Documentation of R Code and Data for "Measuring the Natural Rate of Interest: International Trends are Determinants"*

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February 6, 2017

This note documents the R code used for the estimation of the natural rate of interest, potential GDP, and its trend growth rate for the United States, Canada, the Euro Area, and the UK, presented in "Measuring the Natural Rate of Interest: International Trends and Determinants" (Holston, Laubach, Williams 2017; henceforth HLW). It also catalogues steps to download and prepare data used in the estimation. The code documented here reflects the final December 2016 version of the paper, with an estimation sample of 1961:Q1 through 2016:Q3.

1 Code Layout and Directory Structure

There is one main R file, *run.hlw.R*, which does the following:

1. Prepares data to be used in the HLW estimation;
2. Defines the sample period, constraints, and variables to be used throughout the estimation;
3. Runs the three-stage HLW estimation for each economy;
4. Saves output.

This file calls multiple R functions and files, each of which are described in this guide. To run the code without modification, use the following structure:

1. A subdirectory titled "rawData" should contain all data described below, downloaded and saved as CSV files;

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2. An empty subdirectory titled "inputData" should be created and will populate with prepared data;
3. An empty subdirectory titled "output" should be created and will populate with estimates and model output.
4. Optional: create a subdirectory titled "Rpackages" to use as the library location for downloaded packages.

For the US, raw data will be downloaded automatically. For all other economies, the user must manually download the data and save it in the rawData subdirectory. For reference, we are using R Version 3.2.2 at the time of release.

2 Accompanying Jupyter Notebook

In addition to providing our code, we make public a Jupyter notebook. The Jupyter Notebook is a web application that enables users to combine text, code, and output in one file. Our notebook contains the documentation in this guide as well as all of our R programs, so it can be used as a standalone reference. Running the notebook is equivalent to running our main R file, *run.hlw.R*. Instructions for installing the Jupyter Notebook application and a quickstart guide to running an existing notebook can be found in the documentation at jupyter.org.

3 Raw Data

A description of the data we require can be found in the Data Appendix to the paper. For each economy, we require data for real GDP, inflation, and the short-term nominal interest rate, as well as a procedure to compute inflation expectations to calculate the ex ante real short-term interest rate. The inflation measure is the annualized quarterly growth rate of the specified consumer price series. Interest rates are expressed on a 365-day annualized basis.

3.1 United States

We use real GDP and core PCE data published by the Bureau of Economic Analysis. The short-term interest rate is the annualized nominal federal funds rate, available from the Board of Governors. Because the federal funds rate frequently fell below the discount rate prior to 1965, we use the Federal Reserve Bank of New York's discount rate, part of the IMF's International Financial Statistics Yearbooks (IFS), prior to 1965. All US data can be downloaded from the St. Louis

Fed's Federal Reserve Economic Data (FRED) website, and will be automatically downloaded in *prepare.rstar.data.us.R.*

FRED Mnemonics:

- Real GDP: GDPC1
- Core PCE: PCEPILFE
- Federal Funds Rate: FEDFUNDS
- FRBNY Discount Rate: INTDSRUSM193N

3.2 Canada

We use real GDP from the IMF's IFS. The short-term nominal interest rate is the Bank of Canada's target for the overnight rate, taken as the end-of-period value for each month and aggregated to quarterly frequency. Prior to May 2001, we use the Bank of Canada's bank rate as the short-term interest rate. We use the BoC's core consumer price index to construct our inflation series. Prior to 1984, we use CPI containing all items. With the exception of GDP, all data is from Statistics Canada.

