right) \text{ and} \tag{8}$$

$$\dot{w}_t = \lambda_w\left(w_t^* - w_t\right). \tag{9}$$

As shown in the online appendix, we then obtain the following expressions:

$$\rho\pi_t = \Lambda_p\left(\omega_t - mpl_t\right) + \dot{\pi}_t \text{ and} \tag{10}$$

$$\rho\pi_t^w = \Lambda_w\left(mrs_t - \omega_t\right) + \dot{\pi}_t^w, \tag{11}$$

where we use the notation $\pi_t \equiv \dot{p}_t$ and $\pi_t^w \equiv \dot{w}_t$ for price and wage inflation and $\omega_t \equiv w_t - p_t$ for the real wage, and the coefficients Λ_p and Λ_w are

$$\Lambda_p = \lambda_p\left(\rho + \lambda_p\right) \text{ and } \Lambda_w = \lambda_w\left(\rho + \lambda_w\right).$$

Real wage dynamics are given by

$$\dot{\omega}_t = \pi_t^w - \pi_t. \tag{12}$$

Equations (10) and (11) can be interpreted in terms of a conflict between the real wage aspirations of workers and firms, an interpretation we develop in Lorenzoni and Werning (2022). In the context of the New Keynesian model, the workers' aspiration is given by the marginal rate of substitution mrs_t at which the representative worker is willing to exchange labor for goods, and the firms' aspiration is the marginal product of labor mpl_t.[8] As in Lorenzoni and Werning (2022), a discrepancy between the aspirations mpl_t and mrs_t is the proximate cause of inflation.

Equations (10) and (11) can also be expressed as traditional Phillips curves because the expressions $\omega_t - mpl_t$ and $mrs_t - \omega_t$ can be written in terms of gaps between equilibrium objects and their "natural" level. Focusing on the wage equation, we can write

$$\begin{aligned} mrs_t - \omega_t &= mrs_t - mrs_t^* - \left(\omega_t - \omega_t^*\right) \\ &= \left(\sigma s_L + \eta\right)\left(n_t - n_t^*\right) - \left(\omega_t - \omega_t^*\right), \end{aligned}$$

where ω_t^* is the flexible-price wage rate and n_t^* is the natural level of employment.[9] Substituting this expression in equation (11) we obtain a wage Phillips curve that connects wage inflation to the employment gap $n_t - n_t^*$. An analogous derivation can be done for the price equation. The crucial observation here is that in both Phillips curves there is an additional term, given by the deviation between the real wage and its flexible-price level ω_t^*. Notice that ω_t is a state variable of our system because both w_t and p_t move only gradually due to stickiness—at a given moment in time ω_t given by the history of past shocks.

Given an initial condition ω_0 and given paths for mpl_t and mrs_t for $t \geq 0$, the three equations (10)–(12) give unique paths for price and wage inflation.

Our approach in the rest of the paper is to split the analysis into two steps: (1) from the paths for fundamental shocks and aggregate real activity derive the paths of mpl_t and mrs_t; and (2) from the paths of mpl_t and mrs_t derive inflation. In general, in a full-blown general equilibrium model, the paths of mpl_t and mrs_t are endogenous and this way of splitting the analysis is somewhat artificial. However, a central point of this paper is to show that this decomposition helps understand the mechanisms underlying inflation in equilibrium.

8. The variable ϕ_t in the notation of Lorenzoni and Werning (2022) corresponds to mpl_t here and the variable γ_t corresponds to mrs_t.

9. This derivation applies because at the natural allocation the real wage is equalized to the workers' *mrs*. The detailed derivations are in the online appendix.

The next section focuses on step 2. We then go back to step 1 in the following section.

II. From Aspirations to Inflation, with and without a Spiral

In general, shocks to the economy translate into endogenous changes in the variables *mpl* and *mrs*, which, as argued above, reflect the real wage aspirations of firms and workers. In this section, we take the paths of *mpl* and *mrs* as given and focus on deriving inflation as a function of them. This part of the analysis isolates how staggered price setting produces inflation for given aspirations and allows us to identify the wage-price spiral mechanism. The next section shows how shocks and policies determine *mpl* and *mrs* and thus completes the analysis. A reader mostly interested in our interpretation of the post-pandemic high inflation episode can skip this section without loss.

Throughout the paper, we mostly focus on exponentially decaying paths of *mpl* and *mrs* that take the following form.[10] Before $t = 0$, the economy is in steady state: all variables expressed in log deviations from the steady state are equal to zero. At $t = 0$, there is an unexpected shock and mpl_0 and mrs_0 jump discretely to values different from zero (at least for one of them). From then on, they both converge back to the original steady state at constant speed δ, so $mpl_t = mpl_0 e^{-\delta t}$ and $mrs_t = mrs_0 e^{-\delta t}$. The demand and supply shocks analyzed in the next section produce paths with this shape, so the analysis here will immediately apply.

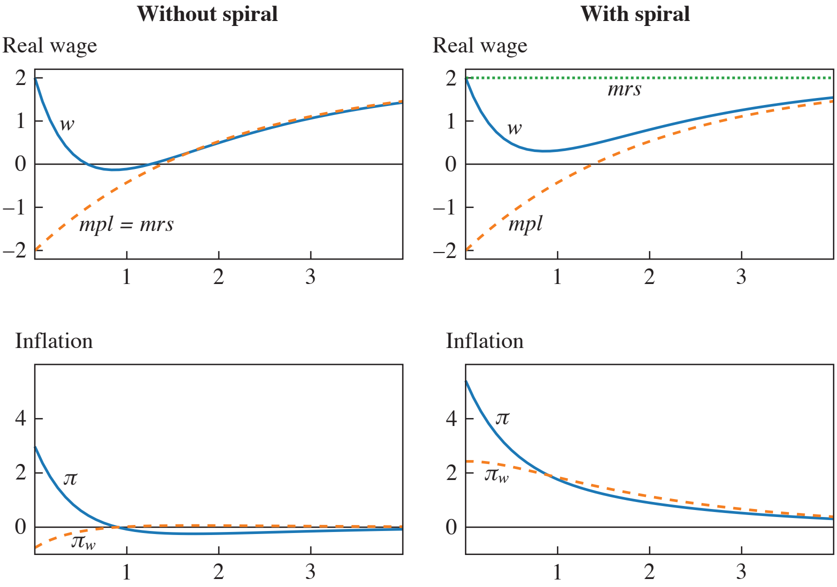
Deriving price and wage inflation from equations (10) and (11) requires solving first the endogenous path of the real wage ω_t. In other words, as mentioned earlier, the real wage is a necessary state variable in our inflation equations. The solution for the real wage in terms of *mpl* and *mrs* comes from solving a second-order ordinary differential equation (ODE); the details are provided in the online appendix. Once we have ω_t, equations (10) and (11) can be solved forward to get

$$\pi_t = \Lambda_p \int_0^\infty e^{-\rho s}\left(\omega_s - mpl_s\right)ds \text{ and} \tag{13}$$

$$\pi_t^w = \Lambda_w \int_0^\infty e^{-\rho s}\left(mrs_s - \omega_s\right)ds. \tag{14}$$

10. In the online appendix, we provide a general analytical characterization of the relation between the paths $\{mpl_t, mrs_t\}$ and price and wage inflation.

Figure 2. Aspirations and Inflation, with and without a Spiral



Source: Authors' calculations. The parameters for the examples are $\lambda_p = 2, \lambda_w = 1, \rho = 0.04, \delta = 0.5$.

Price and wage inflation are driven by current and anticipated gaps between the real wage and firms' and workers' aspirations. These two equations are used to provide intuition in this section.

II.A. Two Examples

Consider the two numerical examples plotted in figure 2.

In the first, *mpl* and *mrs* fall by the same amount at date 0, that is, $mrs_0 = mpl_0 < 0$. On impact, the reduction in *mpl* increases firms' marginal costs, leading firms to increase nominal prices, while the reduction in *mrs* lowers workers' aspirations and workers reduce nominal wages. In the top left panel of figure 2, we see that this leads to $\pi_0 > 0 > \pi_0^w$. The real wage starts falling, as shown in the lower left panel. As time goes by, the force of the initial shock goes away while, at the same time, the real wage is lower. Both forces reduce $\omega - mpl$ in the price inflation equation and increase $mrs - \omega$ in the wage inflation equation: the gap between aspirations and the real wage fall for both. After some date, when *mpl* and *mrs* are small enough and the real wage has fallen enough, both inflation rates π_t and π_t^w flip sign and we have $\pi_t < 0 < \pi_t^w$. From then on, the real wage starts growing and converges back to its initial level.

In this example, even though wage setters and price setters respond to each other's prices (current and anticipated), this does not produce generalized inflation or deflation, because the two parties are aiming to achieve the same relative price adjustment, so their actions tend to dampen each other. The fact that firms increase prices tends to remove the deflationary impulse on the workers' side. The fact that workers lower their wages tends to remove the inflationary impulse on the firms' side. In this case a wage-price spiral is not present.

In the second example, only the aspirations of firms change, with $mpl_0 < 0$, but mrs_0 is unchanged at zero. In this case there is a positive gap $mrs_0 - mpl_0$. This case is illustrated in the two panels on the right in figure 2.

On impact, the reduction in *mpl* increases firms' marginal costs as in the first example. Now there is no direct effect of *mrs* on the workers' side; workers anticipate a future reduction in real wages and react at date zero by raising their nominal wage demand.[11] Therefore, we get both wage and price inflation, $\pi_0 > \pi_0^w > 0$. In general, in every case where there is a unilateral change in *mpl*, with no change in *mrs*, it is possible to show that price inflation is larger than wage inflation at $t = 0$, given that the price equation is affected directly by the change in *mpl*, while the wage equation is only affected indirectly through the future equilibrium adjustment in ω.[12]

Notice the back and forth between price and wage inflation that amplifies the initial shock. The shock originates in the inflation equation but produces an undesirable relative price adjustment for workers, creating a positive gap between workers' aspirations and the real wage path, inducing wage setters to respond. This causes price inflation to spill over into wage inflation. The wage setters' response in turn dampens the adjustment in the real wage, relative to what happens in our first example: comparing the two lower panels in figure 2, the real wage ω_t falls less in the panel on the right. Therefore, the presence of wage inflation, slowing the fall in real wages, reinforces the price inflation response as firms, anticipating a weaker reduction in real wages, keep price inflation higher.[13]

11. In equation (14), $mrs_s = 0$ and $\omega_s < 0$ for all s. Why the real wage falls in this example is explained below.

12. See proposition 5 in the online appendix.

13. If nominal wages were perfectly sticky, this amplification would not be present and price inflation would be lower throughout. We go back to the relation between stickiness and amplification at the end of this section.

The expression “wage-price spiral” is used to describe these mutually reinforcing dynamics between price and wage inflation. In the first example there is no wage-price spiral, in the second there is.

II.B. Spiral Dynamics and Conflict Inflation

In the two examples above, we just argued that the first example shows no spiral while the second does. But how can we distinguish more formally the spiral force in the second from the relative price adjustment mechanism that drives nominal prices and wages in the first?

The crucial difference is that in the second example, the attempt of each side to move the relative price in its preferred direction leads to a protracted period of high inflation in both prices and wages. Let us measure the spiral effect in terms of the cumulated effect on price and wage inflation over the entire episode. Since the real wage always mean reverts to zero and cumulated price and wage inflation are the same, we can define

$$\Pi^{Spiral} \equiv \int_0^\infty \pi_t \, dt = \int_0^\infty \pi_t^w \, dt.$$

In the online appendix, we prove that

$$\Pi^{Spiral} = \frac{\Lambda_p \Lambda_w}{\Lambda_p + \Lambda_w} \frac{1}{\delta(\rho+\delta)} \left(mrs_0 - mpl_0 \right).$$

Notice the symmetric role of Λ_p and Λ_w in this expression: For the spiral effect to be present, we need prices and wages to respond to each other. If one side has fixed nominal prices, for example $\lambda_w = 0$, then the spiral is completely absent. On the other hand, if we vary λ_p and λ_w and hold fixed the total degree of nominal rigidity in the economy $\lambda_w + \lambda_p$, then the maximum power of the spiral arises when $\lambda_p = \lambda_w$, that is, when each side responds to the other with equal speed.

The spiral measure just introduced, immediately connects spiral dynamics to the notion of conflict inflation proposed in Lorenzoni and Werning (2022), which is defined as follows:

$$\Pi_t^{Conflict} \equiv \frac{\Lambda_p \Lambda_w}{\Lambda_p + \Lambda_w} \int_0^\infty e^{-\rho s} \left(mrs_{t+s} - mpl_{t+s} \right) ds,$$

and with exponentially decaying shocks, yields

$$\Pi_t^{Conflict} = \frac{\Lambda_p \Lambda_w}{\Lambda_p + \Lambda_w} \frac{1}{\rho + \delta} \left(mrs_0 - mpl_0 \right).$$

We then conclude that

$$\Pi^{Spiral} = \frac{1}{\delta} \Pi_0^{Conflict},$$

which means that conflict inflation at date zero fully captures the underlying forces that lead to a protracted period of joint price and wage inflation.

Notice that from equations (13) and (14), we get

$$\Pi_0^{Conflict} = \alpha \pi_0 + \left(1 - \alpha \right) \pi_0^w, \tag{15}$$

where α is a coefficient of relative stickiness, defined as

$$\alpha \equiv \frac{\frac{1}{\Lambda_p}}{\frac{1}{\Lambda_p} + \frac{1}{\Lambda_w}}.$$

We then have a "forecasting" interpretation of the result above. Consider an econometrician who does not observe the underlying shocks mrs_0 and mpl_0 at $t = 0$ but only the current inflation rates π_0 and π_0^w. Conflict inflation is the linear combination of π_0 and π_0^w that provides the best estimate of the cumulated future effect of the underlying shocks on inflation.[14]

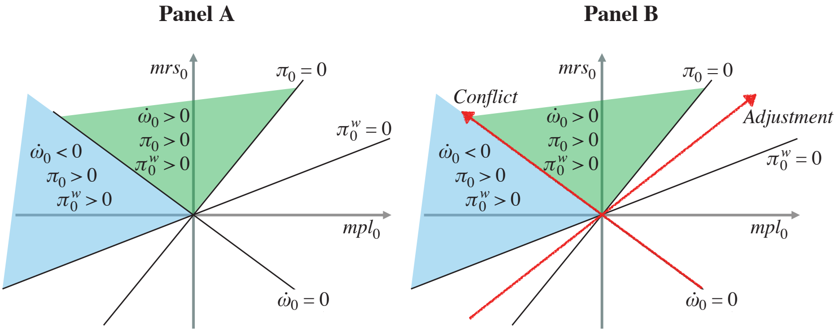
From equation (15) and $\dot{\omega}_0 = \pi_0^w - \pi_0$, we get the decomposition

$$\pi_0 = \Pi_0^{Conflict} - \left(1 - \alpha \right) \dot{\omega}_0 \text{ and}$$

$$\pi_0^w = \Pi_0^{Conflict} + \alpha \dot{\omega}_0.$$

14. This result relies on the simple joint AR(1) structure of the shocks to mrs_0 and mpl_0. It is an open important question how to extend the connection between conflict inflation and inflation forecasting to richer structures.

Figure 3. Regions for *mpl_0* and *mrs_0*



Source: Authors' calculations.

Conflict inflation captures the underlying common component of price and wage inflation due to the gap between the aspirations on the two sides of the market (mpl_0 and mrs_0). The presence of the gap is crucial to set in motion mutually reinforcing responses on the two sides. When there is no gap, there can be no generalized inflation, π_t and π_t^w have opposite sign, and the mutual responses tend to dampen the initial shock, consistent with our first example.

Notice that in the New Keynesian model considered here, conflict inflation $\Pi_t^{conflict}$ is proportional to the output gap as we shall see in the next section. This implies that conflict inflation coincides with the notion of divine-coincidence inflation in Rubbo (2020) and with the composite inflation index in the optimal inflation-targeting rule of Giannoni and Woodford (2005).[15]

A GRAPHICAL REPRESENTATION A graphical representation can help interpret the decomposition above.

In panel A of figure 3 we divide the space (mpl_0, mrs_0) into six regions, depending on the sign of the three variables π_0, π_0^w, and $\dot{\omega}_t$.

Proposition 1 shows that the configuration in figure 3 is general and independent of parameters, given exponentially decaying shocks. The proposition gives conditions in terms of the coefficient ψ, which is a function of the parameters Λ_p, Λ_w, ρ, and δ, and defined in the online appendix.

15. See chapter 6, section 4 of Galí (2015) for a textbook discussion.

PROPOSITION 1. Given exponentially decaying paths for *mpl* and *mrs*, at date $t = 0$, price and wage inflation satisfy

$$\pi_0 > 0 \text{ iff } (1-\alpha)\psi \cdot mrs_0 > (1-\alpha\psi) \cdot mpl_0,$$

$$\pi_0^w > 0 \text{ iff } \left(1-(1-\alpha)\psi\right) \cdot mrs_0 > \alpha\psi \cdot mpl_0,$$

and

$$\pi_0^w - \pi_0 = \dot{\omega}_0 > 0 \text{ iff } \alpha mpl_0 + (1-\alpha) mrs_0 > 0.$$

The slope of the boundary of the $\pi_0 > 0$ region is always steeper than that of the $\pi_0^w > 0$ region.

The shaded regions in figure 3 are those in which the economy features positive price and wage inflation. Both $mrs_0 > 0$ and $mpl_0 < 0$ are inflationary forces and produce inflation as long as one of them is present and strong enough.

A positive value for mrs_0 acts directly on wage inflation, a negative mpl_0 acts directly on price inflation. Both also act indirectly through their effects on ω_t. A high mrs_0, by pushing future real wages up, tends to increase expected marginal costs and price inflation at $t = 0$. A low mpl_0, by pushing future real wages down, tends to increase wage demands and wage inflation at $t = 0$. The fact that *mrs* acts directly on wages, while *mpl* acts directly on prices gives some intuition for why the slope of the $\pi_0 = 0$ line is steeper than that of the $\pi_0^w = 0$ line.

The difference between the two shaded regions is that in the region to the left, the real wage falls at $t = 0$ while it increases in the region to the right. The reason for the difference is the relative strength of the pressure on price setters and wage setters.

Panel B of figure 3 is identical to panel A but adds two axes that represent the conflict and adjustment components of inflation.

The adjustment axis is simply given by the 45 degree line, $mrs_0 = mpl_0$, given that along that line conflict inflation is zero.

The conflict axis is the boundary between the shaded regions: it is the locus where the power of a wage-price spiral is stronger because the aspirations of workers and firms are opposite and of equal force once we adjust for the frequency of price adjustment, that is, where

$$(1-\alpha) mrs_0 = -\alpha mpl_0.$$

Along that locus there is zero adjustment inflation: the opposite efforts of workers and firms produce no movement in the real wage and only socially wasteful price dispersion.[16]

To clarify the connection between the figure and the analysis above, it is useful to remember that the figure only shows the impact effect on π_0 and π_0^w. As time goes by and ω_t changes, the same figure applies but with the origin of the conflict and adjustment axes (and of the $\pi_t = 0$ and $\pi_t^w = 0$ loci) shifting along the 45 degree line. So, for example, we can have a shock in the upper-right quadrant that initially produces $\pi_0 < 0$ and $\pi_0^w > 0$, but also gives positive conflict inflation $\Pi_0^{Conflict} > 0$. As time goes by, we will have $\omega_t > 0$ and the origin will shift to the right along the 45 degree line while, at the same time, mrs_t and mpl_t move linearly toward the (0, 0) origin. This will at some point produce a combination $\pi_t > 0$ and $\pi_t^w > 0$, consistent with the fact that the shock will eventually produce positive cumulated inflation in both prices and wages.[17]

II.C. Stickiness and Amplification

Consider now a different exercise: fix the size of two initial shocks $mrs_0 > 0$ and $mpl_0 < 0$ and change the economy's parameters λ_w and λ_p to vary the degree by which the shocks get amplified through the wage-price responses.

As we increase the speed at which either prices or wages are reset, the wage-price spiral mechanism gets stronger. This is shown in figure 4, where we plot level curves for π and π_w. The relatively steeper curves (in absolute value) correspond to π, the flatter ones to π_w. A higher frequency

16. Projecting any point (mpl_0, mrs_0) on the two axes, the conflict coordinate gives conflict inflation Π_0, while the adjustment coordinate gives $\dot{\omega}_0$. The two coordinates measure adjustment and conflict inflation if we scale the axes as follows: on the adjustment axis, the unit vector is

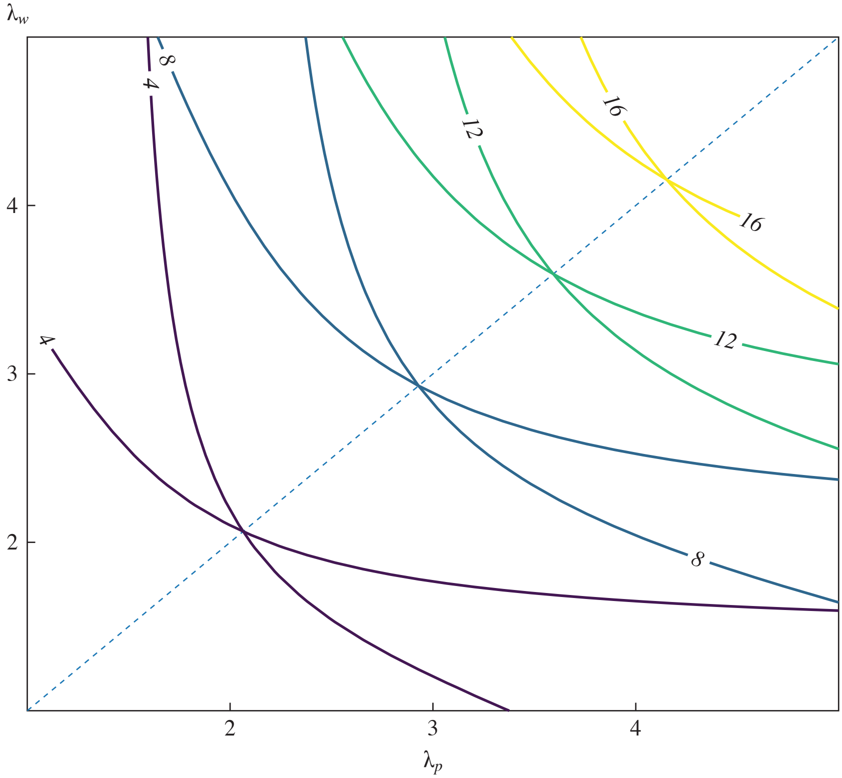
$$\begin{pmatrix} mpl_0 \\ mrs_0 \end{pmatrix} = \frac{r_2 + \delta}{\Lambda_p + \Lambda_w} \begin{pmatrix} 1 \\ 1 \end{pmatrix},$$

where r_2 is the positive eigenvalue of the real wage ODE, as defined in the online appendix; and on the conflict axis, the unit vector is

$$\begin{pmatrix} mpl_0 \\ mrs_0 \end{pmatrix} = \frac{\Lambda_p + \Lambda_w}{\Lambda_p \Lambda_w} (\rho + \delta) \begin{pmatrix} -(1-\alpha) \\ \alpha \end{pmatrix}.$$

17. Notice also that there is a t—the t at which $\dot{\omega}_t = 0$—where $\pi_t = \pi_t^w = \Pi_t^{Conflict} > 0$.

Figure 4. Price and Wage Inflation Contours for Different Degrees of Stickiness



Source: Authors' calculations.

of price adjustment λ_p increases both π and π_w but has a stronger effect on the former. The reverse holds for λ_w. For ease of illustration, we consider an economy hit by a symmetric shock $mrs_0 = -mpl_0$. This implies that when $\lambda_p = \lambda_w$, proposition 1 gives $\dot{\omega}_0 = 0$ and $\pi_0 = \pi_0^w$. In the figure, the contour levels corresponding to equal price and wage inflation meet on the 45 degree line.

