Online Appendix

Accounting for Macro-Finance Trends: Market Power, Intangibles, and Risk Premia

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This appendix first presents some additional empirical results, then presents and estimates a model with explicit intangible capital.

1 Data appendix

Our data and programs are available online. This appendix details our variable construction. We give Haver mnemonics. In many cases we have considered alternative series. However, because we focus on the medium-run trends, differences in cyclical behavior among series have little effect on our target moments.

- Real risk-free interest rate: in our baseline, we use the one-year Treasury constant maturity less current inflation (fcm1@usecon minus ypcuslfe@usecon), though results are nearly identical if other proxies for inflation are used, such as ex-post realized total or core inflation or the median 1-year ahead SPF expected inflation. In the extension, we use also the AAA/AA FTSE index for corporate bond yields minus the median 10 year ahead CPI expected inflation (syct5a@usecon - asacx10@surveys), and we also use the 10 year interest rate minus SPF expected inflation minus the term premium measured by Adrian, Crumb and Moench (fcm10@usecon minus asacx10@surveys minus facm10tv@usecon).

- Price-dividend ratio: we use the cum-dividend and ex-dividend annual returns from CRSP to construct the price-dividend ratio.

- Labor share: we use the gross labor share for nonfinancial corporations, defined as bncomp@usna divided by (bngdp@usna minus bnytpix@usna minus bnbtrn@usna).

- Investment-capital ratio: for investment, we use total nominal fixed investment in private assets over the corresponding capital stock measured at current cost from the Fixed Asset Tables of the BEA (zpt@capstock over ep@capstock). These measures include both nonresidential and residential, and the non-residential part includes equipment, structures and intellectual property products.

- Profitability: to ensure consistency between our measures, we construct profitability as the ratio

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of (one - our measure of labor share) to the ratio of the capital stock (measured at current cost, ep@capstock) to GDP (nominal, gdp@usna).

- Employment-population ratio: the ratio of civilian employment of people 16 years and over to civilian noninstitutional population of people 16 years old and over, i.e. le@usecon to lnn@usecon. These data are originally from the Current Population Survey.

- Population growth: the growth rate of lnn@usecon.

- TFP growth: we use Fernald's unadjusted total factor productivity for the business sector. We have also experimented with other TFP measures, with only minor effects on our results.

- Investment price growth: the growth of the ratio of the chained index for fixed investment (jf@usna) to the chained index of nondurable consumption and services.

- Leverage: for the extension with leverage, we use data from S&P to construct aggregate net market leverage as the sum of current debt and long-term debt less cash equivalents, divided by the close price, all on a per share basis, i.e. (lq500@spah + lt500@spah - aq500@spah)/pc500@spah.

- The empirical estimates of the equity premium in section 7.1 are constructed using monthly data from Shiller. Following Campbell and Thompson (2009), we construct the payout ratio as the ratio of a five-year centered moving average of dividends to earnings; and we use a three-year centered moving average of earnings to book equity as the return on equity. We use CRSP daily data to calculate realized volatility. The Gordon equity risk premium (ERP) is estimated as the average $D/P + G_D - RF$ where G_D is the growth rate of dividend. The Fama-French ERP is estimated as $D/P + G_E - RF$ where G_E is the growth rate of earnings. The Campbell-Gordon ERP is $\lambda(.5D/P + .5E/P) + (1-\lambda)ROE - RF$ where λ is the smoothed payout ratio.

- The data from section 7.2 are obtained from the Federal Reserve Board for the Gilchrist-Zakrajsek series; and Haver for the BAA, AAA, and 10 year interest rate, and VIX. Realized volatility is calculated using daily data from CRSP.

2 Model Appendix

The first subsection discusses aggregation. The second subsection lists the equations characterizing the equilibrium. The third subsection shows how to solve the model. The fourth subsection provides some formulas for the moments of the macroeconomic shock under various distributional assumptions.

2.0.1 Aggregation

Given our assumptions that capital and labor can be reallocated frictionlessly across firms at the beginning of each period, and given the constant-return-to-scale technology, firms face a constant (common) marginal cost mc_t. Each firm sets its price p_{it} and output y_{it} to maximize profits, subject to its demand curve:

$$\max_{y_{it},\, p_{it}} \left(p_{it} - mc_t\right) y_{it},$$
$$s.t. \quad : \quad y_{it} = Y_t \left(\frac{p_{it}}{P_t}\right)^{-\varepsilon},$$

where P_t is the price index, which we can normalize to one as a numeraire. This program leads to the optimal markup, equal to the inverse of the demand elasticity:

$$\frac{p_{it} - mc_t}{p_{it}} = \frac{1}{\varepsilon}.$$

Hence all firms set the same price, and in equilibrium we obtain that $n_{it} = N_t$, $k_{it} = K_t$, $y_{it} = Y_t$, $p_{it} = P_t = 1$ and marginal cost is

$$mc_t = \frac{\varepsilon - 1}{\varepsilon}.$$

Marginal cost can be calculated as the cost of expanding production using either labor or capital, or

$$mc_t = \frac{w_t}{MPN_t} = \frac{R_t}{MPK_t},$$

where w_t is the real wage, R_t the rental rate of capital, and MPN_t and MPK_t are the marginal products of labor and capital respectively. This leads to the first order conditions

$$\begin{aligned} (1-\alpha)\frac{Y_t}{N_t} &= \frac{\varepsilon}{\varepsilon - 1} w_t, \\ \alpha \frac{Y_t}{K_t} &= \frac{\varepsilon}{\varepsilon - 1} R_t. \end{aligned}$$

2.0.2 System of equations characterizing the equilibrum

Utility recursion:

$$V_t = \left((1-\beta) L_t c_{pc,t}^{1-\sigma} + \beta E_t \left(V_{t+1}^{1-\theta}\right)^{\frac{1-\sigma}{1-\theta}}\right)^{\frac{1}{1-\sigma}}.$$

Utility per capita (since L_t is deterministic):

$$V_{pc,t} \equiv \frac{V_t}{L_t^{\frac{1}{1-\sigma}}} = \left((1-\beta) c_{pc,t}^{1-\sigma} + \beta \frac{L_{t+1}}{L_t} E_t \left(V_{pc,t+1}^{1-\theta}\right)^{\frac{1-\sigma}{1-\theta}}\right)^{\frac{1}{1-\sigma}}.$$

Stochastic discount factor:

$$M_{t+1} = \beta \left(\frac{c_{pc,t+1}}{c_{pc,t}}\right)^{-\sigma} \left(\frac{V_{pc,t+1}}{E_t(V_{pc,t+1}^{1-\theta})^{\frac{1}{1-\theta}}}\right)^{\sigma-\theta}.$$

Stochastic trend:

$$S_{t+1} = S_t e^{\chi_{t+1}}.$$

Production function:

$$Y_t = Z_t K_t^{\alpha} (S_t N_t)^{1-\alpha}.$$

Capital accumulation:

$$K_{t+1} = \left(\left(1-\delta\right) K_t + Q_t X_t\right) e^{\chi_{t+1}}.$$

First-order conditions:

$$\begin{aligned}(1-\alpha)\frac{Y_t}{N_t} &= \mu w_t, \\ \alpha\frac{Y_t}{K_t} &= \mu R_t.\end{aligned}$$

Euler equation:

$$E_t\left[M_{t+1}R^K_{t+1}\right] = 1,$$

Return on capital:

$$R^K_{t+1} = \left(R_{t+1} + (1-\delta)\frac{1}{q_{t+1}}\right) q_t e^{\chi_{t+1}}.$$

Resource constraint:

$$C_t + X_t = Y_t.$$

2.0.3 Solution method

To solve the model involves solving for the constants y^*, x^*, k^*, etc. that give the (log) intercept of the solution for each of the macro variables, e.g.:

$$\log Y_t = \log y^* + \log S_t + \log T_t.$$

To obtain these constants, we follow the standard procedure to solve the neoclassical model. We first detrend the equations by T_t and S_t, taking into account the relation between g_T, g_L, g_Q, and g_Z. The Euler equation is hence rewritten along the risky balanced growth path as:

$$\frac{1}{\beta^*} = \left(\frac{\alpha}{\mu}Q^*\left(\frac{k^*}{\overline{N}}\right)^{\alpha-1}\frac{1}{1+g_Q} + \frac{1-\delta}{1+g_Q}\right),$$

which provides an equation in k^* given the parameters. We then obtain $y^* = k^{*\alpha}\overline{N}^{1-\alpha}$ from the detrended production function and x^* from the detrended capital accumulation equation:

$$x^* = k^*\left((1+g_Q)(1+g_T) - (1-\delta)\right),$$

and finally $c^* = y^* - x^*$. The calculation of the risk-free rate and price-dividend ratio follows immediately from the formula for the SDF.