Mnemonics:

- Real GDP: IFS series "Gross Domestic Product, Real, Seasonally adjusted, Index"
- Core CPI: v41690926 (Table 326-0022); Source: Statistics Canada
- CPI: v41690914 (Table 326-0022); v41690973 (Table 326-0020); Source: Statistics Canada
- Bank Rate: v122530 (Table 176-0043); Source: Statistics Canada
- Target Rate: v39079 (Table 176-0048); Source: Statistics Canada

3.3 Euro Area

All data is from the ECB's Area-Wide Model, which is available from the Euro Area Business Cycle Network (Fagan et al. 2001). We use the core price index beginning in 1988 and the total price index prior; because data availability is longer for the non-seasonally adjusted series, we use those and seasonally adjust them. The nominal short-term interest rate is the three-month rate.

Mnemonics:

- Real GDP: YER
- Price Index: HICP (not seasonally adjusted)
- Core Price Index: HEX
- Nominal Short-term Rate: STN

Since the AWM is released annually, we update each series using data from the ECB's Statistical Data Warehouse. For example, at the time of publication we used AWM data through 2015:Q4 and updated the series using ECB data for the first three quarters of 2016.

ECB Statistical Data Warehouse Mnemonics:

- Real GDP: MNA.Q.Y.I8.W2.S1.S1.B.B1GQ._Z._Z._Z.EUR.LR.N
- Core Price Index: ICP.M.U2.N.XE0000.4.INX
- Nominal Short-term Rate: FM.Q.U2.EUR.RT.MM.EURIBOR3MD_.HSTA

3.4 United Kingdom

GDP data are taken from the Office of National Statistics (ONS). Our inflation measure is core CPI; prior to 1970 we use all-items CPI; both are from the OECD. The short-term interest rate is the Bank of England's Official Bank Rate.

Mnemonics:

- Real GDP: Series ABMI; Source: ONS
- CPI: Series "MEI Prices: Consumer prices - all items"; Source: OECD
- Core CPI: Series "MEI Prices: Consumer prices - all items non-food, non-energy"; Source: OECD
- Nominal Short-term Rate: "Official Bank Rate History"; Source: Bank of England Statistical Interactive Database

4 Basic Functions used Throughout HLW Programs

In the accompanying set of code, these functions are stored in *utilities.R*.

Function: *shiftQuarter*
Description: This function takes in a (year, quarter) date in time series format and a shift number, and returns the (year, quarter) date corresponding to the shift. Positive values of shift produce leads and negative values of shift produce lags. For example, entering 2014q1 with a shift of -1 would return 2013q4. Entering 2014q1 with a shift of 1 would return 2014q2. In each case, the first argument of the function must be entered as a two-element vector, where the first element corresponds to the year and the second element corresponds to the quarter. For example, 2014q1 must be entered as "c(2014, 1)".

Function: *shiftMonth*
Description: This function takes in a (year, month) date in time series format and a shift number, and returns the (year, month) date corresponding to the shift. Positive values of shift produce leads and negative values of shift produce lags. For example, entering 2014m1 with a shift of

-1 would return 2013m12. Entering 2014m1 with a shift of 1 would return 2014m2. In each case, the first argument of the function must be entered as a two-element vector, where the first element corresponds to the year and the second element corresponds to the month. This function is analogous to shiftQuarter().

Function: *getFRED*
Description: This function downloads data from FRED. It returns quarterly data. User must provide the FRED url.

Function: *splice*
Description: This function splices two series, with the series s2 beginning at splice.date and extended back using the growth rate at the splice.date times series s1. The freq argument accepts two values - 'quarterly' and 'monthly' - but it could be modified to take more.

Function: *gradient*
Description: This function computes the gradient of a function f given a vector input x.

5 R Packages

The "tis" package is used to manage time series data. The "mFilter" packages contains the hpfilter() function. We use the "nloptr" package for optimization. The "seasonal" package is an interface to the US Census Bureau software X-13-ARIMA-SEATS. It uses X-13 binaries which are installed by the dependent "x13binary" packages; users may need to manually specify the path to the binaries.

6 Data Preparation

We use four R scripts, one for each economy titled *prepare.rstar.data.XX.R*, to prepare the data to be used in the estimation. Inputs are the raw data described above, and files are described in this section. In the accompanying set of code, they are sourced in *run.hlw.R*. In each file, the user must manually specify the start and end dates for the prepared data to be used in the estimation. The variable *data.start* should be 4 quarters prior to the estimation start date.