Increasing either price or wage flexibility increases *both* price and wage inflation. This is the total force of the wage-price mechanism. At the same time, what happens to the real wage depends on the relative force on the two sides. Increasing λ_p tends to move us to the region below the 45 degree line, where real wages fall. Increasing λ_w has the opposite effect.

III. Demand and Supply Shocks

We now go back to the full model and trace price and wage inflation back to the general equilibrium effect of two shocks: a demand shock and a supply shock.

We show that if the economy is in an initial state that is sensitive to supply constraints, in a sense to be made precise, a positive demand shock and a negative supply shock have qualitatively similar implications on inflation. Namely, there will be a dynamic response in three phases: first, a fast increase in noncore inflation, captured here by the price of the scarce input X; then a period of sustained general inflation in prices and wages with price inflation stronger than wage inflation and real wages falling; and finally, a period of persistent wage inflation with price inflation lower than wage inflation and real wages growing back. As argued in the introduction, these dynamics seem to capture the recent post-pandemic inflationary experience well.

III.A. A Demand Shock

Consider an expansionary demand shock driven by easy monetary policy. In particular, suppose the shock is such that real spending increases to $y_0 > 0$ at date $t = 0$, and after that, it decays exponentially at rate δ, so

$$y_t = y_0 e^{-\delta t}.$$

We have not explicitly modeled monetary policy, which could be done by solving the consumers' intertemporal optimization problem and adding an interest rule to the model. However, it can be shown that the shock above translates immediately into a shock that reduces temporarily the real interest rate below its natural level (here ρ), hence stimulating consumer spending. A demand shock coming from a fiscal impulse or consumer sentiment would also have similar implications.

III.B. An Inequality for Supply-Constrained Demand Shocks

The responses of the aspirations mpl_t and mrs_t are easily derived from equations (3) and (6):

$$mpl_t = -\frac{s_X}{\epsilon} e^{-\delta t} \frac{1}{s_L} y_0 < 0, \; mrs_t = \left(\sigma s_L + \eta\right) e^{-\delta t} \frac{1}{s_L} y_0 > 0.$$

The response of the relative price of the X input (expressed in terms of labor) also follows immediately from equation (7):

$$p_{Xt} - w_t = \frac{1}{\epsilon} e^{-\delta t} n_0 > 0.$$

Given the sign of these responses, proposition 1 immediately tells us that both price and wage inflation are positive following this shock. Firms would like to pay lower real wages, given that the marginal product of labor has fallen. Consumers would like to be paid higher real wages because they are spending more and working more, so the income and substitution effects both push for a higher real marginal compensation of labor. These opposing forces produce spiral inflation, that is, conflict inflation, as discussed in the previous section.

What happens to the real wage is generally ambiguous, but proposition 1 gives us an easy condition to check and establish the sign of its response. Proposition 2 provides this condition.

PROPOSITION 2. In response to a monetary shock leading to a transitory, exponentially decaying increase in real output, price and wage inflation are both positive. Price inflation is higher than wage inflation, and consequently real wages fall at $t = 0$, if and only if the following condition is satisfied:

$$\Lambda_p \frac{s_X}{\epsilon} > \Lambda_w \left(\sigma s_L + \eta \right). \tag{16}$$

When an economy satisfies inequality (16), we say that it is supply-constrained or sensitive to supply constraints because, as we shall see, the relative scarcity of the X input driven by the ratio N_t/X_t, plays a central role in price and wage inflation dynamics.

The intuition for inequality (16) is as follows.

Consider first the expression on the left-hand side, $\Lambda_p \frac{s_X}{\epsilon}$. The ratio $\frac{s_X}{\epsilon}$ captures the effect of an increase in employment on the marginal product of labor. To increase output, the economy must increase the labor input, with a fixed supply of the input X. The ratio $\frac{N_t}{X_t}$ goes up, making the X factor relatively scarcer and labor relatively abundant. How much this lowers the marginal product of labor depends on how important the input X is in the production of the final good—the share s_X—and how elastically labor can substitute for X—the elasticity ϵ. If s_X is high and ϵ is low, we get a large effect. Finally, the coefficient Λ_p captures how quickly firms can respond to lower marginal productivity, that is, to higher marginal costs by raising nominal prices.

The expression on the right-hand side, $\Lambda_w(\sigma s_L + \eta)$, comes from the workers' side. In particular, the expression $\sigma s_L + \eta$ captures how income and substitution effects change how much workers would like to be compensated on the margin, while Λ_w captures how quickly a higher *mrs* leads to increasing nominal wages.

As we discussed in the previous section, both impulses, to *mpl* on the firms' side and to *mrs* on the workers' side, lead to mutual reactions, that is, to indirect effects: an impulse on firms' marginal costs also leads to increasing nominal wages, and an impulse on workers' marginal rate of substitution also leads to nominal price inflation. However, proposition 1 shows that the indirect effects are always weaker than the direct effects and that the presence of indirect effects does not change the relative size of the effects on the two sides. Therefore, focusing on the relative strength of the direct effects, we can safely conclude that price inflation will be higher in equilibrium than wage inflation if and only if the direct impulse on prices—the left-hand side of equation (16)—is stronger than the direct impulse on wages—the right-hand side.

III.C. An Example

Having unpacked analytically the effect of the shock at date $t = 0$, let us turn to a numerical example to look at the full dynamics and get a sense of the magnitudes involved. We focus on an example that satisfies inequality (16).

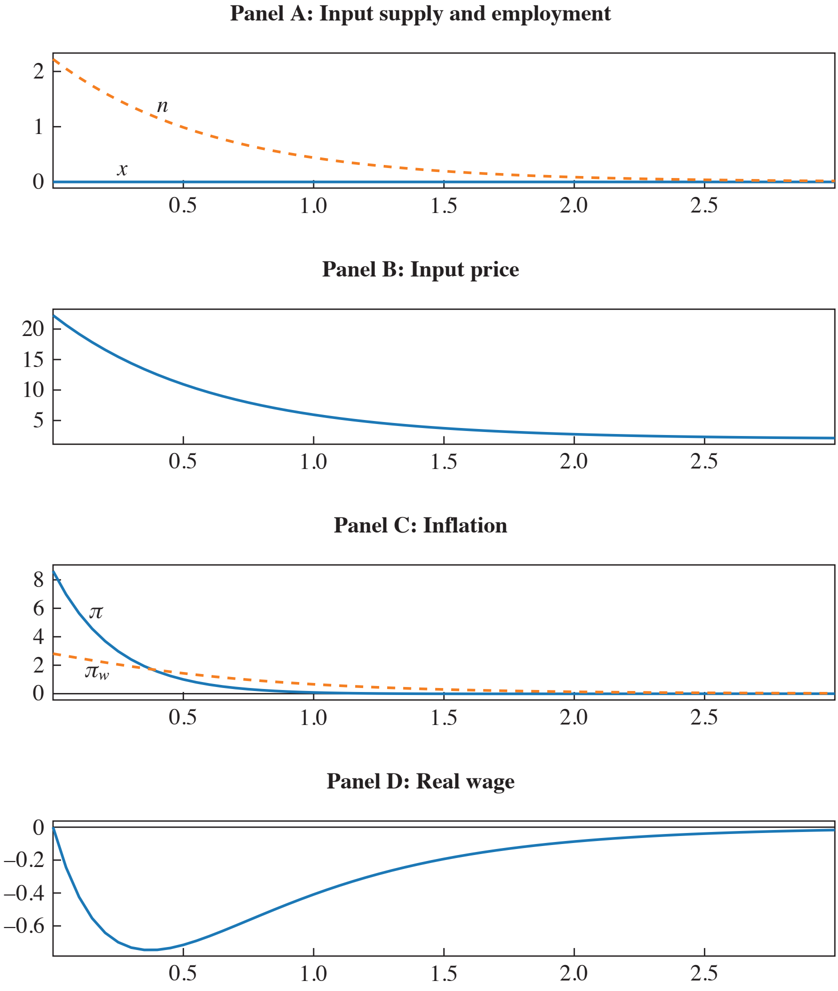
In figure 5, we plot the response to a temporary expansionary shock that increases y above potential by 2 percent on impact and converges back to potential at the rate $\delta = 1$. The parameters used are in table 1.[18]

Panel A shows the path of employment n, which is proportional to output, and the path of x, which by assumption is constant at zero. The remaining panels show the responses of different prices.

The input price is flexible, so it jumps on impact and then gradually goes back to its initial level as the shock goes away. This is shown in panel B of the figure. Notice that this panel shows the level of the input price, not its inflation rate. Inflation for that price is infinite at $t = 0$ and negative

18. All plots show log deviations from a steady state times 100 or, approximately, percentage deviations from a steady state.

Figure 5. A Supply-Constrained Demand Shock



Source: Authors' calculations.

Table 1. Parameters

Preferences	$\sigma = 1$	$\eta = 1/2$	$\rho = 0.04$
Technology	$s_X = 0.1$	$\epsilon = 0.1$	
Stickiness	$\lambda_p = 4$	$\lambda_w = 1$	

Source: Authors' calculations.

afterward. Due to perfect flexibility, P_X jumps by more than 20 percent at $t = 0$. This large increase is due to our assumption of a low elasticity of substitution between labor and the input X ($\epsilon = 0.1$), so when employment is growing too fast relative to the supply of X, the price of X reacts strongly.

The effect of the increase in the input price is to increase firms' marginal costs. The impact effect on the nominal marginal cost $w_0 - mpl_0$ is +2 percent, as the input represents 10 percent of the cost in a steady state, $s_X = 0.1$, and the elasticity is also $\epsilon = 0.1$, so the ratio $s_X/\epsilon = 1$. As we see in panel C of figure 5, this increase in marginal costs translates into fast inflation on impact: 10 percent above its steady-state level (so 12 percent inflation if we assume the central bank is keeping inflation at 2 percent in steady state).[19] This large response to a relatively small increase in marginal costs is due to our assumption of relatively flexible prices ($\lambda_p = 4$; i.e., prices reset on average every quarter), to the firms' having rational expectations and a long horizon (captured by the discount rate ρ), and, of course, to the wage response, that is, to the presence of a wage-price spiral.

On the wage side, the direct impact effect on the mrs is $(\sigma s_L + \eta) \times 2\% = 2.8\%$ and is close in magnitude to the effect on the marginal cost of goods, both are 2 percent. However, wages are more sticky ($\lambda_w = 1$), so the effect on wage inflation is weaker. Wage inflation is also plotted in panel C of figure 5.

The real wage falls on impact, as shown in panel D. However, as time goes by, the lower level of the real wage pushes workers to ask for nominal wage increases larger than price inflation. Wage growth eventually reverses sign and the real wage converges back to trend.

Figure 5 illustrates the three phases of adjustment mentioned in the introduction. First, very fast inflation in the sector where the supply constraints are binding, here the market for input X. Second, a phase in which price inflation is faster than wage inflation. Third, at some point wage inflation crosses price inflation and we enter the third phase in which real wages recover.

We will discuss in more depth the connection between this example and current developments at the end of this section. But first, let us look at a supply shock.

19. Notice that π_t is an instantaneous rate of inflation, expressed in annual terms. Since inflation falls relatively quickly in our example, measured quarterly inflation in the first quarter after the shock is lower than 12 percent.

III.D. A Supply Shock

Consider the same economy's response to a temporary reduction in the endowment of input X. Suppose, for now, that the central bank responds in such a way as to keep employment constant at its initial steady-state level, $n_t = 0$.

Again, the reaction of monetary policy is left implicit in the path of quantities. Since X falls, constant employment corresponds to a reduction in real output. It can be shown that this means that the central bank is increasing the real interest rate. However, as we shall see, the real rate increase that produces $n_t = 0$ is not large enough to achieve the natural allocation, given our chosen parameters.

The responses of *mpl* and *mrs* are now

$$mpl_t = \frac{s_X}{\epsilon} e^{-\delta t} x_0 < 0, \; mrs_t = \sigma s_X e^{-\delta t} x_0 < 0,$$

while the response of the price of good X is

$$p_{Xt} - w_t = \frac{1}{\epsilon} e^{-\delta t} n_0 > 0.$$

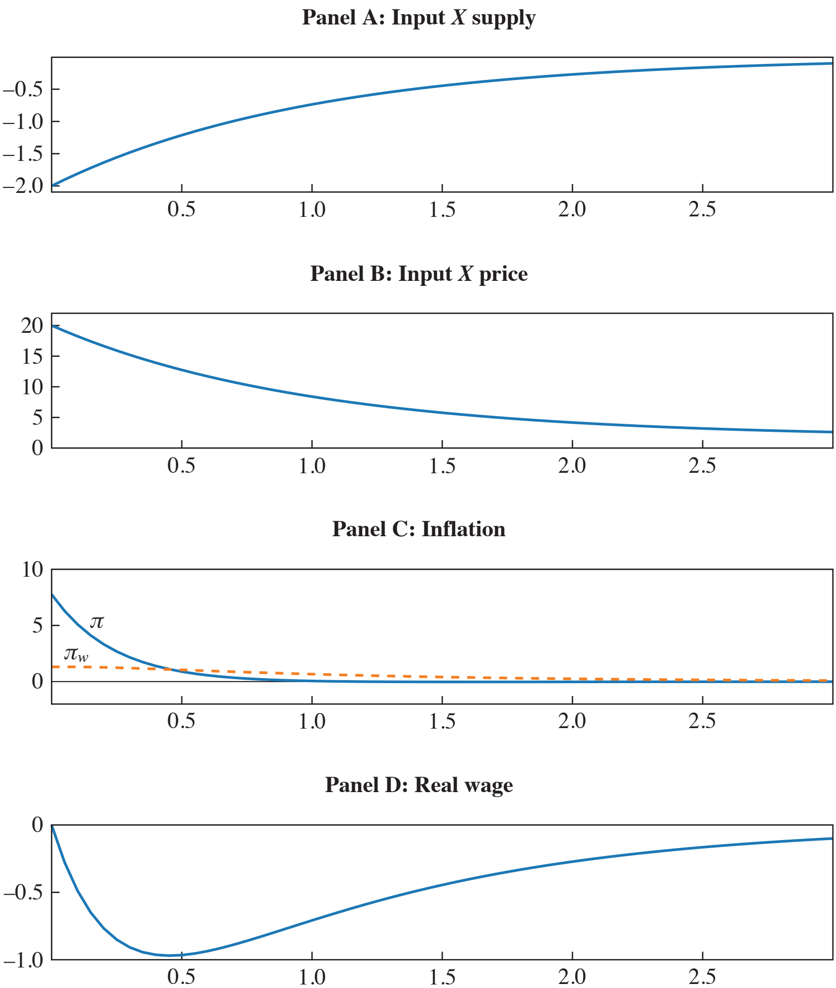
The main difference is that now the reduction in output reduces workers' *mrs* via an income effect. This weakens real wage demands. Given the parameter choices in table 1, the inflationary forces on the firms' side are still strong enough that we obtain positive wage and price inflation. In the representation in figure 3, we are in the portion of the shaded region on the left that intersects the lower left quadrant. From proposition 1, we also know that $mpl_0 < 0$ and $mrs_0 < 0$ imply that the real wage falls on impact for any parameter configuration.

The responses are illustrated in figure 6. For ease of comparison, we pick a negative shock to x_0 that produces the same increase in the input price as the positive shock to y_0 in figure 5.

While nominal wages are growing less and the real wage drop is larger than in figure 5, the overall shapes and magnitudes are not very different from the demand shock. The crucial observation here is that if we scale shocks so that the input price response is the same, we are pinning down the change in the labor-to-X ratio, as

$$p_{X0} - w_0 = \frac{1}{\epsilon}\left(n_0 - x_0\right),$$

Figure 6. A Supply Shock



Source: Authors' calculations.

and the same ratio $n_0 - x_0$ determines

$$mpl_0 = \frac{s_X}{\epsilon}\left(n_0 - x_0\right).$$

Once we choose the quantitative size of the fall in $n_0 - x_0$, we have pinned down the inflationary impulse on the firms' side.

The main difference is that in this case the wage-price spiral mechanism is weaker as workers' aspirations fall instead of increasing in the case of a supply shock. This explains why both price and wage inflation are lower in this case.

III.E. Supply Shocks and the Monetary Response

The response to the supply shock depends on how monetary policy adjusts. So far, we assumed a policy that keeps the employment path unchanged at $n_t = 0$. However, the natural level of employment depends in general on x_t. In particular, keeping employment and output at their natural levels requires that $mrs_t = mpl_t$, and n_t^* can be derived from the condition

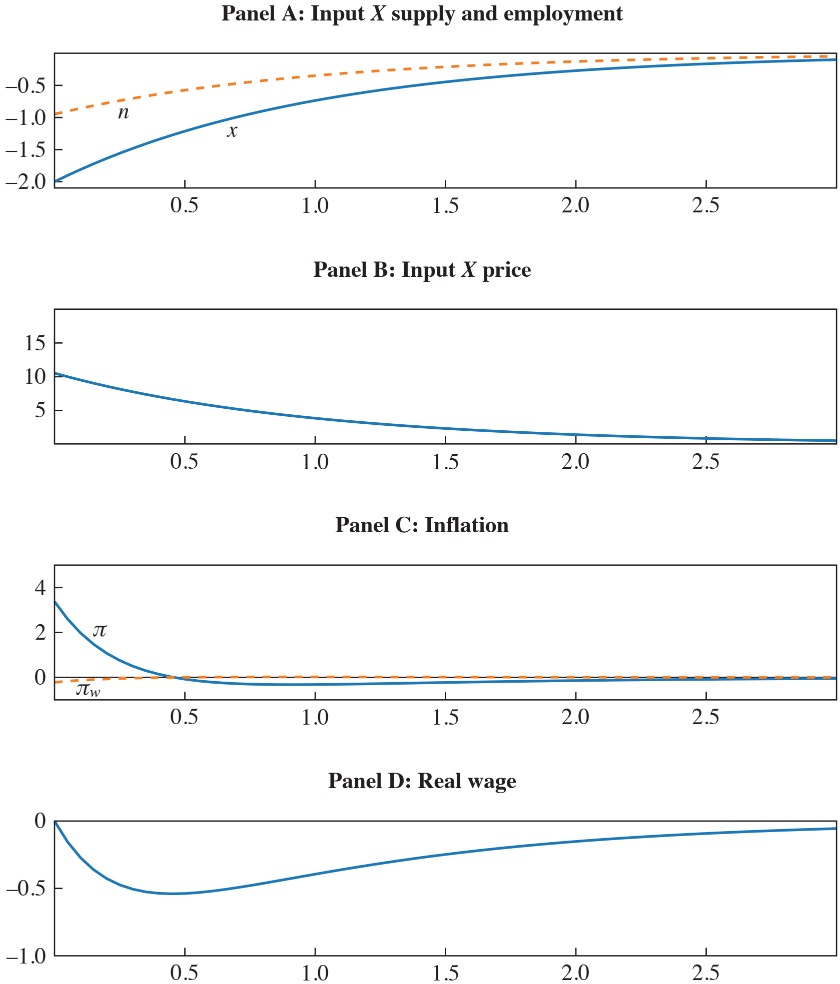
$$\sigma\left(s_L n_t^* + s_X x_t\right) + \eta n_t^* = \frac{s_X}{\epsilon}\left(x_t - n_t^*\right).$$

The responses of price and wage inflation when

$$n_t = n_t^* = \frac{\frac{1}{\epsilon} - \sigma}{\sigma s_L + \frac{s_X}{\epsilon} + \eta} s_X x_t$$

are plotted in figure 7. Since our parameterization features a low degree of substitutability between labor and the input X, we have $\frac{1}{\epsilon} - \sigma > 0$, and a reduction in x_t lowers the natural level of employment, as shown in panel A. The natural level of output $y_t^* = s_X x_t + s_L n_t^*$ is then lower for two reasons: the direct effect of a lower x_t and the lower level of natural employment. There is a clear difference in the inflation paths when quantities are at their natural levels: we see positive price inflation but negative wage inflation. This goes on as long as the real wage falls; once the real wage starts growing again, the signs of price and wage inflation flip. In other words, real wage adjustments always take place with nominal prices and wages moving in opposite directions.

Figure 7. A Supply Shock with Quantities on Their Natural Path



Source: Authors' calculations.

