

Savings Lost: The Hight Cost of Early Refinancing¹

Summary

We conduct a “counterfactual” analysis of select taxable advance refundings which were executed between 2018 and 2021.^{2 3} Instead of advance refunding their not-yet-callable tax-exempt bonds with taxable bonds, municipal issuers could have waited until the call date, and then refunded these bonds with tax-exempt bonds. We compare the actual interest cost savings from taxable advance refundings to the counterfactual interest costs savings from refunding with tax-exempt bonds on the call date. Although the savings from taxable advance refundings were significant, waiting until the call date would have been far more beneficial. According to our analysis, waiting would have provided almost 60% more savings. Based on our results, we estimate that in aggregate taxable advance refundings have cost taxpayers billions of dollars.

The potentially wasteful practice of municipal issuers is to focus on savings, without consideration to the value of the forfeited refunding option. Prudent debt managers should quantify the value of this option, and compare it to the saving. The ratio of the savings to the option value, the so-called refunding efficiency, provides guidance (Kalotay, 2007; Kalotay, 2011). This guidance argues that refunding is not advisable unless the efficiency is least 90%. The efficiency of the typical taxable refunding was barely 70%, indicating that the *expected* waste was 30% of the option value (Kalotay, 2021). Because interest rates subsequent to the taxable advance refundings declined, the actual waste turned out to be much larger than expected.

We believe that counterfactual analyses, such as the ones conducted here, would lead to more disciplined debt management practices in general, and to more efficient refunding decisions in particular.

Background on the municipal landscape

In recent years, the municipal market has been dominated by tax-exempt 5% bonds callable at par in Year 10. Because tax-exempt interest rates have been significantly below 5%, 5% bonds are sold at substantial premiums over par. The above-par price appeals to institutional investors, who understandably want to avoid the underperformance of bonds purchased near par in the event rates rise, due to the *de minimis* tax treatment (Kalotay and Davidson, 2021).

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² In the parlance of the municipal market, a “refunding” refers to a refinancing of a government or tax-exempt borrower’s outstanding debt

³ We use the term “counterfactual” to describe what did not “factually” happen with the refinancing but could have happened (i.e., the “counterfactual”) if the municipal issuer waited until the call date to refund the outstanding bonds on a tax-exempt basis.

Because in recent times 5% has been far the borrowing cost of investment-grade issuers, the par call in Year 10 has virtually guaranteed that 5% bonds would be called and refunded even if rates increased, as long as they stayed below 5%. This is evident by observing that there are virtually no outstanding 5% bonds more than 10 years old.

Until 2017, tax-exempt municipal bonds were eligible for advance refunding with tax-exempt bonds. In a tax-exempt advance refunding transaction, the municipality would issue new tax-exempt bonds, use the proceeds to purchase an escrow to defease the outstanding bonds to the call date, and then call the outstanding bonds. Due to the above-market 5% coupon, such advance refundings generated significant savings (Kalotay, 2012). We note that advance refunding resulted in the proliferation of tax-exempt bonds, because the refunded bonds remained outstanding until their call date.

Elimination of tax-exempt advance refundings

- The Tax Cuts and Jobs Act of 2017 signed into law on December 22, 2017 prohibited the issuance of tax-exempt advance refunding bonds (Kalotay, 2018).⁴ A possible alternative has been to advance refund with taxable bonds, instead of tax-exempt bonds. Taxable bonds are not prohibited from advance refunding tax-exempt bonds. Because interest rates remained relatively low from 2018 until 2022, investment-grade municipal borrowers were able to issue taxable bonds well below the 5% coupon rate of their outstanding tax-exempt bonds and report large savings.

The municipality's taxable rate is higher than its tax-exempt rate, usually by 50 to 100 basis points, depending on maturity. Consequently, if rates remained near their level at the time of the taxable transaction, refunding with tax-exempt bonds at the call date would result in significantly greater savings. The likely rationale for advance refunding with taxable bonds was the concern that by the time the bonds became callable interest rates could substantially rise, thus reducing the potential savings from refunding.

Overview of taxable advance refundings

Between 2018 and 2021 well over 200 tax-exempt issues were advance refunded with taxable bonds. The typical refunding occurred roughly two years prior to the call date. The taxable refunding bonds were sold near par, and some of them are callable in ten years (with some bonds sold with a make-whole call). During this period Treasury rates were exceptionally low, resulting in very low escrow yield, typically well below 1%. Low yields increased the size of the escrow, thus reducing the savings. (Kalotay, 2019)

⁴ Municipal issuers could still refund with tax-exempt bonds on a "current refunding" basis. A current refunding occurs when the tax-exempt refunding bonds are sold 90 days or less from the call date of the refunded bonds.