Each file does the following, in addition to country-specific steps described below:

- Takes the log of real GDP.
- Creates an annualized (inflation and) core inflation series from the price index(es).
- Constructs a measure of inflation expectations as a four-quarter moving average of past inflation.
- Expresses interest rate data on a 365-day basis.

6.1 *prepare.rstar.data.us.R*

This file compiles and prepares the data used in HLW for the US. Data will be imported using the getFRED() function in *utilities.R*. For this connection to work, the user must have the *wget* command line utility. Alternatively, one can manually download the data from FRED.

In addition to the steps above, this file splices FRBNY discount rates with the federal funds rate in 1965:Q1. Output is saved to *inputData/rstar.data.us.csv*.

6.2 *prepare.rstar.data.ca.R*

This file compiles and prepares the data used in HLW for Canada. Manually specify the start dates for the two CPI series and the core CPI series. Code is set up such that each CANSIM series is stored in a separate CSV with time as rows.

Import GDP Data

1. Download data from the International Financial Statistics (IFS) Database on the IMF website: Series: "Gross Domestic Product, Real, Seasonally adjusted, Index"
2. Save data as a CSV and specify the file name in gdp.file.

Import Bank Rate Data

1. Download data from the Statistics Canada (CANSIM) website: Series: v122530.
2. Save data as a CSV and specify the file name in bank.rate.file.

Import Target Rate Data

1. Download data from the Statistics Canada (CANSIM) website: Series: v39079.
2. Save data as a CSV and specify the file name in target.rate.file.
3. Manually specify start date of downloaded target rate data.

Import Core CPI and CPI Data

1. Download data from the Statistics Canada (CANSIM) website: Series: v41690926, v41690914, v41690973 (core CPI, CPI, and CPI to be used for backcasting, respectively).
   - Series v41690914 begins in Jan. 1992 and will be used as the CPI series thereafter.
   - Growth rates from series v41690973 are used to extend the CPI series back to 1959.
   - Series v41690914 and v41690926 are already seasonally adjusted but v41690973 is not, so we seasonally adjust it.
2. Save data as a CSV and specify the file name in core.cpi.file, cpi.file, cpi.back.file
3. Manually specify start dates for each series.

Additional steps to prepare data:

- Creates an all-items price index using series v41690914 from Jan. 1992 to present. Uses growth rates of series v41690973 to extend the series back to Jan. 1959.
- Creates annualized inflation and core inflation series using the spliced CPI price index and core CPI price index, respectively.
- Our inflation measure uses core inflation beginning in 1984Q2 and all-items inflation prior.
- Creates an interest rate series using the target rate beginning in May 2001 and the bank rate prior, and aggregates this series to quarterly frequency.

Output is saved to *inputData/rstar.data.ca.csv*.

6.3 *prepare.rstar.data.ea.R*

This file compiles and prepares the data used in HLW for the Euro Area. Note that in HLW the estimation sample begins later for the Euro Area than for other economies due to data availability. Manually specify the start date for the core CPI series (Mnemonic: HEX).

Import Area Wide Model Data

1. Download data from the Euro Area Business Cycle Network website.
2. Save data as a CSV and specify the file name in awm.file.

Import ECB Data (for extending series past AWM end date):

1. Download data from the ECB's Statistical Data Warehouse (see Sec. 3.3 for mnemonics).
2. Save data as CSV files and specify names in variable.file.
3. Ensure data is sorted with dates ascending.
4. Specify start dates and splice dates where applicable.
5. Splice GDP and interest series.

We seasonally adjust the non-seasonally-adjusted CPI and core CPI series (HICP and HEX) rather than using the provided seasonally adjusted series (HICPSA and HEXSA) because the latter series are only available for part of the sample. Our inflation measure uses core inflation beginning in 1988Q1 and all-items inflation prior. Output is saved to *inputData/rstar.data.ea.csv*.