This is not just an outcome of our choice of parameters. When quantities are at their natural levels, we have $mrs_t = mpl_t$ and both are equal, by definition, to the natural real wage ω_t^*. The inflation equations then become

$$\pi_t = \Lambda_p \int_t^{\infty} e^{-\rho(s-t)} \left(\omega_s - \omega_s^*\right) ds \text{ and}$$

$$\pi_t^w = \Lambda_w \int_t^{\infty} e^{-\rho(s-t)} \left(\omega_s^* - \omega_s\right) ds.$$

The general result in proposition 3 follows immediately.

PROPOSITION 3. If quantities are at their natural levels, price and wage inflation π_t and π_t^w are either both zero or have opposite sign.

This result can be visualized in figure 3 by noticing that the regions where π and π^w have the same sign are either entirely above or entirely below the 45 degree line, where $mrs = mpl$.

Using the concepts introduced in section II, we can then say that if the output gap is always zero, conflict inflation is zero, that is, a wage-price spiral is not present.[20]

Behind the similar adjustment patterns illustrated in figures 5 and 6, there is a similar problem of excess demand producing positive conflict inflation. Excess demand can be caused either by a positive demand shock or a negative supply shock coupled with an insufficient monetary policy response.

However, notice also that, as is well known, an economy with both price and wage rigidities does not feature divine coincidence, so a policy of keeping the output gap at zero, that is, of keeping quantities at their flexible price levels, is not necessarily optimal in our environment. We analyze optimal policy in the next section.

Comparing figures 6 and 7 also shows that while employment falls more at the natural allocation, real wages fall less. This may seem surprising, but it is due to the fact that the dynamics of the real wage are more strongly affected by *mpl* than by *mrs*, and *mpl* is higher along the path with lower employment. A different intuition for the same phenomenon is that lower employment reduces the pressure on the market for the scarce input, as seen in panel B of both figures, weakening price inflation due to the high *X* price and increasing the real wage. Yet another intuition is that due to the

20. This result explains why conflict inflation in this model is equal to the divine-coincidence inflation of Rubbo (2020).

fact that prices of goods and nonlabor inputs are relatively more flexible than wages, the relation between real wages and employment is dominated by the labor demand side, so higher employment levels push down real wages.

III.F. Interpretation and Connections

This adjustment pattern shows both price and wage inflation, with price inflation stronger early on and wage inflation catching up later. If the central bank keeps the economy always at its flexible price allocation, this pattern will not be present, as price and wage inflation have opposite sign.

The examples presented are clearly just numerical simulations with parameters chosen mostly for clarity of exposition. Nonetheless, we believe there are some useful lessons and some interesting connections with recent experience.

DEMAND SHOCKS AND WAGE INFLATION Our model helps clarify that excess demand does not necessarily need to show up primarily through a tight labor market and high wage inflation. A commonly held view is that excessive demand works its way from a tight labor market to higher wages through the wage Phillips curve and, eventually, to higher prices. A demand shock then should produce increasing real wages. As we just showed, this is not necessarily the case. In the model, price and wage rigidities interact with general equilibrium forces on both goods and labor markets, and the direction of adjustment of the real wage is in general ambiguous. At a general level, the notion that real wages can potentially fall is obvious and commonly noted in the extreme case where nominal wages are fully rigid: in that case, the real wage must fall whenever inflation is positive.[21] Our analysis gives an easy way to interpret condition for real wages to fall or rise, clarifying the economic forces at play.

An intuitive way of making our point here is to observe that inflation is in general caused by some form of scarcity on the supply side, relative to existing demand pressures. But there are multiple inputs on the supply side, labor inputs and nonlabor inputs. Depending on the episode, scarcity can manifest itself more strongly in labor inputs or in nonlabor inputs. When nonlabor input scarcity dominates, price inflation will be faster than wage inflation.

SMALL AND LARGE ECONOMIES Many papers measure supply shocks directly in terms of changes in input prices.[22] In this paper, we emphasize the general equilibrium nature of the price shock by making the price p_X fully endogenous.

21. See, for example, figure 6.3 in Galí (2015).
22. For example, this is the strategy in the model used by Bernanke and Blanchard (2023).

It is important to remark that the degree to which p_X should be treated as endogenous or exogenous depends on the size of the economy relative to the world economy. For a small open economy that trades X frictionlessly with the rest of the world (a reasonable approximation for some energy inputs), it makes sense to redo the analysis by taking p_{Xt} as given and deriving x_t endogenously instead of shocking x_t and deriving p_{Xt} endogenously. The results for a supply shock would be similar. However, the effects of a demand shock that is completely idiosyncratic to the small open economy (that is, not correlated with a global demand shock) would be very different, as the relative scarcity of X in the world at large would not be affected by a localized shock to demand. On the other hand, a demand expansion in a large country would transmit to smaller economies as a supply shock, via the price p_X.

PASS-THROUGH FROM NONCORE TO CORE INFLATION We can identify the first phase of our three-phase responses as an initial period of high noncore inflation. Technically, the price p_X in our model does not appear directly in the Consumer Price Index (CPI), because X is only used as an input, not as a final good. Therefore, there is no distinction between core and noncore inflation in the model. However, it is easy to modify the model to allow for direct consumption of X, or for multiple sectors, some of which use X more intensively than others, and to make the distinction between core and noncore more explicit. The fact that the response of p_t lags the response of p_{Xt} shows that our model features a clear mechanism for pass-through from noncore inflation to core inflation. Recent work by Ball, Leigh, and Mishra (2022) shows empirically that this pass-through has been high in the post-pandemic period.

A related observation is that the fact that p_{Xt} is falling after jumping at $t = 0$ is not in contradiction with the fact that supply constraints are crucial for the inflation episode. It is the level of p_{Xt}, not its rate of change, that reflects the underlying scarcity in the economy, that is, a high labor to nonlabor inputs ratio $n_t - x_t$, and this scarcity is a crucial driver of the high inflation rate in goods through its effects on mpl_t.

NONLINEAR PHILLIPS CURVES Many economists have pointed out the potentially important role of a nonlinear Phillips curve in explaining recent experience.[23] Our model is linearized, but it is linearized around a steady state that captures the economy's state at the moment the shock hits. Therefore, we can easily see the effect on nonlinearities through the parameter s_X

23. See, for example, Benigno and Eggertsson (2023).

in the linearized model. That parameter is not a model's constant but depends on initial conditions. In particular, s_X is higher if the initial steady state features a relatively high initial ratio N_t/X_t. In other words, if the X input is already relatively scarce when the shock hits, the effects of the shock on inflation will be magnified. It would be interesting to explore model extensions in which the elasticity ϵ is also endogenous and depends on the state of the economy.

Notice that the nonlinearity we are pointing out here is not nonlinearity in the wage Phillips curve, which is the one that has received more attention, but rather nonlinearity in the response of nonlabor input prices, which affects the price Phillips curve.[24]

PROFITS A possible interpretation of the scarce input X is not as a market-supplied input but rather as capturing fixed production capacity and other bottlenecks at the firm level. The formal analysis is slightly different when the input is fixed at the firm level instead of being fixed economy-wide and frictionlessly traded.[25] But the qualitative responses are similar.

There is, however, a marked difference in interpretation between a model with a market-supplied input X and a model with fixed capacity. In the first model, observed profit margins at the firm level fall in response to the shocks analyzed because nominal prices increase less than marginal costs due to stickiness. In the second model, observed profit margins increase because firm profits include the shadow price of the scarce input X, which increases sharply in all our examples.

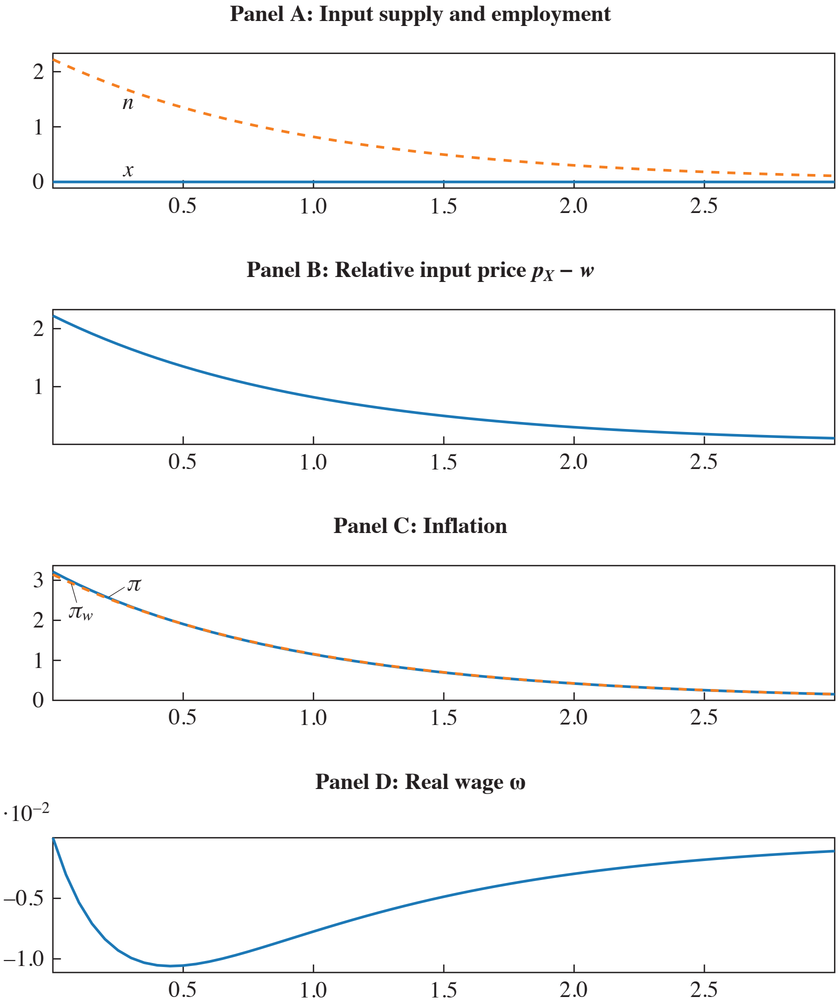
THE ROLE OF ϵ In our examples, we have used a low elasticity $\epsilon = 0.1$. This low elasticity plays two roles: it magnifies the response of p_{Xt}, explaining the initial jump in noncore inflation, and it magnifies the response of *mplt*, explaining the prolonged inflation episode. To see the central role of this parameter, consider an example with all the same assumptions of our demand shock in figure 5 but assume a Cobb-Douglas production function, with $\epsilon = 1$. The responses are plotted in figure 8.

Two differences stand out compared to our baseline parametrization. First, there is a smaller response of the relative price of the X input in panel B. With higher elasticity, the relative scarcity of X has a smaller price effect (the effect is proportional to $1/\epsilon$, so it falls by a factor of 10).

24. Comin, Johnson, and Jones (2023) use occasional binding constraints to study a model with a similar nonlinearity in the price Phillips curve.

25. In particular, a model with firm-specific, non-traded X is a model with decreasing returns to labor at the firm level, which produces strategic complementarity in pricing that is absent in our model with constant returns.

Figure 8. A Demand Shock with Higher Elasticity of Substitution



Source: Authors' calculations.

This implies a smaller overall inflation response. Second, the responses of wage and price inflation are almost indistinguishable, and consequently, the real wage is not affected. This is because the response of *mpl* is weaker while the response of *mrs* is unchanged (as we keep the value of *sL* unchanged in the two examples).

This suggests that, at the aggregate level, to capture episodes in which the relative scarcity of nonlabor inputs triggers an inflationary episode, with a lagged response of wage inflation, a low degree of elasticity at the aggregate level is a needed ingredient.

IV. Optimal Policy

In the previous section, we looked at economies in which the central bank unnecessarily stimulates the economy (demand shock) or the central bank responds weakly to a supply shock, so as to allow for both price and wage inflation (the supply shock with $n_t = 0$). The first example is a policy mistake by construction. Of course, due to imperfect information and lags in the effects of monetary policy, mistakes can happen. However, in this section, we focus on the second shock, a supply shock, and ask what the optimal response is. Throughout, we assume monetary policy has perfect information on the underlying shocks and instantaneous control on the level of real activity.

The questions we address in this section are two. Is it possible that, following a supply shock, the optimal response is to let the economy overheat, that is, to choose a positive output gap $y_t - y_t^* > 0$? Is it possible that the optimal response entails both positive price and wage inflation?

It is well known that divine coincidence fails in our environment. But that is just a statement about feasibility: an outcome with no inflationary distortions, $\pi_t = \pi_t^w = 0$, and a zero output gap, $y_t = y_t^*$, are not feasible in our model. The real wage needs to move in the flexible price equilibrium and that is incompatible with zero nominal inflation in p_t and w_t. Our contribution here is to characterize the signs of the deviations of π_t, π_t^w, and $y_t - y_t^*$ from zero under optimal policy.

In particular, proposition 3 above tells us that if the central bank chooses $y_t = y_t^*$, then the signs of π_t and π_t^w will always be opposite. In other words, with a zero output gap, the adjustment in the real wage never requires *both* price and wage inflation. Therefore, one could conjecture that generalized inflation, that is, inflation in both prices and wages, is never optimal. However, a zero output gap is not necessarily optimal, so that conjecture is not generally correct.

IV.A. Optimal Policy Problem

Following standard steps, the objective function of the central bank can be derived as a quadratic approximation to the social welfare function:

$$\int_0^\infty e^{-\rho t}\frac{1}{2}\left[-\left(y_t - y_t^*\right)^2 - \Phi_p \pi_t^2 - \Phi_w\left(\pi_t^w\right)^2\right]dt. \tag{17}$$

Deviations from first-best welfare come from two distortions: output deviations from its natural level, that is, from the level that equalizes the marginal benefit of producing goods with its marginal cost in terms of labor effort; and inflation in prices and wages that causes inefficient dispersion in relative prices of different varieties. The terms in equation (17) reflect these distortions. The values of the coefficients Φ_p and Φ_w depend on the model parameters and are derived in the online appendix.

The natural level of the real wage following a supply shock is

$$\omega_t^* = \frac{s_X}{\epsilon}\frac{\sigma + \eta}{\sigma s_L + \frac{s_X}{\epsilon} + \eta} x_t.$$

We can then express *mpl* and *mrs* in terms of the natural real wage and deviations of employment from its natural path

$$mpl_t = \omega_t^* - \frac{s_X}{\epsilon}\left(n_t - n_t^*\right) \text{ and} \tag{18}$$

$$mrs_t = \omega_t^* + \left(\sigma s_L + \eta\right)\left(n_t - n_t^*\right). \tag{19}$$

The optimal policy problem is to maximize equation (17), subject to the constraints coming from the price-setting (10) and (11), the real wage dynamic equation

$$\dot{\omega}_t = \pi_t^w - \pi_t$$

and the aggregate production function expressed as

$$y_t = s_L n_t + s_X x_t.$$

The optimality conditions that characterize an optimal policy are derived in the online appendix.

IV.B. Examples

We now consider examples that illustrate a variety of possible outcomes.

It helps the interpretation of the policy trade-offs to focus on the simple case of a permanent shock to x_t. With this shock, in all our examples, in the long run, the real wage is permanently lower and so are *mpl* and *mrs*, so that the economy eventually reaches a new steady state with zero inflation and zero output gap. To reach that new steady state requires ω_t to fall. This can be achieved by many combinations of price and wage inflation or deflation, as long as price inflation is larger than wage inflation. The question is, what is the optimal way to get there?

EXAMPLE 1. A SYMMETRIC CASE Our first example is an economy with parameters that have the following properties: the welfare costs of wage and price inflation enter symmetrically the objective function, $\Phi_p = \Phi_w$; wages and prices are equally sticky, $\Lambda_p = \Lambda_w$; and the output gap has symmetric effects on *mpl* and *mrs*.[26]

Figure 9 illustrates optimal policy outcomes in this example. Given the symmetry of the problem, the reduction in real wages is achieved by spreading the adjustment equally between nominal wage deflation and nominal price inflation. The output gap is kept exactly at zero. This example is clearly a knife-edge case and relies on the symmetry of the parameters. As soon as we abandon this symmetry things get more interesting.

EXAMPLE 2. A HOT ECONOMY In the second example, the parameters chosen imply that the welfare cost of wage inflation is larger than that of price inflation, $\Phi_p < \Phi_w$, and wages are more sticky than prices, $\Lambda_p > \Lambda_w$.[27] We still have a set of parameters that implies roughly symmetric effects of the output gap on *mpl* and *mrs*, but the differences are sufficient to obtain a quite different result. Figure 10 illustrates optimal policy outcomes in

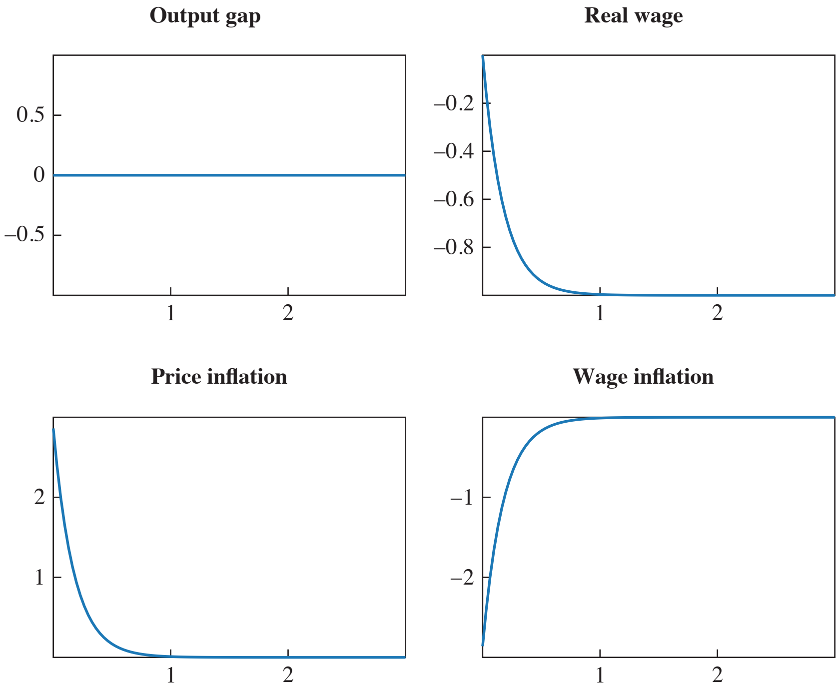
26. The following parameters satisfy these conditions and are used in the numerical example:

$\sigma = 1$	$\eta = 0$	$\rho = 0.05$	
$s_X = 1/2$	$\epsilon = 1$	$\varepsilon_C = 1.5$	$\varepsilon_L = 3$
$\lambda_p = 4$	$\lambda_w = 4$		

27. The parameters are as follows:

$\sigma = 1$	$\eta = 0$	$\rho = 0.05$	
$s_X = 0.1$	$\epsilon = 1$	$\varepsilon_C = 1.5$	$\varepsilon_L = 4$
$\lambda_p = 4$	$\lambda_w = 2$		

Figure 9. A Symmetric Example



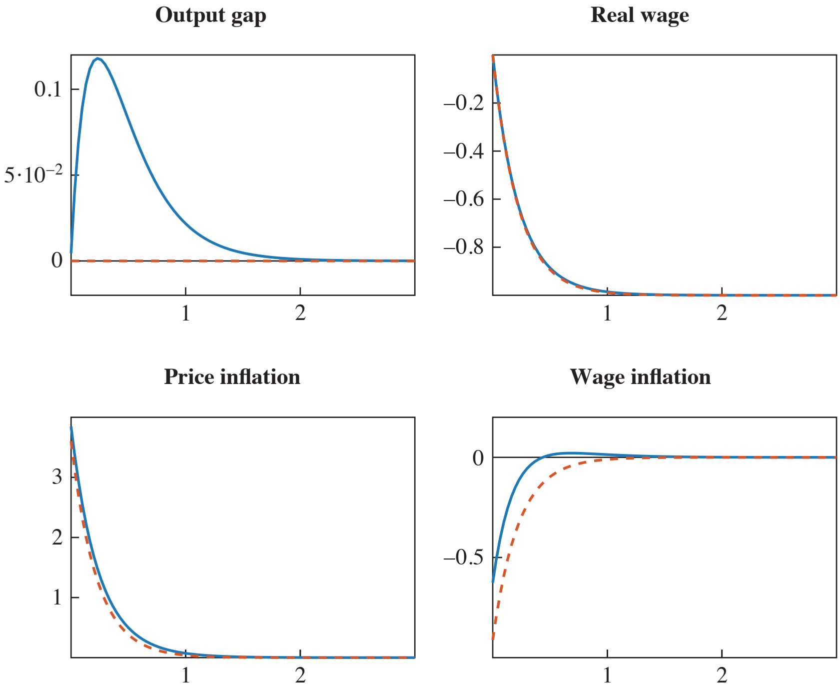
Source: Authors' calculations.

this case. For comparison, in the figure we also plot outcomes under a zero output gap policy (dashed lines).

In this second example, it is optimal to have a positive output gap throughout the transition. Recall from equations (10)–(11) and (18)–(19) that increasing the output gap has two direct effects: by decreasing *mpl*, it leads to higher price inflation; and by increasing *mrs*, it leads to higher wage inflation. If we start at a zero output gap policy with positive price inflation and negative wage inflation, the effect can be welfare improving because the welfare cost of price inflation is smaller than the welfare cost of wage deflation.

The role of $\Lambda_p > \Lambda_w$ is subtler and has to do with dynamics. With $\Lambda_p > \Lambda_w$, a higher output gap also implies a faster declining real wage. Since a lower real wage in the future requires less adjustment, lowering the real wage today is welfare improving from a dynamic point of view. Therefore, a parameterization with $\Lambda_p > \Lambda_w$ makes it easier to obtain examples with a welfare-improving positive output gap.

Figure 10. An Optimal Hot Economy



Source: Authors' calculations.

