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Mikkel Plagborg-Møller, Princeton University
Lucrezia Reichlin, London Business School
Giovanni Ricco, University of Warwick
Thomas Hasenzagl, University of Minnesota

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When is Growth at Risk?*

Mikkel Plagborg-Møller[1], Lucrezia Reichlin[2], Giovanni Ricco[3], and Thomas Hasenzagl[4]

[1] *Princeton University*
[2] *London Business School, Now-Casting Economics, and CEPR*
[3] *University of Warwick, OFCE-SciencesPo, and CEPR*
[4] *University of Minnesota*

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Abstract: This paper empirically evaluates the potentially non-linear nexus between financial indicators and the distribution of future GDP growth, using a rich set of macroeconomic and financial variables covering 13 advanced economies. We evaluate the out-of-sample performance including a fully real time exercise based on a flexible non parametric model and then use a parametric model for estimating the moments of the distribution of GDP conditional on financial variables and evaluating their in-sample estimation uncertainty. Our overall conclusion is pessimistic: moments other than the conditional mean are poorly estimated and no predictors we consider provide robust and precise advance warnings of tail risks or indeed about any features of the GDP growth distribution other than the mean. In particular, financial variables contribute little to such distributional forecasts, beyond the information contained in real indicators.

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Following the Great Recession, there has been an increasing interest in understanding the relationship between financial fragility and the business cycle. Having failed to predict the crash, the economics profession has been trying to understand what was missing in standard macroeconomic models and what are the key indicators of stress in financial markets which may help forecast crises and identify the build-up of macroeconomic risks ahead of time. The research agenda does not only involve prediction but also a revisitation of the earlier literature on financial frictions and the business cycle, pioneered by Bernanke and Gertler (1989), Kiyotaki and Moore (1997), and Bernanke et al. (1999), on the basis of the experience of the 2008 Great Recession.

This research goes beyond academia since it is potentially informative for macro-prudential policies, which indeed focus on the interaction between financial institutions, markets and the wider economy. Such policies need to be grounded in theoretical and empirical knowledge on what are the appropriate tools for strengthening the resilience of the financial system to macroeconomic shocks and *vice versa*. Early warnings of growth fragility would allow monetary and fiscal policy-makers to respond proactively to budding crises.

The structural literature has focused on two alternative classes of variables: those capturing the effect of an external financial premium (in line with models based on the financial accelerator) and those capturing balance sheet constraints such as household or bank credit, reflecting the idea that leverage is a main indicator of the accumulation of financial instabilities (see Gertler and Gilchrist, 2018, for a review).

Price variables such as credit spreads are typically used as proxies for the external financial premium. In fact, there is some consensus that measures derived from different types of interest rate spreads can have predictive power for future economic conditions. For the US, for example, the influential work of Gilchrist and Zakrajšek (2012) has proposed a measure of an excess bond premium that has been widely adopted in both academic and policy work.

A different but related line of research, pioneered by the BIS, has stressed the importance of the leverage cycle as an indicator of risk and used *excess private credit* as a measure of macrofinancial imbalances (see Basel Committee for Banking Supervision, 2010). Some studies have pointed at a correlation of excess growth in leverage and financial crises (see Jorda et al., 2011, Schularick and Taylor, 2012, Jorda et al., 2013 and related literature) and found that recessions preceded by financial crises are deeper and followed by slower recoveries (e.g. Reinhart and Rogoff, 2009, Laeven and Valencia, 2012 and related literature).[1] However, this literature mainly focuses on long-term features of the nexus between finance and the macroeconony and on financial crises rather than recessions. At business cycle frequency, growth rates of credit aggregates are found to be pro-cyclical and lagging (see for example Giannone et al., 2019b). In a recent paper, Brunnermeier et al. (2019) have pointed out that credit "moves passively with output" but that the negative correlation between credit spreads and output is mostly explained by the endogenous response of monetary policy.

Although the literature is very rich, few robust results have emerged from empirical studies about the extent to which financial variables can be used to predict economic activity. This confirms the conclusions of a literature that preceded the crisis (see, for example, Stock and Watson, 2003, Forni et al., 2003 and Hatzius et al., 2010). In particular, three features of financial variables provide challenges to probing both the predictive and the causal relationships connecting them to the real variables. First, movements in financial variables are largely endogenous to the business cycle. Second, the dynamics of financial variables – and spreads in particular – are potentially non-linear and may be related to the higher moments of the GDP distribution rather than just the central tendency. Finally, there is a great degree of heterogeneity among financial indicators. Different types of financial variables capture different mechanisms through which financial markets and the macroeconomy interact.

The idea that financial and economic conditions may be correlated non-linearly has recently inspired a line of research which uses non-parametric methods in order to study the predictive distribution of GDP and its evolution in relation to financial conditions. Giglio et al. (2016) and Adrian et al. (2019) estimate the predictive GDP distribution conditional on a synthetic index of financial conditions. This index aggregates variables capturing financial risk, leverage and credit quality. For the US, such an index is constructed by the Chicago Fed (the National Financial Conditions Index, NFCI). Both papers, focusing on US data, found that the lower quantiles of GDP growth vary with financial conditions while the upper quantiles are stable over time, therefore pointing to an asymmetric and non-linear relationship between financial and real variables. New research is building on these ideas. Recent contributions are in Boyarchenko et al. (2019), Loria et al. (2019), Brownlees and Souza (2019), Figueres and Jarociński (2019) and Delle Monache et al. (2019).

[1] A related but different line of research has identified a financial cycle with different characteristics than the business cycle but leading it and found that financial cycle booms either end-up in crises or weaken growth (see Borio and Lowe, 2002 for early work and more recently Drehmann et al., 2012, Claessens et al., 2012 and many other papers).

As proposed by Adrian et al. (2018), the evaluation of the predictive GDP distribution can be used to define the concept of *growth at risk*, defined as the value of GDP growth at the lower fifth percentile of the predicted growth distribution, conditional on an index of financial stress. This concept has been adopted by policy institutions in many different countries to monitor risks (see, for example, Prasad et al., 2019 for a description of the use of this method at the IMF). The appeal of this approach to policy work, in particular macro-prudential, is that it provides a framework in which forecasting can be thought of as a risk managing exercise (see Kilian and Manganelli, 2008, for the first development of this idea).

The value of this framework for policy in practice rests on whether the dynamics of the moments of the conditional distribution of GDP can be captured with some degree of precision and on whether there is some out-of-sample predictability for moments other than the mean. In a recent paper, Reichlin et al. (2020) evaluate the out-of-sample performance of an aggregate indicator of financial stress and of some key financial variables for the GDP distribution, using the non-parametric approach of Adrian et al. (2019), and found little evidence of predictability beyond what can be achieved using timely indicators of the real economy. In this paper we broaden this analysis in several directions by asking three questions.

First, we want to assess the marginal role of financial variables in estimating and predicting the conditional distribution of GDP once we condition appropriately on available monthly macroeconomic information. Our conjecture is that monthly macroeconomic and financial variables co-move strongly at the contemporaneous level and that a large part of what is revealed by the NFCI reflects some joint information. This of course would not be the case if financial markets primarily reflected forward-looking information, a feature which cannot be assumed and must be tested.

Second, we want to evaluate whether non-linearities in the predictive distribution can be effectively exploited for forecasting and whether the dynamics of moments other than the mean can be precisely estimated. We believe that both evaluations are important to understand whether the growth-at-risk framework can be used in practice for macro-prudential policy. The out-of-sample evaluation takes in consideration overall uncertainty: stochastic, estimation and model uncertainty. Parameter uncertainty – that is, uncertainty conditional on a particular assumed model – can be evaluated in-sample. For the first purpose we use the non-parametric method proposed by Giglio et al. (2016) and Adrian et al. (2019), while for the second purpose we use a fully parametric implementation of their approach. The motivation for using two different models is that the non-parametric approach very flexibly captures non-linearities without relying on particular functional forms, but, unlike the parametric method, it cannot easily be used to assess the statistical uncertainty surrounding the estimation of the moments of the growth distribution. We view the two approaches as complementary.

Third, we assess the potential different roles of individual financial variables in estimating the moments of the conditional distribution by considering a variable selection algorithm. The motivation here is that – as has been observed by Reichlin et al. (2020) – financial variables have very different dynamic properties so that, by aggregating via factor extraction, some information can be lost. An approach that allows individual variables to enter the model in a flexible way may therefore be of interest. Moreover, understanding which specific economic variables carry information about the distribution of GDP growth would allow policy-makers and academics to hone in on specific mechanisms of growth fragility.

We consider both U.S. data and a panel of twelve other OECD countries. This allows us to consider more than a few recessionary events in our sample. For the U.S., for which we have a richer data set, we perform the analysis both separately and in combination with other countries' data.

The overall conclusion of our analysis is pessimistic on the ability of the data to tell us something more than the evolution of the conditional mean. All other moments are imprecisely estimated. Moreover, although we find that financial information has some limited ability to inform conditional mean forecasts at very short horizons, both the out-of-sample analysis and the in-sample results point to very little additional predictive power of financial variables for other moments and for all moments at longer horizons. This remains true in a now-casting exercise where data on financial variables are allowed to have the realistic advantage of being publicly available at an earlier date than data on macroeconomic variables. Finally, when single variables are allowed to enter flexibly in the model, these results are confirmed for both credit spreads variables (prices) and credit variables (quantities), although our methods cannot rule out that some interaction between spreads and credit is at work.

At a more general level, our analysis confirms the older literature's results of the lack of predictive power of financial variables for the real economy, but we show that this finding carries over to an approach that in principle is capable of capturing non-linearities and tail risks. Our findings suggest that markets do not anticipate the timing of the recession and they price the risk only once they see it. In other words, the onset of a recession comes as a surprise to seemingly all agents in the economy. This blindness can be interpreted as revealing that information is rapidly available to all, but rare events such as recessions are fundamentally unforecastable. However, our results do not imply that macro-prudential policies should give up on limiting the accumulation of financial fragilities, since it is likely that those fragilities amplify the damage to the real economy once recessions do occur. However, this is not a feature that we can evaluate using the methods in this paper.

The order of the sections of the paper is organized around the questions we ask. After presenting some motivating facts in Section I, Section II asks the question of whether financial variables have specific forward looking information that can inform an *out-of-sample* predictive relationship with the mean or higher moments of the GDP distribution. We also assess whether financial variables have predictive power for the GDP distribution during the *now-casting* period, where we consider their timeliness advantage with respect to real economic indicators. Section III asks how precisely the moments of the predictive distribution of GDP growth, conditional on real and financial factors, can be estimated *in-sample*. As in Section II, we use as predictors both a *global* factor that includes joint real and financial information and a *financial* factor that includes the financial information orthogonal to the global factor. Section IV abandons the factor-based predictors and instead asks whether there are any specific individual economics variables that are able to explain the dynamics of GDP growth moments. Section V concludes.

I. A Few Motivational Charts

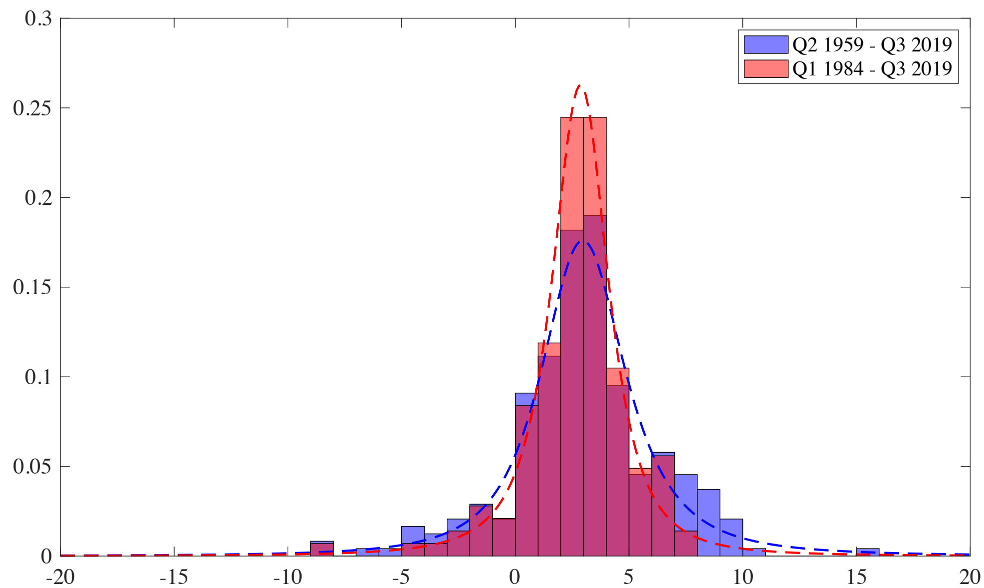
In this section we present a few facts that motivate the analysis of the paper.

Fact 1: Economic fluctuations are asymmetric over the business cycle. Figure 1 shows that the distribution of U.S. GDP growth exhibits some skewness and fat tails. The figure plots the histograms of annual real GDP growth over the samples 1959Q2-2019Q3 (in blue) and 1984Q1-2019Q3 (pink) and the associated fitted distributions. The dark red area describes the overlapping segments. Growth in both subsamples exhibits skewness and heavy tails, although arguably to varying degrees. Indeed the literature has suggested that recessions can be described as a combination of a negative first-moment (mean) shock and a positive second-moment (uncertainty) shock (e.g. Bloom, 2014) or as negative third-moment (skewness) shocks (e.g., Bloom et al., 2016) and fat tails have been found to be a feature of GDP distribution in many advanced economies (see, for example, Fagiolo et al., 2008).

This fact motivates an analysis which is based on estimation and forecasting of moments other than the mean of the predictive GDP distribution.

Fact 2: Financial condition indicators and spreads are highly negatively correlated with output growth at the time of recessions. Figure 2 shows a clear negative correlation between spreads and GDP growth around recessions (although the relation is unstable over the sample). The figure plots quarterly annualized GDP growth for the

Figure 1: Annual real GDP growth.[a]



Sources: FRED-QD and authors' calculations.

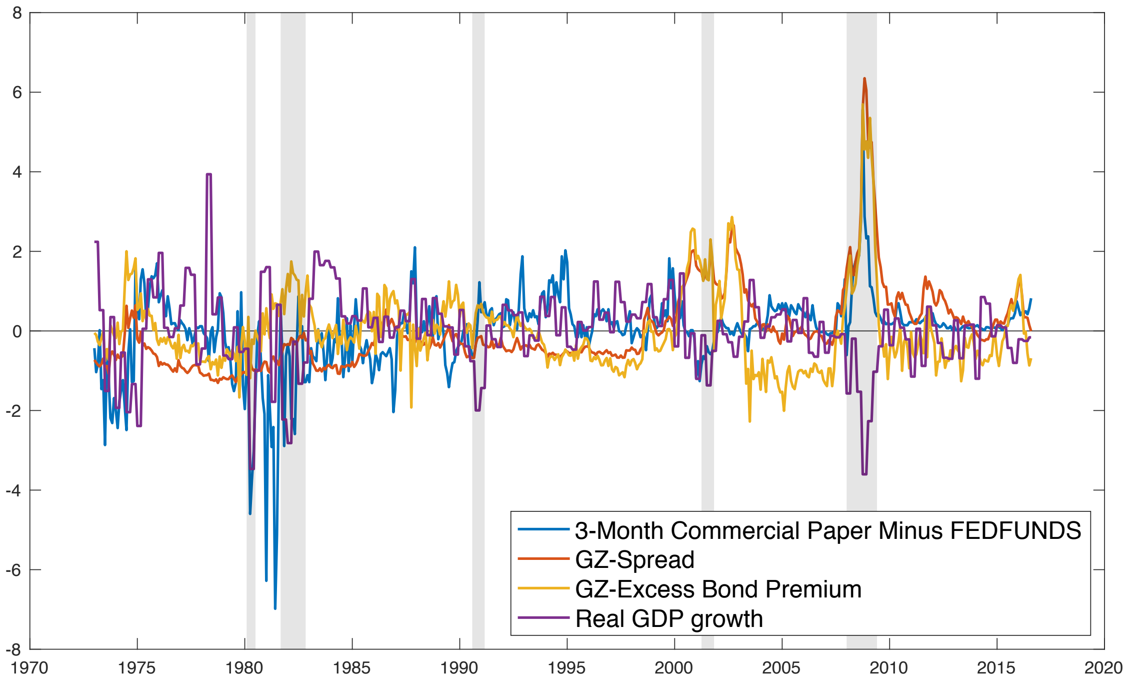
[a] Histograms of annual real GDP growth over the samples 1959q2-2019q3 and 1984q1-2019q3. The fitted distribution are computed by adopting the flexible skew-t distribution developed by Azzalini and Capitanio (2003).

period from 1973q1 to 2015q1 against three credit spreads that have been considered in the literature as measures of financial risk (see Gilchrist and Zakrajšek (2012)).

This chart suggests that the asymmetry in the business cycle for output growth is associated with the asymmetry in the behavior of credit spreads. The latter increase sharply in coincidence or just prior to an economic contraction, while there is no symmetric movement in these variables during booms. The intriguing suggestion is that, by conditioning on these variables, it would be possible to capture higher moments of the GDP conditional distribution. As discussed in the Introduction, this idea has been the inspiration for the literature that has explored the predictive power of financial variables for moments other than the mean, and which we seek to evaluate in this paper.

Fact 3: Movements in financial indicators are largely endogenous and related to output growth. Financial time series and macroeconomic variables share a pronounced contemporaneous common component. Figure 3 reports the quarterly average of the monthly Chicago Fed's National Financial Conditions Index (NFCI)

Figure 2: Financial stress indicators and GDP growth rates.[a]



Sources: FRED-MD, FRED-QD, and Gilchrist and Zakrajšek (2012).

[a] 3-Month Commercial Paper minus Federal funds rate spread, Gilchrist and Zakrajšek (2012) spread and excess bond premium, and real GDP growth from 1973q1 to 2016q3.

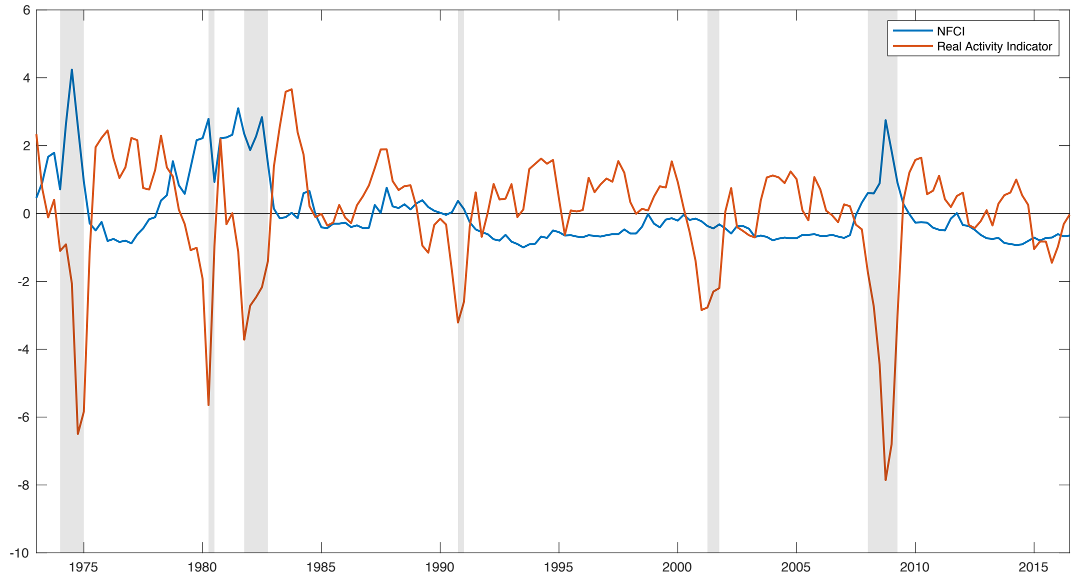
and of a business cycle index computed from a large set of monthly macroeconomic indicators.[2,3]

It shows that the two synthetic aggregate indicators of financial and macroeconomic variables exhibit a very clear pattern of comovement. The strong correlation

[2]The NFCI index is a synthetic indicator computed as a common factor extracted from 105 mixed-frequency – weekly, monthly and quarterly – financial variables. It averages four categories of data: credit quality, risk, non-financial and financial leverage. All variables are transformed to stationarity and standardized. For a description of the NFCI (variables considered and methodology), see Brave and Butters (2012) and the Chicago Fed's dedicated website: `https://www.chicagofed.org/publications/nfci/index`. Both factors are estimated by maximum likelihood following Doz et al. (2012) and averaged across quarters.

[3]The business cycle index is computed as the first common factor to all of the variables in the FRED-MD dataset, except the ones classified as financial. Appendix S.A and Appendix S.B provide details on the estimation of the factor.

Figure 3: Business cycle and financial condition indices.[a]



Sources: authors' computation.

[a] The charts plots an index of real activity extracted as a common factor from a large set of macroeconomic variables and excluding financial variables against the Chicago Fed's National Financial Condition Index (NFCI). The time sample is from 1975q1 to 2015q1.

emerging from the plot indicates that movements in financial indicators are possibly endogenous and contemporaneous to business cycles fluctuations.

This fact suggests that, in order to establish the role of financial variables for predicting the GDP distribution, one should control for the common and contemporaneous component (what we define as the "global factor") and focus on the additional "marginal" information available in the financial indicators (the "financial factor"). This is what our analysis will do.

Fact 4: Different types of financial variables have heterogenous dynamics along the business cycle. Figure 4 provides a more disaggregated view of financial stress by plotting the NFCI and its components. The chart suggests that the NFCI aggregates components with heterogeneous dynamic characteristics, potentially reflecting different forms of fragility in the financial system. It shows that the aggregate NFCI dynamics reflect mainly the risk and credit components, while non-financial leverage follows a smoother cyclical pattern, and financial leverage exhibits some higher-frequency idiosyncratic dynamics.

Figure 4: Heterogenous dynamics of financial indicators.[a]

Sources: FRED-QD.

[a] Chicago Fed's National Financial Condition Index (NFCI) underlying components from 1973q1 to 2015q1.

Indeed, different indicators of stress capture different aspects of financial frictions, which may be relevant at different moments in time – either preceding, contemporaneous to, or following the financial crisis (see Bernanke, 2018 for an analysis of the 2008 recession in the U.S.).

This fact motivates our analysis of the role of individual variables in predicting the moments of the conditional distribution of GDP growth.

II. Predicting Growth at Risk

In this section we assess whether financial variables aggregate forward-looking information that helps predict the distribution of future GDP growth. In particular, we are interested in teasing out information about the future path of output and its moments *in excess* of the contemporaneous information provided by other macroeconomic indicators. Toward this aim, we consider the marginal gain in the predictive distributions for GDP growth (and its moments) when financial-specific information is incorporated, relative to baseline models that only condition on the *global* common component in real and financial data.

We provide both an out-of-sample exercise – one and four quarters ahead – and a fully real-time monitoring of risks to GDP growth with a realistic data release calendar, encompassing macroeconomic and financial variables. It is worth observing that the out-of-sample exercise provides an overall summary of the performance of the model by factoring in several types of uncertainty, excluding the uncertainty about data itself, that is a component of the flow of revised data releases. The real-time exercise takes the latter dimension of uncertainty partially into account since it is based on a realistic calendar of data releases mimicking the information flow.

The results are overall negative. The inclusion of financial-specific information does not improve the mean squared forecast error of the model, nor does it help capture the dynamics of any of its moments. However, financial variables appear (very marginally) to help in pinning down the common contemporaneous information, in real time.