Table 1 displays the ten largest taxable advance refunding transactions. Looking at this table, one gets a sense of the relative import of this financial strategy in realizing savings by observing the large size of the transactions (\$539 million to \$1.2 billion), usage across multiple states (California, Arizona, Ohio, New York, Texas, Florida and Massachusetts) and diversity in the types of borrowers (state, city, county, toll road, community college, public utility, and building authorities both school and dormitory).

Methodology

Our goal is to compare the savings from the actual taxable advance refunding transaction to the counterfactual savings from refunding with tax-exempt bonds at the call date. Because the savings are quantified in present value terms they depend on the assumptions, which are listed below. In addition to the dollar amounts, we are also interested in their relative size, as a percentage. Our analytical approach follows.

Assumptions on present value calculations

- The common present value date is the date of the taxable refunding
- Municipal issuers should use their taxable cost of borrowing to determine the present value of their debt service, whether the bonds being valued are taxable or tax-exempt (Kalotay and Tuckman, 1999). Therefore, we use the true interest cost (TIC) of the taxable advance refunding bonds for discounting purposes. We parenthetically observe that TIC is actually a ‘callable’ rate, because the portfolio on which it is based may include callable bonds. Option-adjusted TIC (IPREO Newsletter, 2012) would be preferable to conventional TIC.

Savings from taxable advance refunding

The size of the transaction can be ambiguous, because some of the funds deployed in the taxable transaction may have come from sources other than the taxable issue. For example, in the MSBA transaction featured below, the issuer deployed the debt service reserve fund of the refunded bonds. In order for an ‘apples to apples’ comparison, we exclude such funds from the analysis.

The proceeds of the taxable issue is known, and this allows us to determine how much of the outstanding tax-exempt issue was refunded with taxable bonds. This calculation is based on the size of the escrow, which is reported in the official statement. We determine the percentage of the escrow which was funded with taxable bonds, and then scale down the outstanding tax-exempt bonds to determine the amount that was refunded with the taxable issue.

Based on the amount that was refunded with the taxable issue, we determine the cashflow saving from the taxable advance refunding in the usual manner. Calculate both the present value of leaving these bonds outstanding until maturity, and the present value of the taxable advance refunding bonds. The present value savings from taxable advance refunding is the difference.

If some of the bonds in the taxable issue are callable, we estimate the value of the call option (“Option Value Acquired”) and increase the savings by that amount. Option Value Acquired assumes the bonds may be refunded with taxable bonds, with the options valued at 20% interest rate volatility. The possibility of refunding the taxable bonds with tax-exempt bonds could be considered in a future study.

Savings from counterfactual tax-exempt current refunding

What if instead of advance refunding with taxable bonds, the outstanding issue was left outstanding until the call date, and then refunded with tax-exempt bonds? The resulting debt service has two components: the known payments on the refunded bonds until the call date, and the payments on the tax-exempt refunding bonds after the call date.

The issuer’s tax-exempt borrowing rates (yield curve) as of the pricing date (usually a couple weeks before the closing date which is also the call date of the refunded bonds) can be estimated from the MSRB’s EMMA database. This yield curve is based on the yields to 5% bonds callable at par in Year 10. If we refund with 5% bonds we also have to account for their option values – as discussed earlier, 5% bonds are virtually certain to be refunded at the end of Year 10.

Instead of refunding with 5% callable bonds, we have taken a conceptually simpler approach, one which provides an apples-to-apples comparison with non-callable taxable bonds. Our approach eliminates the need to include option values in the calculation of savings, and provides identical savings. The key is to estimate the par coupon optionless bonds of the relevant maturities. This is accomplished by converting the 5% callable curve from EMMA into a par AAA optionless curve, at the specified interest volatility. Given the exceptionally low interest rates during this period, 30% interest rate volatility was a reasonable choice. Kalotay (2017) describes the process of stripping out the call option to create the optionless bond yield curve. We then adjust this AAA optionless curve to the appropriate credit rating yield curve for each counterfactual issue based on MMD yield spreads (AAA, AA and A) by maturity on the hypothetical pricing date of the counterfactual bonds. In constructing this curve, we use the credit rating of the issuer at the time of the taxable advance refunding.

In summary, we structure par optionless tax-exempt bonds so that their maturities and amounts resemble those of the outstanding issue. We combine the resulting cashflows with the flows of the outstanding issue prior to the call date, and determine the present value of the combined flows. The estimated savings from the counterfactual strategy is the difference between the present value of the refunded bonds debt service to maturity and the present value of the counterfactual tax-exempt refunding bonds to maturity (i.e., the ‘waiting to the call date’ strategy). In order to adjust for transaction costs, we reduce the present value savings by 0.50%.

Case Study: Massachusetts School Building Authority (MSBA)

On November 20, 2019, the MSBA refunded its outstanding 2011 Series B Bonds (the “2011B Bonds”), callable at par on October 15, 2021. Table 2 details the portfolio of refunded 2011B Bonds. The face amount of the 2011B Bonds was \$747.69MM; the coupons of these bonds varied between 5% and 5.25%. According to the official statement for the 2019 taxable advance refunding bonds (the “2019 Bonds”), the cost of the escrow required to defease the 2011B Bonds to the call date was \$798.89 million.