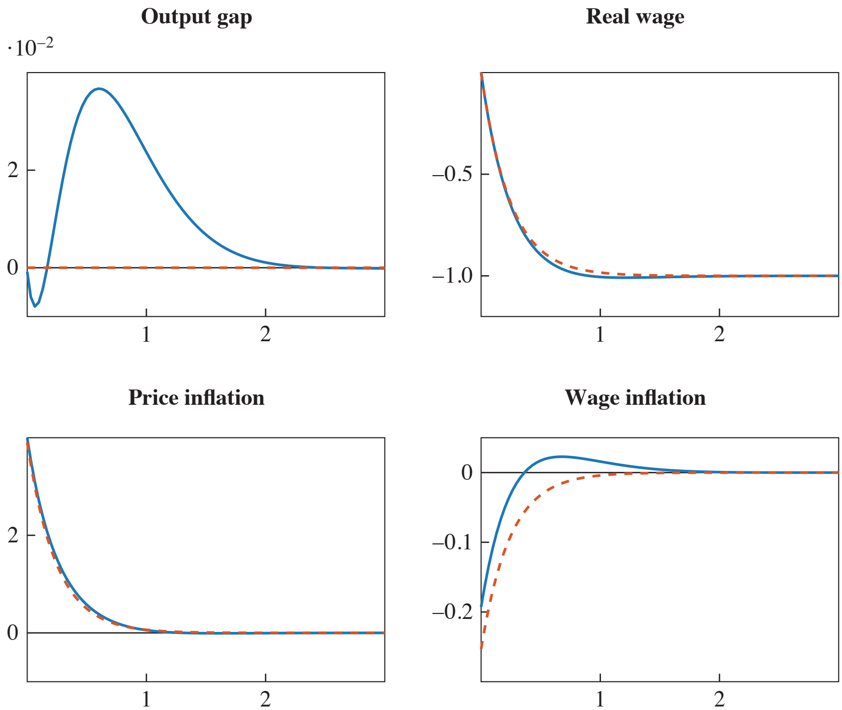
Notice that it is also possible to choose parameters that imply that the welfare costs of price inflation are relatively larger than those of wage inflation, and to obtain examples in which it is optimal to run a negative output gap in the transition.

EXAMPLE 3. GENERALIZED INFLATION AND A HOT ECONOMY Our third example is a variant on the second example, with an even larger welfare cost associated to wage dispersion (a larger Φ_w), a larger distance between price and wage stickiness, and a smaller value of the elasticity of substitution between labor and the X input, ϵ, which implies that running a hot economy has larger benefits in terms of lowering the real wage by having a larger effect on firms' marginal costs and thus on price inflation.[28]

28. The parameters are as follows:

$\sigma = 1$	$\eta = 0$	$\rho = 0.05$	
$s_X = 0.1$	$\epsilon = 0.1$	$\varepsilon_C = 1.5$	$\varepsilon_L = 8$
$\lambda_p = 4$	$\lambda_w = 1$		

Figure 11. An Example with Generalized Inflation and a Hot Economy



Source: Authors' calculations.

The parametric choices above amplify the forces we saw in example 2, and they imply that there is an interval during the transition in which the optimal policy yields both a hot economy ($y_t > y_t^*$) and generalized price and wage inflation ($\pi_t > 0$ and $\pi_t^w > 0$).[29]

This result is surprising from a static point of view (see figure 11). Given the welfare function (17), at any point in time in which $y_t > y_t^*$, $\pi_t > 0$, and $\pi_t^w > 0$, it is welfare improving, from a static point of view, to reduce y_t, as it unambiguously lowers π_t and π_t^w and leads to an increase in the current payoff. However, from a dynamic perspective, there is an additional argument. Increasing y_t at time t has the effect of increasing π_s and π_s^w in all previous periods due to the forward-looking element in price setting. This entails welfare gains in early periods in the transition in which

29. Notice that these qualitative features can actually be seen in example 2 too, but it is useful to choose an example where they are more clearly visible.

$\pi_s^w < 0$. Through this forward-looking force, a positive output gap later in the transition can be beneficial even if, at that point, $\pi_t^w > 0$.

Now, while this example is theoretically interesting, it does have the flavor of an overly sophisticated form of forward guidance. Therefore, we do not think it provides a strong argument in favor of policies that deliver $y_t > y_t^*$, $\pi_t > 0$, and $\pi_t^w > 0$ at the same time. In the context of the present model, given the distortions it captures, it is hard to make a compelling practical case that the combination of a hot economy with positive wage and price inflation is a desirable outcome, even in response to a supply shock and even in the presence of inelastic supply constraints.[30]

V. Adaptive Expectations and Real Rigidities

The model with rational expectations analyzed so far has two embedded features: the effect of any shock tends to be front-loaded, as agents perfectly anticipate its future effects on prices; and there is no room for persistent deviations of inflation expectations from target, as agents anticipate the economy will go back to its initial steady state. We now explore variants of the model that deviate from rational expectations and allow for more inertial responses by introducing two ingredients: adaptive expectations on expected inflation and a gradual adjustment of price setters' and wage setters' relative price objectives. For this second ingredient we use the label "real rigidities."

The objective of this section is twofold. First, by allowing for inertial responses, we allow the feedback between prices and wages to play out more explicitly over time: shocks that produce high prices in the goods market only gradually lead to higher wage demands in the labor market. In other words, the wage-price spiral, instead of playing out in the "virtual time" of best responses, plays out in the observed dynamics of prices and wages. Second, by allowing for deviations of inflation expectations from target, we capture the common concern of central bankers that prolonged episodes of high inflation may lead to de-anchoring of inflation expectations.

From an empirical perspective, we show that adaptive expectations and inertia reinforce the main prediction of the baseline model in section III: there is a lagged and persistent increase in wage inflation following a large increase in price inflation. However, the medium-term implications are

30. This does not mean that such a case could not maybe be made in richer models, which capture, just to make an example, the benefits of labor reallocation as in Guerrieri and others (2021). But that is clearly outside the scope of this paper.

different depending on the sources of inertia: if inertia is mostly due to de-anchoring, inflation can take a long time to go back to target, absent a recession; if instead inertia is mostly due to real rigidities, then a path of immaculate disinflation is possible.

Let us begin by rewriting the price-setting conditions making explicit agents' expectations. Letting E_t^f and E_t^w denote firms' and workers' expectations, we can write

$$p_t^* = (\rho + \lambda_p) E_t^f \int_t^\infty e^{-(\rho+\lambda_p)(\tau - t)} \left(w_\tau + s_X \left(p_{X\tau} - w_\tau \right) \right) d\tau$$

$$= w_t + (\rho + \lambda_p) E_t^f \int_t^\infty e^{-(\rho+\lambda_p)(\tau - t)} s_X \left(p_{X\tau} - w_\tau \right) d\tau$$

$$+ E_t^f \int_t^\infty e^{-(\rho+\lambda_p)(\tau - t)} \dot{w}_\tau d\tau.$$

Reset prices are decomposed in three components: the current nominal wage, the expected path of the relative price of input X versus labor, and the expected path of future wage inflation.

We assume that agents expect a constant inflation rate over the future horizon

$$E_t^f \dot{w}_t = \pi_t^{w,e},$$

and expected inflation is driven by the simple adaptive, constant-gain rule

$$\dot{\pi}_t^{w,e} = \gamma \left(\dot{w}_t - \pi_t^{w,e} \right). \tag{20}$$

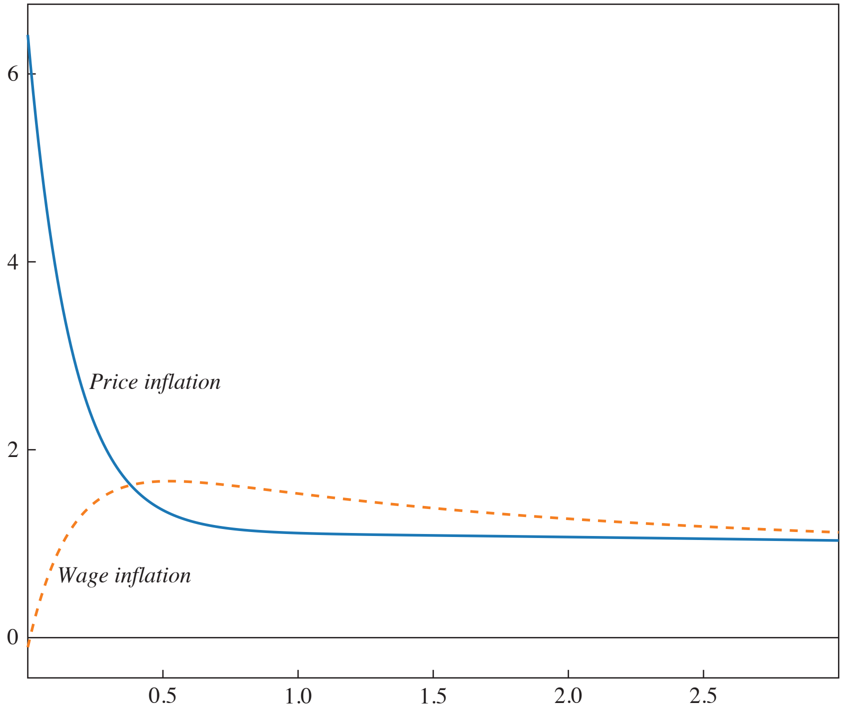
Moreover, we assume that agents perfectly anticipate the path of real variables n_t, x_t, and y_t, and can deduce the path of the relative price $p_{Xt} - w_t$ from the equilibrium condition in factor markets

$$x_t - n_t = -\epsilon \left(p_{Xt} - w_t \right).$$

Combining these assumptions with exponentially decaying, one-time shocks at date zero, as in section III, we can substitute in the expression above for p_t^*, substitute in the inflation equation (8), and obtain the following:

$$\dot{p}_t = \lambda_p \left[\frac{s_X}{\epsilon} \frac{\rho + \lambda_p}{\rho + \lambda_p + \delta} \left(n_t - x_t \right) - \left(p_t - w_t \right) \right] + \frac{\lambda_p}{\rho + \lambda_p} \pi_t^{w,e}. \tag{21}$$

Figure 12. A Supply Shock with Adaptive Expectations



Source: Authors' calculations.

Similar steps on the wage-setting side of the model lead to

$$\dot{w}_t = \lambda_w \left[\frac{\rho + \lambda_w}{\rho + \lambda_w + \delta} \left(\sigma y_t + \eta n_t \right) - \left(w_t - p_t \right) \right] + \frac{\lambda_w}{\rho + \lambda_w} \pi_t^e, \tag{22}$$

where price inflation follows the adaptive rule

$$\dot{\pi}_t^e = \gamma \left(\dot{p}_t - \pi_t^e \right). \tag{23}$$

Equations (20)–(23) can be solved forward for any given initial condition w_0, p_0.

AN EXAMPLE OF DE-ANCHORING Figure 12 shows the response of inflation to a supply shock in a numerical example analogous to the one shown in figure 6, except for the assumption of adaptive expectations. The parameters are the same as in table 1, and we set $\gamma = 1$. There are two main differences from the case of rational expectations. First, wage inflation is weaker on impact and only picks up gradually, as initially workers do not anticipate higher prices and so do not start trying to catch up until their purchasing power has actually been eroded by past inflation.[31] Second, there is a very persistent effect on inflation, due to the learning dynamics. Since ρ is small, the coefficients on the expected inflation terms on the right-hand side of equations (21)–(22) are close to one. This implies that even though all quantities and all relative price targets for workers and firms have gone back to steady state, we can have a prolonged period of self-sustaining inflation. This is a case of de-anchoring in which the only way to go back to target inflation faster is for the central bank to keep activity low for some time.

The wage-price spiral is active in the self-sustaining phase of prolonged inflation, but it is exactly balanced on the two sides, so real wages remain constant.

AN EXAMPLE WITH REAL RIGIDITIES We now consider a different source of inertia, due to a gradual adjustment of the relative price targets of price and wage setters. In particular, we assume that changes in real marginal costs and the marginal rate of substitution between consumption and leisure only gradually change the behavior of price and wage setters. We replace the inflation dynamics above, equations (21)–(22), with the following equations:

$$\dot{p}_t = \lambda_p \left[a_t^p - \left(p_t - w_t \right) \right] + \frac{\lambda_p}{\rho + \lambda_p} \pi_t^{w,e} \text{ and}$$

$$\dot{w}_t = \lambda_w \left[a_t^w - \left(w_t - p_t \right) \right] + \frac{\lambda_w}{\rho + \lambda_w} \pi_t^e.$$

The real aspirations of price setters and wage setters, a_t^p and a_t^w, follow the adjustment equations

$$\dot{a}_t^p = \xi_p \left[\frac{s_X}{\epsilon} \frac{\rho + \lambda_p}{\rho + \lambda_p + \delta} \left(n_t - x_t \right) - a_t^p \right] \text{ and}$$

31. Notice that given that n is kept on its pre-shock path ($n = 0$) and that output falls due to the supply shock ($y_0 = s_X x_0 < 0$), there is an income effect that depresses the real wage demands of workers on impact, causing a very small initial nominal wage deflation, which is barely visible in the figure.

$$\dot{a}_t^w = \xi_w \left[\frac{\rho + \lambda_w}{\rho + \lambda_w + \delta} \left(\sigma y_t + \eta n_t \right) - a_t^w \right].$$

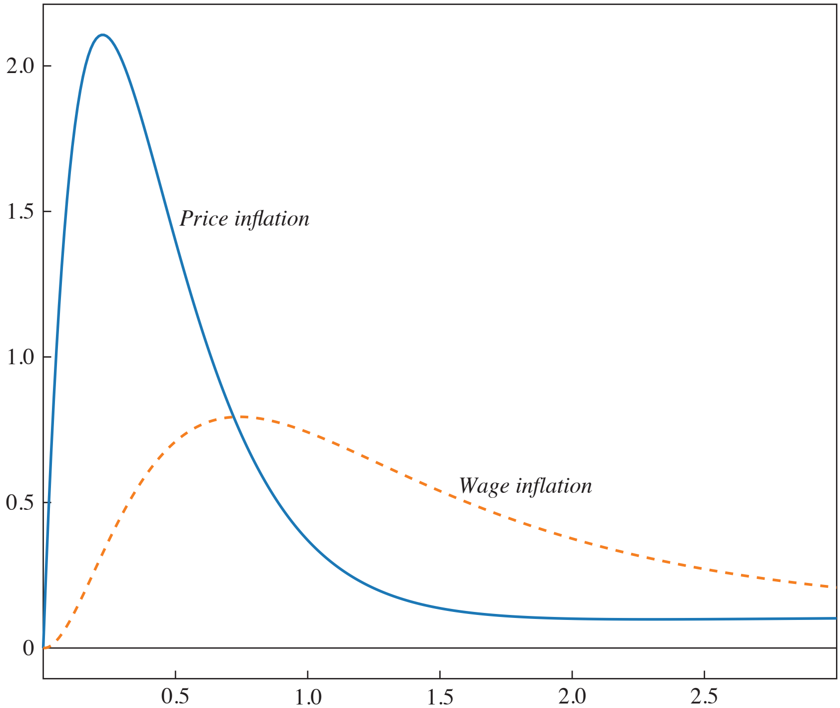
Aspirations are driven by the same forces that drive them in the baseline model, which in the case of firms are anticipated real input prices captured by the term $\frac{s_X}{\epsilon} \frac{\rho + \lambda_p}{\rho + \lambda_p + \delta} \left(n_t - x_t \right)$, and in the case of workers are anticipated marginal rates of substitution between consumption and leisure captured by $\frac{\rho + \lambda_w}{\rho + \lambda_w + \delta} \left(\sigma y_t + \eta n_t \right)$. However, these forces only gradually modify the aspirations of firms in terms of the desired margins ($p_t - w_t$ for the firms and $w_t - p_t$ for the workers).

We assume that the inflation expectations $\pi_t^{w,e}$ and π_t^e still follow the learning processes equations (20) and (23), so this version of the model includes both inertia caused by slow adjustment of inflation expectations and inertia caused by real rigidities. The choice to combine the two is because an interpretation of the real rigidities here is also some form of bounded rationality in processing observed changes in input prices and changes in labor market conditions, and combining that with perfect foresight on future price paths seems less natural. However, to focus on the role of real rigidities, we choose a parameterization with a lower $\gamma = 0.1$, relative to the parameterization used for figure 12, so inflation expectations play a more limited role. For the parameters ξ_p and ξ_w, we experiment with values equal to four and one, so the degree of real rigidity in the goods and labor market mirror the degree of nominal rigidity (captured by λ_p and λ_w). The inflation responses to the same supply shock used above are reported in figure 13.

In this economy, both price and wage inflation display hump-shaped responses, and the wage response is more delayed and more persistent than in the rational expectations baseline. The delay in the wage response is essentially due to the same reason as in the model with only adaptive inflation expectations: wage setters only start to demand higher nominal wages when price inflation has been going on for a while and has moved real wages away from their aspirations. The additional delay here is because of the fact that prices also take longer to respond due to the real rigidity in price setting.[32]

32. The real rigidity in wage setting does not really play an important role in this simulation because with a pure supply shock to x, the effect on $\sigma y + \eta n$ is very small, so workers' aspirations are essentially constant at zero. In line with this observation, simulations with larger and smaller values of ξ_w produce responses very similar to those in figure 13. Of course, in the case of other shocks this is no longer the case.

Figure 13. A Supply Shock with Adaptive Expectations



Source: Authors' calculations.

The example in figure 13 comes closest to capturing an immaculate inflation-disinflation scenario. The shock causes persistent responses of prices and wages. The persistence is purely due to the fact that price setters take some time to respond and wage inflation follows with further delay because wage setters only start responding after price setters have increased the price level enough to lower $w - p$. The persistence of wage inflation in this scenario is not a symptom of persistent overheating in the labor market but of a gradual return to pre-shock trends for the real wage.

VI. Conclusion

We explored the wage-price spiral in a canonical model of price and wage setting.

Interpreting inflation as the outcome of inconsistent aspirations for the real wage (or other relative prices) opens the door to many theoretical and empirical questions. We are especially interested in extending our work to explore potential sources of inertia in the inflation process, expanding the models explored in section V.

In the model analyzed here there is an instantaneous connection between the output gap and the real wage aspirations of workers and firms. However, it is plausible that workers' real wage aspirations respond gradually to changes in labor market conditions. Similarly, changes in goods market conditions could slowly affect firms' expected profit margins. These are sources of inertia in inflation that come from agents' views on relative prices and so are different from sources of inertia tied to future inflation expectations, which most research has focused on. Even if inflation expectations are well anchored, it is possible for inflation to persist if the disagreement between firms and workers is inertial. On the empirical front, while there is a lot of literature measuring inflation expectations, there has been limited effort so far at measuring workers' and firms' aspirations for real pay and real profit margins.

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Comments and Discussion

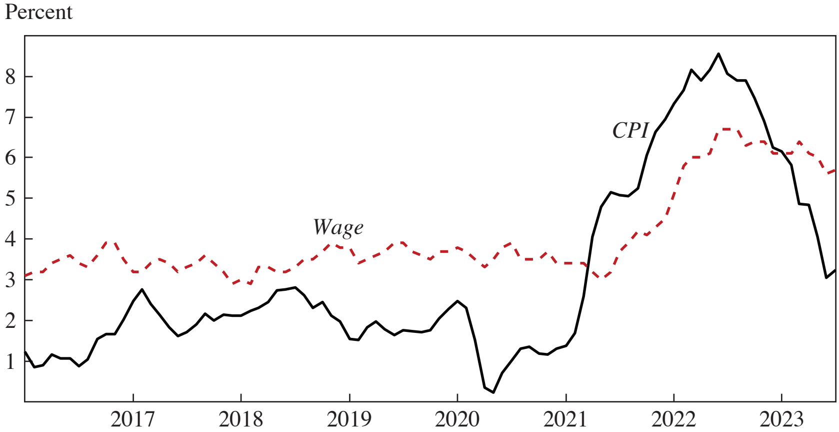
COMMENT BY

JORDI GALÍ Lorenzoni and Werning deal with a subject that is central to macroeconomics: the sources and mechanisms behind inflation fluctuations. Interest in that subject has only been enhanced by the recent high inflation episode. More specifically, they revisit the potential role of wage-price spirals as a factor of inflation persistence using a New Keynesian model with staggered price and wage setting à la Erceg, Henderson, and Levin (2000) as a reference framework. Their analysis yields a number of interesting results, including a connection between wage-price spirals and the concept of "conflict inflation," which they introduced in earlier work (Lorenzoni and Werning 2022). The paper contains many insights, of which I will single out the discussion of the potential role of two departures from the standard model as sources of inflation persistence, namely, the introduction of expectations de-anchoring and real rigidities.

My discussion is organized as follows. Firstly, I raise a caveat regarding the authors' characterization of the recent wage and price developments that motivate the paper. I then contrast the notion of inflation as conflict proposed in the paper with a more conventional interpretation of wage spirals. Next, I will discuss the connection between wage-price spirals and conflict inflation and relate some of the paper's normative findings to the existing literature. Finally, I will discuss the extensions of the model incorporating adaptive expectations and real rigidities.

RECENT WAGE AND PRICE DEVELOPMENTS REVISITED While the focus of Lorenzoni and Werning's paper is theoretical, its motivation is driven by the wage and price developments observed in the wake of the COVID-19

Figure 1. CPI and Wage Inflation

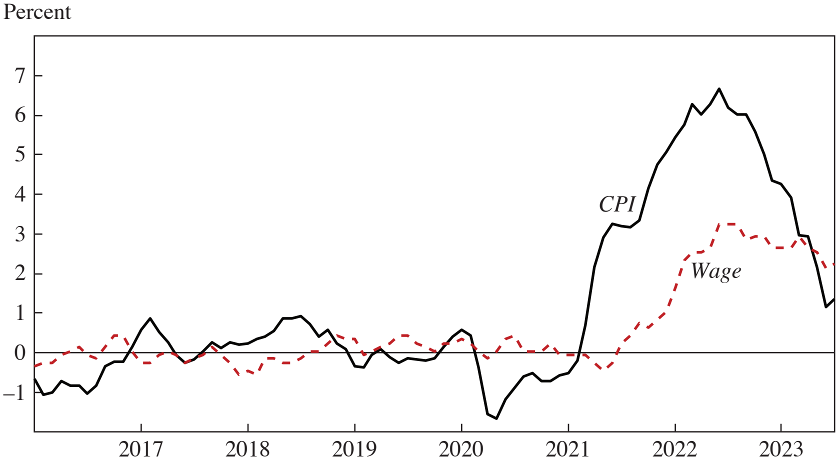


Source: CPI data from Bureau of Labor Statistics, retrieved from FRED; and wage inflation data from Wage Growth Tracker, Atlanta Fed.

pandemic and the war in Ukraine. Figure 1 summarizes these developments by displaying year-on-year US price and wage inflation from 2016 onward, using the Consumer Price Index (CPI) and the Federal Reserve Bank of Atlanta's wage index, respectively. The figure reveals the temporal pattern stressed by the authors, with wage inflation lagging price inflation both on the way up—with the real wage declining as a result—and on the way down—with wage inflation remaining roughly unchanged over the past year even in the face of a marked decline in price inflation—with the consequent increase in the real wage. That observation had led, in the authors' words, to "the concern . . . that higher wage growth would prevent inflation from going back to target, or even set off an out-of-control wage-price spiral." A central message of the paper is that such a concern is likely to be unwarranted, for the observed pattern is precisely the one that a standard model, calibrated in a way consistent with the evidence on the relative stickiness of prices and wages, would predict in response to either an expansionary demand shock or an adverse supply shock (both persistent, but not permanent) in an environment in which the monetary policy rule guarantees the return of price inflation to its intended target.