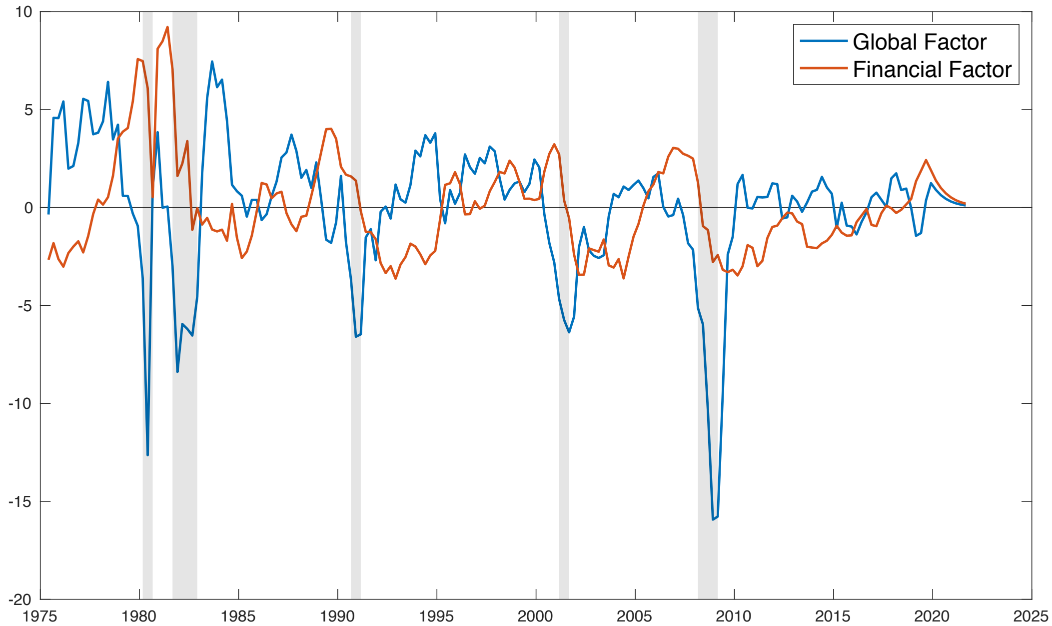
II.A. The Evolution of Out-of-Sample Growth Moments

We first ask the following questions: How do the moments of the predictive distributions vary over time? Does financial variables capture shifts in the predictive mean, variance, or higher moments of the GDP distribution? Is it possible to predict an increase in GDP growth vulnerability out of sample? This exercise focuses on short-to-medium horizons and tries to gauge the overall abilities of the models in assessing risks to GDP growth. Importantly, while providing an assessment of the models' performance against the several sources of uncertainty – stochastic, estimation and model uncertainty – it abstracts form the data uncertainty that characterizes data releases in real time. We integrate this last source of uncertainty in the subsequent real-time exercise.

DATA AND MODEL The first step in our exercise is the estimation of common factors from a large panel of variables. Specifically, we extract two indices of *commonalities*. The first factor, which we refer to as the *global factor*, is common to all the variables in the McCracken and Ng (2016) FRED-MD dataset, including real, financial, monetary, and price variables. The second factor, which we refer to as the *financial factor*, is only common to the financial variables and is orthogonal to the global factor. Figure 5 plots the two factors over the sample period. Appendix S.A provides details on the factor models adopted to estimate the factors.[4] Table 6 in

[4]Figure 18 in the appendix reports the estimated loadings for the factor model with a global and a financial factor.

Figure 5: Global and financial factors.[a]



Sources: authors' computation.

[a] The Global and Financial factors, for the period from 1975q1 to 2019q3.

Appendix S.B provides details on the dataset and on the assumptions adopted to estimated the factor.

The main difference from the analysis of Adrian et al. (2019) is that, while they adopt the NFCI as the main indicator of financial conditions, we separate the information contained in the global factor and the orthogonal financial factor. Reichlin et al. (2020) observe that the NFCI is largely endogenous to economic conditions in the U.S., and that it has high correlation with a factor extracted from non-financial variables only (as also shown in Figure 3). This observation motivates our choice to adopt a global indicator of economic conditions as well as a financial-specific factor that could, in principle, capture forward-looking information on the moments of the predictive distribution of GDP growth that is not obtainable from current economic conditions.

We employ the factors as predictors in the non-parametric quantile regression framework of Adrian et al. (2019). To compare the predictive content of the two factors, we consider three empirical specifications. We model annualized cumulative GDP growth at the one-quarter-ahead and four-quarter-ahead horizons as being driven by, respectively:

(*model 1*) GDP growth at time t;

(*model 2*) GDP growth at time t and the economic activity global factor at time t;

(*model 3*) GDP growth at time t and both the global and the financial factors at time t.

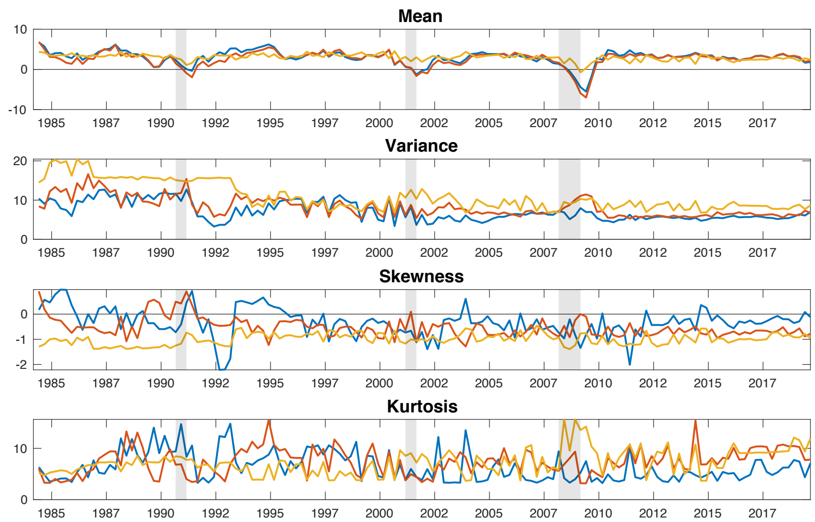
We first estimate the factor model using data from 1975q3 to 1984q1. We then iteratively estimate the predictive distributions of GDP growth one and four quarters ahead, expanding the estimation sample, one quarter at a time, until the end of the sample in 2019q3. In every quarter of the out-of-sample period, we apply the non-parametric prediction approach of Adrian et al. (2019). This involves first estimating the relationship between the percentiles of future GDP growth and the predictors using quantile regressions. Then we smooth out the predictive distribution by fitting a flexible family of distributions to the estimated conditional percentiles, allowing for both skewness and heavy tails. The details of the prediction procedure are described in Appendix S.A.

RESULTS Regardless of the predictors used, the models fail to provide noticeable advance out-of-sample signals of the likelihood or severity of recessions. Figure 6 shows the first four moments of the forecast distribution of GDP growth at horizons $h = 1$ and $h = 4$. By breaking down the predictive distribution into different moments, we aim to show what features of the distribution of GDP growth are predictable, if any. The figure compares the models that condition on (i) the global factor, the financial factor, and GDP (blue line), (ii) the global factor and GDP (red line), and (iii) lagged GDP only (yellow line).

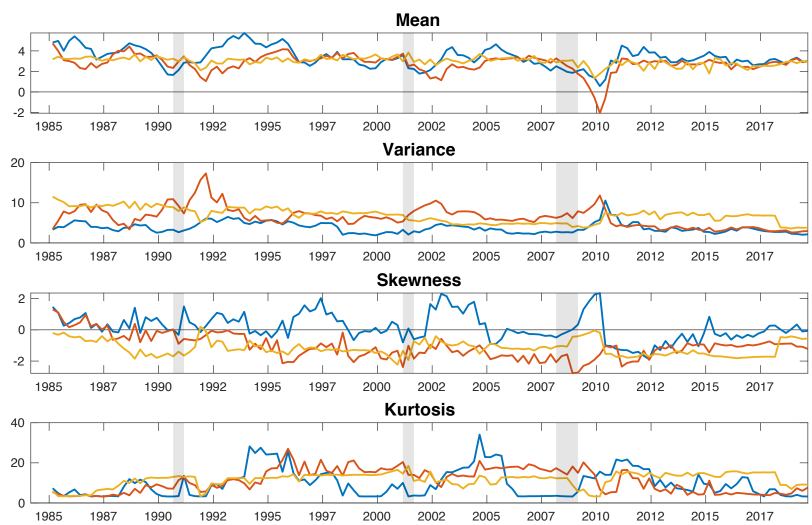
At the the one-quarter-ahead horizon ($h = 1$) shown in panel (A), the distributions of both models that incorporate factors show a sharp decrease in the mean around the period of the Great Recession, but importantly, the model incorporating the financial factor does not seem to have an informational advantage. Strangely, the model not incorporating the financial indicator seems to capture an increase in the variance related to the Great Recession, albeit with some delay. In fact, the movement in the variance lags the 2008 recession by a few quarters and it results from the incorporation into the model, with a quarter of delay, of the spike in spreads in the fourth quarter of 2008. Also, the increase is not remarkable when compared to the level of the forecast variance in the '90s. Skewness and kurtosis apparently move over the sample but with patterns that are not easy to interpret or to relate to economic contractions.

At the four-quarter-ahead horizon ($h = 4$) shown in panel B, the findings are in line with those discussed for $h = 1$ but even more delayed. Interestingly, only

Figure 6: Out-of-sample forecasts: Time evolution of the predictive distribution of GDP growth.[a]



Panel (A): One-quarter ahead predictive distribution of GDP growth.



Panel (B): Four-quarter ahead predictive distribution of GDP growth.

Sources: authors' computation.

[a] Time evolution of the four moments of the one-quarter ahead predictive distribution of GDP growth, from 1993q1 to 2015q4, for the models including (i) the Global factor, Financial factor, and GDP (blue), (ii) the Global factor and GDP (red), and (iii) GDP only (yellow).

the model with the real factor forecasts substantial contractions in GDP at the four-quarter horizon around recessionary periods, although with long delay. Higher moments do not exhibit interpretable patterns. This raises doubts on the ability of the models to correctly capture the dynamics of these moments, at least out-of-sample, an issue we will return to in Section III.

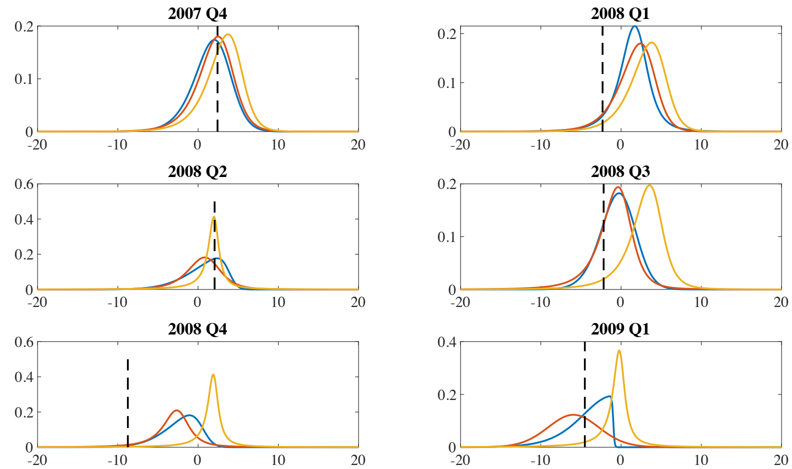
We now zoom in on the Great Recession period. Figure 7 reports the two predictive distributions at different points in time, for $h = 1$ and $h = 4$, before and during the Great Recession (2007q4-2009q1), for the three different models. None of the model seems to predict the crisis. At horizon $h = 1$, panel (A) shows that all the models fail to capture the onset of the economic downturn in 2008q1, and they all assign a low probability to it. As financial stress spikes up in the fourth quarter of 2008, the conditional forecast of both models including the global factor fans out, attaching higher likelihood to a wider range of events. At horizon $h = 4$, panel (B) shows that all models seems to do equally bad in capturing the shift in economic conditions. Although the model that only conditions on lagged GDP performs particularly poorly, the two models incorporating factors yield very similar predictive distributions. Indeed, the model that also incorporates financial variables seems to have little informational advantage.

A more systematic evaluation of the distributional forecast accuracy confirms the minuscule predictive content of the financial factor. Figure 7 shows the *predictive scores* of the two models that incorporate factors. The predictive score is high if a model attaches a high likelihood to the value of GDP growth that is actually realized (see the formal definition in Appendix S.A). While at $h = 1$ the two models have nearly indistinguishable predictive scores, at $h = 4$, the model incorporating the financial factor seems to have a very small advantage over the model with the global factor only. Yet its performances do not uniformly dominate the second model, over the sample.

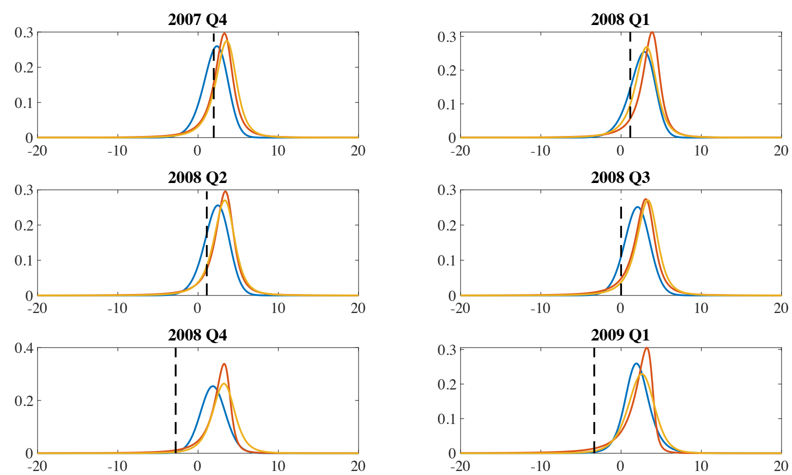
SUMMARY An explorative out-of-sample analysis of the framework of Adrian et al. (2019) indicates that the financial variables help only very marginally in improving the performance of a model that already includes a real activity indicator computed as the common factor of a large panel of real macroeconomic variables. Interestingly, the movements in higher moments seem to be not very informative.[5] In particular, skewness and kurtosis do not show any interpretable movement around recessions. This suggests that growth vulnerability is a story about the mean and possibly volatility of growth, rather than about time-variation in the probability of extreme events. We return to this issue in Section III, where we will be able to

[5] This is consistent with the findings of Adrian et al. (2019).

Figure 7: Out-of-sample forecasts: Predictive distributions during the Great Recession.[a]



Panel (A): One-quarter ahead predictive distribution of GDP growth.

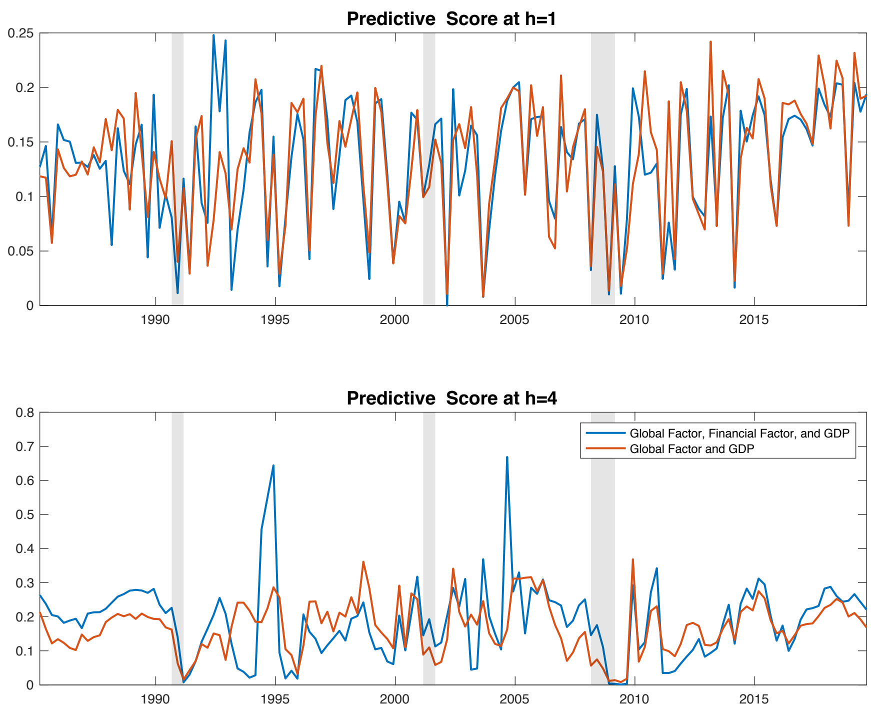


Panel (B): Four-quarter ahead predictive distribution of GDP growth.

Sources: authors' computation.

[a] Quarter by quarter evolution of the predictive distributions in the period of the Great Recession, from 2007q4 to 2009q1, for the models including (i) the global factor, financial factor, and GDP (blue), (ii) the global factor and GDP (red), and (iii) GDP only (yellow). The charts report also the realization of annualised GDP growth one and (cumulative) four quarters ahead, respectively.

Figure 8: Out-of-sample forecasts: Predictive scores.[a]



Sources: authors' computation.

[a] Time evolution of the predictive scores of the one- and four-quarter ahead predictive distribution of GDP growth, from 1993q1 to 2015q4, for the models including (i) the Global factor, Financial factor, and GDP (blue) and (ii) the Global factor and GDP (red). Higher values indicate better forecast performance, in the sense of attaching higher likelihood to the realized events.

characterize the statistical uncertainty associated with the estimation of each time-varying moment. In the next subsection we explore the specific informational content of financial indicators and their relations with real variables, their timeliness, and the heterogeneity across financial variables.

II.B. Real-Time Monitoring of Risks to Growth

To assess the predictive ability of the quantile regression model in real time, we now turn to *nowcasts*, i.e., predicting the current-quarter value of GDP growth ($h = 0$). We will also continue to consider the one-quarter-ahead forecast horizon ($h = 1$). Although these horizons are too short-term for the practical implementation of macro-prudential policies, they are relevant for prediction since the literature has shown that, generally, there is very little predictability for the mean of GDP growth beyond one quarter (see, for example, Giannone et al., 2008). Additionally, monetary and fiscal policy may be able to respond within the quarter in some cases. Finally, our results so far seem to indicate that the model has limited predictive ability at longer horizons anyway.

DATA AND MODEL In this exercise we update the factors and hence the forecast and nowcast in relation to a calendar of data releases, in the tradition of the now-casting literature. As we did above, we extract a number of common factors from a subset of the variables in the monthly FRED-MD dataset. Beyond the global factor (common to all the variables) and the financial factor (common to the financial variables only and orthogonal to the global one), we also consider a *non-financial* factor, from the subset of the data set that exclude financial variables.

Specifically, we construct a calendar of data releases using the average release lag for each variable. In the out-of-sample exercise, we then iterate over the release calendar, position ourselves at each release date, and perform the following three-step procedure:

(*Step 1*) We estimate the factors using an EM algorithm. Then we average the monthly factors to get quarterly factors.

(*Step 2*) We apply the nonparametric forecast approach of the previous subsection to quarterly data up to the current quarter. Using this approach, we construct predictive distributions for current-quarter and next-quarter GDP growth.

We consider the following three sets of predictor variables.

(*model 1*) Global factor only;

Table 1: Groups of variables used in the nowcast exercise and their release lags.[a]

Variable Group	Release lag
Consumer Sentiment	15
Interest Rate Spreads, Stock indices, and exchange rates	30
Unemployment	37
Monetary Aggregates	42
IP & subcomponents	47
Housing Starts & subcomponents	46
CPI & subcomponents	48
New Private housing	54
Personal Consumption Expenditure & Real Personal Income	60

[a] The lag variable is measured as the average number of days between the first day of the reference month and the publication date.

(*model 2*) Global factor and financial factor;

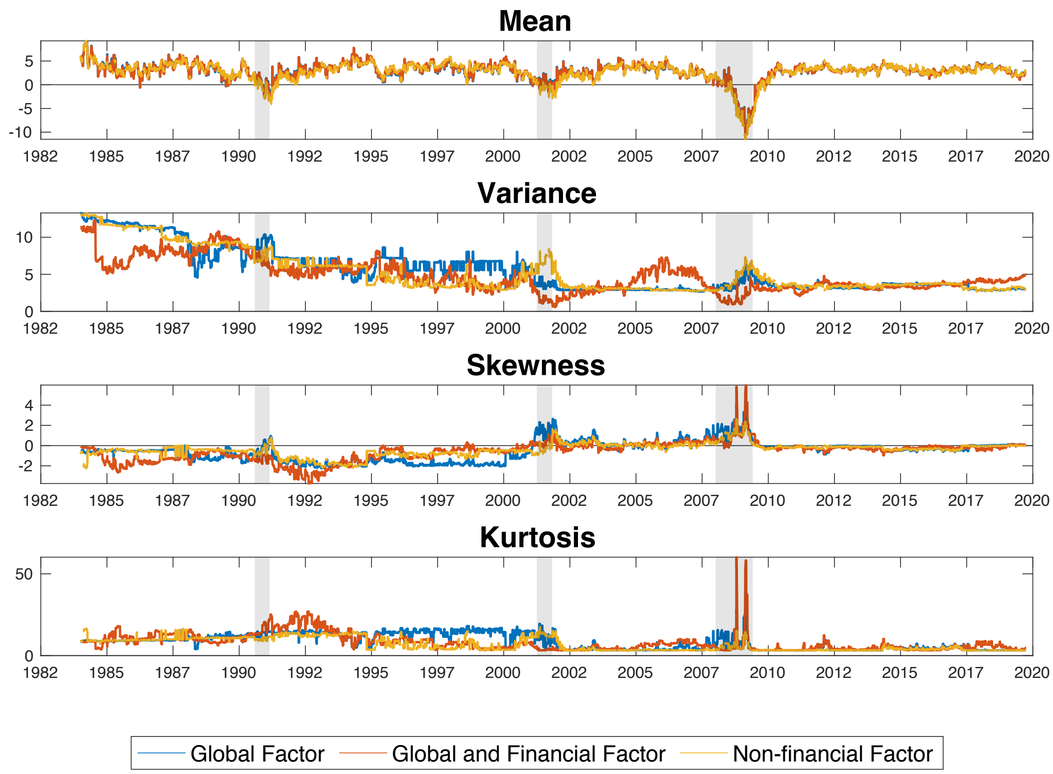
(*model 3*) Non-financial factor only.

Note that, although the non-financial index is quarterly, we have constructed it out of a mixed-frequency model that includes monthly variables by averaging the monthly indices for each quarter. This is indeed how it is used for nowcasting GDP (see Giannone et al., 2008). We begin the out-of-sample forecasting exercise in 1984Q1. For each data release we estimate the factors and the quantile regression parameters using an expanding dataset starting in 1975Q2.

Our exercise allows for the possibility that financial variables have an informational advantage when used in a forecasting framework that takes the real-time data flow into account. Table 1 shows the average lag of the release of the most important groups of variables that we use in the exercise. Table 6 in the appendix shows all the variables included in the dataset, their average release lag, and the factors on which they load. It is worth noting that the average release lag for many of the financial variables is significantly shorter than the average release lag for real, monetary, or price variables. In an additional exercise, for which the results are available on request, we give financial variables a larger advantage by assuming that they are available at the beginning of the month but the results are very similar to what reported here. By employing the growth-at-risk framework, our methodology also allows for financial variables to affect higher moments of the GDP forecast, which could be particularly important in determining tail risks.

Comparing the short-term forecasting performance of a model that contains only the global factor and a model that contains both the global and financial factors allows us to study the additional information content of financial variables over and

Figure 9: Nowcast of the moments of GDP growth.[a]



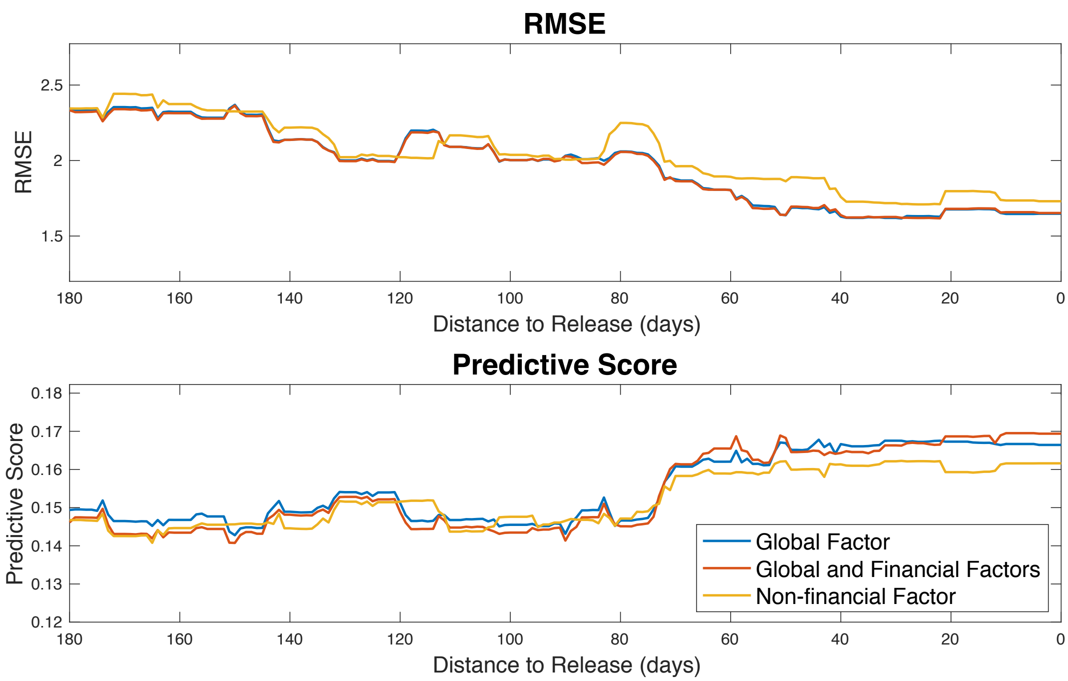
Sources: authors' computation.

