Superhuman science: How artificial intelligence may impact innovation

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Abstract

New product innovation in fields like drug discovery and material science can be characterized as combinatorial search over a vast range of possibilities. Modeling innovation as a costly multi-stage search process, we explore how improvements in Artificial Intelligence (AI) could affect the productivity of the discovery pipeline in allowing improved prioritization of innovations that flow through that pipeline. We show how AI-aided prediction can increase the expected value of innovation and can increase or decrease the demand for downstream testing, depending on the type of innovation, and examine how AI can reduce costs associated with well-defined bottlenecks in the discovery pipeline. Finally, we discuss the critical role that policy can play to mitigate potential market failures associated with access to and provision of data as well as the provision of training necessary to more closely approach the socially optimal level of productivity enhancing innovations enabled by this technology.
1. Introduction

In November 2020, Google DeepMind’s AlphaFold won the 14th round of the CASP\(^1\) protein folding competition. They used a type of AI, machine learning using deep neural networks, to help predict the 3D structure of target proteins based on their amino acids sequences. The “protein folding problem” is challenging due to the vast number of potential shapes that a protein could take for a particular amino acid sequence. Knowing the shape of proteins is, among other uses, critical for identifying targets for drugs to bind to in order to produce therapeutic effects.

“This will change medicine. It will change research. It will change bioengineering. It will change everything,” says Andrei Lupas, an evolutionary biologist at the Max Planck Institute for Developmental Biology in Tübingen, Germany, who assessed the performance of different teams in CASP. AlphaFold has already helped him find the structure of a protein [in half an hour] that has vexed his lab for a decade, and he expects it will alter how he works and the questions he tackles… (from Nature, November 2020)\(^2\)

This is just one example of the use of AI in scientific discovery and innovation. Machine-learning-based AI tools are being increasingly applied where innovators must search over large and complex search spaces, promising to improve the productivity of the innovation process in various domains that have proved challenging for researchers, such as materials science, proteomics, genomics, and drug discovery.

Although there is extensive debate over the prospects for artificial general intelligence (AGI), the AI we consider in this paper is narrow, helping the innovator on the specific task of predicting which innovations are most likely to succeed. On this narrow task the AI can be superhuman in the sense of surpassing human-level performance. Notwithstanding this superhuman capacity at specific tasks, we do not assume that the AI replaces the human scientist, but rather explore how the combination of the AI and scientists can increase discovery through improved prioritization.

\(^{1}\) Community Wide Experiment on the Critical Assessment of Techniques for Protein Structure Prediction (CASP).

\(^{2}\) https://www.nature.com/articles/d41586-020-03348-4
We develop a model of innovation that includes two stages: prediction and testing.\(^3\) Prediction by human scientists is characterized as formulating hypotheses based on theory and prior empirical evidence. Prediction by AIs is characterized as pattern recognition based on prior experimental data. In our setting, testing remains fully in the domain of human scientists and involves running experiments to test the predictions from the first stage. In other words, AIs only impact the first stage: prediction.

The model is motivated by five main ideas. First, innovation can be viewed as search over a (potentially vast) combinatorial search space. Second, innovation results from a costly multi-stage process. Third, the uncertainty innovators face as to the location of valuable combinations in this space can be partly reduced by having a prediction model – or map – of the underlying search landscape. Fourth, the output of the prediction model can usefully be summarized in the form of a ranking function that allows prioritization in the context of the costly multi-stage search process. And fifth, breakthroughs in AI are a potential source of improvement in the performance of these maps.

The main contribution of this paper is to develop a framework for linking an AI-based improvement in prediction technology to the economic effects on innovation in combinatorial-type discovery problems. Using the device of the ranking function, we show the effects of the improved technology on the expected marginal values and costs of potential innovations. However, the model also shows how the impact on innovation depends in subtle ways on features of the innovation search process such as whether the search is parallel or sequential, whether the potential innovations are independent or are substitutes, and the extent of bottlenecks in the discovery pipeline.

Scholars have long modeled innovation as a process of combining existing knowledge to produce new knowledge (Usher, 1929; Schumpeter, 1939; Nelson and Winter, 1982; Weitzman, 1998; Fleming and Sorenson, 2004; Arthur, 2009). The idea of new knowledge as new combinations has received particular attention where the mapping from existing knowledge inputs to new knowledge is highly complex. Examples of such domains of discovery include biotechnology, molecular and materials science, and particle physics. Recent advances in machine

\(^3\) We later extend this to multiple stages by including intermediate screening stages between prediction and testing.
learning – and in particular advances in artificial neural networks such as deep learning – have led to optimism that these advances provide a new general purpose technology (GPT) for discovery (Agrawal et al., 2019a; Cockburn et al., 2019). These tools are already altering innovation practice in fields such as genomics, proteomics, small molecule drug discovery, and materials science, even if there is skepticism about their ultimate impact on R&D productivity.

To help understand how AI might affect innovation we build on the classic innovation function approach to economic growth to develop a model of AI-aided innovation. This approach models innovation as a function of research effort and existing knowledge stocks (Romer, 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992; Jones, 1995). In our model, the innovator must search over a vast space of potential combinations. Innovators use knowledge in the form of data on past successes and failures to develop a prediction model for the “fitness landscape” that maps combinations to the probability of success of those combinations. In essence, the model of the fitness landscape aids their search in the context of a multi-stage search process by helping to predict which combinations out of possibly billions have the greatest likelihood of success. We thus treat a key part of the discovery process as a prediction problem that can benefit from recent advances in AI.

A summary measure of the output of the prediction model is a logistic ranking function that shows how the probability of success declines as we move from better- to worse-ranked combinations. This function plays a key role in the prioritization process for a multi-stage discovery pipeline where the later stages – screening and testing – are costly, and access to improved technologies for prediction can better prioritize the use of costly R&D resources. In the context of a multi-stage discovery pipeline with potentially multiple intermediate screens between initial prediction and final (determinative) testing, we examine how bottlenecks in the pipeline in the form of high costs and ineffective screens can reduce the number of potential innovations that enter the pipeline. We also explore how such bottlenecks can reduce the return to the adoption of AI-based prediction and the possibilities for AI improving – or even substituting for – later costly stages of the pipeline.

The challenge for policy makers is to ensure that the AI-driven technological gains felt across numerous scientific domains – in particular the biological and information sciences – translates into aggregate productivity growth. While the advances have been large, future
improvements will be stymied by market failures with respect to 1) a lack of access to data due to private sector competition, poor incentives to publish failed experiments, and privacy regulation, 2) a lack of experiments in areas of the search space that are sparsely populated and thus have a lower chance of generating successful innovation but provide high social value by contributing data to the sparsely populated parts of the search space, and 3) an under-provision of training for multi-disciplinary skills that combine AI and domain specific expertise in areas such as chemistry and biology. A central role – and challenge – of policy makers will be to ensure that these bottlenecks do not arise so that the full benefits of AI as a GPT are able to be realized.

This paper is related to a number of literatures. First, our paper is inspired by growing literature describing the use of machine learning in scientific discovery and innovation. Rapid advances in hardware, software, and data availability have driven this growth, with applications of deep neural networks showing notably rapid growth across a number of domains. For a sampling of reviews see Chen et al. (2018) and Vamathevan et al. (2019) [drug discovery], Angermueller et al. (2016) and Tang et al. (2019) [computational biology], Wainberg et al. (2018) and Zou et al. (2019) [genomics], Goh et al. (2017) and Nature Communications (2020) [computational chemistry], and Butler et al. (2018) and Keith et al. (2021) [materials science]. The apparent success of deep learning in providing predictive models of highly complex combinatorial spaces explains the rising interest in it as a GPT for discovery. This potential – together with the power of viewing innovation as a prediction-model-aided combinatorial search process – motivates the paper.

Second, we draw on and extend the innovation production function that has been at the core of developments in endogenous growth theory (see, e.g., Jones 2005; Trammell and Korinek, 2021). As already noted, central to the innovation production function approach is the idea that existing knowledge is an input into the production of new knowledge – the “standing on the shoulders of giants” or spillover effect. Typically, papers in this literature do not explicitly adopt a combinatorial view of the process through which existing knowledge is turned into new knowledge (although see Romer, 2003, Weitzman, 1998, and Jones (2021) for important exceptions). In an elegant recent paper, Jones (2021) combines the insights of Kortum (1997) and Weitzman (1998) to explore the links between combinatorial growth in the size of search and exponential economic growth. The driver of economic growth is that the innovator makes draws
from a known distribution and growth results from an increase in the best alternative drawn. Jones shows how exponential growth results for certain distributions with thin tails when there is combinatorial growth in the number of draws. A key difference between his model and the one in this paper is that we assume that search (i.e., testing) is costly. This forces the innovator to prioritize instead of exhaustively searching the known landscape. Another important difference is the information available to our innovator is in the form of a fitness function over the search landscape, which contrasts with knowledge of the distribution in Jones’ model. We introduce AI as a means to improve the model of the landscape and thus improve prioritization. We also highlight an important source of spillovers: data on past successes and failures. These data are used to develop improved prediction models and thus can be a source of growth-sustaining productivity improvement in the innovation search process.

Third, our paper draws on literature in economics that applies the ideas of fitness landscapes to the study of innovation. As used in economics and management science, this work is typically situated in evolutionary economics and builds on the work of Nelson and Winter (1982). Important contributions include Levinthal (1997), Goretti and Levinthal (2000), Kauffman et al. (2000), Rivkin (2000), Fleming (2001), and Fleming and Sorenson (2004). Fleming and Sorenson (2004) introduced the idea of science as a map to aid technological search, an idea that is central to our approach. In our model, innovators use (imperfect) knowledge of the fitness landscape (the prediction model) to identify a promising subset of potential combinations followed by screening/testing of that subset.

Fourth, we draw on the literature on optimal search where information is imperfect and search is costly. This literature originated with Stigler (1961) with influential developments in McCall (1975). We draw in particular on a special case of the “Pandora’s box” model developed in Weitzman (1979). The sequential search problem examined by Weitzman involves boxes that

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4 Sewall Wright (1932) first introduced the fitness landscape concept in evolutionary biology, and Stuart Kauffman (see Kauffman, 1993) extensively developed it.

5 Drawing on the evolutionary approach, these papers model innovation (or imitation) as a “walk” or “hill climb” towards a local optimum on the fitness landscape. Innovators typically search one-mutant neighbors and adopt fitter variants until a local optimum is reached, although “long-jumps,” in which innovators jump longer distances across the landscape, are also studied as a process of exploratory search. The evolutionary approach has proved extremely fruitful and provides rich dynamics for the search process. However, to better connect with the innovation function approach that has been standard in the endogenous growth literature, we do not use an evolutionary approach in this paper.
vary in the distribution of potential outcomes, search costs, and the time that elapses before the value of the box is revealed. He shows that a “reservation price” can be attached to each box. Ranking the boxes in descending order of reservation prices, the optimal search rule – “Pandora’s rule” – is to continue down the ranking until the maximum value obtained is greater than the reservation prices of all remaining unopened boxes.

We utilise a special case of the Weitzman model where there are just two outcomes that can be revealed in the test of a given combination – success or failure – and there is an unbiased estimate of the probability of success available to a risk-neutral innovator from the prediction model. We also assume that costs are the same for all tests and that there is no time discounting. This allows the innovator to prioritize solely based on the ranking of combinations given by the prediction model, and we make a strong assumption about the functional form of the ranking function that is the output of the prediction stage of the discovery process.

Using this tractable search set up, we examine two cases of parallel (or simultaneous) search where (1) the innovator seeks to discover all combinations with an expected net value greater than or equal to zero and (2) the innovator seeks a single success but must choose in advance which combinations are to go for testing;\(^\text{6}\) we also explore the case of a Weitzman sequential search process with single innovation target. Drawing on the real options literature, we extend our two-stage search process to a more general multi-stage search process where there is an option to abandon an advancing combination at the end of each screening stage (Roberts and Weitzman, 1981; Dixit and Pindyck, 1994) and the probability of success is sequentially revised using Bayesian updating.

Finally, our paper draws on contributions to the emerging literature on the economics of artificial intelligence. A key breakthrough in AI has followed from the shift from rules-based systems to a statistical approach that emphasizes prediction (see, e.g., Athey, 2017; Mullainathan and Spiess, 2017; Agrawal et al., 2018; and Taddy, 2019). As emphasized by Arrow (1962), uncertainty is a pervasive feature of the innovation process and hence the value of prediction

\(^{6}\) Chade and Smith (2006) provide a more general treatment of the simultaneous search problem.
technologies that help reduce it. We can view AI – and machine learning in particular – as a GPT for prediction and make extensive use of this idea in our paper.7

We organize the remainder of the paper as follows. Section 2 briefly illustrates how AI is altering the discovery process in genomics, proteomics, drug discovery, and materials science. Section 3 sets out the basic conceptual building blocks of search-based innovation model over a vast combinatorial space and introduces the idea of a ranking function. Section 4 then develops a simple two-stage example of “search with a map” in which the first stage is the development of a prediction model (the output of which is captured by a logistic ranking function) combined with identification of a probability threshold for determinative testing, and the second stage is costly testing of all combinations with a probability of success at or above the threshold. Section 5 extends the model to a multi-stage setting to allow for a sequential refinement by Bayesian updating of predictions through possibly multiple intermediate screening stages and where there is an option to abandon at the end of each stage. Section 6 examines the interaction between an AI-based improvement in the prediction model and bottlenecks in terms of the ultimate impact of the improved prediction technology on the productivity of innovation. Section 7 reflects on AI as a method of generating improved prediction models (or maps) of the fitness landscape that has undergone rapid recent development. Section 8 discusses possible policy and managerial implications. Section 9 concludes with a recap of the main ideas and possible directions for future research.

