Supplement: When is Growth at Risk?

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S. Supplementary Appendix

S.A. Econometric Methods

We here describe the econometric methods we use to analyze the tail risk – and specifically downside risk – to GDP growth. Beyond estimating the mean and volatility, we aim to understand how much the available time series data can tell us about the skewness and kurtosis of growth. If we were able to characterize the dynamics of these higher moments, it would go a long way toward providing a complete understanding of tail risks. Intuitively, however, such higher moments are sensitive to the occurrence of rare events, and so may be hard to pin down from time series data that only goes back to the 1970s at best.

To strike a balance between flexibility and statistical precision, we consider both nonparametric and fully parametric estimation approaches. We first adopt the nonparametric approach proposed by Adrian et al. (2019), who use quantile regressions to estimate time series of the conditional variance, skew, and kurtosis of U.S. real GDP growth, as well as corresponding measures of downside risk. Then we consider parametric methods which, unlike the nonparametric method, are able to quantify the potentially high uncertainty surrounding these estimates and also allow for a richer set of predictor variables.

Additionally, at the end of this subsection we describe the factor model used to extract the global and financial factors that serve as predictors in Sections II and III.

NONPARAMETRIC APPROACH The nonparametric approach to estimating growth tail risk developed by Adrian et al. (2019) consists of two steps. First, quantile regressions are used to estimate the conditional quantiles of GDP growth as a function of predictors. Second, a flexible family of probability distributions is fitted to the conditional quantiles. We now describe each of these steps in turn.

Let y_t denote the quarter-over-quarter real GDP log growth rate between time $t-1$ and t. Let $y_{t,t+h} = \sum_{\ell=1}^{h} y_{t+\ell}$ denote the cumulative log growth in real GDP between time t and $t+h$. Finally, let x_t denote a p-dimensional vector of predictor variables.

Quantile regression. In the first step we estimate the conditional quantile function (CQF) of $y_{t,t+h}$ given x_t at quantile τ:

$$Q_\tau\left(y_{t,t+h}|x_t\right) = \inf\{y : F_{y_{t,t+h}|x_t}\left(y|x_t\right) \geqslant \tau\},$$

where $F_{y_{t,t+h}|x_t}\left(y_{t+h}|x_t\right)$ is the conditional cumulative distribution of $y_{t,t+h}$ given x_t. The CQF solves the following maximization problem:

$$Q_\tau\left(y_{t,t+h}|x_t\right) = \text{argmin}_{q(x_t)}\mathbb{E}\left[\rho_\tau\left(y_{t,t+h} - q(x_t)\right)\right], \tag{S.1}$$

where $\rho_\tau(u) = (\tau - \mathbb{1}(u \leqslant 0))u$ is a function which weights positive and negative terms asymmetrically.

Under the assumption that the CQF is linear, $Q_\tau\left(y_{t,t+h}|x_t\right) = \beta_\tau' x_t$, we have

$$\beta_\tau = \text{argmin}_b\mathbb{E}\left(\rho_\tau\left(y_{t,t+h} - x_t' b\right)\right). \tag{S.2}$$

The quantile regression estimator $\hat{\beta}_\tau$ is defined as the sample analogue of β_τ and can be found as the solution to a linear programming problem. The estimator of the CQF at quantile τ is then given by

$$\hat{Q}_\tau(y_{t,t+h}|x_t) = x_t'\hat{\beta}_\tau. \tag{S.3}$$

Fitted distribution. In order to compute other features of the conditional distribution than just quantiles, Adrian et al. (2019) fit a flexible family of probability distributions to the estimated 5th, 25th, 75th, and 95th percentiles from the first step. That is, they select the parameters of the chosen distribution family to match as closely as possible the estimates $\hat{Q}_\tau(y_{t,t+h}|x_t)$ at those percentiles (conditional on the realized values of x_t).

The specific family of distributions used by Adrian et al. (2019) is the *skew-t* distribution of Azzalini and Capitanio (2003), which generalizes the usual symmetric

Student-t distribution. To define this distribution, consider first a random variable U that has the standard *skew-normal* distribution with density function

$$p_U(x;\alpha) = 2\varphi(x)\Phi(\alpha x), \quad x \in \mathbb{R}, \tag{S.4}$$

where $\varphi(\cdot)$ and $\Phi(\cdot)$ are the density function and distribution function of the standard normal distribution. The skew-normal density is unimodal, and it reduces to the standard normal density for $\alpha = 0$. The parameter α governs the skewness of the density, with $\alpha > 0$ implying right-skew and $\alpha < 0$ implying left-skew.[1] The skew-t$(\mu, \sigma, \alpha, \nu)$ distribution is defined as the distribution of the random variable

$$S = \mu + \sigma \frac{U}{\sqrt{V/\nu}},$$

where U has the skew-normal distribution with parameter α, V is χ^2-distributed with ν degrees of freedom, and U and V are independent.[2] If $\alpha = 0$ with ν fixed, this reduces to the usual scaled Student-t distribution; if $\nu \to \infty$ with α fixed, this reduces to the scaled skew-normal distribution. More generally, ν governs the kurtosis of the distribution, with smaller values corresponding to fatter tails.

The parameters of the skew-t distribution are chosen to fit the quantile regression estimates at each realized value of the covariates x_t, generating a sequence of parameters $(\hat{\mu}_t, \hat{\sigma}_t, \hat{\alpha}_t, \hat{\nu}_t)$, $t = 1, \ldots, T$. Then moments of the fitted distribution at each point in time are calculated. Following Adrian et al. (2019), we report the mean, variance, skewness, and kurtosis, as well as a measure of tail risk: expected shortfall. The *5% expected shortfall* is given by the conditional expectation of GDP growth, conditional on a growth realization that is below the 5th percentile of the conditional growth distribution.

Finally, to measure out-of-sample predictive accuracy of distributional forecasts, we consider the *predictive score* that defined as the predictive distribution generated by the model and evaluated at the outturn value of the time series. Higher values of the predictive scores indicate more accurate predictions because they show that the model assigns higher likelihood to realized outcomes.

PARAMETRIC APPROACHES Because the nonparametric method is data-hungry and only applicable when the number of predictors is small, we additionally consider two parametric models of the time-variation in the volatility and/or skewness. We estimate these models using a fully Bayesian approach, allowing us to

[1] Azzalini (1985) plots the density function $p_U(x;\alpha)$ for different values of α.

[2] The PDF of S is given by $f(s;\mu,\sigma,\alpha,\nu) = \frac{2}{\sigma} t\left(\frac{s-\mu}{\sigma};\nu\right) T\left(\alpha \frac{s-\mu}{\sigma}\sqrt{\frac{\nu+1}{\nu+(\frac{s-\mu}{\sigma})^2}};\nu+1\right)$, where $t(\cdot;\nu)$ and $T(\cdot;\nu)$ are the PDF and CDF of the Student-t distribution with ν degrees of freedom.

(i) consider a large number of predictor variables x_t simultaneously and (ii) easily summarize uncertainty about all parameters of interest.

Dynamic skew-t model. First, we consider a dynamic model with innovations that have the skew-t distribution of Azzalini and Capitanio (2003), with mean, volatility, and skewness parameters being functions of observed predictor variables. Whereas the nonparametric approach of Appendix S.A uses the skew-t distribution as a pedagogical tool for interpreting the results of the quantile regressions, here we instead employ the distribution in a fully specified model of GDP growth dynamics.

Recall that y_{t+1} denotes the q/q log growth rate of GDP between time t and $t+1$. Let $\mathcal{F}_t$ denote all available data up to time t. We then assume that

$$y_{t+1} = \mu_t + \sigma_t \varepsilon_{t+1}, \tag{S.5}$$

where the conditional distribution of the innovations is skew-t, as defined above:

$$(\varepsilon_{t+1} \mid \mathcal{F}_t) \sim \text{skew-t}(0, 1, \alpha_t, \nu).$$

The time-varying location μ_t, scale σ_t, and shape α_t parameters are assumed to be driven by the explanatory variables x_t, as follows:

$$\mu_t = \gamma_\mu + \rho_\mu y_t + \beta_\mu' x_t, \tag{S.6}$$

$$\sigma_t = \exp\left(\gamma_\sigma + \rho_\sigma y_t + \beta_\sigma' x_t\right), \tag{S.7}$$

$$\alpha_t = \gamma_\alpha + \rho_\alpha y_t + \beta_\alpha' x_t. \tag{S.8}$$

Since we allow lagged GDP growth to enter into these time-varying parameters, any predictive power of the variables in x_t must go beyond the informational content embodied in lagged GDP growth itself. The heavy-tailedness parameter $\nu > 0$ is assumed constant over time.

When considering the implications of the model for prediction at horizons $h > 1$, we must specify a dynamic model for the predictor variables x_t. For simplicity, we assume that x_t evolves as a VAR(1) model with i.i.d. normal innovations that are independent of the innovation ε_t in the equation (S.5) for GDP growth. We omit the intercept from the VAR model, as we studentize each predictor variable x_{jt} before running the entire estimation procedure.

Conditional heteroskedasticity model. We also consider a more parsimonious version of the above dynamic skew-t model that only allows for time-variation in first and second moments. Adrian et al. (2019, Section III.B) find that a simple conditionally Gaussian time series model delivers results that are broadly in line with their nonparametric quantile regressions. We will therefore also consider this model, although with an expanded set of predictor variables.

The conditional heteroskedasticity model is obtained as the special case of the skew-t model where we set $\alpha_t = 0$ for all t and let the degrees of freedom $\nu \to \infty$. Thus, the conditional distribution of GDP growth is assumed to be normal, with potentially time-varying conditional mean μ_t and conditional standard deviation σ_t. This model features a symmetric conditional forecast distribution, but it is potentially consistent with *unconditional* skewness (and heavy tails) in GDP growth, depending on the distribution of the predictor variables x_t.

Priors. We consider two types of prior distribution in our Bayesian estimation routine, depending on whether interest centers on variable selection or merely prediction.

Our baseline prior for prediction is a conventional hierarchical normal shrinkage prior on all coefficients:

$$\beta_{\mu,j} \overset{iid}{\sim} N(0, \tau_\mu^2),\ j = 1, \dots, p, \quad \tau_\mu \sim \text{Cauchy}^+(0, 1),$$

and similarly for β_σ and β_α (if applicable). Here "$\text{Cauchy}^+(0, c)$" denotes the Cauchy distribution restricted to $[0, \infty)$ with location parameter 0 and scale parameter c. The coefficients are *a priori* independent across the μ_t, σ_t, and α_t equations. The prior on the degrees of freedom parameter ν is a Gamma distribution with shape parameter 1.5 and rate parameter 0.1, implying a prior mean of 15 and standard deviation of 12.2. Lest we bias the analysis against finding a large predictive role for lagged GDP growth, we impose highly diffuse Cauchy priors on the intercepts and lagged-growth coefficients in equations (S.6)–(S.8). Their prior Cauchy scale parameter is set to 5.

We adopt an alternative prior when our interest centers on variable selection and discovering parsimonious, interpretable models. For computational convenience, we only impose this prior on the conditional heteroskedasticity model in Section IV.B. This prior must impose a belief in approximate sparsity. To that end, we employ the "horseshoe prior" of Carvalho et al. (2010) on the mean and volatility coefficients, β_μ and β_σ. This prior assumes

$$(\beta_{\mu,j} \mid \lambda_{\mu,j}, \tau_\mu) \overset{indep}{\sim} N(0, \lambda_{\mu,j}^2), \quad (\lambda_{\mu,j} \mid \tau_\mu) \overset{iid}{\sim} \text{Cauchy}^+(0, \tau_\mu), \quad \tau_\mu \sim \text{Cauchy}^+(0, 1),$$

and similarly for β_σ. Note that – crucially – there is a separate scale parameter $\lambda_{\mu,j}$ corresponding to *each* coefficient $\beta_{\mu,j}$, $j = 1, \dots, p$.[3] Carvalho et al. (2010) show that

[3]We actually restrict the prior distributions of τ_μ and τ_β to the interval $[1/p, \infty)$, where p is the number of predictors; this improves numerical convergence without affecting the final substantive results.

this prior specification implies a belief in approximate sparsity: The "signal-to-noise" ratio $\frac{1}{1+\lambda_{\mu,j}^2}$ for coefficient $\beta_{\mu,j}$ has a U shaped prior density (or "horseshoe shape"), which causes the posterior distribution for $\beta_{\mu,j}$ to *either* shrink the coefficient heavily towards zero *or* hardly shrink the coefficient at all.[4] The typical empirical result is a model with only a few selected predictor variables whose coefficients are not biased by excessive shrinkage.