[a] Time evolution of the four moments of nowcast predictive distribution of GDP growth at $h = 0$ of quantile regressions with the global factor only (blue line) and the model with the global and financial factors (red line), from 1984q2 to 2019q3.

above what is common to all the other economic variables. Additionally, comparing the short-term forecasting performance of the model that contains only the non-financial factor helps assess the effects of financial variables on imputing the global factor.

RESULTS While the global factor captures all the movements in the mean of the GDP nowcast, financial variables are potentially informative about the higher moments of the nowcast distribution. Figure 9 reports the evolution over time of the four moments of the predictive growth distribution at horizon $h = 0$. The top panel shows that the conditional means of the predictive distributions in all models are nearly identical. This is not surprising, since the global factor already captures the co-movement between all variables, including the financial variables, and thus

Figure 10: Nowcast evaluation.[a]



Sources: authors' computation.

[a] Top panel: Root mean squared error of the nowcast predictive distribution of GDP growth. Bottom Panel: Predictive score of the nowcast predictive distribution of GDP growth. Both charts show the values over the 1984q1 to 2019q3 sample, averaged of the distance to the release data of GDP.

adding the orthogonal, financial factor is not expected to have a large effect on the mean of the predictive distribution. This intuition is also confirmed by observing that the model with the factor estimated using only non-financial variables provides a forecast for the mean that is nearly identical to the other models'. Hence financial variables help only marginally in estimating the common factor more precisely. The models disagree more about the variance, skewness, and kurtosis of the predictive distributions. For example, in the middle of the Great Recession, the model with the financial factor shows a sharp spike in skewness and kurtosis in the density nowcast for the first quarter of 2009. This may be an indication that the real-time model that incorporates financial variables captures some downside risks to growth, but they do it with a delay.

Figure 10 shows that the early availability of financial variables does not translate into more accurate forecasts of the *mean* of the GDP distribution at short horizons.

The top panel of the chart shows the root mean squared errors of the three models, which are entirely due to changes in the mean of the predictive distributions. We make the following observations: (i) The root mean squared forecast errors of all three models are on a slightly downwards sloping path throughout the forecasting period. This indicates that the data released over the forecasting period marginally improves the forecasting performance of the model. (ii) The root mean squared forecast errors of model 1 and model 2 are nearly identical, which indicates that including the financial factor into the model does not improve the forecasting performance of the mean of the predictive distribution. (iii) Model 3, in which we only condition on the non-financial factor, performs slightly worse than the other two models, which indicates that including financial variables into the global factor leads to a slight improvement in forecasting performance.

Figure 10 also shows that financial variables do not improve the short-term forecasting performance of the model, even when account for the effect of financial variables on the entire predictive *distribution* of GDP growth. This is apparent from the bottom panel of the figure, which shows the predictive scores of the three models. Again, we notice a slight improvement of the forecasting performance of all models over the forecasting horizon. However, in the case of the predictive score, all three models perform equally well.

SUMMARY Our out-of-sample test of the predictive ability of a nowcasting model in which we augment the standard global factor with an orthogonal financial factor reaches a disappointing conclusion: The performance of the model with both the global and financial factor is indistinguishable – in terms of root mean squared error and predictive score – from a model with only the global factor. However, the inclusion of financial variables into the global factor does lead to a small improvement relative to a model with only a non-financial factor.

III. How Does the Distribution of GDP Growth Change Over Time?

The previous section demonstrated that there may be some limited out-of-sample information about the time-varying forecast distribution of GDP growth, although most of the predictive information comes from a global factor, not specifically financial variables. However, the method used there did not allow us to quantify the uncertainty surrounding any putative time-variation in the conditional moments. In this section, we estimate a full statistical model of post-1975 U.S. GDP growth that allows conditional moments to vary flexibly over time. Crucially, we will be able to quantify the uncertainty about the parameters in the model and thus the implied uncertainty about the evolution of the conditional moments of GDP growth. Unlike the previous section, we focus on *in-sample* results in this section. Thus, the only uncertainty is about the parameters of the model, which is assumed to be correctly specified. Even then, we find that the data is only informative about the conditional mean; the time-variation of the conditional variance and higher moments are very imprecisely estimated. As a result, the time-variation in the conditional recession probability and in the potential severity of recessions is driven almost exclusively by movements in the mean.

III.A. Data and Model

We model quarterly GDP growth as being driven by lagged GDP growth, as well as the global and financial factors estimated in Section II. We use the final estimates of these factors. In this section we merely use these factors as a convenient set of low-dimensional explanatory variables, whereas the next section will attempt to attribute any explanatory power to individual variables with more direct economic interpretation. The sample period for estimation is 1975q2–2019q2, similar to Adrian et al. (2019).

We assume that the one-quarter-ahead conditional distribution of GDP growth is given by the flexible *skew-t* distribution developed by Azzalini and Capitanio (2003). The distribution is indexed by four parameters: location μ, scale σ, shape α, and heavy-tailedness ν. These parameters influence—but do not directly equal—the conditional mean, variance, skewness, and kurtosis of the distribution. If $\alpha = 0$, the distribution reduces to the usual symmetric Student-t distribution with ν degrees of freedom, which in turn reduces to the normal distribution when $\nu \to \infty$. If $\alpha > 0$, the distribution is positively skewed (higher probability of above-average growth than of below-average growth), while $\alpha < 0$ implies the opposite. Smaller values of ν correspond to fatter tails of the growth distribution (higher probability of abnormally low or high growth).

To allow the explanatory variables to influence several features of the GDP distribution, we model the location parameter $\mu = \mu_t$, the logarithm of the scale parameter $\log \sigma = \log \sigma_t$, and the shape parameter $\alpha = \alpha_t$ as being *time-varying*. These parameters are each assumed to depend linearly on an intercept, lagged GDP growth, and the lagged global and financial factors. The heavy-tailedness parameter ν is assumed constant over time. This parameter mainly influences the kurtosis of the conditional growth distribution, and we will show below that there is little information in the data about time-variation in higher moments anyway. We apply a Bayesian estimation procedure with weakly informative priors on the parameters.

The model and estimation procedure are described in detail in Appendix S.A. As discussed in the appendix, our model can be viewed as a fully Bayesian implementation of the estimation approach developed by Adrian et al. (2019) and used in Section II. An advantage of our approach is that we can easily summarize the posterior uncertainty about time-varying parameters and moments.

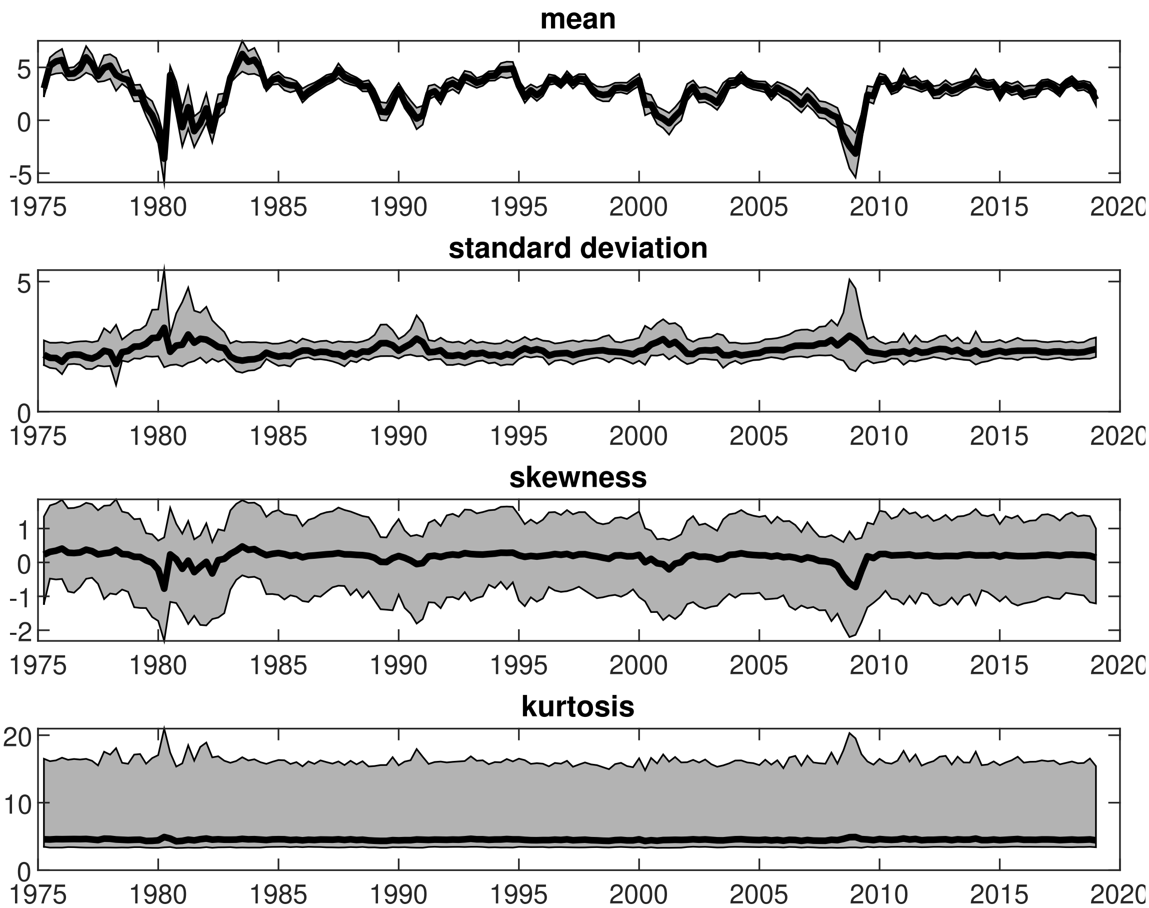
III.B. Time-Variation in U.S. Moments and Tail Risk

Figure 11 shows that the data is only able to accurately pin down the time-variation in the *mean* of the one-quarter-ahead conditional distribution of GDP growth. The standard deviation, skewness, and kurtosis of the forecast distribution are much less precisely estimated. The figure shows the posterior median and 90% credible interval for the moments at each point in time. The uncertainty is due to the fact that the underlying model parameters are estimated with varying degrees of precision in the post-1975 data. As is clear from the figure, the implied uncertainty about higher moments is large. Although the posterior median of the conditional standard deviation does fluctuate, quarters with potentially large swings are also associated with high uncertainty. The time paths of skewness and kurtosis are even more imprecisely estimated.

As one might expect, Figure 12 shows that there is also substantial uncertainty attached to the conditional moments of the *four-quarter-ahead* forecast distribution. As in the previous section, we here seek to forecast the cumulative growth between time t and $t + 4$. Very little can be said with certainty about the time-variation of any of the forecast moments, other than the mean, at the 1-year horizon.

How does the uncertainty about higher moments affect inferences about the left tail of the growth distribution? The top panel of Figure 13 shows the time-varying implied one-quarter-ahead conditional probability of a recession (i.e., negative growth in the following quarter). We see that the recession probability varies substantially over time and is reasonably precisely estimated. However, this is purely due to movements in the conditional mean of next-quarter GDP growth, as opposed to movements in the other moments: The second panel of the figure shows the conditional probability of GDP growth falling below the conditional mean; this probability does not vary much over time and is imprecisely estimated. The third panel of the figure shows the *5% expected shortfall*, which is a measure of the severity of a recession, should it materialize (specifically, it equals expected growth conditional on growth falling below the 5th percentile of its conditional distribution). The expected shortfall moves

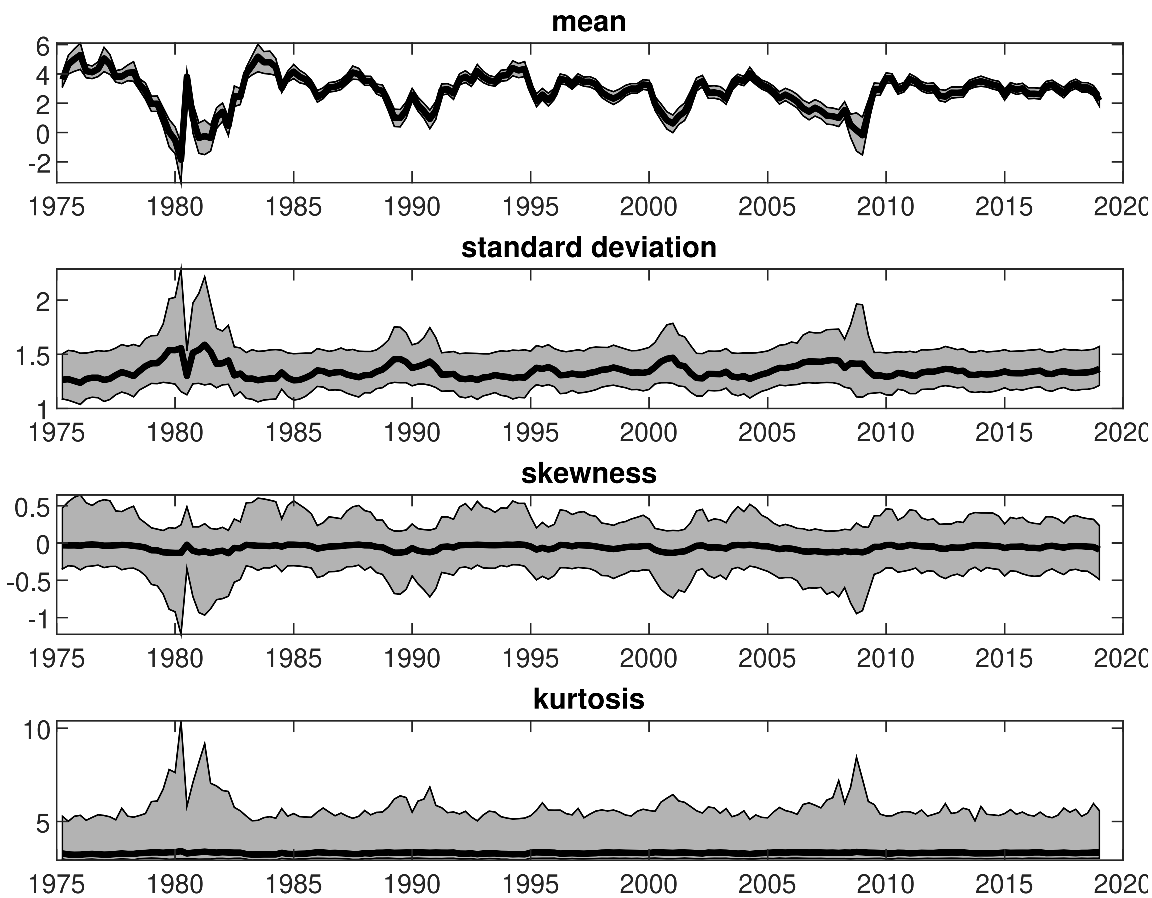
Figure 11: U.S. factor model: Time-varying moments, one quarter ahead.[a]



Sources: FRED-QD, FRED-MD, and authors' calculations.

[a] Time-varying moments of the one-quarter-ahead forecast distribution of GDP growth (annualized). The thick line is the posterior median (across parameter draws) at each point in time. The gray shaded band is the pointwise 90% posterior credible band (across parameter draws) at each point in time. The time axis shows the quarter in which the forecast is made.

Figure 12: U.S. factor model: Time-varying moments, four quarters ahead.[a]



Sources: FRED-QD, FRED-MD, and authors' calculations.

[a] Time-varying moments of the four-quarter-ahead forecast distribution of cumulative GDP growth between time t and $t+4$. The thick line is the posterior median (across parameter draws) at each point in time. The gray shaded band is the pointwise 90% posterior credible band (across parameter draws) at each point in time. The time axis shows the quarter in which the forecast is made.

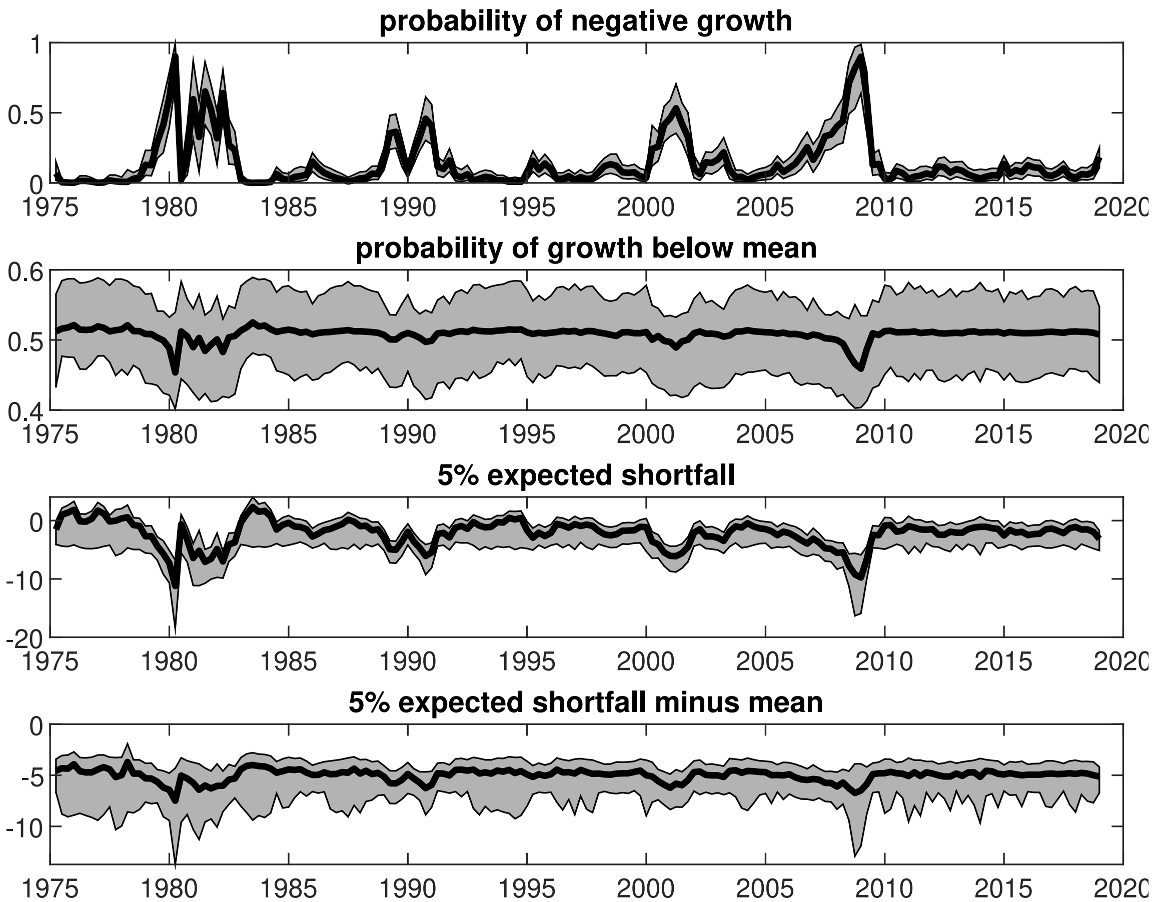
around over time, but the fourth panel—which subtracts off the conditional mean—shows that this movement is almost entirely due to movement in the mean. We report analogous results for four-quarter-ahead forecasts in Appendix S.D; these are qualitatively similar.

Thus, there appears to be little exploitable time-variation in the conditional GDP growth distribution apart from the mean. Although knowing the conditional standard deviation and higher moments would be very helpful for characterizing the risks to GDP growth, it appears that the available data for the U.S. is simply not sufficiently informative about these moments. On the positive side, movements in the conditional mean do appear to be partially predictable, at least in sample. Note that if we are interested in estimating the probability of recessions, and we shut down movement in all moments except for the mean, our model reduces to a probit forecasting model, which is a commonly used specification in applied work.

The financial factor contributes very little to the growth forecasts, whereas the global factor plays a larger role for the conditional mean. Appendix S.D shows the posterior distribution of the model coefficients. The mean coefficients on both factors are statistically significant at conventional levels, but the coefficient on the global factor is estimated to be larger in magnitude. In the appendix we also investigate how the time-varying forecast moments shown in Figure 11 change if we remove the global factor or the financial factor from the conditioning set when producing forecasts. Removing the financial factor has almost no discernible effect on any of the moments, whereas removing the global factor does lead to substantial changes in the path of the conditional mean, especially around the Great Recession period. Thus, as in the out-of-sample results in the previous section, the orthogonal financial factor plays a very minor role in short-term forecasting even *in-sample*.

Figure 11 suggests that the *unconditional* skewness of U.S. GDP growth is indistinguishable from zero, but this result masks a subtle feature of the posterior distribution of the underlying model parameters. In Appendix S.D we show that the marginal posterior distributions for the *intercepts* in the equations for the scale parameter σ_t and shape parameter α_t both exhibit a marked bimodality. These two parameters are highly negatively correlated in the posterior. In essence, the data cannot distinguish whether U.S. GDP growth features (i) a low mean but positive skewness, or (ii) a high mean but negative skewness. Notice that this is not a statement about variation in skewness *over time*, but simply a statement about posterior uncertainty about the nature of the unconditional GDP growth distribution. However, we show in Appendix S.D that if the model is estimated on the post-1980 sample, the positive skewness mode disappears. Figure 2 shows that U.S. GDP growth was especially erratic in the late 1970s, and indeed growth from 1975–1979 has a

Figure 13: U.S. factor model: Recession probability and expected shortfall, one quarter ahead.[a]



Sources: FRED-QD, FRED-MD, and authors' calculations.

[a] Recession probability, probability of growth below the conditional mean, expected shortfall, and expected shortfall minus conditional mean for the one-quarter-ahead conditional distribution of GDP growth (annualized). The thick line is the posterior median (across parameter draws) at each point in time. The gray shaded band is the pointwise 90% posterior credible band (across parameter draws) at each point in time. The time axis shows the quarter in which the forecast is made.

positive sample skewness. Yet the post-1980 data points quite clearly towards negative unconditional skewness. We return to the estimation of unconditional skewness and kurtosis in Section IV.

CROSS-COUNTRY EVIDENCE The fact that time-variation in moments other than the mean is imprecisely estimated holds up in data for other OECD countries. We relegate the discussion of the cross-country data set to the next section, where this data is used more intensively. We estimate a global and financial factor separately for each of 12 other OECD countries, using the same method as we used for the U.S. Appendix S.D shows the estimated time-varying forecast moments for Australia, Italy, and Japan, which are representative of other countries as well. In all cases, the conditional mean of GDP growth is estimated quite precisely, but posterior uncertainty about the model parameters translates into substantial uncertainty about the time paths of the conditional standard deviation, skewness, and kurtosis.

SUMMARY When using lagged GDP growth, a global factor, and a financial factor as predictors, it appears to be highly challenging to accurately estimate the time-variation in the conditional variance, skewness, and kurtosis of GDP growth. The conditional mean, however, is reasonably precisely estimated, and it does appear to vary substantially over time. This is true in data for the U.S. and for other OECD countries. Hence, at least if we ignore out-of-sample forecasting issues, GDP growth forecasting is not a completely futile exercise at short horizons—though all the action is in the mean and none in the tails. More generally, our results demonstrate the importance of taking parameter uncertainty into account when making inferences about rare events from relatively short time series.

However, because we focused on factors as predictors, it remains a possibility that *individual* economic variables might provide strong signals about risks to GDP growth. We turn to this question in the next section.

