Local Government Debt Valuation*

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Abstract

We construct a novel data set on the fiscal position of municipalities in the United States and document a secular decline in their financial health. Our data combines financial data from the Annual Comprehensive Financial Reports (ACFRs) of municipalities along with Census data of their revenue and expenditure cash flows. We find that a large share of municipalities operate with a negative net position—akin to a negative book equity position in the corporate context. We find that most of the decline originates from the accumulation of legacy obligations, i.e., pensions and other post-employment benefits (OPEBs); this is recognized by municipal bond markets through higher credit spreads. While accounting values from the ACFRs are informative, they are based on book valuations which potentially convey limited information about the economic value of assets and liabilities. Thus, we turn to the market valuation of local governments' equity by estimating an SDF that matches the valuation of a wide range of assets in the economy to prices future tax and expenditure claims. Using market prices for tax and expenditure claims, and market valuations of liability positions we find that the market values of equity are highly correlated with the book values. The negative equity position—in terms of book and market values—for some local governments suggests the presence of implicit insurance by the state and federal governments.

Keywords: Municipal finance, financial distress, public finance, fiscal sustainability, sovereign default, asset pricing.

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1 Introduction

A growing literature has questioned the sustainability of sovereign debt in the United States (Rubin, Orszag, and Sinai, 2004; Jiang, Lustig, Van Nieuwerburgh, and Xiaolan, 2019). Federal debt as a percent of gross domestic product has approximately doubled between the onset of the great financial crisis (GFC) and 2021. Relatively little, however, is known about the financial health of local governments. Local government are significant economic entities as they account for $1.6 trn—8.1% of GDP—in public expenditures and 10.0% of employment.[1] They perform a number of important functions in public works, public safety, and other local public amenities. Despite their economic importance even basic financial statistics on their assets and liabilities are unavailable.[2] Unlike the federal government, local governments operate under more constrained conditions. Local governments are subject to budgetary institution that limits their tax authority and ability to finance operating expenditures with additional debt issuance (Alt and Lowry, 1994; Bohn and Inman, 1996; Krane, Rigos, and Hill, 2001; Poterba, 1994, 1995; Reschovsky, 2019). While fiscally more constrained, local governments receive quantitatively large intergovernmental transfers from the state and federal government.

Using a novel data set on the fiscal position of municipalities in the U.S., we document the status quo and the trend of municipal finances in the United States. We document three important facts. First, a substantial share of municipalities operate with a negative net position—akin to a negative book equity position in the corporate context—and the share is increasing over the past ten years. Second, most of this decline comes from the accumulation of legacy obligations; such as, pensions and other post-employment benefits. Third, markets recognize this deterioration in the fiscal position by demanding higher yields on municipal debt securities.

These facts cast doubt on the solvency of local governments in the United States. While we acknowledge that accounting values from the ACFRs provide only a *backward* looking perspective, we take it as motivation to study the *forward* looking market valuation.

[1]Public expenditures are derived from the ASSLGF in fiscal year 2017 and employment is obtained from the 05/2019 BLS Occupational Employment and Wage Statistics (OES).

[2]The Financial Accounts of the United States, L.107, provide aggregate statistics for state and local governments. As of Q2 2021, state and local governments have accumulated total financial liabilities of 8.15 trn USD, whereas have only 4.23 trn USD in total financial assets in possession. This does not account for other post employment benefits which, according to estimates by Joffe (2021), amount to about 1.23 trn USD as of the end of fiscal year 2019.

Answering this question requires a disciplined way of valuing revenue and expenditure claims of local governments. For that we estimate a stochastic discount factor (SDF) that prices a broad set of assets in the economy. We find that a exponentially affine stochastic discount factor can fit a broad set of asset prices in the economy relatively well, including a broad index in the municipal bond market. Using a long time series for the revenue and expenditure claims of a rich panel of 388 local governments, we estimate the cross-sectional exposure to systematic risk. This allows us to capture the cross-sectional valuations of local governments. We use the estimated price-dividend ratios for government expenditure and revenues to calculate the market value of equity for local governments. More precisely, we calculate the present value of revenues plus cash holdings on the asset side, and subtract out the present value of expenditures, the value of pension obligations and OPEBs, and the present value of debt on the liabilities side. Our results suggest a non-trivial number of municipalities with *negative* market value of equity. On the face of it, this would suggest a number of municipalities across the U.S. are insolvent. But if this were true, corresponding municipal yields should not be low and stable, as we see in the data. In fact, credit spread differences between positive equity value municipalities and negative equity value municipalities are small.

To resolve this conundrum, we argue that a simple accounting of *visible* cash flows misses the important role of implicit insurance provided by federal and state governments. Our approach assumes that there is no mis-pricing in the municipal bond market and that market participants incorporate the implicit insurance when valuing the municipalities' debt position. For example, the low and stable yields in the municipal bond market reflect the expectations of market participants of the default probability taking into account state and federal insurance. We conceptualize transfers by state and federal government as an option on the underlying position of the local government, which leads to a rich set of predictions in the cross-section. For instance, states with proactive policies for supporting local governments provide greater insurance than states with Chapter 9 bankruptcy provisions for municipalities (see Spiotto (2012)).

Apart from the *absolute* valuation similar to the valuation of government debt at the federal level (Jiang, Lustig, Van Nieuwerburgh, and Xiaolan, 2019), we additionally obtain *relative* valuations in the cross-section of municipalities. The relative valuations are particularly insightful since they remove the potential of other *common* omitted factors to affect the valuation. This includes e.g. the impact of inflation (Hilscher, Raviv, and Reis, 2021) or common convenience yield on sub-national debt analogously to the convenience yield observed for federal debt (Krishnamurthy and Vissing-Jorgensen, 2012; Jiang, Lustig, Van Nieuwerburgh, and Xiaolan, 2019), or common rational bubbles (Samuelson, 1954; Diamond, 1965; Brunnermeier, Merkel, and Sannikov, 2020).

Related Literature: We build on the extensive literature on debt sustainability in macroeconomics starting with the seminal work of Hansen and Sargent (1980), Hansen, Roberds, and Sargent (1991), and Sargent (2012). This literature assumes mostly a constant discount rate and ignores the differential cyclical properties of claims. We adopt risk adjusted returns from the asset pricing literature (Alvarez and Jermann, 2005; Hansen and Scheinkman, 2009; Backus, Boyarchenko, and Chernov, 2018) that results from a stochastic discount factor that prices states of the world, while drawing on the extensive literature in public finance to value pension and other post employment benefits (Novy-Marx and Rauh, 2009, 2011a; Brown and Wilcox, 2009; Lucas and Zeldes, 2006).

For the valuation we build on a mature literature on dynamic asset pricing which combines a vector auto-regression model for the state variables as in Campbell (1991, 1993, 1996). We further use a no-arbitrage model for the stochastic discount factor as in Duffie and Kan (1996), Dai and Singleton (2000), Ang and Piazzesi (2003), Lustig, Van Nieuwerburgh, and Verdelhan (2013), Jiang, Lustig, Van Nieuwerburgh, and Xiaolan (2019), Gupta and Van Nieuwerburgh (2021). Jiang, Lustig, Van Nieuwerburgh, and Xiaolan (2019) price tax and expenditure claims to value the market value of federal government debt. This paper takes a similar approach but for a broad cross-section of local governments.

Further, we contribute to the literature of local finances. Adelino, Cunha, and Ferreira (2017) studies the re-calibration of credit ratings and the associated change in fiscal capacity; Shoag, Tuttle, and Veuger (2019), Chava, Malakar, and Singh (2021a) study the fiscal implications of large bankruptcies on local communities; Cellini, Ferreira, and Rothstein (2010) study the effect of bond elections; Yi (2021) studies the credit supply shock in the municipal bond market due to financial regulation; Haughwout, Hyman, and Shachar (2021), Green and Loualiche (2020) and Clemens and Veuger (2021) study the fiscal implications of COVID-19 on state and local governments, Chava, Malakar, and Singh (2021b) studies the impact of business subsidies on municipal bond yields and Giesecke and Mateen (2021), Chernick, Reschovsky, and Newman (2021) study the effect of a decline in the local tax base. Myers (2022) shows that municipal governments option to file for fiscal emergencies leads to perverse incentives into how they manage spending and borrowing, leading to excessive risk-taking.

The paper is organized as follows. Section 2 introduces our novel dataset along with pre-existing dataset that we use in our study. Section 3 documents important facts on the status quo and the trajectory of the financial conditions of municipalities across the United States and provides evidence of the cross-sectional pricing in the municipal bond market. Section 4 discusses the structure of local governments' balances sheets and its implications for the re-pricing. Section 5 introduces the pricing methodology, Section 6 conducts the market valuation of equity of U.S. municipalities and Section 7 concludes.

2 Data

We assemble a comprehensive dataset on the financial position of municipalities across the United States. We summarize the main components of our data here and provide further details in the Data Appendix.

2.1 Data and Sample Selection

Annual Survey of State and Local Government Finances The Annual Survey of State and Local Government Finances (ASSLGF) serves as a key input for the tabulation of the national accounts pertaining to the revenues and expenditures of all governmental units. The Census Bureau conducts a full census in years ending with "2" and "7"; and a survey of a subset in between. This includes the so called "certainty sample" which is surveyed in every year. The certainty sample constitutes the main sample for our analysis. We confirm that the information from ASSLGF are consistent with the national account from the Bureau of Economic Analysis (BEA). The validation for a selected set of items is shown in the Appendix in Figure DA.3, Figure DA.1, and Figure DA.2.

Moody's Investors Service Data We augment the ASSLGF data with more detailed data on assets and liabilities from annual comprehensive financial reports (ACFRs). While municipalities are legally required to file ACFRs annually, the publication is irregular at best.[3] Even after obtaining the reports, any comprehensive study is difficult because the reports are provided in an unstructured data format. We overcome the data scarcity by drawing on a large dataset on key financial indicators for a broad sample of local governments across the United States from Moody's Investors Service. Moody's prepares these financial indicators as part of its rating service by drawing directly from the ACFRs and subsequently harmonizing the records for comparison. The financial indicators serve as a primary input into the rating of municipal debt securities.[4] We further augment the Moody's data with manually collected data to obtain the best possible coverage for the

[3]For example, the state auditor of California launched a project in 2019 that—for the first time—collected the annual comprehensive financial reports from all its municipalities. This initiative has been started to create transparency and identify financial distress early—a recognition of increasing pension and other post employment obligations.

[4]We validated the records for a random sample of municipalities and found them to be consistent with available ACFRs.

certainty sample.

Municipal Bond Data We complement the data set on balance sheet conditions of local governments with municipal bond yields in the primary and secondary market. Primary market information are obtained from Mergent Municipal Bond Database which records issuer characteristics and a large set of bond characteristics. In addition, we obtain secondary market data from MSRB EMMA. MSRB EMMA is a trade repository which records every trade in municipal bond securities since 2005. MSRB EMMA includes the trade time, trade price and implied yield to maturity, as well as, information whether it was a broker to customer trade or broker to broker trade. We only include broker to customer trades in our analysis. One challenge is to establish the connection between the bond issuance and the bond issuer. While Mergent Bond Database records the issuer name, the match to the financial information of the issuing entity is not straightforward. We overcome part of the challenge by using Moody's historical linkage table. This table is created as part of Moody's ratings activity and documents in great detail whether a local government is the direct issuer or the financial obligor for an issuance. While this linking table covers many debt securities, it does so only for a subset. For the remainder we draw on the universe of debt security disclosures under the U.S. Securities and Exchange Commission (SEC) Rule 15c2-12 as provided by MSRB.

Local Government Shape Files As local governments are not defined homogeneously across the United States, we create new shape files that represent the jurisdictional boundaries of local governments. More details on the historical reason for the heterogeneity across the United States, as well, as details on the construction are provided in Section DA.7.2 of the Data Appendix.

3 Financial Conditions Across U.S. Municipalities

How did the financial conditions of local governments evolve over the last decade? We document the secular development of financial health for a broad sample of local governments across the United States. As the accounting and reporting of financial indicators differs from corporations we provide some institutional background on the accounting methodologies and introduce the two main financial indicators that we focus on in Section 3.1. Section 3.2 presents the secular development in association with the market based measures of financial health for the national sample.

3.1 Institutional Background

Accounts and Accounting Local governments manage their finances typically based on governmental funds. The general fund covers most of the operational revenues and expenditures; other funds—such as, the capital project, debt service, internal service, and enterprise fund—often exist and take on a more specialized role. Every state except for Vermont imposes a statutory or constitutional balanced budget provision on the general fund (NCSL, 2010).[5]

The accounting basis for these funds is *fund accounting* or *modified accrual accounting*. Fund accounting emphasizes cash-flows over solvency and resembles the cash flow statement rather than the profit and loss statement or balance sheet in the corporate context. While the accounts are the primary basis for decision making, local governments are required to publish the statement of net position in their comprehensive annual financial report ("ACFR"). The methodology for the statement of net positions is closer to conventional accrual accounting. The statement of net positions represents assets and liabilities more comprehensively. However, the funds receive most of the attention in the administrative decision-making process. In principle, this hybrid accounting framework allows for large deficits on an accrual basis, i.e. in the ACFRs, as long as it does not materially affect the (cash) balance in the general fund. Pension and other post employment benefit commitments are two examples for which the expenditures and the cash flow impact

[5] The balanced budget provision applies with varying degree of stringency as e.g. discussed in Bohn and Inman (1996) and Poterba (1995).

occurs with a large time gap.[6] Thus, the difference between operating expenses and the incrementally accrued liability can be large.