The horseshoe prior is more computationally tractable and arguably more economically meaningful than the "spike-and-slab" prior, which assumes that coefficients are exactly 0 with positive prior probability (e.g., Giannone et al., 2019). Since the horseshoe prior distribution for all parameters is absolutely continuous, we can employ highly computationally efficient posterior sampling software, as described below. Follett and Yu (2017) also employ the horseshoe prior of Carvalho et al. (2010) for variable selection in a time series context, but they only impose this prior on the slope variables of a VAR rather than on the volatility component.

In contrast with the low-dimensional approaches to growth-at-risk of Adrian et al. (2019, Appendix A.2) and Carriero et al. (2019), our estimation method is designed to perform variable selection from a large set of candidate mean and volatility predictor variables. Mazzi and Mitchell (2019) estimate a Bayesian time series quantile regression model with shrinkage priors, but their Laplace prior does not impose a prior belief in approximate sparsity, as emphasized by Carvalho et al. (2010, Section 1.3). Manzan (2015) performs variable selection for distributional forecasts using a Lasso-like version of quantile regression, but he is interested in measures of forecast performance rather than in quantifying the uncertainty surrounding the variables selected. Our fully Bayesian approach facilitates the reporting of uncertainty about individual parameters.

When we consider h-step-ahead forecasting for $h > 1$, we require a prior on the VAR model for the predictors x_t. Here we use the conventional choice of a maximally diffuse normal-inverse-Wishart prior. We impose prior independence of these VAR parameters from the parameters in the model for GDP growth. Hence, the posterior for the VAR parameters is of normal-inverse-Wishart form and can be drawn from independently of the posterior draws for the rest of the model.

Posterior computation. We sample from the posterior distribution of the dynamic skew-t model and conditional heteroskedasticity model using the automated Markov Chain Monte Carlo (MCMC) software Stan (Carpenter et al., 2017), specifically the MatlabStan interface. Despite the large number of parameters (more than

[4]Thus, the posterior medians behave loosely like post-selection Lasso, but the fully Bayesian approach here makes uncertainty quantification straight-forward.

100 for some specifications), we are able to reliably and quickly explore the posterior distributions. For each model and specification, we do the following. We run four parallel MCMC chains, starting from rough least-squares estimates of the parameters.[5] We confirm convergence using the $\hat{R}$ convergence metric of Gelman and Rubin (1992) and by visual inspection of the parameter trace plots. Each of the four chains do 5,000 warm-up iterations and then 5,000 further iterations. This yields 20,000 stored parameter draws from all chains. The effective sample sizes (i.e., adjusting for serial correlation in the chain) of the parameters of interest almost all exceed 1,000.

Running the entire algorithm takes about 3–6 minutes per specification for the conditional heteroskedasticity model with many predictors, and less than an hour per specification for the dynamic skew-t model with many predictors, on a PC with 3.6 GHz processor and four cores. We have verified that the algorithm accurately recovers important predictors in simulated data of sample size $T = 200$ with 20–50 predictor variables.

When computing moments of the h-step-ahead forecast distribution in Section III, we proceed as follows. Due to computational constraints, we select a random subset of 2,000 posterior parameter draws. For each of these, and for each point in time t, we simulate 5,000 h-quarter-ahead paths of (y_t, x_t) by iterating on the model equations; we then compute various moments of the distribution of cumulative growth $\sum_{\ell=1}^{h} y_{t+\ell}$ from time t to $t+h$. In the case $h = 1$, we do not need to resort to simulation, since the one-step-ahead skew-t density is available in closed form, as discussed above (see also the formulas for the cumulative distribution function and moments in Azzalini and Capitanio, 2003).

FACTOR ESTIMATION We now describe the factor estimation procedure used to generate the predictors in Sections II and III.

Let $z_t = (z_{1,t}, z_{2,t}, \ldots, z_{n,t})'$ denote a standardized time series process at time t. In our application, z_t contains the variables in Table S.1. We assume that z_t admits the following factor model representation and that the $r \times 1$ vector of common factors f_t follow a VAR(p) process:

$$
\begin{aligned}
z_t &= \Delta f_t + \epsilon_t, \\
f_t &= A_1 f_{t-1} + A_2 f_{t-2} + \cdots + A_p f_{t-p} + u_t, \quad u_t \sim i.i.d.N(0, \Sigma_u).
\end{aligned}
$$

[5] The μ_t coefficients are estimated by OLS as usual. The $\log \sigma_t$ coefficients are then estimated by OLS, using the logarithm of the absolute values of the first-step residuals as left-hand side variable. The α_t coefficients are initialized as random draws near 0. ν is initialized at 10.

Δ is the $n \times r$ matrix of factor loadings and the $n \times 1$ vector ϵ_t contains the idiosyncratic components. We allow for serial correlation in the idiosyncratic components, specifically we assume that ϵ_t follows an AR(1) process:

$$\epsilon_t = \alpha \epsilon_{t-1} + e, \quad e \overset{i.i.d.}{\sim} N(0, \Sigma_e).$$

In the application with the global and financial factors we use $r = 2$ and $p = 2$ and apply appropriate zero restrictions on Δ and the coefficients of the factor VAR, so that the financial factor is specific to the subset of financial variables. In the application with the non-financial factor we use $r = 1$ and $p = 2$ and apply the restriction that the single factor only loads on non-financial variables.

The model is estimated using maximum likelihood estimation via an EM-algorithm, which is initialized using principal components (Doz et al., 2012). In order to estimate the principal components, all missing observations are first replaced via spline interpolation.

Table S.1 below reports the list of variables employed in the exercises and whether they load on the global, the financial and/or the non-financial factors.

S.B. Data: Details

Here we provide details on the construction of the U.S. and multi-country data sets.

MONTHLY U.S. DATA Table S.1 lists the predictor variables in the monthly US dataset. Before further analysis we transform all series to stationarity, following the recommendations of McCracken and Ng where possible. The series are available over the sample period of 01/1959–12/2019 but we restrict our analysis to the 04/1975–09/2019 sample (01/1980–09/2019 sample for the real-time exercise).

Table S.1: Monthly US dataset.[a]

Code	Description	Lag	Factors: Global	Fin	Non-fin
RPI	Real Personal Income	30	x		x
W875RX1	RPE ex transfer receipts	30	x		x
DPCERA3M086SBEA	Real personal consumption expenditures	30	x		x
CMRMTSPLx**	Real Manufacturing and Trade Industries Sales	60	x		x
RETAILx**	Retail and Food Services Sales		x		x
INDPRO	IP Index	16	x		x
IPFPNSS	IP: Final Products and Nonindustrial Supplies	16	x		x
IPFINAL	IP: Final Products (Market Group)	16	x		x
PCONGD	IP: Consumer Goods	16	x		x
IPDCONGD	IP: Durable Consumer Goods	16	x		x
IPNCONGD	IP: Nondurable Consumer Goods	16	x		x
IPBUSEQ	IP: Business Equipment	16	x		x
IPMAT	IP: Materials	16	x		x
IPDMAT	IP: Durable Materials	16	x		x
IPNMAT	IP: Nondurable Materials	16	x		x
IPMANSICS	IP: Manufacturing (SIC)	16	x		x
IPB51222S	IP: Residential Utilities	16	x		x
IPFUELS	IP: Fuels	16	x		x
CUMFNS*	Capacity Utilization	47	x		x
HWI*	Help-Wanted Index		x		x
HWIURATIO*	Ratio of Help Wanted/Unemployed		x		x
CLF16OV	Civilian Labor Force	5	x		x
CE16OV	Civilian Employment	5	x		x
UNRATE	Civilian Unemployment Rate	5	x		x
UEMPMEAN	Average Duration of Unemployment	5	x		x
UEMPLT5	Civilians Unemployed: <5 Weeks	5	x		x
UEMP5TO14	Civilians Unemployed: 5-14 Weeks	5	x		x
UEMP15OV	Civilians Unemployed: 15+ Weeks	5	x		x
UEMP15T26	Civilians Unemployed: 15-26 Weeks	5	x		x
UEMP27OV	Civilians Unemployed: 27+ Weeks	5	x		x
CLAIMSx*	Initial Claims		x		x
PAYEMS	All Employees: Total nonfarm	5	x		x
USGOOD	All Employees: Goods-Producing	5	x		x
CES1021000001	All Employees: Mining and Logging	5	x		x
USCONS	All Employees: Construction	5	x		x
MANEMP	All Employees: Manufacturing	5	x		x

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Table S.1 – continued from previous page

Code	Description	Lag	Factors: Global	Fin	Non-fin
DMANEMP	All Employees: Durable goods	5	x		x
NDMANEMP	All Employees: Nondurable goods	5	x		x
SRVPRD	All Employees: Service Industries	5	x		x
USTPU	All Employees: TT&U	5	x		x
USWTRADE	All Employees: Wholesale Trade	5	x		x
USTRADE	All Employees: Retail Trade	5	x		x
USFIRE	All Employees: Financial Activities	5	x		x
USGOVT	All Employees: Government	5	x		x
CES0600000007	Hours: Goods-Producing	5	x		x
AWOTMAN	Overtime Hours: Manufacturing	14	x		x
AWHMAN	Hours: Manufacturing	5	x		x
HOUST	Housing Starts	18	x		x
HOUSTNE	Housing Starts, Northeast	18	x		x
HOUSTMW	Housing Starts, Midwest	18	x		x
HOUSTS	Housing Starts, South	18	x		x
HOUSTW	Housing Starts, West	18	x		x
PERMIT	New Private Housing Permits	18	x		x
PERMITNE	New Private Housing Permits, Northeast	18	x		x
PERMITMW	New Private Housing Permits, Midwest	18	x		x
PERMITS	New Private Housing Permits, South	18	x		x
PERMITW	New Private Housing Permits, West	18	x		x
ACOGNO***	Orders: Consumer Goods	35	x		x
AMDMNOx*	New Orders for Durable Goods		x		x
ANDENOx**	New Orders for Nondefense Capital Goods	26	x		x
AMDMUOx**	Unfilled Orders for Durable Goods	26	x		x
BUSINVx**	Total Business Inventories	45	x		x
ISRATIOx**	Total Business: Inventories to Sales Ratio	45	x		x
M1SL	M1 Money Stock	15	x		x
M2SL	M2 Money Stock	15	x		x
M2REAL*	Real M2 Money Stock		x		x
M3SL***	M3 Money Stock	15	x		x
AMBSL	St. Louis Adjusted Monetary Base	15	x		x
TOTRESNS	Total Reserves of Depository Institutions	8	x		x
NONBORRES	Reserves Of Depository Institutions	8	x		x
BUSLOANS	Commercial and Industrial Loans	12	x	x	
REALLN	Real Estate Loans at All Commercial Banks	12	x	x	
NONREVSL	Total Nonrevolving Credit	37	x	x	
CONSPI*	Nonrevolving consumer credit to Personal Income		x	x	
S&P 500	S&PâĂŹs Stock Price Index: Composite	1	x	x	
S&P: indust*	S&PâĂŹs Stock Price Index: Industrials	1	x	x	
S&P div yield	S&PâĂŹs Composite Stock: Dividend Yield	1	x	x	
S&P PE ratio	S&PâĂŹs Composite Stock: Price-Earnings Ratio	1	x	x	
FEDFUNDS	Effective Federal Funds Rate	1	x	x	
CP3Mx	3-Month AA Financial Commercial Paper Rate	1	x	x	
TB3MS	3-Month Treasury Bill Rate	1	x	x	
TB6MS	6-Month Treasury Bill Rate	1	x	x	
GS1	1-Year Treasury Rate	1	x	x	
GS5	5-Year Treasury Rate	1	x	x	
GS10	10-Year Treasury Rate	1	x	x	
AAA	Moody's Seasoned Aaa Corporate Bond Yield	1	x	x	
BAA	Moody's Seasoned Baa Corporate Bond Yield	1	x	x	
COMPAPFFx*	3-Month Commercial Paper Minus FEDFUNDS		x	x	

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Table S.1 – continued from previous page