IV. Which Variables Predict Growth Risk?

Do real activity and financial conditions indices represent the best way to predict and describe growth vulnerability? Policy-makers and academics alike may additionally be interested in which specific economic variables carry most predictive power, for several reasons. First, when designing macro-prudential policies or when explaining such policies to the public, it would be useful to know the most important economic predictor variables, narrowly defined. Second, financial indices – such as the Chicago Fed index used by Adrian et al. (2019) – are usually not constructed to explicitly optimize the ability to forecast *tail risk* in GDP growth. Thus, it is possible that additional predictive power can be gleaned from considering predictor variables individually. Finally, detailed results on the performance of individual predictor variables may shine light on mechanisms that can guide theoretical model-building.

In this section we complement the factor-based analysis of Section III by performing a variable selection exercise to find those specific economic time series that best forecast various moments of GDP growth. We do this by estimating a conditional heteroskedasticity model and the dynamic skew-t model considered in the previous section on U.S. and cross-country data sets, with a wide array of candidate predictor variables. Rather than focusing directly on tail risks, we break down our results by the conditional moments of GDP growth, since this sheds more light on potential mechanisms. Our fully Bayesian approach allows us to describe the uncertainty surrounding the variable selection. For simplicity and clarity, we restrict attention to *one-quarter-ahead* forecasting in this section.

Relative to the literature, our contribution here is to select individual variables – among a large set of candidate variables – that predict GDP growth, its volatility, and higher moments, in data for the U.S. and for 12 other OECD countries. In contrast to the multi-country analyses of Adrian et al. (2018) and Brownlees and Souza (2019), our focus is on variable selection and on characterizing cross-country heterogeneity in growth dynamics. Unlike these papers, we do not explore the role of the forecast horizon.

IV.A. Data

We employ two different data sets: a quarterly U.S. data set and a multi-country data set for 13 OECD countries. In addition to GDP growth (the outcome variable), both data sets contain an extensive set of possible predictor variables. The U.S. data set is especially rich and extends back to 1975, while the predictors in the multi-country data set are slightly more limited in scope and extend back to 1980.

The quarterly U.S. data set is based on the FRED-QD data set constructed by Michael W. McCracken and Serena Ng, building on earlier work by Stock and Watson (2012).[6] This data set is frequently used for high-dimensional prediction in macroeconomics due to its broad scope, reliable data quality, and ease of availability. We select series from various categories of real, price, and financial variables. Though the selected financial series do not cover the full universe used to construct the Chicago Fed's NFCI, as used by Adrian et al. (2019), we include both corporate spreads; government bond yields; credit and loan volume; federal, corporate, and household

[6] https://research.stlouisfed.org/econ/mccracken/fred-databases/

balance sheet variables; stock price, dividends, and trading volume; implied volatility; exchange rates; and commodity prices. We supplement with data from Global Financial Data and Haver Analytics on commodities prices; consumer, business, and purchasing manager surveys; and stock trading volume. This yields a total of 43 predictor variables.

The multi-country data set covers 13 OECD countries, with up to 34 predictor variables for each country. As in the U.S. data described previously, the potential predictor variables include a variety of real, price, survey, and financial variables. Our overarching goal is to ensure that variable definitions and samples are comparable across countries, so that any cross-country heterogeneity can be interpreted in a straight-forward way. The 13 countries are Australia (AUS), Belgium (BEL), Canada (CAN), Switzerland (CHE), Germany (DEU), Spain (ESP), France (FRA), United Kingdom (GBR), Italy (ITA), Japan (JPN), Netherlands (NLD), Sweden (SWE), and United States (USA).[7] Our primary data source is the OECD Economic Outlook and Main Economic Indicators databases. We supplement with data from the BIS on house prices and credit, financial data from Global Financial Data, and household and business surveys from Haver Analytics.

Exploiting data from several countries could in principle ameliorate the inevitable data limitations when estimating the effect of financial indicators on real growth vulnerability (Adrian et al., 2018). According to Carmen Reinhart's classification,[8] the U.S. has only undergone two banking crises since 1980: the savings and loan crisis in the late 1980s and the global financial crisis of 2007–2010. However, every year from 1980–2014, with the exception of 2002–2006, has witnessed a new or ongoing banking crisis in at least one of the 13 countries in our data set. If we include currency crises in the calculation, only the years 2004 and 2006 were crisis-free in all 13 countries. In an average year, 3.7 countries experience a crisis (standard deviation 2.7). From 1980–2016 there have been a total of 99 country-years of banking crises and 47 country-years of currency crises for the countries in our data set (just 9 country-years experienced both types of crisis at once).

The full list of all U.S. and multi-country predictor variables (and their abbreviations) can be found in Appendix S.B.

To make coefficients comparable across different predictor variables, we standardize all predictors (but not GDP growth) to have sample mean zero and variance 1, separately for each country.

[7] Adrian et al. (2018) consider the same countries, excluding Belgium and the Netherlands.

[8] `https://www.hbs.edu/behavioral-finance-and-financial-stability/data/Pages/global.aspx`

IV.B. Which Variables Forecast Growth and Its Volatility?

We first attempt to identify important predictors of the *mean* and *volatility* of GDP growth. We will initially restrict attention to a more parsimonious version of the dynamic skew-t model from Section III. Specifically, we assume that only the mean and variance can vary over time, shutting down any potential time-variation in higher moments. This *conditional heteroskedasticity model* was also analyzed by Adrian et al. (2019). Because we are interested in selecting the relevant predictor variables among a large set of candidates, we employ a Bayesian prior distribution on the model parameters that imposes approximate prior sparsity, that is, it prefers parsimonious (and thus interpretable) models. We give further details about the estimation procedure in Appendix S.A.

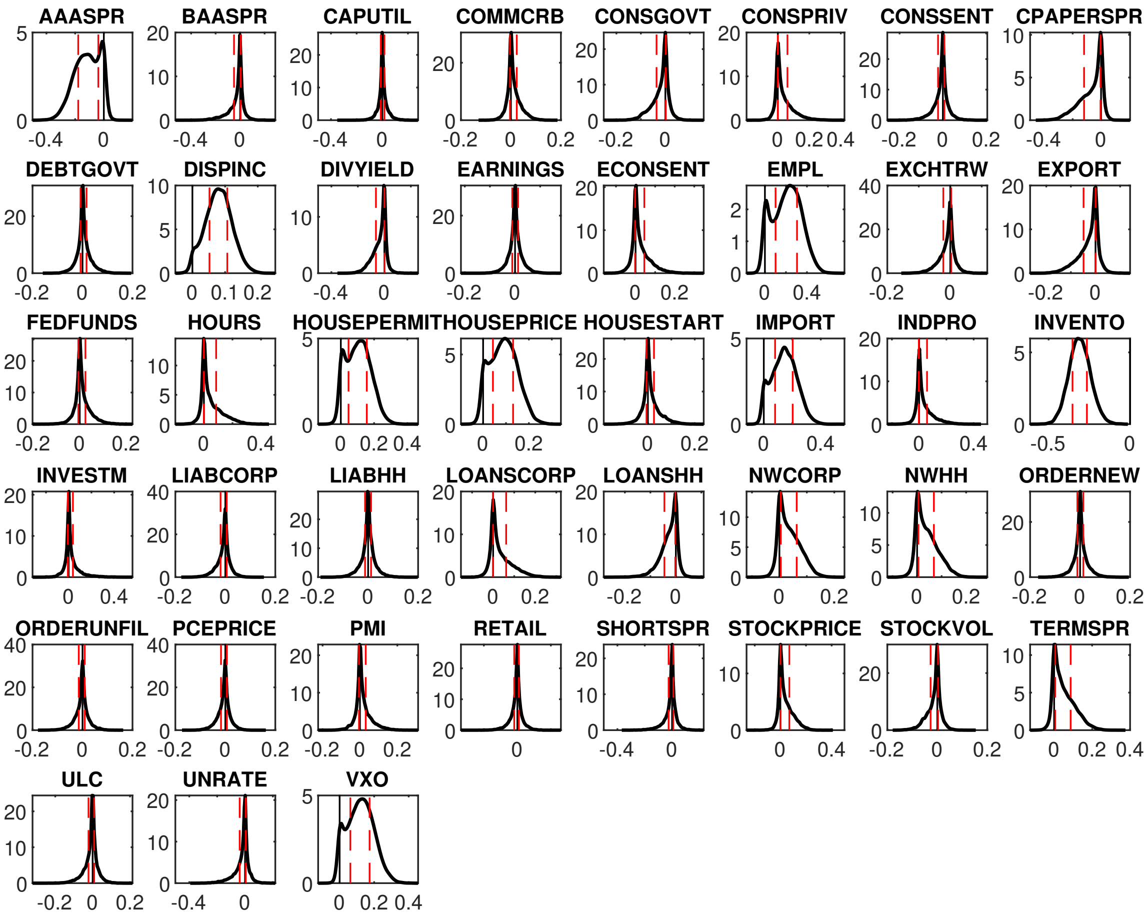
RESULTS: U.S. DATA We first estimate the model on the quarterly U.S. data set from 1975q2–2019q2. Lagged GDP growth turns out to not be especially important for either the conditional mean or volatility, conditional on the other predictors variables discussed below. Hence, we report the results for the lagged growth coefficients and the intercepts in Appendix S.E.

Mean forecasting. Which variables help predict the mean of GDP growth? Figure 14 shows the posterior densities for the mean predictor coefficients. Recall that all predictors have been standardized, so that the magnitudes of different coefficients are immediately comparable. About a third of the variables are found to have high posterior probability of being at least somewhat economically important. There is especially high posterior probability of inventories (INVENTO) being an economically important predictor of the mean of GDP growth, with statistically significant roles also played by disposable income (DISPINC), employment (EMPL), new housing permits (HOUSEPERMIT), house prices (HOUSEPRICE), and imports (IMPORT).

The only two financial variables that have a high probability of being important for the mean are implied volatility (VXO) and the spread between AAA corporate bonds and 10-year Treasuries (AAASPR). Perhaps surprisingly, the coefficient on the term spread (TERMSPR) is estimated to be small. There is only weak evidence that credit aggregates may play some role, although business loans (LOANSCORP), business net worth (NWCORP), and household net worth (NWHH) cannot be entirely ruled out.

Volatility forecasting. When it comes to volatility forecasting, there is strong evidence of predictive power for only a few variables. Figure 15 shows the posterior densities of the volatility coefficients. The coefficient on the AAA corporate bond spread (AAASPR) has substantial posterior mass at values in the range $[-0.3, -0.1]$

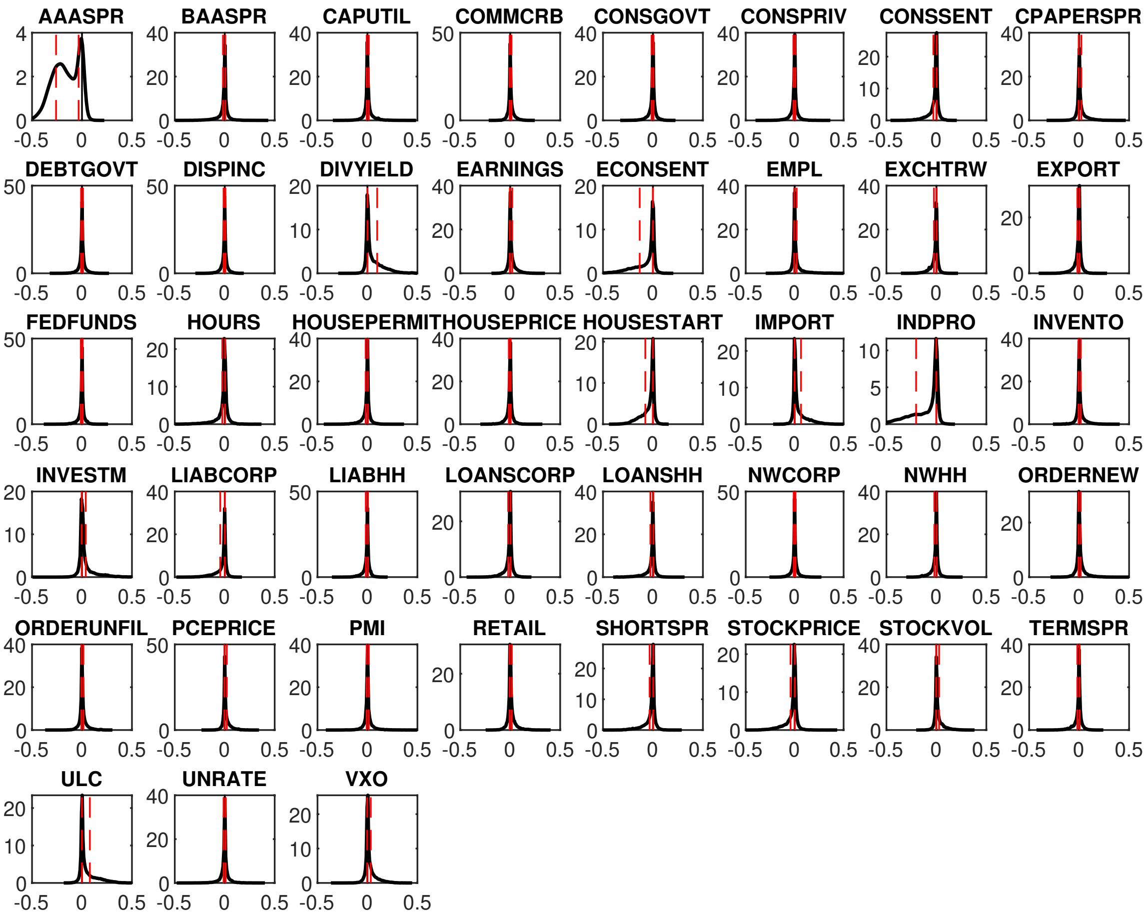
Figure 14: U.S. conditional heteroskedasticity model: Posterior of mean coefficients.[a]



Sources: FRED-QD, Global Financial Data, Haver Analytics, and authors' calculations.

[a] Posterior densities of the coefficients on mean predictor variables in the conditional heteroskedasticity model. Vertical red dashed lines indicate posterior interquartile ranges. A coefficient value of 0.1 means that an increase in the predictor by one standard deviation is associated with a 0.1 percentage point increase in the conditional mean of q/q GDP growth.

Figure 15: U.S. conditional heteroskedasticity model: Posterior of volatility coefficients.[a]



Sources: FRED-QD, Global Financial Data, Haver Analytics, and authors' calculations.

[a] Posterior densities of the coefficients on volatility predictor variables in the conditional heteroskedasticity model. Vertical red dashed lines indicate posterior interquartile ranges. A coefficient value of 0.1 means that an increase in the predictor by one standard deviation is associated with a 10% increase in the conditional volatility of q/q GDP growth.

(the posterior median is -0.16), indicating that a *ceteris paribus* one standard deviation increase in this spread is associated with a 10%–30% increase in GDP growth volatility, a potentially substantial effect. Yet the bimodal nature of the posterior density reflects the fact that the data, combined with our prior belief in sparsity, cannot entirely rule out that even this coefficient may be close to 0.

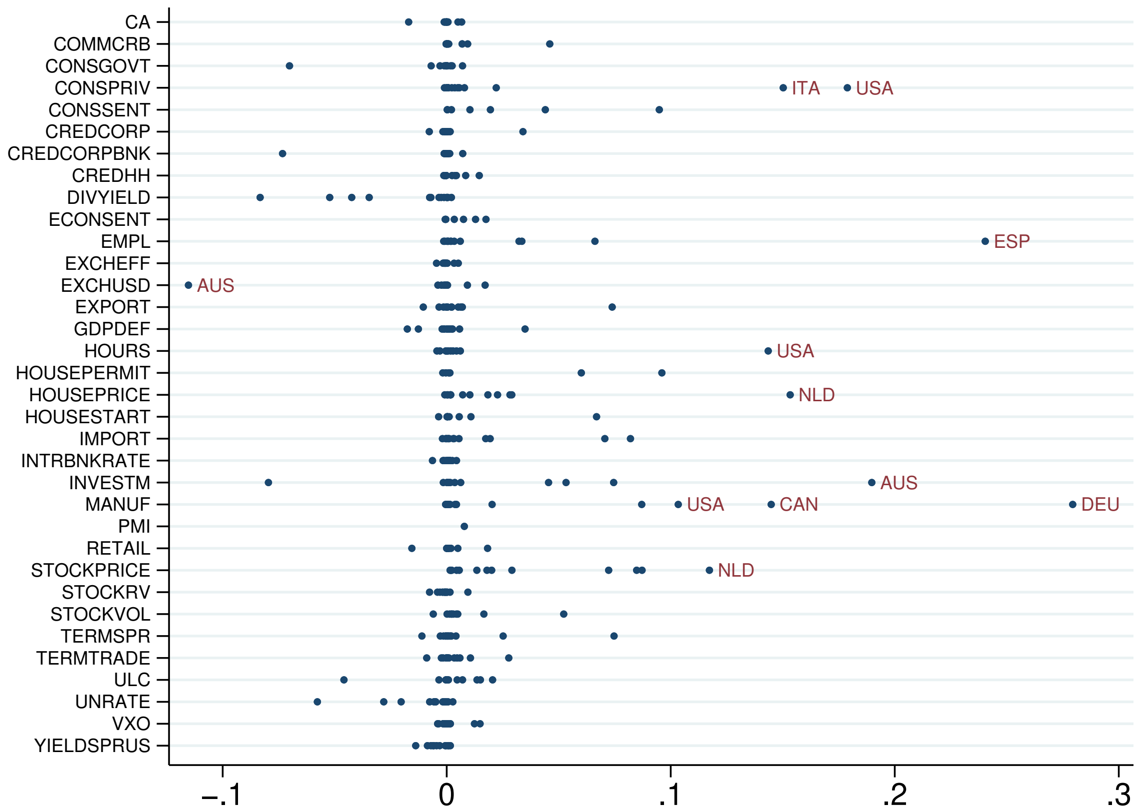
None of the other predictor variables are unambiguously important for volatility forecasting. Other than the AAA spread and lagged GDP growth, no coefficient has a posterior median greater than 0.05 in magnitude. There are five other variables for which the posterior probability of their coefficients exceeding 0.05, or being below -0.05, lies in the range 30–50%: business condition surveys (ECONSENT), housing starts (HOUSESTART), and industrial production (INDPRO) all possibly have a negative association with volatility, while the S&P 500 dividend yield (DIVYIELD) and unit labor cost index (ULC) possibly have a positive association with volatility. Of these variables, the one with the highest degree of posterior certainty is industrial production, for which the posterior probability of lying below -0.05 is a still modest 48%.

RESULTS: CROSS-COUNTRY DATA Are the predictors of GDP growth and its volatility robustly identifiable across several developed countries? Estimating the conditional heteroskedasticity model separately on 13 OECD countries from 1980q1–2018q4, we find that the answer to this question is a resounding no.

Mean forecasting. Although we found encouraging in-sample results on mean forecasting in U.S. data, the precise identities of the relevant predictor variables appear to be highly heterogeneous across the 13 OECD countries. The Table 2 shows summary statistics of the posterior distributions of the mean predictor coefficients across countries. Other than lagged GDP growth, only the national stock index (STOCKPRICE) is significant at the 50% level for more than half the countries (in the sense that the posterior interquartile range excludes 0). The coefficients on consumer sentiment (CONSSENT) and the manufacturing production index (MANUF) also have posterior probability greater than 20% (on average across countries) of being larger than 0.1, meaning that a one-standard-deviation increase is associated with 10 basis points higher q/q GDP growth. Other than the stock index, no other financial variables seem important for more than a few countries, including various financial spreads and credit aggregates.

Figure 16 confirms that there are indeed some predictor variables are economically important predictors of the means for a few countries, but no predictor is important for the majority of countries. For example, the manufacturing index (MANUF) has a coefficient above 0.1 for Canada, Germany, and the U.S, and private consumption

Figure 16: Cross-country conditional heteroskedasticity model: Posterior medians of mean coefficients.[a]



Sources: OECD, BIS, Global Financial Data, Haver Analytics, and authors' calculations.

[a] Posterior medians of the coefficients on mean predictor variables. Each row in the plot corresponds to a variable, while the dots in each row correspond to different countries.

(CONSPRIV) is an important predictor in Italy and the U.S. Financial variables generally do not appear to be economically important mean predictors in most countries, with the possible exception of the stock index (STOCKPRICE).

Volatility forecasting. Cross-country heterogeneity is even more pervasive in volatility forecasting. Table 3 shows summary statistics of the posterior distributions of the volatility predictor coefficients across countries. The only volatility predictor variable that is significant at the 50% level for more than 5 countries is the term spread (TERMSPR). Turning to economic significance, it is only the coefficients on S&P 100 implied volatility (VXO, a global variable) and on lagged GDP growth itself (ylag) that have non-negligible posterior probability of being larger than 0.05 in

Table 2: Cross-country conditional heteroskedasticity model: Posterior of mean coefficients.[a]

Variable	#[b]	Average across countries: median[c]	signif[d]	P>.1[e]	P<−.1[e]
CA	13	-0.0006	0.08	0.01	0.01
COMMCRB	13	0.0055	0.15	0.03	0.00
CONSGOVT	13	-0.0054	0.08	0.00	0.03
CONSPRIV	13	0.0289	0.23	0.15	0.00
CONSSENT	7	0.0245	0.43	0.17	0.00
CREDCORP	13	0.0019	0.08	0.03	0.02
CREDCORPBNK	13	-0.0052	0.08	0.02	0.04
CREDHH	12	0.0024	0.00	0.04	0.01
DIVYIELD	13	-0.0178	0.31	0.01	0.11
ECONSENT	6	0.0067	0.33	0.06	0.01
EMPL	13	0.0296	0.31	0.15	0.00
EXCHEFF	13	-0.0003	0.00	0.01	0.01
EXCHUSD	12	-0.0081	0.08	0.02	0.05
EXPORT	13	0.0063	0.08	0.05	0.01
GDPDEF	13	0.0010	0.15	0.01	0.01
HOURS	12	0.0126	0.08	0.07	0.00
HOUSEPERMIT	6	0.0261	0.33	0.14	0.00
HOUSEPRICE	13	0.0211	0.46	0.11	0.00
HOUSESTART	8	0.0102	0.13	0.06	0.01
IMPORT	13	0.0155	0.23	0.10	0.00
INTRBNKRATE	13	0.0003	0.00	0.01	0.01
INVESTM	13	0.0227	0.38	0.15	0.03
MANUF	13	0.0497	0.38	0.21	0.00
PMI	1	0.0079	0.00	0.07	0.00
RETAIL	12	0.0011	0.17	0.02	0.02
STOCKPRICE	13	0.0352	0.54	0.20	0.00
STOCKRV	13	-0.0007	0.00	0.01	0.02
STOCKVOL	10	0.0081	0.20	0.06	0.00
TERMSPR	13	0.0072	0.23	0.05	0.01
TERMTRADE	13	0.0032	0.08	0.02	0.01
ULC	12	0.0010	0.25	0.05	0.02
UNRATE	13	-0.0103	0.23	0.00	0.08
VXO	13	0.0015	0.00	0.01	0.01
YIELDSPRUS	12	-0.0039	0.08	0.00	0.03
ylag	13	0.1449	0.77	0.59	0.13

Sources: OECD, BIS, Global Financial Data, Haver Analytics, and authors' calculations.
[a] Summary statistics of the mean coefficient posterior distributions for the 13 OECD countries.
[b] Number of non-missing countries.
[c] Posterior median of coefficient.
[d] Indicator for whether posterior interquartile range for coefficient excludes 0.
[e] Posterior probability that coefficient is > 0.1 or < -0.1, respectively.

magnitude for more than a handful of countries. Recall that a coefficient magnitude of 0.05 means that a one-standard-deviation change in the variable predicts a 5% change in volatility, a modest amount.

Very few of the posterior medians of the volatility coefficients are economically significant, as shown in Figure 17. The only three variables whose posterior medians are large in magnitude for two or more countries are stock prices (STOCKPRICE), S&P 100 implied volatility (VXO), and the 10-year government bond spread vis-à-vis the U.S. (YIELDSPRUS). However, with the exception of VXO, the signs of the estimated effects of these variables differ across countries. If interest centers on specific countries, however, we do find strong evidence of substantial predictive power for a small number of additional variables, such as economic sentiment surveys (ECONSENT) and the term spread (TERMSPR) for the Netherlands, and house prices (HOUSEPRICE) for Japan.