2.0.4 Explicit formulas for some distributions of macro shocks

The expressions for the "big ratios" involve expectations of the macro shocks χ_{t+1}. It is useful to spell out these expectations in some interesting special cases. Technically, following Martin (2013), we recognize that we can rewrite these moments using the moment-generating function, defined for $x \in \mathbb{R}$ as $\phi(x) = E\left(e^{x\chi_{t+1}}\right)$. In particular, defining $\widehat{\beta} = \beta(1+g_{PC})^{-\sigma}$, we have:

$$\log\beta^* = \log\widehat{\beta} + \frac{1-\sigma}{1-\theta}\log\phi(1-\theta),$$

$$\log RF = -\log\widehat{\beta} - \frac{\theta-\sigma}{1-\theta}\log\phi(1-\theta) - \log\phi(-\theta),$$

$$\log ERP = \log\phi(1) + \log\phi(-\theta) - \log\phi(1-\theta).$$

While in our paper we will focus on the case where χ is a rare disaster, nothing in our analysis precludes using a different distribution. One that is particularly tractable is the lognormal case, i.e. χ is normal with mean μ_χ and variance σ_χ^2. In particular, setting $\mu_\chi = -\sigma_\chi^2/2$, an increase in σ_χ is a pure risk shock (i.e. in the sense of second order stochastic dominance). In that case, we have $\phi(x) = e^{-x(1-x)\frac{\sigma_\chi^2}{2}}$, and hence, as noted in section 3.4.1,

$$\begin{aligned}
\log\beta^* &= \log\widehat{\beta} - (1-\sigma)\theta\frac{\sigma_\chi^2}{2}, \\
\log RF &= -\log\widehat{\beta} - (1+\sigma)\theta\frac{\sigma_\chi^2}{2}, \\
\log ERP &= \theta\sigma_\chi^2.
\end{aligned}$$

Another tractable case is the compound Poisson process. Suppose that for $j \geq 0$,

$$\Pr(\chi_{t+1} = -jb) = \frac{\lambda^j}{j!}e^{-\lambda},$$

i.e. instead of at most a single disaster realization per period, there are potentially several of these shocks, and that the number of shocks follows a Poisson distribution, with intensity $\lambda \approx p$. (Because p is small, this compound Poisson process case is very close quantitatively to the simple binomial case, but leads to somewhat more elegant formulas.) The moment generating function is $\phi(x) = e^{\lambda\left(e^{-xb}-1\right)}$, and the objects of interest are:

$$\begin{aligned}
\log\beta^* &= \log\widehat{\beta} + \frac{1-\sigma}{1-\theta}\lambda\left(e^{-(1-\theta)b} - 1\right), \\
\log RF &= -\log\widehat{\beta} + \frac{\theta-\sigma}{1-\theta}\lambda\left(e^{-(1-\theta)b} - 1\right) - \lambda\left(e^{\theta b} - 1\right), \\
\log ERP &= \lambda\left(e^{-b} + e^{\theta b} - e^{-(1-\theta)b} - 1\right).
\end{aligned}$$

It is straightforward to extend this calculation to the case of random size of shocks b, as in Kilic and Wachter (2017).

In our application we will assume thatχ_{t+1} follows a three-point distribution, i.e.

$$\begin{aligned}
\chi_{t+1} &= 0 \text{ with probability } 1-2p, \\
\chi_{t+1} &= \log(1-b) \text{ with probability } p, \\
\chi_{t+1} &= \log(1+b_H) \text{ with probability } p,
\end{aligned}$$

where b and b_H and are chosen so that $E\left(e^{\chi+1}\right) = 1$. The second state is a "disaster": output and consumption fall permanently by a factor $1-b$. The third state is a "windfall" or "bonanza" state that offsets the mean effect of the disaster. One could also use a more traditional two-point process, without the third state, which would then not satisfy $E\left(e^{\chi+1}\right) = 1$, and in that case a change in p would have both a first moment and second moment effect.

	Pi/K	Pi/Y	RF	PD	I/K	gr. TFP	gr. invt price	gr. pop.	Emp/Pop
β	0.00	-0.02	-0.20	0.04	-0.00	-0.37	0.11	-0.74	-0.00
μ	1.88	0.28	0.00	0.15	-2.48	-0.09	0.09	-0.06	-0.00
p	-0.00	0.07	-1.28	-0.07	0.00	1.81	-0.54	1.27	0.00
δ	0.00	-5.94	0.00	0.00	100.00	-145.15	146.88	-101.77	-0.00
α	-1.32	0.88	-0.00	-0.10	1.74	0.06	-0.06	0.04	0.00
g_P	-0.00	0.00	-0.00	0.00	0.00	-0.00	0.00	100.00	0.00
g_Z	5.36	0.73	0.00	0.42	-7.09	105.34	-7.05	-1.87	-0.00
g_Q	0.00	-0.00	-0.00	0.00	0.00	-0.00	-100.00	-0.00	0.00
N	0.00	0.00	0.00	0.00	-0.00	0.00	-0.00	0.00	100.00

Table 1: Sensitivity matrix for the baseline model.