Financial Indicators We use two main financial indicators to describe the development of financial conditions. First, the unrestricted net position as a percentage of operating revenues and second, the total debt as a percentage of the full value.[7] The unrestricted net position is directly reported in the statement of net position of the ACFR and is an important input into the credit rating.[8] The unrestricted net position consists of three major parts: (i) long-term debt that is not directly associated with capital assets, (ii) pension obligations, and (iii) other post employment benefits ("OPEB"). The portion of long-term debt that is not directly associated with capital assets can be understood as debt that has been issued to fund operating expenses. The use of the unrestricted net position derives its justification under the premise that most of the capital assets are highly illiquid and thus cannot be used to serve the liabilities. It excludes the fraction of liabilities that are directly associated with the capital assets—revenue bonds to fund capital projects is one such example. Further, the unrestricted net position is calculated under an accounting framework that is closer to accrual accounting, that is, the expenditures are accounted for at the time of accrual, not at the time of the cash outflow. The total debt as percentage of the full value captures the indebtedness relative to the maximum amount of all taxable properties that could be drawn upon for taxation and is another prominent input into the rating of municipal securities.

We consider the aforementioned financial indicators to represent the overall financial position most accurately. For the purpose of the most direct comparison with the corporate balance sheet, we also consider the net position. The net position is the difference of all assets minus liabilities and is analogous to definition of book equity in the corporate

[6] For a detailed discussion about the actuarial recognition of pension liabilities see e.g. Novy-Marx and Rauh (2011b). There is an active debate in the academic literature to what extent the actuarial treatment reflect the economic liability (Novy-Marx and Rauh, 2009, 2011a; Brown and Wilcox, 2009; Lucas and Zeldes, 2006).

[7] The full value stands for the full market value of properties in the tax jurisdiction. In comparison to the grand list or assessment value which is used as the basis for property taxation, the full value is unaffected by the methodology that is used for the tax assessment.

[8] We obtained this information from a conversation with the public finance credit analyst from a major rating agency. Apart from this, the independent think tank "Truth in Accounting" emphasizes the relevance of the unrestricted net position to assess the financial situation of local governments.

context.[9]

3.2 Nationwide Sample

This section describes the state and the trend of financial conditions of local governments for a nationwide sample. We start by introducing the sample and the sample selection.

The nationwide sample is dictated by the data availability of financial data. The data is collected as an input in Moody's rating methodology. As such the sample covers predominantly local governments that are active participants in the municipal bond market. Ex-ante we would expected that this selection favors local governments that are large in terms of its population. We find that the median population size is 21,187 and the mean population size is 59,787.[10] While the observed selection on size may compromise the overall representativeness for local governments, we suspect that we capture the economically most relevant local governments. In addition, we restrict the sample to those observations that have non-missing values for the unrestricted net position over operating expenditures both in 2007 and 2018. This allows us to make inter-temporal comparisons without the concerns about composition effects. We obtain a total sample of 1,803 local governments across the United States for which we tabulate summary statistics in Table A.2. We further show the geographic distribution in Figure 1.

The financial conditions for the nationwide sample show a deterioration over time. We present histograms for the two aforementioned financial indicators for the years 2007 and 2018. Figure 2a overlays the histograms of the unrestricted net position for the years 2007 and 2018. While in 2007 the distribution is centered and fairly symmetric around zero, the distribution shifts markedly to the left in 2018. Furthermore, the unrestricted net position shows a long and fat left tail. Concretely, the median decreases from 28.40% to -18.97% and the 5% percentile of the unrestricted net position over operating revenue distribution decreases from -25.02% to -190.62%. At the very left tail the unrestricted net position is about 5 times its annual revenues in 2018. The observation about deteriorat-

[9] Figure A.1 in the Appendix shows a schematic balance sheet and Table A.1 presents summary statistics of the most salient items in the national sample.

[10] The median population is 1,030 and the mean population is 7,393 in the Census of Government in 2017. We have to conduct further validations of those numbers as the city with the smallest population is currently listed with zero population. While the anticipated sample selection towards larger local governments is present, we consider the median population of 21,187 as modest.

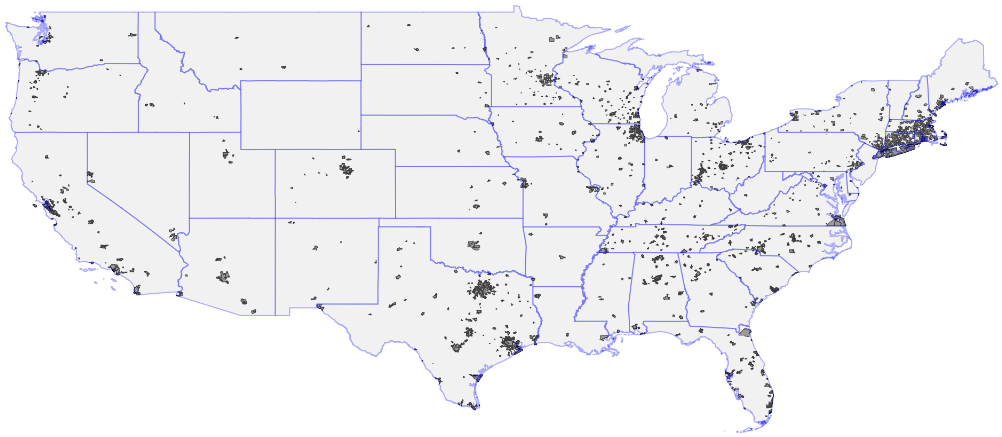


Figure 1 – Nationwide Sample

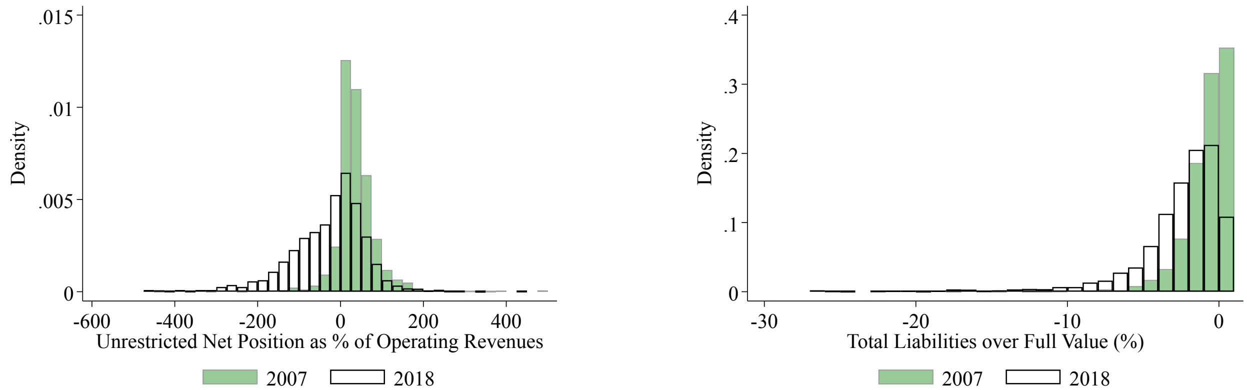
Notes: The sample contains all local governments for which information on the unrestricted net position is available in 2007 and 2018. Shape files for local governments are self-constructed as described in the Data Appendix DA.7.2. Data is obtained from the Census of Governments, the Census Bureau and Moody's Investor Services.

ing financial conditions is not specific to the unrestricted net position. Figure 2b shows the two histograms for total debt over full value. Between 2007 and 2018 the median decreases from -0.49% to -1.99% and the 5% percentile of the total debt over full value distribution decreases from -3.37% to -8.86%.

Next, we show the discrepancy between the general fund balance—following fund accounting—and the unrestricted net position—based on accrual accounting. Figure 3a plots the general fund balance over operating revenues against the unrestricted net position over operating revenues. While only 5/1,803 (0.27%) local governments operate with a negative general fund balance, 1,099/1,803 (60.95%) operate with a negative unrestricted net position at the end of fiscal year 2018.

Further, we show the strong relationship of the unrestricted net position to legacy obligations. Figure 3b plots the unrestricted net position over operating revenues against net pension and net OPEB liabilities over operating revenues.

Lastly, we investigate to what extent the financial indicators from the financial reports align with market signals in the municipal bond market. We find that the unrestricted net position over operating revenues and total debt over the full value—is associated strongly with municipal bond spreads as shown in Figure 4a. The cross-sectional variation could

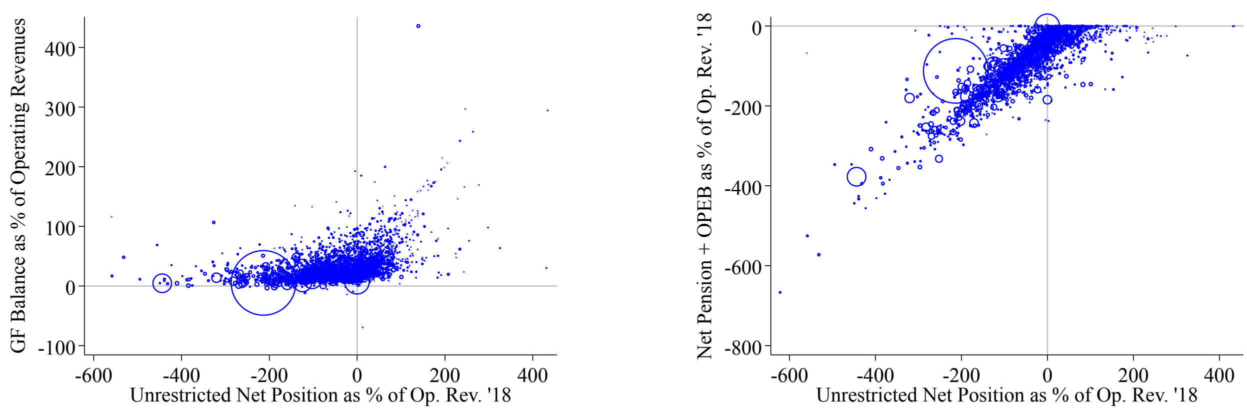


(a) Unrestricted Net Position

(b) Total Liability over Full Value

Figure 2 – Financial Conditions Indicators

Notes: Panel (a) shows the unrestricted net position as a share of the the general fund total revenue in 2007 (green) and 2018 (transparent). Panel (b) plots the total liabilities (total bonded debt + unfunded OPEB liabilities + unfunded pension liabilities) over full value. The plot follows the convention that liabilities are expressed with a negative sign. Data is obtained from Moody's Investor Service.



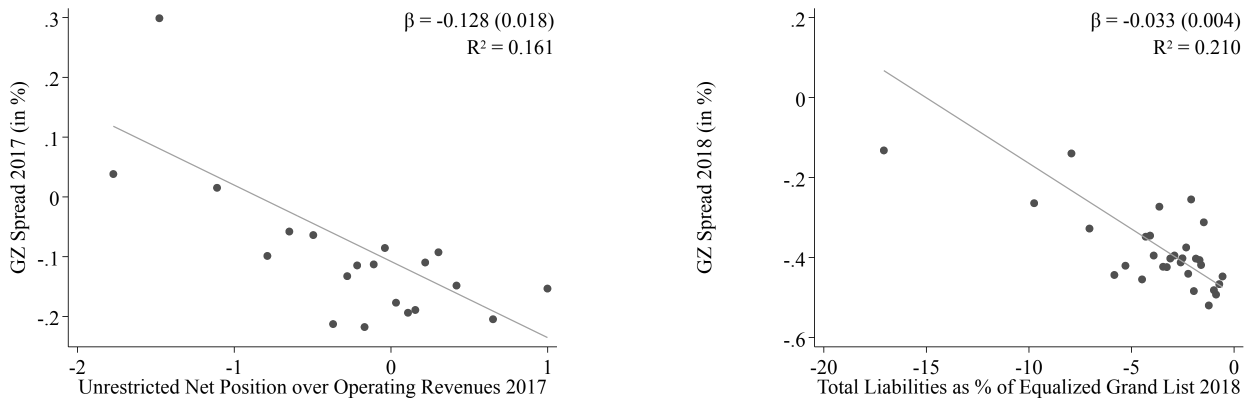
(a) Budget Balance

(b) Liability Composition

Figure 3 – Budget Balance and Liability Composition in 2018

Notes: Panel (a) plots the budget balance of the general fund (end of fiscal year) and the unrestricted net position over operating revenues. Panel (b) plots the unfunded OPEB and pension liability and the unrestricted net position over operating revenues. The size of the circle corresponds to the total operating revenues of the corresponding local government. The sample contains all local governments for which the unrestricted net position is available in both 2007 and 2018. Data is obtained from Moody's Investor Services.

in principle reflect relative differences in default risk and/or liquidity risk. Schwert (2017) argues that default risk accounts for 74% to 84% of the average yield spread after adjusting for the tax-exempt status. Thus, we interpret the cross-sectional variation of yields to reflect the difference in default risk rather than a compensation of liquidity risk.