SUMMARY We arrive at a negative conclusion: Though it is possible to find strong evidence of a few important mean predictors and (less frequently) volatility predictors for individual countries—such as for the U.S.—generalizing to other countries seems fraught with danger. There is little agreement across countries about the identity and sign of important mean and volatility predictors, despite our efforts to construct a data set with comparable variable definitions and data availability.

Contrary to the conjecture mentioned in Section I that financial spreads and credit aggregates might carry different information about growth vulnerability, we do not find a robust role for either type of variable in mean or volatility forecasting. No financial variable in our data set plays a statistically and economically significant role in forecasting GDP growth at short horizons for more than a handful of the 13 countries we consider. We stress, though, that our data set does not contain a measure of corporate borrowing spreads due to data availability. Thus, our analysis does not overturn the existing literature discussed in the introduction, although it does caution against putting too much faith in single-country analyses.

IV.C. Which Variables Are Informative About Higher Moments?

Can we go beyond the mean or volatility and characterize the predictors of time-variation in skewness and kurtosis of U.S. GDP growth? To answer this question, we turn again to the full dynamic skew-t model described in Section III, but instead

Table 3: Cross-country conditional heteroskedasticity model: Posterior of volatility coefficients.[a]

Variable	#[b]	Average across countries median[c]	signif[d]	P>.05[e]	P<−.05[e]
CA	13	-0.0073	0.31	0.06	0.17
COMMCRB	13	-0.0134	0.23	0.07	0.18
CONSGOVT	13	0.0031	0.00	0.11	0.05
CONSPRIV	13	0.0012	0.15	0.13	0.11
CONSSENT	7	-0.0159	0.14	0.04	0.27
CREDCORP	13	-0.0013	0.00	0.07	0.12
CREDCORPBNK	13	-0.0059	0.08	0.06	0.14
CREDHH	12	-0.0012	0.00	0.07	0.12
DIVYIELD	13	0.0018	0.00	0.11	0.05
ECONSENT	6	-0.0745	0.33	0.04	0.32
EMPL	13	-0.0010	0.00	0.08	0.11
EXCHEFF	13	0.0039	0.08	0.11	0.08
EXCHUSD	12	-0.0007	0.00	0.08	0.10
EXPORT	13	-0.0019	0.00	0.05	0.10
GDPDEF	13	-0.0008	0.08	0.07	0.07
HOURS	12	0.0073	0.25	0.13	0.11
HOUSEPERMIT	6	-0.0076	0.17	0.07	0.17
HOUSEPRICE	13	0.0064	0.23	0.11	0.14
HOUSESTART	8	-0.0051	0.25	0.06	0.14
IMPORT	13	0.0079	0.08	0.15	0.05
INTRBNKRATE	13	-0.0014	0.08	0.09	0.09
INVESTM	13	0.0017	0.00	0.09	0.08
MANUF	13	-0.0107	0.15	0.07	0.16
PMI	1	-0.0008	0.00	0.04	0.09
RETAIL	12	-0.0021	0.17	0.10	0.10
STOCKPRICE	13	0.0019	0.23	0.11	0.16
STOCKRV	13	0.0025	0.08	0.13	0.07
STOCKVOL	10	0.0010	0.20	0.11	0.11
TERMSPR	13	-0.0379	0.54	0.04	0.31
TERMTRADE	13	0.0106	0.15	0.14	0.07
ULC	12	0.0057	0.17	0.15	0.05
UNRATE	13	0.0106	0.08	0.15	0.07
VXO	13	0.0596	0.38	0.40	0.01
YIELDSPRUS	12	0.0321	0.42	0.25	0.12
ylag	13	-0.0283	0.38	0.34	0.42

Sources: OECD, BIS, Global Financial Data, Haver Analytics, and authors' calculations.

[a] Summary statistics of the volatility coefficient posterior distributions for the 13 OECD countries.

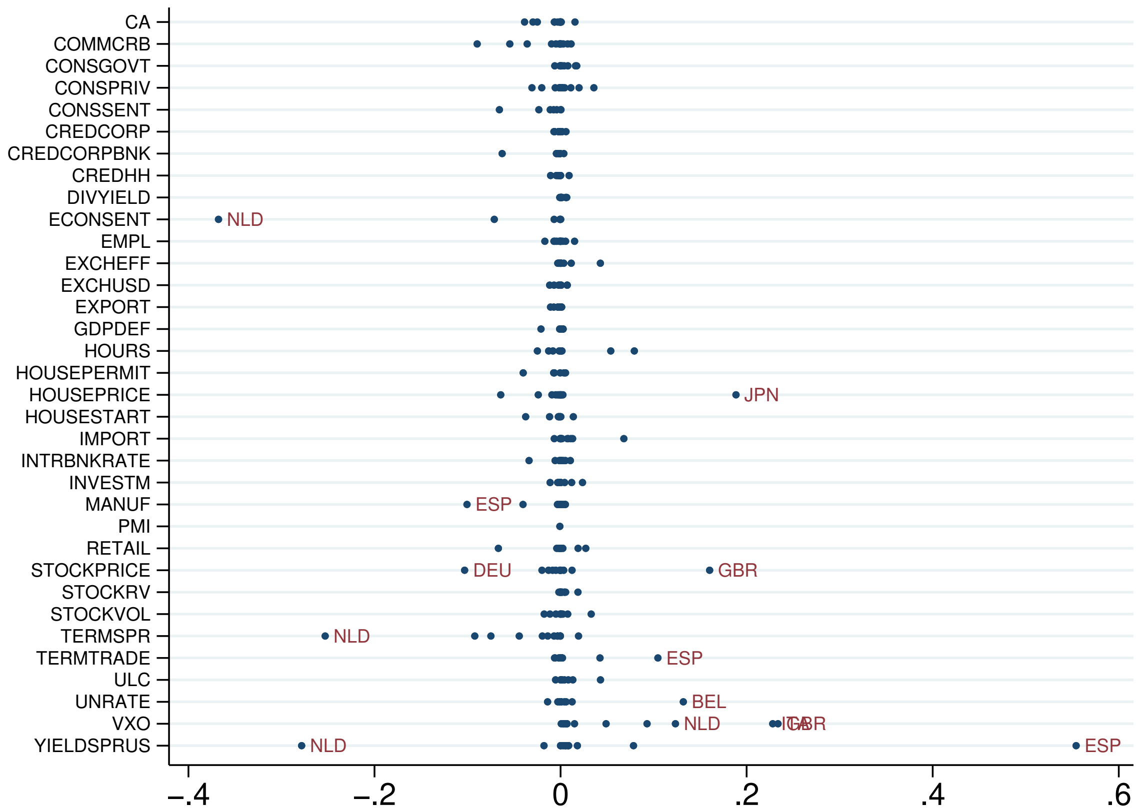
[b] Number of non-missing countries.

[c] Posterior median of coefficient.

[d] Indicator for whether posterior interquartile range for coefficient excludes 0.

[e] Posterior probability that coefficient is > 0.05 or < -0.05, respectively.

Figure 17: Cross-country conditional heteroskedasticity model: Posterior medians of volatility coefficients.[a]



Sources: OECD, BIS, Global Financial Data, Haver Analytics, and authors' calculations.
[a] Posterior medians of the coefficients on volatility predictor variables. Each row in the plot corresponds to a variable, while the dots in each row correspond to different countries.

of using a small number of factors as explanatory variables, we use our full set of individual economic predictor variables.[9]

QUANTIFYING SKEWNESS Our analysis requires us to quantify the skewness of the GDP growth distribution. Since the units of the skew-t scale parameter α itself are not easily interpretable, we adopt an approach suggested by Dette et al. (2018). The Total Variation Distance (TVD) measure of skewness measures

[9] It turns out to be computationally difficult to impose a prior belief in sparsity in the full dynamic skew-t model, unlike in the conditional heteroskedasticity model considered in Section IV.B. Hence, we here instead use conventional normal shrinkage priors. See Appendix S.A for details.

the distance between the skewed distribution and a symmetric counterpart of the distribution. The units of TVD are probabilities: A TVD of 0 indicates that the two distributions agree fully about the probabilities of all events, while a TVD of 1 indicates that one of the distributions is 100% certain about some event that the other distribution attaches 0% probability to. We consider the limit $\nu \to \infty$ to isolate the effect of α. Letting $TVD(\alpha_t)$ denote the TVD given parameter $\alpha = \alpha_t$, we report below the Average Partial Effect (APE) of each predictor variable $x_{j,t}$ on the TVD:

$$APETVD_j = \frac{1}{T}\sum_{t=1}^{T} \frac{\partial TVD(\alpha_t)}{\partial x_{j,t}}. \tag{1}$$

This measures the effect of a one unit (i.e., one standard deviation) increase in $x_{j,t}$ on the TVD, holding all other predictors constant, averaged over all observations in the sample. Formulas for $TVD(\alpha_t)$ and $APETVD_j$ are given in Appendix S.E.

RESULTS: CROSS-COUNTRY DATA The data for our cross-country analysis is precisely the same as the global data set in Section IV.B. Due to numerical convergence issues, we drop results for Spain and Japan. Standard diagnostic checks confirm that results are reliable for the other countries. We omit the corresponding separate U.S. analysis for brevity.

Table 4 shows that the data is essentially uninformative about which variables contribute to time-variation in conditional skewness. The table lists summary statistics of the posterior distribution of $APETVD_j$, defined in (1), across countries and variables. Although some of the variables do come out as statistically significant at the 50% level, the cross-country average posterior median is very close to 0. Moreover, the posterior probability (averaged across countries) that $APETVD_j$ is greater than 2.5% in magnitude is vanishingly small for all predictors j.

The distribution of GDP growth does exhibit clear *unconditional* skewness as well as moderate kurtosis in many countries. Table 5 displays, for each of the 11 countries, posterior summaries of α_t, $TVD(\alpha_t)$, and ν. Based on time-averaged TVD, most countries exhibit substantial skewness, as values of TVD around 25–40% indicate substantial departures from symmetry. From the time-averaged α_t values it is clear, however, that the direction of skewness varies across countries: GDP growth tends to be negatively skewed in Switzerland, Germany, France, Netherlands, and U.S., and positively skewed in the other countries. As expected based on the above results, there does not appear to be substantial time-variation in the extent of the skew, as can be seen by comparing the average and standard deviation of TVD over

Table 4: Cross-country skew-t model: Posterior of APETVD.[a]

Variable	#[b]	Average across countries median[c]	signif[d]	P>.025[e]	P<−.025[e]
CA	11	0.0004	0.00	0.03	0.02
COMMCRB	11	-0.0000	0.09	0.02	0.02
CONSGOVT	11	-0.0022	0.09	0.02	0.04
CONSPRIV	11	0.0011	0.27	0.05	0.03
CONSSENT	5	0.0056	0.60	0.07	0.01
CREDCORP	11	-0.0006	0.18	0.03	0.04
CREDCORPBNK	11	0.0012	0.09	0.04	0.03
CREDHH	10	0.0008	0.10	0.04	0.03
DIVYIELD	11	0.0002	0.27	0.04	0.03
ECONSENT	6	0.0030	0.17	0.05	0.02
EMPL	11	0.0001	0.00	0.02	0.02
EXCHEFF	11	-0.0018	0.27	0.02	0.04
EXCHUSD	10	-0.0014	0.20	0.02	0.03
EXPORT	11	0.0002	0.27	0.03	0.03
GDPDEF	11	0.0007	0.18	0.03	0.02
HOURS	10	-0.0005	0.20	0.02	0.04
HOUSEPERMIT	6	0.0038	0.33	0.07	0.01
HOUSEPRICE	11	0.0018	0.09	0.03	0.02
HOUSESTART	6	0.0012	0.50	0.05	0.05
IMPORT	11	-0.0009	0.09	0.02	0.04
INTRBNKRATE	11	0.0007	0.09	0.02	0.02
INVESTM	11	0.0033	0.27	0.07	0.03
MANUF	11	-0.0000	0.09	0.03	0.03
PMI	1	0.0002	0.00	0.03	0.04
RETAIL	11	0.0037	0.36	0.05	0.01
STOCKPRICE	11	0.0014	0.09	0.04	0.02
STOCKRV	11	-0.0003	0.00	0.02	0.02
STOCKVOL	8	0.0014	0.25	0.04	0.03
TERMSPR	11	0.0028	0.18	0.05	0.01
TERMTRADE	11	0.0009	0.27	0.04	0.02
ULC	11	-0.0018	0.27	0.02	0.04
UNRATE	11	-0.0027	0.18	0.02	0.07
VXO	11	-0.0010	0.00	0.02	0.03
YIELDSPRUS	10	-0.0001	0.20	0.04	0.04
ylag	11	-0.0180	0.36	0.26	0.41

Sources: OECD, BIS, Global Financial Data, Haver Analytics, and authors' calculations.
[a] Summary statistics of the $APETVD$ posterior distributions for the 13 OECD countries.
[b] Number of non-missing countries.
[c] Posterior median.
[d] Indicator for whether posterior interquartile range excludes 0.
[e] Posterior probability that $APETVD$ is > 0.025 or < -0.025, respectively.

Table 5: Cross-country skew-t model: Unconditional skew and kurtosis.[a]

Country	avg(α)[b]	avg(TVD)[c]	std(TVD)[c]	Q1(ν)[d]	med(ν)[d]	Q3(ν)[d]
AUS	5.224	0.387	0.087	12.0	18.0	26.7
BEL	1.747	0.311	0.092	7.6	13.0	21.6
CAN	0.472	0.273	0.101	12.0	18.3	27.4
CHE	-0.821	0.243	0.081	8.5	13.0	20.2
DEU	-5.574	0.363	0.093	13.5	20.1	29.4
FRA	-0.160	0.248	0.100	12.0	18.2	26.9
GBR	1.578	0.307	0.107	4.5	7.1	12.5
ITA	4.229	0.369	0.089	12.8	19.4	28.5
NLD	-4.719	0.392	0.087	10.9	16.7	25.4
SWE	2.381	0.331	0.114	6.5	10.0	16.2
USA	-2.194	0.321	0.096	14.6	21.5	31.0

Sources: OECD, BIS, Global Financial Data, Haver Analytics, and authors' calculations.
[a] Unconditional higher moments of the GDP growth distribution, for 11 OECD countries.
[b] Posterior mean of average (across time) of α_t.
[c] Posterior means of average and standard deviation (across time) of $TVD(\alpha_t)$, respectively.
[d] Posterior first quartile, median, and third quartile of ν, respectively.

time within countries.[10] As for kurtosis, all countries but the United Kingdom have posterior medians of ν in excess of 10, indicating at most moderately fat tails.

SUMMARY Skewness—and to a lesser extent fat tails—do seem to be pervasive features of the unconditional GDP growth distribution in many countries, but attributing the time-variation in these higher moments to specific interpretable economic variables appears challenging given available data. This echoes the result in Section III, which used aggregated factors as predictor variables. In particular, corporate or household credit growth is not robustly associated with negative conditional skewness of GDP growth. Adrian et al. (2018) find evidence for an interaction effect in cross-country data: When credit growth is high, financial conditions are stronger predictors of risks to GDP growth at short horizons. Although we do not have explicit interaction terms in our model, the dynamic skew-t model can in principle generate this empirical pattern if credit growth negatively affects skew while other financial variables affect the mean and/or variance of GDP growth. However, we do not find evidence for this mechanism in our data set. It is an interesting topic

[10] This is consistent with the conclusion of Adrian et al. (2019, p. 1276), who however do not report measures of parameter uncertainty.

for future research to extend the dynamic skew-t model to allow for further state dependence.

V. Summary and Conclusions

The results presented in this paper indicate that financial variables have very limited predictive power for the distribution of GDP growth at short horizons, especially – but not limited to – the tail risk. Two factors drive these results.

First, moments other than the mean are estimated very imprecisely. Although our findings confirm that GDP in many countries features a skewed unconditional distribution, its dynamics conditional on a financial and a global factor are hard to estimate. The same is true for the conditional skewness and conditional variance. In other words, the only moment for which we get tight estimates is the conditional mean. This implies that, when computing the probability of recessions from the estimated moments, we essentially obtain what we could have obtained by using a probit model. These results are confirmed when we allow individual variables to enter the model in a flexible way rather than as aggregate indicators. The variable selection exercise does not point to any stable stylized facts, except for the finding that real indicators are selected more often than financial ones.

Second, information in monthly financial variables is highly correlated with information in real variables. This suggests that, as the economy enters a recession, markets have a sudden change in sentiments which leads to a spike in the spread variables, but this happens contemporaneously with the fall in output. A common factor extracted from financial and real data predicts a fall in the mean of GDP, but it is a poor predictor of other moments, both out-of-sample and in-sample. While our results do not rule out a transmission of shocks from the level of variables to their variance and other moments, as sometimes postulated in stochastic volatility models, this mechanism is empirically tenuous. Only at the nowcasting horizon (within the quarter) does it marginally help to add financial information. This is because it improves the estimation of the common factor which, in turn, improves the root mean squared forecast error. In other word, financial variables have a role in helping predicting GDP only because because they improve the forecast of the central tendency of the distribution, but only at very short horizon and by a narrow margin. The large common component between macroeconomic and financial data also explains why the variable selection algorithm does not point to clear results.

The substantial cross-country heterogeneity in the identities of important predictor variables calls for humility in theoretical model-building: The precise channels of the financial-real vulnerability nexus are difficult to tease out from the available data. In particular, it is likely a mistake to treat broad financial conditions indices as catch-all representations of any arbitrary financial friction that is of theoretical interest.

Lack of predictive power might be the result of time instability between financial variables and GDP, which in turn may caused by changes to the financial system and the conduct of monetary policy. This is something to be investigated further in future research.

Another conjecture is that our methods may fail to capture state dependency and interactions between financial fragility and macroeconomic dynamics. For example, Krishnamurthy and Muir (2017) find that the interaction between credit spreads and pre-crisis credit growth can forecast the severity of the crisis. Aikman et al. (2016) find that, when private non-financial leverage is above trend, an easing of financial conditions predicts an economic expansion in the near term and a contraction in the following quarters. This is an interesting line of research which has implication for policies, as emphasized by Adrian et al. (2018). It implies that, although recessions are fundamentally unpredictable, prudential action can make the system less fragile so that, when they occur, the damage is limited. Although we do not directly investigate the role of such interactions, our results at the very least suggest that empirical analysis of this phenomenon must be fraught with substantial estimation uncertainty.

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S. Supplementary Appendix

S.A. Econometric Methods

We here describe the econometric methods we use to analyze the tail risk—and specifically downside risk—to GDP growth. Beyond estimating the mean and volatility, we aim to understand how much the available time series data can tell us about the skewness and kurtosis of growth. If we were able to characterize the dynamics of these higher moments, it would go a long way toward providing a complete understanding of tail risks. Intuitively, however, such higher moments are sensitive to the occurrence of rare events, and so may be hard to pin down from time series data that only goes back to the 1970s at best.

To strike a balance between flexibility and statistical precision, we consider both nonparametric and fully parametric estimation approaches. We first adopt the nonparametric approach proposed by Adrian et al. (2019), who use quantile regressions to estimate time series of the conditional variance, skew, and kurtosis of U.S. real GDP growth, as well as corresponding measures of downside risk. Then we consider two parametric methods which, unlike the nonparametric method, are able to quantify the potentially high uncertainty surrounding these estimates and also allow for a richer set of predictor variables.

Additionally, at the end of this subsection we describe the factor model used to extract the global and financial factors that serve as predictors in Sections II and III.

NONPARAMETRIC APPROACH The nonparametric approach to estimating growth tail risk developed by Adrian et al. (2019) consists of two steps. First, quantile regressions are used to estimate the conditional quantiles of GDP growth as a function of predictors. Second, a flexible family of probability distributions is fitted to the conditional quantiles. We now describe each of these steps in turn.

Let y_t denote the quarter-over-quarter real GDP log growth rate between time $t-1$ and t. Let $y_{t,t+h} = \sum_{\ell=1}^{h} y_{t+\ell}$ denote the cumulative log growth in real GDP between time t and $t+h$. Finally, let x_t denote a p-dimensional vector of predictor variables.

Quantile regression. In the first step we estimate the conditional quantile function (CQF) of $y_{t,t+h}$ given x_t at quantile τ:

$$Q_\tau\left(y_{t,t+h}|x_t\right) = \inf\{y : F_{y_{t,t+h}|x_t}\left(y|x_t\right) \geq \tau\},$$

where $F_{y_{t,t+h}|x_t}(y_{t+h}|x_t)$ is the conditional cumulative distribution of $y_{t,t+h}$ given x_t. The CQF solves the following maximization problem:

$$Q_\tau\left(y_{t,t+h}|x_t\right) = \operatorname{argmin}_{q(x_t)}\mathbb{E}\left[\rho_\tau\left(y_{t,t+h} - q(x_t)\right)\right], \tag{2}$$

where $\rho_\tau(u) = (\tau - \mathbb{1}(u \leq 0))u$ is a function which weights positive and negative terms asymmetrically.

Under the assumption that the CQF is linear, $Q_\tau\left(y_{t,t+h}|x_t\right) = \beta_\tau' x_t$, we have

$$\beta_\tau = \operatorname{argmin}_b\mathbb{E}\left(\rho_\tau\left(y_{t,t+h} - x_t' b\right)\right). \tag{3}$$

The quantile regression estimator $\hat{\beta}_\tau$ is defined as the sample analogue of β_τ and can be found as the solution to a linear programming problem. The estimator of the CQF at quantile τ is then given by

$$\hat{Q}_\tau(y_{t,t+h}|x_t) = x_t'\hat{\beta}_\tau. \tag{4}$$

Fitted distribution. In order to compute other features of the conditional distribution than just quantiles, Adrian et al. (2019) fit a flexible family of probability distributions to the estimated 5th, 25th, 75th, and 95th percentiles from the first step. That is, they select the parameters of the chosen distribution family to match as closely as possible the estimates $\hat{Q}_\tau(y_{t,t+h}|x_t)$ at those percentiles (conditional on the realized values of x_t).

The specific family of distributions used by Adrian et al. (2019) is the *skew-t* distribution of Azzalini and Capitanio (2003), which generalizes the usual symmetric Student-t distribution. To define this distribution, consider first a random variable U that has the standard *skew-normal* distribution with density function

$$p_U(x;\alpha) = 2\varphi(x)\Phi(\alpha x), \quad x \in \mathbb{R}, \tag{5}$$

where $\varphi(\cdot)$ and $\Phi(\cdot)$ are the density function and distribution function of the standard normal distribution. The skew-normal density is unimodal, and it reduces to the standard normal density for $\alpha = 0$. The parameter α governs the skewness of the density, with $\alpha > 0$ implying right-skew and $\alpha < 0$ implying left-skew.[11] The skew-t$(\mu, \sigma, \alpha, \nu)$ distribution is defined as the distribution of the random variable

$$S = \mu + \sigma\frac{U}{\sqrt{V/\nu}},$$

[11] Azzalini (1985) plots the density function $p_U(x;\alpha)$ for different values of α.

where U has the skew-normal distribution with parameter α, V is χ^2-distributed with ν degrees of freedom, and U and V are independent.[12] If $\alpha = 0$ with ν fixed, this reduces to the usual scaled Student-t distribution; if $\nu \to \infty$ with α fixed, this reduces to the scaled skew-normal distribution. More generally, ν governs the kurtosis of the distribution, with smaller values corresponding to fatter tails.

The parameters of the skew-t distribution are chosen to fit the quantile regression estimates at each realized value of the covariates x_t, generating a sequence of parameters $(\hat{\mu}_t, \hat{\sigma}_t, \hat{\alpha}_t, \hat{\nu}_t)$, $t = 1, \ldots, T$. Then moments of the fitted distribution at each point in time are calculated. Following Adrian et al. (2019), we report the mean, variance, skewness, and kurtosis, as well as a measure of tail risk: expected shortfall. The *5% expected shortfall* is given by the conditional expectation of GDP growth, conditional on a growth realization that is below the 5th percentile of the conditional growth distribution.

Finally, to measure out-of-sample predictive accuracy of distributional forecasts, we consider the *predictive score* that defined as the predictive distribution generated by the model and evaluated at the outturn value of the time series. Higher values of the predictive scores indicate more accurate predictions because they show that the model assigns higher likelihood to realized outcomes.