Code	Description	Lag	Factors Global	Fin	Non-fin
TB3SMFFM	3-Month Treasury C Minus FEDFUNDS	1	x	x	
TB6SMFFM	6-Month Treasury C Minus FEDFUNDS	1	x	x	
T1YFFM	1-Year Treasury C Minus FEDFUNDS	1	x	x	
T5YFFM	5-Year Treasury C Minus FEDFUNDS	1	x	x	
T10YFFM	10-Year Treasury C Minus FEDFUNDS	1	x	x	
AAAFFM	Moody's Aaa Corporate Bond Minus FEDFUNDS	1	x	x	
BAAFFM	Moody's Baa Corporate Bond Minus FEDFUNDS	1	x	x	
TWEXMMTH	Trade Weighted U.S. Dollar Index: Major Currencies	1	x	x	
EXSZUSx**	Switzerland / U.S. Foreign Exchange Rate	1	x	x	
EXJPUSx**	Japan / U.S. Foreign Exchange Rate	1	x	x	
EXUSUKx**	U.S. / U.K. Foreign Exchange Rate	1	x	x	
EXCAUSx**	Canada / U.S. Foreign Exchange Rate	1	x	x	
WPSFD49207*	PPI: Finished Goods		x		x
WPSFD49502*	PPI: Personal Consumption Goods		x		x
WPSID61*	PPI: Processed Goods for Intermediate Demand		x		x
WPSID62*	PPI: Unprocessed Goods for Intermediate Demand		x		x
PPIFGS***	PPI: Finished Goods	16	x		x
PPIFCG***	PPI: Finished Consumer Goods	16	x		x
PPIITM***	PPI: Intermediate Materials	16	x		x
PPICRM***	PPI: Intermediate Materials	16	x		x
OILPRICEx	Crude Oil, spliced WTI and Cushing	1	x		x
PPICMM	PPI: Metals and metal products	16	x		x
CPIAUCSL	CPI : All Items	16	x		x
CPIAPPSL	CPI : Apparel	16	x		x
CPITRNSL	CPI : Transportation	16	x		x
CPIMEDSL	CPI : Medical Care	16	x		x
CUSR0000SAC	CPI : Commodities	16	x		x
CUSR0000SAD	CPI : Durables	16	x		x
CUSR0000SAS	CPI : Services	16	x		x
CPIULFSL	CPI : All Items Less Food	16	x		x
CUSR0000SA0L2	CPI : All items less shelter	16	x		x
CUSR0000SA0L5	CPI : All items less medical care	16	x		x
PCEPI	Personal Cons. Expend.: Chain Index	30	x		x
DDURRG3M086SBEA	Personal Cons. Exp: Durable goods	30	x		x
DNDGRG3M086SBEA	Personal Cons. Exp: Nondurable goods	30	x		x
DSERRG3M086SBEA	Personal Cons. Exp: Services	30	x		x
CES0600000008	Avg Hourly Earnings : Goods-Producing	5	x		x
CES2000000008	Avg Hourly Earnings : Construction	5	x		x
CES3000000008	Avg Hourly Earnings : Manufacturing	5	x		x
UMCSENTx***	Consumer Sentiment Index	-2	x		x
MZMSL	MZM Money Stock	15	x		x
DTCOLNVHFNM	Consumer Motor Vehicle Loans Outstanding	58	x	x	
DTCTHFNM	Total Consumer Loans and Leases Outstanding	58	x	x	
INVEST	Securities in Bank Credit at All Commercial Banks	12	x	x	
VXOCLSx*	Volatility Index		x	x	
ISMC@USECON***	ISM Composite Index	1	x		x
NAPMVDI@USECON***	ISM Mfg: Supplier Deliveries Index	1	x		x
IPMAN***	Industrial Production: Manufacturing	16	x		x
MCUMFN***	Capacity Utilization: Manufacturing	16	x		x
TCU***	Capacity Utilization: Total Industry	16	x		x
DGORDER***	Manufacturers' New Orders: Durable Goods	26	x		x
CPFFM***	3-Month Commercial Paper Minus Federal Funds Rate	1	x	x	

Continued on next page

Table S.1 – continued from previous page

Code	Description	Lag	Factors Global	Fin	Non-fin
PCUOMFGOMFG***	Producer Price Index by Industry: Total Manufacturing	13	x		x

Sources: FRED-MD, ALFRED and Haver Analytics.
Predictor variables in the monthly US factor. The lag variable is the approximate number of days between the last day of the reference month and the date at which the variable becomes available. All variables have been transformed to stationarity following the suggestions in McCracken and Ng (2016) whenever available. A * next to a variable indicate that the variable is not used in the real-time exercise, ** indicate that a variables is used in the real-time exercise, but without the adjustments that are made in the FRED-MD dataset, and *** indicates that a variable is used in the real-time exercise but not in the other exercises.

QUARTERLY U.S. DATA Table S.2 lists the 43 predictor variables and the outcome variable (real GDP growth). Before further analysis we transform all series to stationarity, following the recommendations of McCracken and Ng. All series are available over the full sample period of 1975q2–2019q2.

QUARTERLY MULTI-COUNTRY DATA Table S.3 lists the 35 variables in our data set, comprising GDP growth and 34 potential predictor variables. As indicated in the table, some variables are missing for certain countries, either entirely or because we drop them due to limited sample size. To increase comparability, we do not attempt to find replacement series for each individual country from outside data sources. Even so, most variables are available for at least 12 of the 13 countries, with three exceptions: (i) surveys on consumer sentiment (7 countries), business sentiment (6 countries), or purchasing managers index (1 country); (ii) indices of housing starts (8 countries) or building permits (6 countries); and (iii) stock trading volume (10 countries). We still include these variables in the analysis, as they appear potentially relevant as timely predictors of growth risk. The most notable absences from our list of predictor variables are capacity utilization, corporate bond spreads, and bank lending rates, as it is unfortunately difficult to find comparable series on these variables going back several decades.

Our data is quarterly and covers the period 1980q1–2018q4. All variables are transformed to approximate stationarity. To create a balanced panel for the analysis, we impute missing data points using a dynamic factor model.[6] The imputation is unlikely to substantially affect the results, as the fraction of missing observations

[6]For each country separately, we employ a static dynamic factor model with 8 factors. We first estimate factors by principal components on series with no missing data, then impute all missing observations by regressing on the factors. Then we re-estimate the factors on the observed and imputed data, re-impute the initially missing observations, and so on until numerical convergence.

Table S.2: Variables in U.S. data set.[a]

Code	Description
AAASPR	Spread: AAA corporate bond vs. 10-yr govt yield
BAASPR	Spread: BAA corporate bond vs. 10-yr govt yield
CAPUTIL	Capacity utilization
COMMCRB	CRB commodity price index
CONSGOVT	Government consumption
CONSPRIV	Private consumption
CONSSENT	Consumer confidence (Conference Board)
CPAPERSPR	Spread: 3-mth commercial paper vs. 3-mth govt yield
DEBTGOVT	Federal debt, % GDP
DISPINC	Disposable income
DIVYIELD	S&P 500 dividend yield
EARNINGS	Hourly earnings, production and non-supervisory
ECONSENT	Business outlook (Philadelphia Fed)
EMPL	Employment, non-farm
EXCHTRW	Nominal trade-weighted exchange rate index
EXPORT	Exports
FEDFUNDS	Federal funds rate
GDP	GDP
HOURS	Hours worked, non-farm business
HOUSEPERMIT	New housing permits
HOUSEPRICE	All-transactions house price index
HOUSESTART	Housing starts, new privately owned
IMPORT	Imports
INDPRO	Industrial production
INVENTO	Manufacturing and trade inventories
INVESTM	Private investment
LIABCORP	Nonfinancial corporate liabilities
LIABHH	Household and non-profit liabilities
LOANSCORP	Commercial and industrial loans, all commercial banks
LOANSHH	Consumer loans, all commercial banks
NWCORP	Nonfinancial corporate net worth
NWHH	Household and non-profit net worth
ORDERNEW	New manufacturing orders
ORDERUNFIL	Unfilled manufacturing orders
PCEPRICE	Personal consumption expenditures price index
PMI	Purchasing managers index
RETAIL	Retail sales
SHORTSPR	Spread: 3-mth govt yield vs. Fed funds rate
STOCKPRICE	S&P 500 stock price index
STOCKVOL	Stock trading volume
TERMSPR	Spread: 10-yr vs. 3-mth govt yield
ULC	Unit labor cost, non-farm business
UNRATE	Civilian unemployment rate
VXO	S&P 100 implied volatility

Sources: FRED-QD, Global Financial Data, Haver Analytics, and authors' calculations.

[a] Predictor variables and predicted variable (GDP growth) in the U.S. data set.

Table S.3: Variables in cross-country data set.[a]

Code	Description	Missing countries
CA	Current account, % GDP	
COMMCRB	CRB commodity price index	Not country-specific
CONSGOVT	Government consumption	
CONSPRIV	Private consumption	
CONSSENT	Consumer/household sentiment	CAN CHE DEU FRA ITA SWE
CREDCORPBNK	Credit to firms from banks	
CREDCORP	Credit to firms	
CREDHH	Credit to households	CHE
DIVYIELD	Dividend yield	
ECONSENT	Business/economic sentiment	AUS CAN CHE DEU ESP ITA JPN
EMPL	Employment	
EXCHEFF	Nominal effective exchange rate	
EXCHUSD	Exchange rate versus US$	USA
EXPORT	Real exports	
GDPDEF	GDP deflator	
GDP	GDP	
HOURS	Hours worked	CAN
HOUSEPERMIT	New housing permits	CHE ESP ITA JPN NLD SWE USA
HOUSEPRICE	House price index	
HOUSESTART	Housing starts	CHE DEU FRA ITA NLD
IMPORT	Real imports	
INTRBNKRATE	3-month interbank rate	
INVESTM	Real investment	
MANUF	Manufacturing index	
PMI	Purchasing managers index	Only available for USA
RETAIL	Retail sales index	ESP
STOCKPRICE	Stock price index	
STOCKRV	Daily realized vol of stock price	
STOCKVOL	Stock trading volume	CAN GBR NLD
TERMSPR	Spread: 10- vs. 2-yr govt yield[b]	
TERMTRADE	Terms of trade	
ULC	Unit labor cost index	ESP
UNRATE	Unemployment rate	
VXO	S&P 100 implied volatility	Not country-specific
YIELDSPRUS	Spread: 10-yr govt yield vs. US	USA

Sources: OECD, BIS, Global Financial Data, Haver Analytics, and authors' calculations.

[a] Predictor variables and predicted variable (GDP growth) in the cross-country data set. The third column indicates the countries for which the variable in question is not available.

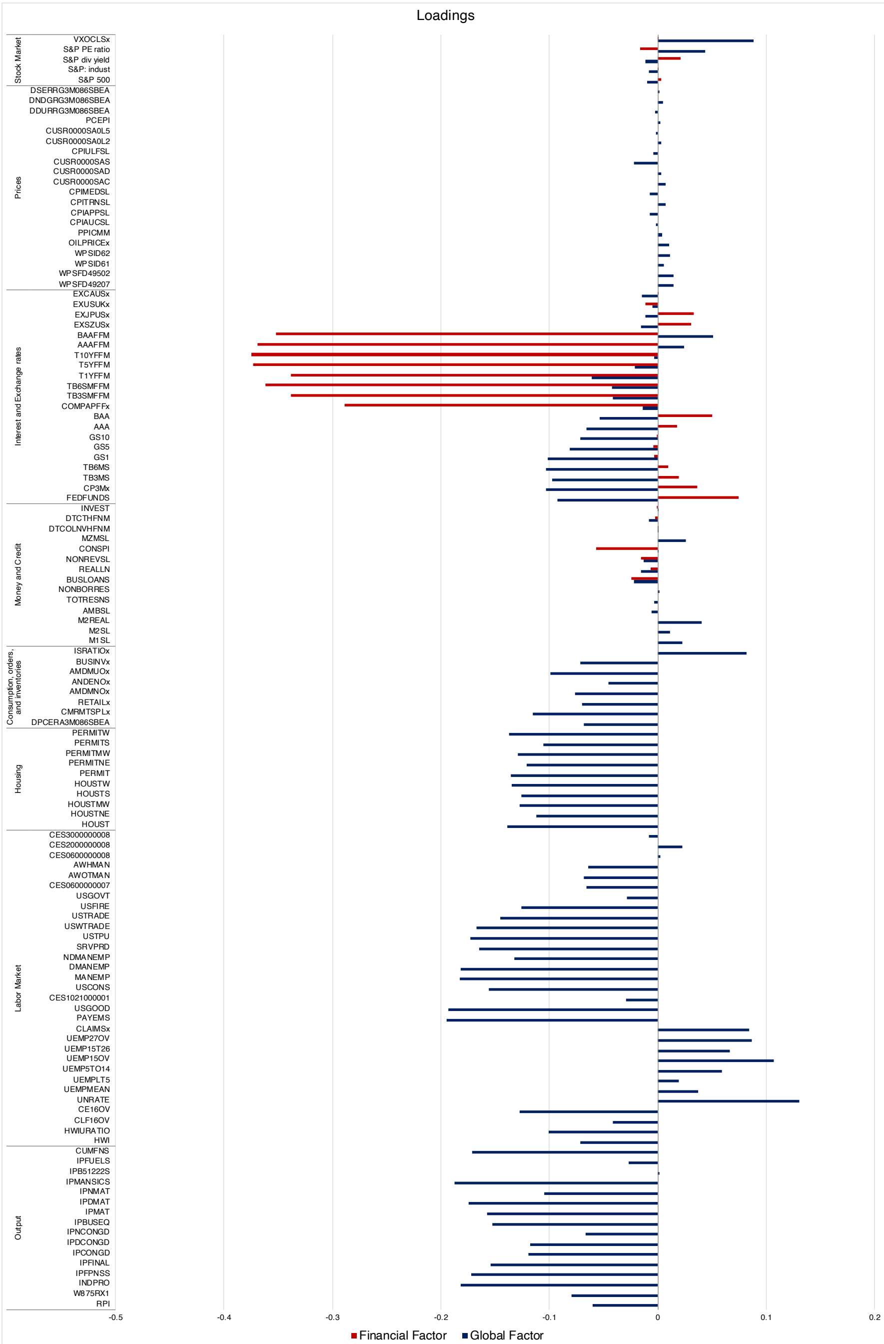
[b] 1-yr or 3-yr yield used if 2-yr yield not available for sufficiently long sample.