Here I would like to point out a caveat in the authors' analysis: the fact that price inflation and wage inflation display different underlying trends may distort the interpretation of figure 1 and its connection with the subsequent model simulations (which abstract from those differential trends). More

Figure 2. CPI and Wage Inflation (Demeaned)

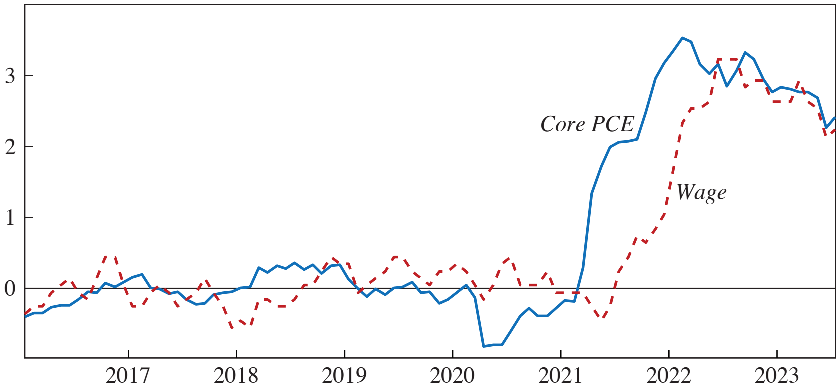


Source: CPI data from Bureau of Labor Statistics, retrieved from FRED; and wage inflation data from Wage Growth Tracker, Atlanta Fed.

specifically, and as figure 1 makes clear, wage inflation is, on average, higher than price inflation (equivalently, the real wage displays an upward trend). When using a simple plot to ascertain the impact of a shock on both variables, it is important to subtract their respective means. This is shown in figure 2, which displays the US price and wage inflation net of their (pre-COVID-19) means. The picture that emerges is significantly different, with more limited evidence of persistently higher wage inflation than price inflation (both relative to trend) at the end of the sample period. In other words, there is no evidence of a tendency for the real wage to revert back to its initial trend. That caveat appears even stronger when one uses core Personal Consumption Expenditures (PCE) data to construct the series for price inflation, as illustrated in figure 3.

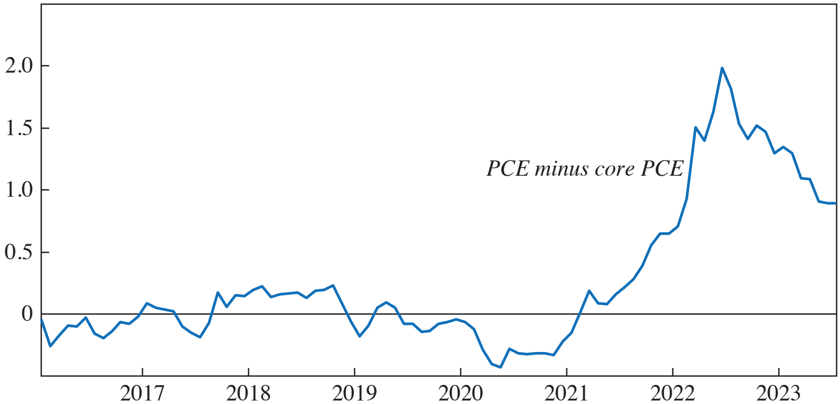
The resulting picture does not accord easily with the model simulations shown later in the paper, which imply trend reversion of the real wage. A possible explanation for the apparent absence of such trend reversion in the data is that the shock experienced by the US economy may have warranted a permanent fall in the real wage. Through the lens of the paper's model, this would be the case in the face of a permanent decline in the energy input endowment. Figure 4 displays some evidence consistent (if nothing else) with the hypothesis of a permanent supply shock: the log deviation between the PCE and core PCE indexes—which can be interpreted

Figure 3. Core PCE and Wage Inflation (Demeaned)



Source: Core PCE data from Bureau of Economic Analysis, retrieved from FRED; and wage inflation data from Wage Growth Tracker, Atlanta Fed.

Figure 4. Transitory or Permanent Shocks?



Source: Bureau of Economic Analysis, retrieved from FRED.

as a proxy for the relative price of noncore components (energy and food)—displays a seemingly permanent increase in the post-COVID-19 period relative to its stable pre-COVID-19 values.

A correct diagnosis of the forces behind the evidence is key to assess the challenges posed by wage developments in the near future, and in particular, by an eventual persistent above-trend wage inflation, possibly motivated by workers' resistance to seeing their real wage eroded. If the hypothesis of a permanent adverse supply shock is correct, that resistance should indeed be a source of concern since, ceteris paribus, it would be inconsistent with the attainment of the Federal Reserve's inflation target. Bringing back inflation to target would require, in that scenario, a recession strong enough to break the downward rigidity in real wages. The extension of the New Keynesian model allowing for real rigidities, developed in section V of the paper, would seem to provide the right framework for analyzing the options facing a central bank in that environment.

ON INFLATION AS CONFLICT As shown in the paper, aggregation of price-setting decisions in the continuous time version of a New Keynesian model yields the following expression for price inflation $\pi_t \equiv \dot{p}_t$:

$$\pi_t = \Lambda_p \int_t^\infty e^{-\rho(s-t)} \left[\left(w_s - p_s \right) - \left(mpl_s - \mu^p \right) \right] ds, \tag{1}$$

where w_s is the (log) average nominal wage, p_s is the (log) price level, mpl_s is the (log) marginal product of labor, and μ^p is the desired (or natural) price markup, assumed to be constant. Note that in contrast with equation (13) in the paper, I do not use demeaned variables, instead showing the constant term explicitly. Coefficient Λ_p, formally defined in the paper, is inversely related to the degree of price stickiness. Parameter $\rho > 0$ is the representative household's time discount rate.

Lorenzoni and Werning use equation (1) as a reference when putting forward their notion of inflation as conflict. Under that perspective, a rise in (price) inflation emerges when firms' real wage aspirations, defined by $mpl_s - \mu^p$, lie below actual real wages, either currently or anticipated. In that case, firms that get a chance to adjust their prices will tend to raise the latter, generating positive inflation.

A similar reasoning carries over to wage inflation, $\pi_t^w \equiv \dot{w}_t$, which is given by

$$\pi_t^w = \Lambda_w \int_t^\infty e^{-\rho(s-t)} \left[\left(mrs_s + \mu^w \right) - \left(w_s - p_s \right) \right] ds, \tag{2}$$

where coefficient Λ_w is inversely related to the degree of wage stickiness. Note that wage inflation is driven by current or anticipated gaps between workers' real wage aspiration, given by the (log) marginal rate of substitution augmented with the desired wage markup, $mrs_s + \mu^w$, and the actual average real wage $w_s - p_s$.

Accordingly, whenever firms' and workers' real wage aspirations are mutually inconsistent, this will necessarily be manifested in either price or wage inflation (or both), thus leading to the authors' view of inflation as conflict. In particular, whenever the path of real wages lies below that of workers' wage aspirations but above that of firms' corresponding aspirations, the implied upward pressure on wages and prices will reinforce each other, giving rise to a wage-price spiral, the focus of the paper.

The previous interpretation of inflation as conflict raises a number of questions, at least when applied to the New Keynesian model. In particular, I believe it gives a somewhat misleading impression about *individual* firms' motives. What drives the pricing decisions of an individual firm is the maximization of its value, which under the model's assumptions is attained by keeping its markup as close as possible (on average) to the optimal (flexible price) markup μ^p. In order to set its price optimally, the individual firm only needs to know its own nominal marginal cost, current and expected. Once that path is known, the real wage of its workers (defined relative to the entire consumption basket) is not of relevance to the price-setting firm. In particular, it does not care if the real wage of its workers goes up as a result of a reduction in other firms' prices.

The markup-based interpretation of an individual firm's motives, which can be read directly from the first-order condition associated with its optimal price-setting decision, is also reflected in inflation equation (1) once we rewrite it as follows:

$$\pi_t = \Lambda_p \int_t^\infty e^{-\rho(s-t)} \left[\mu^p - \left\{ p_s - \left(w_s - mpl_s \right) \right\} \right] ds$$

$$= \Lambda_p \int_t^\infty e^{-\rho(s-t)} \left(\mu^p - \mu_s^p \right) ds,$$

where $\mu_s^p \equiv p_s - (w_s - mpl_s)$ is the average price markup (with $w_s - mpl_s$ measuring the average marginal cost). Similarly, for wage inflation one can write

$$\pi_t^w = \Lambda_w \int_t^\infty e^{-\rho(s-t)} \left(\mu^w - \mu_s^w \right) ds,$$

where $\mu_s^w \equiv (w_s - p_s) - mrs_s$ is the average wage markup. Through this lens, price and wage inflation have a natural interpretation as the result of misalignments between actual and desired price and wage markups, respectively, and the consequent decisions by firms and workers in order to minimize those misalignments (at least in an expected sense), when allowed to do so.[1]

To be clear, the model is what it is, independent of the stories one can tell about its underlying mechanisms, and the wage-price block of the authors' model is fully standard. But to the extent that those stories help us understand the workings of the model, I can see two advantages of the interpretation based on markup misalignments relative to inflation as conflict advocated by the authors. First, while inflation is driven by deviations of a particular variable from some reference target in both cases, under the authors' interpretation that variable is the real wage whose target varies continuously over time and may even be nonstationary. By contrast, under the interpretation I prefer, the driving variable is the markup whose target is constant under standard assumptions. Second, as argued above, the markup misalignment interpretation seems to capture better the perspective of individual firms when making their price-setting decisions.

Finally, it is worth noting that the markup-based interpretation of inflation also provides a simple narrative for wage-price spirals. To see this, consider an adverse supply shock which raises firms' marginal costs and, as a result, lowers price markups relative to target. Firms that have a chance to adjust their prices will, on average, raise them, thus generating positive price inflation. Workers' real wages will be eroded as a result, thus lowering their average wage markup relative to target and inducing nominal wage increases among those workers who have a chance to reset their wage. The resulting wage inflation will in turn raise firms' marginal costs, leading to a second round of upward price adjustments, and so on.

CONFLICT INFLATION AND WAGE-PRICE SPIRALS Lorenzoni and Werning introduce the concept of conflict inflation as a component of price and wage inflation that results from a conflict between the wage aspirations of firms and workers. Formally, they define conflict inflation as follows:

$$\Pi_t^C \equiv \frac{\Lambda_p \Lambda_w}{\Lambda_p + \Lambda_w} \int_t^\infty e^{-\rho(s-t)} \Big[\big(mrs_s + \mu^w \big) - \big(mpl_s - \mu^p \big) \Big] ds, \tag{3}$$

1. See, for example, Galí (2015) for a textbook treatment of the New Keynesian model that stresses this interpretation.

where, once again, I am making explicit the constant terms in the expression. Note that conflict inflation is a discounted integral of current and future gaps between workers' real wage aspirations, $mrs_s + \mu^w$, and the corresponding aspirations for firms, $mpl_s - \mu^p$. A central theme in the paper is the connection between conflict inflation, thus defined, and the presence of a wage-price spiral. What is the nature of that connection?

Note that by combining equations (1) and (2) with the above definition of conflict inflation, one can show:

$$\Pi_t^C = \alpha\pi_t + (1-\alpha)\pi_t^w, \tag{4}$$

where $\alpha \equiv \frac{\Lambda_w}{\Lambda_w + \Lambda_p} \in [0, 1]$. In words, conflict inflation can be expressed as a particular weighted average of price inflation and wage inflation, with the weight of each variable increasing in its relative stickiness.

A straightforward algebraic manipulation of equation (4) allows the authors to obtain the following expressions for price and wage inflation:

$$\pi_t = \Pi_t^C - (1-\alpha)\dot{\omega}_t \text{ and} \tag{5}$$

$$\pi_t^w = \Pi_t^C + \alpha\dot{\omega}_t, \tag{6}$$

where $\dot{\omega}_t \equiv \pi_t^w - \pi_t$ is the change in the real wage. Equations (5) and (6) motivate the authors' intended connection between conflict inflation and wage-price spirals, since Π_t^C can be interpreted, in their words, as the "underlying common component of price and wage inflation due to the gap between the aspirations on the two sides of the market."

However, establishing a rigorous connection between conflict inflation and wage-price spirals requires a formal definition of the latter. What is a wage-price spiral, after all? How can one measure its intensity?

While macroeconomists likely share at least a vague notion of what a wage-price spiral is, as far as I can tell there is no consensus on a formal definition of that phenomenon.[2] A possible definition, and one that the authors adhere to in several instances throughout their paper, is an episode

2. A recent paper by International Monetary Fund economists (Alvàrez and others 2022) seeks to identify wage-price spiral episodes throughout history. They use as a definition the observation of three successive quarters with accelerating price and wage inflation.

in which both price and wage inflation are positive.[3] Note, however, that conflict inflation would not seem to be a good indicator of the intensity of a wage-price spiral under such a definition, for any positive value of conflict inflation is consistent with wage and price inflation values of different sign.[4]

Furthermore, it is not obvious why any arbitrary weighted average of price and wage inflation (defined by a weight α different from $\frac{\Lambda_w}{\Lambda_w + \Lambda_p}$) could not also be thought of as a plausible wage-price spiral indicator, since equations (5) and (6) would also hold for that alternative measure. That measure, however, would bear no simple relation with conflict inflation.

So the question remains: what makes the particular weighted average of price and wage inflation defined by equation (4) with $\alpha = \frac{\Lambda_w}{\Lambda_w + \Lambda_p}$ (and which corresponds to conflict inflation) special or particularly desirable as a measure of wage-price spirals?

To address that question, the authors first propose a formal measure of the intensity of wage-price spirals, which they refer to as "spiral inflation." Formally, they define spiral inflation (in response to a shock at time zero) as:

$$\Pi_0^S = \int_0^\infty \pi_s ds,$$

that is, the cumulative change in price inflation. To the extent that the shock under consideration does not have a long-run effect on the real wage (as assumed by the authors), it follows $\int_0^\infty \pi_s ds = \int_0^\infty \pi_s^w ds$, that is, the cumulative change in wage inflation must equal that of price inflation, with their common value corresponding to spiral inflation, the authors' proposed wage-price spiral indicator.

Next the authors move on to show that, in the particular case that conflict inflation decays exponentially, spiral inflation will be proportional to conflict inflation. To see this, note that

3. More generally, one could define a wage-price spiral episode as one displaying price and wage inflation above their corresponding steady-state values. In the authors' model, those steady-state values are zero by assumption.

4. On the other hand, positive conflict inflation is a necessary condition for a wage-price spiral under that proposed definition. As the authors argue, however, positive conflict inflation necessarily implies positive cumulative price and wage inflation through the adjustment to the steady state, under certain assumptions.

$$\begin{aligned}\Pi_0^S &= \int_0^\infty \pi_s ds \\ &= \int_0^\infty \Pi_s^C ds - (1-\alpha)\int_0^\infty \dot{\omega}_s ds \\ &= \int_0^\infty \Pi_0^C e^{-\delta s} ds - 0 \\ &= \frac{1}{\delta}\Pi_0^C,\end{aligned}$$

where δ is the rate of decay of conflict inflation and $\int_0^\infty \dot{\omega}_s ds = 0$ follows from the stationarity of the real wage. The previous finding is interpreted by the authors as implying that "conflict inflation at date zero fully captures the underlying forces that lead to a protracted period of joint price and wage inflation," thus establishing the desired connection between conflict inflation and wage-price spirals.

The interest of the previous result notwithstanding, it is important to point out some caveats. First, the proportionality between spiral inflation Π_0^S and conflict inflation Π_0^C holds in the particular case of exponential decay, but it will not hold more generally. While such an exponential decay may be supported by an appropriate choice of monetary policy, it is generally not a property of the equilibrium. Furthermore, the coefficient of proportionality between the two variables depends on the rate of decay, which will not be invariant to the persistence of the shock or the policy rule in place. Accordingly, similar readings of conflict inflation at different points in time (or for different economies) may correspond to different levels of spiral inflation. Second, the tight relation between conflict inflation and spiral inflation hinges on the assumption of a stationary real wage, which is needed for $\int_0^\infty \dot{\omega}_s ds = 0$ to hold. Accordingly, the simple relation between spiral inflation and conflict inflation will vanish in the face of shocks with permanent effects on the real wage. Third, and perhaps most important, even in the case of a stationary real wage, the link between spiral inflation and conflict inflation uncovered above holds at time zero, that is, the time of the shock, when the real wage is still at its steady-state level, but it fails to do so on an arbitrary period $t > 0$ when that variable is away from the steady state, for in that case $\int_0^\infty \dot{\omega}_s ds \neq 0$.

CONFLICT, SPIRALS, AND THE DESIGN OF MONETARY POLICY Section IV of Lorenzoni and Werning's paper revisits the problem of optimal policy in the face of supply shocks. Given that the analysis of optimal policy in the New Keynesian model with staggered prices and wages, tracing back to

Erceg, Henderson, and Levin (2000), is generally well understood, some of the authors' findings are not entirely novel, though they are recast here in terms of conflict inflation and, more generally, they are related to the notion of a wage-price spiral. In particular, there are two well-established results in the literature on optimal policy in the model in Erceg, Henderson, and Levin (2000).[5] First, there exists a specific weighted average of wage inflation and price inflation, referred to as "composite inflation" in Galí (2015), for which the divine coincidence holds, that is, full stabilization of that variable implies full stabilization of the output gap. Second, there is a knife-edge parameter configuration for which the optimal policy calls for a full stabilization of the output gap and, hence, of composite inflation. More generally, and for a broad range of parameter values, such a policy is nearly optimal.

The connection between the previous results and some of the findings in the paper becomes clear once we recognize that the weighted average defining conflict inflation in equation (4) matches exactly the one that defines composite inflation in the existing literature. In particular, the symmetric case considered by the authors in their example 1 corresponds to the knife-edge case referred to above, while examples 2 and 3 can be viewed as an illustration of the near optimality of stabilization of the output gap more generally as reflected in the tiny response of that variable (once the scale of the plot is taken into account) under the optimal policy, as displayed in figures 10 and 11 in the paper.

Beyond the connection with the existing literature, the authors' analysis uncovers some results that shed light on a number of issues and that, in my opinion, are not sufficiently stressed in the paper.

First, the authors derive the second-order approximation to the welfare losses for the case of continuous time. The resulting expression is similar to the one for the discrete time case, originally derived in Erceg, Henderson, and Levin (2000). It is worth noting a difference, not emphasized by the authors, related to their use of a CES production function: the coefficient on the output gap Φ_y is inversely related to the elasticity of substitution between energy and labor. Thus, ceteris paribus, a low value for that elasticity will be associated with a higher weight on output gap stability in the central bank's loss function. That result, in a model in which the divine coincidence does not hold, is of great interest and its implications would seem to deserve some further discussion.

5. See proposition 3.9 and section 4.4 in Woodford (2004) and section 6.4.3 in Galí (2015).

Second, the authors note the following result, which follows from equation (4): with a zero output gap (and, hence, zero conflict/composite inflation), the adjustment in the real wage never requires positive inflation for *both* wages and prices. A slight generalization of that result, based on the near-optimality findings mentioned above, would run as follows: the fact that the optimal policy involves, at most, tiny deviations of conflict inflation from zero, rules out non-negligible positive inflation for both wages and prices as an optimal outcome. In their example 3, the authors uncover an instance of coexistence of positive wage and price inflation for a very brief period during the adjustment, but one should note that wage inflation is almost zero during that brief episode.

Under a definition of wage-price spirals as episodes with (non-negligible) positive inflation in wages and prices, the previous discussion would establish an interesting connection between optimal policy and the subject that is the focus of this paper, namely, the observation that wage-price spirals are (almost) always suboptimal. But, as discussed above, this is not the definition of wage-price spirals adopted by the authors, who instead focus on the concept of spiral inflation as an indicator of the intensity of wage-price spiral episodes. Unfortunately, the usefulness of spiral inflation in the context of the authors' optimal policy exercise is limited, since the real wage is permanently affected by the shock considered, implying that the mapping between conflict and spiral inflation is lost. In fact, under the optimal policy, and given the discussion above, we have

$$\Pi_0^S \simeq -\left(1-\alpha\right)\int_0^{\infty}\dot{\omega}_s ds,$$

which may take a large positive value in response to an adverse supply shock even if wage inflation and price inflation co-move negatively during the adjustment period (as in the three examples considered). It is clear that, in that instance, spiral inflation would not be a good indicator of a wage-price spiral.

ADAPTIVE EXPECTATIONS AND REAL RIGIDITIES Section V departs from the standard model in Erceg, Henderson, and Levin (2000) by exploring the implications of two potential sources of inertia, namely, a form of adaptive expectations that implies de-anchoring and the presence of real rigidities. The former is modeled by assuming that firms and workers expect constant inflation at all horizons (at a level that may be different from the steady state, thus the interpretation as a form of de-anchoring), with that variable adjusting slowly in response to variations in realized inflation. The latter assumes that the real wage targets of workers and firms adjust sluggishly in response to changes in mrs_s and mpl_s.

Lorenzoni and Werning show that the introduction of de-anchoring leads to both greater inertia and higher persistence in both price and wage inflation relative to the baseline model, as a result of a strong underlying wage-price spiral mechanism. That prediction is enhanced when real rigidities are added.