	Pi/K	Pi/Y	RF	PD	I/K	gr. TFP	gr. invt price	gr. pop.	Emp/Pop
β^*	0.00	-0.05	0.00	0.05	-0.00	-1.30	0.39	-0.91	0.00
μ	1.88	0.28	-0.00	0.15	-2.48	-0.08	0.09	-0.06	0.00
p	-0.00	0.07	-1.28	-0.07	0.00	1.81	-0.54	1.27	-0.00
δ	0.00	-5.94	0.00	0.00	100.00	-145.15	146.88	-101.77	0.00
α	-1.32	0.88	0.00	-0.10	1.74	0.06	-0.06	0.04	-0.00
g_P	-0.00	0.00	-0.00	-0.00	0.00	0.00	-0.00	100.00	-0.00
g_Z	5.36	0.73	-0.00	0.42	-7.09	105.34	-7.05	-1.87	0.00
g_Q	-0.00	-0.00	-0.00	0.00	-0.00	-0.00	-100.00	0.00	-0.00
N	0.00	0.00	0.00	-0.00	0.00	0.00	0.00	-0.00	100.00

Table 2: Sensitivity matrix. Here the parameters are redefined with $\beta^\star$ instead of β.

3 Additional Empirical Results

3.1 Identification

Table 1 reports the moment sensitivity, as suggested by Andrews, Gentzknow and Shapiro (2017). For each parameter (row), it shows the effect of changing each data moment on the parameter. For instance, increasing the estimate of the profit–capital ratio by 1 percentage point leads to a higher μ by about 1.88 point; or increasing the estimate of the risk-free rate by 1 percentage point leads to a lower β by about 0.20. Table 2 reports the same statistics when the parameters estimated are re-defined to be β^* instead of β. This table illustrates the recursive identification discussed in the text.

3.2 Decomposition: bounds

Table 3 reports the upper bound and lower bound of the effect of each parameter on each moment. This is calculated by consider all possible combinations of orders of changing parameters, as explained in the

	Pi/K	Pi/Y	RF	PD	I/K	gr. TFP	gr. invt price	gr. pop.	Emp/Pop
β	-2.08	0.00	-1.23	19.17	-0.00	0.00	0.00	0.00	0.00
	-1.69	0.00	-1.21	45.66	0.00	0.00	0.00	0.00	0.00
μ	2.30	4.13	0.00	0.00	-0.00	-0.17	0.00	0.00	0.00
	3.21	4.13	0.00	0.00	0.00	-0.12	0.00	0.00	0.00
p	0.68	0.00	-1.64	-26.68	-0.00	0.00	0.00	0.00	0.00
	0.84	0.00	-1.61	-5.69	0.00	0.00	0.00	0.00	0.00
δ	0.62	0.00	0.00	0.00	0.47	0.00	0.00	0.00	0.00
	0.74	0.00	0.00	0.00	0.47	0.00	0.00	0.00	0.00
α	0.00	-0.03	-0.00	-0.06	-0.00	-0.00	0.00	0.00	0.00
	0.01	-0.02	-0.00	-0.01	-0.00	-0.00	0.00	0.00	0.00
g_P	-0.00	0.00	-0.00	-5.06	-0.07	-0.00	0.00	-0.07	0.00
	0.00	0.00	0.00	-0.69	-0.07	-0.00	0.00	-0.07	0.00
g_Z	-0.32	0.00	-0.19	-12.57	-0.39	-0.27	0.00	0.00	0.00
	-0.26	0.00	-0.19	-1.96	-0.39	-0.25	0.00	0.00	0.00
g_Q	-1.27	0.00	-0.10	-7.33	-0.88	0.04	0.64	0.00	0.00
	-1.03	0.00	-0.10	-1.04	-0.87	0.08	0.64	0.00	0.00
N	-0.00	0.00	0.00	0.00	-0.00	0.00	0.00	0.00	-1.51
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.51

Table 3: The table reports for each moment, and for each parameter, a lower bound and an upper bound on the effect of the change in parameter on the moment, where the bounds are obtained by considering all possible orders of changing parameters.

text, footnote 13. For instance, the effect of β on the risk-free rate RF is bounded between -1.23 and -1.21. The effect of β on the PD ratio is bounded between 19.17 and 45.66. As can be seen from the table, the bounds are fairly tight, except for the PD ratio.