does not exceed 3% for any country. Moreover, no individual time series used in our analysis has more than 30% missing observations. For Germany only, we use the shorter sample 1991q2–2018q4, as the OECD data treats West Germany separately from East Germany before 1991.

S.C. Factor Loadings

Figure S.1 reports the estimated loadings for the monthly factor model with a global and a financial factor. Details of the factor estimation are provided in Appendix S.A.

Figure S.1: Loadings of the factor model[a]

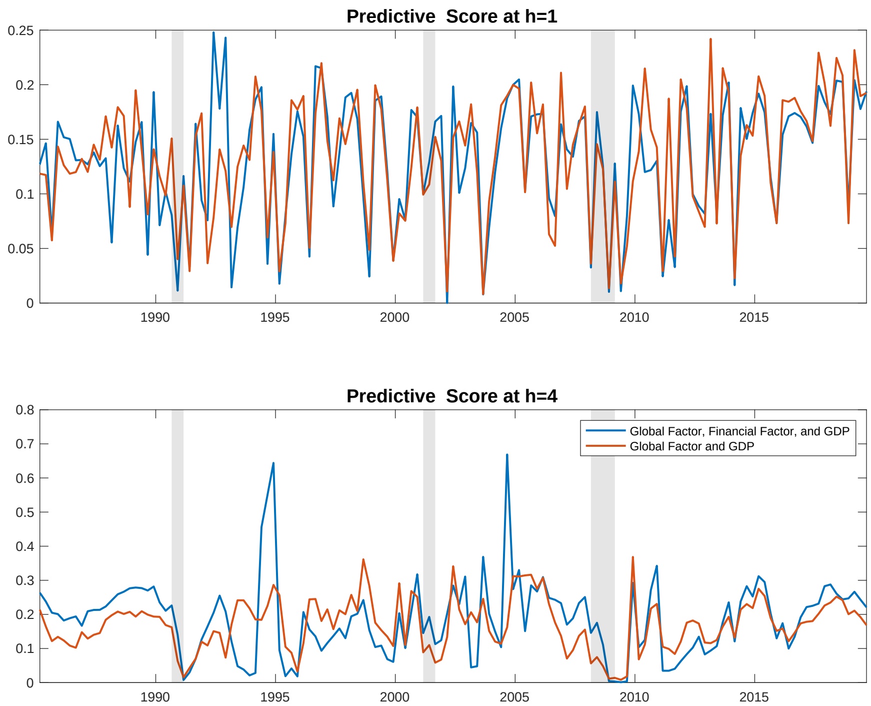


Sources: authors' calculations.

[a] The table reports the loadings of the factor model with global and financial factor.

S.D. Out-of-Sample Forecasts: Additional Figures

Figure S.2: Out-of-sample forecasts: Predictive scores.[a]



Sources: authors' computation.

[a] Time evolution of the predictive scores of the one- and four-quarter ahead predictive distribution of GDP growth, from 1993q1 to 2015q4, for the models including (i) the Global factor, Financial factor, and GDP (blue) and (ii) the Global factor and GDP (red). Higher values indicate better forecast performance, in the sense of attaching higher likelihood to the realized events.

Table S.4: Benchmark Linear Forecasting Regressions.[a]

	h = 1				h = 4			
	Model 1	Model 2	Model 3	Model 4	Model 1	Model 2	Model 3	Model 4
GDP	-0.190*	-0.204**	-0.251**	-0.227**	-0.0263	-0.0463	-0.0567	-0.0707
	(0.112)	(0.0984)	(0.101)	(0.0952)	(0.0829)	(0.0680)	(0.0695)	(0.0679)
global	2.106***	1.999***	3.158***	2.811***	0.885***	0.737***	1.414***	0.921***
	(0.399)	(0.353)	(0.417)	(0.376)	(0.303)	(0.275)	(0.377)	(0.351)
L.global			-0.790**	-0.904**			-0.468*	-0.234
			(0.360)	(0.406)			(0.249)	(0.267)
L2.global			-0.616*	-0.331			-0.129	-0.341*
			(0.342)	(0.434)			(0.214)	(0.191)
L3.global			0.120	0.0261			-0.0401	0.230
			(0.265)	(0.290)			(0.271)	(0.223)
financial		-0.681***		0.123		-0.952***		-0.601***
		(0.189)		(0.353)		(0.146)		(0.199)
L.financial				-0.834**				-0.0389
				(0.414)				(0.244)
L2.financial				0.513*				-0.270
				(0.289)				(0.268)
L3.financial				-0.425*				-0.219
				(0.243)				(0.197)
Observations	176	176	173	173	173	173	170	170
R-squared	0.344	0.396	0.428	0.468	0.178	0.404	0.219	0.432

Sources: FRED-MD, FRED-QD, and authors' calculations.

[a] Coefficients in h-step-ahead forecasting regressions of annualized cumulative GDP growth onto various combinations of predictor variables. Newey-West standard errors in parentheses (bandwidth = 4), *p <0.1, **p<0.05, ***p<0.01.
Model 1: One lag, only GDP growth and global factor.
Model 2: One lag, also includes financial factor.
Model 3: Four lags, only GDP growth and global factor.
Model 4: Four lags, also includes financial factor.

S.E. Simple Regression Benchmark

In Table S.4 we run simple linear forecasting regressions of GDP growth on lagged GDP growth, the global factor, and the financial factor. The left-hand side variable is annualized cumulative GDP growth over the following h quarters. The two factors have been standardized so their standard deviation is 1. The standard errors adjust for serial correlation in the residuals. As in Section III, these results are all in-sample, unlike the out-of-sample results presented in Section II.

S.F. Dynamic Skew-t Model With Factors as Predictors: Details

Here we provide further results for the skew-t model with factors as explanatory variables in Section III.

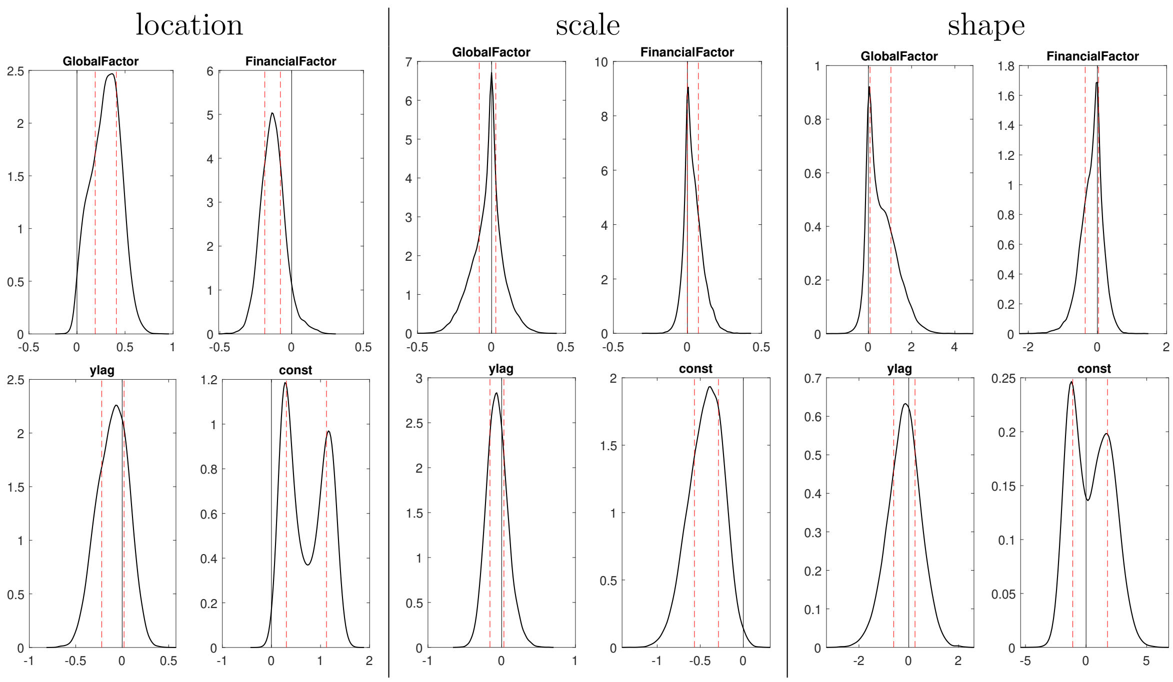
U.S. RESULTS: POSTERIOR DISTRIBUTION OF MODEL COEFFICIENTS We first report the posterior of the underlying model parameters in the U.S. skew-t model with factors as explanatory variables. Figure S.3 shows the posterior densities of the location, shape, and scale coefficients on the global factor and on the orthogonalized financial factor. There is only weak evidence that real or financial conditions meaningfully influence the conditional skewness of U.S. GDP growth. The 50% posterior credible intervals for the shape coefficients β_α either contain 0 or very nearly contain 0 for both factors. The posterior probability that the skewness coefficient on the global factor exceeds 0.05 is 40.6%, while the probability that it is less than -0.05 is 29.5%. The corresponding probabilities for the financial factor are 12.7% and 29.4%. Thus, the data is neither able to decisively pin down the magnitudes nor the signs of the effects that the factors have on the conditional skewness. The same is true of the coefficient on lagged GDP growth.

The GDP growth distribution seems to feature mildly heavy tails. There is, however, substantial posterior uncertainty about the degrees of freedom parameter ν, with a posterior interquartile range of ν is $[6.5, 15.3]$.

Figure S.3 also depicts a pronounced bimodality in the marginal posteriors for the intercepts in the scale and shape parameter equations. As mentioned in Section III, this bimodality is an artifact of the years 1975–1979. Figure S.4 shows the posterior densities of the coefficients in the U.S. two-factor dynamic skew-t model estimated on the shorter 1980q1–2019q2 subsample. The data is the same as in Section III (including the factor estimates), but we only provide the post-1980 data to the posterior sampler. It is evident that the post-1980 period does not exhibit the bimodality in the posterior distribution for the intercepts that we found on the full 1975q2–2019q2 sample. Instead the evidence here mostly points towards negative unconditional skewness, consistent with Section IV.C.

U.S. RESULTS: TIME-VARIATION OF SKEW-T PARAMETERS Figure S.5 shows the evolution over time of the four parameters μ_t, σ_t, α_t, and ν of the dynamic skew-t model. Relative to the posterior uncertainty, there is little discernible time-variation in any of these except for μ_t. This finding is consistent with the results on time-variation of the moments of the GDP distribution reported in Section III.