6.4 *prepare.rstar.data.uk.R*

This file compiles and prepares the data used in HLW for the UK. Manually specify the start date for the core CPI series.

Import GDP Data

1. Download data from the ONS website:
   Series: ABMI: "Gross Domestic Product: chained volume measures: Seasonally adjusted

(Millions of pounds).

2. Save data as a CSV and specify the file name in gdp.file.

Import Core CPI and CPI Data

1. Download data from the OECD website:

   Source: Consumer Prices (MEI); Series: Consumer prices - all items; Consumer prices - all items non-food, non-energy.
2. Save both data series in one CSV and specify the file name in cpi.file.
3. Manually specify start dates for each series.

Import Bank Rate Data

1. Download data from the BOE website: Bank of England Statistical Interactive Database - official Bank Rate history.
2. Manually edit the historical since 1694 series.
   (a) Include only 1960 to present, but enter 11/20/1958 change as 1/1/1960 so series has starting value.
   (b) Populate rows with years. Remove sub-headers and rows of spaces:
       In Excel, select area and choose F5, "Special", "Blanks", Delete.
   (c) Code assume the setup has 4 columns: "year","day","month","rate".

Converts series of changes in bank rate to a daily rate series:

- Variable bank.rate.data contains changes in the BOE bank rate (dates are included only if a change occurs).
- Variable bank.rate.changes.d is a daily time series with NA values inserted for dates on which a change does not occur.
- Variable bank.rate.d is a daily series created by carrying forward the last value when a value is NA using the na.locf() function in the 'xts' package.

Additional steps to prepare data:

- Seasonally adjusts the CPI and core CPI series.
- Our inflation measure uses core inflation beginning in 1970Q2 and all-items inflation prior.
- Aggregates the bank rate series to quarterly frequency and expresses interest rates on a 365-day basis.

Output is saved to *inputData/rstar.data.uk.csv*.

7 Estimation

The results reported in Holston, Laubach, and Williams (2017) are based on our estimation method described in Section 2.2 of the paper. The estimation proceeds in sequential steps through three stages, each of which is implemented in an R program. These and all other R programs are described in this section.

7.1 Main Estimation Program: *run.hlw.estimation.R*

The function *run.hlw.estimation.R* is called by *run.hlw.R* once for each economy. It takes as inputs the key variables for the given economy: log output, inflation, and the real and nominal short-term interest rates, as well as the specified constraints on a_r and b_y. It calls the programs *rstar.stageX.R* to run the three stages of the HLW estimation. Additionally, it calls the programs *median.unbiased.estimator.stageX.R* to obtain the signal-to-noise ratios λ_g and λ_z.

The programs *unpack.parameters.stageX.R* set up coefficient matrices for the corresponding state-space models for the given parameter vectors. In all stages, we impose the constraint $b_y \geq 0.025$. In stages 2 and 3, we impose $a_r \leq -0.0025$. These constraints are labeled as a3.constraint and b2.constraint, respectively, in the code.

7.2 The Stage 1, 2, and 3 State-Space Models

This sections presents the state-space models, and the next section documents the corresponding R programs. Notation matches that of Hamilton (1994) and is also used in the R programs. All of the state-space models can be cast in the form:

$$\mathbf{y}_t = \mathbf{A}' \cdot \mathbf{x}_t + \mathbf{H}' \cdot \xi_t + \mathbf{v}_t \tag{1}$$

$$\xi_t = \mathbf{F} \cdot \xi_{t-1} + \epsilon_t \tag{2}$$

Here, $\mathbf{y}_t$ is a vector of contemporaneous endogenous variables, while $\mathbf{x}_t$ is a vector of exogenous and lagged exogenous variables. ξ_t is vector of unobserved states. The vectors of stochastic disturbances $\mathbf{v}_t$ and ϵ_t are assumed to be Gaussian and mututally uncorrelated, with mean zero and covariance matrices $\mathbf{R}$ and $\mathbf{Q}$, respectively. The covariance matrix $\mathbf{R}$ is always assumed to be diagonal.