MSBA refunded its 2011B bonds with a \$715.42 million principal of the 2019 Bonds. The 2019 Bonds carried AA+/AA3/AA ratings. \$2.65 million in cost of issuance expense reduced the amount available for the escrow to \$712.77 million; the remainder needed for the \$798.89 million escrow was funded by other means, mainly liquidation of the 2011 B bonds debt service reserve fund. Thus, as shown in Table 3, the 2019 Bonds provided $712.77/798.89 = 89.22\%$ of the escrow. Applying 89.22% to the \$747.69 million principal amount of the 2011B Bonds reveals that the proceeds of the taxable issue were sufficient to refund \$667.09 million principal amount of the 2011B Bonds. Accordingly, the savings calculated below are based on \$667.09 million principal amount of the 2011B Bonds.

In order to calculate the present value savings, we first determined that the TIC of the 2019 taxable issue was 3.205%, based on the actual maturity dates, par amounts and coupon rates of the 2019 Bonds as shown in Table 4. Based on this discount rate, the present value of the outstanding 2011B Bonds was \$819.75 million, and the present value of the 2019 Bonds was \$715.42 million. Thus the cashflow savings, on a present value basis, amounted to *\$819.75 million - \$715.42 million, or \$104.33 million.*

The 2019 Bonds were sold with a ten-year par call date of October 15, 2029. The Option Value Acquired of the 2019 Bonds was \$17.30 million, and we increased the savings attributable to the taxable advance refunding by this option value, resulting in total savings *\$104.33 million + \$17.30 million = \$121.63 million.*

As shown in Graph 1, the 10-Year AAA yield at the time of the refunding in 2019 was roughly 1.52%. Contrary to MSBA’s expectations, rates subsequently declined (except for a brief period of time in March and April 2020 as a result of the flight to quality effect at the onset of the COVID-19 pandemic), and by the call date on 11/1/ 2021 the 10-year AAA muni yield fell to 1.22%, about 30 basis points lower than at the time of the advance refunding. Based on the tax-exempt yield curve prevailing as of October 1, 2021, we estimated how much MSBA would have saved by waiting until the call date and then refunding \$667.09 million principal amount of the 2011B Bonds with optionless tax-exempt bonds (the “Counterfactual 2021 Bonds”). Table 5 details the actual 5% tax-exempt 10-year callable yields and the corresponding optionless par bond yields based on 30% interest rate volatility, as of October 1, 2021. The optionless par bond

yields include an adjustment for the credit spread between AAA and AA bonds in years 1 through 20 that ranged from 7 to 20 basis points. In order to cover the issuance expense, we grossed up the refunding issue by \$3.35 million (0.50% of the par amount), to \$670.44 million. Table 6 details the maturity dates, par amounts, yields and coupon rates for the Counterfactual 2021 Bonds. Based on this counterfactual portfolio, we determined that *refunding with tax-exempt bonds at the call date would have resulted in present value savings of \$221.63 million.*

To recap, MSBA actually saved \$121.63 million by advance refunding the 2011B Bonds with its taxable 2019 Bonds. By waiting until the call date in 2021 and then refunding with tax-exempt bonds on a current refunding basis, MSBA would have saved \$221.63 million. The \$100.00 million ‘savings lost’ (\$221.63 - \$121.63) from advance refunding with taxable bonds will be borne by the Massachusetts taxpayers.

It is instructive to consider the ratio of the savings through what we are calling the “proficiency ratio”, which is the percent of actual savings to the counterfactual savings. The proficiency ratio essentially determines how proficient the issuer was in capturing savings that would have been available by waiting to refund at the call date. A higher ratio is preferable, as it indicates a smaller loss in savings relative to delaying the refinancing decision to the call date. The break-even proficiency ratio is 100%. The MSBA proficiency ratio was 54.88% (i.e., \$121.63 million / \$221.63 million), indicating that *MSBA captured less than 55% of the savings available if they waited to the call date.* While the reported present value savings may depend on debatable assumptions pertaining to discounting and option valuation, these ratios are robust, and provide an excellent indication of how well, or how poorly, the issuer’s debt is managed.

Actual Savings for Selected Taxable Advance Refunding Transactions

We applied the approach used in the MSBA case study to 13 more transactions to explore the scope of ‘savings lost’ across other government issuers. We initially focused solely on the top ten largest transactions. However, because four of these refundings had call dates later in 2023, at the time we could not construct the counterfactual refundings for those financings. That left us with six of the top ten taxable advance refundings in our sample. We added eight other clear-cut taxable advance refundings, avoiding complex transactions such as those with several call dates, and issues sold for multiple purposes.