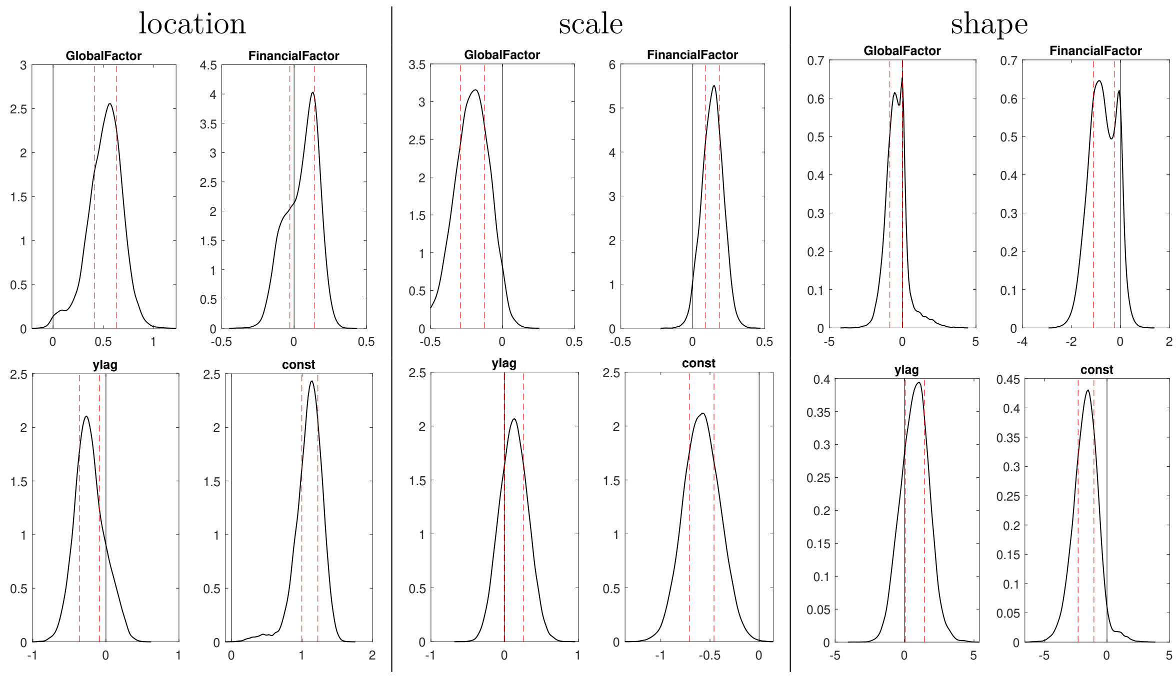
Figure S.3: Two-factor dynamic skew-t model: Posterior on 1975–2019 sample.[a]



Sources: FRED-QD, FRED-MD, and authors' calculations.

[a] Posterior densities of coefficients on global factor, financial factor, lagged GDP growth (ylag), and intercept (const) in the equations for the location parameter μ_t (left panel), scale parameter $\log \sigma_t$ (middle panel), and shape parameter α_t (right panel). Vertical red dashed lines indicate posterior interquartile ranges.

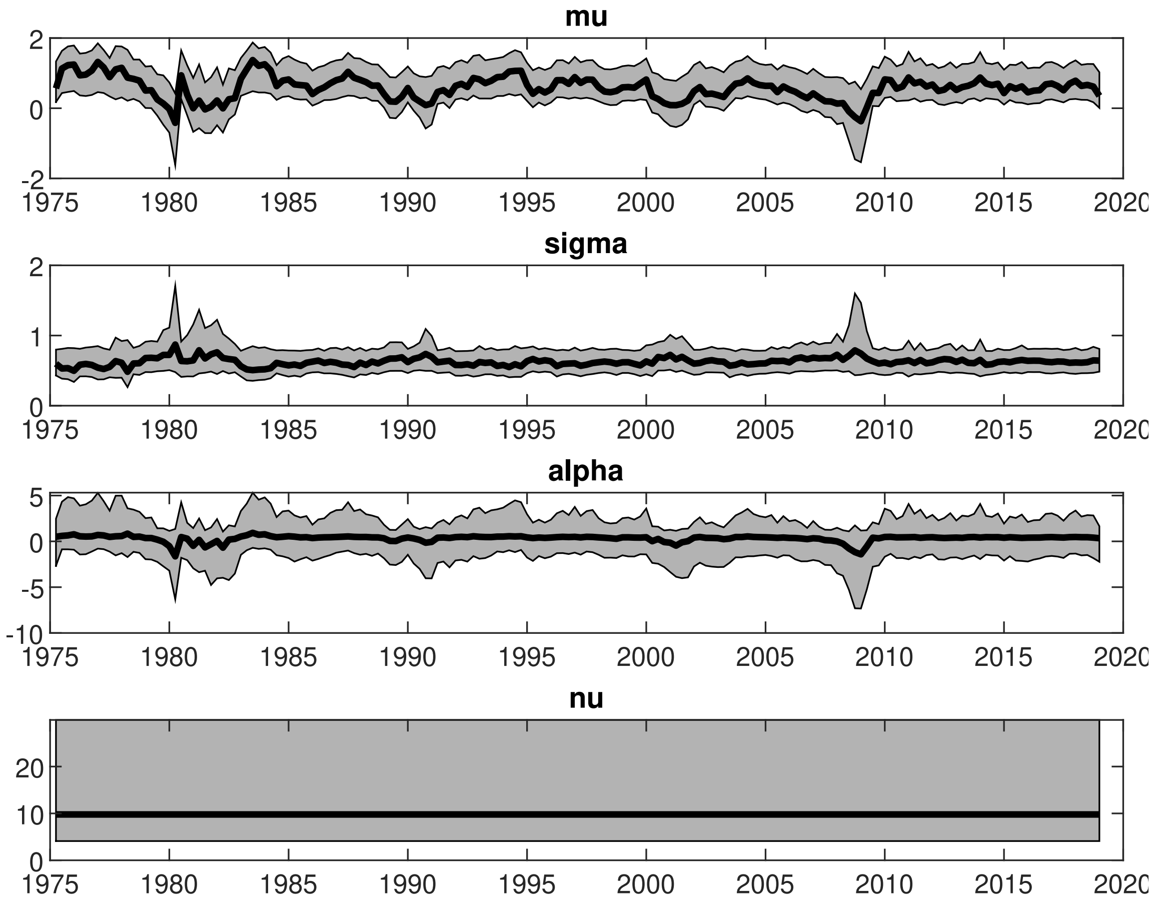
Figure S.4: Two-factor dynamic skew-t model: Posterior on 1980–2019 subsample.[a]



Sources: FRED-QD, FRED-MD, and authors' calculations.

[a] Posterior densities of coefficients on global factor, financial factor, lagged GDP growth (ylag), and intercept (const) in the equations for the location parameter (S.6) (left panel), scale parameter (S.7) (middle panel), and shape parameter (S.8) (right panel). Vertical red dashed lines indicate posterior interquartile ranges.

Figure S.5: U.S. results: Time-varying skew-t parameters.[a]



Sources: FRED-QD, FRED-MD, and authors' calculations.

[a] Time-varying parameters of the skew-t forecast distribution for GDP growth. The thick line is the posterior median (across parameter draws) at each point in time. The gray shaded band is the pointwise 90% posterior credible band (across parameter draws) at each point in time. Recall that the parameter ν is assumed constant over time. The time axis shows the quarter in which the forecast is made.

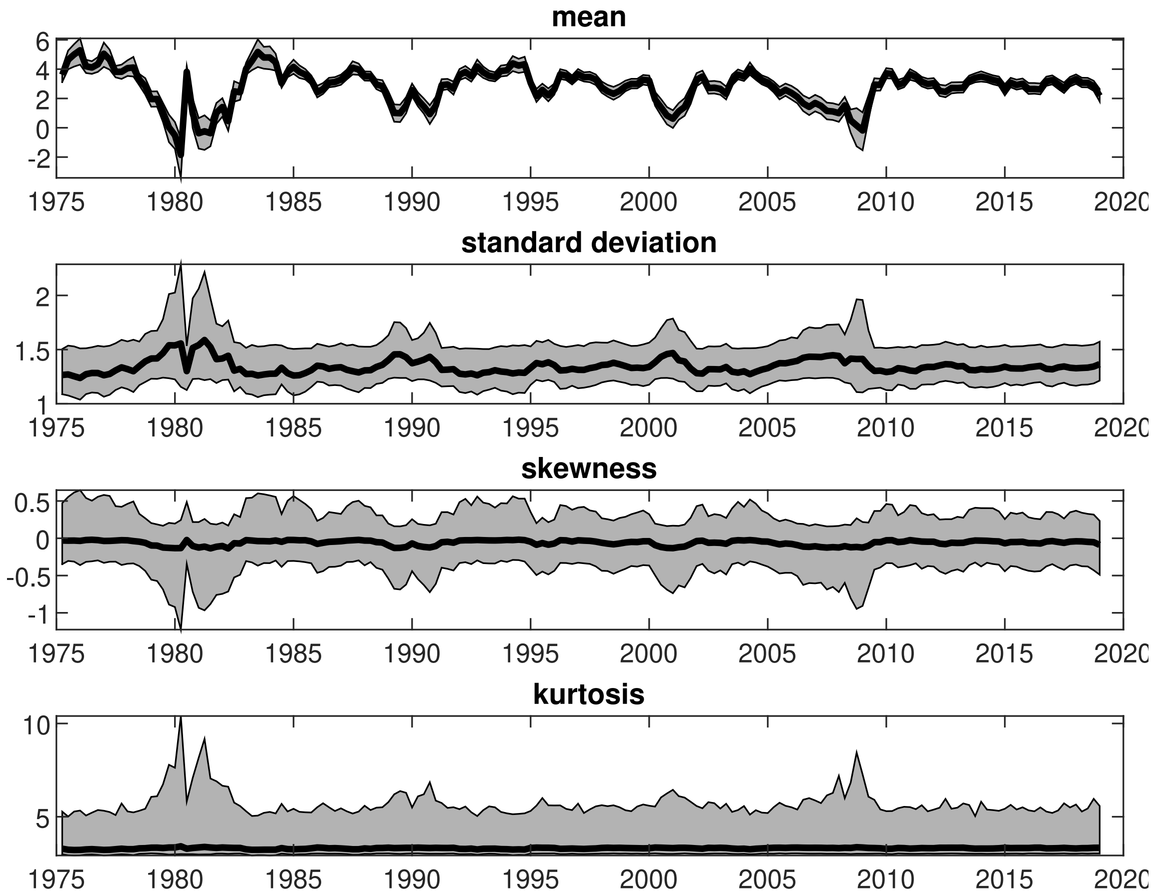
U.S. RESULTS: FOUR-QUARTER-AHEAD MOMENTS Figure S.6 shows that, as for the one-quarter-ahead moments in Figure 10, there is substantial uncertainty attached to the conditional moments of the *four-quarter-ahead* forecast distribution. As in the previous section, we here seek to forecast the cumulative growth between time t and $t + 4$. Very little can be said with certainty about the time-variation of any of the forecast moments, other than the mean, at the 1-year horizon.

U.S. RESULTS: FOUR-QUARTER-AHEAD RECESSION PROBABILITY AND EXPECTED SHORTFALL Figure S.7 shows the four-quarter-ahead recession probability and expected shortfall, to complement the one-quarter-ahead results reported in Figure 11. The four-quarter growth is cumulative, so the first panel, say, reports the probability that the cumulative growth over the following four quarters is negative. Note that the time-variation of the conditional probability in the second panel (the probability of four-quarter cumulative growth falling below the conditional mean of next-quarter annualized growth) is due to the fact that, in a recession, some mean reversion in growth is expected.

U.S. RESULTS: RELATIVE PREDICTIVE ROLE OF GLOBAL AND FINANCIAL FACTORS Figure S.8 shows the time-varying moments of the one-quarter-ahead forecast distribution if we set the *global* factor $x_{1,t}$ equal to 0 when producing every forecast. Figure S.9 shows the corresponding figure if we instead set the *financial* factor $x_{2,t}$ equal to 0 when producing every forecast. Notice that we use the precise same estimated model as in Section III.B, we only change the conditioning variables x_t used to produce the forecasts at each point in time and for each posterior parameter draw. As is clear from these figures, zeroing out the financial factor changes very little relative to the baseline in Figure 10 (which conditioned on the actual data values of both factors), whereas zeroing out the global factor has a noticeable effect on the conditional mean during the Great Recession period. Neither factor has a substantial effect on the other moments, although the posterior median for conditional skewness does change somewhat around 1980 and 2008 when we zero out the global factor (still, the posterior uncertainty about this moment is high).