(a) GZ Spread - Unrestricted Net Position

(b) GZ Spread - Total Liability over Full Value

Figure 4 – Primary Market Bond Spreads

Notes: Panel (a) plots the relationship between the GZ Spread (Gilchrist and Zakrajšek, 2012), a duration matched yield spread, for issuances with maturity of over one year at issuance, tax-exempt status, and classified as full general obligation and the unrestricted net position. Panel (b) plots the relationship between the GZ Spread for issuances with maturity of over one year at issuance, tax-exempt status, and classified as full general obligation and the total liability (total bonded debt + unfunded OPEB liabilities + unfunded pension liabilities) over the full value. Data on municipalities' ACFRs is obtained from Moody's Investor Services and primary bond issuance data is from Mergent Municipal Bond Database. All plots are binscatters with 30 quantiles.

3.3 Census Certainty Sample

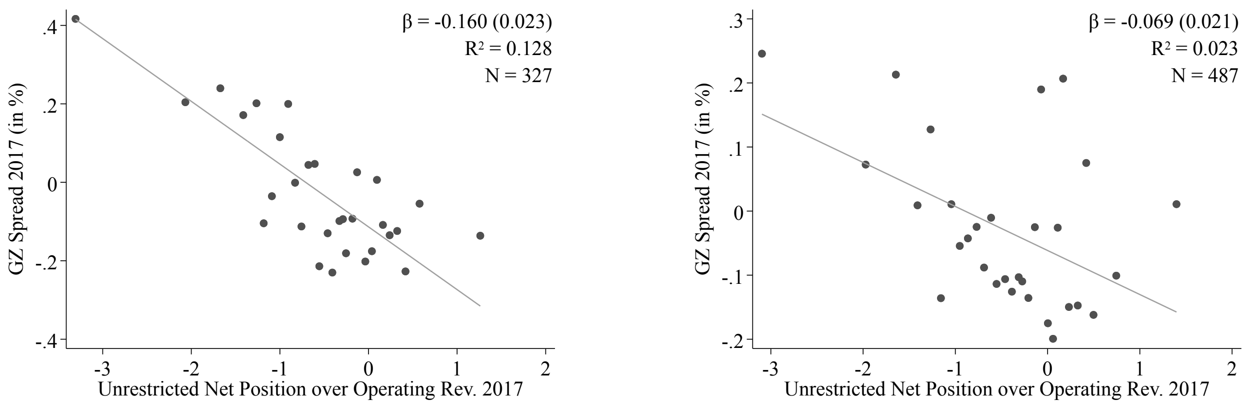
For the remainder of the paper we focus on the so-called census certainty sample which consists of 622 local governments across the United States.[11] This is the primary sample to estimate the governmental sector in BEA NIPA. Further, to the best of our knowledge, this is the broadest sample for which long time series on revenue and expenditures are available in the United States. First, we obtain additional information on book values from the Moody's Investors Service Data and further manually collect book value information from ACFRs if available. The geographic location of the census certainty sample is exhibited in Figure A.2 and the summary statistics in Table A.3.

[11]The intersection of local governments which are both in the census certainty sample and have book value information and other market based information available is 388.

In terms of its fiscal position, the local governments in the census certainty sample show great similarity with those in the broader national sample. The median of the unrestricted net position over operating revenues in 2018 is -29.32% as opposed to -18.97%, the 25-th percentile is -81.11% as opposed to -84.62% and the 75-th percentile is 4.02% as opposed to 22.08%. We find this similarity also for the alternative fiscal indicator, that is, the total liability over full value. In 2018, the median is -3.09% in the census certainty sample compared to -2.34% in the broader national sample. Similarly to the national sample, about 23.1% of local governments operate with a negative net position and about 71.1% operate with a negative unrestricted net position. Overall, the census certainty sample shows similar characteristics to the much broader national sample. This is re-assuring since data limitations will limit us to this sample henceforth.

The similarity of the sample is also reflected in the municipal bond market. We repeat the analysis from above for the census sample in the primary and secondary bond market. Figure 5 shows the relationship between the unrestricted net position over operating revenues and the duration matched spread in the municipal bond market. One of the limitations in this analysis is that the sample is selected by the issuer's activity in the municipal bond market. We can only include an observation if there was at least one issuance at the municipal bond market for the primary market and if there was at least one trade in the bond for the secondary market analysis. Reassuringly, the slope that we find the primary and secondary market brackets the slope of the national sample.

We find similar results, as presented above, for the total liabilities over full value. Figure 6 shows the association between the total liabilities over full value in the primary and secondary market. While the sample in the primary and secondary market once again differs due to the availability of transactions in the respective market, we find slopes that are remarkably similar to each other. Furthermore, the slope is of similar magnitude as for the broader national sample. This re-affirms once more the similarity of these two samples in terms of its financial characteristics.

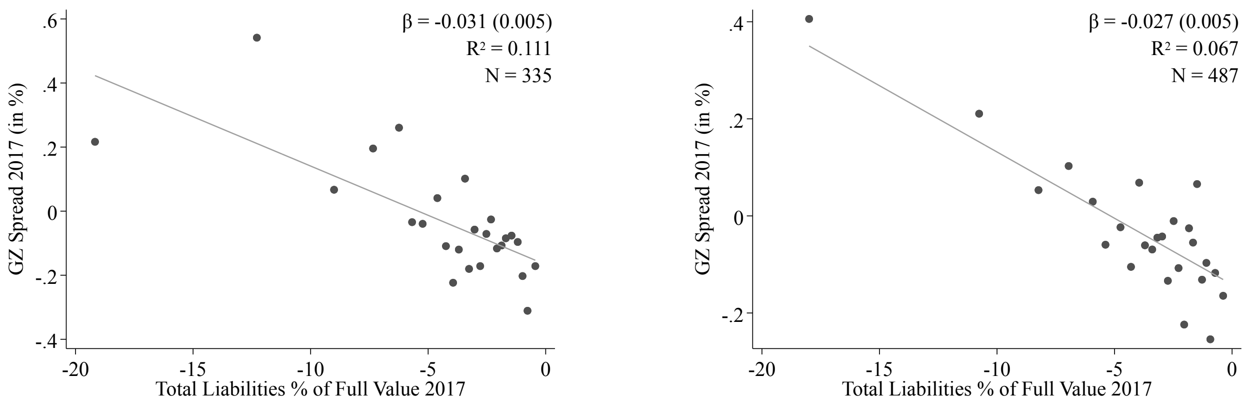


(a) Primary Market

(b) Secondary Market

Figure 5 – Certainty Sample - GZ Spread - Unrestricted Net Position

Notes: Panel (a) plots the relationship between the GZ Spread (Gilchrist and Zakrajšek, 2012), a duration matched yield spread, and the unrestricted net position over operating revenues for issuances with maturity of over one year at issuance and tax-exempted status in the primary market. All spreads are tax-rate adjusted. State specific marginal income taxes are obtained from Babina, Jotikasthira, Lundblad, and Ramadorai (2021). Number of observations are restricted to the sample that had at least one primary issuance in the respective fiscal year. Panel (b) plots the relationship between the GZ Spread and the unrestricted net position over operating revenues in the secondary market. Number of observations are restricted to the sample that had at least one transaction in the secondary market in the respective fiscal year. Data sources are detailed in Section 2. All plots are binscatters with 30 quantiles.



(a) Primary Market

(b) Secondary Market

Figure 6 – Certainty Sample - GZ Spread - Total Liability over Full Value

Notes: Panel (a) plots the relationship between the GZ Spread (Gilchrist and Zakrajšek, 2012), a duration matched yield spread, and the total liability (total bonded debt + unfunded OPEB liabilities + unfunded pension liabilities) over full value. for issuances with maturity of over one year at issuance and tax-exempted status in the primary market. All spreads are tax-rate adjusted. State specific marginal income taxes are obtained from Babina, Jotikasthira, Lundblad, and Ramadorai (2021). Number of observations are restricted to the sample that had at least one primary issuance in the respective fiscal year. Panel (b) plots the relationship between the GZ Spread and total liability (total bonded debt + unfunded OPEB liabilities + unfunded pension liabilities) over full value in the secondary market. Number of observations are restricted to the sample that had at least one transaction in the secondary market in the respective fiscal year. Data sources are detailed in Section 2. All plots are binscatters with 30 quantiles.

4 Pricing the Components of a Local Government Balance Sheet

In this section, we discuss the main components of a local government's balance sheet. In contrast to the discussion in the previous section in which we considered the book values as accounted by governmental accounting standards, we focus on the market values of the main components of the balance sheet. The valuation of assets and liabilities which is consistent with market prices provides a more timely evaluation of local governments' financial position.

In a first step, we are decomposing the balance sheet into its main components by starting with the basic accounting identity for equity.

$$Equity = Assets - Liabilities$$

There is some debate about the assets of local governments. We follow the convention in the rating process which assumes that local governments cannot liquidate their capital assets.[12] Thus, we are decomposing the assets into the present value of the revenue stream and current cash and cash equivalents which can be liquidated at short notice.

$$Assets = PV(Revenues) + Cash$$

We can further unpack the liabilities in its components.

$$Liabilities = PV(Expenditures) + PV(PensionObligations) + PV(OPEB) + PV(Debt)$$

The liabilities include the present value of expenditures plus the value of pension obligations and OPEB obligations, and the present value of outstanding debt. The full equation is:

$$Equity = PV(Revenues) + Cash$$

[12]While we have witnessed transfers of assets into the fiduciary funds in the last decade, we are unaware of liquidations of public assets to overcome financial distress.

$$- PV(Expenditures) - PV(PensionObligations) - PV(OPEB) - PV(Debt) \tag{1}$$

Equation (1) constitutes the main equation for the determination of the market value of the equity position. It remains to show how we calculate each item of the right hand side of Equation (1). Subsection 4.1 will discuss the re-pricing of the market value of pension and OPEB obligations and of outstanding debt obligations. Section 5 introduces the model that we use to price the present value of revenues and expenditures.

4.1 Re-pricing of Balance Sheet Components

We are re-pricing pension obligations, other post employment benefits (OPEB) and long-term debt following some of the seminal papers in the literature.

Long-term Debt Long-term debt are valued in accordance to the market's expectation of the default probability as exhibited by the credit spread over treasuries (after accounting for the tax exemption when applicable).

$$MV_{LTDebt} = BV_{LTDebt} \exp(-cs_\tau \tau) \tag{2}$$

where τ is the duration of the overall long-term debt portfolio of the local government and cs_τ the corresponding credit spread.

Pension Obligations Pension are valued as if they constitute a risk free liability in accordance with some of the seminal papers in the literature (Novy-Marx and Rauh, 2009, 2011a; Brown and Wilcox, 2009; Lucas and Zeldes, 2006).

$$\begin{aligned} MV_{NetPensionLiability} = BV_{NetPensionLiability}[1 + Duration_{NPL}(y_{Pension} - y_{try}) \\ + \frac{1}{2} Convexity_{NPL}(y_{Pension} - y_{try})^2] \end{aligned} \tag{3}$$

where $y_{Pension}$ is the actuarially assumed discount rate and y_{try} is the duration matched treasury yield.

Other Post-Employment Benefits While there is some debate whether other post-employment benefits enjoy the same protections as pension benefits, we value the liability consistent with the pension liabilities.

$$MV_{NetOPEB} = BV_{NetOPEB}[1 + Duration_{NPL}(y_{OPEB} - y_{try}) + \frac{1}{2}Convexity_{NPL}(y_{OPEB} - y_{try})^2] \tag{4}$$

where y_{OPEB} is the actuarially assumed discount rate and y_{try} is the duration matched treasury yield.

5 Present Value of Revenues and Expenditures

In Section 4.1 we discussed the re-pricing of pension and OPEB obligations and long-term debt. The remaining two components are the present value of revenues and expenditures. Unfortunately, revenue and expenditure claims are not traded in the market and thus no market prices are available. Furthermore, revenue and expenditure claims may potentially have risky payoffs which results in a price adjustment from a risk-free annuity. We overcome these limitations by estimating a stochastic discount factor that prices a broad set of assets in the economy. The existence of a (strictly positive) stochastic discount factor allows us to price the revenue and expenditure claims such that there are no arbitrage opportunities between traded assets and non-traded claims.

Our model consists of three components: (i) a VAR that governs the evolution of the state vector $\mathbf{z}_t$; (ii) an exponentially affine asset pricing model that describes the stochastic discount factor $M^{\$}_{t+1}$; (iii) a spanning argument that allows us to price variables outside the state vector, in particular revenue and expenditure claims of local governments.