2. Innovation as search over complex spaces: Some motivating examples

To help motivate our modeling approach, we first provide some illustrations of discovery challenges that involve search of complex combinatorial spaces, challenges for which machine learning-based prediction models appear to be fruitful. The common structure of such challenges is that an innovation involves a combination (e.g., a set of gene interactions or a chemical

7 Another important strand of the economics of AI literature has focused on the effects of AI on the demands for different types of skill. Researchers in this area have used the idea of a task-based production function to allow for the possibility that the introduction of AI (and other new technologies such as robotics) could lower the employment and wages of certain types of workers depending on the tasks they perform (Acemoglu and Autor, 2011; Autor, 2015; Acemoglu and Respeto, 2016, 2017 and 2019b). Such labor demand effects could take place for knowledge production tasks as well (Aghion et al., 2019).
molecule) that has relevant properties or activities (e.g., a relationship to an intermediating cell variable or a ligand-protein binding affinity). The discovery processes we consider are typically multi-staged (e.g., predictive screening, synthesis, testing, etc.). We are especially interested in how researchers use machine learning to screen candidate combinations and thereby allow a ranking of combinations for further exploration along the discovery pipeline.

Our first example is from genomic medicine. A genome is a hugely complex set of instructions for building an organism. This building process can be viewed as a combinatorial problem: genes interact in complex ways including the interaction between protein coding and regulatory regions within the genome. Genomic medicine exploits the relationship between DNA sequences and the risk of various diseases.

A central idea is that of gene expression, the process by which the information in the gene is first transcribed to make messenger RNA (mRNA) and then the mRNA is translated to make a protein (Leung et al., 2016). Researchers can utilize predictive models for various stages of this process. An encompassing approach is to model the relationship between DNA sequences and disease outcomes.\(^8\)

Our second example is proteomics – the study of the structure and function of proteins. Understanding proteins is a complex task given their vast number (far exceeding the number of genes) and interactions with cell and environmental variables. Part of biochemical process of gene expression is the translation via the ribosome of mRNA into the amino acid sequences that comprise the protein. As noted in the introduction, one of the challenges of proteomics is the prediction of the tertiary (or 3D) structure of a protein given its amino acid sequence. Substantial progress is being made this problem with the help of AI tools such as deep learning (Calloway, 2020; Senior et al. (2020). An improved ability to predict protein structure is providing new targets for therapeutic medicine. The most recent dramatic example of the use of AI for therapeutic medicine is in the context of vaccine discovery for COVID-19. AI was used, for example, to

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\(^8\) However, given the complexity of the process, researchers are making significant progress by developing predictive models of the relationship between DNA sequences and various intermediating “cell variables” (Leung et al, 2016). These cell variables can provide potential targets for therapeutic interventions. New measurement methods can give high-throughput data on various cell variables, and advances in machine learning are allowing researchers to take advantage of these abundant data to develop predictive models of their functioning. The combination of breakthrough gene editing technologies (notably CRISPR-Cas9) and a better understanding of the complex links between genes and disease could underpin major medical advances.
predict which of the tens of thousands of different pieces of the virus the immune system was most likely to recognize, enabling immunologists to focus their vaccine designs on a more manageable number of potential targets (Waltz, 2020). In the case of Moderna, a biotechnology company, AI was used to predict the optimal mRNA sequence to provide the information needed to make a protein to attack the virus. Moderna accomplished this by January 13, 2020, only two days after the Chinese authorities posted the genome sequence of the virus online (Iansiti et al, 2021).

Our third example is small molecule drugs – a mainstay of therapeutic medicine. Scholars have estimated the space of potential small organic molecules to contain more than $10^{60}$ possible structures (Virshup et al., 2013).\(^9\) Taking the target (or “lock”) as given, the challenge is then to identify a ligand (or “key”) that binds effectively and leads to the desired therapeutic effect. This screening challenge can be aided by machine learning models of binding efficacy (see, e.g., Chen et al., 2018). Some recent approaches take advantage of knowledge of the three-dimensional structures of both the target proteins and the small molecule ligands. Machine learning models such as convolutional neural networks (CNNs) – initially applied in tasks such as image recognition – are being successfully applied to utilize this information to improve predictions of bioactivity for drug discovery applications (see, e.g., Wallach et al., 2015 and Gomes et al., 2017).

Our final example comes from materials science. As with drug discovery, the space of potential molecules is vast. Researchers have used computational chemistry to virtually screen for the properties of molecules, including methods such as quantum chemistry and molecular mechanics. However, the computational costs of such simulation methods can be prohibitive, leading to increasing interest in statistical approaches such as machine learning to prioritize molecules for simulation- or experiment-based characterization (see, e.g., Pyzer-Knapp et al., 2015). For example, input data on molecular descriptors and output data on molecular properties could be used to develop a machine-learning-based predictive model of a large chemical space that would otherwise be prohibitively costly through computational and experimental methods.

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\(^9\) These chemical combinations (or ligands) interact with targets (e.g., a protein) that regulate biochemical processes within the body. Disease can occur when these targets malfunction. By binding to a malfunctioning target protein, the ligand may be able to alter its adverse bioactivity. As combinations of chemical elements, these small molecules can take a vast number of forms leading to a massive combinatorial search space even for a single protein target.
Materials discovery – for instance, new materials for clean energy technologies or medical devices – fits well with the multi-stage search over a combinatorial search space characterization of the innovation process. The tasks involved in the discovery of new materials include the predictive screening of potential molecules, the making (or synthesis) of those molecules, the testing of the molecules using high-throughput methods and the characterization of their properties. Among the innovation challenges to which machine learning is being applied are the development of new catalysts to convert earth-abundant molecules (such as CO$_2$, H$_2$O and N$_2$) into fuels and chemicals, new photovoltaic and thermoelectric materials, and new forms of batteries for energy storage (Tabor et al., 2018). However, researchers are concerned that bottlenecks in the discovery process severely slow the flow of new discoveries.

3. Conceptual Building Blocks of a Combinatorial Model of Innovation

Our examples of the growing use of machine learning in scientific discovery and innovation reveal a wide range of types of input data, output data, and algorithms to produce prediction models. In this section, we put these details aside and set out the conceptual building blocks of a highly stylized model of the innovation process as search over a vast combinatorial search space.

3.1 The search space

The starting point is to conceptualize innovation as the combination of more basic elements such as genes or molecules (Romer, 1993, Weitzman, 1996, and Arthur, 2009). We represent a combination as a string with $A$ elements. In the simplest case, the string reflects whether an element is present or not in the combination, with a 1 indicating presence and a 0 absence. More generally, each of the elements in a string can have multiple states, with the set of $M$ states denoted as its alphabet. We can then represent the string underlying a given combination by the particular states of the elements that comprise that combination.

For a given innovator, the available elements determine their *combinatorial search space* – that is, the set of all possible combinations that can be formed. In the binary case ($M = 2$), for an innovator with $A$ available elements the number of all possible combinations that can be formed from these elements is $2^A$.\(^\text{10}\) We henceforth assume that the binary case holds.

\(^{10}\) More generally, if each idea can take any one of $M$ states, the total number of possible combinations is $M^A$. 

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Innovation output can now be thought of as a function of the potential combinations: $I = F(2^A)$. More precisely, we think of innovation as resulting from search over the set of potential combinations. In conceptualizing this search space we make use of the idea of a *fitness landscape* (see, e.g., Kauffman, 1993). For a given fitness landscape, we associate each potential combination with a scalar that reflects its (fitness) value according to some particular property of interest.\(^{11}\)

Central to the fitness landscape is a measure of distance. The distance between any two strings (or combinations) is the number of states that differ between those strings (or Hamming distance). For a given string, the 1-neighbor strings are all those strings that differ by just one state. The $d$-neighbor strings are all those strings that differ in exactly $d$ of the states. The correlation structure of the landscape determines how correlated the values of the combinations are across different distances. Low correlations between the values of combinations outside close neighborhoods are associated with more “rugged” fitness landscapes.

The mapping from a combination to its scalar value can be viewed as an index function. For a combination to be successful – i.e., lead to an innovation – we assume its index value must be equal to or greater than some threshold. We can thus recast our landscape in a simplified form so that combinations with a value at or above that threshold have a value of 1 and combinations with a value below the threshold have a value of 0.

### 3.2 The ranking function

Our search process over the combinatorial search space follows a special case of the general (“Pandora’s box”) search model developed in Weitzman (1979). The decision maker searches by deciding on the order to open boxes (which corresponds to a combination advancing along the discovery pipeline in our application) and when to stop the search. In the general model, the decision maker knows ex ante the probability distribution of outcomes, faces boxes with different opening costs and different time lengths before the contents of any opened box is revealed. In the context of sequential search, Weitzman shows that each box can be assigned a “reservation price” that depends on the unique characteristics of that box. He derives “Pandora’s rule” as the optimal search strategy: continue to open boxes in descending order of the reservation price until the value

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\(^{11}\) Value may be multidimensional, so there can be multiple landscapes, each one associated with a particular property of interest. However, we will typically assume a single dimension of technological fitness for convenience.
of the best outcome achieved is greater than the reservation prices of all remaining unopened boxes.

Weitzman considers a special case that matches our basic setup: the outcome for a given box is either a success or a failure with a given probability of success for each box; the cost of opening a box is the same for all boxes; and the length of time before the outcome is revealed is irrelevant as there is no time discounting. In this special case, it is optimal to open all boxes with an expected net value greater than or equal to zero in descending order of the probability of success and to stop the search when a success is achieved (see Appendix 1). In addition to examining sequential versions of the search process, we also examine cases of parallel search, where the innovator must choose all the combinations to be advanced (or boxes to be opened) before any testing can take place (i.e., the actual opening of the boxes).

We now introduce a second resource available to the innovator: data. In addition to the available elements – the stock of which determines the combinatorial search space – the innovator has knowledge of previous successes (i.e., combinations with a value of 1) and previous failures (i.e., combinations with a value of 0). The total number of labelled successes and failures is $D$. These data can be used by innovator to develop a model (or map) of fitness landscape in the form of prediction model that outputs the probability of success of any given potential combination.

Our innovator is faced with the task of searching over a potentially vast combinatorial search landscape of $2^A - D$ potential combinations, which for notational simplicity we denote as $N$. A higher value of $D$ reduces the number of combinations available to be discovered but also provides “training data” for developing a prediction model for new successful combinations.

This innovation search is therefore characterized by great uncertainty over the location of the valuable undiscovered combinations on the landscape, which are assumed to be small in number relative to the size of the search space. However, our innovator also has access to the prediction model to help detect the location of the undiscovered successes. As discussed further in in Section 7, this prediction model could be provided by theory, simulation, parametric statistical data modelling, machine learning, or even educated guesses – although all methods will likely rely to some degree on observations of prior successes and failures.
Combinations are ultimately either successes (1) or failures (0), but without conducting the determinative test the innovator is uncertain as to which and must form an expectation of the probability of success prior to testing. If a successful innovation is brought to market after completion of the necessary screening and testing, it results in a payoff to the innovator of $\pi$, which we normalize to 1 without loss of generality. We seek a decision rule at Stage 0 to determine which combinations to advance in the pipeline. In determining this decision rule, we assume that the innovator is risk neutral and has the objective to maximise expected total value (net of the cost of conducting later screens and tests) of innovation.

There is a “ground truth” – here the truth as revealed by a test – only imperfectly known to the innovator as to the location of undiscovered successes on the landscape. There are $G$ successes in total so that successes as a share of potential combinations is $G/N$, where $G$ is a natural measure of the fecundity of search space. A useful graphical representation of this ground truth is the unit step function shown in Figure 1. Denoting the known probability of success of the $r^{th}$ ranked potential combination as $p_0^r$, the unit step function is:

\[
(1) \quad p_0^r = 1 \quad \text{for } r \leq G \\
= 0 \quad \text{for } r > G.
\]

Figure 1 shows the both case where there is perfect ability to discriminate between successes and failures (where the internal rankings within the subsets of both successes and failure is arbitrary) and also the case where there is no ability to discriminate between successes and failures. In the latter case, the probability of success is simply the probability of finding a success as a result of a random draw from the search space, i.e., $G/N$.