Table 7 details the 14 taxable advance refundings in our sample. It consists of a variety of government issuers across geography, type (city, state, special purpose), credit ratings (AAA, AA and A), size, and timing (issuances in 2018, 2019, and 2020; refunded bond call dates in 2019, 2020, 2021 and 2022). The selection consists of \$5.69 billion in par amount of taxable advance refunding bonds and \$5.36 billion in tax-exempt refunded bonds. The TICs on these financings ranged from 2.46% to 4.10%, with an unweighted average TIC of 3.13%. The aggregate cashflow present value savings on these transactions was \$845.00 million and the

savings from the Option Value Acquired was \$143.37 million, for total option-adjusted present value savings of \$988.37 million. This represents 18.46% savings as a percent of refunded bonds. These savings levels are much higher than the static refunding heuristics employed by many governments, such as 3% or 5% minimum thresholds. While the actual taxable advance refunding savings level were significant on an absolute basis, the focus of this paper is how these savings compare relative to what these governments would have saved if they waited a couple years to refinance on a tax-exempt basis. The next section of this paper focuses on that alternative.

Counterfactual Savings from Tax-Exempt Current Refunding

Table 8 details the results of the counterfactual analysis for the 14 transactions in our sample. The par amount of the counterfactual tax-exempt refunding bonds was \$5.382 billion. As a point of comparison with the 3.133% TIC of the taxable transactions, the unweighted average TIC of the counterfactual transactions was 1.978%. The aggregate cash flow present value savings on these counterfactual transactions was \$1.566 billion, which represents 29.23% savings as a percent of refunded bonds, which should be compared to the 18.46% actual savings from the taxable refundings.

Table 9 compares the results of the actual taxable advance refunding bonds and the counterfactual tax-exempt current refunding bonds. The counterfactual savings (\$1.566 billion) was \$577 million greater than the actual option-adjusted present value savings from the taxable advance refundings (\$988.37 million). The proficiency ratio of every taxable advance refunding transaction was below 100%, indicating that considerably larger savings would have been achieved if these governments waited until the call date to refinance. In fact, most transactions performed very poorly; the arithmetic average proficiency ratio of the transactions was 61.25%. Weighted by size of the counterfactual transactions, the proficiency ratio was 63.13%. (i.e., \$988.37 million in actual savings / \$1.566 billion in counterfactual savings). So, in aggregate, these government issuers realized only 63% of the savings that they would have received if they waited until the call date to refund the bonds. *In summary, the 'savings lost' resulting from these taxable refinancings amount to \$577 million.*

Conclusion

We have considered 14 taxable advance refundings of tax-exempt bonds between 2018 and 2020. Although these transactions resulted in considerable savings, option-based analysis at the time of the transaction suggested that they were premature. The typical refunding efficiency was roughly 70%, indicating that the savings captured only 70% of the savings that would be expected by waiting until the call date. Of course, waiting entails interest rate risk --- the actual rates by the call date could be higher or lower than expected, resulting in smaller or greater savings than indicated by the option value at the time of the advance refunding.

A possible explanation for accepting a 70% refunding efficiency is extreme risk aversion. However, the management of callable bonds entails interest rate risk. Municipal issuers acquire the call option at a cost, although the up-front cost of the call option is seldom recognized or acknowledged. The cost of the call option on a long 5% bond sold at 120 may exceed 20 points, i.e., the price of a like optionless bond would exceed 140. Evidently the cost of the call option and the commensurate interest rate risk is acceptable by a municipality at the time of structuring. So why is this risk not acceptable after the bonds have been issued? It would be insightful for issuers to compare the savings from taxable advance refundings to the cost of the call options at the time of issuance. Relative to the cost of the call options, the savings will be much less impressive than they are in isolation.

Today with hindsight we can observe the tax-exempt rates as of the call dates of the refunded bonds, and determine the counterfactual savings resulting from a current refunding with tax-exempt bonds. We have shown that in each case the issuer would have realized considerably greater savings by waiting. For example, in the sample MSBA transaction, the savings would have been roughly \$100 million more. In aggregate the savings on just these 14 transactions would have been \$577 million more.

The aggregate ‘savings lost’ of taxpayers resulting from the well over 200 taxable advance refunding transactions between 2018 and 2020 is likely to amount to billions of dollars. The municipal finance community and the stakeholders should be aware of this enormous waste and consider how to avoid such in the future (Kalotay, Yang and Fabozzi, 2007).

Although the call option is a common feature of tax-exempt bonds, option value is seldom if ever considered explicitly in structuring and refunding transactions. The rules of thumb for refunding decisions, such as 3% or 5% present value savings, are inadequate even for bonds issued near par, and they are virtually deceitful for 5% bonds issued at a high premium. The lack of attention to option value and the resulting poor managerial decisions have cost taxpayers dearly. Improving the municipal debt management process will require the participation of several parties, including issuers, advisors, regulators, and trade associations.