5.1 Evolution of state variables

There is a $N \times 1$ vector $\mathbf{z}$ of state variables that follows a first order VAR with Gaussian error:

$$\mathbf{z}_t = \mathbf{\Psi}\mathbf{z}_{t-1} + \mathbf{u}_t = \mathbf{\Psi}\mathbf{z}_{t-1} + \Sigma^{\frac{1}{2}}\varepsilon_{\mathbf{t}} \tag{5}$$

where $\mathbf{\Psi}$ is a $N \times N$ companion matrix, $\mathbf{u}_t$ is a Gaussian error $\mathbf{u}_t \sim i.i.d.\ \mathcal{N}(0, \Sigma)$, the Cholesky decomposition of the covariance matrix gives us the lower triangular matrix $\Sigma^{\frac{1}{2}}$, with structural shocks $\varepsilon_{\mathbf{t}} \sim i.i.d.\ \mathcal{N}(0, \mathcal{I})$. The vector $\mathbf{z}$ is demeaned by the sample averages of each individual element.

We include a rich set of state variables: lagged inflation, GDP growth, short yield, the 5-1 year yield spread, the stock market price-dividend ratio and dividend growth. We include both levels and growth of federal taxes and spending following Jiang, Lustig, Van Nieuwerburgh, and Xiaolan (2019). Importantly, we add the municipal credit spread to the VAR. The municipal credit spread is defined as the difference between the 10-year reference municipal yield from Bondbuyer and the 10 year Treasury bond yield. Table 1 provides a list of these variables along with their sample means.

The inclusion of spending and taxes in the state vector means that we assume the federal government commits to a policy that is affine in the state vector. Including the levels of these two variables along with their growth rates means that we capture the idea of automatic stabilizers since any deviation in growth rates leads to a reversion to the long run level relative to GDP. The credit spread of the reference municipal yield captures an important aggregate component of the muni bond market.

Position	Variable	Variable Mean	Sample Mean
0	π_t	π_0	0.03108
1	x_t	x_0	0.029745
2	$y(1)^{\$}_t$	$y(1)^{\$}_0$	0.04329
3	$yspr^{\$}_t$	$yspr^{\$}_0$	0.005838
4	pd_t	pd_0	3.528392
5	Δd_t	Δd_0	0.060559
6	$\Delta \log \tau_t$	$\Delta \log \tau_0$	-0.006712
7	$\log \tau_t$	$\log \tau_0$	-2.236345
8	$\Delta \log g_t$	$\Delta \log g_0$	0.001887
9	$\log g_t$	$\log g_0$	-2.214822
10	$\Delta \log d_t$	$\Delta \log d_0$	0.003952
11	$\log d_t$	$\log d_0$	-1.042491
12	cs_t	cs_0	-0.003064

Table 1 – State Variables

Estimation We estimate the VAR using OLS. The point estimates for $\mathbf{\Psi}$ are reported in Table 2. The point estimates for the Choleski decomposition $\Sigma^{\frac{1}{2}}$ is reported in Table 3.

	π_{t-1}	x_{t-1}	$y(1)^{\$}_{t-1}$	$yspr^{\$}_{t-1}$	pd_{t-1}	Δd_{t-1}	$\Delta \log \tau_{t-1}$	$\log \tau_{t-1}$	$\Delta \log g_{t-1}$	$\log g_{t-1}$	$\Delta \log d_{t-1}$	$\log d_{t-1}$	cs_{t-1}
π_t	0.67	-0.14	-0.11	-0.21	-0.00	0.01	0.03	-0.05	-0.04	-0.01	-0.05	-0.02	-0.29
x_t	-0.01	0.50	0.33	0.53	0.02	0.06	-0.09	-0.02	0.08	0.08	-0.05	0.04	-0.16
$y(1)^{\$}_t$	0.08	0.08	0.57	-0.12	-0.00	0.04	-0.01	-0.03	-0.03	0.00	0.02	-0.00	-0.55
$yspr^{\$}_t$	-0.03	-0.09	0.08	0.40	-0.01	-0.03	0.01	0.02	0.01	-0.01	0.02	-0.00	0.30
pd_t	-0.22	-1.03	-0.31	-3.05	0.69	-0.21	-0.35	0.05	0.08	-0.28	-0.07	0.17	3.90
Δd_t	1.69	-0.01	-0.91	-1.36	0.08	0.29	-0.11	-0.30	-0.33	0.17	-0.02	0.10	-1.42
$\Delta \log \tau_t$	1.67	0.31	-0.22	-3.08	0.17	0.07	-0.01	-0.55	-0.18	0.35	-0.09	0.20	-1.80
$\log \tau_t$	1.67	0.31	-0.22	-3.08	0.17	0.07	-0.01	0.45	-0.18	0.35	-0.09	0.20	-1.80
$\Delta \log g_t$	-0.60	-0.96	-1.65	-1.61	-0.11	-0.14	0.07	0.08	0.11	-0.41	0.12	-0.18	-0.39
$\log g_t$	-0.60	-0.96	-1.65	-1.61	-0.11	-0.14	0.07	0.08	0.11	0.59	0.12	-0.18	-0.39
$\Delta \log d_t$	-0.51	-1.60	0.83	2.01	-0.05	0.07	0.13	-0.03	0.12	-0.19	0.21	-0.08	-0.17
$\log d_t$	-0.76	-1.41	2.70	4.71	-0.10	-0.15	0.28	-0.20	0.42	-0.05	-0.54	0.92	5.45
cs_t	-0.08	-0.16	0.12	0.45	0.01	0.02	0.00	0.01	-0.01	0.01	-0.02	0.00	0.45

Table 2 – VAR Coefficients

	π_{t-1}	x_{t-1}	$y(1)^{\$}_{t-1}$	$yspr^{\$}_{t-1}$	pd_{t-1}	Δd_{t-1}	$\Delta \log \tau_{t-1}$	$\log \tau_{t-1}$	$\Delta \log g_{t-1}$	$\log g_{t-1}$	$\Delta \log d_{t-1}$	$\log d_{t-1}$	cs_{t-1}
π_t	0.54	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
x_t	-0.03	1.45	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$y(1)^{\$}_t$	0.58	0.68	0.93	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$yspr^{\$}_t$	-0.05	-0.16	-0.27	0.39	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
pd_t	-4.56	-1.04	2.80	-0.20	13.89	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Δd_t	0.95	1.01	0.05	-0.37	-0.36	4.72	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\Delta \log \tau_t$	0.61	0.44	0.70	0.23	0.92	0.96	4.08	0.00	0.00	0.00	0.00	0.00	0.00
$\log \tau_t$	0.61	0.44	0.70	0.23	0.92	0.96	4.08	0.00	0.00	0.00	0.00	0.00	0.00
$\Delta \log g_t$	-0.93	-2.62	-0.66	-0.25	-0.45	-0.66	-0.50	0.00	2.67	0.00	0.00	0.00	0.00
$\log g_t$	-0.93	-2.62	-0.66	-0.25	-0.45	-0.66	-0.50	0.00	2.67	0.00	0.00	0.00	0.00
$\Delta \log d_t$	-2.32	-3.73	-2.92	-0.48	-0.98	0.52	-1.33	0.00	0.92	0.00	2.10	0.00	0.00
$\log d_t$	-2.32	-3.73	-2.92	-0.48	-0.98	0.52	-1.33	0.00	0.92	0.00	2.10	0.00	0.00
cs_t	-0.33	-0.59	-0.45	-0.16	-0.04	0.01	-0.04	0.00	-0.03	0.00	0.03	0.00	0.29

Table 3 – VAR Cholesky Decomposition ($\Sigma^{\frac{1}{2}} * 100$)

5.2 Asset Pricing

We use an exponentially affine model (Duffie and Kan (1996)) to price the stochastic discount factor. The advantage of this model is that it only assumes no-arbitrage, prices bond yields well, as well as provides a reasonable equity risk premium. The nominal SDF is conditionally log-normal:

$$m^{\$}_{t+1} = -y^{\$}_t(1) - \frac{1}{2}\mathbf{\Lambda_t}'\mathbf{\Lambda_t} - \mathbf{\Lambda_t}'\varepsilon_{t+1} \tag{6}$$

where $m^{\$}_{t+1} = \log(M^{\$}_{t+1})$ the short rate is $y^{\$}_t(1)$ and the $\mathbf{\Lambda_t}$ vector prices the sources of risk in the structural innovations ε_{t+1}. Further, the $\mathbf{\Lambda_t}$ vector is expressed as the combination of an unconditional price of risk and a time varying component.

$$\mathbf{\Lambda_t} = \mathbf{\Lambda_0} + \mathbf{\Lambda_1} z_t$$

Here, $\mathbf{\Lambda_1}$ is a $N \times N$ matrix that provides the time variation in risk premia while $\mathbf{\Lambda_0}$ is the average price of risk in a $N \times 1$ vector.[13] In what follows, the selector vector is defined as $e_{(.)}$.

Nominal Bond Pricing

The nominal yields are given by:

$$y^{\$}_t(h) = -\frac{A^{\$}(h)}{h} - \frac{B^{\$}(h)'}{h} z_t \tag{7}$$

[13]This approach has been used more recently in Jiang, Lustig, Van Nieuwerburgh, and Xiaolan (2019) and Gupta and Van Nieuwerburgh (2021), among others.

Since we have the 5-1 yield spread, we can use it to obtain the following moment restrictions:

$$e'_{y1} + e'_{yspr} = -\frac{1}{5}B_5^{\$} \tag{8}$$

$$y_{0,1}^{\$} + yspr_0 = -\frac{1}{5}A_5^{\$} \tag{9}$$

where $\tilde{\Psi} = \Psi - \Sigma^{\frac{1}{2}}\Lambda_1$ is the risk neutral companion matrix. Then we use equation (3) to fit yields of 2, 10, 20, 30 years from the data.

Real Bond Pricing

The real bond yields are also affine in the state vector:

$$y_t(h) = -\frac{A(h)}{h} - \frac{B(h)'}{h}z_t \tag{10}$$

We match data for real bond yields for 5, 7, 10, 20 and 30 years using the above expression.

Equity Pricing

The log price-dividend ratios on dividend strips are affine in the state vector:

$$pd_t^m(h) = A^m(h) + (B^m(h))'z_t \tag{11}$$

We take the first 3600 dividend strip horizons to match each date's ratio with the data:

$$\exp(\bar{pd} + (e_{pd})'z_t) = \sum_{h=0}^{\infty}\exp(A^m(h) + (B^m(h))'z_t) \tag{12}$$

where $\bar{pd}$ is the mean log price dividend ratio in the data. We also match the equity risk premium.

Estimation We estimate the model by matching federal government bond prices and stock prices in the data with predictions from the model. Appendix 7 contains the relevant moment conditions. The estimated $\mathbf{\Lambda_0}$ and $\mathbf{\Lambda_1}$ are provided below. As can be seen in Figure A.4, we get very good fit for the nominal bond yields at various maturities. In Figure A.5, we also get a decent fit for real bond yields.

$$\mathbf{\Lambda_0} = \begin{bmatrix} 0.00 & 3.65 & 6.30 & -10.65 & 0.00 & 3.12 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 33.00 \end{bmatrix}'$$

$$\mathbf{\Lambda_1} = \begin{bmatrix}
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.00 & -14.94 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & -33.23 & -7.78 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
19.25 & 13.36 & -149.72 & -18.24 & 7.70 & -5.80 & -184.75 & 18.01 & -7.89 & 7.77 & -6.93 & -1.23 & 52.89 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
170.78 & 4.86 & 238.60 & -96.60 & -6.08 & 8.80 & 50.07 & -12.06 & 14.16 & 3.63 & -5.92 & 20.50 & -51.32 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 12.81
\end{bmatrix}$$

5.3 Pricing Local Government Claims

A local government claim W can be decomposed into strips. Each strip is defined by its horizon j at time t, that pays off W_{t+j} at time $t+j$ and nothing otherwise. There will be a relevant holding period return $R^{w,j}_{t,t+j}$ on these strips. The local government component is not part of the state vector z_t. The nominal growth rate of this component is

$$\Delta \log W_{t+1} = \log \frac{W_{t+1}}{W_t} = \log \left[\frac{W_{t+1}}{W_t} \times \frac{Y_{t+1}}{Y_t} \times \frac{Y_t}{Y_{t+1}} \right]$$

Define $w = W/Y$, where Y is the GDP. x is the log GDP growth rate, π is the log inflation.