CROSS-COUNTRY RESULTS: TIME-VARIATION OF MOMENTS Figures S.10 to S.12 show the time-variation in the moments of the GDP growth distribution in Australia, Italy, and Japan, respectively. As was the case for the U.S. results discussed in Section III, there is little evidence of predictable time-variation in the standard deviation, skewness, or kurtosis. Results for the other OECD countries in our data set are qualitatively similar.

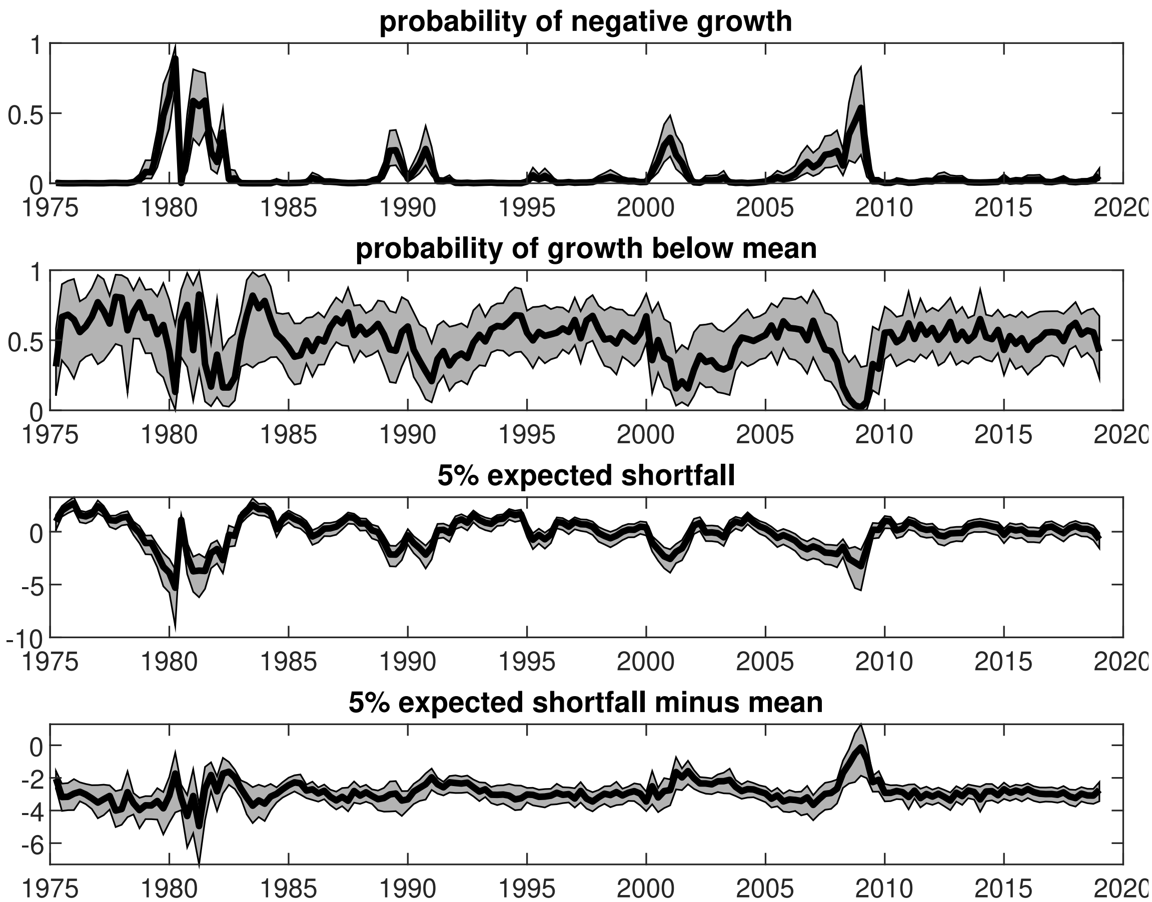
Figure S.6: U.S. factor model: Time-varying moments, four quarters ahead.[a]



Sources: FRED-QD, FRED-MD, and authors' calculations.

[a] Time-varying moments of the four-quarter-ahead forecast distribution of cumulative GDP growth between time t and $t+4$. The thick line is the posterior median (across parameter draws) at each point in time. The gray shaded band is the pointwise 90% posterior credible band (across parameter draws) at each point in time. The time axis shows the quarter in which the forecast is made.

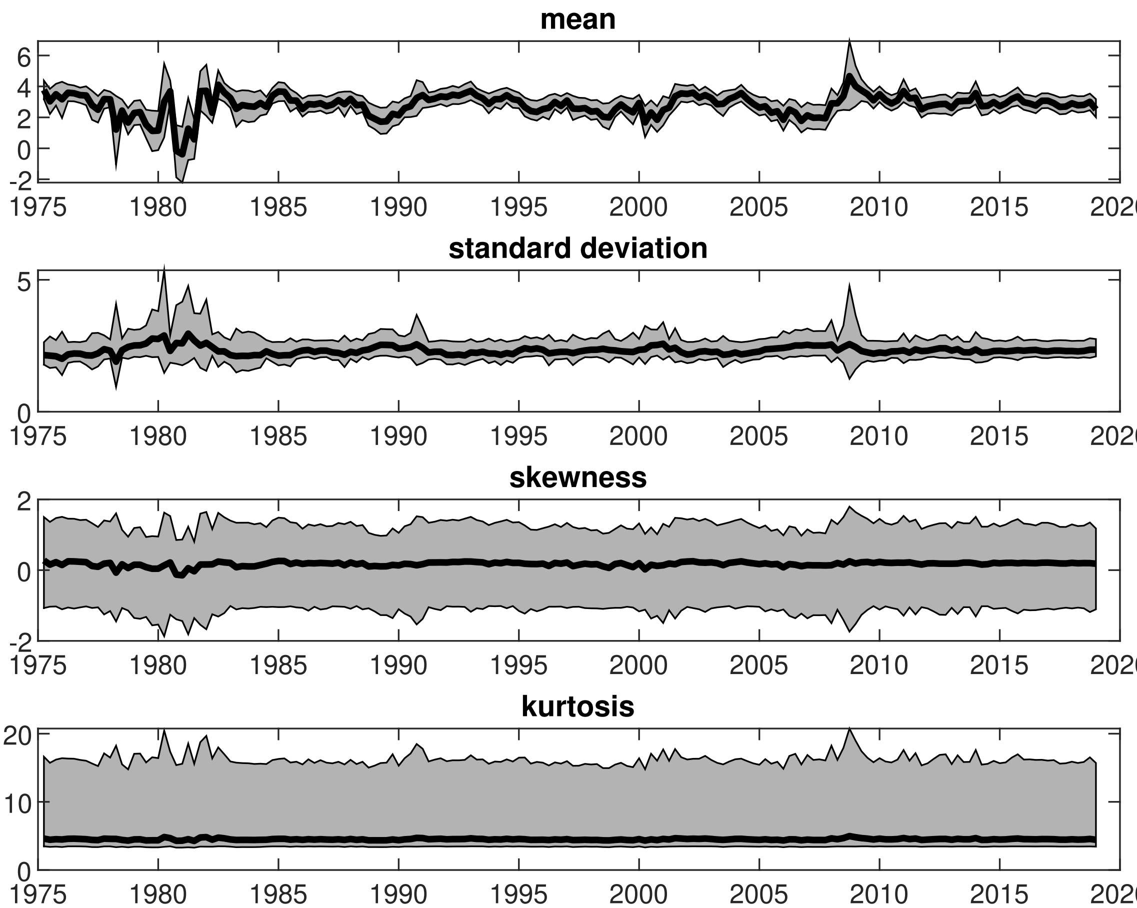
Figure S.7: U.S. factor model: Recession probability and expected shortfall, four quarters ahead.[a]



Sources: FRED-QD, FRED-MD, and authors' calculations.

[a] Probability of negative growth, probability of growth below the current conditional mean, expected shortfall, and expected shortfall minus current conditional mean for the four-quarter-ahead conditional distribution of cumulative GDP growth between time t and $t+4$. "Current conditional mean" refers to the conditional expectation of *next-quarter* GDP growth (annualized). The thick line is the posterior median (across parameter draws) at each point in time. The gray shaded band is the pointwise 90% posterior credible band (across parameter draws) at each point in time. The time axis shows the quarter in which the forecast is made.

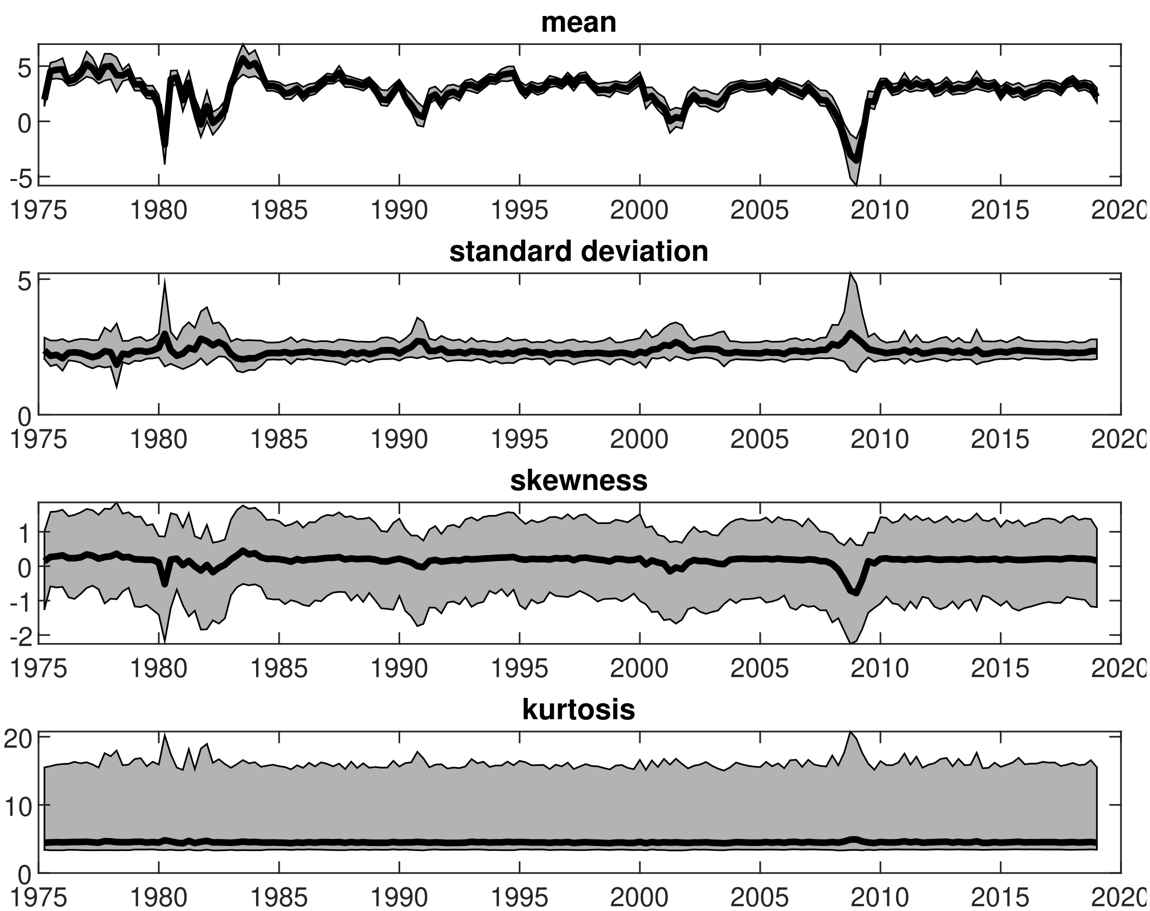
Figure S.8: U.S. factor model: Time-varying moments, one quarter ahead, zeroing out the global factor.[a]



Sources: FRED-QD, FRED-MD, and authors' calculations.

[a] Time-varying moments of the one-quarter-ahead forecast distribution of GDP growth (annualized), but setting the global factor equal to 0 when computing forecasts. The thick line is the posterior median (across parameter draws) at each point in time. The gray shaded band is the pointwise 90% posterior credible band (across parameter draws) at each point in time. The time axis shows the quarter in which the forecast is made.