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TABLES AND GRAPHS

TABLE 1
Top Ten Largest Taxable Advance Refundings
(January 1, 2018 – January 1, 2021)

	Issuer	Description	Series	Size (\$MM)	Issue Date
1	State of California	GO Refunding Bonds	Series 2018	1,200.00	04/25/18
2	NYS Thruway Authority	General Revenue Bonds	Series M	857.63	10/30/19
3	Broward County, Florida	Airport System Revenue Refunding Bonds	Series 2019 C	719.94	11/21/19
4	Massachusetts School Building Authority	Subordinate Dedicated Sales Tax Bonds	2019 Series B	715.42	11/20/19
5	San Diego Community College District	GO Refunding Bonds	2019 Series A & B	693.44	10/16/19
6	San Francisco City/County Public Utility Commission	Water Revenue Bonds	2019 Subseries A, B and C	656.96	01/09/20
7	NYS Dorm Authority	Facilities Revenue Bonds	Series 2019B	560.80	12/03/19
8	Houston City, Texas	Utility Revenue Refunding Bonds	Series 2019C	539.14	09/17/19
9	Arizona Transportation Board	Highway Revenue Refunding Bonds	Series 2020	510.28	02/12/20
10	Ohio Turnpike & Infrastructure Authority	Senior and Junior Lien Revenue Refunding Bonds	Series 2020A	472.47	02/13/20
			Total	6,926.08	

TABLE 2 Massachusetts School Building Authority 2011 Series B Bonds (Tax-Exempt) Bonds Refunded by 2019 Series B Bonds (Federally Taxable)				
Maturity Date	Refunded Principal	Coupon Rate	Call Date	Call Price
10/15/2022	20,000,000	5.000%	10/15/2021	100.000%
10/15/2023	20,000,000	5.000%	10/15/2021	100.000%
10/15/2027	40,750,000	5.000%	10/15/2021	100.000%
10/15/2031	46,630,000	5.000%	10/15/2021	100.000%
10/15/2032	49,025,000	5.000%	10/15/2021	100.000%
10/15/2033	25,345,000	5.000%	10/15/2021	100.000%
10/15/2033	26,190,000	5.250%	10/15/2021	100.000%
10/15/2034	26,645,000	5.000%	10/15/2021	100.000%
10/15/2034	27,535,000	5.250%	10/15/2021	100.000%
10/15/2035	28,010,000	5.000%	10/15/2021	100.000%
10/15/2035	28,950,000	5.250%	10/15/2021	100.000%
10/15/2036	59,880,000	5.000%	10/15/2021	100.000%
10/15/2037	62,950,000	5.000%	10/15/2021	100.000%
10/15/2038	66,180,000	5.000%	10/15/2021	100.000%
10/15/2039	69,570,000	5.000%	10/15/2021	100.000%
10/15/2040	73,140,000	5.000%	10/15/2021	100.000%
10/15/2041	76,890,000	5.000%	10/15/2021	100.000%
Total	747,690,000			

TABLE 3 \$715,420,000 Massachusetts School Building Authority Subordinated Dedicated Sales Tax Refunding Bonds 2019 Series B (Federally Taxable) 2011B Bonds Escrow Fund Sources	
Sale proceeds of 2019B Bonds	715,420,000
Less: Cost of issuance (including underwriters discount) of 2019B Bonds	(2,650,200)
Other available funds	86,119,184
Total deposit to 2011B Bonds Escrow Fund	798,888,984
Percentage of 2011 Bonds Escrow Fund allocable to 2019B bond proceeds	89.22%

TABLE 4
\$715,420,000 Massachusetts School Building Authority
Subordinated Dedicated Sales Tax Refunding Bonds
2019 Series B (Federally Taxable)
Debt Service Payments and True Interest Cost (TIC) Calculation