$$\Delta \log W_{t+1} = \Delta \log w_{t+1} + x_{t+1} + \pi_{t+1}$$

We postulate that the growth rate in local government claim is spanned by the state vector

$$\Delta \log w_{t+1} = w_0 + T' z_{t+1} + U' \eta_{t+1}$$

where w_0 is the average value of w in the data, η_{t+1} is an added shock vector that is orthogonal to those included in ε_{t+1}. With the spanning assumption, we conjecture that the log price-dividend ratio for a given horizon h is affine in the state vector:

$$pd_t^w(h) = A^w(h) + B^w(h)'z_t$$

The Euler equation for the price dividend is given by

$$PD_t^w(h+1) = \mathbb{E}_t \left[M_{t+1} PD_{t+1}(h) \frac{W_{t+1}}{W_t} \right]$$

$$= \mathbb{E}_t \left[\exp\{ m_{t+1} + \Delta \log w_{t+1} + x_{t+1} + \pi_{t+1} + pd_{t+1}^w(h) \} \right]$$

$$= \exp\{-y_0^{\$}(1) - e_{y1}' z_t - \frac{1}{2}\mathbf{\Lambda_t}'\mathbf{\Lambda_t} + A^w(h) + w_0 + x_0 + \pi_0 + (T + e_x + e_\pi + B^w(h))'\mathbf{\Psi} z_t\}$$

$$\times \mathbb{E}_t[\exp\{-\mathbf{\Lambda_t}'\varepsilon_{t+1} + (T + e_x + e_\pi + B^w(h))'\Sigma^{1/2}\varepsilon_{t+1} + U'\eta_{t+1}\}]$$

$$= \exp\{-y_0^{\$}(1) - e_{y1}' z_t + A^w(h) + w_0 + x_0 + \pi_0 + (T + e_x + e_\pi + B^w(h))'\mathbf{\Psi} z_t$$

$$+ \frac{1}{2}(T + e_x + e_\pi + B^w(h))'\Sigma(T + e_x + e_\pi + B^w(h))$$

$$-(T + e_x + e_\pi + B^w(h))'\Sigma^{1/2}(\mathbf{\Lambda_0} + \mathbf{\Lambda_1} z_t) + \frac{1}{2}U'U\}$$

Collecting terms

$$A^w(h+1) = -y_0^{\$}(1) + A^w(h) + w_0 + x_0 + \pi_0 + \frac{1}{2}(T + e_x + e_\pi + B^w(h))'\Sigma(T + e_x + e_\pi + B^w(h)) + \frac{1}{2}U'U$$

$$-(T + e_x + e_\pi + B^w(h))'\Sigma^{1/2}\mathbf{\Lambda_0}$$

and

$$B^w(h+1)' = (T + e_x + e_\pi + B^w(h))'\mathbf{\Psi} - e_{y1}' - (T + e_x + e_\pi + B^w(h))'\Sigma^{1/2}\mathbf{\Lambda_1}$$

Here, we add the appropriate boundary conditions, $A^w(0) = B^w(0) = 0$. The price-dividend ratio of the cum-dividend government claim is

$$\sum_{h=0}^{\infty} \exp(A^w(h+1) + B^w(h+1)'z_t) \tag{13}$$

Pricing: We price revenue and expenditure claims by first projecting its growth rate onto the state variables of our model. The validity of the pricing rests on the assumption that all pricing relevant information is spanned by the state variables. Our model includes 13 state variables, including the municipal credit spread, that captures several sources of pricing relevant risk. The spanning assumption is common in many cross-sectional asset pricing models. In our next iteration, additional tests for e.g. a common component in the residuals will be done. The absence of a common component in residuals would provide some evidence against the possibility of an omitted risk factor with non-negative price of risk. We use equation 6.2 to estimate the price dividend ratio of the claim at each time t.

6 Market Valuation

One of the limitations of the analysis which is based on accounting values is that accounting values are mostly backward looking. For instance, capital projects are often valued at the purchasing cost minus depreciation rather than at the present value of expected generated cash-flow. Further, some of the revenue potential for local governments is not tied to a specific asset; instead it is the result of the privilege to raise taxes. While we think that accounting values carry some merit, we acknowledge its limitations. Thus, we complement the analysis from Section 3 with a market based valuation of local governments' equity.

One of the challenges is that market prices for revenue and expenditure claims are not directly observable in the market. We thus estimate a stochastic discount factor as detailed in Section 5. The stochastic discount factor prices a large set of assets in the economy as shown in Figure A.4 and A.5. This includes municipal debt securities as shown by the close fit with one of the main indices in the municipal bond market as shown in Figure A.3. The existence of a stochastic discount factor is sufficient to price those claims consistently with other assets for which prices are observable (Cochrane, 2009).

6.1 Cross-Sectional Exposure

In the absence of risk the value of a claim is given by its present value discounted at the risk free rate. While this is a convenient benchmark, it is equally unrealistic. Local governments' sales and excise taxes are mechanically tied to the turnover of goods and services in the economy which tend to be pro-cyclical. Similarly, property taxes are to some extent related to the valuations in the local housing market.[14] Hence, local governments' receipts are exposed to aggregate risk which should be reflected in the asset's valuation. An analogous argument applies to government expenditures. While local governments have much less discretion about the timing of expenditures due to the balanced budget requirement than the federal government, we observe a counter-cyclical pattern

[14]The extent to which the property tax revenues follow the valuation depends from state to state due to the autonomy to set property taxes (Reschovsky, 2019). For instance, Giesecke and Mateen (2021) show that local government offset the decline in house prices by increasing the property tax rate in Connecticut. As a result, the effect on total property tax revenues is muted.

in expenditures.

For illustrative purposes let us consider a one factor model first in which the real GDP growth rate is the only risk factor in the economy. While the real GDP growth rate is most likely not the only risk factor, it tends to be one of them in canonical asset pricing models.

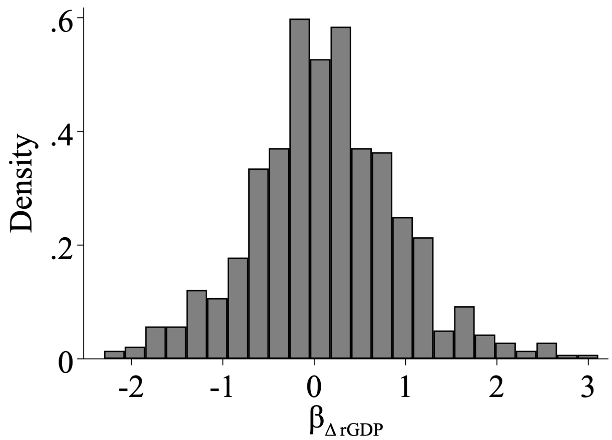
	(1)	(2)
Real GDP growth rate	0.126***	0.563***
	(0.0340)	(0.0844)
Share property tax rate		0.00207
		(0.00201)
Real GDP growth rate × Share property tax rate		-0.635***
		(0.114)
R^2	0.001	0.002
City FE	✓	✓
Observations	26094	26094

Table 4 – Risk Exposure

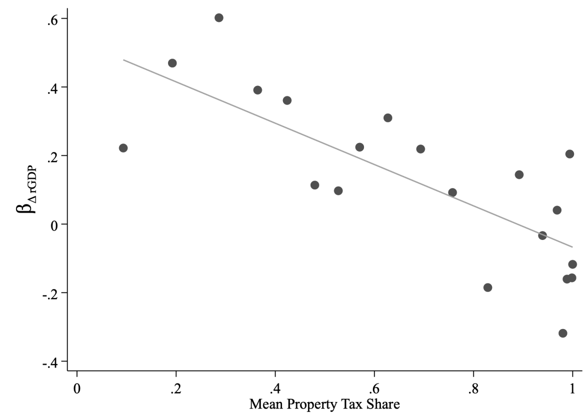
Notes: The tables shows the estimates of the following specification. Column (1) $\Delta lnOwnSource_{it+1} = \alpha_i time_{t+1} + \beta \Delta lnOrGDP_{it+1} + \epsilon_{it+1}$ and column (2) Column (1) $\Delta lnOwnSource_{it+1} = \alpha_i time_{t+1} + \beta_1 Sharepropertytax_i + \beta_2 \Delta lnOrGDP_{it+1} + \beta_3 \Delta lnOrGDP_{it+1} \times Sharepropertytax_i + +\epsilon_{it+1}$ where $Sharepropertytax_i$ is the average share of property taxes as of total own source revenues over the sample horizon for local government i and $time_{t+1}$ is a deterministic time trend.

Column 1 of Table 4 shows that the de-trended growth rate of own source receipts shows exposure to the real GDP growth rate. Importantly, there is substantial cross-sectional heterogeneity in the exposure; column (2) explores the heterogeneity with respect to the share of own source receipts that originate from property taxes. Consistent with the hypothesis above, local governments that receive a large share from property taxes are relatively less exposed to the business cycle fluctuations than local governments with other sources of revenue.

Alternatively, we can estimate the exposure for each local government separately. Figure 7a shows the full distribution of the estimated exposure in the cross-section of local governments. We find a large range of cross-sectional exposure estimates with a mode slightly above zero. The relationship between this individually estimated exposure and the mean share of property taxes of own source revenues is shown in Figure 7b.



(a) CX Exposure Real GDP



(b) Exposure and Mean Property Tax Share

Figure 7 – Exposure Heterogeneity

Notes: The histogram in Panel (a) shows the cross-sectional exposure of $\Delta lnOwnSourceRevenue_{it+1}$ to real GDP growth rate $\Delta lnrGDP_{it+1}$. Specifically, it tabulates the estimates of the exposure $\hat{\beta}_i$ from the following specification: $\Delta lnOwnSourceRevenue_{it+1} = \alpha time_{t+1} + \beta_i \Delta lnrGDP_{it+1} + \epsilon_{it+1}$ which is estimated for each local government i separately and where $time_{t+1}$ is a deterministic time trend. Panel (b) shows a binscatter with 30 bins of the cross-sectional relationship between the exposure estimates $\hat{\beta}_i$ and the average share of property taxes as of total own source revenues over the sample horizon for each local government i.

6.2 Price-to-Dividend Ratios

In the previous Subsection 6.1 we showed the exposure to a single risk factor for illustrative purposes and to create intuition for the origin of some of the heterogeneities. For the pricing we use the full set of state variables as detailed in Section 5. Ultimately, the price-dividend ratio is given by Equation which is exponentially affine in the exposure to the state variables and the corresponding risk premia. Figure 8 summarizes the estimation results by tabulating the price-to-dividend ratios for revenues and expenditures. Interestingly, and to some extent mirroring the insights from Subsection 6.1, we find substantial heterogeneity across local governments in terms of their price-to-dividend ratios.

To interpret the exact numbers of price-to-dividend ratios we find it is useful to start from a risk-free benchmark. If revenues were determinstic and just for the sake of the argument, assuming that the relevant risk-free interest rate is 3% (for reference, if we take the 30 year treasury which hovered around 3% during this sample period), one would expected a price-to-dividend ratio of 1/0.03 = 33.3. Estimated price-to-dividend ratios of revenues are smaller–for some much smaller–than this risk-free benchmark which points

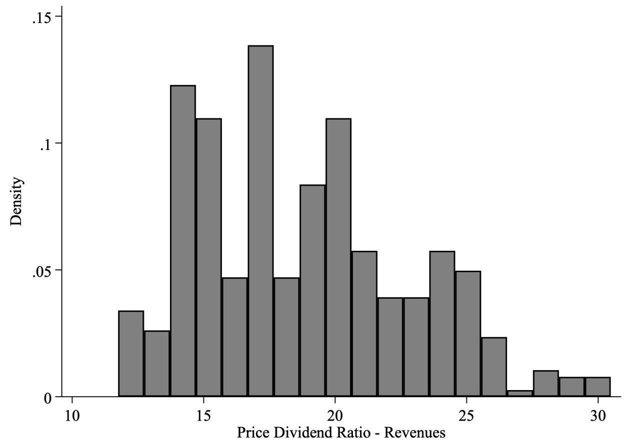


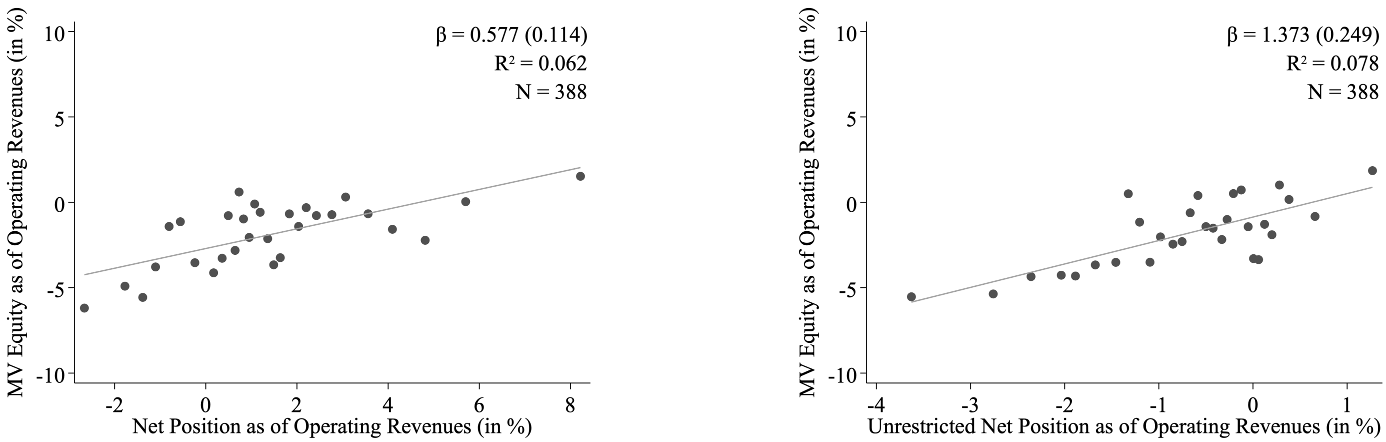
Figure 8 – Price-to-Dividend Ratio Revenues

to pro-cyclical revenues of local governments. This pattern of revenues mirrors the procyclicality of federal government revenues as reported in Jiang, Lustig, Van Nieuwerburgh, and Xiaolan (2019).