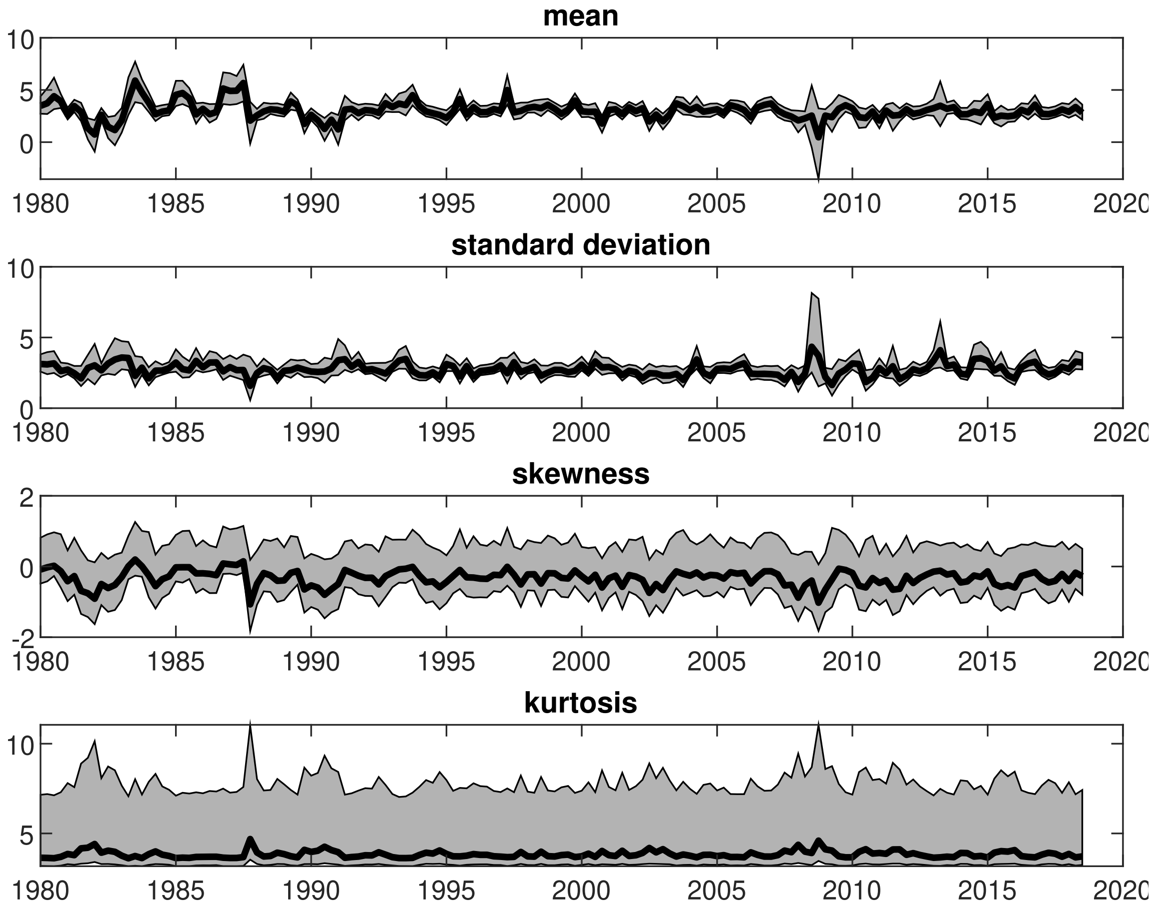
Figure S.9: U.S. factor model: Time-varying moments, one quarter ahead, zeroing out the financial factor.[a]



Sources: FRED-QD, FRED-MD, and authors' calculations.

[a] Time-varying moments of the one-quarter-ahead forecast distribution of GDP growth (annualized), but setting the financial factor equal to 0 when computing forecasts. The thick line is the posterior median (across parameter draws) at each point in time. The gray shaded band is the pointwise 90% posterior credible band (across parameter draws) at each point in time. The time axis shows the quarter in which the forecast is made.

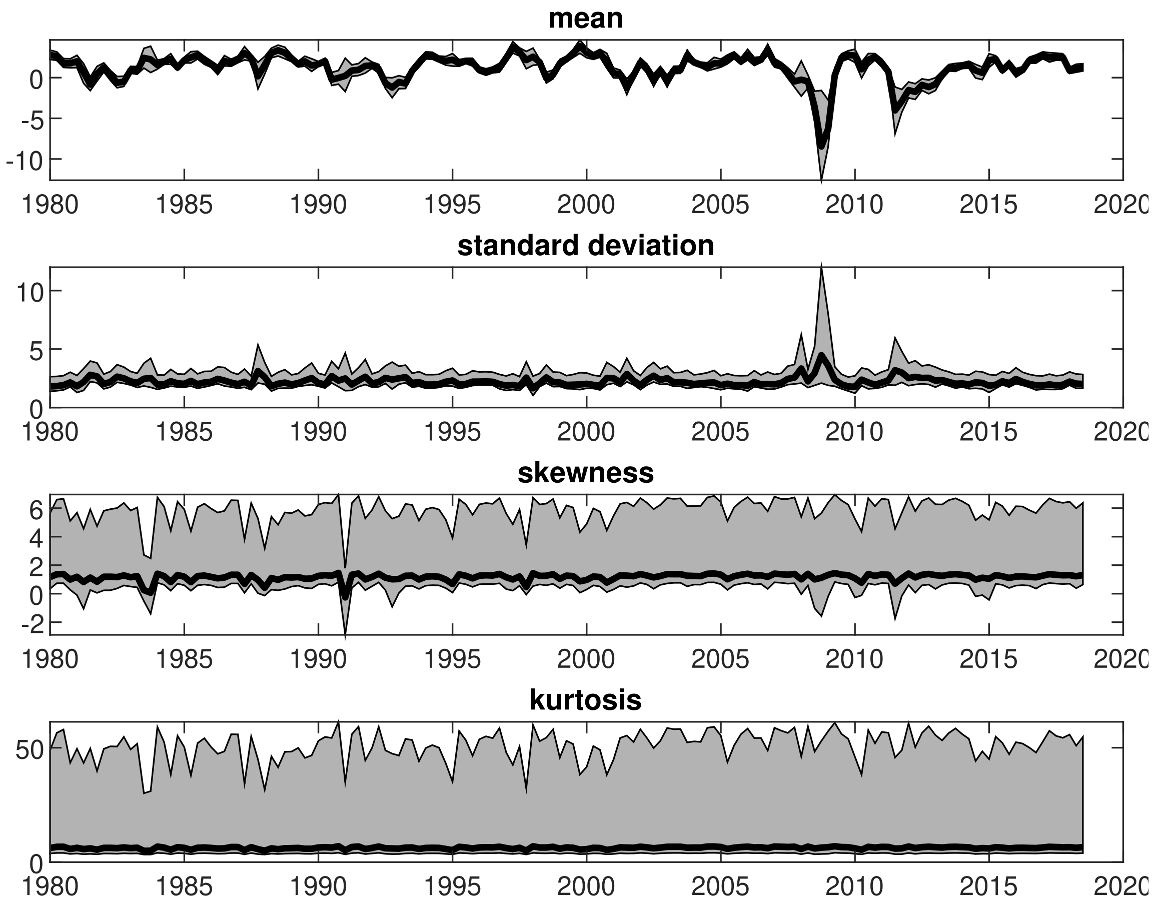
Figure S.10: Factor model, Australia: Time-varying moments, one quarter ahead.[a]



Sources: OECD, BIS, Global Financial Data, Haver Analytics, and authors' calculations.

[a] Time-varying moments of the one-quarter-ahead forecast distribution of GDP growth (annualized). The thick line is the posterior median (across parameter draws) at each point in time. The gray shaded band is the pointwise 90% posterior credible band (across parameter draws) at each point in time. The time axis shows the quarter in which the forecast is made.

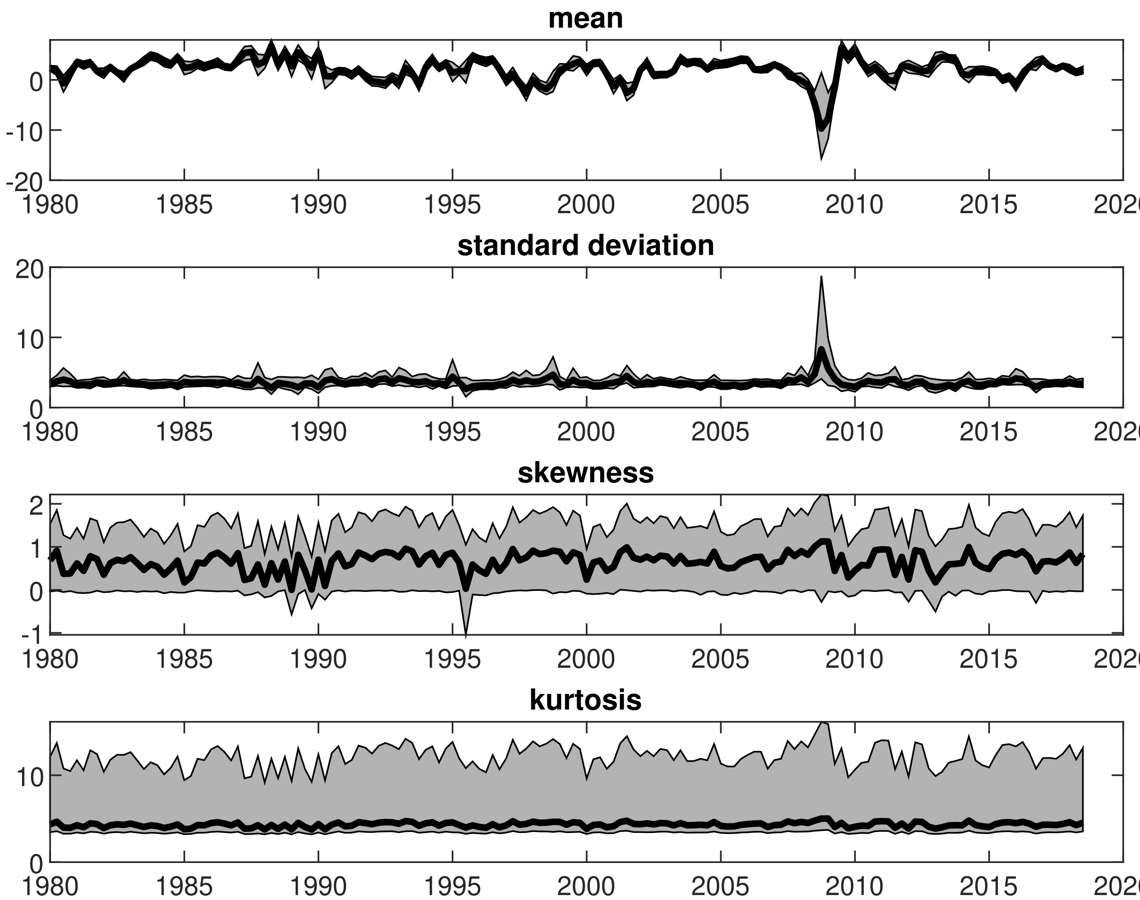
Figure S.11: Factor model, Italy: Time-varying moments, one quarter ahead.[a]



Sources: OECD, BIS, Global Financial Data, Haver Analytics, and authors' calculations.

[a] Time-varying moments of the one-quarter-ahead forecast distribution of GDP growth (annualized). The thick line is the posterior median (across parameter draws) at each point in time. The gray shaded band is the pointwise 90% posterior credible band (across parameter draws) at each point in time. The time axis shows the quarter in which the forecast is made.

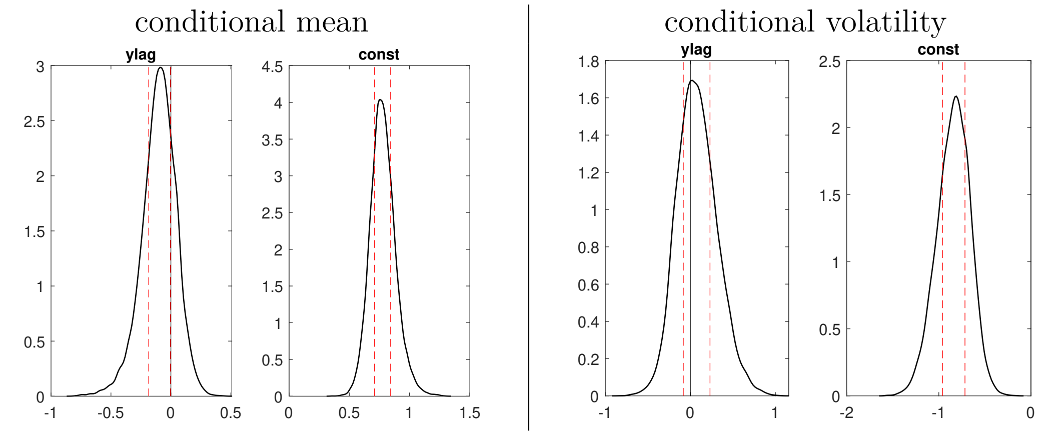
Figure S.12: Factor model, Japan: Time-varying moments, one quarter ahead.[a]



Sources: OECD, BIS, Global Financial Data, Haver Analytics, and authors' calculations.