PARAMETRIC APPROACHES Because the nonparametric method is data-hungry and only applicable when the number of predictors is small, we additionally consider two parametric models of the time-variation in the volatility and/or skewness. We estimate these models using a fully Bayesian approach, allowing us to (i) consider a large number of predictor variables x_t simultaneously and (ii) easily summarize uncertainty about all parameters of interest.

Dynamic skew-t model. First, we consider a dynamic model with innovations that have the skew-t distribution of Azzalini and Capitanio (2003), with mean, volatility, and skewness parameters being functions of observed predictor variables. Whereas the nonparametric approach of Appendix S.A uses the skew-t distribution as a pedagogical tool for interpreting the results of the quantile regressions, here we instead employ the distribution in a fully specified model of GDP growth dynamics.

Recall that y_{t+1} denotes the q/q log growth rate of GDP between time t and $t+1$. Let $\mathcal{F}_t$ denote all available data up to time t. We then assume that

$$y_{t+1} = \mu_t + \sigma_t \varepsilon_{t+1}, \tag{6}$$

[12]The PDF of S is given by $f(s; \mu, \sigma, \alpha, \nu) = \frac{2}{\sigma} t\left(\frac{s-\mu}{\sigma}; \nu\right) T(\alpha \frac{s-\mu}{\sigma} \sqrt{\frac{\nu+1}{\nu+(\frac{s-\mu}{\sigma})^2}}; \nu + 1)$, where $t(\cdot; \nu)$ and $T(\cdot; \nu)$ are the PDF and CDF of the Student-t distribution with ν degrees of freedom.

where the conditional distribution of the innovations is skew-t, as defined above:

$$(\varepsilon_{t+1} \mid \mathcal{F}_t) \sim \text{skew-t}(0, 1, \alpha_t, \nu).$$

The time-varying location μ_t, scale σ_t, and shape α_t parameters are assumed to be driven by the explanatory variables x_t, as follows:

$$\mu_t = \gamma_\mu + \rho_\mu y_t + \beta_\mu' x_t, \tag{7}$$

$$\sigma_t = \exp\left(\gamma_\sigma + \rho_\sigma y_t + \beta_\sigma' x_t\right), \tag{8}$$

$$\alpha_t = \gamma_\alpha + \rho_\alpha y_t + \beta_\alpha' x_t. \tag{9}$$

Since we allow lagged GDP growth to enter into these time-varying parameters, any predictive power of the variables in x_t must go beyond the informational content embodied in lagged GDP growth itself. The heavy-tailedness parameter $\nu > 0$ is assumed constant over time.

When considering the implications of the model for prediction at horizons $h > 1$, we must specify a dynamic model for the predictor variables x_t. For simplicity, we assume that x_t evolves as a VAR(1) model with i.i.d. normal innovations that are independent of the innovation ε_t in the equation (6) for GDP growth. We omit the intercept from the VAR model, as we studentize each predictor variable x_{jt} before running the entire estimation procedure.

Conditional heteroskedasticity model. We also consider a more parsimonious version of the above dynamic skew-t model that only allows for time-variation in first and second moments. Adrian et al. (2019, Section III.B) find that a simple conditionally Gaussian time series model delivers results that are broadly in line with their nonparametric quantile regressions. We will therefore also consider this model, although with an expanded set of predictor variables.

The conditional heteroskedasticity model is obtained as the special case of the skew-t model where we set $\alpha_t = 0$ for all t and let the degrees of freedom $\nu \to \infty$. Thus, the conditional distribution of GDP growth is assumed to be normal, with potentially time-varying conditional mean μ_t and conditional standard deviation σ_t. This model features a symmetric conditional forecast distribution, but it is potentially consistent with *unconditional* skewness (and heavy tails) in GDP growth, depending on the distribution of the predictor variables x_t.

Priors. We consider two types of prior distribution in our Bayesian estimation routine, depending on whether interest centers on variable selection or merely prediction.

Our baseline prior for prediction is a conventional hierarchical normal shrinkage prior on all coefficients:

$$\beta_{\mu,j} \overset{iid}{\sim} N(0, \tau_\mu^2),\ j = 1, \dots, p, \quad \tau_\mu \sim \text{Cauchy}^+(0, 1),$$

and similarly for β_σ and β_α (if applicable). Here "$\text{Cauchy}^+(0, c)$" denotes the Cauchy distribution restricted to $[0, \infty)$ with location parameter 0 and scale parameter c. The coefficients are *a priori* independent across the μ_t, σ_t, and α_t equations. The prior on the degrees of freedom parameter ν is a Gamma distribution with shape parameter 1.5 and rate parameter 0.1, implying a prior mean of 15 and standard deviation of 12.2. Lest we bias the analysis against finding a large predictive role for lagged GDP growth, we impose highly diffuse Cauchy priors on the intercepts and lagged-growth coefficients in equations (7)–(9). Their prior Cauchy scale parameter is set to 5.

We adopt an alternative prior when our interest centers on variable selection and discovering parsimonious, interpretable models. For computational convenience, we only impose this prior on the conditional heteroskedasticity model in Section IV.B. This prior must impose a belief in approximate sparsity. To that end, we employ the "horseshoe prior" of Carvalho et al. (2010) on the mean and volatility coefficients, β_μ and β_σ. This prior assumes

$$(\beta_{\mu,j} \mid \lambda_{\mu,j}, \tau_\mu) \overset{indep}{\sim} N(0, \lambda_{\mu,j}^2), \quad (\lambda_{\mu,j} \mid \tau_\mu) \overset{iid}{\sim} \text{Cauchy}^+(0, \tau_\mu), \quad \tau_\mu \sim \text{Cauchy}^+(0, 1),$$

and similarly for β_σ. Note that—crucially—there is a separate scale parameter $\lambda_{\mu,j}$ corresponding to *each* coefficient $\beta_{\mu,j}$, $j = 1, \dots, p$.[13] Carvalho et al. (2010) show that this prior specification implies a belief in approximate sparsity: The "signal-to-noise" ratio $\frac{1}{1+\lambda_{\mu,j}^2}$ for coefficient $\beta_{\mu,j}$ has a U shaped prior density (or "horseshoe shape"), which causes the posterior distribution for $\beta_{\mu,j}$ to *either* shrink the coefficient heavily towards zero *or* hardly shrink the coefficient at all.[14] The typical empirical result is a model with only a few selected predictor variables whose coefficients are not biased by excessive shrinkage.

The horseshoe prior is more computationally tractable and arguably more economically meaningful than the "spike-and-slab" prior, which assumes that coefficients are exactly 0 with positive prior probability (e.g., Giannone et al., 2019a). Since the

[13] We actually restrict the prior distributions of τ_μ and τ_β to the interval $[1/p, \infty)$, where p is the number of predictors; this improves numerical convergence without affecting the final substantive results.

[14] Thus, the posterior medians behave loosely like post-selection Lasso, but the fully Bayesian approach here makes uncertainty quantification straight-forward.

horseshoe prior distribution for all parameters is absolutely continuous, we can employ highly computationally efficient posterior sampling software, as described below. Follett and Yu (2017) also employ the horseshoe prior of Carvalho et al. (2010) for variable selection in a time series context, but they only impose this prior on the slope variables of a VAR rather than on the volatility component.

In contrast with the low-dimensional approaches to growth-at-risk of Adrian et al. (2019, Appendix A.2) and Carriero et al. (2019), our estimation method is designed to perform variable selection from a large set of candidate mean and volatility predictor variables. Mazzi and Mitchell (2019) estimate a Bayesian time series quantile regression model with shrinkage priors, but their Laplace prior does not impose a prior belief in approximate sparsity, as emphasized by Carvalho et al. (2010, Section 1.3). Manzan (2015) performs variable selection for distributional forecasts using a Lasso-like version of quantile regression, but he is interested in measures of forecast performance rather than in quantifying the uncertainty surrounding the variables selected. Our fully Bayesian approach facilitates the reporting of uncertainty about individual parameters.

When we consider h-step-ahead forecasting for $h > 1$, we require a prior on the VAR model for the predictors x_t. Here we use the conventional choice of a maximally diffuse normal-inverse-Wishart prior. We impose prior independence of these VAR parameters from the parameters in the model for GDP growth. Hence, the posterior for the VAR parameters is of normal-inverse-Wishart form and can be drawn from independently of the posterior draws for the rest of the model.

Posterior computation. We sample from the posterior distribution of the dynamic skew-t model and conditional heteroskedasticity model using the automated Markov Chain Monte Carlo (MCMC) software Stan (Carpenter et al., 2017), specifically the MatlabStan interface. Despite the large number of parameters (more than 100 for some specifications), we are able to reliably and quickly explore the posterior distributions. For each model and specification, we do the following. We run four parallel MCMC chains, starting from rough least-squares estimates of the parameters.[15] We confirm convergence using the $\hat{R}$ convergence metric of Gelman and Rubin (1992) and by visual inspection of the parameter trace plots. Each of the four chains do 5,000 warm-up iterations and then 5,000 further iterations. This yields 20,000

[15]The μ_t coefficients are estimated by OLS as usual. The $\log \sigma_t$ coefficients are then estimated by OLS, using the logarithm of the absolute values of the first-step residuals as left-hand side variable. The α_t coefficients are initialized as random draws near 0; we avoid precise zeros since the Jacobian of the log likelihood is singular at $\alpha_t = 0$. ν is initialized at 10.

stored parameter draws from all chains. The effective sample sizes (i.e., adjusting for serial correlation in the chain) of the parameters of interest almost all exceed 1,000.

Running the entire algorithm takes about 3–6 minutes per specification for the conditional heteroskedasticity model with many predictors, and less than an hour per specification for the dynamic skew-t model with many predictors, on a PC with 3.6 GHz processor and four cores. We have verified that the algorithm accurately recovers important predictors in simulated data of sample size $T = 200$ with 20–50 predictor variables.

When computing moments of the h-step-ahead forecast distribution in Section III, we proceed as follows. Due to computational constraints, we select a random subset of 2,000 posterior parameter draws. For each of these, and for each point in time t, we simulate 5,000 h-quarter-ahead paths of (y_t, x_t) by iterating on the model equations; we then compute various moments of the distribution of cumulative growth $\sum_{\ell=1}^{h} y_{t+\ell}$ from time t to $t+h$. In the case $h = 1$, we do not need to resort to simulation, since the one-step-ahead skew-t density is available in closed form, as discussed above (see also the formulas for the cumulative distribution function and moments in Azzalini and Capitanio, 2003).

FACTOR ESTIMATION We now describe the factor estimation procedure used to generate the predictors in Sections II and III.

Let $z_t = (z_{1,t}, z_{2,t}, \dots, z_{n,t})'$ denote a standardized time series process at time t. In our application, z_t contains the variables in Table 6. We assume that z_t admits the following factor model representation and that the $r \times 1$ vector of common factors f_t follow a VAR(p) process:

$$\begin{aligned} z_t &= \Delta f_t + \epsilon_t, \\ f_t &= A_1 f_{t-1} + A_2 f_{t-2} + \cdots + A_p f_{t-p} + u_t, \quad u_t \sim i.i.d. N(0, \Sigma_u). \end{aligned}$$

Δ is the $n \times r$ matrix of factor loadings and the $n \times 1$ vector ϵ_t contains the idiosyncratic components. We allow for serial correlation in the idiosyncratic components, specifically we assume that ϵ_t follows an AR(1) process:

$$\epsilon_t = \alpha \epsilon_{t-1} + e, \quad e \overset{i.i.d.}{\sim} N(0, \Sigma_e).$$

In the application with the global and financial factors we use $r = 2$ and $p = 2$ and apply appropriate zero restrictions on Δ and the coefficients of the factor VAR, so that the financial factor is specific to the subset of financial variables. In the application with the non-financial factor we use $r = 1$ and $p = 2$ and apply the restriction that the single factor only loads on non-financial variables.

The model is estimated using maximum likelihood estimation via an EM-algorithm, which is initialized using principal components (see Doz et al. (2012)). In order to estimate the principal components all missing observations are first replaced via spline interpolation.

Table 6 below reports the list of variable employed in the exercises and whether they load on the global, the financial and the non-financial factors.

S.B. Data: Details

Here we provide details on the construction of the U.S. and multi-country data sets.

MONTHLY U.S. DATA Table 6 lists the predictor variables in the monthly US dataset. Before further analysis we transform all series to stationarity, following the recommendations of McCracken and Ng where possible. The series are available over the sample period of 01/1959–12/2019 but we restrict our analysis to the 02/1973–09/2019 sample.

Table 6: Monthly US dataset.[a]

			Factors		
Code	Description	Lag	Global	Fin	Non-fin
RPI	Real Personal Income	60	x		x
W875RX1	RPE ex transfer receipts	60	x		x
DPCERA3M086SBEA	Real personal consumption expenditures	60	x		x
CMRMTSPLx	Real Manufacturing and Trade Industries Sales	60	x		x
RETAILx	Retail and Food Services Sales	44	x		x
INDPRO	IP Index	47	x		x
IPFPNSS	IP: Final Products and Nonindustrial Supplies	47	x		x
IPFINAL	IP: Final Products (Market Group)	47	x		x
PCONGD	IP: Consumer Goods	47	x		x
IPDCONGD	IP: Durable Consumer Goods	47	x		x
IPNCONGD	IP: Nondurable Consumer Goods	47	x		x
IPBUSEQ	IP: Business Equipment	47	x		x
IPMAT	IP: Materials	47	x		x
IPDMAT	IP: Durable Materials	47	x		x
IPNMAT	IP: Nondurable Materials	47	x		x
IPMANSICS	IP: Manufacturing (SIC)	47	x		x
IPB51222S	IP: Residential Utilities	47	x		x
IPFUELS	IP: Fuels	47	x		x
CUMFNS	Capacity Utilization	47	x		x
HWI	Help-Wanted Index	37	x		x
HWIURATIO	Ratio of Help Wanted/Unemployed	37	x		x
CLF16OV	Civilian Labor Force	37	x		x
CE16OV	Civilian Employment	37	x		x
UNRATE	Civilian Unemployment Rate	37	x		x
UEMPMEAN	Average Duration of Unemployment	37	x		x
UEMPLT5	Civilians Unemployed: <5 Weeks	37	x		x
UEMP5TO14	Civilians Unemployed: 5-14 Weeks	37	x		x
UEMP15OV	Civilians Unemployed: 15+ Weeks	37	x		x
UEMP15T26	Civilians Unemployed: 15-26 Weeks	37	x		x
UEMP27OV	Civilians Unemployed: 27+ Weeks	37	x		x
CLAIMSx	Initial Claims	37	x		x
PAYEMS	All Employees: Total nonfarm	37	x		x
HOUST	Housing Starts	46	x		x
HOUSTNE	Housing Starts, Northeast	46	x		x
HOUSTMW	Housing Starts, Midwest	46	x		x
HOUSTS	Housing Starts, South	46	x		x

Continued on next page

Table 6 – continued from previous page

Code	Description	Lag	Factors Global	Fin	Non-fin
HOUSTW	Housing Starts, West	46	x		x
PERMIT	New Private Housing Permits	54	x		x
PERMITNE	New Private Housing Permits, Northeast	54	x		x
PERMITMW	New Private Housing Permits, Midwest	54	x		x
PERMITS	New Private Housing Permits, South	54	x		x
PERMITW	New Private Housing Permits, West	54	x		x
AMDMNOx	New Orders for Durable Goods	65	x		x
ANDENOx	New Orders for Nondefense Capital Goods	65	x		x
AMDMUOx	Unfilled Orders for Durable Goods	65	x		x
BUSINVx	Total Business Inventories	74	x		x
ISRATIOx	Total Business: Inventories to Sales Ratio	74	x		x
M1SL	M1 Money Stock	42	x		x
M2SL	M2 Money Stock	42	x		x
M2REAL	Real M2 Money Stock	42	x		x
AMBSL	St. Louis Adjusted Monetary Base	42	x		x
TOTRESNS	Total Reserves of Depository Institutions	42	x		x
NONBORRES	Reserves Of Depository Institutions	42	x		x
BUSLOANS	Commercial and Industrial Loans	43	x	x	
REALLN	Real Estate Loans at All Commercial Banks	43	x	x	
NONREVSL	Total Nonrevolving Credit	65	x	x	
CONSPI	Nonrevolving consumer credit to Personal Income	65	x	x	
S&P 500	S&P's Stock Price Index: Composite	30	x	x	
S&P: indust	S&P's Stock Price Index: Industrials	30	x	x	
S&P div yield	S&P's Composite Stock: Dividend Yield	30	x	x	
S&P PE ratio	S&P's Composite Stock: Price-Earnings Ratio	30	x	x	
FEDFUNDS	Effective Federal Funds Rate	30	x	x	
CP3Mx	3-Month AA Financial Commercial Paper Rate	35	x	x	
TB3MS	3-Month Treasury Bill Rate	30	x	x	
TB6MS	6-Month Treasury Bill Rate	30	x	x	
GS1	1-Year Treasury Rate	30	x	x	
GS5	5-Year Treasury Rate	30	x	x	
GS10	10-Year Treasury Rate	30	x	x	
AAA	Moody's Seasoned Aaa Corporate Bond Yield	30	x	x	
BAA	Moody's Seasoned Baa Corporate Bond Yield	30	x	x	
COMPAPFFx	3-Month Commercial Paper Minus FEDFUNDS	30	x	x	
TB3SMFFM	3-Month Treasury C Minus FEDFUNDS	30	x	x	
TB6SMFFM	6-Month Treasury C Minus FEDFUNDS	30	x	x	
T1YFFM	1-Year Treasury C Minus FEDFUNDS	30	x	x	
T5YFFM	5-Year Treasury C Minus FEDFUNDS	30	x	x	
T10YFFM	10-Year Treasury C Minus FEDFUNDS	30	x	x	
AAAFFM	Moody's Aaa Corporate Bond Minus FEDFUNDS	30	x	x	
BAAFFM	Moody's Baa Corporate Bond Minus FEDFUNDS	30	x	x	
TWEXMMTH	Trade Weighted U.S. Dollar Index: Major Currencies	30	x	x	
EXSZUSx	Switzerland / U.S. Foreign Exchange Rate	30	x	x	
EXJPUSx	Japan / U.S. Foreign Exchange Rate	30	x	x	
EXUSUKx	U.S. / U.K. Foreign Exchange Rate	30	x	x	
EXCAUSx	Canada / U.S. Foreign Exchange Rate	30	x	x	
WPSFD49207	PPI: Finished Goods	44	x		x
WPSFD49502	PPI: Personal Consumption Goods	44	x		x
WPSID61	PPI: Processed Goods for Intermediate Demand	44	x		x
WPSID62	PPI: Unprocessed Goods for Intermediate Demand	44	x		x
OILPRICEx	Crude Oil, spliced WTI and Cushing	30	x		x

Continued on next page

Table 6 – continued from previous page

Code	Description	Lag	Factors Global	Fin	Non-fin
PPICMM	PPI: Metals and metal products	44	x		x
CPIAUCSL	CPI : All Items	48	x		x
CPIAPPSL	CPI : Apparel	48	x		x
CPITRNSL	CPI : Transportation	48	x		x
CPIMEDSL	CPI : Medical Care	48	x		x
CUSR0000SAC	CPI : Commodities	48	x		x
CUSR0000SAD	CPI : Durables	48	x		x
CUSR0000SAS	CPI : Services	48	x		x
CPIULFSL	CPI : All Items Less Food	48	x		x
CUSR0000SA0L2	CPI : All items less shelter	48	x		x
CUSR0000SA0L5	CPI : All items less medical care	48	x		x
PCEPI	Personal Cons. Expend.: Chain Index	60	x		x
DDURRG3M086SBEA	Personal Cons. Exp: Durable goods	60	x		x
DNDGRG3M086SBEA	Personal Cons. Exp: Nondurable goods	60	x		x
DSERRG3M086SBEA	Personal Cons. Exp: Services	60	x		x
CES0600000008	Avg Hourly Earnings : Goods-Producing	37	x		x
CES2000000008	Avg Hourly Earnings : Construction	37	x		x
CES3000000008	Avg Hourly Earnings : Manufacturing	37	x		x
UMCSENTx	Consumer Sentiment Index	15	x		x
MZMSL	MZM Money Stock	42	x	x	x
DTCOLNVHFNM	Consumer Motor Vehicle Loans Outstanding	30	x	x	
DTCTHFNM	Total Consumer Loans and Leases Outstanding	30	x	x	
INVEST	Securities in Bank Credit at All Commercial Banks	30	x	x	
VXOCLSx	Volatility Index	30	x	x	

Sources: FRED-MD.
Predictor variables in the monthly US factor. The lag variable is measured as the average number of days between the first day of the reference month and the publication date. All variables have been transformed to stationarity following the suggestions in McCracken and Ng (2016).

QUARTERLY U.S. DATA Table 7 lists the 43 predictor variables and the outcome variable (real GDP growth). Before further analysis we transform all series to stationarity, following the recommendations of McCracken and Ng. All series are available over the full sample period of 1975q2–2019q2.

QUARTERLY MULTI-COUNTRY DATA Table 8 lists the 35 variables in our data set, comprising GDP growth and 34 potential predictor variables. As indicated in the table, some variables are missing for certain countries, either entirely or because we drop them due to limited sample size. To increase comparability, we do not attempt to find replacement series for each individual country from outside data sources. Even so, most variables are available for at least 12 of the 13 countries, with three exceptions: (i) surveys on consumer sentiment (7 countries), business sentiment (6 countries), or purchasing managers index (1 country); (ii) indices of housing starts (8 countries) or building permits (6 countries); and (iii) stock trading volume (10 countries). We still include these variables in the analysis, as they appear

Table 7: Variables in U.S. data set.[a]

Code	Description
AAASPR	Spread: AAA corporate bond vs. 10-yr govt yield
BAASPR	Spread: BAA corporate bond vs. 10-yr govt yield
CAPUTIL	Capacity utilization
COMMCRB	CRB commodity price index
CONSGOVT	Government consumption
CONSPRIV	Private consumption
CONSSENT	Consumer confidence (Conference Board)
CPAPERSPR	Spread: 3-mth commercial paper vs. 3-mth govt yield
DEBTGOVT	Federal debt, % GDP
DISPINC	Disposable income
DIVYIELD	S&P 500 dividend yield
EARNINGS	Hourly earnings, production and non-supervisory
ECONSENT	Business outlook (Philadelphia Fed)
EMPL	Employment, non-farm
EXCHTRW	Nominal trade-weighted exchange rate index
EXPORT	Exports
FEDFUNDS	Federal funds rate
GDP	GDP
HOURS	Hours worked, non-farm business
HOUSEPERMIT	New housing permits
HOUSEPRICE	All-transactions house price index
HOUSESTART	Housing starts, new privately owned
IMPORT	Imports
INDPRO	Industrial production
INVENTO	Manufacturing and trade inventories
INVESTM	Private investment
LIABCORP	Nonfinancial corporate liabilities
LIABHH	Household and non-profit liabilities
LOANSCORP	Commercial and industrial loans, all commercial banks
LOANSHH	Consumer loans, all commercial banks
NWCORP	Nonfinancial corporate net worth
NWHH	Household and non-profit net worth
ORDERNEW	New manufacturing orders
ORDERUNFIL	Unfilled manufacturing orders
PCEPRICE	Personal consumption expenditures price index
PMI	Purchasing managers index
RETAIL	Retail sales
SHORTSPR	Spread: 3-mth govt yield vs. Fed funds rate
STOCKPRICE	S&P 500 stock price index
STOCKVOL	Stock trading volume
TERMSPR	Spread: 10-yr vs. 3-mth govt yield
ULC	Unit labor cost, non-farm business
UNRATE	Civilian unemployment rate
VXO	S&P 100 implied volatility

Sources: FRED-QD, Global Financial Data, Haver Analytics, and authors' calculations.

[a] Predictor variables and predicted variable (GDP growth) in the U.S. data set.

potentially relevant as timely predictors of growth risk. The most notable absences from our list of predictor variables are capacity utilization, corporate bond spreads, and bank lending rates, as it is unfortunately difficult to find comparable series on these variables going back several decades.