Unfortunately, the authors do not carry out an analysis of optimal policy using the modified model. I believe it would be interesting to explore whether the two sources of inertia considered in this section could overturn the result derived for their baseline model, regarding the impossibility of non-negligible positive inflation coexisting for both wages and prices as an optimal outcome. I hope the authors (or someone else) undertake that analysis in future work.

Here is a minor quibble I have on this section: when considering the calibration with real rigidities (the second source of inertia), the authors maintain the assumption of adaptive expectations (the first source of inertia), but they lower the setting of γ from 1 to 0.1, which is justified on the grounds that "inflation expectations play a more limited role." This may be somewhat confusing to the reader since, as far as I understand, lowering γ makes inflation expectation even more sluggish (and thus further from rational expectations than in their first exercise where they only considered adaptive expectations as a source of inertia). In any event, I believe the authors should have gone back to rational expectations when studying real rigidities, in order to insulate the independent role played by this second source of inertia.

As a final comment, I would encourage the authors to discuss the connection between the two sources of inertia and wage indexation, a feature that is often incorporated in estimated versions of the standard model.[6] Wage indexation is typically modeled by having the nominal wages that are not re-optimized to be adjusted automatically in proportion to past price inflation. That mechanism is a source of real wage rigidity whose implications would be worth contrasting with the type of real rigidity assumed by the authors.

CONCLUDING REMARKS Recent price and wage developments in the United States and other advanced economies have rekindled fears of a wage-price spiral that may hinder central banks' efforts to control inflation. Lorenzoni and Werning's paper seeks to understand those developments through the lens of a New Keynesian model with sticky prices and wages. The first

6. See, for example, Smets and Wouters (2007).

challenge is to come up with an operational definition and measure of a wage-price spiral. The authors' proposed measure, spiral inflation, seems to be useful under certain conditions but not generally. The authors also explore the usefulness of conflict inflation, a concept they introduced in earlier work (Lorenzoni and Werning 2022), in accounting for wage-price spirals, and its connection with spiral inflation. In the context of the New Keynesian model, conflict inflation turns out to coincide with the particular weighted average of price and wage inflation (composite inflation), the stabilization of which implies the stabilization of the output gap; thus, conflict inflation inherits all the normative implications associated with composite inflation. Furthermore, conflict inflation is shown to be proportional to spiral inflation under certain conditions. In my discussion, I have raised some caveats about the usefulness of both conflict inflation and spiral inflation to help us understand and measure wage-price spirals. That skepticism notwithstanding, I found the paper to be thought-provoking and insightful along many dimensions. The likely inefficiency of wage-price spirals is an implication of their analysis that I found particularly interesting. It would be interesting to explore the type of changes in the environment that would allow that result to be overturned. An analysis of the normative implications of the sources of inertia introduced by Lorenzoni and Werning would seem to be a natural starting point in that endeavor.

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COMMENT BY

AYŞEGÜL ŞAHIN The onset of the COVID-19 pandemic in early 2020 led to a brief yet deep economic downturn. Following a significant decline in economic activity, the economy experienced a resurgence, accompanied by an abrupt and unexpected rise in inflation. After lying dormant for two decades, inflation surged, with the Consumer Price Index (CPI) climbing from 1.4 percent in January 2021 to 8.9 percent in June 2022. The evolution of wage inflation followed a different pattern. The Employment Cost Index increased at a lower pace than the price inflation initially and real wages declined. Most recently, as price inflation declined, wage growth surpassed price inflation, resulting in a boost in real wages. The rise in real wages triggered concerns about a potential wage-price spiral, which may impede the return of inflation to its target level of 2 percent.

Lorenzoni and Werning provide a careful examination of price and wage inflation dynamics through the lens of the New Keynesian framework. Their analysis yields several important insights about the drivers and consequences of the recent high inflation episode. This comment reviews and interprets Lorenzoni and Werning's findings and suggests extensions for future research.

FRAMEWORK The authors consider a New Keynesian framework with both wage and price rigidities. An important addition to the standard model is a nonlabor input (X) with a flexible price and inelastic supply. This input, X, is the second input to production besides labor L. It broadly captures supply chain disruptions and the rise in the price of energy and raw materials reflecting pandemic-related factors that adversely affected production. An important assumption is the low substitutability between X and L, which is important for the initial surge in inflation. This is because when demand increases, the price of the nonlabor input X rises, leading to scarcity. Consequently, the marginal product of labor (MPL) declines, given the low substitutability between X and L. This scarcity contributes to a rise in noncore inflation, creating a distributional tension between workers and firms, potentially initiating a wage-price spiral. Notably, real wages initially decrease as price inflation picks up. The key takeaway from these dynamics is that the fact that nominal wage growth is currently exceeding price inflation could be given an optimistic interpretation. In particular, it might be interpreted as a sign of real wages going back to trend and not necessarily as a concern of an ongoing wage-price spiral.

Key conditions that the framework requires to match the price and wage dynamics since 2021 are summarized in proposition 2 in the paper. When an economy satisfies the condition stated in proposition 2, the authors refer to it as a "supply-constrained" economy. This condition is met if some key assumptions are satisfied, specifically: X is inelastically supplied with flexible price, which allows its price to adjust rapidly; X plays a significant role in production with a high share (denoted as s_X) and acts as a complement to labor, characterized by low substitutability (ϵ); and wages are relatively more rigid than prices ($\Lambda_w < \Lambda_p$).

To summarize, the elasticity of substitution between the nonlabor input and labor, along with the relative rigidity of wages and prices, plays a crucial role in the joint dynamics of wages and prices. If the nonlabor input is less important in production and can be readily substituted by labor, the increase in inflation would be subdued. Moreover, the rigidity of wages in comparison to prices significantly influences the joint dynamics of price and wage inflation.

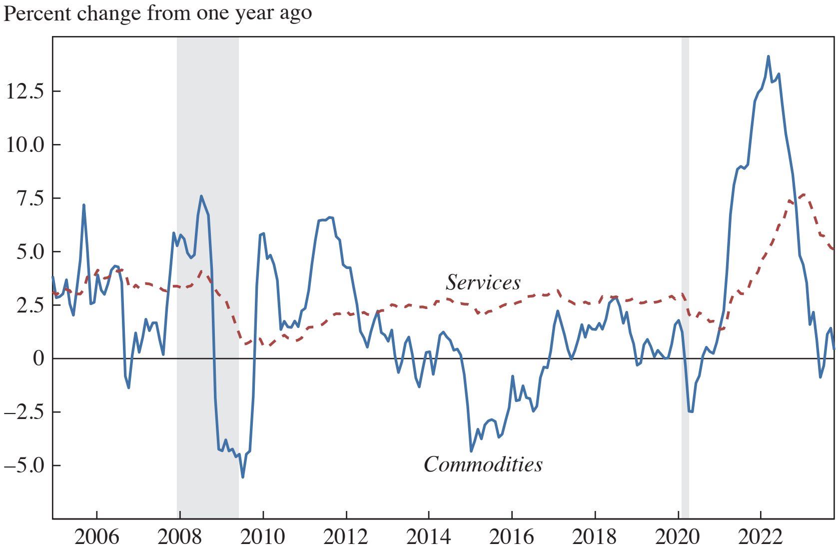
These key parameters that are highlighted in proposition 2 are likely to vary across different sectors of the economy. More specifically, goods-producing and service-providing sectors use different production technologies. The literature finds complementarity between intermediates, which supports the low elasticity of substitution assumption for the goods sector.[1] However, the elasticity of substitution is likely to be higher in the services sector, and wage rigidities are less likely to play an important role for services since labor turnover has been very high in the recent period. That is why the framework in the paper is likely to be more relevant for accounting for inflation dynamics in the goods sector.

GOODS AND SERVICES INFLATION Examining the goods-producing and service-providing sectors would be useful for digging deeper into inflation dynamics since the initial surge in the US inflation was almost solely driven by goods inflation. The pickup in services inflation has also been significant, but it has been more modest, and it lagged inflation in the goods sector as shown in figure 1. This is a reversal of the typical inflation dynamics in the last twenty years, which were characterized by pro-cyclical services price inflation and essentially zero goods price inflation over the past ten years.

An important factor that is often cited for the surge in goods prices is supply chain bottlenecks. Figure 2 shows that the price of industrial supplies and materials has risen sharply, increasing by more than 50 percent at the onset of the pandemic. This increase coincided with the emergence of goods inflation and is often referred to as the main driver of a rise in prices.[2]

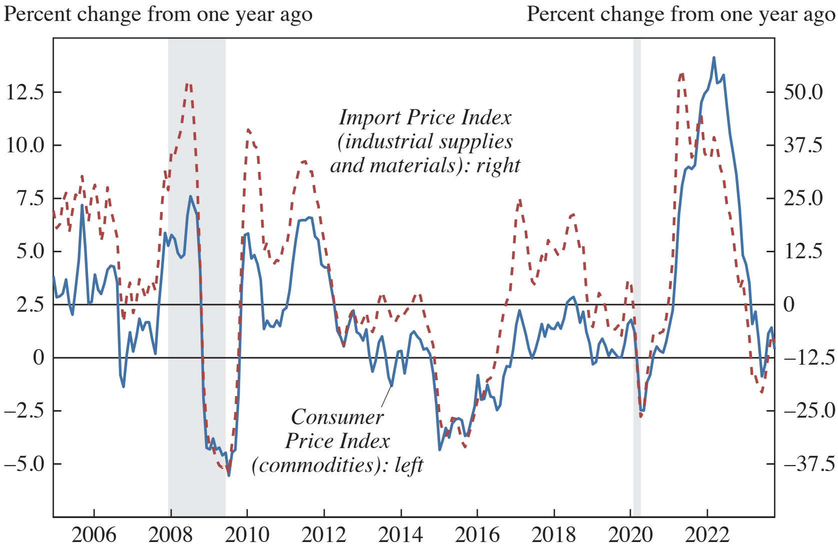
1. See, for example, Boehm, Flaaen, and Pandalai-Nayar (2019).
2. See, for example, Amiti and others (2023).

Figure 1. Consumer Price Indexes for Commodities and Services



Source: Bureau of Labor Statistics, series CUSR0000SAC and CUSR0000SAS, retrieved from FRED.
Note: Consumer Price Indexes are for all urban consumers, US city average. Shaded areas indicate US recessions.

Figure 2. Consumer Price Index for Commodities and Import Price Index (End Use) for Industrial Supplies and Materials



Source: Bureau of Labor Statistics, series CUSR0000SAC and IR1, retrieved from FRED.
Note: Consumer Price Index is for all urban consumers, US city average. Shaded areas indicate US recessions.

Figure 3. Consumer Price Index for Commodities and Employment Cost Index for Wages and Salaries for Private Industry Workers in Goods-Producing Industries

Source: Bureau of Labor Statistics, series CUSR0000SAC and CIS202G000000000I, retrieved from FRED.

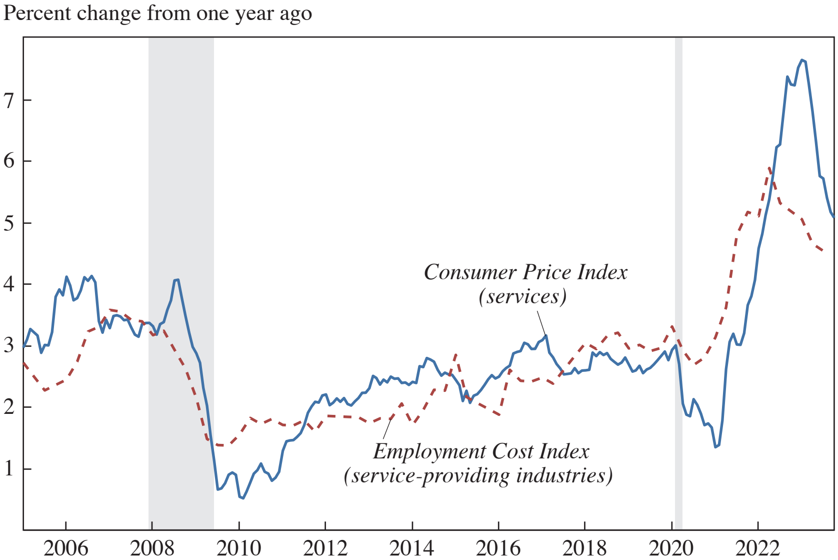
Note: Consumer Price Index is for all urban consumers, US city average. Shaded areas indicate US recessions.

These observations all support the authors' modeling choices. The introduction of the nonlabor input allows the model to account for the rise in prices of industrial supplies and raw materials. In addition, the assumption that it is inelastically supplied with a flexible price—which prevents their quantities from adjusting to relieve price pressures—mimics the supply chain disruptions related to the pandemic. Figure 3 shows the time series of price inflation in the goods sector along with wage growth in the sector. The evolution of price and wage inflation is similar to dynamics generated by the model.

However, price and wage inflation dynamics in the services sector look very different: wage inflation picks up before prices, and it also starts to retreat before prices, as shown in figure 4. While the model does a good job of accounting for joint price-wage dynamics in the goods sector, it is less applicable to the services sector.

WORKERS' AND FIRMS' ASPIRATIONS The authors define an interesting concept of inflation that they refer to as "conflict inflation." Fundamentally,

Figure 4. Consumer Price Index for Services and Employment Cost Index for Wages and Salaries for Private Industry Workers in Service-Providing Industries



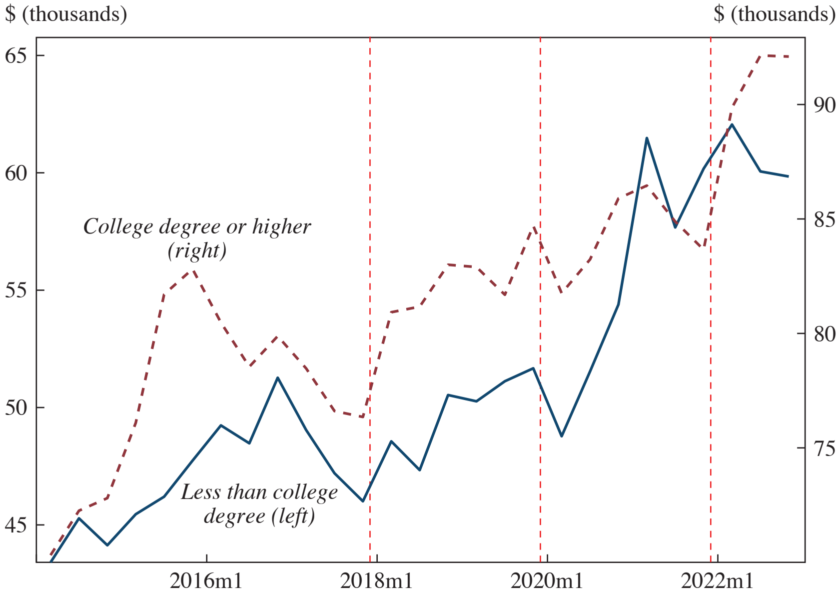
Source: Bureau of Labor Statistics, series CUSR0000SAS and CIS202S000000000I, retrieved from FRED.

Note: Consumer Price Index is for all urban consumers, US city average. Shaded areas indicate US recessions.

the economic intuition behind the wage-price spiral mechanism lies in the divergence of views between workers and firms regarding the relative price of goods and labor, represented by the real wage *W/P*. When firms adjust nominal prices, they do so with a specific target for *W/P* in mind. However, workers may demand nominal wages with the aim of achieving a higher real wage. This conflict in aspirations leads to inflation in both prices and wages. This definition of a wage-price spiral emphasizes the disagreement or conflict as a key driver of inflation, as analyzed in a companion paper (Lorenzoni and Werning 2022).

While it is hard to measure the degree of disagreement between workers' and firms' aspirations, some new data sources provide us with some information regarding the evolution of these aspirations. A useful metric for summarizing workers' aspirations is the reservation wage of workers. The Federal Reserve Bank of New York's Survey of Consumer Expectations provides a measure of the reservation wages obtained from the following survey question: "Suppose someone offered you a job today in a line of

Figure 5. Reservation Wages by Educational Attainment



Source: Survey of Consumer Expectations, Federal Reserve Bank of New York.

work that you would consider. What is the lowest wage or salary you would accept (BEFORE taxes and other deductions) for this job?"

Figure 5 shows the reservation wages by educational attainment starting in 2014. Reservation wages started to rise for both workers with college education and those without in 2017, but the rise was much steeper for workers without a college degree. This increase is in line with the authors' characterization of the inflationary episode.

What about firms' aspirations or willingness to pay workers? Wages posted by employers with job openings provide a direct measure of firms' wage aspirations for the workers they plan on hiring. Crump and others (2022) utilize data from Burning Glass Technologies on posted job vacancies to examine posted wages. They find that, on average, posted wages for jobs with salaries below $75,000 grew at a rate of about 12 percent from 2019 to 2021 compared to about 8 percent from 2017 to 2019. The strong posted wage growth at lower salary positions over the last two years coincided with the stark rise in reservation wages of workers without college education.

Although there has been a rise in workers' wage aspirations, as indicated by their reservation wages, posted wages indicate that firms have met these aspirations when posting job openings. Though these measures are only suggestive, they point to a less important role for conflict between firms and workers in driving inflation dynamics.

OTHER DEVELOPMENTS IN THE LABOR MARKET While the paper provides an intriguing explanation for wage and price inflation dynamics, several developments in the labor market point to the existence of other factors. Arguably, the most striking development in the labor market has been the so-called Great Resignation: the quits rate for employed workers reached 3 percent in 2021, almost 50 percent higher than in 2019.[3] Moreover, the Beveridge curve exhibited a wide loop and a vertical shift, unlike its commonly observed horizontal movements. The behavior of wages during the recovery from the pandemic recession also deviated from historical patterns. While high-wage workers typically experience faster wage growth during recoveries, leading to an increase in the wage gap between high- and low-wage workers, the opposite occurred after the pandemic, leading to wage compression, as documented by Autor, Dube, and McGrew (2023). One possibility is that the shift in worker preferences toward more flexible jobs coupled with a rapid recovery triggered an increase in quits by workers in search of more flexible job opportunities and put downward pressure on wages in high-amenity jobs, as argued by Bagga and others (2023). Under this interpretation, as reallocation from low- to high-amenity jobs subsides, job-to-job transitions could be more inflationary.

CONCLUDING REMARKS Lorenzoni and Werning provide a timely paper on an important topic with rich insights. They focus on conflict inflation as a key driver of the post-pandemic inflation surge. They also carefully study the interplay of supply chain disruptions and disagreement between workers and firms. For future research, exploring a multi-sector model and distinguishing between goods and services with different production technologies and degrees of wage and price rigidities, could provide valuable insights. Additionally, incorporating measures of workers' and firms' expectations about wages and prices would help improve the model's quantitative implications.

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GENERAL DISCUSSION Jason Furman suggested that the authors look into empirical evidence on wage-price spirals outside of the motivating case of the United States to decide the usefulness of the proposed model. He pointed specifically to the recent, and quite different, experience in Europe: there was a larger shock to what the authors refer to as X, and real wages declined significantly, as did nominal wages after an initially modest response. This would have implications for the pro- and countercyclicality of wages in different situations. Referring to the way the authors define a wage spiral as following the logic of a conflict, Furman also proposed an approach where countries could be grouped according to institutions that maximize conflict and institutions that minimize conflict, looking at how their impulse responses differ.

Martin Baily contemplated what wage-setting model the authors had in mind, noting that there are different labor markets, including a very small part that is unionized and a part that is not. Given that many wages are set in a spot market or something close to it, how do aspirations fit into the picture? Baily was skeptical of the notion that workers with an aspiration for higher wages would simply be able to ask for them. To better gauge this part of the model, he advised the authors to look at how labor market institutions differ across countries or within the United States over time.

Guido Lorenzoni clarified that the central issue is not who sets the wages; rather, the general equilibrium problem is that once the nominal wage is set, the firm and the worker negotiating take the price of all other goods as a given. The firm can set the real wage in terms of the goods they produce but not in terms of the goods that the worker consumes; thus, there is still a coordination problem to be solved.

Betsey Stevenson provided two examples that have arguably had an impact on workers' aspirations in terms of wages. Stevenson explained that many nurses quit their jobs during the labor shortage and signed up as travel nurses—often being assigned back to the same hospital but making a substantially higher wage. As a second example, she highlighted the fact that unionized workers have seen greater real wage declines than their non-unionized peers—an unusual development that is likely to leave unionized workers frustrated. Furthermore, Stevenson speculated that the widespread anger among workers, beyond those who have experienced falling real wages and despite a strong economy, will fuel expectations around the labor market for the next year or two.

David Romer asked the authors to elaborate on how their reinterpretation of inflation—conflict inflation—differs from standard accounts. In a standard New Keynesian model, for example, one could interpret an episode of inflation when output is above normal as a result of the set of monopolistically competitive firms having a form of conflict because they have mutually incompatible goals, as each would like its relative price to be above average. Romer also argued against focusing only on inflation expectations. Workers may demand higher wages not just because they expect inflation will be higher in the future but also because their real wages have fallen, pointing to the recent United Auto Workers strike as an example.[1] Romer therefore wondered whether aspirations might be a variable with a life of its own and, if added to the models, could provide additional insights.

Also raising options for expansions to the authors' model, Şebnem Kalemli-Özcan mentioned her own work using a production network in which one can identify both sectoral labor supply shocks—pointing specifically to the service sector as relevant here—as well as nonlabor, goods shocks.[2] To Furman's point, Kalemli-Özcan stated that the timing and

1. Reuters, "UAW to Expand Strike at Ford, General Motors," September 29, 2023, https://www.reuters.com/business/autos-transportation/uaw-expand-strike-ford-general-motors-2023-09-29/.

2. Cem Çakmakli, Selva Demiralp, Şebnem Kalemli-Özcan, Sevcan Yeşiltaş, and Muhammed A. Yildirim, "The Economic Case for Global Vaccinations: An Epidemiological Model with International Production Networks," working paper 28395 (Cambridge, Mass.: National Bureau of Economic Research, 2022), https://www.nber.org/papers/w28395; Julian di Giovanni, Şebnem Kalemli-Özcan, Alvaro Silva, and Muhammed A. Yildirim, "Pandemic-Era Inflation Drivers and Global Spillovers," working paper 31887 (Cambridge, Mass.: National Bureau of Economic Research, 2023), https://www.nber.org/papers/w31887.

intensity of the nonlabor goods, the labor supply, and labor demand shocks were very different across countries, noting as an example how there is growing consensus in the United Kingdom that the labor supply shock is of a permanent nature, that is, it is expected to permanently reduce the labor force.