We seek a functional form for the ranking function such that the probability of discovering a success is equal to $G/N$ when the prediction model has zero discriminating power and approaches the ground truth as the model approaches perfect discrimination. The following logistic decay function has the property that the ground truth is approached as $\beta \to \infty$:
\[
(2) \quad p_0^r = \frac{1}{1 + Ke^{\beta(r-G-1)}}
\]

where \( \beta \geq 0 \). The value of \( K \) can be chosen so that the probability of success is equal to \( G/N \) when the prediction model has zero discriminating power (i.e., \( \beta = 0 \)):

\[
(3) \quad p_0^r = \frac{1}{1 + K} = \frac{G}{N'}
\]

\[
=> K = \frac{N - G}{G}. 
\]

We thus chose this (appropriately restricted) logistic function as an analytically convenient representation of the ranking function. Furthermore, we assume that \( \beta \) monotonically increases with the performance of the prediction model. As shown in Figure 2, the shape of ranking function curve varies from a horizontal line at \( G/N \) when \( \beta = 0 \), and converges to the ground truth unit step function as \( \beta \to \infty \). Thus the performance of the prediction model is controlled by a single parameter. Increases in \( \beta \) cause the ranking function curve to rotate in a clockwise direction around the point \((G + 1, G/N)\).\(^{12}\)

\(^{12}\) Numerous measures of performance exist for binary dependent variable models. Ideally, the measures of performance would be applied to a held back test sample given the risk of overfitting, especially for models that allow for highly flexible functional forms. One particularly intuitive measure for evaluating predictive performance both in and out of estimation sample is the Tjur Coefficient of Discrimination (Tjur, 2009): \( E[p(success) | success] - E[p(success) | failure] \). This coefficient varies from 0 to 1, with a coefficient of 1 indicating a perfectly discriminating model with the mean probability of success given the combination is an actual success equal to 1 and the mean probability of success given the combination is an actual failure equal to 0. When the mean probability of success estimated by the model is the same for actual successes and actual failures the coefficient has a value of zero. The area under the receiver operating characteristic curve (AUC) is also a widely used measure of the performance of a binary dependent variable model that can be usefully related to the accuracy of the probability of success rankings: the AUC can be interpreted as the probability that a randomly chosen actual success is ranked better (a lower number given our model) than a randomly chosen actual failure. In general, there is of course no reason that the probability rankings produced by a prediction model will follow a logistic curve. However, it is intuitive that a better performing model will tend to increase the estimated probabilities associated with better ranked combinations and decrease the estimated probabilities associated with poorly ranked combinations. In our model, an increase in the parameter \( \beta \) brings about the required clockwise rotation in the ranking function curve. This fact combined with the analytical convenience of the logistic form leads us to treat \( \beta \) as useful parameter to control the performance of the prediction model.
Although an increase in $\beta$ causes the probability of success to increase at any given rank from 1 to $G$, there is no presumption that the identity of the combinations at any given rank remains the same. Thus, for example, with the move to an improved prediction model we assume that the probability of success of the top-ranked combination will increase; however, the identity of the top-ranked combination could change, and indeed it is possible the probability of the previously top-ranked combination will fall.

A desirable feature of a ranking function is that the expected total number of successes equals $G$ when aggregated over the entire space of $N$ potential combinations. This is clearly the case when $\beta = 0$ (random search) and is approximately true as $\beta$ goes to infinity (perfect discrimination). However, it is not generally true for intermediate values of $\beta$. We thus consider an augmented ranking function where a normalizing constant, $\theta(\beta)$, is added to equation (2), where the value of the necessary constant will depend on $\beta$ (for given values of $G, A$ and $D$) such that the sum of the probabilities of success equals $G$.

$$\sum_{r=1}^{N} \left( \frac{1}{1 + \frac{N - G}{G} e^{\beta(r-1)}} + \theta(\beta) \right) = G$$

$$\Rightarrow \theta(\beta) = \frac{1}{N} \left[ G - \sum_{r=1}^{N} \frac{1}{1 + \frac{N - G}{G} e^{\beta(r-1)}} \right].$$

Given that the term in square brackets converges to a constant as $N$ goes to infinity, the division by $N$ ensures that $\theta(\beta)$ converges to zero. It follows that the non-normalized ranking function provides a good approximation to the augmented (normalized) ranking function for a sufficiently large search space. The non-normalized function also ensures that the probability of success lies in the $[0,1]$ range and so we assume this function in what follows, recognizing that we are implicitly assuming a sufficiently large value of $N$ so that any approximation error is negligible.

For considering the effects of improved prediction on expected search outcomes it is useful to introduce an idea of dominance in comparing ranking functions. The swivel property of the ranking function ensures that as we increase $\beta$ the probability of success at all ranks 1 to $G$ will increase. Provided that the relevant range of ranking function is for probabilities of success greater
than $G/N$, we can say that in comparing two ranking functions $A$ and $B$, where $A$ has a higher value of $\beta$ than $B$, then $A$ dominates $B$ over the relevant range. In the next section, we examine in the context of some simple search structures how an improvement in the prediction model – i.e., an increase in $\beta$ – affects the search and the expected net value of innovation resulting from that search.

4. A Two-Stage (Predict-Test) Example of Searching with a Map

In this section, we explore in the context of a two-stage example how access to an improved prediction model could affect the innovation process and its outcomes without for the moment considering the source of the improvement. We label the stages 0 and 1. Stage 0 (prediction) is the development (or “training”) of the prediction mode and the choice of probability threshold at or above which a combination is sent for testing (Stage 1). We conveniently represent the output of the prediction model as a ranking function mapping the rank (1 for the top ranked combination to $N$ for the bottom ranked combination) to the model-based probability that the combination is a success. Stage 1 (testing) involves the conducting of the test, where we assume that the testing is a regulatory requirement that cannot be bypassed by taking a promising combination straight to market. The cost of a test is $c_1$.

Three cases of two-stage innovation search are now considered. In Case 1, the value of innovations is independent of which other valuable innovations are discovered, combinations are advanced in parallel, and our risk-neutral innovator seeks to discover all valuable combinations with a positive expected value net of the cost of testing. In Case 2, the value to our innovator of finding additional successful combinations once one success is achieved is then zero so that combinations that match the target are perfect substitutes. Combinations are again advanced in parallel but the innovator takes into account the probability that a success will be found with other combinations in the portfolio to be tested in deciding to add an additional combination to that portfolio. Finally, Case 3 most closely matches the Weitzman framework with sequential search. We explore both a version with identical payoffs conditional on success and identical testing costs and a version that allows for heterogeneity along one or both of these dimensions. We consider each case in turn focusing on how an improvement in the prediction model affects the decision to select combinations for testing and ultimately the expected total net value of innovation.
Case 1: Parallel search for innovations and innovations are independent

In our first case, our innovator seeks to discover as many as possible successes for a single target that is hidden in the search space. Using the analogy of “locks” and “keys” familiar from drug discovery, there is a single lock and \( G \) keys hidden in boxes spread across the search landscape. However, the boxes are costly to open, which forces the innovator to prioritize. We assume the keys are differentiated and it is possible to find multiple valuable keys to the lock with negligible competition between them, say different versions of a drug with the same therapeutic effect but appeal to different segments of the market.\(^{13}\) It follows that the expected value of innovation is additive. We will dispense with the assumption of no-redundancy in Case 2 below.

To evaluate the expected value impact of an improved prediction technology it is useful to cast the testing decision in terms of a comparison of the expected marginal gross value and marginal cost of testing. We denote the optimal number of combinations to send for testing as \( r^* \). Assuming an ordering of tests in terms of decreasing expected marginal gross value of the innovation, the expected marginal gross value of the \( r^{th} \) test is simply:

\[
(5) \quad MV_r^e = p_r^0.
\]

The marginal cost of the \( r^{th} \) test is a constant, \( c_1 \).

The expected marginal gross value and marginal cost curves are illustrated in Figure 3a, where we ignore the discrete nature of the problem for graphical simplicity. The subset of combinations that will be sent for testing comprises those combinations with an expected marginal gross value greater than or equal to marginal cost. If we make the additional simplifying assumption that at the last combination sent for testing marginal value and marginal cost are equated, then the probability threshold is given by:

\[
(6) \quad p_r^0 = c_1.
\]

\(^{13}\) A more technical motivation comes from using a Dixit-Sliglitz-Ethier type preferences/production function. With an additive objective function of this type, the demand curve for a new variety is unaffected by the discovery of an additional variety given the preference for variety built into these functions.
Using equation (2) – our logistic ranking function – allows us to solve for the optional number of tests. All tests where the expected marginal gross value is greater than or equal marginal cost will be conducted:

\[
(7) \quad \frac{1}{1 + \left( \frac{N - G}{G} \right) e^{\beta(r^* - G - 1)}} = c_1
\]

\[
=> \quad r^* = G + 1 - \frac{\ln \left( \frac{N - G}{G} \right) - \ln \left( \frac{1}{c_1} - 1 \right)}{\beta}
\]

More generally, the probability threshold (with associated \( r^* \)) will be the lowest probability in the ranking at which \( MV^e_r \geq MC \).

Expected total net value is then:

\[
(8) \quad V^e = -c_1 r^* + \sum_{r=1}^{r^*} p^0_r
\]

\[
= -c_1 \left( G + 1 - \frac{1}{\beta} \left[ \ln \left( \frac{N - G}{G} \right) - \ln \left( \frac{1}{c_1} - 1 \right) \right] \right)
\]

\[
+ \sum_{r=1}^{r^*} \left( \frac{1}{1 + \left( \frac{N - G}{G} \right) e^{\beta(r - G - 1)}} \right).
\]

From Figure 3b we can see the impact on the number of combinations that will be sent for testing of an improvement in the prediction model – i.e., an increase in \( \beta \). Provided \( c_1 > (G/N) \), the \( MC \) curve will intersect the \( MV^e \) curves above the crossing point of the \( MV^e \) curves at \( G/N \). We assume that this condition holds, recalling that the number of undiscovered combinations is assumed to be small compared to the size of the combinatorial search space. The number of combinations sent for testing will be a non-decreasing function of \( \beta \) and strictly increasing for a large enough increase in \( \beta \) for it to be optimal to send at least one additional combination for testing.
Of most interest is what happens to the expected total net value of innovation, $V^e$, when there is an improvement in the prediction model. At the original optimal number of tests, $r^*$, the marginal expected gross value will be greater with the new (higher $\beta$) prediction model than with the original model (see Figure 3b). It follows that $V^e$ must be higher even if the number of combinations going for testing did not change. However, any increase in the optimal number of tests from $r^*$ to $r^{**}$ as a result of the improved prediction model will lead to a further increase in $V^e$. The (approximate) overall increase in $V^e$ is shown as the relevant area under the curve in Figure 3b.

Case 2: Parallel search for innovations and Innovations are perfect substitutes

Case 1 made the strong assumption that the value of alternative innovations are independent and our risk neutral innovator seeks all innovations that yield a positive expected net value. However, in many innovation search problems the innovator may be looking for a single combination that meets a particular target such as a small molecule drug that binds with a target protein to improve its functioning or a material for a battery with a desired property. Bringing multiple innovations to market that achieve the same target will be wasteful to the extent that these innovations are perfect substitutes.

Returning to our analogy of locks and keys, there is now a single target and $G$ possible keys that open that lock. As with Case 1, the innovator must choose which boxes to open prior to opening any of the boxes (so it remains a case of parallel search), but in adding an additional box to the portfolio of boxes to be opened, the innovator takes into account that already included boxes in the portfolio may lead to the finding of the key that opens the box, making the additional box redundant ex post.\textsuperscript{14}

The difference from Case 1 comes in the position and shape of the expected marginal gross value curve: as our innovator evaluates the expected marginal gross value of an additional test they

\textsuperscript{14} We assume that the model of the search landscape is given. However, if the model could be rerun before choosing the next combination to be added to the testing portfolio test (i.e., choosing the next additional box to open), the innovator could rerun the model based on the assumption that the previous combinations in the portfolio were failures and thereby improve the portfolio selected through incremental additions of combinations.
must consider the probability that a combination meeting the target would have been discovered with the existing tests. The expected marginal gross value curve is now:

\[
(9) \quad MV_r^e = p_r^0 \quad \text{for } r = 1
\]

\[
= (\prod_{j=1}^{r-1} (1 - p_{r-j}^0)) p_r^0 \quad \text{for } r = 2, 3, \ldots, 2^A - D.
\]

For the marginal test, \(r\), the term inside the first round brackets gives the probability that the desired target combination will not have been discovered in the previous \(r - 1\) tests. For example, for the 3rd test the probability that the target will not have been discovered in tests 1 and 2 is \((1 - p_1^0)(1 - p_2^0)\). The probability \((1 - p_1^0)(1 - p_2^0)p_3^0\) is thus the probability that target will be found in the third test and not before.

The marginal cost of a test is again equal to \(c_1\). As illustrated in Figure 4a, the optimal number of combinations to send for testing is identified by moving down the probability ranking and testing all combinations for which the expected marginal gross value is greater than or equal to the marginal cost. This gives the optimal number of tests, \(r^*\), and the expected total net value is:

\[
(10) \quad V^e = -c_1 r^* + \sum_{r=1}^{r^*} \left( \prod_{j=1}^{r-1} (1 - p_{r-j}^0) \right) p_r^0.
\]

Figures 4b and 4c show the impact on the optimal number of tests when the innovator gets access to an improved prediction model, again captured by an increase in \(\beta\). As can be seen from the figures, the marginal value curves (pre and post improvement) will cross at a certain point in the ranking. The impact of an improvement in the prediction model will depend on whether marginal cost, \(c_1\), is above or below marginal value at this crossover point. Figure 4b shows a case where the crossover point occurs below \(c_1\). In this case the optimal number of tests will increase. Figure 4c shows a case where the crossover occurs above \(c_1\) and the optimal number of tests will decrease.

What is the effect of an increase in \(\beta\) on \(V^e\)? Provided \(p_r^0\) is greater than \(G/N\), we can show that \(V^e\) increases regardless of whether the optimal number of tests rises or falls. To see why,
note that for a higher value of $\beta$ the probability of success is higher for each position in the ranking. Comparing the probability that a success will have been found at the $r^{th}$ ranked combination with and without the improvement in the prediction (where a prime indicates the post-improvement probability at a given rank):

$$\left(11\right) \ 1 - \prod_{j=1}^{r} \left(1 - p^{0}_{r-j}\right) > 1 - \prod_{j=1}^{r} \left(1 - p^{0}_{r-j}\right).$$

Given the improved prediction model, the probabilities on the left-hand-side all greater than the probabilities on the right-hand-side at a given rank. Thus, the cumulative probability that a success will have been discovered is also higher for every position in the ranking. In Figure 4b, $V^e$ is thus higher with the higher $\beta$ at the original optimal number of tests, $r^*$. The move to the higher optimal number of tests $r^{**}$ increases $V^e$ still further. A similar situation arises in Figure 4c even though the optimal number of tests now falls rather than rises. At the original number of tests $V^e$ is again higher with the higher $\beta$. The move to the optimal number of tests (fewer tests in this situation) further increases this gain relative to the situation with the inferior prediction model (i.e., lower $\beta$).