Our data is quarterly and covers the period 1980q1–2018q4. All variables are transformed to approximate stationarity. To create a balanced panel for the analysis, we impute missing data points using a dynamic factor model.[16] The imputation is unlikely to substantially affect the results, as the fraction of missing observations does not exceed 3% for any country. Moreover, no individual time series used in our analysis has more than 30% missing observations. For Germany only, we use the shorter sample 1991q2–2018q4, as the OECD data treats West Germany separately from East Germany before 1991.

[16]For each country separately, we employ a static dynamic factor model with 8 factors. We first estimate factors by principal components on series with no missing data, then impute all missing observations by regressing on the factors. Then we re-estimate the factors on the observed and imputed data, re-impute the initially missing observations, and so on until numerical convergence.

Table 8: Variables in cross-country data set.[a]

Code	Description	Missing countries
CA	Current account, % GDP	
COMMCRB	CRB commodity price index	Not country-specific
CONSGOVT	Government consumption	
CONSPRIV	Private consumption	
CONSSENT	Consumer/household sentiment	CAN CHE DEU FRA ITA SWE
CREDCORPBNK	Credit to firms from banks	
CREDCORP	Credit to firms	
CREDHH	Credit to households	CHE
DIVYIELD	Dividend yield	
ECONSENT	Business/economic sentiment	AUS CAN CHE DEU ESP ITA JPN
EMPL	Employment	
EXCHEFF	Nominal effective exchange rate	
EXCHUSD	Exchange rate versus US$	USA
EXPORT	Real exports	
GDPDEF	GDP deflator	
GDP	GDP	
HOURS	Hours worked	CAN
HOUSEPERMIT	New housing permits	CHE ESP ITA JPN NLD SWE USA
HOUSEPRICE	House price index	
HOUSESTART	Housing starts	CHE DEU FRA ITA NLD
IMPORT	Real imports	
INTRBNKRATE	3-month interbank rate	
INVESTM	Real investment	
MANUF	Manufacturing index	
PMI	Purchasing managers index	Only available for USA
RETAIL	Retail sales index	ESP
STOCKPRICE	Stock price index	
STOCKRV	Daily realized vol of stock price	
STOCKVOL	Stock trading volume	CAN GBR NLD
TERMSPR	Spread: 10- vs. 2-yr govt yield[b]	
TERMTRADE	Terms of trade	
ULC	Unit labor cost index	ESP
UNRATE	Unemployment rate	
VXO	S&P 100 implied volatility	Not country-specific
YIELDSPRUS	Spread: 10-yr govt yield vs. US	USA

Sources: OECD, BIS, Global Financial Data, Haver Analytics, and authors' calculations.

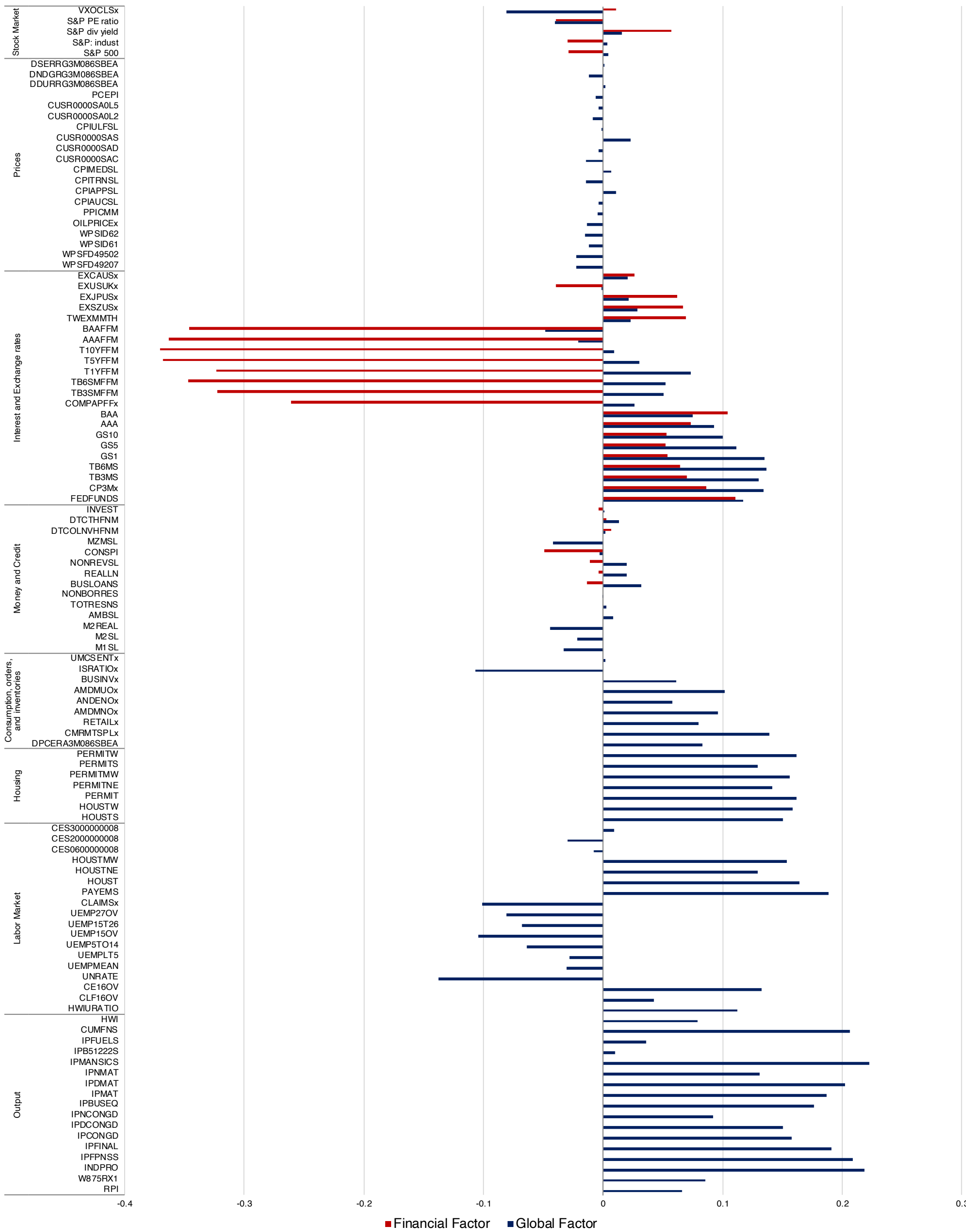
[a] Predictor variables and predicted variable (GDP growth) in the cross-country data set. The third column indicates the countries for which the variable in question is not available.

[b] 1-yr or 3-yr yield used if 2-yr yield not available for sufficiently long sample.

S.C. Factor Loadings

Figure 18 reports the estimated loadings for the monthly factor model with a global and a financial factor. Details of the factor estimation are provided in Appendix S.A.

Figure 18: Loadings of the factor model[a]



Sources: authors' calculations.

[a] The table reports the loadings of the factor model with global and financial factor.

S.D. Dynamic Skew-t Model With Factors as Predictors: Details

Here we provide further results for the skew-t model with factors as explanatory variables in Section III.

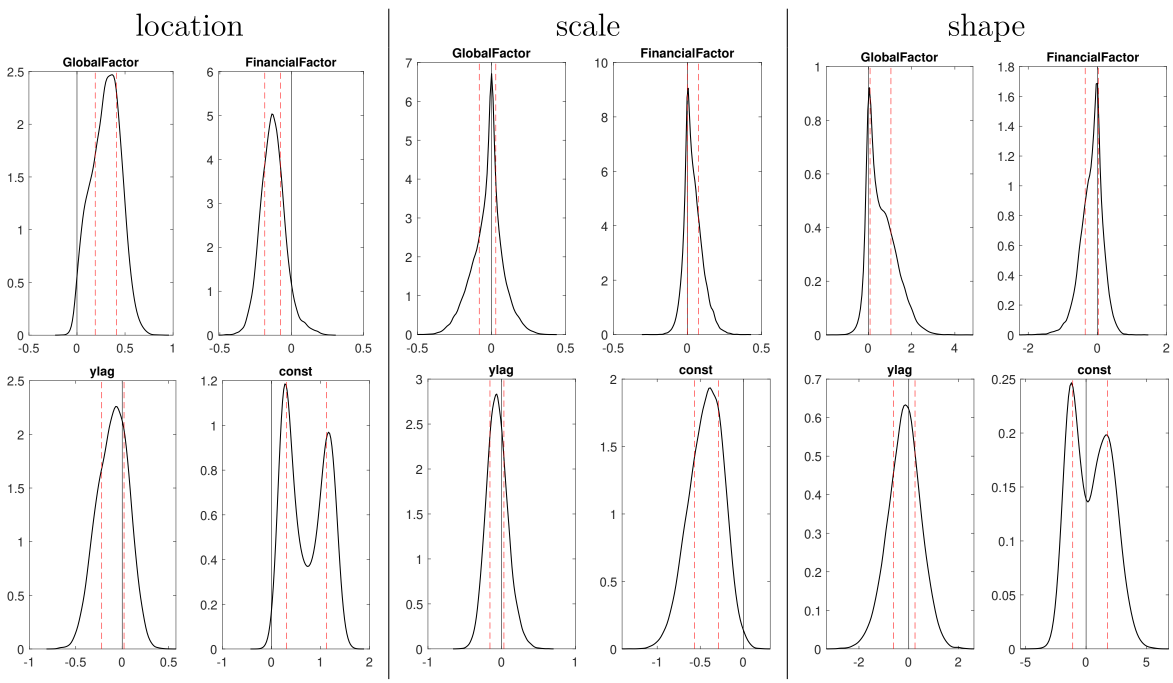
U.S. RESULTS: POSTERIOR DISTRIBUTION OF MODEL COEFFICIENTS We first report the posterior of the underlying model parameters in the U.S. skew-t model with factors as explanatory variables. Figure 19 shows the posterior densities of the location, shape, and scale coefficients on the global factor and on the orthogonalized financial factor. There is only weak evidence that real or financial conditions meaningfully influence the conditional skewness of U.S. GDP growth. The 50% posterior credible intervals for the shape coefficients β_α either contain 0 or very nearly contain 0 for both factors. The posterior probability that the skewness coefficient on the global factor exceeds 0.05 is 40.6%, while the probability that it is less than -0.05 is 29.5%. The corresponding probabilities for the financial factor are 12.7% and 29.4%. Thus, the data is neither able to decisively pin down the magnitudes nor the signs of the effects that the factors have on the conditional skewness. The same is true of the coefficient on lagged GDP growth.

The GDP growth distribution seems to feature mildly heavy tails. There is, however, substantial posterior uncertainty about the degrees of freedom parameter ν, with a posterior interquartile range of ν is $[6.5, 15.3]$.

Figure 19 also depicts a pronounced bimodality in the marginal posteriors for the intercepts in the scale and shape parameter equations. As mentioned in Section III, this bimodality is an artifact of the years 1975–1979. Figure 20 shows the posterior densities of the coefficients in the U.S. two-factor dynamic skew-t model estimated on the shorter 1980q1–2019q2 subsample. The data is the same as in Section III (including the factor estimates), but we only provide the post-1980 data to the posterior sampler. It is evident that the post-1980 period does not exhibit the bimodality in the posterior distribution for the intercepts that we found on the full 1975q2–2019q2 sample. Instead the evidence here mostly points towards negative unconditional skewness, consistent with Section IV.C.

U.S. RESULTS: TIME-VARIATION OF SKEW-T PARAMETERS Figure 21 shows the evolution over time of the four parameters μ_t, σ_t, α_t, and ν of the dynamic skew-t model. Relative to the posterior uncertainty, there is little discernible time-variation in any of these except for μ_t. This finding is consistent with the results on time-variation of the moments of the GDP distribution reported in Section III.

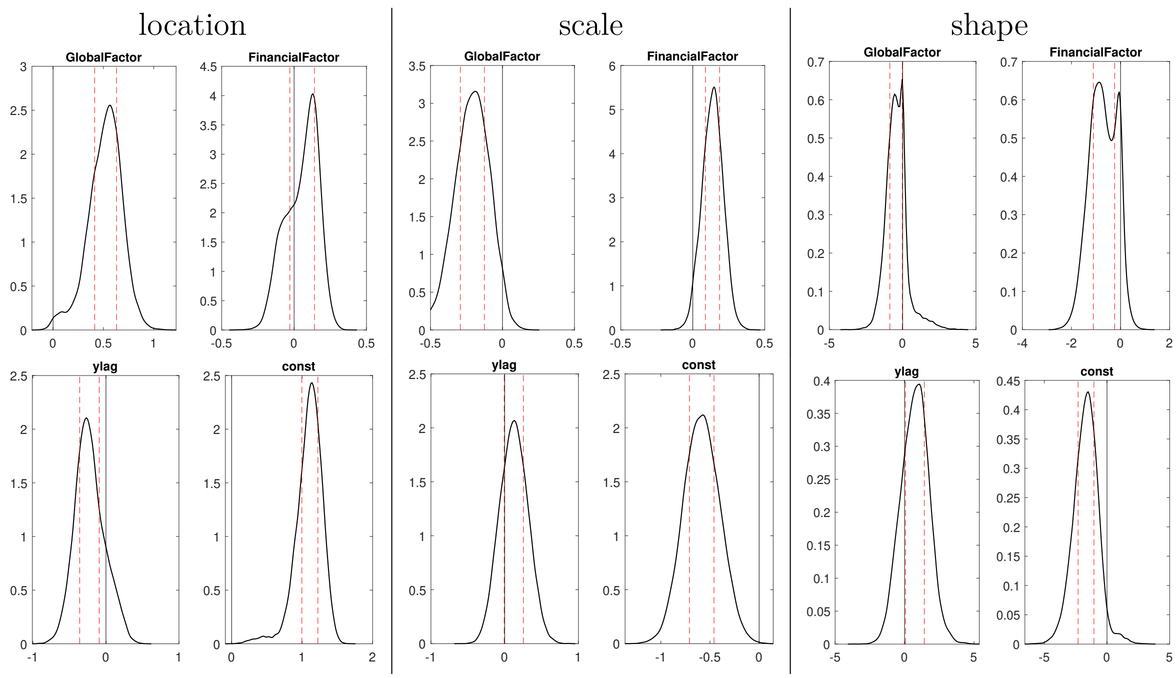
Figure 19: Two-factor dynamic skew-t model: Posterior on 1975–2019 sample.[a]



Sources: FRED-QD, FRED-MD, and authors' calculations.

[a] Posterior densities of coefficients on global factor, financial factor, lagged GDP growth (ylag), and intercept (const) in the equations for the location parameter μ_t (left panel), scale parameter $\log \sigma_t$ (middle panel), and shape parameter α_t (right panel). Vertical red dashed lines indicate posterior interquartile ranges.

Figure 20: Two-factor dynamic skew-t model: Posterior on 1980–2019 subsample.[a]



Sources: FRED-QD, FRED-MD, and authors' calculations.

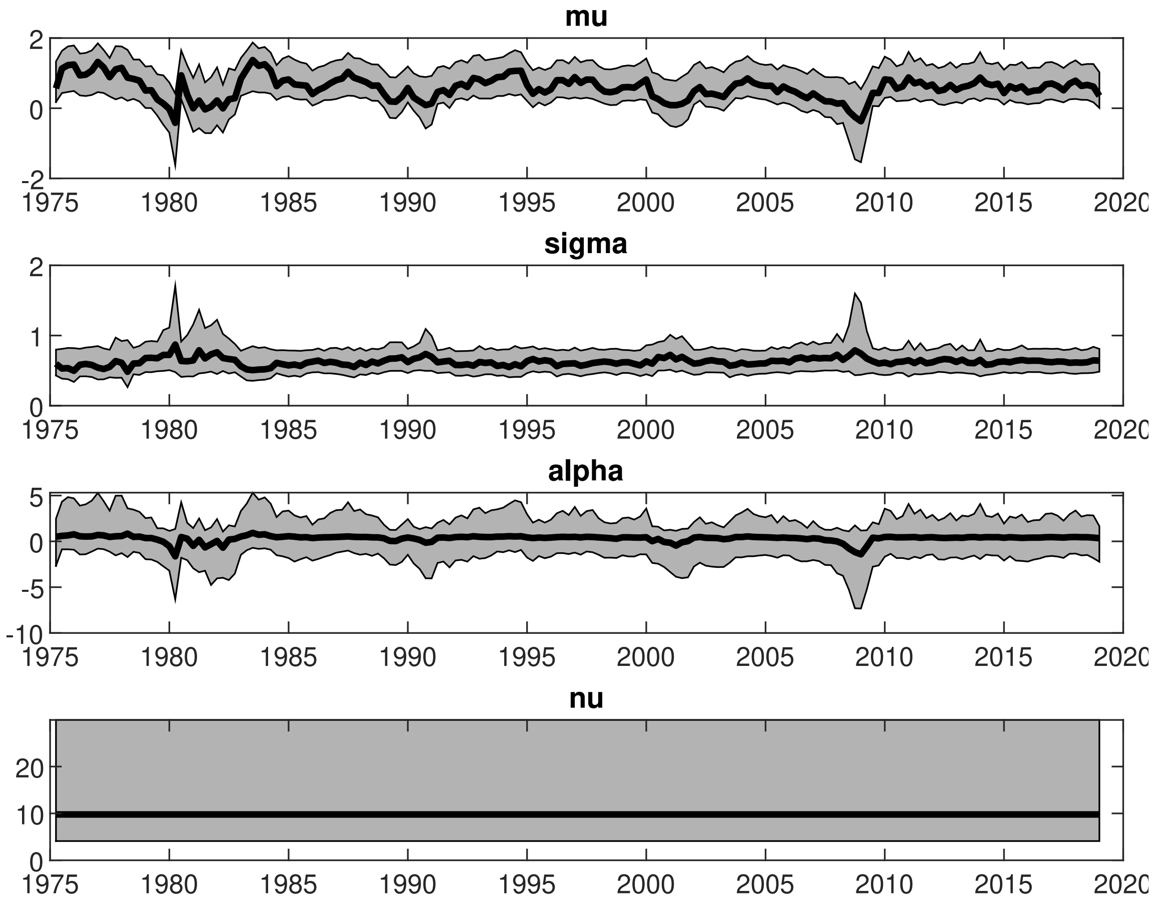
[a] Posterior densities of coefficients on global factor, financial factor, lagged GDP growth (ylag), and intercept (const) in the equations for the location parameter (7) (left panel), scale parameter (8) (middle panel), and shape parameter (9) (right panel). Vertical red dashed lines indicate posterior interquartile ranges.

U.S. RESULTS: FOUR-QUARTER-AHEAD RECESSION PROBABILITY AND EXPECTED SHORTFALL Figure 22 shows the four-quarter-ahead recession probability and expected shortfall, to complement the one-quarter-ahead results reported in Figure 13. The four-quarter growth is cumulative, so the first panel, say, reports the probability that the cumulative growth over the following four quarters is negative. Note that the time-variation of the conditional probability in the second panel (the probability of four-quarter cumulative growth falling below the conditional mean of next-quarter annualized growth) is due to the fact that, in a recession, some mean reversion in growth is expected.

U.S. RESULTS: RELATIVE PREDICTIVE ROLE OF GLOBAL AND FINANCIAL FACTORS Figure 23 shows the time-varying moments of the one-quarter-ahead forecast distribution if we set the *global* factor $x_{1,t}$ equal to 0 when producing every forecast. Figure 24 shows the corresponding figure if we instead set the *financial* factor $x_{2,t}$ equal to 0 when producing every forecast. Notice that we use the precise same estimated model as in Section III.B, we only change the conditioning variables x_t used to produce the forecasts at each point in time and for each posterior parameter draw. As is clear from these figures, zeroing out the financial factor changes very little relative to the baseline in Figure 11 (which conditioned on the actual data values of both factors), whereas zeroing out the global factor has a noticeable effect on the conditional mean during the Great Recession period. Neither factor has a substantial effect on the other moments, although the posterior median for conditional skewness does change somewhat around 1980 and 2008 when we zero out the global factor (still, the posterior uncertainty about this moment is high).

CROSS-COUNTRY RESULTS: TIME-VARIATION OF MOMENTS Figures 25 to 27 show the time-variation in the moments of the GDP growth distribution in Australia, Italy, and Japan, respectively. As was the case for the U.S. results discussed in Section III, there is little evidence of predictable time-variation in the standard deviation, skewness, or kurtosis. Results for the other OECD countries in our data set are qualitatively similar.

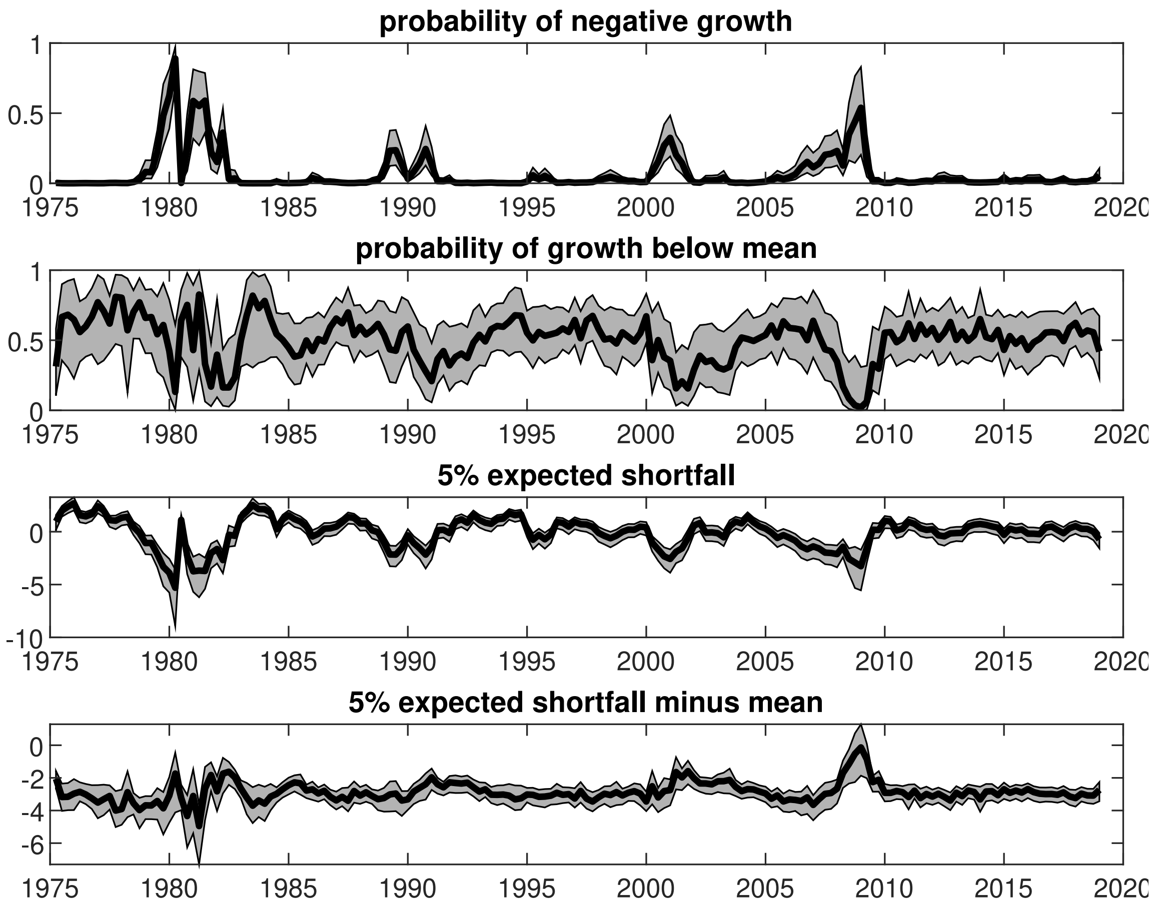
Figure 21: U.S. results: Time-varying skew-t parameters.[a]



Sources: FRED-QD, FRED-MD, and authors' calculations.

[a] Time-varying parameters of the skew-t forecast distribution for GDP growth. The thick line is the posterior median (across parameter draws) at each point in time. The gray shaded band is the pointwise 90% posterior credible band (across parameter draws) at each point in time. Recall that the parameter ν is assumed constant over time. The time axis shows the quarter in which the forecast is made.