For each model, there is a corresponding vector of parameters to be estimated by maximum likelihood. Because maximum likelihood estimates of the innovations to g and z, σ_g and σ_z, are likely to be biased towards zero (see Section 2.2 of HLW for explanation), we use Stock and Watson's (1998) mediun unbiased estimator to obtain estimates of two ratios, $\lambda_g \equiv \frac{\sigma_g}{\sigma_{y^*}}$ and $\lambda_z \equiv \frac{a_r \sigma_z}{\sigma_{\tilde{y}}}$. We impose these ratios when estimating the remaining model parameters by maximum likelihood.

7.3 The Stage 1 Model

The first-stage model, which corresponds to the *rstar.stage1.R* program, can be represented by the following matrices:

$$\mathbf{y}_t = [y_t, \pi_t]' \tag{3}$$

$$\mathbf{x}_t = [y_{t-1}, y_{t-2}, \pi_{t-1}, \pi_{t-2,4}]' \tag{4}$$

$$\xi_t = \left[y_t^*, y_{t-1}^*, y_{t-2}^*\right]' \tag{5}$$

$$\mathbf{H}' = \begin{bmatrix} 1 & -a_{y,1} & -a_{y,2} \\ 0 & -b_y & 0 \end{bmatrix}, \quad \mathbf{A}' = \begin{bmatrix} a_{y,1} & a_{y,2} & 0 & 0 \\ b_y & 0 & b_\pi & 1-b_\pi \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad \mathbf{Q} = \begin{bmatrix} \sigma_{y^*}^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The vector of parameters to be estimated by maximum likelihood is as follows:

$$\theta_1 = \left[a_{y,1}, a_{y,2}, b_\pi, b_y, g, \sigma_{\widetilde{y}}, \sigma_\pi, \sigma_{y*}\right]$$

7.4 The Stage 2 Model

The second-stage model, which corresponds to the *rstar.stage2.R* program, can be represented by the following matrices:

$$\mathbf{y}_t = [y_t, \pi_t]' \tag{6}$$

$$\mathbf{x}_t = [y_{t-1}, y_{t-2}, r_{t-1}, r_{t-2}, \pi_{t-1}, \pi_{t-2,4}, 1]' \tag{7}$$

$$\xi_t = \left[y_t^*, y_{t-1}^*, y_{t-2}^*, g_{t-1}\right]' \tag{8}$$

$$\mathbf{H}' = \begin{bmatrix} 1 & -a_{y,1} & -a_{y,2} & a_g \\ 0 & -b_y & 0 & 0 \end{bmatrix}, \quad \mathbf{A}' = \begin{bmatrix} a_{y,1} & a_{y,2} & \frac{a_r}{2} & \frac{a_r}{2} & 0 & 0 & a_0 \\ b_y & 0 & 0 & 0 & b_\pi & 1-b_\pi & 0 \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{Q} = \begin{bmatrix} \sigma_{y^*}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & (\lambda_g \sigma_{y^*})^2 \end{bmatrix}$$

The vector of parameters to be estimated by maximum likelihood is as follows:

$$\theta_2 = \left[a_{y,1}, a_{y,2}, a_r, a_0, a_g, b_\pi, b_y, \sigma_{\widetilde{y}}, \sigma_\pi, \sigma_{y*}\right]$$