6.3 Book values vs. Market values

The market price-to-dividend ratios allow us to compute the market value of equity. In particular, we multiply the contemporaneous revenues and expenditures with the corresponding price-to-dividend ratio and deduct the value of the outstanding liabilities.[15] This provides us with a measure of market equity for each local government. Table 9 shows the cross-sectional relationship between the market value of equity and the two accounting measure of market values from the ACFRs, that is, the net position and the unrestricted net position. For comparability across local governments all values are normalized by the operating revenues. We find a positive relationship between the book values and the market values for the net position and a stronger positive relationship for the unrestricted net position. The stronger relationship for the unrestricted net position is not surprising since the inclusion of capital assets into the net position adds additional variation which is not necessarily reflected in its revenue generating potential.

[15]We avoid double counting of cash-flows to debt, other post employment benefits and pension by deducting the interest expenses, the pension and OPEB contributions from current expenditures.



(a) Net Position

(b) Unrestricted Net Position

Figure 9 – Market vs. Book Values

Notes: Panel (a) shows the cross sectional relationship between the market value as of operating revenues and the net position as of operating revenues in 2019. Panel (b) shows the cross sectional relationship between the market value as of operating revenues and the unrestricted net position as of operating revenues in 2019.

6.4 Equity Value of Local Governments and Role of Implicit Insurance

Is the market value of equity really negative for certain local governments? If that were the case, it would mean that these local governments are insolvent, and debt markets should not be issuing debt to these entities. But we do not see this. In fact, municipal yields even for negative equity value governments have remained quite low and stable.

In this section, we offer a possible resolution of the contrasting evidence. The accounting framework in Equation 1 does not account for the insurance offered by governments at the state and federal level. For example, it may be argued that federal and state governments are implicit guarantors of local government debt.[16]

Define IS as the insurance:

$$\begin{aligned} IS = \max\{0, &PV(Expenditures) + PV(PensionObligations) \\ &+PV(OPEB) + PV(Debt) - PV(Revenues) - Cash\} \end{aligned} \tag{14}$$

We have defined insurance as the amount that makes market value of equity exactly

[16]Local governments also have taxing power, and with the caveat that any increase in taxes may lower the tax base, local governments may offer implicit insurance to any outstanding debt through their taxing power. However, by using 40 years of data of the stochastic properties of local government expenditure, we are already capturing these patterns, including their covariance with the stochastic discount factor.

zero. Naturally, the actual insurance could be higher than this; as such our definition is a *lower bound* on the actual insurance. Our approach assumes that there is no mispricing by the municipal bond market and that a simple accounting of *visible* components of cash flow does not consider the implicit insurance value of governmental support. Start with debt. We take debt at its face value and we assume that the yields represent the expectations of market participants as to the default probability of the municipality, *all factors considered*, including insurance. On the pensions side, there are constitutional safeguards protecting the claims of pensioners, with court judgements ruling that these claims are senior to bond holder debt (see Spiotto (2013)). We discount these claims by the risk free rate, thereby following a rich literature investigating the riskiness of pension payouts (see Novy-Marx and Rauh (2011a)) that concludes that pension payouts are unrelated to the business cycle and therefore should be discounted at the risk free rate.

It is difficult to directly measure the insurance cash flow because of the absence of sufficient counterfactuals of bailouts of local governments, barring a handful of examples like Detroit and San Bernandinho. What can be observed, however, are constraints on the amount of insurance that can be given out. For example, certain U.S. states give Chapter 9 protection to their municipalities. Other states have proactive policies to assist municipalities, a form of insurance. Gao, Lee, and Murphy (2019) find that Chapter 9 states have higher bond yields in the muni market. Under our interpretation, we expect to find a less negative value of market equity in Chapter 9 states, precisely because the "hidden" insurance payout is smaller in these municipalities. On the level of local municipalities themselves, we expect to find a more negative value of market equity in states which give more autonomy to municipalities to set their taxes and revenue policies. The autonomy has been measured in different ways in the literature. Shoag, Tuttle, and Veuger (2019) differentiate between home rule and non-home rule states. Home rule states give more autonomy to municipalities and we expect a more negative market value of equity, all else equal. Another method of measuring autonomy is offered by Reschovsky (2019) who ranks states based on a methodology from the OECD.

7 Conclusion

Using a novel data set on the fiscal position of municipalities across the US we document the deterioration of municipalities' fiscal positions. Besides the overall deterioration, a substantial share of municipalities operate with a negative net position—akin to a negative book equity position in the corporate context. Book valuations may provide an incomplete assessment of local government's solvency as it follows a rigid set of governmental accounting standards which are predominantly backward looking. Thus, we assess the market value of equity by pricing the main components of local governments' balance sheet. We do this by estimating a exponentially affine term-structure model that prices a broad array of assets in the economy. We use the model to price a broad cross-section of municipal claims. To the best of our knowledge, this is the first paper to be able to price such a broad array of assets. By being able to provide a market value measure of municipalities equity, we show that book values are positively related to the market valuations which further supports the initial assessment of local governments' fiscal position. Somewhat surprisingly, the municipal bond market discriminates the differences in the fiscal position only to a limited extent. We attribute the limited distinction in credit to the belief in implicit insurance by the state and federal government. Future work includes considering potentially different specifications for our pricing kernel and further validating our estimated valuation multiplier.

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Tables and Figures:

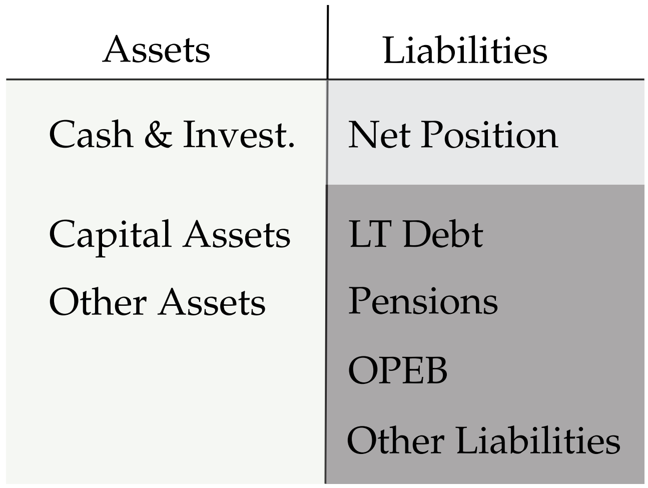


Figure A.1 – Schematic Balance Sheet

Assets	mean	p25	p50	p75	count
Share Cash Investments	0.24	0.16	0.21	0.29	1,803
Share Receivables	0.07	0.02	0.05	0.10	1,803
Share LT. Illiquid Assets	0.06	0.03	0.05	0.08	1,803
Share Capital Assets	0.63	0.56	0.65	0.73	1,803
Liabilities	mean	p25	p50	p75	count
Share Net OPEB	0.15	0.01	0.04	0.17	1,803
Share Net Pension	0.15	0.03	0.09	0.21	1,803
Share Lt. Debt	0.25	0.11	0.21	0.32	1,803
Share Other Current Liabilities	-0.04	-0.05	-0.03	-0.02	1,803
Share Other Non-Current Liabilities	0.08	0.03	0.06	0.11	1,803
Share Notes and Loans	-0.00	0.00	0.00	0.00	1,803
Share Net Position	0.35	0.18	0.46	0.65	1,803

Table A.1 – National Sample - Balance Sheet 2018

Notes: The table tabulates the main asset and liability positions of the Statement of Net Asset Position in the ACFR of municipalities across the United States. The sample contains all local governments for which information on the unrestricted net position is available in 2007 and 2018. Data is obtained from Moody's Investor Services.

	mean	p25	p50	p75	count
Operating Revenues 2018 (in '000)	161871.62	16231.00	36396.78	83926.94	1,803
GF Balance as of Op. Rev 2007 (%)	25.94	11.89	20.58	34.74	1,802
GF Balance as of Op. Rev 2018 (%)	33.40	16.83	26.93	42.74	1,802
Total liability over EGL 2007 (%)	-1.12	-1.83	-0.98	-0.45	1,726
Total liability over EGL 2018 (%)	-3.22	-3.97	-2.34	-1.18	1,784
Δ Total liability over EGL 07-18 (%)	-1.88	-2.49	-1.10	-0.29	1,719
Unr. Net. Pos. as of Op. Rev 2007 (%)	32.54	11.50	28.40	54.43	1,803
Unr. Net. Pos. as of Op. Rev 2018 (%)	-34.99	-84.62	-18.97	22.08	1,803
Δ Unr. Net. Pos. as of Op. Rev 07-18 (%)	-67.53	-112.86	-59.18	-14.73	1,803
Fraction Negative Unr. Net. Pos. 2018	0.61	0.00	1.00	1.00	1,803
Fraction Negative Net Position 2018	0.15	0.00	0.00	0.00	1,803
Net OPEB as of Op. Rev 2018 (%)	-34.89	-50.61	-11.96	-2.12	1,803
Net Pension as of Op. Rev 2018 (%)	-43.04	-62.04	-28.77	-8.93	1,803
Population (Census 2010)	59435.05	10292.00	21193.00	46746.00	1,803
Median House Value (Census2010)	266039.45	135700.00	210800.00	330600.00	1,803
Per Capita Income (ACS 2010)	31609.13	22418.00	27941.00	36467.00	1,802
Share 65+ Age (Census2010)	0.14	0.11	0.14	0.17	1,803
Share White (Census2010)	0.81	0.74	0.87	0.93	1,803
Share Black (Census2010)	0.08	0.01	0.03	0.10	1,803
Share Asian (Census2010)	0.04	0.01	0.02	0.04	1,803
Home Ownership (Census2010)	0.67	0.56	0.66	0.78	1,803

Table A.2 – Summary Statistics National Sample

Notes: The sample contains all local governments for which information on the unrestricted net position is available in 2007 and 2018. Counts of less than 1803 observations indicate missing data. The table follows the sign convention that liabilities are expressed as a negative values. Data is obtained from Moody's Investor Services.

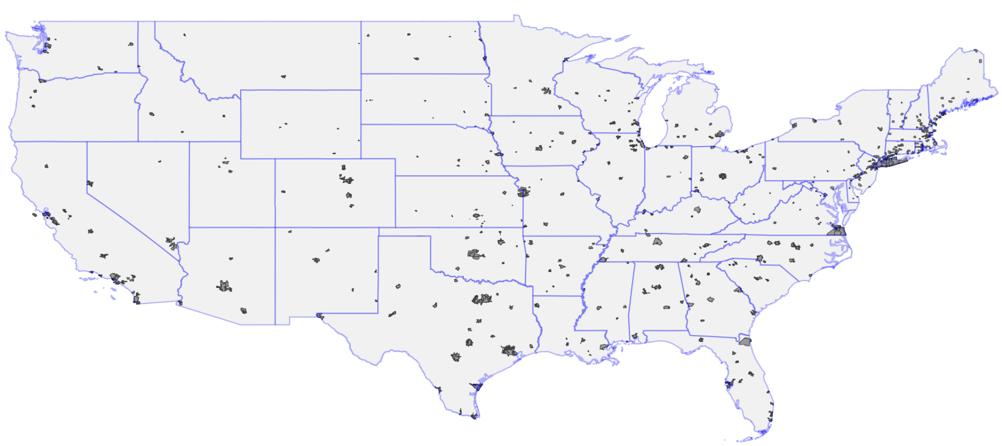


Figure A.2 – Census Certainty Sample

Notes: The sample contains all cities that are part of the Census certainty sample. It comprises a total of Shape files for local governments are self-constructed as described in the Data Appendix DA.7.2. Data is obtained from the Census of Governments, the Census Bureau and Moody's Investor Services.

	mean	p25	p50	p75	count
Population (Census 2010)	152868.87	27692.00	68132.50	140768.00	622
Median House Value (Census 2010)	225489.71	114900.00	176900.00	279100.00	622
Per Capita Income (ACS 2010)	27081.30	20906.00	24478.00	30949.00	621
Share 65+ Age (Census 2010)	0.13	0.11	0.13	0.15	622
Share White (Census 2010)	0.72	0.59	0.76	0.88	622
Share Black (Census 2010)	0.14	0.02	0.06	0.20	622
Share Asian (Census 2010)	0.04	0.01	0.02	0.05	622
Home Ownership (Census 2010)	0.59	0.51	0.59	0.66	622
Unr. Net Position as of Op. Rev 2017 (%)	-42.96	-90.41	-30.13	13.70	560
Unr. Net Position as of Op. Rev 2018 (%)	-42.00	-79.60	-26.30	6.55	560
Fraction Negative Unr. Net. Pos. 2018	0.69	0.00	1.00	1.00	560
Fraction Negative Net. Pos. 2018	0.22	0.00	0.00	0.00	560
Total Liabilities of Full Value 2017 (%)	-4.00	-4.72	-2.93	-1.57	530
Total Liabilities of Full Value 2018 (%)	-4.63	-5.30	-3.05	-1.70	530
Net Position as of Op. Rev 2017 (%)	185.94	49.01	138.36	283.73	560
Net OPEB as of Op. Rev 2017 (%)	-24.35	-34.27	-8.39	-2.24	567
Net Pension as of Op. Rev 2017 (%)	-65.67	-99.57	-52.14	-20.60	567
Long Term Debt as of Op. Rev 2017 (%)	-96.58	-126.57	-76.15	-42.07	567

Table A.3 – Summary Statistics Census Certainty Sample

Notes: The table tabulates the summary statistics for the Census certainty sample. Counts of less than 622 observations indicate missing data. The table follows the sign convention that liabilities are expressed as a negative values. Data is obtained as described in Section 2.