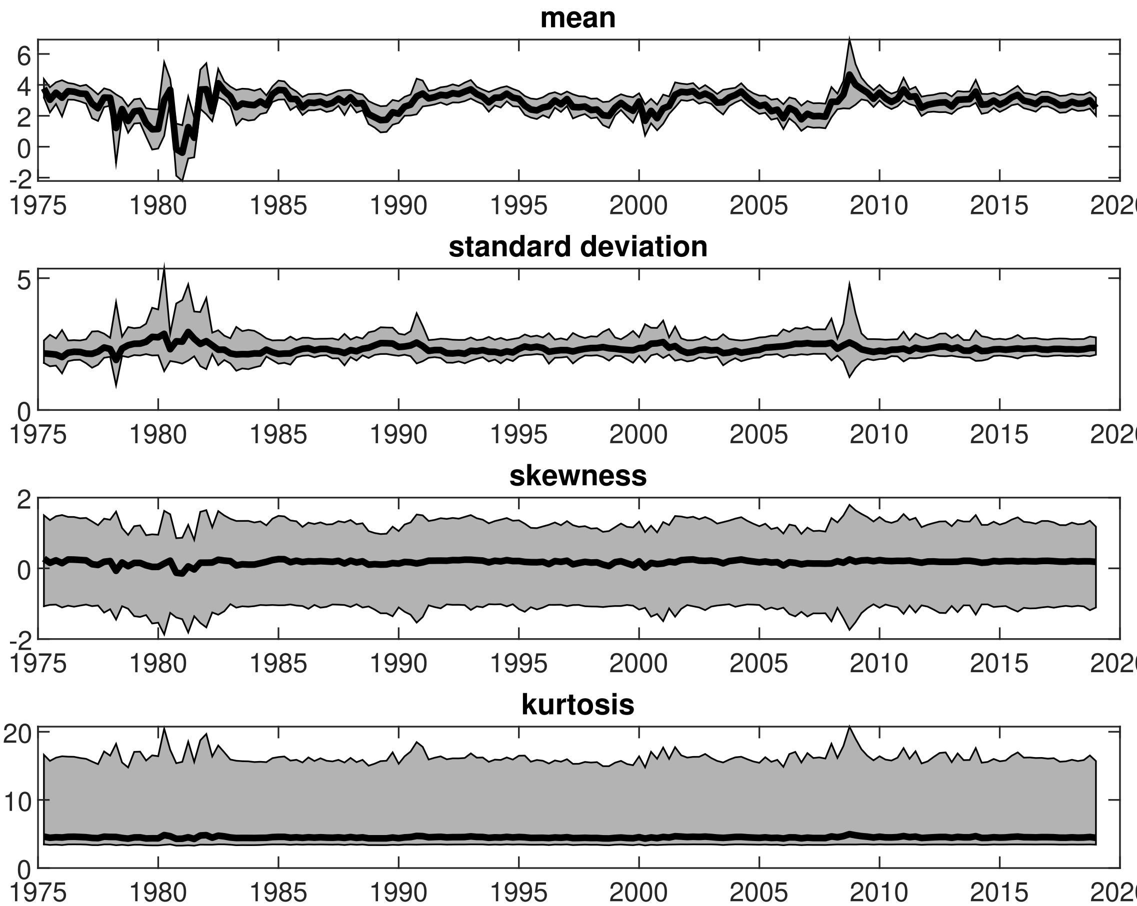
Figure 22: U.S. factor model: Recession probability and expected shortfall, four quarters ahead.[a]



Sources: FRED-QD, FRED-MD, and authors' calculations.

[a] Probability of negative growth, probability of growth below the current conditional mean, expected shortfall, and expected shortfall minus current conditional mean for the four-quarter-ahead conditional distribution of cumulative GDP growth between time t and $t+4$. "Current conditional mean" refers to the conditional expectation of *next-quarter* GDP growth (annualized). The thick line is the posterior median (across parameter draws) at each point in time. The gray shaded band is the pointwise 90% posterior credible band (across parameter draws) at each point in time. The time axis shows the quarter in which the forecast is made.

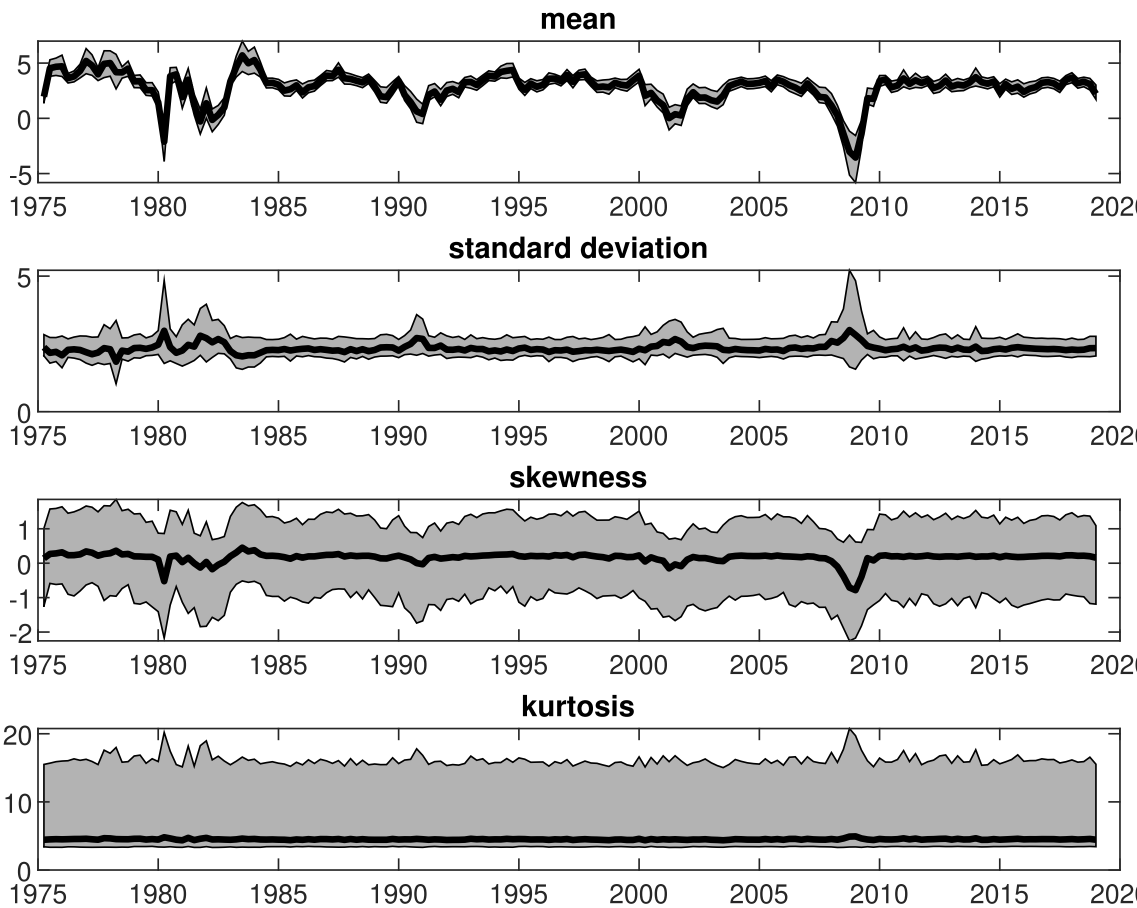
Figure 23: U.S. factor model: Time-varying moments, one quarter ahead, zeroing out the global factor.[a]



Sources: FRED-QD, FRED-MD, and authors' calculations.

[a] Time-varying moments of the one-quarter-ahead forecast distribution of GDP growth (annualized), but setting the global factor equal to 0 when computing forecasts. The thick line is the posterior median (across parameter draws) at each point in time. The gray shaded band is the pointwise 90% posterior credible band (across parameter draws) at each point in time. The time axis shows the quarter in which the forecast is made.

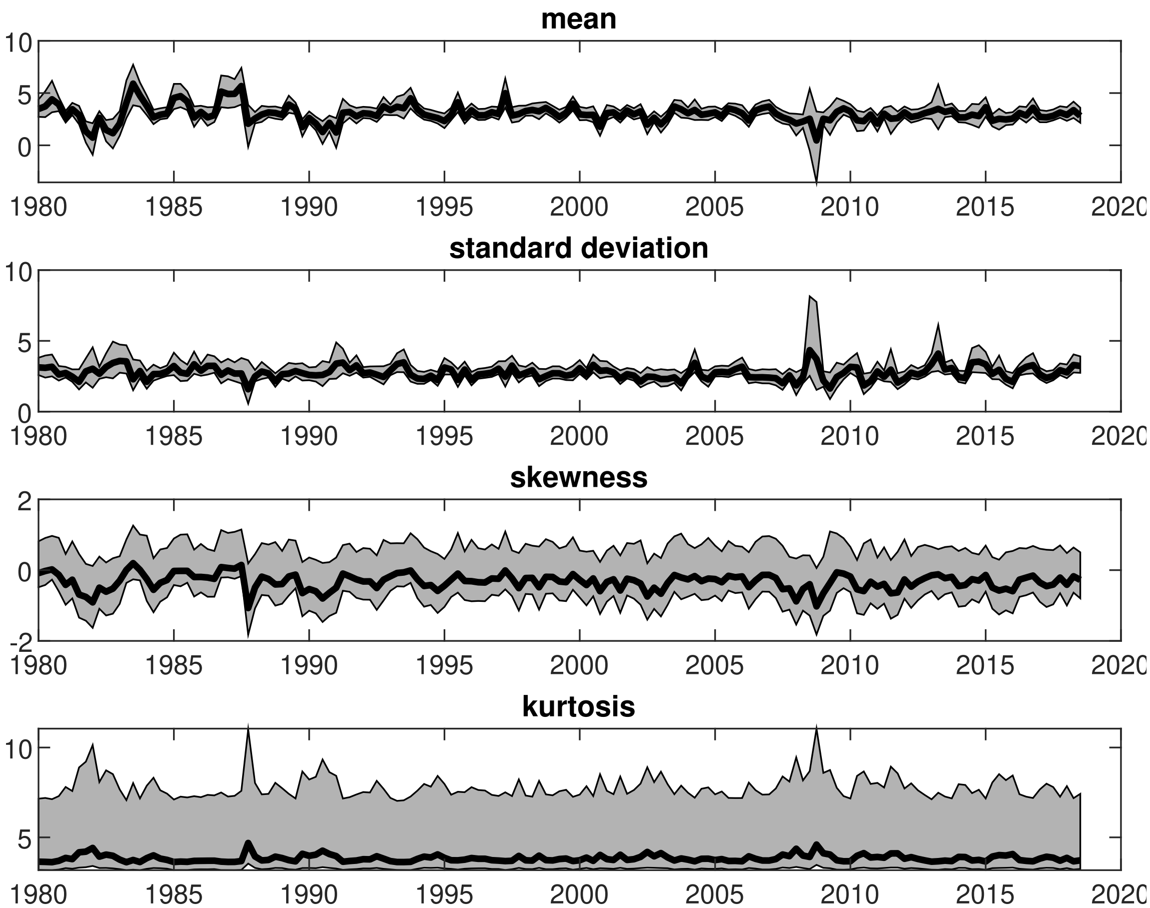
Figure 24: U.S. factor model: Time-varying moments, one quarter ahead, zeroing out the financial factor.[a]



Sources: FRED-QD, FRED-MD, and authors' calculations.

[a] Time-varying moments of the one-quarter-ahead forecast distribution of GDP growth (annualized), but setting the financial factor equal to 0 when computing forecasts. The thick line is the posterior median (across parameter draws) at each point in time. The gray shaded band is the pointwise 90% posterior credible band (across parameter draws) at each point in time. The time axis shows the quarter in which the forecast is made.

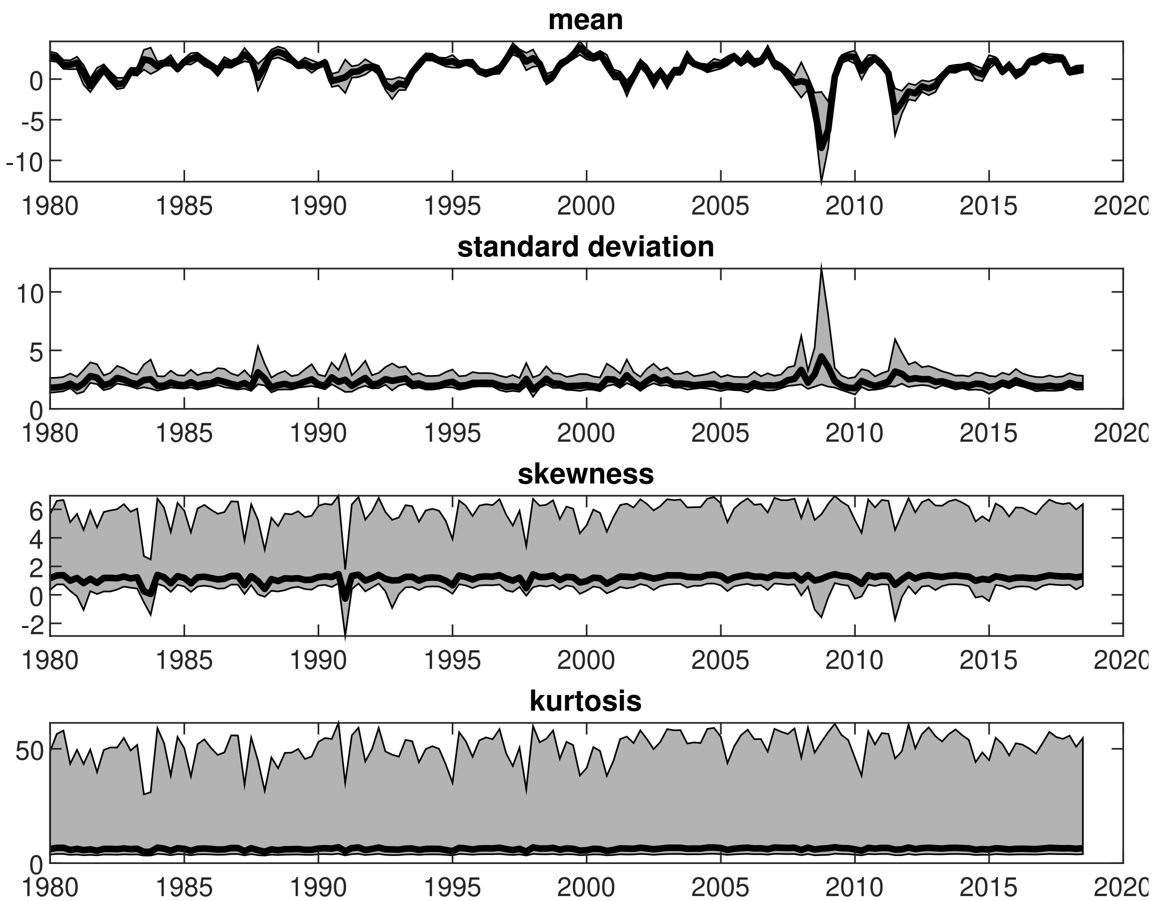
Figure 25: Factor model, Australia: Time-varying moments, one quarter ahead.[a]



Sources: OECD, BIS, Global Financial Data, Haver Analytics, and authors' calculations.

[a] Time-varying moments of the one-quarter-ahead forecast distribution of GDP growth (annualized). The thick line is the posterior median (across parameter draws) at each point in time. The gray shaded band is the pointwise 90% posterior credible band (across parameter draws) at each point in time. The time axis shows the quarter in which the forecast is made.

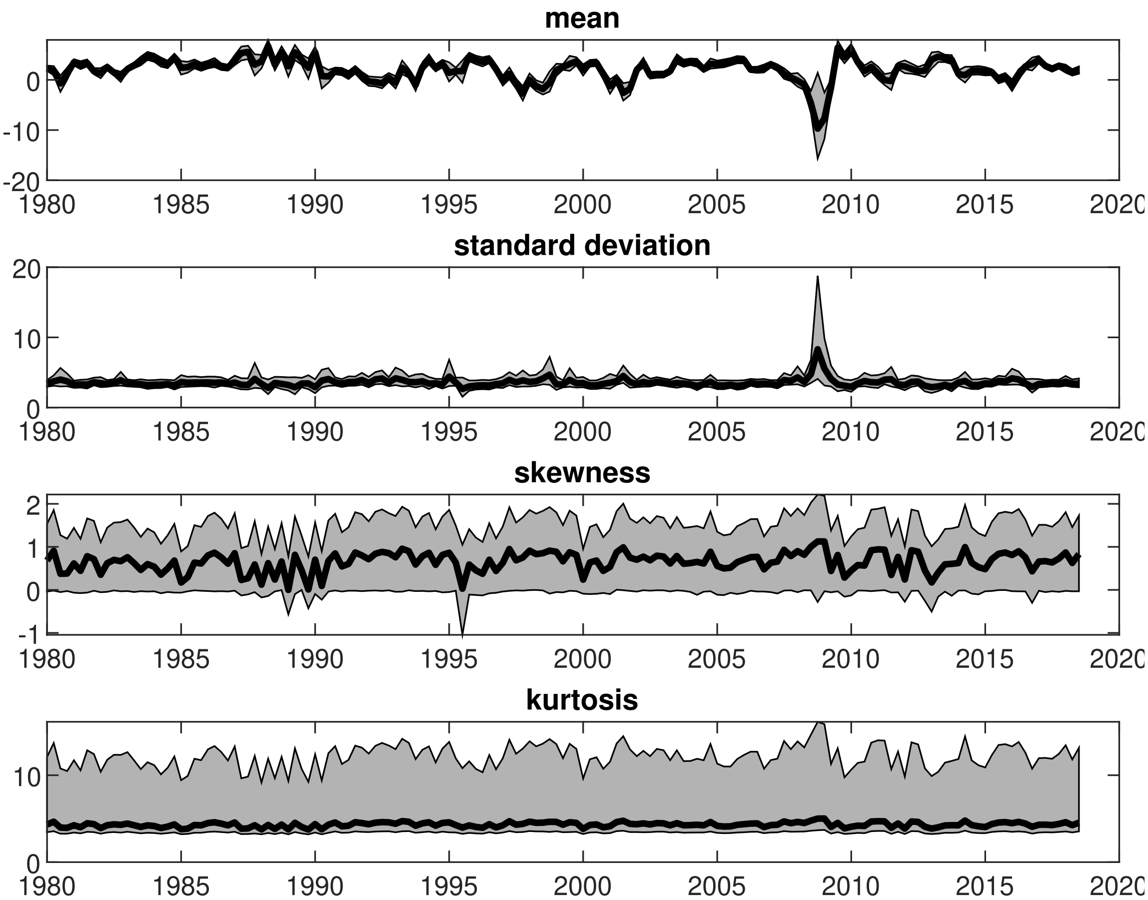
Figure 26: Factor model, Italy: Time-varying moments, one quarter ahead.[a]



Sources: OECD, BIS, Global Financial Data, Haver Analytics, and authors' calculations.

[a] Time-varying moments of the one-quarter-ahead forecast distribution of GDP growth (annualized). The thick line is the posterior median (across parameter draws) at each point in time. The gray shaded band is the pointwise 90% posterior credible band (across parameter draws) at each point in time. The time axis shows the quarter in which the forecast is made.

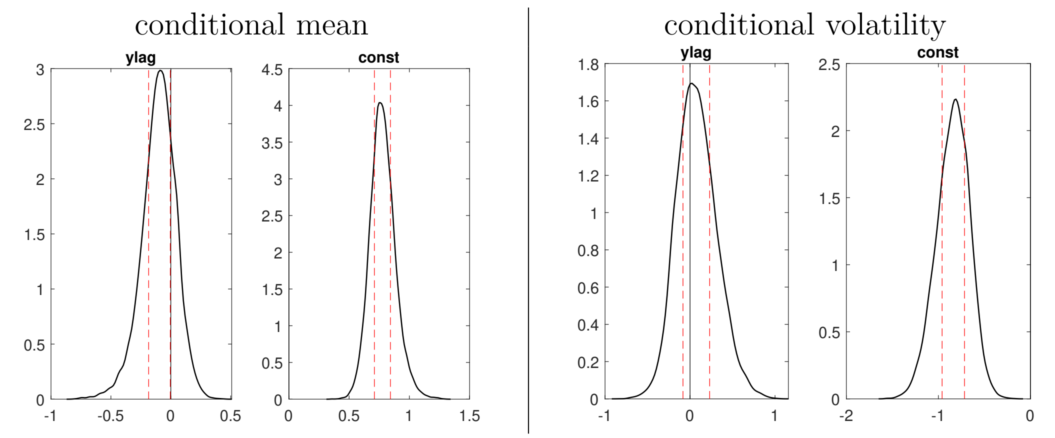
Figure 27: Factor model, Japan: Time-varying moments, one quarter ahead.[a]



Sources: OECD, BIS, Global Financial Data, Haver Analytics, and authors' calculations.

[a] Time-varying moments of the one-quarter-ahead forecast distribution of GDP growth (annualized). The thick line is the posterior median (across parameter draws) at each point in time. The gray shaded band is the pointwise 90% posterior credible band (across parameter draws) at each point in time. The time axis shows the quarter in which the forecast is made.

Figure 28: U.S. conditional heteroskedasticity model: Posterior of unpenalized coefficients.[a]



Sources: FRED-QD, Global Financial Data, Haver Analytics, and authors' calculations.

[a] Posterior densities of coefficients on lagged growth (ylag) and intercept (const) in the conditional mean equation (7) (left panel) and conditional volatility equation (8) (right panel). Vertical red dashed lines indicate posterior interquartile ranges.

S.E. Variable Selection: Details

Here we provide further empirical results for the variable selection exercises discussed in Section IV, and we define the Total Variation Distance measure of skewness.

U.S. CONDITIONAL HETEROSKEDASTICITY MODEL: POSTERIOR FOR OTHER PARAMETERS Figure 28 shows the posterior densities for lagged GDP growth and the intercept in the conditional mean and volatility equations. GDP growth exhibits slight mean reversion, holding constant all other predictor variables. There is no strong evidence that lagged GDP growth is an important predictor of volatility, conditional on the other predictors.

QUANTIFYING THE SKEWNESS OF THE SKEW-NORMAL DISTRIBUTION The total variation distance (TVD) between two absolutely continuous random variables X_1 and X_2 with densities $p_1(x)$ and $p_2(x)$, respectively, is given by

$$TVD(X_1, X_2) = \sup_{\mathcal{A}} |P(X_1 \in \mathcal{A}) - P(X_2 \in \mathcal{A})| = \frac{1}{2} \int |p_1(x) - p_2(x)|\, dx,$$

where the supremum is taken over all Borel sets. By definition, the TVD lies between 0 and 1, where 0 means total agreement and 1 means total disagreement.

We quantify the skewness of the skew-t distribution by computing the TVD between a skew-normal random variable α and a standard normal distribution (thus, we effectively let the degrees of freedom $\nu \to \infty$, in order to focus on α). Let U denote a standard skew-normal distributed random variable with density (5) and shape parameter α, and let X denote a standard normal random variable. Then Dette et al. (2018) show that[17]

$$TVD(U, X) = \frac{\arctan(|\alpha|)}{\pi}.$$

Let $TVD(\alpha)$ denote the above expression as a function of the skewness parameter α. With α_t defined as in (9), the Average Partial Effect on the TVD of the j-th predictor variable $x_{j,t}$ is given by

$$APETVD_j = \frac{1}{T}\sum_{t=1}^{T} \frac{\partial TVD(\alpha_t)}{\partial x_{j,t}} = \frac{1}{T}\sum_{t=1}^{T} \frac{\operatorname{sign}(\alpha_t)}{\pi(1+\alpha_t^2)}\beta_{\alpha,j}.$$

[17]Alternative derivation: Define $Z \sim N(0,1)$ independent of X. Then

$$\begin{aligned} TVD(U, X) &= \frac{1}{2}\int |2\Phi(\alpha x) - 1|\varphi(x)\, dx = \frac{1}{2}\int P(-|\alpha x| \le Z \le |\alpha x|)\varphi(x)\, dx \\ &= \frac{1}{2}E\left[P(|Z| \le |\alpha X| \mid X)\right] = \frac{1}{2}P(|Z/X| \le |\alpha|). \end{aligned}$$

Finally, use the fact that $Z/X \sim \text{Cauchy}(0,1)$ with distribution function $\frac{1}{\pi}\arctan(x) + \frac{1}{2}$.