Gerald Cohen proposed that thinking about inflation requires a high level of sophistication, including separating goods and services. To properly assess a model of inflation, we ought to investigate how the various markups from different parts of the economy are translated into the inflation numbers.

Iván Werning responded to the calls for a more sophisticated model by explaining that the intent was to generate a very stylized, general model. The authors wanted to capture the fact that the wages are sluggish in the simplest possible way. Werning further argued that the policy debate is often even simpler than the model they proposed, missing simple aspects of the inflation issue that the authors wanted to point to using their model. In terms of the shocks, Werning said that other types of shocks—a permanent one, for example—could easily be incorporated into the model, as could aspirations. He emphasized that they had not been omitted because the authors did not believe in them. Werning noted that the strength of their model is precisely the fact that it is very general, and the purpose of their paper is to provide a different perspective using a standard model, perhaps with a tweak to the parameters. Responding to questions about introducing aspirations into the model, Werning referred to a paper by Olivier Blanchard and Jordi Galí as a source of inspiration for authors' other work where they introduce the possibility that workers demand higher wages for reasons that go beyond nominal wage rigidities.

Lorenzoni explained that their model is completely compatible with a multi-sector approach in which, as we saw in the most recent episode, service inflation lags behind goods inflation, for example. In the paper, the simplest case with two sectors is presented: one produces goods and the other "produces" labor. But even in this simple case, the model provides the same, important intuition: different sectors react with a different lag in response to a shock, which gives rise to a sort of ripple effect that travels through the economy. Lorenzoni pondered the necessary preconditions for such a ripple effect to take place, noting that a price increase is not always enough to create a wave. But if we collectively were to lose faith in the stability of the unit of account, the ripple effect that follows would see the higher cost being passed along from the goods-producing sector to firms, and then to consumers—the wage earners—who would negotiate for a higher wage. This is the source of conflict that the paper highlights.

Justin Wolfers made the point that the 6 percent private-sector unionization rate in the United States perhaps does not lend itself very well to the frame of the proposed model but would fit a labor market like that in Australia quite well.[3] Offering suggestions for future additions, Wolfers encouraged the authors to add a third player to their model: central banks. He was curious whether there would be distributional consequences as a result of central banks adopting an inflation-targeting regime as opposed to a nominal wage target.

Caroline Hoxby was struck by how the authors' findings seemed to have a clear analogue in the public finance literature. The shock in this case would be a tax reform, and the findings typically indicate a response that is quite fast for workers whose earnings depend on prices—a car dealer, for example. The response is significantly smaller for workers whose earnings depend on wages. This produces a strikingly similar pattern to figure 1 in the paper.

Michael Kiley offered a different perspective from what he interpreted as the authors' conclusion: wage-price feedback is currently limited, which suggests there is no cause for concern. Referring to the empirical literature on Phillips curves, Kiley highlighted two stylized facts. Wage-price feedback was limited from the 1990s to the 2010s but was much more pronounced in the 1970s and the 1980s. Kiley argued that the data tend to support that we are currently in a situation that more closely resembles the 1970s and the 1980s, citing his own work and suggesting that a lack of anchoring of inflation expectations or the really big shock we just experienced are the two most plausible explanations for the more apparent wage-price feedback we are seeing now.[4] While the data seem to support the latter explanation, Kiley emphasized that the real concern is the possibility that it is indeed inflation expectations that are drifting.

Benjamin Moll wondered if the authors could talk about how the results would change if the assumption in the model about perfect foresight was relaxed: for example, if workers were more myopic, and perhaps firms were more forward-looking than the workers.

3. US Department of Labor, Bureau of Labor Statistics, "Union Members—2022," news release, January 19, 2023, https://www.bls.gov/news.release/pdf/union2.pdf.

4. Michael T. Kiley, "The Role of Wages in Trend Inflation: Back to the 1980s?" Finance and Economics Discussion Series (Washington: Board of Governors of the Federal Reserve System, 2023), https://www.federalreserve.gov/econres/feds/the-role-of-wages-in-trend-inflation-back-to-the-1980s.htm.

Bringing us back to 2020, when real wages were higher and then started falling, Wendy Edelberg made the point that the effects of the supply shock—abstracting from demand—had to be absorbed somewhere: a drop in productivity and real income was inevitable. But how much of the cumulative real wage loss can be attributed to the supply shock?

A Appendix

A.1 Derivations for Section 1

Derivation of equations (10) and (11)

Differentiate both sides of (4) and (8) with respect to time to get

$$\dot{p}_t^* = -\left(\rho + \lambda_p\right)\left(w_t - mpl_t\right) + \left(\rho + \lambda_p\right) p_t^*,$$

and

$$\ddot{p}_t = \lambda_p \left(\dot{p}_t^* - \dot{p}_t\right).$$

Substituting $\dot{p}_t^*$ from the first equation on the right-hand side of the second equation and changing notation for inflation, yields

$$\dot{\pi}_t = \lambda_p \left(-\left(\rho + \lambda_p\right)\left(w_t - p_t - mpl_t\right) + \left(\rho + \lambda_p\right)\left(p_t^* - p_t\right) - \pi_t\right).$$

Using $\lambda_p \left(p_t^* - p_t\right) = \pi_t$ and rearranging gives

$$\dot{\pi}_t = -\lambda_p \left(\rho + \lambda_p\right)\left(w_t - p_t - mpl_t\right) + \rho \pi_t,$$

which corresponds to (10). Equation (11) is derived in a similar way.

A.2 Additional Material for Section 2

Real wage dynamics

Combining equations (10)-(12) gives the second order ordinary differential equation

$$\ddot{\omega}_t = \rho \dot{\omega}_t + \Lambda \left(\omega_t - \tilde{\omega}_t\right), \tag{24}$$

where

$$\Lambda = \Lambda_p + \Lambda_w,$$

and where

$$\tilde{\omega}_t = \alpha mpl_t + (1 - \alpha)\, mrs_t,$$

is the average of the aspirations of workers and firms, weighted by the relative degree of price rigidity

$$\alpha = \frac{\Lambda_p}{\Lambda_p + \Lambda_w}.$$

The next proposition provides the saddle-path stable solution of (24).

Proposition 4. *The real wage satisfies the first order ODE*

$$\dot{\omega}_t = r_1 \omega_t + \Lambda \int_t^{\infty} e^{-r_2(\tau - t)} \tilde{\omega}_\tau d\tau, \tag{25}$$

where $r_1$and r_2 are the roots of the quadratic equation

$$r(r-\rho)=\Lambda,$$

and satisfy $r_1<0<\rho<r_2$. The solution of (25) *is*

$$\omega_t = e^{r_1 t}\omega_0 + \int_0^\infty H_{s,t}\tilde{\omega}_s ds, \tag{26}$$

where $H_{s,t}$ is defined as

$$H_{s,t}=\frac{\Lambda}{r_2-r_1}\left(e^{\min\{r_1(t-s),-r_2(s-t)\}}-e^{r_1 t-r_2 s}\right).$$

Proof. Since $\Lambda>0$ there are two real eigenvalues r_1,r_2 that solve

$$r^2-\rho r-\Lambda=0.$$

The ODE can then be written as

$$(\partial-r_1)(\partial-r_2)\omega_t=-\Lambda\tilde{\omega}_t$$

where ∂ is the time-derivative operator. Integrating forward gives

$$(\partial-r_1)\omega_t=-\frac{1}{\partial-r_2}\Lambda\tilde{\omega}_t=\Lambda\int_t^\infty e^{-r_2(\tau-t)}\tilde{\omega}_\tau d\tau,$$

which gives (25). Integrating backward gives

$$\omega_t=e^{r_1 t}\omega_0+\Lambda\int_0^t e^{r_1(t-s)}\int_s^\infty e^{-r_2(\tau-s)}\tilde{\omega}_\tau d\tau ds.$$

Changing the order of integration, the double integral on the right-hand side becomes

$$\int_0^t\int_0^\tau e^{r_1(t-s)}e^{-r_2(\tau-s)}\tilde{\omega}_\tau ds d\tau+\int_t^\infty\int_0^t e^{r_1(t-s)}e^{-r_2(\tau-s)}\tilde{\omega}_\tau ds d\tau$$

which gives

$$\omega_t=e^{r_1 t}\omega_0+\Lambda\int_0^t\frac{e^{r_1(t-s)}-e^{r_1 t-r_2 s}}{r_2-r_1}\tilde{\omega}_\tau ds+\Lambda\int_t^\infty\frac{e^{-r_2(s-t)}-e^{r_1 t}e^{-r_2 s}}{r_2-r_1}\tilde{\omega}_\tau ds,$$

which can be written compactly as (26). □

The second term in (24) shows that real wage dynamics are driven by a forward-looking expression, capturing the anticipated levels of the average aspiration $\tilde{\omega}_t$.

The first term in (25) shows that the real wage tends to mean revert, since $r_1<0$. The intuition for the mean-reversion is that a higher ω_t increases $\omega_t - mpl_t$, i.e., the distance between the real wage and the firms' aspiration mpl_t, pushing up price inflation. It also

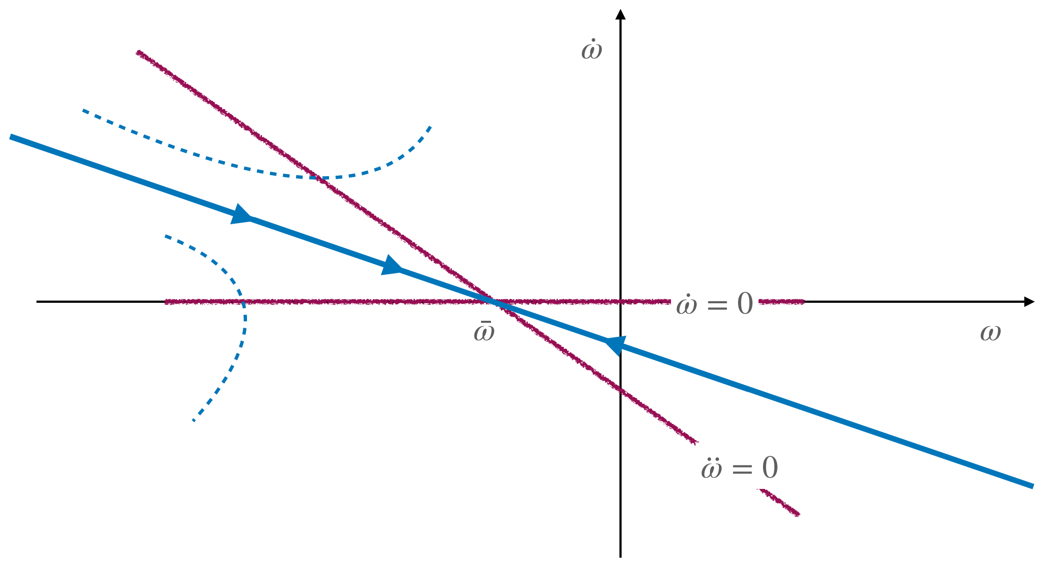

Figure 14: A permanent shock

reduces $mrs_t - \omega_t$, i.e., the distance between the workers' aspiration mrs_t and the real wage, which pushes down wage inflation. Higher price inflation and lower wage inflation reduce the real wage.

Graphical analysis

Suppose the economy is in steady state with all variables equal to 0. At date 0, unexpectedly, there is a one time, permanent reduction in mpl, which goes to $\overline{mpl} < 0$. The level of mrs remains unchanged at 0.

The phase diagram in Figure 14 represents the second order ODE (24). The stationary locus $\dot{\omega} = 0$ coincides with the horizontal axis. The stationary locus $\ddot{\omega} = 0$ is downward sloping. Both are drawn in purple. The saddle path, in blue, is given by the equation

$$\dot{\omega}_t = r_1 \left(\omega_t - \overline{\omega} \right),$$

where

$$\overline{\omega} = \frac{\Lambda_p}{\Lambda_p + \Lambda_w} \overline{mpl}$$

is the constant value of $\tilde{\omega}_t$ after the shock and is also the long-run level of the real wage. The expression for the saddle path comes from 25, using the condition $-r_1 r_2 = \Lambda_p + \Lambda_w$.

The diagram shows that starting at $\omega_0 = 0$, we initially have $\dot{\omega}_t < 0$. Gradually, as the real wage reaches its new long-run level $\overline{\omega}$, this effect goes away.

There are initially two forces pushing up price inflation: a permanently higher conflict component, plus a temporarily positive adjustment component, reflecting the initial fall in the real wage. On the wage inflation side, adjustment inflation has the opposite effect

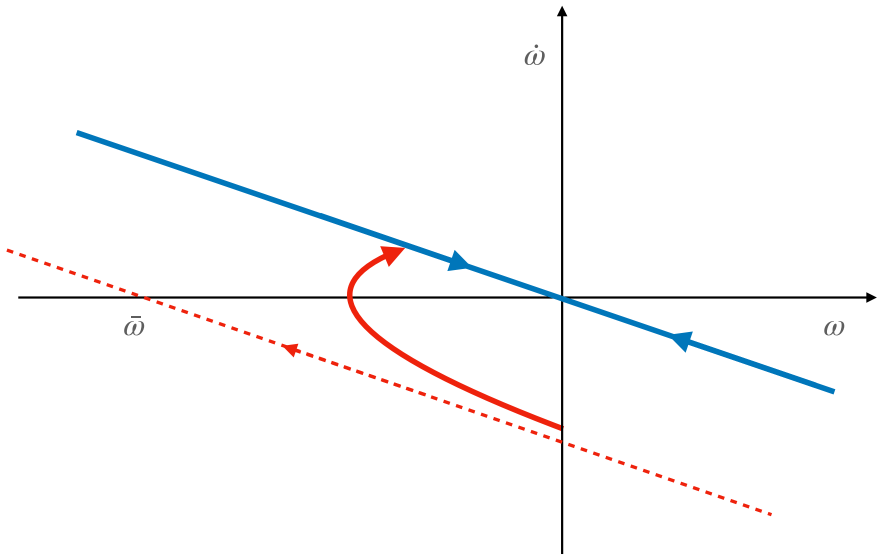

Figure 15: A transitory shock

and initially keeps wage inflation lower than Π^C.[34]

In the long run, the adjustment component goes away, and wage and price inflation converge to the same level, equal to the conflict component.

A similar graphical analysis can be applied to a temporary shock. Consider an economy in steady state with all variables at 0. At $t = 0$, unexpectedly, firms realize that for a finite time interval $[0, T]$ they will face $\overline{mpl} < 0$. At T, mpl goes back to zero. The value of mrs remains at zero throughout. Figure 15 illustrates the phase diagram in this case, with the real wage first moving towards $\overline{\omega}$ and then gradually reverting towards zero.

Proposition 5 in the appendix provides formal derivations for a general class of experiments like the two just analyzed, in which only one side of the labor market is affected, that is, where only mpl or only mrs deviate from zero.

General Result for Unilateral Changes in mrs and mpl

Proposition 5. *Suppose there is no change in $mrs_t = 0$ and the path for mpl_t is negative for all $t \in [0, \infty)$. Then the impact responses at $t = 0$ are*

$$\pi_0 > \pi_0^w > 0.$$

[34]It is easy to prove that despite the presence of the adjustment component, wage inflation is always positive in this experiment. From (11) we get

$$\pi_t^w = \int_t^\infty e^{-\rho(\tau - t)} \left(mrs_\tau - \omega_\tau \right) d\tau,$$

and notice that $mrs_t = 0$ and $\omega_t < 0$ for all $t > 0$, from the phase diagram.

Suppose there is no change in $mpl_t = 0$ and the path for mrs_t is positive for all $t \in [0, \infty)$. Then the impact responses at $t = 0$ are

$$\pi_0^w > \pi_0 > 0.$$

Proof. Recall the expression for real wages from Proposition 4:

$$\omega_t = e^{r_1 t}\omega_0 + \int_0^\infty H_{s,t}\tilde{\omega}_s ds.$$

If $mpl_t < 0$ and $mrs_t = 0$ for all t, it follows that $\tilde{\omega}_s < 0$ for all s on the right-hand side, so $\omega_t < 0$ for all t. From equation (14), wage inflation at date 0 is then

$$\pi_t^w = -\Lambda_w \int_0^\infty e^{-\rho s}\omega_s ds > 0.$$

Moreover, from equation (25) at $t = 0$, we have

$$\dot{\omega}_0 = \Lambda \int_0^\infty e^{-r_2(\tau - t)}\tilde{\omega}_\tau d\tau < 0,$$

which then implies

$$\pi_0 = \pi_0^w - \dot{\omega}_0 > \pi_0^w > 0.$$

Symmetric derivations prove the other case. □

Proof of Proposition 1

The coefficient ψ in the statement of the proposition is defined as follows

$$\psi = \frac{r_2}{r_2 + \delta}\frac{-r_1}{-r_1 + \rho}.$$

We first derive the real wage path using (26) in the proof of Proposition 4. Solving the integrals gives

$$\begin{aligned}\omega_t =& \Lambda \int_0^t e^{r_1(t-s)} \int_s^\infty e^{-r_2(\tau - s)} e^{-\delta\tau} d\tau ds = \Lambda \frac{1}{r_2 + \delta} \int_0^t e^{r_1(t-s) - \delta s} ds = \\ =& \frac{e^{r_1 t} - e^{-\delta t}}{(r_2 + \delta)(r_1 + \delta)} \left(\Lambda_p mpl_0 + \Lambda_w mrs_0\right).\end{aligned}$$

Write price inflation as

$$\pi_t = \int_t^\infty e^{-\rho(\tau - t)} \left(\omega_\tau - mpl_\tau\right) d\tau,$$

substituting ω_t and integrating gives

$$\pi_t = \frac{1}{r_1 + \delta}\frac{1}{r_2 + \delta}\left(\frac{e^{r_1 t}}{\rho - r_1} - \frac{e^{-\delta t}}{\delta + \rho}\right)\left[\Lambda_p mpl_0 + \Lambda_w mrs_0\right] - \frac{e^{-\delta t}}{\rho + \delta} mpl_0.$$

We then get that $\pi_t > 0$ if and only if

$$\frac{1}{r_1+\delta}\frac{1}{r_2+\delta}\left(\frac{e^{r_1 t}}{\rho-r_1}-\frac{e^{-\delta t}}{\delta+\rho}\right)\left[\Lambda_p mpl_0+\Lambda_w mrs_0\right] > \frac{e^{-\delta t}}{\rho+\delta}mpl_0,$$

which can be rewritten using $-r_1 r_2 = \Lambda_p + \Lambda_w$ (from the proof of Proposition (4)), to get

$$\frac{r_2}{r_2+\delta}\frac{-r_1}{r_1+\delta}\left(\frac{e^{r_1 t}}{\rho-r_1}-\frac{e^{-\delta t}}{\delta+\rho}\right)\frac{\Lambda_p mpl_0+\Lambda_w mrs_0}{\Lambda_p+\Lambda_w} > \frac{e^{-\delta t}}{\rho+\delta}mpl_0.$$

$$\frac{1}{r_2+\delta}\frac{1}{\rho-r_1}\left(\Lambda_p mpl_0+\Lambda_w mrs_0\right) > mpl_0.$$

Setting $t=0$ and rearranging gives the condition for $\pi_0 > 0$ in the statement of the proposition, with

Write wage inflation as

$$\pi_t^w = \int_t^\infty e^{-\rho(\tau-t)}\left(mrs_\tau-\omega_\tau\right)d\tau.$$

Similar steps as those above yield the following condition for $\pi_t^w > 0$

$$\frac{r_2}{r_2+\delta}\frac{-r_1}{r_1+\delta}\left(\frac{e^{r_1 t}}{\rho-r_1}-\frac{e^{-\delta t}}{\delta+\rho}\right)\frac{\Lambda_p mpl_0+\Lambda_w mrs_0}{\Lambda_p+\Lambda_w} < \frac{e^{-\delta t}}{\rho+\delta}mrs_0.$$

The last statement follows because

$$\frac{1-\alpha\psi}{(1-\alpha)\psi} > \frac{\alpha\psi}{1-(1-\alpha)\psi}.$$

Deriving Π^{Spiral}

Recall the definition

$$\Pi^{Spiral} = \int_0^\infty \pi_t dt.$$

Using the decomposition of π_t in the text

$$\int_0^\infty \pi_t = \int_0^\infty \Pi_t^{Conflict}dt + \int_0^\infty \dot{\omega}_t dt$$

and since $\omega_t \to 0$ the second term on the right-hand side is zero. Moreover, since

$$\Pi_t^{Conflict} = \frac{\Lambda_p\Lambda_w}{\Lambda_p+\Lambda_w}\frac{1}{\rho+\delta}\left(mrs_0-mpl_0\right)e^{-\delta t}$$

the expression in the text follows.

A.3 Optimal Policy

Quadratic Approximation of the Welfare Function

The welfare of the representative consumer is

$$\int_0^\infty e^{-\rho t}\left(\frac{1}{1-\sigma}Y_t^{1-\sigma}-\frac{1}{1+\eta}N_t^{1+\eta}\right)dt. \tag{27}$$

We will first derive the expression in parenthesis in terms of relative price distortions in prices and wages.

Labor demand for variety j is

$$L_{jt}=\left(\frac{W_{jt}}{W_t}\right)^{-\varepsilon_L}L_t$$

and imposing market clearing in the labor market we obtain

$$N_t=\int_0^1 L_{jt}dj=\Delta_t^w L_t, \tag{28}$$

where

$$\Delta_t^w\equiv\int_0^1\left(\frac{W_{jt}}{W_t}\right)^{-\varepsilon_L}dj, \tag{29}$$

which is a measure of allocative distortions due to wage dispersion.