**Case 3. Sequential search where successful innovations are perfect substitutes**

Our first two cases assume identical payoffs (gross value) across combinations conditional on a successful test and identical testing costs for all combinations. Both cases also involve parallel rather than sequential search. Our third case more closely follows the Weitzman structure with potentially combination-specific payoffs, $\pi_i$, conditional on a successful test and combination-specific second stage testing costs, $c_{1,i}$. The innovator is seeking a match for a single target and search is sequential, so that a test must be completed before moving on to the next combination in the search order. As shown in Weitzman (1979), the optimal ranking of combinations (or boxes) in a sequential search problem is based in this setting on their “reservation price”, $z_i$, given by:

$$\left(12\right) \ \ p^0_i \pi_i + \left(1 - p^0_i\right)z_i - c_{1,i} = z_i$$

$$=> \ \ z_i = \pi_i - \frac{c_{1,i}}{p^0_i}.$$
The first equation can be viewed as a comparison between a “sure thing” available to the innovator with payoff, $z_l$, and a lottery where the sure thing is available as a backup option. For a given combination, $z_l$ is determined as the value that makes the innovator indifferent between the two alternatives. The innovator will sequentially search in decreasing order of reservation prices and stop the search when a success is achieved (see Appendix 1).

For ease of comparison with Cases 1 and 2, we first consider the special situation where combinations differ only in their probabilities of success (i.e., payoffs conditional on success and testing costs are the same across combinations). As with the other cases, we examine how an improvement in the prediction model (i.e., an increase in $\beta$) affects the combinations that advance for testing and the expected value of innovation. However, in contrast to parallel search, the number of combinations that the innovator that will advance for testing (or, equivalently, the duration of search) is uncertain at the outset of testing.

The expected duration of search (i.e., the expected number of combinations to be tested) is equal to the probability that the search will involve exactly $r$ combinations times $r$, summed from $r = 1, ..., r_{max}$, where $r_{max}$ is the maximum number of combinations with an expected net value greater than or equal to zero:

$$L^e = \sum_{r=1}^{r_{max}} \left( \prod_{j=1}^{r} (1 - p_{j-1}^0)p_j^0 \right) r. \tag{13}$$

How does an improvement in the prediction model affect the expected total net value of search? Assuming the improvement increases the maximum number of combinations that would be tested, the expected total value of the search will increase. Given the increased likelihood of an early success with improved predictions, it is also likely the expected duration of the search will fall, which would lower expected costs and reinforce the positive effect on expected total net value. However, with the increased maximum possible duration of the search, the possibility of an increase in the expected duration of the search cannot be ruled out. Without identification of the precise parameters, it is therefore not possible to be definitive on the overall effect of an improvement in the prediction model on expected total net value.

With sequential search, it is interesting to see the possible effects of relaxing the homogeneity assumption and instead allow for combinations to vary in terms of the payoff
conditional on success. (We continue to assume the testing cost is the same for all combinations.)

To see the potential implications of an improvement in the prediction model under such heterogeneity, note first that the partial derivative of the reservation price that determines the search ordering with respect to the probability of success is:

$$\frac{\partial z_i}{\partial p_i^0} = \frac{c_{1,i}}{(p_i^0)^2}.$$ (14)

The impact of a small change in the probability of success for a combination is thus decreasing in the initial probability of success.

To see how an improvement in the prediction model could lead to a shift in the sequential search ordering towards “riskier” combinations, we consider two combinations, $i$ and $j$, with identical initial (i.e., pre-improvement in the prediction model) reservation prices (i.e., $z_i = z_j$), but where $\pi_i < \pi_j$ and $p_i^0 > p_j^0$. Thus, box $j$ is riskier in the sense that it has a lower probability of success but a higher payoff conditional on success. To bias things against the riskier combination we further assume that $\Delta p_i^0 > \Delta p_j^0 > 0$ as a result of the improved prediction model.

The following approximations hold for relatively small increases in the predicted probabilities:

$$\Delta z_i \approx \frac{c_1}{(p_i^0)^2} \Delta p_i^0.$$ (15)

$$\Delta z_j \approx \frac{c_1}{(p_j^0)^2} \Delta p_j^0.$$ (16)

However, despite combination $i$ having the larger increase in the probability of success, combination $j$ – the riskier combination – will actually move ahead in the ranking when:

$$\Delta z_j > \Delta z_i,$$ (17)

$$\implies \frac{p_i^0}{p_j^0} > \frac{\Delta p_i^0}{\Delta p_j^0}.$$
That is, box $j$ will move ahead in the ranking if the ratio of proportionate changes in the probability of success is less than the initial ratios of the probabilities of success. Therefore, it is possible for riskier combinations to move ahead in the ranking despite experiencing a smaller increase in their probability of success as a result of the improved prediction model. This suggests another possible effect of improved prediction: improved prediction leads to riskier combinations moving along the discovery pipeline, enhancing the chances of more radical innovations being discovered.

4.3 Extensions

*Multi-objective-optimization (MOO)*

Our basic model implicitly assumed a single objective (e.g., the binding efficacy of a small molecule drug with a given protein target). More realistically, a successful combination will have to meet multiple objectives – e.g., the drug is non-toxic in addition to efficacious binding. One straightforward extension is to assume that a combination must meet all requirements to be considered a success. That is, it must achieve a value of 1 on all necessary dimensions so that the overall indicator of success is the product of the individual success indicators. We then assume that the logistic ranking function applies to the overall probability of success.

A less strict approach where objectives are measured continuously is to identify a Pareto frontier whereby one objective (say non-toxicity) cannot be increased without sacrificing another objective (say efficacy). The trade-offs between objectives might then be considered at a later stage of the discovery process (e.g. lead optimization). We discuss a multi-stage extension of our two-stage model that allows for multiple intermediate screening stages prior to final determinative testing below.

*Multi-task and transfer learning*

One challenge in applying AI-based prediction to tasks such as discovering a small-molecule drug that is effective against a given target is the sparsity of relevant data. This has led to interest in techniques such as multi-task learning where the prediction model is estimated based on a vector of outcome variables.

Transfer learning refers to the ability to transfer knowledge from the use of data in related domains in the generation of the prediction model for new innovation tasks. We have already implicitly
used the idea of transfer learning in the cases analysed above in that related successes must be used in generating the prediction model for targets that by definition have not yielded a success.

Active learning

In our model, the innovator is only focused on the exploitation of the prediction model of the search space so as to discover new valuable combinations. Therefore, the choice of combinations to send for testing in both the cases discussed is solely based on the predicted probabilities of success of those combinations. However, another goal in selecting combinations to send for testing might be the improvement of the prediction model for future innovation search tasks. The optimal exploration might lead to different choices in terms of the ranking of combinations to send for testing. For example, the innovator may want to explore relatively unknown parts of the search space even if the benefit for current tasks is limited. In the context of an ongoing innovation effort, there can therefore be a trade-off between the goals of exploitation and exploration. An “active learning” strategy directly addresses this trade-off between the immediate innovation value and future information value of testing.

A possible limitation of passive learning strategies based on exploitation of the existing prediction model is that there will be overconcentration on known parts of the combinatorial search space. In the context of, say, drug or materials design, active learning strategies have included techniques such as uncertainty sampling – testing combinations predicted with low confidence by the model – and model-improvement – testing combinations based on predictions of the how the new data point will improve the performance of the prediction model (Reker and Schneider, 2014). Active learning strategies “assist the selection process by focusing on areas of chemical space that have the greatest chance of success while considering structural novelty. The core feature of these algorithms is their ability to adapt the structure-activity landscapes through feedback” (Reker and Schneider, 2014, p. 458).

Discriminative versus generative models.

We have modelled the prioritization process as an effort to discriminate between potential combinations based on their predicted probability of success. A complementary approach aims to directly “design” a combination – say a small molecule drug – that meets specific requirements using a generative approach (see, e.g., Merk et al., 2018). The generative approach effectively
inverts the process, whereby the researcher starts with the desired properties and uses the model to identify the combination that comes closest to achieving these properties. Such generative modelling using AI falls into the broader category of de novo design. These approaches may be especially important in the context of refinements of rankings (such as lead optimization in the case of drug discovery):

While compound elimination by appropriate scoring models discards the bulk of the designs (“negative design”) with acceptable accuracy, the selection of the best or most promising (“positive design”) remains prone to error. More accurate activity prediction models that extend the capabilities of existing approaches could originate from advanced machine learning methods. (Schneider, 2018, p. 109.)

5. A Multi-Stage Discovery Process and with Bottlenecks

5.1. Extension of the model to include multiple intermediate screening stages with Bayesian updating between stages

We assumed in Section 4 a highly simplified two-stage innovation process involving prediction (development of a prediction model choosing a testing threshold) and testing (testing all combinations with a probability of success at or above a threshold value that depended on the market value of an innovation success and the cost of conducting the test). The motivation for this simple set-up is that the discovery pipeline often involves the production of a priority list for testing where such testing is expensive. AI can be introduced as a possible way to improve the list.

Of course, in reality innovation processes are more complex and involve multiple stages rather than just two. The typical stages of the drug discovery process include target identification, hit generation, generation of lead compounds from the hits (“hits to lead”), optimization of the lead compounds, pre-clinical trials (animal studies), and Phase I, II and III human clinical trials. In materials discovery, the stages can involve prediction of molecule properties, synthesis of the molecules, and characterization of the actual properties through testing, but this can be followed by further stages such as investigating the ability to synthesize at scale and testing the molecule under the different environmental conditions that could be observed in the field.

We therefore extend our two-stage prediction-test model in this section to a multi-stage setting. Our main focus is on the implications of bottlenecks in later stages of the discovery
pipeline. The basic two-stage model already highlights one cause of bottlenecks – the costliness of later-stage testing. This has the implication that higher testing costs will reduce the number of combinations that advance to testing. However, another important source of bottlenecks is the poor efficacy of the various screens that make up the pipeline between the initial prediction stage and final testing stage. In the context of an option to abandon combinations following a negative screening result, we show that less effective screens also reduce the number of combinations that enter the pipeline. We discuss how such bottlenecks could limit the benefit from improved AI-based prediction and how AI might itself be applied to later stages in the pipeline to help alleviate those bottlenecks.

More specifically, we extend the basic two-stage model of prediction and testing to a discovery pipeline that involves potentially multiple intermediate screening stages in addition to the prediction and final (determinative) testing stages. As in Case 1 above, we assume that the innovator is seeking to find all combinations with a positive net value. Although we continue to assume that the final testing stage is determinative, we allow for imperfections in the intermediate screening stages in the form of the possibility of false negatives and false positives in the screens. For simplicity, we assume the false negative rates and the false positive rates are constant across each of the intermediate stages. We denote the false negative rate as \( x \) and the false positive rate as \( y \); the corresponding true positive and true negative are \( 1 - x \) and \( 1 - y \) respectively.

To minimize notation we again assume, without loss of generality, that the gross payoff from a success, \( \pi \), is equal to 1. Thus, the expected gross value of a combination at the completion of any given stage is just the estimated probability of success at that stage. We assume initially that all stages must be completed to launch an innovation on the market so that it is not possible to skip a stage. At the completion of the final (test) stage, the gross value is equal to 1 (a success) or 0 (a failure), with the estimated probability of a success going into the testing stage equal to the estimated probability of success at the completion of the last intermediate stage.

There are \( S \) stages, \( s = 0, \ldots, S \), including the Stage 0 prediction stage, the final Stage \( S \) testing stage and \( S - 1 \) intermediate stages. For any given intermediate stage, \( s \in \{1, \ldots, S - 1\} \),

\( ^{15} \) As applied to a specific candidate combination, our multi-stage discovery process is an example of what Roberts and Weitzman (1981) call a *Sequential Development Project*: costs are additive across stages; value is received only at the end of the project; and there is a possibility of abandoning the project at the end of each stage.
the probability of success is optimally calculated using Bayes rule given the prior probability inherited from the previous stage. We thus model the pipeline as a series of screens between the initial prediction stage and the final determinative test that lead to Bayesian updating of the prior probability of success. Applying Bayes rule, this updating process is given by:

\[
(18) \quad p_r^s | \text{positive screen} = \frac{(1 - x)p_r^{s-1}}{(1 - x)p_r^{s-1} + y(1 - p_r^{s-1})},
\]

\[
(19) \quad p_r^s | \text{negative screen} = \frac{xp_r^{s-1}}{xp_r^{s-1} + (1 - y)(1 - p_r^{s-1})},
\]

where the denominators in (18) and (19) are the probability of a positive screen and a negative screen respectively.

As a combination advances along the discovery pipeline, it is assumed to follow a discrete-stage Markov process as illustrated in Figure 5. The assumption of constant values of \(x\) and \(y\) at each stage means that we obtain the binomial lattice (or recombinant) structure shown in the figure, where the branches recombine to limit the number of possible values for the probability of success at each stage in the process. For example, the lattice structure implies that at Stage 3 the expected probability of success is the same for a given Stage 0 probability where there is a scenario of two positive screens followed by a negative screen, a scenario of a negative screen followed by two positive screens, or a scenario of a positive screen followed by a negative screen followed by another positive screen. Conveniently, the lattice structure implies that number of values that this probability can take at Stage \(s\) is \(2^s\), so that the number of possible values rises only linearly with the number of the stage.\(^{16}\)

For a given \(p_r^0\), the probability of a reaching a given node at Stage \(s\) for a given number of positive and negative screens is:

\[
(20) \quad q_r^s (h) = (1 - x)^h x^{s-h} p_r^0 + y^h (1 - y)^{s-h} (1 - p_r^0).
\]

\(^{16}\) With stage-specific values of \(x\) and \(y\), the number of possible values for the probability of success would rise exponentially with the number of stages so that the number of possible values at stage \(s\) would be \(2^s\). Therefore the lattice/recombinant structure dramatically simplifies the computational burden when there are multiple intermediate stages.
In the case where any negative screen leads the combination to be abandoned, we obtain a simple expression for the probability that the combination survives $s$ stages:

$$q^s_r(s) = (1 - x)^s p^0_r + y^s (1 - p^0_r).$$

Figure 6 shows how the probability of surviving for a combination falls through multiple intermediate stages (assuming a combination is always abandoned following a negative screen). Thus, in addition to showing how the probability of success evolves as a combination travels through the pipeline, the model also allows us to make predictions about the survival rates along the pipeline.