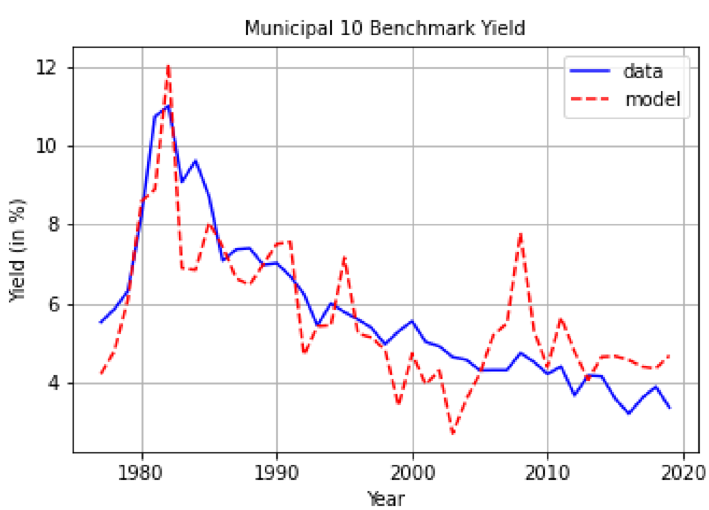


Figure A.3 – SDF - Municipal Bond Yield Index

Notes: The figure plots the model implied and the actual municipal bond yield. The municipal bond yield is the 10 year reference yield from BondBuyer for the time horizon 1977-2019. The data comes from Bloomberg.

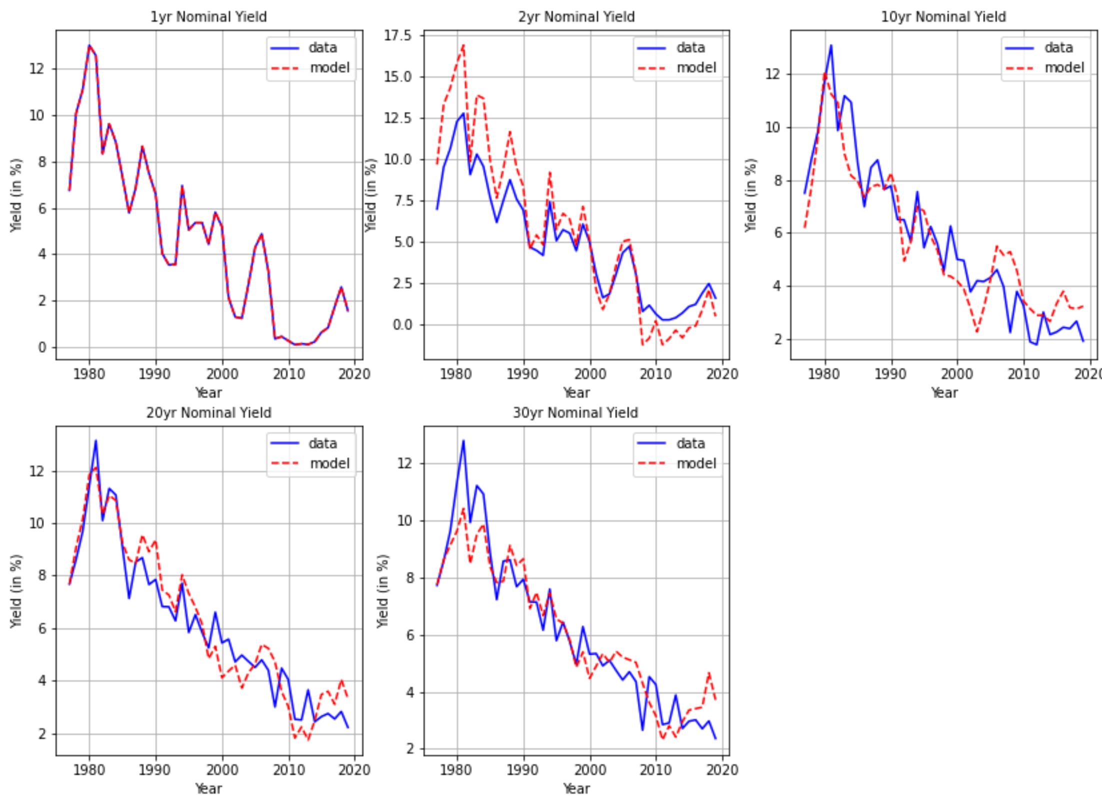


Figure A.4 – SDF - Nominal Yields

Notes: The figure plots the model implied and the data for the 1yr, 2yr, 10yr, 20yr, and 30yr treasury yield for the time horizon 1977-2019. The data comes from FRED and FRASER.

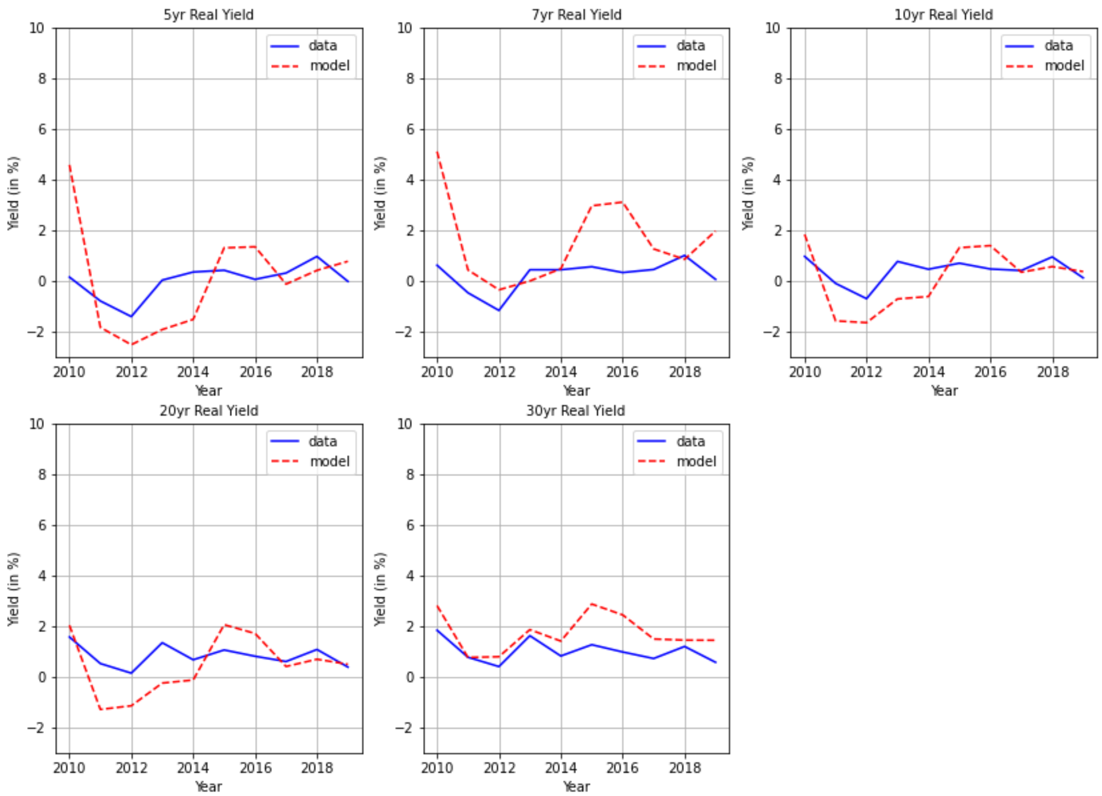


Figure A.5 – SDF - Real Yields

Notes: The figure plots the model implied and the data for the 5yr, 7yr, 10yr, 20yr, and 30yr TIPS yield for the time horizon 2010-2019. The data comes from FRED.

Appendix B: Moment Conditions

Call Ψ the companion matrix, Σ the covariance matrix, and $\Sigma^{\frac{1}{2}}$ the cholesky decomposition of the covariance matrix. The selector vector is defined as $e_{(.)}$.

B.7.1 Moment Conditions

Nominal Bond Pricing
The nominal yields are given by:

$$y_t^{\$}(h) = -\frac{A^{\$}(h)}{h} - \frac{B^{\$}(h)'}{h} z_t \tag{15}$$

Since we have the 5-1 yield spread, we can use it to obtain the following moment restriction:

$$e'_{y1} + e'_{yspr} = \frac{1}{5} e'_{y1}(I - \tilde{\Psi}^5)(I - \tilde{\Psi})^{-1} \tag{16}$$

$$y_{0,1}^{\$} + yspr_0 = -\frac{1}{5} A_5^{\$} \tag{17}$$

where $\tilde{\Psi} = \Psi - \Sigma^{\frac{1}{2}} \Lambda_1$. Then we use equation (1) to fit yields of 2, 10, 20, 30 years from the data. For that we need to calculate the $A^{\$}, B^{\$}$.

$$A_{\tau+1}^{\$} = -y_{0,1}^{\$} + A_{\tau}^{\$} + \frac{1}{2}(B_{\tau}^{\$})' \Sigma (B_{\tau}^{\$}) - (B_{\tau}^{\$})' \Sigma^{\frac{1}{2}} \Lambda_0 \tag{18}$$

$$(B_{\tau+1}^{\$})' = (B_{\tau}^{\$})' \Psi - e'_{y1} - (B_{\tau}^{\$})' \Sigma^{\frac{1}{2}} \Lambda_1 \tag{19}$$

Thus, we have 12 restrictions from equation (2), 1 from equation (3), and $4 \times T$ restrictions from equations (1),(4),(5). We overweight the 30 year yield.

Real Bond Pricing
The real bond yields are also affine in the state vector:

$$y_t(h) = -\frac{A(h)}{h} - \frac{B(h)'}{h} z_t \tag{20}$$

We match data for real bond yields for 5, 7, 10, 20 and 30 years using the above expression and using:

$$A(h+1) = -y_0(1) + A(h) + \frac{1}{2}(B(h))' \Sigma (B(h)) - (B(h))' \Sigma^{\frac{1}{2}} (\Lambda_0 - \Sigma^{\frac{1}{2}\prime} e_{\pi}) \tag{21}$$

$$(B(h+1))' = -(e_{y1})' + (e_\pi + B(h))'(\Psi - \Sigma^{\frac{1}{2}}\Lambda_1) \tag{22}$$

where the real yield is given by:

$$y_0(1) = y_0^{\$}(1) - \pi_0 - \frac{1}{2}(e_\pi)'\Sigma e_\pi + (e_\pi)'\Sigma^{\frac{1}{2}}\Lambda_0 \tag{23}$$

So we have another set of moment conditions, $T_2 \times 5$. We overweight the 30 year yield.