7.5 The Stage 3 Model

The third-stage model, which corresponds to the *rstar.stage3.R* program, can be represented by the following matrices:

$$\mathbf{y}_t = [y_t, \pi_t]' \tag{9}$$

$$\mathbf{x}_t = [y_{t-1}, y_{t-2}, r_{t-1}, r_{t-2}, \pi_{t-1}, \pi_{t-2,4}]' \tag{10}$$

$$\xi_t = \left[y_t^*, y_{t-1}^*, y_{t-2}^*, g_{t-1}, g_{t-2}, z_{t-1}, z_{t-2}\right]' \tag{11}$$

$$\mathbf{H}' = \begin{bmatrix} 1 & -a_{y,1} & -a_{y,2} & \frac{-a_r}{2} & \frac{-a_r}{2} & \frac{-a_r}{2} & \frac{-a_r}{2} \\ 0 & -b_y & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{A}' = \begin{bmatrix} a_{y,1} & a_{y,2} & \frac{a_r}{2} & \frac{a_r}{2} & 0 & 0 \\ b_y & 0 & 0 & 0 & b_\pi & 1-b_\pi \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad \mathbf{Q} = \begin{bmatrix} \left(1+\lambda_g^2\right)\sigma_{y^*}^2 & 0 & 0 & (\lambda_g \sigma_{y^*})^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ (\lambda_g \sigma_{y^*})^2 & 0 & 0 & (\lambda_g \sigma_{y^*})^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \left(\frac{\lambda_z \sigma_{\tilde{y}}}{a_r}\right)^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The vector of parameters to be estimated by maximum likelihood is as follows:

$$\theta_3 = \left[a_{y,1}, a_{y,2}, a_r, b_\pi, b_y, \sigma_{\tilde{y}}, \sigma_\pi, \sigma_{y*}\right]$$

7.6 R Programs to Run the State-Space Models

The programs *rstar.stageX.R* run the models in stages 1-3 of the HLW estimation.

7.7 R Programs for Median Unbiased Estimators

The function *median.unbiased.estimator.stage1.R* computes the exponential Wald statistic of Andrews and Ploberger (1994) for a structural break with unknown break date from the first difference of the preliminary estimate of the natural rate of output from the stage 1 model to obtain the median unbiased estimate of λ_g.

The function *median.unbiased.estimator.stage2.R* applies the exponential Wald test for an intercept shift in the IS equation at an unknown date to obtain the median unbiased estimate of λ_z, taking as input estimates from the stage 2 model.

7.8 Kalman Filter Programs

Within the program *kalman.states.R*, the function kalman.states() calls kalman.states.filtered() and kalman.states.smoothed() to apply the Kalman filter and smoother. It takes as input the coefficient matrices for the given state-space model as well as the conditional expectation and covariance matrix of the initial state, xi.tm1tm1 $\left(\xi_{t-1|t-1}\right)$ and P.tm1tm1 $\left(P_{t-1|t-1}\right)$, respectively. *kalman.states.wrapper.R* is a wrapper function for *kalman.states.R* that specifies inputs based on the estimation stage.

7.9 Log Likelihood Programs

The function *kalman.log.likelihood.R* takes as input the coefficient matrices of the given state-space model and the conditional expectation and covariance matrix of the initial state and returns the log likelihood value and a vector with the log likelihood at each time t. *log.likelihood.wrapper.R* is a wrapper function for *kalman.log.likelihood.R* that specifies inputs based on the estimation stage.

7.10 Standard Error Program

The function *kalman.standard.errors.R* computes confidence intervals and corresponding standard errors for the estimates of the states using Hamilton's (1986) Monte Carlo procedure that accounts for both filter and parameter uncertainty. See footnote 7 in HLW.

7.11 Miscellaneous Programs

The function *calculate.covariance.R* calculates the covariance matrix of the initial state from the gradients of the likelihood function. The function *format.output.R* generates a dataframe to be written to a CSV containing one-sided estimates, parameter values, standard errors, and other statistics of interest.

8 Run Estimation for Each Economy

For each economy, the final section of *run.hlw.R* reads in prepared data from *inputData/rstar.data.XX.csv*, runs the HLW estimation by calling *run.hlw.estimation.R*, and saves one-sided estimates and a spreadsheet of output.

9 References

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