Demand for variety i is

$$\frac{Y_{it}}{Y_t}=\left(\frac{P_{it}}{P_t}\right)^{-\varepsilon_C}.$$

Since all goods producers use the same inputs ratio X_{it}/L_{it}, we also have

$$Y_{it}=\left[a_L+a_X\left(\frac{X_t}{L_t}\right)^{1-\frac{1}{\varepsilon}}\right]^{\frac{1}{1-\frac{1}{\varepsilon}}}\frac{L_{it}}{L_t}L_t.$$

Combining these conditions we obtain

$$\left(\frac{P_{it}}{P_t}\right)^{-\varepsilon_C}=\frac{Y_{it}}{Y_t}=\left[a_L+a_X\left(\frac{X_t}{L_t}\right)^{1-\frac{1}{\varepsilon}}\right]^{\frac{1}{1-\frac{1}{\varepsilon}}}\frac{L_{it}}{L_t}\frac{L_t}{Y_t}.$$

Integrating both sides, using $\int L_{it}=L_t$ and rearranging, yields

$$Y_t\Delta_t=\left[a_L L_t^{1-\frac{1}{\varepsilon}}+a_X X_t^{1-\frac{1}{\varepsilon}}\right]^{\frac{1}{1-\frac{1}{\varepsilon}}}, \tag{30}$$

where

$$\Delta_t=\int_0^1\left(\frac{P_{it}}{P_t}\right)^{-\varepsilon_C}di \tag{31}$$

is a measure of allocative distortions due to price dispersion.

Substituting (28) and (30) in the welfare function (27), we can then write it as

$$\int_0^\infty e^{-\rho t} \mathcal{U}\left(l_t, x_t, \delta_t, \delta_t^w\right) dt \tag{32}$$

where

$$\mathcal{U}\left(l, x, \delta, \delta^w\right) \equiv U\left(\frac{F\left(L^{ss} e^l, X^{ss} e^x\right)}{e^\delta}\right) - V\left(e^{\delta^w} L^{ss} e^l\right)$$

and

$$U(C) \equiv \frac{1}{1-\sigma} C^{1-\sigma},$$
$$F(L, X) \equiv \left(a_L L^{1-\frac{1}{\varepsilon}} + a_X X^{1-\frac{1}{\varepsilon}}\right)^{\frac{1}{1-\frac{1}{\varepsilon}}},$$
$$V(L) \equiv \frac{1}{1+\eta} L^{1+\eta}.$$

Consider the second-order approximation (first-order in δ and δ^w as they are already second-order variables), where arguments are omitted for brevity on the right-hand side.

$$\mathcal{U}\left(l, x, \delta, \delta^w\right) \approx \frac{1}{2}\left[\mathcal{U}_{ll} l^2 + 2U_{lx} x l + \frac{1}{2} U_{xx} x^2\right] + \mathcal{U}_\delta \delta + \mathcal{U}_{\delta^w} \delta^w.$$

By definition, the natural level of employment satisfies the first-order condition $\mathcal{U}_l = 0$, and, taking a first order approximation to this first-order condition near the steady state, the linearized natural level of employment $l^*(x)$ must satisfy, by the implicit function theorem,

$$\mathcal{U}_{ll} l^*(x) + \mathcal{U}_{lx} x = 0.$$

Therefore, the approximation for $\mathcal{U}$ above can be rewritten as

$$\frac{1}{2}\left[\mathcal{U}_{ll} l^2 + 2U_{ll} l^* l + \frac{1}{2} U_{xx} x^2\right] + \mathcal{U}_\delta \delta + \mathcal{U}_{\delta^w} \delta^w,$$

and ignoring constant terms not controlled by the planner, as

$$\frac{1}{2} \mathcal{U}_{ll}\left(l - l^*\right) + \mathcal{U}_\delta \delta + \mathcal{U}_{\delta^w} \delta^w.$$

To derive the values of the terms $\mathcal{U}_{ll}, \mathcal{U}_\delta, \mathcal{U}_{\delta^w}$, proceed as follows. Differentiating $\mathcal{U}$ gives

$$\mathcal{U}_l = U'\left(\frac{F(L, X)}{\Delta}\right) \frac{F_L(L, X)}{\Delta} L - V'(L) \Delta^w L,$$

$$\mathcal{U}_{ll} = U''\left(\frac{F(L,X)}{\Delta}\right)\left(\frac{F_L(L,X)}{\Delta}L\right)^2 + U'\left(\frac{F(L,X)}{\Delta}\right)\frac{F_{LL}(L,X)}{\Delta}L^2 +$$
$$+ U'\left(\frac{F(L,X)}{\Delta}\right)\frac{F_L(L,X)}{\Delta}L - \Delta^w V''(L)L^2 - \Delta^w V'(L)L$$

$$\mathcal{U}_\delta = -U'\left(\frac{F(L,X)}{\Delta}\right)\frac{F(L,X)}{\Delta^2}\Delta,$$
$$\mathcal{U}_{\delta^w} = -V'(L)\Delta^w L.$$

Computing these expressions at frictionless steady state gives $\mathcal{U}_l = 0$ and

$$\mathcal{U}_{ll} = U''(Y)(F_L L)^2 + U' F_{LL} L^2 - V'' L^2,$$
$$\mathcal{U}_\delta = -U'(Y)Y,$$
$$\mathcal{U}_{\delta^w} = -V'(L)L.$$

Rearranging the expression for $\mathcal{U}_{ll}$

$$\mathcal{U}_{ll} = U' F_L L\left[\frac{U''}{U'}Y\frac{F_L L}{F} + \frac{F_{LL}}{F_L}L - \frac{V''}{V'}L\right] = U' F_L L\left[-\sigma s_L + \frac{F_{LL}}{F_L}L - \eta\right] =$$
$$= -U' F_L L\left[\sigma s_L + \frac{s_X}{\epsilon} + \eta\right].$$

The second order approximation of $\mathcal{U}$ is then

$$-\frac{1}{2}U' Y s_L\left(\sigma s_L + \frac{s_X}{\epsilon} + \eta\right)(l - l^*)^2 - U' Y\delta - V'(L)L\delta^w.$$

In steady state, we have $V'(L) = F_L U'(Y)$, or, multiplying both sides by L and using the definition of s_L,

$$V'(L)L = \frac{F_L L}{Y}U'(Y)Y = s_L U'(Y)Y.$$

Finally, using

$$y = s_L l + s_X x, \quad y^* = s_L l^* + s_X x,$$

we have

$$y - y^* = s_L(l - l^*).$$

Therefore, removing the multiplicative constant $U'(Y)Y$, the second order approximation of the social welfare function takes the form

$$\int_0^\infty e^{-\rho t}\left[-\frac{1}{2}\left(\sigma + \frac{1}{\epsilon}\frac{s_X}{s_L} + \frac{\eta}{s_L}\right)(y_t - y_t^*)^2 - \delta_t - s_L\delta_t^w.\right]$$

Our last step is to express the last two terms, in δ and δ^w, in terms of inflation rates.

Differentiating (31) with respect to time gives

$$\frac{\dot{\Delta}_t^p}{\Delta_t^p} = \varepsilon_C \pi_t + \lambda_p \left[\frac{(P_t^*/P_t)^{-\varepsilon_C}}{\Delta_t^p} - 1 \right]. \tag{33}$$

The exact relation between P_t^*/P_t and price inflation is

$$\pi_t = \frac{\dot{P}_t}{P_t} = \frac{\lambda_p}{1-\varepsilon_C} \left(\left(\frac{P_t^*}{P_t} \right)^{1-\varepsilon_C} - 1 \right),$$

which can be rewritten as

$$\frac{P_t^*}{P_t} = \left(1 + \frac{1-\varepsilon_C}{\lambda_p} \pi_t \right)^{\frac{1}{1-\varepsilon_C}}.$$

Substituting in (33) and using the notation $\delta_t^p = \log \Delta_t^p$ gives

$$\dot{\delta}_t^p = \varepsilon_C \pi_t + \lambda_p \left[e^{-\delta_t^p} \left(1 + \frac{1-\varepsilon_C}{\lambda_p} \pi_t \right)^{-\frac{\varepsilon_C}{1-\varepsilon_C}} - 1 \right].$$

A second order Taylor approximation of the right-hand side at $\pi_t = 0$ and $\Delta^p = 1$ yields

$$\dot{\delta}_t^p = -\lambda_p \delta_t^p + \frac{1}{2} \frac{\varepsilon_C}{\lambda_p} \pi_t^2,$$

where we approximate to the first order in δ_t and to the second order in π_t. Solving this equation backward in time starting at $\delta_0 = 0$, we obtain

$$\delta_t^p = \frac{1}{2} \int_0^t \frac{\varepsilon_C}{\lambda_p} e^{-\lambda_p(t-s)} \pi_s^2 ds.$$

Computing the present value of distortions gives

$$\int_0^\infty e^{-\rho t} \delta_t^p dt = \frac{1}{2} \frac{\varepsilon_C}{\lambda_p (\lambda_p + \rho)} \int_0^\infty e^{-\rho t} \pi_t^2 dt.$$

Analogous steps for wage distortions gives

$$\int_0^\infty e^{-\rho t} \delta_t^w dt = \frac{1}{2} \frac{\varepsilon_L}{\lambda_w (\lambda_w + \rho)} \int_0^\infty e^{-\rho t} (\pi_t^w)^2 dt.$$

In conclusion, the social welfare function is approximated to the second order by the expression

$$-\frac{1}{2} \int_0^\infty e^{-\rho t} \left[\left(\sigma + \frac{1}{\epsilon} \frac{s_X}{s_L} + \frac{\eta}{s_L} \right) (y_t - y_t^*)^2 + \frac{\varepsilon_C}{\lambda_p (\lambda_p + \rho)} \pi_t^2 - \frac{s_L \varepsilon_L}{\lambda_w (\lambda_w + \rho)} \pi_t^w \right] dt,$$

which corresponds to the quadratic objective in the text with coefficients

$$\Phi_y = \sigma + \frac{1}{\epsilon}\frac{s_X}{s_L} + \frac{\eta}{s_L},$$
$$\Phi_p = \frac{\varepsilon_C}{\lambda_p\left(\lambda_p + \rho\right)},$$
$$\Phi_w = \frac{s_L \varepsilon_L}{\lambda_w\left(\lambda_w + \rho\right)}.$$

Optimal Policy Problem

Define the coefficients

$$\xi_p = \frac{1}{\epsilon}\frac{s_X}{s_L},$$
$$\xi_w = \frac{\sigma s_L + \eta}{s_L}.$$

and let hats denote deviations from first best allocations. The optimal policy problem can then be written compactly as follows

$$\min \int_0^\infty e^{-\rho t}\frac{1}{2}\left[\Phi_y \hat{y}_t^2 + \Phi_p \pi_t^2 + \Phi_w \left(\pi_t^w\right)^2\right] dt$$

subject to

$$\rho\pi_t = \Lambda_p\left(\hat{\omega}_t + \xi_p \hat{y}_t\right) + \dot{\pi}_t, \tag{34}$$
$$\rho\pi_t^w = \Lambda_w\left(\xi_w \hat{y}_t - \hat{\omega}_t\right) + \dot{\pi}_t^w, \tag{35}$$
$$\dot{\hat{\omega}}_t = \pi_t^w - \pi_t - \dot{\omega}_t^*, \tag{36}$$

taking ω_0 as given. Form the Hamiltonian

$$\begin{aligned} -\frac{1}{2}\left[\Phi_y \hat{y}_t^2 + \Phi_p \pi_t^2 + \Phi_w \left(\pi_t^w\right)^2\right] + \\ -\Lambda_p\left(\hat{\omega}_t + \xi_p \hat{y}_t\right)\nu_t + \pi_t \dot{\nu}_t + \\ -\Lambda_w\left(\xi_w \hat{y}_t - \hat{\omega}_t\right)\nu_t^w + \pi_t^w \dot{\nu}_t^w + \\ \left(\pi_t^w - \pi_t - \dot{\omega}_t^*\right)\mu_t - \rho\omega_t\mu_t + \omega_t\dot{\mu}_t, \end{aligned}$$

where ν_t, ν_t^w, μ_t denote the Lagrange multipliers on the three constraints, and derive the following first-order conditions for $\hat{y}_t, \hat{\omega}_t, \pi_t, \pi_t^w$:

$$-\Phi_y \hat{y}_t - \Lambda_p \xi_p \nu_t - \Lambda_w \xi_w \nu_t^w = 0, \tag{37}$$
$$-\Lambda_p \nu_t + \Lambda_w \nu_t^w - \rho\mu_t + \dot{\mu}_t = 0, \tag{38}$$
$$-\Phi_p \pi_t + \dot{\nu}_t - \mu_t = 0, \tag{39}$$
$$-\Phi_w \pi_t^w + \dot{\nu}_t^w + \mu_t = 0. \tag{40}$$

Given that the initial inflation rates π_0 and π_0^w are free variables we have

$$\nu_0 = \nu_0^w = 0,$$

which, using (37) and (38), implies

$$\hat{y}_0 = 0,$$

and

$$\dot{\mu}_0 = \rho \mu_0. \tag{41}$$

We will use these two as initial conditions for $\hat{y}_0$ and μ_0 and derive the paths of these two variables from the following two ODEs, that come from differentiating (37) and (38) with respect to time:

$$\Phi_y \dot{\hat{y}}_t + \Lambda_p \xi_p \dot{\nu}_t + \Lambda_w \xi_w \dot{\nu}_t^w = 0,$$
$$-\Lambda_p \dot{\nu}_t + \Lambda_w \dot{\nu}_t^w - \rho \dot{\mu}_t + \ddot{\mu}_t = 0.$$

Using (39) and (40) to substitute for $\dot{\nu}_t$ and $\dot{\nu}_t^w$, the ODEs above become:

$$\begin{aligned} \Phi_y \dot{\hat{y}}_t + \Lambda_p \xi_p \left(\Phi_p \pi_t + \mu_t \right) + \Lambda_w \xi_w \left(\Phi_w \pi_t^w - \mu_t \right) = 0, \\ \ddot{\mu}_t - \rho \dot{\mu}_t - \left(\Lambda_p + \Lambda_w \right) \mu_t - \Lambda_p \Phi_p \pi_t + \Lambda_w \Phi_w \pi_t^w = 0. \end{aligned} \tag{42}$$

A useful observation here is that the ODE for the Lagrange multiplier μ_t is a second order ODE with exactly the same structure as the ODE for the real wage, analyzed in Proposition 4. Therefore, by analogy with (26), the solution can be written as follows

$$\mu_t = e^{r_1 t} \mu_0 + \int_0^\infty H_{s,t} \left(\Lambda_w \Phi_w \pi_t^w - \Lambda_p \Phi_p \pi_t \right) ds. \tag{43}$$

To derive μ_0, we use the analog of equation 25 in Proposition 4, evaluated at time $t = 0$

$$\dot{\mu}_0 = r_1 \mu_0 + \int_0^\infty e^{-r_2 t} \left(\Lambda_w \Phi_w \pi_t^w - \Lambda_p \Phi_p \pi_t \right) dt$$

and the initial condition (41), to obtain

$$\mu_0 = \frac{1}{\rho - r_1} \int_0^\infty e^{-r_2 t} \left(\Lambda_w \Phi_w \pi_t^w - \Lambda_p \Phi_p \pi_t \right) dt. \tag{44}$$

In summary, an optimal policy is found finding a pair of paths $\{\mu_t, \hat{y}_t, \pi_t, \pi_t^w, \omega_t\}_{t=0}^\infty$ that satisfy $\hat{y}_0 = 0$, the optimality conditions (42), (43), and (44) and the equilibrium conditions (34)-(35).

Algorithm

This algorithm solves for optimal policy exploiting a simple finite difference method to express all differential and integral equations as linear equations.

Choose a vector $\mathbf{t} = (t_1, t_2, ..., t_K)$ of K equi-spaced dates in the interval $[0, T]$ for some horizon T. Notice that

$$\Lambda_p mpl_t + \Lambda_w mrs_t = \left(\Lambda_p + \Lambda_w\right)\omega_t^* + \left(\Lambda_w \xi_w - \Lambda_p \xi_p\right)\hat{y}_t.$$

Real wages are then obtained from the following matrix version of (26):

$$\boldsymbol{\omega} = \omega_0 e^{r_1 \mathbf{t}} + \mathbf{H}\left(\left(\Lambda_p + \Lambda_w\right)\boldsymbol{\omega}^* + \left(\Lambda_w \xi_w - \Lambda_p \xi_p\right)\hat{\mathbf{y}}\right),$$

where boldface variables represent vectors of the corresponding variable at the times $\mathbf{t}$ and $\mathbf{H}$ is a matrix with elements $H_{t_i,t_j}\Delta t$ (using the expression for $H_{s,t}$ in Proposition 4). The inflation equations can also be written in matrix form

$$\boldsymbol{\pi} = \Lambda_p \mathbf{A}\left(\boldsymbol{\omega} - \boldsymbol{\omega}^* + \xi_p \hat{\mathbf{y}}\right),$$
$$\boldsymbol{\pi}^w = \Lambda_w \mathbf{A}\left[\xi_w \hat{\mathbf{y}} - (\boldsymbol{\omega} - \boldsymbol{\omega}^*)\right],$$

where the matrix $\mathbf{A}$ has elements A_{t_i,t_j} with

$$A_{t,s} = e^{-\rho(s-t)}\Delta t \text{ if } s \geq t,$$

and

$$A_{t,s} = 0 \text{ if } s < t.$$

Substituting the solution for $\boldsymbol{\omega}$ in the inflation equations above gives, after some rearranging

$$\boldsymbol{\pi} = \omega_0 \Lambda_p \mathbf{A} e^{r_1 \mathbf{t}} + \Lambda_p \mathbf{A}\left[\xi_p \mathbf{I} - \left(\Lambda_p \xi_p - \Lambda_w \xi_w\right)\mathbf{H}\right]\hat{\mathbf{y}} + \Lambda_p \mathbf{A}\left[\left(\Lambda_p + \Lambda_w\right)\mathbf{H} - \mathbf{I}\right]\boldsymbol{\omega}^* \quad (45)$$

$$\boldsymbol{\pi}^w = -\omega_0 \Lambda_w \mathbf{A} e^{r_1 \mathbf{t}} + \Lambda_w \mathbf{A}\left[\xi_w \mathbf{I} + \left(\Lambda_p \xi_p - \Lambda_w \xi_w\right)\mathbf{H}\right]\hat{\mathbf{y}} - \Lambda_w \mathbf{A}\left[\left(\Lambda_p + \Lambda_w\right)\mathbf{H} - \mathbf{I}\right]\boldsymbol{\omega}^* \quad (46)$$

Translating in matrix forms condition (43) we obtain

$$\boldsymbol{\mu} = \mu_0 e^{r_1 \mathbf{t}} + \mathbf{H}\left(\Lambda_w \Phi_w \boldsymbol{\pi}^w - \Lambda_p \Phi_p \boldsymbol{\pi}\right)$$

and substituting for μ_0 using the matrix version of (44), we can write

$$\boldsymbol{\mu} = \mathbf{M}_w \boldsymbol{\pi}^w - \mathbf{M}_p \boldsymbol{\pi} \quad (47)$$

where

$$\mathbf{M}_p = \Lambda_p \Phi_p \mathbf{M},$$
$$\mathbf{M}_w = \Lambda_w \Phi_w \mathbf{M},$$

and

$$\mathbf{M} = \frac{1}{\rho - r_1} \begin{bmatrix} e^{r_1 t_1} \\ e^{r_1 t_2} \\ e^{r_1 t_3} \\ \dots \\ e^{r_1 t_K} \end{bmatrix} \begin{bmatrix} e^{-r_2 t_1} & e^{-r_2 t_2} & e^{-r_2 t_3} & \dots & e^{-r_2 t_K} \end{bmatrix} \Delta t + \mathbf{H}.$$

Finally the ODE for $\hat{y}_t$ in equation (42) can be written in matrix form as

$$\Phi_y \mathbf{D} \cdot \hat{\mathbf{y}} + \Lambda_p \xi_p \left(\Phi_p \boldsymbol{\pi} + \boldsymbol{\mu} \right) + \Lambda_w \xi_w \left(\Phi_w \boldsymbol{\pi}^w - \boldsymbol{\mu} \right) = 0$$

where

$$\mathbf{D} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ & \dots & & \dots \\ & & & \end{bmatrix} \frac{1}{\Delta t},$$

(the form of the first row imposes the condition $y_0 = 0$). Substituting for $\boldsymbol{\mu}$, from (47), we can rewrite the ODE for $\hat{y}_t$ as:

$$\Phi_y \mathbf{D} \cdot \hat{\mathbf{y}} + \mathbf{G}_p \boldsymbol{\pi} + \mathbf{G}_w \boldsymbol{\pi}^w = 0,$$

where

$$\mathbf{G}_p = \Lambda_p \xi_p \Phi_p \mathbf{I} - \left(\Lambda_p \xi_p - \Lambda_w \xi_w \right) \mathbf{M}_p,$$
$$\mathbf{G}_w = \Lambda_w \xi_w \Phi_w \mathbf{I} + \left(\Lambda_p \xi_p - \Lambda_w \xi_w \right) \mathbf{M}_w.$$

Substituting for $\boldsymbol{\pi}$ and $\boldsymbol{\pi}^w$, from (45)-(46), we obtain a linear equation in $\hat{\mathbf{y}}$ and exogenous variables, which can be written as

$$\mathbf{C}\hat{\mathbf{y}} = \mathbf{b},$$

where

$$\mathbf{C} = \Phi_y \mathbf{D} + \mathbf{G}_p \Lambda_p \mathbf{A} \left[\xi_p \mathbf{I} - \left(\Lambda_p \xi_p - \Lambda_w \xi_w \right) \mathbf{H} \right] + \mathbf{G}_w \Lambda_w \mathbf{A} \left[\xi_w \mathbf{I} + \left(\Lambda_p \xi_p - \Lambda_w \xi_w \right) \mathbf{H} \right],$$

and

$$\mathbf{b} = \left(\Lambda_w \mathbf{G}_w - \Lambda_p \mathbf{G}_p \right) \left\{ \omega_0 \mathbf{A} e^{r_1 \mathbf{t}} + \mathbf{A} \left[\left(\Lambda_p + \Lambda_w \right) \mathbf{H} - \mathbf{I} \right] \boldsymbol{\omega}^* \right\}.$$