Note that the probabilities for possible nodes that can be reached at a given stage $s$ sum to 1:

$$\sum_{h=0}^{s} \binom{s}{h} [(1 - x)^h x^{s-h} p^s_r + y^h (1 - y)^{s-h} (1 - p^0_r)] = 1,$$

where the number of paths to a given node is $\binom{s}{h}$. For example, at stage 3, there are three paths to the nodes involving a total of two successes and one failure: $\binom{3}{2} = \frac{3!}{(3-2)!2!} = 3$. At Stage $s$, the probability of success given $h$ positive screens and $s - h$ negative screens (i.e., the probability of success at the given node) is:

$$p^s_r|h \text{ positive screens} = \frac{(1 - x)^h x^{s-h} p^0_r}{(1 - x)^h x^{s-h} p^0_r + y^h (1 - y)^{s-h} (1 - p^0_r)}.$$  

We can verify that when viewed from the end of the Stage 0 (i.e., the prediction stage) the expected probability of success at Stage $s$ remains equal to $p^0_r$:

$$p^s_r = \sum_{h=0}^{s} \binom{s}{h} [(1 - x)^h x^{s-h} p^0_r + y^h (1 - y)^{s-h} (1 - p^0_r)] - p^0_r \right] \frac{(1 - x)^h x^{s-h} p^0_r}{(1 - x)^h x^{s-h} p^0_r + y^h (1 - y)^{s-h} (1 - p^0_r)}.$$
\[= \sum_{h=0}^{s} \binom{s}{h} (1-x)^h x^{s-h} p_r^0 = p_r^0.\]

5.2 A three-stage “predict-screen-test” example

We now examine the implications of the existence of the option to abandon a combination upon the realization of a negative screen. We illustrate the implications for the number of combinations that initially enter the discovery pipeline with a three-stage example – that is a pipeline with an initial prediction stage (Stage 0) a single intermediate screening stage (Stage 1) and the final determinative testing stage (Stage 2). The cost of the intermediate screen is \(c_1\) and the cost of final test is \(c_2\). We assume that \(c_1 + c_2 \leq 1\) and that both \(c_1\) and \(c_2\) are strictly positive. Upon receiving a negative result on the intermediate screen, the option to abandon will be exercised if:

\[
\frac{xp_r^0}{xp_r^0 + (1-y)(1-p_r^0)} < c_2. \tag{25}
\]

\[
\Rightarrow p_r^0 < \frac{(1-y)c_2}{x + (1-x-y)c_2}
\]

Given the availability of the option to abandon, we now move attention back to the end of Stage 0. We assume that the innovator is operating in the range where the combination will be abandoned on the realization of a negative screen in Stage 1. We can therefore determine the cut-off Stage 0 probability below which combinations will not advance to intermediate screening, with a higher cut-off probability implying that fewer combinations advance given the ranking function. Equating expected marginal value with expected marginal cost allows us to identify the cut-off probability for advancing a combination to the intermediate stage. Moreover, given the logistic ranking function, the probability of success declines monotonically with the rank of the combination, so identifying the cut-off probability is identical to identifying the number of combinations to advance in the pipeline.

The equation of expected marginal value and expected marginal cost yields:
\[
(26) \quad [(1 - x)p_r^0 + y(1 - p_r^0)] \left[ \frac{(1 - x)p_r^0}{(1 - x)p_r^0 + y(1 - p_r^0)} \right] = c_1 + [(1 - x)p_r^0 + y(1 - p_r^0)]c_2,
\]
\[
=> \quad p_r^0 = \frac{c_1 + yc_2}{(1 - x)(1 - c_2) + yc_2}.
\]

Note that for this cut-off probability to be strictly less than 1 we require that \(c_1 < (1 - x)(1 - c_2)\). Moreover, for the denominator to be positive (and thus that the cut-off probability is greater than zero), we require that \((1 - x)(1 - c_2) + yc_2 > 0\). Finally, note that for the probability of a successful screen to increase with the prior probability of success we require that \(1 - x - y > 0\).

The innovator will choose to advance all combinations with a positive expected net value recognizing that Stage 2 cost will not be incurred (i.e., the option to abandon will be exercised) in the event of a negative screen. Figure 7 examines this decision by relating the expected net value of the \(r^{th}\) ranked combination, \(v_r^e\), to \(p_r^0\). The line with the intercept\(−(c_1 + c_2)\) shows how the expected net value changes with \(p_r^0\) in the absence of an option to abandon or where that option is not exercised. The line with the intercept\(−(c_1 + yc_2)\) shows how expected net value evolves with \(p_r^0\) assuming the option to abandon is always exercised when there is a negative result on the Stage 1 intermediate screen. The bold line shows the evolution of expected net value given the optimal advancement of combinations from Stage 0 as given by equation (21) and the optimal exercise of the option to abandon after the screening result given by equation (20). For low values of \(p_r^0\) the combination will not advance to the screening stage and the expected net value is zero. For intermediate values of \(p_r^0\) the combination will advance to screening but will be abandoned in the event of a negative screen, saving on Stage 2 testing costs. For high enough values of \(p_r^0\) the combination will not be abandoned even in the event of a negative screen. However, we assume here that the screen must take place due to it being an indispensable part of the final testing stage so that it needs to be conducted regardless of whether the combination advances to the final stage. In the next section, we examine the implications of being able to skip the screening stage altogether where Stage 0 predictions are sufficiently high and go straight to determinative testing.
We finally examine how a change in either the cost of the screening or testing stages, or in the accuracy of the screen (which is affected by both $x$ and $y$), impacts the number of combinations that advance from Stage 0. Starting with a change in either of the costs, we find:

$$\frac{\partial p_r^0}{\partial c_1} = \frac{1}{(1 - x)(1 - c_2) + yc_2} > 0.$$  \hfill (27)

$$\frac{\partial p_r^0}{\partial c_2} = \frac{y[(1 - x)(1 - c_2) + yc_2] + (1 - x - y)c_1 + yc_2}{[(1 - x)(1 - c_2) + yc_2]^2} > 0.$$  \hfill (28)

The accuracy of the test will decrease with either a rise in the false negative rate (i.e., an increase in $x$) or a rise in the false positive rate (i.e., an increase in $y$) on the screen. In both cases, the effect is to raise the cut-off probability and thus decrease the number of combinations advancing in the pipeline:

$$\frac{\partial p_r^0}{\partial x} = \frac{(1 - c_2)[c_1 + yc_2]}{[(1 - x)(1 - c_2) + yc_2]^2} > 0.$$  \hfill (29)

$$\frac{\partial p_r^0}{\partial y} = \frac{c_2[(1 - x)(1 - c_2) - c_1]}{[(1 - x)(1 - c_2) + yc_2]^2} > 0.$$  \hfill (30)

Therefore, in addition to the bottlenecks that result from the costliness of later stages in the discovery pipeline, this simple model of the pipeline shows how an additional source of bottlenecks can result from poorly performing screens as captured by high false negative rates and/or high false positive rates. As an increase in either $x$ or $y$ will effectively decrease the uncertainty associated with a given screen, it decreases the value of the option to abandon. A bottleneck is then a reflection of low uncertainty relating to the screen given the poor discriminating power of that screen. As is familiar with options pricing in the financial and real options contexts, this reduction in uncertainty as a result of low efficacy screens will reduce the option value component of the total net expected value.

The effect of an increase in bottlenecks (i.e., some combination of higher values of $c_1$ and/or $c_2$ and higher values of $x$ and/or $y$) on the number of combinations advancing from Stage
0 is shown in Figure 8. A worsening of any of the determinants of the bottlenecks will shift the horizontal line upwards, leading to an increase in the cut-off Stage 0 probability of success and thus a reduction in the number of combinations that enter the pipeline (i.e., a fall in \( r^* \)).

The logic of the three-stage example can be extended to a discovery process with \( S - 1 \) intermediate stages. Entering the penultimate stage (i.e., the last intermediate stage) the analysis is identical at any given node to the three-stage example given above. Any positive option value for this sub-problem will result in positive option value for the decision problem overall. A similar argument applies to any other node along the decision tree, where the decision maker assumes they will act optimally in terms of any decision to abandon at any subsequent node. Therefore, if the option to abandon has value at any node in the decision tree, then the existence of the option has value for the decision process overall.

The challenge faced by innovators is sometimes thought of in terms of the high failure rate along the discovery pipeline. However, this analysis points to the importance of “failing” early before significant costs are incurred in pursuing ultimately unsuccessful innovations. Being able to identify poor combinations early increases the incentive to allow combinations to advance from the AI-assisted prediction stage. The goal might be thought of as: “fail early, fail often” (Babineaux and Krumboltz, 2003). We can think of an important adverse effect from the bottleneck as being combinations that stay too long in the pipeline but eventually fail, decreasing the incentive to enter combinations into the post-prediction pipeline to begin with.

6. Interaction of bottlenecks with an AI-based improvement in prediction

6.1 Examples of Negative Interactions

Our model of the discovery process has shown: (i) that an AI-based improvement in first-stage prediction leads to an increase in the expected net value of innovation; and (ii) for any given combination that enters the discovery pipe, that bottlenecks in the pipeline due to costly screening and testing and/or the poor efficacy of screens reduces the expected net value of that combination. A remaining question is how the existence of bottlenecks interacts with the AI-based improvement in prediction. While the interaction is generally quite complex, depending, inter alia, on the nature of the innovation task, we illustrate the possibility of a negative interaction for the independent
innovations case where the innovator is seeking all possible positive expected value innovations. We examine both a two-stage and a three-stage example.

A two-stage (predict-test) example

We begin with a simple two stage example where the only source of bottleneck is the cost of Stage 1 testing. Figure 9 shows the expected net gain from the adoption of the AI-prediction technology. Ignoring the discrete nature of the problem, the area between the ranking functions up to the number of combinations that advance for testing is the total expected gain from the access to AI. The left panel shows the case of a low cost of second-stage testing (i.e., less severe bottleneck) and the right panel shows the case of high cost second-stage testing (i.e., more severe bottleneck). The increase in expected total net value from access to the improved prediction technology is negatively affected where the extent of the bottleneck is greater. In addition to reducing the positive impact of the improved prediction technology once adopted, the existence of the more severe bottleneck could affect the decision to invest in the AI-based prediction technology to begin with. Such negative interaction could therefore lead to a limited impact on the innovation process of the availability of the improved prediction technology, even though in expectation the impact remains positive.

A three stage (predict-screen-test) example

We next consider the interaction between an improved prediction model and the wider set of potential bottlenecks in the three-stage model. We make the additional simplifying assumption that the false positive rate, \( y \), is zero, leaving three sources of potential bottlenecks – high values of \( c_1, c_2 \) or \( x \). Using equation (26), we can write the expected net value of a combination at a given rank, \( r \), as:

\[
(31) \quad v^e_r = (1 - x)p^0_r - c_1 - [(1 - x)p^0_r]c_2 \\
= [(1 - x)(1 - c_2)]p^0_r - c_1.
\]

We can therefore visualize the expected total net value, \( V^e \), as the area between the two curves shown in Figures 10a and 10b up to the optimal number of combinations to be sent for testing. Figure 10a shows the impact of an improved prediction model on \( V^e \) for both high and low values of \( c_1 \). The increase in \( V^e \) is higher where \( c_1 \) is low rather than where it is high, indicating again a
negative interaction between the improvement in the prediction model and the extent of the bottleneck. Figure 10b compares the impact of an improved prediction model in the case where either $c_2$ or $x$ is low compared to where either $c_2$ or $x$ is high. The size of both $c_2$ or $x$ will affect how much the downward-sloping curve will shift following an improvement in the prediction model. As with the first example, we see a negative interaction between the improved model and the size of the bottleneck. More substantial bottlenecks will again make it less likely that an investment in the improved prediction technology will be undertaken.

We hasten to add that both these examples assume independent innovations (Case 1). As we have seen in our analysis of Case 1 in Section 4, an improvement in the prediction technology leads to more combinations advancing in the pipeline. This explains why higher costs and less effective screens tend to blunt the positive impact of the improved technology. However, as was evident in Cases 2 and 3, it is possible that the improved technology leads to more targeted search with fewer combinations advancing. With less combinations advancing the existence of bottlenecks becomes less of a concern (e.g. fewer costly tests need to be conducted), so that the improvement in the prediction technology and the extent of the bottlenecks interact positively rather than negatively. Nevertheless, putting aside the interactions, it remains true that if AI can be applied to later stages of the pipeline to reduce bottlenecks, there will be a direct gain in expected value (e.g. lower costs for any advancing combination). We finally briefly discuss how AI could be applied beyond the prediction stage and consider both the possible direct and indirect benefits for innovation.