Date	Principal	Coupon Rate	Interest	Total Debt Service	PV of Total Debt Service @ 3.205%
11/20/2019					(715,420,000)
4/15/2020			8,853,290	8,853,290	8,740,639
10/15/2020	3,830,000	1.885%	10,990,291	14,820,291	14,400,956
4/15/2021			10,954,768	10,954,768	10,476,925
10/15/2021	5,110,000	1.905%	10,954,768	16,064,768	15,121,720
4/15/2022			10,906,095	10,906,095	10,103,971
10/15/2022	25,155,000	1.963%	10,906,095	36,061,095	32,881,966
4/15/2023			10,659,199	10,659,199	9,566,201
10/15/2023	24,910,000	2.078%	10,659,199	35,569,199	31,418,474
4/15/2024			10,400,384	10,400,384	9,041,833
10/15/2024	4,720,000	2.128%	10,400,384	15,120,384	12,937,965
4/15/2025			10,350,163	10,350,163	8,716,586
10/15/2025	4,825,000	2.278%	10,350,163	15,175,163	12,578,495
4/15/2026			10,295,207	10,295,207	8,398,978
10/15/2026	4,935,000	2.378%	10,295,207	15,230,207	12,229,064
4/15/2027			10,236,529	10,236,529	8,089,771
10/15/2027	45,280,000	2.436%	10,236,529	55,516,529	43,181,914
4/15/2028			9,685,019	9,685,019	7,414,402
10/15/2028	4,115,000	2.566%	9,685,019	13,800,019	10,398,038
4/15/2029			9,632,224	9,632,224	7,143,225
10/15/2029	4,225,000	2.666%	9,632,224	13,857,224	10,114,399
4/15/2030			9,575,904	9,575,904	6,879,229
10/15/2030	4,340,000	2.766%	9,575,904	13,915,904	9,839,373
4/15/2031			9,515,882	9,515,882	6,622,182
10/15/2031	50,590,000	2.866%	9,515,882	60,105,882	41,168,508
4/15/2032			8,790,927	8,790,927	5,926,236
10/15/2032	52,090,000	2.966%	8,790,927	60,880,927	40,394,437
4/15/2033			8,018,433	8,018,433	5,236,316
10/15/2033	53,650,000	3.066%	8,018,433	61,668,433	39,636,505
4/15/2034			7,195,978	7,195,978	4,552,169
10/15/2034	55,280,000	3.166%	7,195,978	62,475,978	38,898,929
4/15/2035			6,320,896	6,320,896	3,873,462
10/15/2035	57,055,000	3.395%	6,320,896	63,375,896	38,224,414
4/15/2036			5,352,387	5,352,387	3,177,315
10/15/2036	58,985,000	3.395%	5,352,387	64,337,387	37,589,997
4/15/2037			4,351,117	4,351,117	2,502,106
10/15/2037	61,025,000	3.395%	4,351,117	65,376,117	37,001,569
4/15/2038			3,315,218	3,315,218	1,846,754
10/15/2038	63,135,000	3.395%	3,315,218	66,450,218	36,432,550
4/15/2039			2,243,501	2,243,501	1,210,641

10/15/2039	65,310,000	3.395%	2,243,501	67,553,501	35,878,408
4/15/2040			1,134,864	1,134,864	593,233
10/15/2040	66,855,000	3.395%	1,134,864	67,989,864	34,980,146
Total	715,420,000		337,712,974	1,053,132,974	0

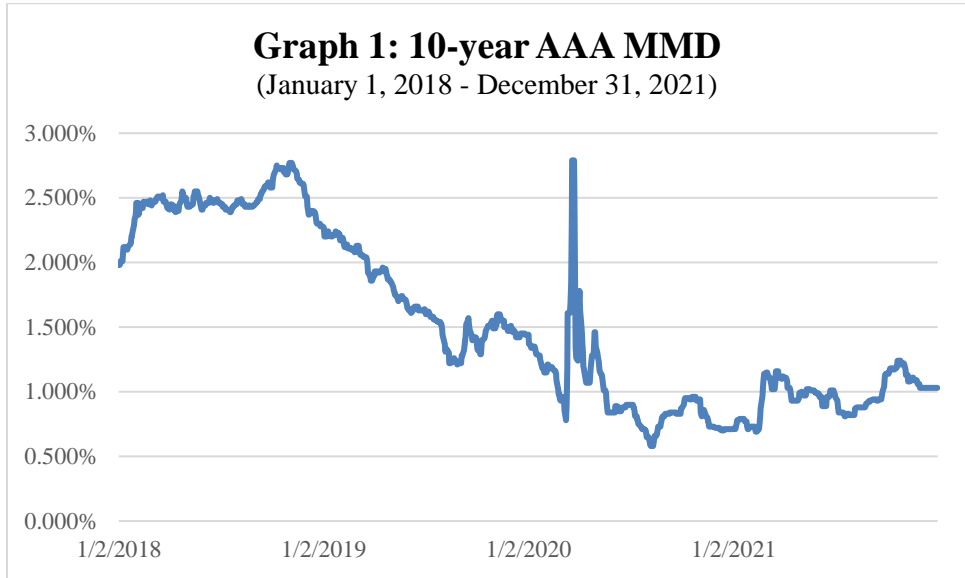


TABLE 5
Massachusetts School Building Authority
***Counterfactual* Dedicated Sales Tax**
Refunding Bonds 2021 Series (Tax-
Exempt)
Actual 5% Callable Yields and Par
Optionless Yields
October 1, 2021

Year	5% Callable Yields	Par Optionless Yields
1	0.153%	0.173%
2	0.161%	0.181%
6	0.676%	0.809%
10	1.117%	1.298%
11	1.178%	1.485%
12	1.219%	1.606%
13	1.253%	1.687%
14	1.286%	1.764%
15	1.315%	1.824%
16	1.347%	1.881%
17	1.381%	1.947%