[a] Time-varying moments of the one-quarter-ahead forecast distribution of GDP growth (annualized). The thick line is the posterior median (across parameter draws) at each point in time. The gray shaded band is the pointwise 90% posterior credible band (across parameter draws) at each point in time. The time axis shows the quarter in which the forecast is made.

Figure S.13: U.S. conditional heteroskedasticity model: Posterior of unpenalized coefficients.[a]



Sources: FRED-QD, Global Financial Data, Haver Analytics, and authors' calculations.
[a] Posterior densities of coefficients on lagged growth (ylag) and intercept (const) in the conditional mean equation (S.6) (left panel) and conditional volatility equation (S.7) (right panel). Vertical red dashed lines indicate posterior interquartile ranges.

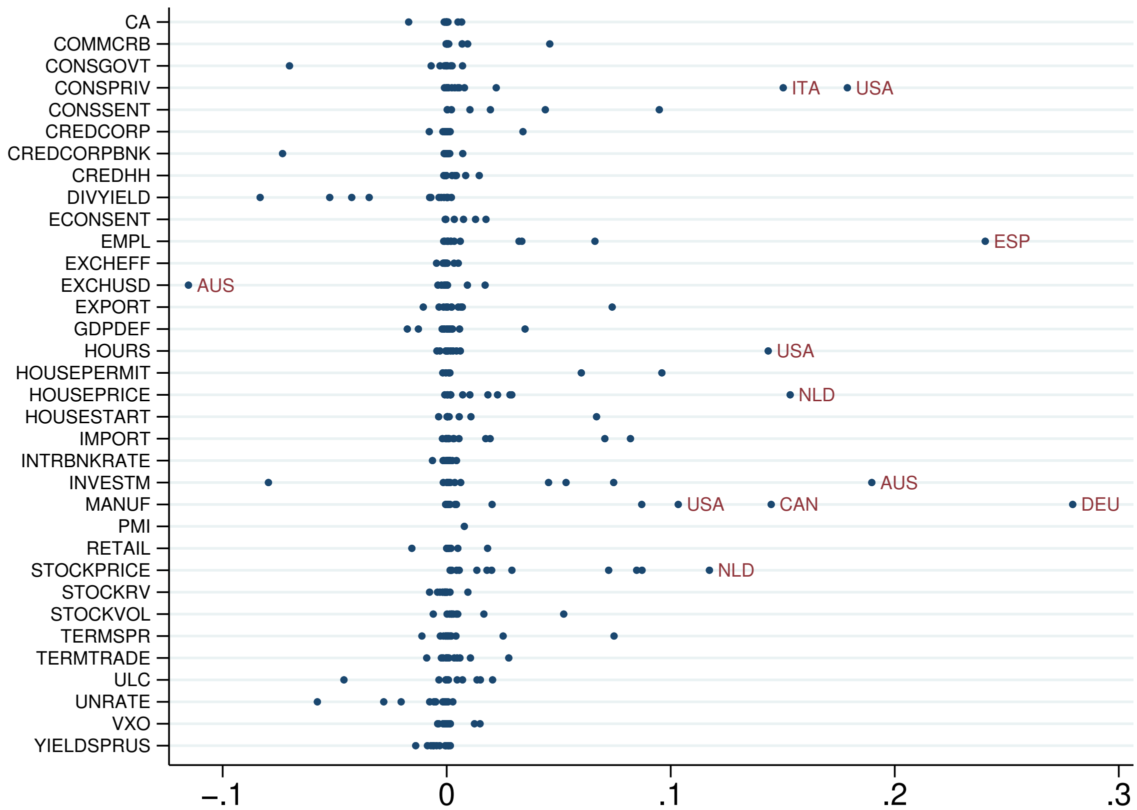
S.G. Variable Selection: Details

Here we provide further empirical results for the variable selection exercises discussed in Section IV, and we define the "TVD" measure of skewness.

U.S. CONDITIONAL HETEROSKEDASTICITY MODEL: POSTERIOR FOR OTHER PARAMETERS Figure S.13 shows the posterior densities for lagged GDP growth and the intercept in the conditional mean and volatility equations. GDP growth exhibits slight mean reversion, holding constant all other predictor variables. There is no strong evidence that lagged GDP growth is an important predictor of volatility, conditional on the other predictors.

CROSS-COUNTRY CONDITIONAL HETEROSKEDASTICITY MODEL: POSTERIOR OF MEAN COEFFICIENTS Figure S.14 confirms that there are indeed some predictor variables that are economically important predictors of the means for a few countries, as argued in Section IV.B. For example, the manufacturing index (MANUF) has a coefficient above 0.1 for Canada, Germany, and the U.S, and private consumption (CONSPRIV) is an important predictor in Italy and the U.S. However, no predictor is important for the majority of countries. Financial variables generally do not appear to be economically important mean predictors in most countries, with the possible exception of the stock index (STOCKPRICE).

Figure S.14: Cross-country conditional heteroskedasticity model: Posterior medians of mean coefficients.[a]



Sources: OECD, BIS, Global Financial Data, Haver Analytics, and authors' calculations.

[a] Posterior medians of the coefficients on mean predictor variables. Each row in the plot corresponds to a variable, while the dots in each row correspond to different countries.

CROSS-COUNTRY DYNAMIC SKEW-T MODEL We here give details on the cross-country dynamic skew-t analysis mentioned in Section IV.C. The data is precisely the same as the global data set in Section IV.B. Due to numerical convergence issues, we drop results for Spain and Japan. Standard diagnostic checks confirm that results are reliable for the other countries. We omit the corresponding separate U.S. analysis for brevity.

Our analysis requires us to quantify the skewness of the GDP growth distribution. Since the units of the skew-t scale parameter α itself are not easily interpretable, we adopt an approach suggested by Dette et al. (2018). The Total Variation Distance (TVD) measure of skewness measures the distance between the skewed distribution and a symmetric counterpart of the distribution. The total variation distance (TVD) between two absolutely continuous random variables X_1 and X_2 with densities $p_1(x)$ and $p_2(x)$, respectively, is given by

$$TVD(X_1, X_2) = \sup_{\mathcal{A}} |P(X_1 \in \mathcal{A}) - P(X_2 \in \mathcal{A})| = \frac{1}{2} \int |p_1(x) - p_2(x)|\, dx,$$

where the supremum is taken over all Borel sets. The units of TVD are probabilities: A TVD of 0 indicates that the two distributions agree fully about the probabilities of all events, while a TVD of 1 indicates that one of the distributions is 100% certain about some event that the other distribution attaches 0% probability to.

Specifically, we quantify the skewness of the skew-t distribution by computing the TVD between a skew-normal random variable α and a standard normal distribution (thus, we effectively let the degrees of freedom $\nu \to \infty$, in order to focus on α). Let U denote a standard skew-normal distributed random variable with density (S.4) and shape parameter α, and let X denote a standard normal random variable. Then Dette et al. (2018) show that[7]

$$TVD(U, X) = \frac{\arctan(|\alpha|)}{\pi}.$$

Let $TVD(\alpha)$ denote the above expression as a function of the skewness parameter α. With α_t defined as in (S.8), the Average Partial Effect (APE) on the TVD of the j-th predictor variable $x_{j,t}$ is given by

$$APETVD_j = \frac{1}{T} \sum_{t=1}^{T} \frac{\partial TVD(\alpha_t)}{\partial x_{j,t}} = \frac{1}{T} \sum_{t=1}^{T} \frac{\text{sign}(\alpha_t)}{\pi(1 + \alpha_t^2)} \beta_{\alpha,j}.$$

This measures the effect of a one unit (i.e., one standard deviation) increase in $x_{j,t}$ on the TVD, holding all other predictors constant, averaged over all observations in the sample.

Table S.5 shows that the data is essentially uninformative about which variables contribute to time-variation in conditional skewness. The table lists summary

[7] Alternative derivation: Define $Z \sim N(0, 1)$ independent of X. Then

$$\begin{aligned} TVD(U, X) &= \frac{1}{2} \int |2\Phi(\alpha x) - 1| \varphi(x)\, dx = \frac{1}{2} \int P(-|\alpha x| \leqslant Z \leqslant |\alpha x|) \varphi(x)\, dx \\ &= \frac{1}{2} E\left[P(|Z| \leqslant |\alpha X| \mid X)\right] = \frac{1}{2} P(|Z/X| \leqslant |\alpha|). \end{aligned}$$

Finally, use the fact that $Z/X \sim \text{Cauchy}(0, 1)$ with distribution function $\frac{1}{\pi} \arctan(x) + \frac{1}{2}$.

statistics of the posterior distribution of $APETVD_j$ across countries and variables. Although some of the variables do come out as statistically significant at the 50% level, the cross-country average posterior median is very close to 0. Moreover, the posterior probability (averaged across countries) that $APETVD_j$ is greater than 2.5% in magnitude is vanishingly small for all predictors j.

Table S.5: Cross-country skew-t model: Posterior of APETVD.[a]

		Average across countries			
Variable	#[b]	median[c]	signif[d]	P>.025[e]	P<−.025[e]
CA	11	0.0004	0.00	0.03	0.02
COMMCRB	11	-0.0000	0.09	0.02	0.02
CONSGOVT	11	-0.0022	0.09	0.02	0.04
CONSPRIV	11	0.0011	0.27	0.05	0.03
CONSSENT	5	0.0056	0.60	0.07	0.01
CREDCORP	11	-0.0006	0.18	0.03	0.04
CREDCORPBNK	11	0.0012	0.09	0.04	0.03
CREDHH	10	0.0008	0.10	0.04	0.03
DIVYIELD	11	0.0002	0.27	0.04	0.03
ECONSENT	6	0.0030	0.17	0.05	0.02
EMPL	11	0.0001	0.00	0.02	0.02
EXCHEFF	11	-0.0018	0.27	0.02	0.04
EXCHUSD	10	-0.0014	0.20	0.02	0.03
EXPORT	11	0.0002	0.27	0.03	0.03
GDPDEF	11	0.0007	0.18	0.03	0.02
HOURS	10	-0.0005	0.20	0.02	0.04
HOUSEPERMIT	6	0.0038	0.33	0.07	0.01
HOUSEPRICE	11	0.0018	0.09	0.03	0.02
HOUSESTART	6	0.0012	0.50	0.05	0.05
IMPORT	11	-0.0009	0.09	0.02	0.04
INTRBNKRATE	11	0.0007	0.09	0.02	0.02
INVESTM	11	0.0033	0.27	0.07	0.03
MANUF	11	-0.0000	0.09	0.03	0.03
PMI	1	0.0002	0.00	0.03	0.04
RETAIL	11	0.0037	0.36	0.05	0.01
STOCKPRICE	11	0.0014	0.09	0.04	0.02
STOCKRV	11	-0.0003	0.00	0.02	0.02
STOCKVOL	8	0.0014	0.25	0.04	0.03
TERMSPR	11	0.0028	0.18	0.05	0.01
TERMTRADE	11	0.0009	0.27	0.04	0.02
ULC	11	-0.0018	0.27	0.02	0.04
UNRATE	11	-0.0027	0.18	0.02	0.07
VXO	11	-0.0010	0.00	0.02	0.03
YIELDSPRUS	10	-0.0001	0.20	0.04	0.04
ylag	11	-0.0180	0.36	0.26	0.41

Sources: OECD, BIS, Global Financial Data, Haver Analytics, and authors' calculations.

[a] Summary statistics of the $APETVD$ posterior distributions for the 13 OECD countries.

[b] Number of non-missing countries.

[c] Posterior median.

[d] Indicator for whether posterior interquartile range excludes 0.

[e] Posterior probability that $APETVD$ is > 0.025 or < -0.025, respectively.

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