6.2 Could AI Reduce Bottlenecks Over Time?

As they relate to the innovation process, one characterization of recent advances in AI-based prediction is as a new general purpose technology (GPT) for invention (Agrawal et al., 2018a; Cockburn et al., 2018). As has been identified in other contexts, new GPTs may only have their full effect after a significant elapse of time due to the need for complementary upstream and downstream investments. In a classic paper, Paul David (1990) has explored the lagged productivity effects of both the invention of electricity and the computer, and similar lagged effects might be operating for AI as a technology for discovery.\footnote{Drawing on historical analogies of delayed productivity effects, David cautioned against undue pessimism due to the apparent limited impact of computers on productivity, notwithstanding their growing prevalence:} Brynjolfsson et al. (2017) explore four
potential causes of the general (not discovery-specific) productivity paradox in AI - false hopes, mismeasurement, redistribution, and implementation lags - and conclude that precisely this issue, implementation lags due to under developed complements, is the most likely culprit.

In the example of drug discovery, the benefit of AI in generating and ranking a large number of leads may be compromised, say, by a bottleneck following lead identification stage. It is worth considering a David-type solution that might evolve over time. As AI-assisted prediction develops, it might also be applied directly in other stages, including substituting for human judgment in the lead optimization (see Agrawal et al., 2019b on the roles of prediction and judgement in a multi-stage decision process). One possibility is that AI is used to predict the outcome of later screens – say the toxicity of a drug. This could lead to the early abandonment of unpromising combinations before significant costs are incurred.

Although it could be some distance in the future, with well-enough performing AI it might be possible to eliminate certain screening stages altogether and replace them with AI-based predictions, eliminating, say, the need for animal-based safety screens to allow a candidate drug to advance to clinical trials.

In the context of our three-stage example of predict-screen-test, we have already seen that if the Stage 0 predicted probability of success is high enough then the innovator knows that combination will not be abandoned even with a negative screen. We previously assumed that the costly screen had to be conducted (possibly for regulatory reasons). However, in the case where the screen is redundant, we can imagine a situation where the innovator is allowed to skip the screening stage. It follows if $p^0_\tau$ is high enough we return essentially to the two-stage case of predict and test.

"Closer study of some economic history of technology, and familiarity with the story of the dynamo revolution in particular, should help us avoid the pitfall of undue sanguinity and the pitfall of unrealistic impatience into which current discussions of the productivity paradox seem to plunge all too frequently," (p. 359-360).

18 Much of the recent interest in machine learning in the econometrics and policy evaluation literature has focused on the value of machine learning in causal inference and policy evaluation (see Athey, 2019, for a recent survey). Machine learning is being applied to help control for confounding variables to estimate average treatment effects and also to estimate heterogeneous treatment effects (Athey, 2019; Athey and Imbens, 2019). Such tools may be especially relevant in later stages of a multi-stage discovery process as complements, say, to random and natural experiments. Thus machine learning may itself help relieve the bottleneck problem over time.
Figure 11 amends Figure 7 to allow for this possibility. If the Stage 0 predicted probability of success is high enough, we see a discontinuous jump in the bold line that links that probability to the expected net value of the combination, $v^S$. Therefore, if the use of AI-based prediction is successful enough in terms of generating high probability of success candidate combinations, it may directly obviate the requirement for costly (and time consuming) downstream parts of the discovery pipeline.

One additional active area of interest in terms of complementary investments to improved AI-based prediction technologies is the development of “autonomous discovery” systems (Aspuru-Guzik and Persson, 2018). Such systems are particularly relevant where it is possible to have rapid feedback in terms of success data on combinations that advance along the pipeline, where those data can be used to improve the prediction model as part of an active learning strategy. For example, machine-learning-based predictions determine which candidates are tested using robotic high throughput screening (HTP) methods.19

Lamenting the slow speed and high cost of the development and deployment of advanced materials using the traditional approach – new materials typically “reach the market after 10-20 years of basic and applied research” – Tabor et al. (2018, p.5) outline what they see as required for an autonomous (closed-loop) innovation process:

To fully exploit the advances in autonomous robotics, machine learning, high-throughput virtual screening, combinatorial methods and in situ or in operando characterization, we must close the loop in the research process. This means that humans must partner with autonomous research robots to design experimental campaigns and that the research robots perform experiments, analyze the results, update our understanding and then use AI and machine learning to design new experiments optimized to the research goals, thus completing one experimental loop.

19 An early example of such an autonomous system is the Robot Scientist (Sparkes et al., 2010). The first prototype Robot Scientist, Adam, generated hypotheses and carried out experiments related to the functional genomics of a yeast. While it is unlikely that truly closed-loop systems will lead to a mass replacement of scientists, the potential exists for the greater use of AI and automation to ease bottlenecks across the discovery pipeline.
Current processes are affected by the bottleneck problem, which Tabor et al. (2018, p.16) suggest is in the “experimental synthesis, characterization and testing of theoretically proposed materials,” but that if an autonomous approach could be implemented “the bottleneck will move to AI.”

7. Making Predictions: Advances in AI as a Shock to the Innovation Process

In this section, we further discuss AI as the source of the shock to the innovation process. Our generic workflow in the multi-stage model is initial prediction followed by various screens and tests based the sequential refinement of the predicted probability of success. There are a number of ways to generate such Stage 0 prediction models for the landscape. A useful distinction is between theory- and simulation-based approaches on one hand and data-based approaches on the other. For reasons to be explained below, we treat machine learning as a subset of the data-based approach. While data is obviously also central in the other stages (say through the use of controlled or natural experiments), the focus here is on how data is used to generate prediction models.

In order to better delineate the role of machine learning as a prediction tool, we first underline the role of theory in generating predictions in Popper’s classic account of the scientific method. However, we take it that the central requirement for science is that predictions can be tested and leave open the source of those predictions to include data-based approaches. Theory, of course, remains a major source of predictions even with the advance of AI. In theoretical chemistry, for example, the Schrödinger equation remains central to the predictions of molecular properties. Given the complexity of predictions beyond extremely small molecules, various approximations such as the Born-Oppenheimer (1927) approximation or Hartree-Fock theory are used (Szabo and Ostlund, 1996). However, even with these approximations the complexity of the calculations requires the use of (typically costly) computer simulations.

The second broad method of prediction prior to testing is data-based prediction generation. As noted, we think of machine learning as a subset of data-based prediction – a subset for which there has been recent rapid progress. In distinguishing this subset, we find it useful to make use of Leo Breiman’s (2001) contrast between “two cultures” of statistical modeling, which he labels the “data modeling culture” and the “algorithmic modeling culture.” We identify the latter with what is commonly referred to as machine learning, although we recognize that there is no widely accepted division between what is part of traditional statistics and what is part of machine learning.
Breiman outlines what he sees are the broad differences between the two approaches, but a key feature for our purposes is that the former, typically informed explicitly by scientific theory, uses a parametric data model. This model will typically specify the dependent variable, the predictor variables, the functional form and the stochastic form of the disturbance term in the model.

We now consider the effect of having access to machine learning as an alternative tool for statistical modeling of the landscape. Breiman (2001, p. 205) describes the alternative “algorithmic” approach as follows:

The approach is that nature produces data in a black box whose insides are complex, mysterious, and, at least, partly unknowable. What is observed is a set of x’s that go in and a subsequent set of y’s that come out. The problem is to find an algorithm f(x) such that for a future x in a test set, f(x) will be a good predictor of y.

The data-generating processes in many combinatorial-type problems – drug discovery, materials science, genomics, etc. – do appear to fit the description of “complex, mysterious and, at least, partly unknowable.” To the extent that the machine learning approach provides a better prediction model (in at least some circumstances) for valuable new combinations that are distributed over a vast and complex search space, it would appear to have the potential to boost the productivity of the innovation search process.20

How does machine learning fit our generic predict-screen-test workflow? We assume that part of the knowledge base consists of data on previous experiments that we treat for simplicity as indicating tested combinations were successes or failures. These binary outcome data together with input data on the combinations (amino acid sequences, molecular descriptors, etc.) are the training data for our supervised machine learning prediction model. The measure of a good prediction model will be how well it predicts (or generalizes) outside of the training sample.

Of course, there is a vast – and rapidly growing – array of available machine learning algorithms. To give a flavor of their use in generating prediction models we note just a selection here and relate them to our generic workflow of classifying potential combinations into predicted

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20 It is important to note that textbooks on machine learning typically subsume parametric regression and classification models as forms of machine learning. However, we find Breiman’s distinction to be useful in highlighting the shock to the innovation process that the rapid advance in machine learning has engendered, and take his narrower category to be what is meant by machine learning (see also Athey and Imbens, 2019).
successes and failures. Probably the most intuitive algorithm is k-nearest neighbors. The predicted probability of success of a candidate combination is simply the average success rate of the k nearest neighbors in the search space. Decision tree methods work by instead segmenting the search space into success and failure regions. The more complex decision tree methods used in practice (e.g. random forests) involve multiple trees that are combined together to produce the predicted probability of success. A third example is the naïve Bayes classifier. In contrast to the approaches noted already, this approach concentrates on the individual “features” – the states of an element of the string describing a combination in our generic example – which are each assumed to have an independent effect on the success of a combination. The output is an estimate of the probability of success conditional on the states all the elements in the string describing that candidate combination.

In addition to the increased availability of data and computing power, much of the recent excitement concerning machine learning is due to rapid improvement in algorithms. Of particular note is the rapid improvement in prediction models based on artificial neural networks, most notably so-called deep learning algorithms.

Deep learning is making major advances in solving problems that have resisted the best attempts of the artificial intelligence community for many years. It has turned out to be very good at discovering intricate structures in high-dimensional data and is therefore applicable to many domains of science, business and government. (LeCun, et al., 2015, p. 436.)

Although an in-depth discussion of the technical advances underlying deep learning is beyond the scope of this paper, three aspects are worth highlighting. First, the development and optimization of multilayer neural networks allows for substantial improvement in the ability to predict outcomes in high-dimensional spaces with complex non-linear interactions (LeCun et al., 2015). Second, given that previous generations of machine learning were constrained by the need

\[\text{\footnotemark}\]

\footnotetext{For example, a review of the use of deep learning in computational biology notes that the “rapid increase in biological data dimension and acquisition rate is challenging conventional analysis strategies,” and that “[m]odern machine learning methods, such as deep learning, promise to leverage very large data sets for finding hidden structure within them, and for making accurate predictions” (Angermueller et al., 2016, p.1). Another review of the use of deep learning in computational chemistry highlights how deep learning has a “ubiquity and broad applicability to a wide range of challenges in the field, including quantitative structure...}\]
to extract features (or explanatory variables) by hand before statistical analysis, a major advance in machine learning involves the use of “representation learning” to automatically extract the relevant features. And third, recent optimism about developments in deep learning relates to demonstrated out-of-sample performance of deep learning models across a range of tasks, including image recognition, speech recognition, language processing, and autonomous vehicles.

Notwithstanding that the most publicized successes of deep learning have been in areas such as games of strategy including chess and go and the winning of image recognition competitions such as ImageNet, parallels to the way in which the new methods work on unstructured data are increasingly being identified in many fields with similar data challenges to produce research breakthroughs. While these new general purpose research tools will certainly not replace displace traditional theory, simulation, and statistical data modeling – nor the researcher’s intuition – as methods for developing predictive models of fitness landscapes, machine learning methods such as deep learning appear to offer powerful new tools for prediction, especially where the complexity of the underlying phenomena present obstacles to more traditional methods.

8. Policy Implications
We focus on policies arising from three distinct market failures. First, the gap between the socially optimal and privately optimal levels of data sharing may be severe. Data is one of the primary ingredients for building AI systems. Policies that either regulate or incentivize the sharing of privately held data may yield significant social welfare dividends under conditions where those data enable the development of AI models that support productivity enhancing innovations.

Second, the private sector may severely under invest in the running of experiments that generate data that could subsequently be used for training AI models to support productivity enhancing innovations. This is likely to be especially salient as lab work is automated via robotics, lowering the marginal cost of experiments. Thus, subsidies and other incentives for running experiments in order to generate data in sparse areas of the search space may generate outsized positive externalities for discovery.

Finally, there may be path dependency associated with the location of market leaders in markets that become highly dependent on AI for innovation, such as drug discovery and materials discovery. Leadership in these domains may depend on access to labor with cross-disciplinary skills (e.g., biology and machine learning). As universities are slow to internalize market forces, regions may benefit from implementing funding and other incentives to accelerate the provision of new cross-disciplinary programs that create a local supply of talent that is suitably trained to commercialize opportunities at the intersection of AI and scientific discovery. We explain these three market failures below, but first put them in context as novel due to the shifting frontier of technological advance.

8.1 Policy and the shifting frontier

The technological frontier has shifted over the past half century. Advances in AI are both a result of that shift, which has favored information sciences and biology over energy, transportation, and chemicals, and are now also a cause of that shift because AI increases returns to the types of discovery that are enabled via prediction predicated on big data. So, AI-related policy must be considered in the context of innovation frontiers that are different today than when many innovation policies were established in the period immediately following WWII.

In terms of productivity, despite indications of rapid technical advance in areas such as information technology and genomics, growth in recent decades has been disappointing (see, e.g.,
Various explanations have been given including increased mismeasurement and lags between breakthroughs in general purpose technologies and productivity impacts (Brynjolfsson et al., 2019). As argued by Gordon, it is also possible that the low hanging fruit in terms of useful productivity improvements have been picked, notwithstanding the appearances of rapid technical advance.

The late 19th century and first half of the 20th century was a period of rapid improvement in the understanding of physical and chemical phenomena. These improvements in turn led to theory-based prediction models that had wide practical application in the middle decades of the 20th century, including in areas such as energy (e.g., electricity), transportation (e.g., the internal combustion engine) and chemicals (e.g., fertilizers).