18	1.406%	1.987%
19	1.442%	2.059%
20	1.461%	2.063%

TABLE 6
Massachusetts School Building Authority
Counterfactual Dedicated Sales Tax Refunding Bonds
2021 Series (Tax-Exempt)
Current Refunding Bonds

Date	Principal	Coupon Rate	Yield	Price (%)	Price (\$)
10/15/2022	17,935,000	0.173%	0.173%	100.000%	17,935,000
10/15/2023	17,935,000	0.181%	0.181%	100.000%	17,935,000
10/15/2027	36,540,000	0.809%	0.809%	100.000%	36,540,000
10/15/2031	41,810,000	1.298%	1.298%	100.000%	41,810,000
10/15/2032	43,960,000	1.485%	1.485%	100.000%	43,960,000
10/15/2033	22,725,000	1.606%	1.606%	100.000%	22,725,000
10/15/2033	23,485,000	1.606%	1.606%	100.000%	23,485,000
10/15/2034	23,890,000	1.687%	1.687%	100.000%	23,890,000
10/15/2034	24,690,000	1.687%	1.687%	100.000%	24,690,000
10/15/2035	25,115,000	1.764%	1.764%	100.000%	25,115,000
10/15/2035	25,960,000	1.764%	1.764%	100.000%	25,960,000
10/15/2036	53,695,000	1.824%	1.824%	100.000%	53,695,000
10/15/2037	56,445,000	1.881%	1.881%	100.000%	56,445,000
10/15/2038	59,345,000	1.947%	1.947%	100.000%	59,345,000
10/15/2039	62,380,000	1.987%	1.987%	100.000%	62,380,000
10/15/2040	65,585,000	2.059%	2.059%	100.000%	65,585,000
10/15/2041	68,945,000	2.063%	2.063%	100.000%	68,945,000
Total	670,440,000				670,440,000

TABLE 7
Actual Taxable Advance Refunding Bonds
Transaction Details and Refunding Results

	Issuer	Description	Refunding Bond Size (\$MM)	Refunding Issue Date	Refunded Bonds Call Date(s)	Refunded Bond Size (\$MM)	Credit Ratings	True Interest Cost (TIC)	Cashflow PV Savings (\$MM)	Option Value Acquired (\$MM)
1	State of California	GO Refunding Bonds	1,200.00	4/25/18	4/1/19	1,209.29	Aa3/AA/AA-	4.097%	254.10	60.30
2	NYS Thruway Authority	General Revenue Bonds, Series M	857.63	10/30/19	1/1/22	784.87	A1/A	3.162%	91.71	13.16
3	Massachusetts School Building Authority	Subordinated Dedicated Sales Tax Bonds, Series 2019B	715.42	11/20/19	10/15/21	667.08	AA+/Aa3/AA	3.205%	104.33	17.30
4	San Francisco City/County Public Utility Commission	Water Revenue Bonds, Subseries 2019A	656.96	1/9/20	11/1/20 11/1/21 5/1/22	611.44	AA-/Aa2	3.322%	85.90	24.50
5	Houston, Texas	Utility Revenue Refunding Bonds, Series 2019C	539.14	8/19/20	11/15/21 11/15/22	491.02	Aa2/AA	2.633%	93.91	7.48
6	Arizona Transportation Board	Highway Revenue Refunding Bonds, Series 2020	510.28	2/12/20	7/1/21 7/1/22	472.96	AA+/Aa1	2.464%	45.57	2.58
7	Harris County Metro Transportation Authority	Sales & Use Tax Refunding Bonds, Series 2020A	304.13	2/27/20	11/1/21	281.33	AAA/AAA	2.732%	68.27	5.88
8	California State University Trustees	Systemwide Revenue Bonds, Series 2020B	207.76	2/27/20	11/1/21	192.35	Aa2/AA-	2.700%	39.09	4.27
9	Pennsylvania State Public School Building Authority	School Lease Revenue Refunding Bonds, Series 2019	188.29	11/20/19	4/1/22	172.07	A2/A+	3.018%	9.40	0 (MWC)
10	Kent State University	General Receipts Bonds, Series 2020B	172.83	1/29/20	5/1/2022	158.17	Aa3/A+	2.993%	20.38	2.68
11	City of Philadelphia	GO Refunding Bonds, Series 2020A	118.03	1/16/20	7/15/21	111.05	A-/A/A2	2.748%	14.63	0.95
12	South Central Connecticut Reg Water Authority	Water System Revenue Bonds, 34 th Series B	83.43	7/2/19	8/1/22	74.07	Aa3/AA-	3.037%	7.04	1.58
13	Miami-Dade Co-Florida	Prof Sport Franchise Facilities Bonds, Series 2018	77.15	9/5/18	10/1/19	72.50	AA/A+	3.908%	6.21	1.12
14	Virginia Port Authority	Commonwealth Port Fund Refunding Bonds, Series 2018	60.35	7/26/18	7/1/20	57.14	AA+/Aa1/AA+	3.836%	4.46	1.57
Aggregate Results			5,691.40			5,355.34		3.133%	845.00	143.37