Equity Pricing
The log price-divident ratios on dividend strips are affine in the state vector:

$$pd_t^m(h) = A^m(h) + (B^m(h))'z_t \tag{24}$$

We take the first 3600 dividend strip horizons to match each date's ratio with the data:

$$\exp(\bar{pd} + (e_{pd})'z_t) = \sum_{h=0}^{\infty} \exp(A^m(h) + (B^m(h))'z_t) \tag{25}$$

where $\bar{pd}$ is the mean log price dividend ratio in the data, and the $A^m(h), B^m(h)$ are:

$$\begin{aligned} A^m(h+1) = A^m(h) + \mu^m - y_0(1) + \frac{1}{2}(e_{dgr} + B^m(h))'\Sigma(e_{dgr} + B^m(h)) \\ - (e_{dgr} + B^m(h))'\Sigma^{\frac{1}{2}}(\Lambda_0 - (\Sigma^{\frac{1}{2}})'e_\pi) \end{aligned} \tag{26}$$

$$B^{m'}(h+1) = (e_{dgr} + e_\pi + B^m(h))'\Psi - e'_{y1} - (e_{dgr} + e_\pi + B^m(h))'\Sigma^{\frac{1}{2}}\Lambda_1 \tag{27}$$

where μ^m is the mean dividend growth rate in the data. Finally, we match the equity risk premium using the restriction:

$$(e_{dgr} + \kappa_1^m e_{pd} + e_\pi)'\Psi - e'_{pd} - e'_{y1} = (e_{dgr} + \kappa_1^m e_{pd})'\Sigma^{\frac{1}{2}}\Lambda'_1 + e'_\pi\Sigma^{\frac{1}{2}}\Lambda_1 \tag{28}$$

where $\kappa_1^m = \frac{\exp(\bar{pd})}{\exp(\bar{pd})+1}$. So there are 12 restrictions from equation (14), and $T \times 1$ restrictions from (11), (12), (13).
The unconditional risk premium can be determined in the following way:

$$r_0^m + \pi_0 - y_{0,1}^{\$} + \frac{1}{2}e'_\pi\Sigma e_\pi + \frac{1}{2}(e_{divm} + \kappa_1^m e_{pd})'\Sigma(e_{divm} + \kappa_1^m e_{pd}) + e'_\pi\Sigma(e_{divm} + \kappa_1^m e_{pd}) = (e_{divm} + \kappa_1^m e_{pd} + e_\pi)\Sigma^{\frac{1}{2}}\Lambda_0$$

where r_0^m is the unconditional mean log real stock return in the data.

B.7.2 Regularization Conditions

Sharpe Ratio

We know that the SDF is exponentially affine and has the following form:

$$m_{t+1}^{\$} = -y_t^{\$}(1) - \frac{1}{2}\Lambda_t'\Lambda_t - \Lambda_t'\varepsilon_{t+1}$$

It is easy to see that:

$$E_t m_{t+1}^{\$} = -y_t^{\$}(1) - \frac{1}{2}\Lambda_t'\Lambda_t$$

$$Var_t(m_{t+1}^{\$}) = \Lambda_t'\Lambda_t$$

It can be shown that the log price of a n-period bond is given by

$$p_{nt} = p_{1t} + E_t p_{n-1,t+1} + \frac{1}{2} var_t p_{n-1,t+1} + cov_t(m_{t+1}^{\$}, p_{n-1,t+1})$$

The Sharpe ratio is defined as the ratio of the expected excess return over the square root of the variance of the excess return. The expected excess return is

$$E_t p_{n-1,t+1} - p_{nt} + p_{1t} + \frac{1}{2} var_t(p_{n-1,t+1}) = -cov_t(m_{t+1}^{\$}, p_{n-1,t+1})$$

Therefore, the Sharpe ratio is given by

$$\theta_t = \frac{-cov_t(m_{t+1}^{\$}, p_{n-1,t+1})}{\sqrt{var_t(p_{n-1,t+1})}} = -cor_t(m_{t+1}^{\$}, p_{n-1,t+1})\sqrt{Var_t(m_{t+1}^{\$})}$$

The constraint on the maximum Sharpe ratio is (when the correlation term is -1)

$$\sqrt{Var_t(m_{t+1}^{\$})} = \sqrt{\Lambda_t'\Lambda_t} < 1.5$$

Nominal and Real Yields

We impose that:

$$y_t^{\$}(h) - y_t(h) \geq (e_\pi)'\Sigma^{\frac{1}{2}}\Lambda_0$$

where we use equations (1) and (6) for the LHS. This restriction is to be tested at maturities of 100, 500, 1000, 2000, 3000, 4000 years.

The following three restrictions are also imposed for the 6 long-term maturities:

1. Nominal yields exceed nominal GDP growth rate. Real yields exceed real GDP

growth rate.

$$y_t^{\$}(h) \geq .0623$$

$$y_t(h) \geq .0304$$

2. The difference between nominal and real yields must exceed the long run inflation.

$$y_t^{\$}(h) - y_t(h) \geq .0318$$

Bond Return Volatilities
The bond return is given by:

$$\begin{aligned} r_{t+1}^{b,\$}(h) &= log(P_{t+1}^{\$}(h)) - log(P_t^{\$}(h+1)) \\ &= A^{\$}(h) - A^{\$}(h+1) + B^{\$}(h)z_{t+1} - B^{\$}(h+1)z_t \end{aligned}$$

Therefore, the bond return volatility:

$$Var(r_{t+1}^{b,\$}(h))$$

We impose the following condition on bond return volatilities for maturities of 100, 500, 1000, 2000, 3000, 4000 years:

$$Var(r_{t+1}^{b,\$}(h)) \geq .2$$

This can be shown to be equal to

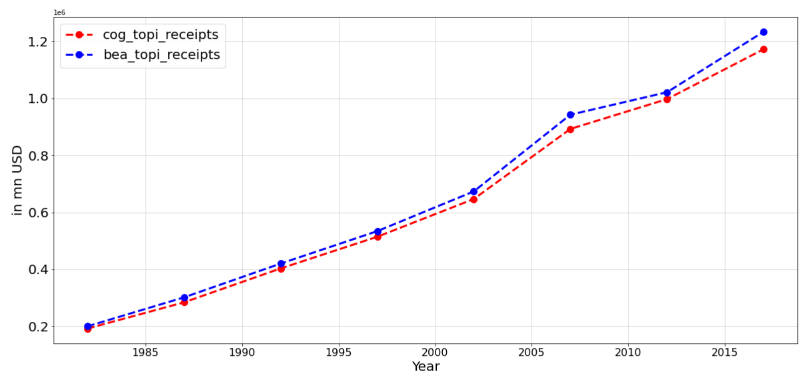
$$Var(r_{t+1}^{b,\$}(h)) = B^{\$}(h)'\Sigma B^{\$}(h) \geq .2$$

Eigenvalue
The maximum eigenvalue of the risk neutral companion matrix $\Psi - \Sigma^{\frac{1}{2}}\Lambda_1 < 1$.

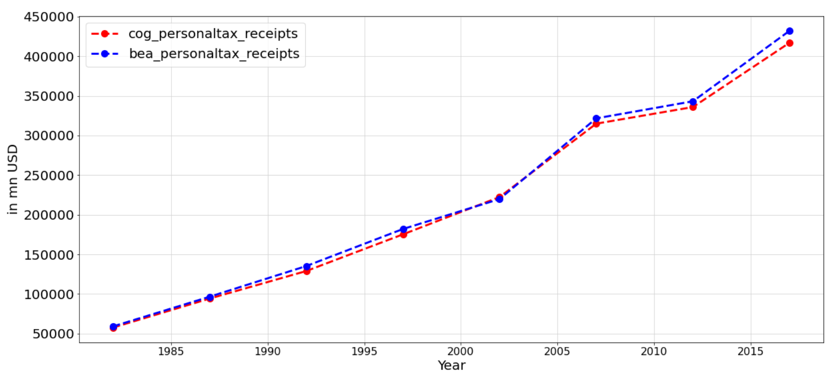
Data Appendix

DA.7.1 Annual Survey of State and Local Government Finances

We perform extensive comparisons of the time series as constructed from the Annual Survey of State and Local Government Finances (ASSLGF) and BEA NIPA. For that we aggregate the cross-section of state and local governments and compare the resulting time series against the National Accounts Table 3.3 from the U.S. Bureau of Economic Analysis. Below the most relevant items are shown. The full set of comparisons can be obtained from the author upon request.



(a) Taxes on Production and Imports



(b) Personal Tax Receipts

Figure DA.1 – NIPA and ASSLGF Tax Revenue

Notes: Panels plot the time series of tax revenues from the Census Bureau Annual Survey of State and Local Government Finances against the National Accounts Table 3.3 from the U.S. Bureau of Economic Analysis. Years ending on "2" and "7" are full census years. In the intermediate period only a subset of observations are observed; remaining missing values are interpolated according to the Census Bureau's interpolation method. Taxes on production and imports include property tax, sales tax, excise tax, and other taxes on production and imports. Personal taxes subsume personal income tax and personal other taxes.

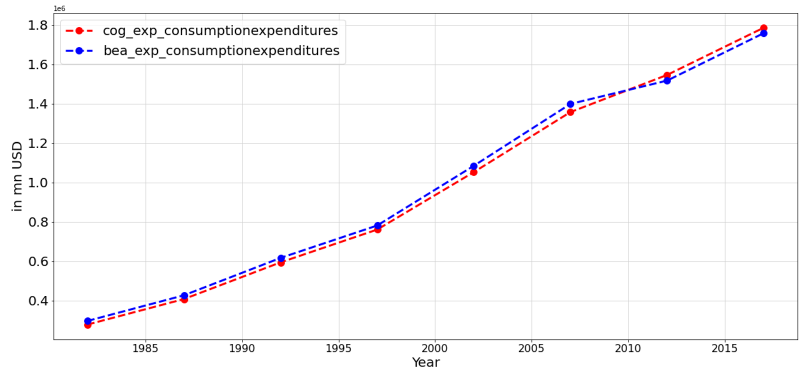


Figure DA.2 – NIPA and ASSLGF Consumption Expenditures

Notes: The figure plots the time series of consumption expenditures from the Census Bureau Annual Survey of State and Local Government Finances against the National Accounts Table 3.3 from the U.S. Bureau of Economic Analysis. Years ending on "2" and "7" are full census years. In the intermediate period only a subset of observations are observed; remaining missing values are interpolated according to the Census Bureau's interpolation method. Consumption expenditures include current expenditures on fire protection, parks and recreation, natural resources, corrections, hospitals, health expenditures, other current expenditures, primary and secondary education, higher education, education n.e.c., central staff expenditures, judicial, libraries, financial administration, solid waste, general building, police, and protective inspection

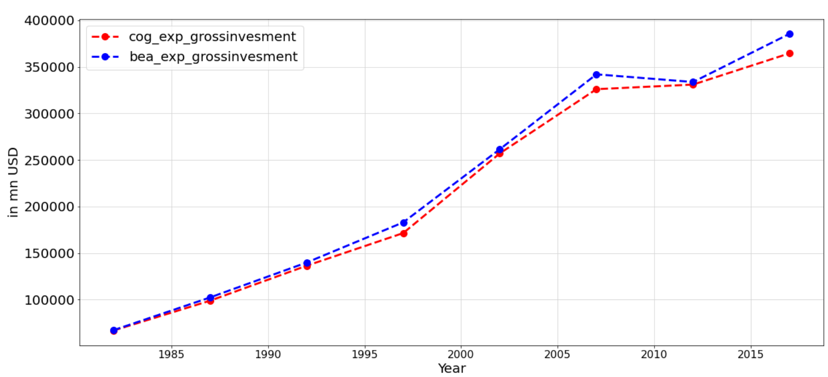


Figure DA.3 – NIPA and ASSLGF Gross Investment

Notes: The figure plots the time series of consumption expenditures from the Census Bureau Annual Survey of State and Local Government Finances against the National Accounts Table 3.3 from the U.S. Bureau of Economic Analysis. Years ending on "2" and "7" are full census years. In the intermediate period only a subset of observations are observed; remaining missing values are interpolated according to the Census Bureau's interpolation method. Gross investment includes capital investments for port facilities, water utilities, highways, air transport, and capital expenditures n.e.c., natural resources, parks and recreation, education, protective and inspection, solid waste, corrections, libraries, general buildings, parking facilities, liquor stores, transit utilities, sewage, electric utilities, fire protection, central staff, health infrastructure, policy, housing, judicial, financial administration, and gas utilities.

DA.7.2 Municipal Shapefiles

Shapefiles for municipalities are not readily available. We construct shape files at the municipal level across the United States by combining information from the Census of Government and shape files from the Census Bureau for places and county subdivisions.

We proceed as follows: First, we select all city and town governments from the Census of Government, that is, units with unit indicator 2 and 3. These city and town governments have a self-governing structure which allow the execution of governmental and administrative functions. As cities were founded and developed throughout the history of the United States, the Census Bureau added additional statistical by necessity.[17] As a result, there is no uniform statistical unit that reflects all city and town governments. Nevertheless, some patterns have emerged.[18]

In the northeast and midwest incorporated townships often correspond to county subdivisions.[19] In the remainder of the United States, the local governments typically correspond to Census places–that is, urban agglomerations with a self-governing structure.

	Count
Census Place	19,393
Census County Subdivision	16,113
Total	35,506

Table DA.1 – Summary Statistics Geographies

[17]The Census Bureau defines two major statistical/geographical areas at the sub-county level: (i) minor civil divisions (MCDs) and (ii) census county divisions (CCDs). While minor civil divisions have legal boundaries and names, as well as, governmental functions or administrative purposes specified by state law, census county divisions are county division mainly for statistical purposes. Many states in the southern and western parts of the United States had few sub-county governmental units; as a result, census county division were introduced starting in the 1950s.

[18]A detailed description and chronology is provided in https://www2.census.gov/geo/pdfs/reference/GARM/Ch8GARM.pdf

[19]A detailed correspondence is tabulated in https://www2.census.gov/geo/pdfs/reference/GARM/Ch8GARM.pdf, Table 8-2. In the northeast and midwest, local governments simultaneously correspond to a Census place and a Census subdivision. In those cases, we found that the geographical delineation coincides