In contrast, many of the most important breakthroughs in recent decades have been in the biological and information sciences. Advances in these areas have also interacted as the complexity of biological systems necessitates complex statistical models, increased computing power, and huge amounts of data. Examples already discussed include the prediction of small molecule drug binding with target proteins, predictions of protein folding to better understand target proteins, and the identification of a broader range of intervention targets through improved understanding of gene regulatory networks. Even in domains where there remains significant potential for theory and simulation-based prediction models – as in materials science – the computational burden of applying these prediction tools over large search spaces can be prohibitive, leading to increased interest in AI-aided approaches that support the search process.

There has therefore been an apparent shift in the technological frontier and in the parts of that frontier that are amenable to further advances using AI-based prediction tools. This changing frontier suggests the need for policymakers to support the redirection of resources to areas where the potential for new discoveries is greatest. In the remainder of this section, we consider three challenges that policymakers must grapple with as they respond to these changes: access to (big) data; bottlenecks in increasingly AI-aided discovery processes; and the policy implications of the changing market for scientist skills.

8.2 Policies to Incentivize Data Sharing
Perhaps the most consequential policy implication of our combinatorial search model relates to the importance of data. Access to data – findings on prior successes and failures in our simplified setting – is essential to building effective prediction models. Greater access to data allows for better exploitation of the fruitful parts of the space by building better maps of that space. The model also suggests the importance of data spillovers in addition to more traditional idea spillovers for sustaining economic growth.

Scientists, funders, and policymakers are increasingly recognizing the importance of open data, and numerous collective efforts have arisen to improve data access. Funding agencies, for example, are more often requiring open access as a condition for funding. Scientists themselves are mobilizing to solve the collective action problem of access to data: the number of open-source datasets is growing. Examples include *The 1000 Genomes Project* (genomics), *PubChem* (chemistry) and *The Materials Project* (materials science).  

A significant concern is a lack of access to data on failures, which are more likely to remain hidden in the notebooks of experimentalists given the strong incentives to publish data on successes and missing incentives to publish data on failures (Raccuglia et al., 2016). The successful training of supervised machine learning models requires knowledge of what failed as well as what succeeded in the past. While access to data on failures has always been valuable for scientific research, the necessity of failure data to train AI models greatly increases the value of failure data. As the extant system of science that favors the publication of successful experiments, new incentives are required to increase the propensity of scientists to publish their failed experiments. Policies designed increase the incentives and decrease the frictions associated with sharing data on failed experiments will likely generate significant positive externalities in a new era where AI models play an important role in productivity enhancing innovation.

Another policy concern stems from the importance of large private companies in the innovation process. Private companies may have strong incentives to limit access to proprietary data. This gives larger companies an advantage that can become self-reinforcing as they

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26 AI is itself being applied to help discover data. For example, *BenchSci* is an AI-based search technology for identifying antibodies that act as reagents in scientific experiments. It attempts to deal with the challenge that reports on compound efficiency are scattered through millions of scientific papers with limited standardization on how the reports are provided. *BenchSci* extracts compound efficiency information thus allowing scientists to more effectively identify compounds for experiments.
monopolize access to expanding datasets. However, many firms are recognizing the advantage of open innovation models (see Chesbrough et al., 2006, for an early discussion) as they gain access as well as provide access by engaging in open knowledge sharing networks. Firms may also have an incentive to make data (and algorithms) accessible as they attempt to stave off regulation and anti-trust enforcement. Even so, recognizing the relative power of leading technology (including social networking) companies in accessing data compared to universities and smaller firms, policies that increase the cost of innovation-retarding monopolization of key emerging sectors will likely yield significant social benefits by way of enabling productivity enhancing innovation.

Finally, national and regional approaches to data protection policies could also affect the geographical evolution of AI-related activity. For example, the relatively stringent approach to data protection under the European General Data Protection Regulation could hinder the development of certain types of AI-based discovery and innovation in the EU. The increasing salience of privacy issues could limit access to certain forms of data such as patient health records, although privacy issues should be less of an issue where personal data is not involved (e.g., datasets on molecular descriptors and properties). China’s less protective approach to personal data (e.g., facial recognition data) could give it a competitive advantage in certain settings. Policymakers will have to make difficult choices on how to balance the trade-off between data access and privacy protection in the context of international competition for leadership in important emerging economic sectors.

8.3. Policies to Incentivize the Creation of Data

AI models are much better at predicting successful combinations from the parts of the search space that are well populated with experimental data compared to those parts that are sparsely populated. Profit-maximizing firms will understandably focus on harvesting results from heavily populated parts of the search space. The difference between the private versus social benefits of running experiments, that are less likely to succeed but will add data to the sparsely populated parts of the search space, may be very significant due to the private costs of running experiments and the public benefits of populating the search space. Therein lies another opportunity for welfare enhancing policies in the context of AI and innovation.

Our model highlights the problem of bottlenecks in the discovery process that could limit the productivity benefits of machine learning as a GPT for prediction. The issue of bottlenecks has
probably received most attention in the context of materials discovery. The long timelines between
the initiation of research and the launch of new materials have led to interest in more autonomous
(or “self-driving) discovery processes (Aspuru-Guzik and Persson, 2018). The slow speed and high
failure rates of discovery pipelines plagued with bottlenecks has increased interest in more
autonomous processes that utilize a combination of AI and robotics. For example, machine-
learning-based predictions determine which candidates are tested using robotic high throughput
screening (HTP) methods. While it seems unlikely that truly closed-loop systems will lead to a
mass replacement of scientists, the potential exists for the greater use of AI and automation to ease
bottlenecks across the discovery pipeline.

The promise of autonomous systems is expressed by Tabor et al. (2018, p.5) as follows:
“[P]latforms that integrate AI with automated and robotized synthesis and characterization have
the potential to accelerate the entire materials discovery and innovation process to reduce this time
[10-20 years] by an order of magnitude.” To deliver on this promise, they call on efforts and
leadership from academia, governments, and industry in building the required multi-disciplinary
workforce and supporting collective action efforts such as the Material Genome Initiative and the
multi-country Mission Innovation Initiative.

The prediction model in our two-task process played a basic role: predicting the probability
of success of any given candidate combination and thus the combinations that should go for testing.
In a truly closed-loop process, an additional source of value from tests needs to be considered: how
the data from those tests can be used to improve the prediction model. For example, an additional
test observation that allows for more accurate discrimination for candidate combinations in the
neighborhood of the probability threshold may have a particularly high information value as the
innovator seeks to exploit promising parts of the combinatorial space.

27 Tabor et al. (2018, p.15) outline what they see as required for an autonomous (closed-loop) innovation
process: “To fully exploit the advances in autonomous robotics, machine learning, high-throughput virtual
screening, combinatorial methods and in situ or in operando characterization, we must close the loop in the
research process. This means that humans must partner with autonomous research robots to design
experimental campaigns and that the research robots perform experiments, analyze the results, update our
understanding and then use AI and machine learning to design new experiments optimized to the research
goals, thus completing one experimental loop.”
In contrast, test observations whose neighbors are in parts of the space that have very high or very low probabilities of success may have low information value. On the other hand, when the cost of autonomous experimental tests is low, it may make sense to conduct more exploratory testing of regions of the space where data on successes and failures is sparse, potentially opening truly novel approaches (say in materials for energy harvesting or storage). While the best experimental design will be context dependent, the important point is that a truly autonomous process will require closing the loop by choosing which experiments to conduct based on both the direct expected innovation value and the indirect expected data value of the test. To the extent that the exploration benefits spillover to other users in the form of generally available data, there is a role for policy to offset the resulting bias towards exploitation over exploration.

8.4. Demand for and supply of scientist skills

Much of the AI-related policy analysis focuses on its potential implications for labor demand and consequently for wages, labor shares, and employment. Concerns of adverse effects on scientists, engineers, and others working in innovation intensive sectors are heightened if we move towards fully autonomous discovery systems. We therefore next consider policy implications of the increased use of machine learning in discovery on the demand for various types of R&D labor.

In a series of papers, Daron Acemoglu and Pascual Restrepo extensively analyze the effects of AI and robotics on labor demand (e.g., Acemoglu and Restrepo, 2018 and 2019a). We adopt the useful framework set out in Acemoglu and Restrepo (2019b). This framework decomposes the labor demand effects into displacement effects, countervailing productivity effects, and new task (or reinstatement) effects.

Our multi-stage model shows the potential for human skill displacement as a result of breakthroughs in machine learning for both tasks. As outlined in Section 7, prediction can be achieved through various methods including theory, computer-based simulation, and the use human intuition to achieve educated guesses. Furthermore, these forms of prediction can be present together as, say, intuition guides theory development and predictions are generated from complex theoretical models using computer aided simulations as in the use of density functional theory in chemistry. Statistical data modeling can also be the basis for prediction by highlighting
associations in the data and can also inform theory and simulation (e.g., by aiding the initial calibration of simulation models).

By providing an alternative source of predictions, there is an obvious potential for machine learning to displace certain existing skills. In fact, we might normally expect that improved prediction would increase the demand for testing through an increased flow of promising combinations through the discovery pipeline. Yet, as Case 2 (search for a single successful combination) of the two-stage model of Section 4 shows, the effect of an upward shift in the ranking function on the amount of testing depends on the relative cost of testing compared to the value of a successful innovation. At sufficiently low levels of this ratio, a more discriminating prediction model actually leads to fewer tests performed as the expected value of the marginal test declines. The reason is that improved prediction increases the probability that the target will be achieved by the other tests performed, lowering the expected value of the marginal test.

While the potential for displacement is clearly present, there may also be countervailing productivity effects that increase the demand for certain skills. Most obviously, where improved prediction does increase the flow of combinations through the pipeline, the demand for the skills of testers (say experts in experimental design or lab technicians) will increase. Acemoglu and Restrepo (2019b) also note additional sources of countervailing effect through increased capital investments and the “deepening of automation.” These processes may be particularly relevant as investments and technological development takes place to remove the bottlenecks in the system in order to better take advantage of the new prediction GPT.

Acemoglu and Restrepo (2019b) stress the importance of the demand for new tasks (and related skills) that come into being as a result of AI. The AlphaFold case with which we open the paper provides a good example: this protein folding prediction effort brought together an interdisciplinary team from structural biology, physics, and machine learning.

Critical to the success of teams at the cutting edge of scientific discovery and innovation is likely to be the ability of members to communicate with other specialists: machine learning specialists need to have a sufficiently deep understanding of biology to work with the structural biologists; the structural biologists will need to know enough machine learning, for tasks such as preparing the data for building well-performing prediction models, to complement the machine-learning specialists. Skill sets that combine expertise such as computer programming or statistics
with knowledge of specific scientific domains are likely to be in especially heavy demand. Such evolving demands will lead to incentives for specific upskilling and career transitions and also changes in the curricula (and hiring priorities) in universities and other training programs.

This creates challenges for academia, government, and industry as they engage in workforce planning. Such integration challenges have long been recognized in innovation-intensive industries such as biotechnology.

To perform well the [biotechnology] sector requires appropriate mechanisms for bringing together and integrating the right mix of cross-disciplinary talents, skills and capabilities. These mechanisms include organizational structures and strategies and the means by which different types of organizations (large firms, small start-ups, universities, etc.) interact. However, the sector also requires micro-organizational mechanisms for creating truly integrated problem solving and avoiding islands of specialization. And, perhaps most importantly, it requires ways of getting together the right mix of people from different scientific and functional backgrounds to collaborate and exchange information. (Pisano, 2006, p.76.)

Universities are understandably slow to internalize market signals for skills newly rewarded by the labor market. Yet, there is often geographic path dependency in the leadership of markets in transition (see e.g., Zucker, Darby, and Brewer (1998) for an example from biotechnology in the 1980s). In other words, the rewards for being early to train graduates with novel skills may be partially appropriated by the local region. So, for the purpose of regional economic development, there may be a case for policy intervention to increase the incentives for universities to accelerate the evolution of their programs for training students with specific skills. In the case of AI, this includes the skills to take advantage of (or indeed build) the new GPTs for discovery. It also includes incentives to ensure appropriate research-led teaching by frontier researchers so that the innovation workers of the future have the (integrated) skills that will be newly in demand.

9. Concluding Comments
AI has achieved well-documented recent successes in tasks such as image recognition, speech recognition, language translation, recommendation systems, and autonomous vehicles. It is now viewed as a GPT for prediction tasks that can be combined with other components to produce novel products and services (Agrawal et al., 2018). The machine-learning based prediction models have the common feature that they can deal with vast combinatorial spaces (e.g. pixels in an image). This paper has focused on machine learning as a GPT for use in the discovery process itself. Conceptualizations of the innovation process as search over a combinatorial space suggests the value of machine learning in helping to map this space in the form of a prediction model.

The burgeoning scientific literature applying machine learning tools suggests the practical value of being able to make predictions when the number of potential combinations can be in the billions. But prediction is just one part of the discovery process. We thus embedded the prediction task as just one part of a costly multi-stage process. In our model of AI-aided discovery, the prediction model produced a ranking function that essentially provided a priority list for later (costly) screening and testing. Improvements in the prediction model – say as a result of the availability of a better performing algorithm – allowed for a more discriminating prioritization and ultimately for a more productive discovery pipeline.