TABLE 8
Counterfactual Tax-Exempt Current Refunding Bonds
Transaction Details and Refunding Results

	Issuer	Description	Refunding Bond Size (\$MM)	Refunding Issue Date	True Interest Cost (TIC)	Cashflow PV Savings (\$MM)
1	State of California	GO Refunding Bonds	1,215.37	4/1/19	2.827%	444.21
2	NYS Thruway Authority	General Revenue Bonds, Series M	788.82	1/1/22	1.635%	226.91
3	Massachusetts School Building Authority	Subordinated Dedicated Sales Tax Bonds, Series 2019B	670.44	10/15/21	1.819%	221.63
4	San Francisco City/County Public Utility Commission	Water Revenue Bonds, Subseries 2019A	614.51	11/1/20, 11/1/21 5/1/22	0.220% 1.904% 2.627%	179.81
5	Houston, Texas	Utility Revenue Refunding Bonds, Series 2019C	493.47	11/15/21 11/15/22	1.686% 3.526%	142.56
6	Arizona Transportation Board	Highway Revenue Refunding Bonds, Series 2020	475.33	7/1/21 7/1/22	0.360% 2.856%	53.03
7	Harris County Metro Transportation Authority	Sales & Use Tax Refunding Bonds, Series 2020A	282.75	11/1/21	1.734%	109.30
8	California State University Trustees	Systemwide Revenue Bonds, Series 2020B	193.31	11/1/21	1.731%	61.07
9	Pennsylvania State Public School Building Authority	School Lease Revenue Refunding Bonds, Series 2019	172.93	4/1/22	2.201%	22.88
10	Kent State University	General Receipts Bonds, Series 2020B	158.97	5/1/22	2.785%	28.68
11	City of Philadelphia	GO Refunding Bonds, Series 2020A	111.60	7/15/21	1.259%	28.14
12	South Central Connecticut Reg Water Authority	Water System Revenue Bonds, 34 th Series B	74.45	8/1/22	2.709%	12.26
13	Miami-Dade Co-Florida	Prof Sport Franchise Facilities Bonds, Series 2018	72.86	10/1/19	2.039%	17.44
14	Virginia Port Authority	Commonwealth Port Fund Refunding Bonds, Series 2018	57.44	7/1/20	1.680%	17.62
Aggregate Results			5,382.25		1.978%	1,565.54

TABLE 9
Actual Taxable Advance Refunding Results

vs.

Counterfactual Tax-Exempt Current Refunding Results

	Issuer	Description	Counter-factual Cashflow PV Savings (\$MM)	Actual Option - adjusted PV Savings (\$MM)	Savings Lost (\$MM)	Proficiency Ratio
1	State of California	GO Refunding Bonds	444.21	314.40	129.81	70.78%
2	NYS Thruway Authority	General Revenue Bonds, Series M	226.91	104.87	122.04	46.22%
3	Massachusetts School Building Authority	Subordinated Dedicated Sales Tax Bonds, Series 2019B	221.63	121.63	100.00	54.88%
4	San Francisco City/County Public Utility Commission	Water Revenue Bonds, Subseries 2019A	179.81	110.40	69.41	61.40%
5	Houston, Texas	Utility Revenue Refunding Bonds, Series 2019C	142.56	101.39	41.17	71.12%
6	Arizona Transportation Board	Highway Revenue Refunding Bonds, Series 2020	53.03	48.15	4.88	90.80%
7	Harris County Metro Transportation Authority	Sales & Use Tax Refunding Bonds, Series 2020A	109.30	74.15	35.15	67.84%
8	California State University Trustees	Systemwide Revenue Bonds, Series 2020B	61.07	43.36	17.71	71.00%
9	Pennsylvania State Public School Building Authority	School Lease Revenue Refunding Bonds, Series 2019	22.88	9.40	13.48	41.08%
10	Kent State University	General Receipts Bonds, Series 2020B	28.68	23.06	5.60	80.47%
11	City of Philadelphia	GO Refunding Bonds, Series 2020A	28.14	15.58	12.56	55.37%
12	South Central Connecticut Reg Water Authority	Water System Revenue Bonds, 34 th Series B	12.26	8.62	3.64	70.31%
13	Miami-Dade Co-Florida	Prof Sport Franchise Facilities Bonds, Series 2018	17.44	7.33	10.11	42.03%
14	Virginia Port Authority	Commonwealth Port Fund Refunding Bonds, Series 2018	17.62	6.03	11.59	34.22%
Aggregate Results			1,565.54	988.37	577.17	61.25%