The main testable hypothesis from the model is therefore that access to AI will increase the productivity of the innovation process for combinatorial-type problems through improved prioritization. Although there has been rapid growth in the use of machine learning – and a great deal of optimism expressed about its effects – the productivity benefits have yet to be well established (Brown et al., 2020). Indeed, there is skepticism about its ultimate benefits, in part reflecting the previous waves of optimism and pessimism associated with AI. There have also been well-publicized cases where early optimistic scientific findings have been reversed and exits of high-profile research groups. Skeptics have a number of concerns, including: limited and error-prone data leading to poorly performing models; failure to replicate results; excessive concentration on already well explored regions of the search space where data are plentiful; difficulties of interpretation of “black box” models and associated issues of trust; and models that appear to work well on test data sets but ultimately perform poorly due to redundancy between training and test data (Bender and Cortés-Cirano, 2020; Wallach and Heifets, 2018).
The next step in our research program is therefore to look for evidence of the productivity effects of increased access to AI-aided discovery. This is obviously complicated by the relative newness of some of the main developments (including the improved performance of deep neural network algorithms). However, we hope it will be possible to exploit plausibly exogenous variation in the access to AI tools. One idea we are exploring is to use variation in historic expertise at the department level in machine learning in computer science and engineering departments, and to see how it relates to later output in application domains within the university (medicine, chemistry, etc.) based on assumption of local knowledge spillovers. Another avenue is to look for access shocks to AI-prediction technologies and see how they differentially affect the productivity of researchers.

The paper also considered the policy implications of the rise in AI-aided discovery. Part of the background is recent disappointing productivity growth despite apparent rapid technological advance, particularly in the biological and information sciences. These advances are opening new frontiers of discovery that have the potential to significantly improve welfare through impacts on areas such as health, agriculture, energy, and climate. If pessimism about future growth in part reflects concerns that the “low-hanging fruit” has been picked, AI may both open new orchards and – the focus of this paper – provide new tools to pick the heretofore hard to reach fruit. However, any beneficial effects of AI on the discovery process could be slowed by barriers to data access, missing incentives to run experiments that will generate data in sparse areas of the search space, and the under provision of training in skills that combine domain-specific knowledge and AI expertise. Policymakers thus face a range of challenges to address these market failures in order to ensure that the potential for AI-aided discovery is realized.

Inspired by burgeoning scientific work using AI tools for discovery (see, e.g. Ramsundar et al. 2019), the type of AI in the model developed in this paper is of the task-specific rather than the general variety. More precisely, the model is motivated by the observed use of AI to help prioritize costly search over physical spaces (e.g. the space of molecules, gene networks, protein shapes, etc.). In contrast, an important idea in modern growth theory is the hypothesis that new ideas are generated by combining existing ideas in a cumulative process (Weitzman, 1998; Romer, 1993; Arthur, 2009; Agrawal et al., 2019). Cognitive scientists have also extensively studied the mental processes underlying such combinatorial processes (Langley et al., 1987; Boden, 2004;
Thagard, 2012). If done by machines, search over idea space would appear to require more human-like artificial general intelligence (AGI) rather than the more specific task-focused tools we have assumed.

The likelihood of the emergence of AGI is extensively debated among AI researchers, with many skeptical of its prospects (see, e.g., Boden, 2016 and Mitchell, 2019). Others, while recognizing the challenges in developing AGI, believe that the implications could be sufficiently disruptive that is important to begin preparing now (Tegmark 2017; Russell, 2019; Acemoglu, 2021). Going back to Poincaré (1913) and Hadamard (1945), human discovery has been conceptualized as a search process over combinatorial search spaces, but discoveries were thought to often require inspiration that comes from the unconscious rather than the conscious mind leaving it primarily in the domain of human intelligence. Others, however, have viewed it as more of a rational process that could in principle be done by machines. Langley et al. (1987), for example, offer the example of Planck’s discovery of a prediction formula for blackbody radiation as resulting from a conscious search process across the space of possible equations that are consistent with the given available experimental data — a task that could, in principle, be replicated by an AI.

While we think that a significant replacement of human intelligence with AGI is unlikely in the coming decades, we note the continued rapid pace of advance in AI technologies. Just to note two examples that are relevant to discovery: first, the development of AI Feynman has shown the power a neural network to use “symbolic regression” to rediscover many of the equations from classical physics (Udrescu and Tegmark, 2020; Udrescu et al. 2020); and second, transformer/attention models (Vaswani, 2017) are leading to significant breakthroughs in natural language processing (e.g. in OpenAI’s GPT-3) and are providing “foundation models” (Bommasani, et al., 2021) that support transfer learning – an important element of more general tools – between domains. But whatever the progress towards AGI for discovery, the recent explosion in research using more task-specific AI tools for prioritization in costly search as modelled in this paper suggests AI is already having important effects on the process of scientific discovery and innovation.

28 A fascinating derivative of GPT-3 is OpenAI Codex, which predicts software code from natural language prompts from the creator.
References


Appendix 1. Optimality of the Weitzman Ordering Over Bernoulli Boxes

In this appendix, we provide a simple proof of the optimality of the Weitzman ordering for searching Bernoulli boxes. Each box, $i$, is described by a probability that it contains a success, $p_i^0$, a payoff conditional on success, $\pi_i$, and a cost of searching the box, $c_{1,i}$. We assume that $p_i^0$ and $c_{1,i}$ are strictly greater than zero and that there is no time discounting. As set out in the text, each box has a “reservation price,” $z_i = \pi_i - \frac{c_{1,i}}{p_i^0}$. The reservation price has the interpretation of the “sure-thing” that has an expected payoff equal to the lottery of opening the box (at a cost) where there is a back-up option equal to the sure-thing that is payable in the event that the opening of the box does not yield a success. In our setting, we interpret the opening of a box as the conduct of a costly test that determines whether it contains a success or not. Applied to Bernoulli boxes, the Weitzman result is that the expected value of the search for the best alternative is maximized by sequentially searching the boxes with an expected net value greater than zero in declining order of reservation prices and stopping the search once a success is achieved.

The optimality of this search strategy can be demonstrated by considering the optimal pairwise ordering of any two boxes $i$ and $j$, where we initially assume that $z_i > z_j$. From the definition of the reservation price we can immediately see that this implies that $\pi_i > \pi_j$. Therefore, if a success is found on opening box $i$ the search will stop. The intuition is that the payoff from box $i$ becomes the sure-thing for the lottery of opening box $j$ and the sure-thing is greater than required for indifference between taking the sure thing and playing the (costly) lottery of opening box $j$.

The key question is whether it is optimal to open box $i$ first. Contrariwise, consider the strategy of opening box $j$ first. There are two possibilities assuming a success is found on opening the box. First, if $\pi_j < z_i$ it will not be optimal to stop the search and the search will continue to box $i$. Of course, since $\pi_i > \pi_j$ given $z_i > z_j$ and $\pi_j < z_i$, box $i$ would always be chosen if a success is achieved with both boxes. Therefore, it would always be optimal to first open box $i$ and only continuing to box $j$ if a success is not found and otherwise saving on the search cost for box $j$ if a success is found for box $i$. 
Second, and more interesting, is the case where \( \pi_j > z_i \) so that search would stop if a success is found on first opening box \( j \). The choice facing the innovator is now whether to first search box \( i \) and stop if a success is found or to first search box \( j \) and stop if a success is found. The optimal order to open these boxes is found by comparing the expected value of the two strategies. The expected value of opening box \( i \) will be greater when:

\[
(A.1) \quad p_i^0(\pi_i - c_{1,i}) + (1 - p_i^0)(p_j^0\pi_j - c_{1,i}) - p_j^0(\pi_j - c_{1,j}) - (1 - p_j^0)(p_i^0\pi_i - c_{1,j}) > 0
\]

\[
\iff z_i > z_j,
\]

where the latter implication is derived by simple algebraic manipulation of the first inequality and the definition of the reservation price. Therefore, the strict ordering of the reservation prices in favor of box \( i \) is a necessary and sufficient for a strict preference for first searching box \( i \). In the case where the reservation prices are equal, the expected value will be independent of the order in which the boxes are searched and the innovator will be indifferent as to the order of search. In this case, we assume that the order of the search is chosen randomly. (Note that \( \pi_i > z_j \) and \( \pi_j > z_i \) under indifference so that it will be optimal to stop the search upon finding a success no matter which of the boxes is searched first.) As such pairwise comparisons can be applied to every pair of boxes, we obtain, as required, a complete (and transitive) optimal search ordering of the full list of relevant boxes based on their reservation prices with search stopping once a success is achieved.
Notes: Where the innovator can perfectly discriminate between successes and failures, the ranking function takes the form of a unit step function. Within the subsets of successes and failures, the ranking is arbitrary. At the other extreme, where there is no ability to discriminate between successes and failures prior to testing, the ranking function is horizontal at the probability of finding a success based on a random draw from the search space.
Figure 2: Ranking Function Curves for Different Values of the Discrimination Parameter, $b$

Notes: The figure shows how the logistic ranking function changes as a result of an increase in $\beta$, which we equate with an improvement in Stage 0 prediction. The logistic ranking function is horizontal at the probability $G/N$ when $b$ is equal to zero. Increases in $b$ cause the logistic ranking function to swivel in a clockwise direction around the point $(G + 1, G/N)$. As $\beta$ goes to infinity the ranking function will converge towards the unit step function representing perfect ability to discriminate between successes and failures.
Figure 3a: Determination of the Optimal Number of Combinations to Advance to Testing

\[ p_r^0 = MV_r^e, MC \]

Figure 3b: Impact of an Improvement in the Prediction Model on the Optimal Number of Combinations to Advance to Testing

\[ p_r^0 = MV_r^e, MC \]

Gain in expected total net value due to improvement in the prediction model.
Figure 4a: Optimal Number of Tests when the Innovator a Single Innovation Target

\[ p_r^0 = MV_r^e, MC \]

Figure 4b: Impact of an Improvement in the Prediction Model on the Optimal Number of Tests when the Innovator has a Single Innovation Target and the Crossover Probability is below \( c_1 \)

\[ p_r^0 = MV_r^e, MC \]

Figure 4c. Impact of an Improvement in the Prediction Model on the Optimal Number of Tests when the Innovator has a Single Innovation Target Crossover Probability is above \( c_1 \)

\[ p_r^0 = MV_r^e, MC \]
Figure 5. Evolution of the Probability of Success through a Multi-Stage Discovery Pipeline

Notes: The figure shows an example of a 5-stage discovery pipeline (where the final determinative testing stage is not shown). In addition to the initial Stage 0 prediction and final Stage 4 testing stages, there are three intermediate stages. The assumption of common values of both $x$ (false negative rate) and $y$ (false positive rate) for each of the screening stages produces the binomial lattice (or recombinant) structure of the possible evolutions of the probability of success of a given combination as it advances along the pipeline. The initial node gives the probability of success following the initial prediction stage ($p^0_r = 0.25$ in the example above). At each intermediate stage, the node gives the probability of success given the outcome of the screen. The slopes of the lines on the graph are determined by ex ante probability of success or failures going into a given screening stage with depends on the relevant prior probability of success and the values of $x$ and $y$. For a given stage, $s$, the number of possible values for the probability of success rises linearly with the number of the stage according to $s + 1$. Viewed from Stage 0, the rational expectation of the probability of success at any subsequent state is $p^0_r$. 
Figure 6. Probability of Survival Following Completion of Stage, $s$

Example of the case where the combination is always abandoned following a negative screen

The figure shows that survival probability through the pipeline under the assumption that the combination will be abandoned in the event of a negative screen. It thus gives the cumulative probability of $s$ successful screens.
Figure 7: Expected Net Value of a Combination with Option to Abandon and Screen Must be Completed to Advance a Combination to Testing

Notes: The figure shows how the expected net value of a combination varies with the post-Stage 0 probability of success for a three-stage discovery pipeline with a single intermediate screening stage. The line with intercept \(-c_1 + c_2\) shows how the expected net value would evolve if both the intermediate screening and final testing stage have to be completed. The line with intercept \(-c_1 + yc_2\) shows how the expected net value would evolve if a combination is always abandoned if there is a negative screen. The bold line reflects optimal behaviour on the part of the innovator where: (i) negative expected net value combinations will not advance from Stage 0 even after taking into account the optional to abandon after the screening stage; and (ii) for high enough values of the Stage 0 probability of success the combinations will abandoned even after receiving a negative result at the intermediate screening stage. The first upward pointing arrow indicates the cut-off value for the Stage 0 probability of success for advancement to the intermediate state. The second upward pointing arrow indicates the Stage 0 probability of success below which the option to abandon would be exercised on receiving a negative screen. The gap between the dashed line and non-bold solid line gives the value of the option to abandon for combinations where the option to abandon would be exercised upon receipt of a negative screen.
Figure 8. Impact of an Increase in Bottlenecks on the Number of Combinations Advancing to the Screening Stage in the Discovery Pipeline
Figure 9: Expected Gain in Total Net Value from Access to Improved AI-Based Prediction Technology in the Two-Stage Model (Case 1)
Figure 10a: Expected Gain in Total Net Value from Access to Improved AI-Based Prediction Technology in the Three-Stage Model (Low versus High Values of $c_1$)

Figure 10b: Expected Gain in Total Net Value from Access to Improved AI-Based Prediction Technology in the Three-Stage Model (Low versus High Values of $c_2$ or $x$)
Figure 11: Expected Net Value of a Combination with Option to Abandon and a Redundant Screen can be Skipped

Notes: This figure modifies Figure 7 to allow for the screening stage to be skipped when even a negative result on the screen does not lead to the exercise of the option to abandon the combination. Such an outcome will only result where the Stage 0 probability of success is sufficiently high: \( p^0_r \geq \frac{(1 - y)c_2}{x + (1 - x - y)c_2} \). There is then a discontinuous jump in the bold line that relates a given Stage 0 probability of success for a combination to the expected net value of that combination. If an AI-based improvement in prediction results in such high probabilities, it would have an additional effect of altering the subsequent downstream discovery pipeline.