3.3 Results with different IES values

Our baseline results, presented in the paper, assume an IES equal to 2. (The IES is not identified given our estimation procedure, so we must set it a priori.) As we discuss in the paper, this value does not matter for some of our results, including the estimated values of several parameters (notably α or μ) or the equity risk premium estimate. It does matter however for the estimate of β and to understand the decompositions of moment changes into parameter changes such as Table 3 in the paper. We now present detailed results when the IES is set to 1 or 0.5 instead of 2. Table 4 presents the model estimates for the baseline model (i.e. $IES = 2$) as well as with $IES = 1$ or 0.5. Tables 5 and 6 present the decompositions of the target moments for $IES = 1$ and $IES = 0.5$. Table 7 presents additional moment decompositions for the baseline model and the cases with $IES = 1$ and $IES = 0.5$ and verifies that these are not

	Baseline			$IES = 1$			$IES = 0.5$		
	1984-'00	2001-'16	Diff.	1984-'00	2001-'16	Diff.	1984-'00	2001-'16	Diff.
β	0.961	0.972	0.012	0.966	0.970	0.004	0.976	0.965	-0.011
μ	1.079	1.146	0.067	1.079	1.146	0.067	1.079	1.146	0.067
p	0.034	0.065	0.031	0.034	0.065	0.031	0.034	0.065	0.031
δ	2.778	3.243	0.465	2.778	3.243	0.465	2.778	3.243	0.465
α	0.244	0.243	-0.000	0.244	0.243	-0.000	0.244	0.243	-0.000
g_P	1.171	1.101	-0.069	1.171	1.101	-0.069	1.171	1.101	-0.069
g_Z	1.298	1.012	-0.286	1.298	1.012	-0.286	1.298	1.012	-0.286
g_Q	1.769	1.127	-0.643	1.769	1.127	-0.643	1.769	1.127	-0.643
$\overline{N}$	62.344	60.838	-1.507	62.344	60.838	-1.507	62.344	60.838	-1.507

Table 4: The table reports the estimated parameter values in each of the two subsamples 1984-2000 and 2001-2016, for the baseline model, the baseline model with IES=1, and the baseline model with IES=0.5.

affected by the choice of the IES. As can be seen from these tables, the main substantive issue affected is the decomposition of the risk-free-rate and the PD ratio. Assuming a low IES does not reduce the importance of risk in the decompositions.

4 A model with intangible accumulation

We now present an extension of our baseline model that incorporates explicitely intangible capital. We will use our estimation framework to examine how the presence of intangible capital affects our results. The extended model makes the following changes compared to the baseline model. First, the production function is now a Cobb-Douglas over both tangible and intangible capital, with respective shares α_T and α_U :

$$Y_t = Z_t K_{T,t}^{\alpha_T} K_{U,t}^{\alpha_U} (S_t N_t)^{1-\alpha_T-\alpha_U}.$$

Second, tangible and intangible capitals are separately accumulated, and subject to potentially different rates of depreciation and of technical progress:

$$\begin{aligned} K_{T,t+1} &= ((1-\delta_T) K_{T,t} + Q_{T,t} X_{T,t}) e^{\chi_{t+1}}, \\ K_{U,t+1} &= ((1-\delta_U) K_{U,t} + Q_{U,t} X_{U,t}) e^{\chi_{t+1}}. \end{aligned}$$

Note our assumption that both types of capital are equally risky, i.e. have the same exposure to the macroeconomic shock χ_{t+1}. Relatively little is known about the relative riskiness of tangible and intangible capital, leading us to make this assumption. Finally, the resource constraint is modified to $C_t + X_{T,t} + X_{U,t} = Y_t$.

	β	μ	p	δ	α	g_P	g_Z	g_Q	$\overline{N}$
Gross profitability	-0.67	2.76	0.00	0.68	0.00	-0.00	-0.58	-1.31	-0.00
Capital share	0.00	4.13	0.00	0.00	-0.03	0.00	0.00	0.00	0.00
Risk-free rate	-0.43	0.00	-2.12	0.00	-0.00	-0.00	-0.38	-0.21	0.00
Price-dividend ratio	9.30	0.00	0.00	0.00	-0.00	-1.52	0.00	-0.00	0.00
Investment-capital	0.00	0.00	0.00	0.47	-0.00	-0.07	-0.39	-0.88	0.00
Growth of TFP	0.00	-0.14	0.00	0.00	-0.00	-0.00	-0.26	0.06	0.00
Growth of invt. price	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.64	0.00
Growth population	0.00	0.00	0.00	0.00	0.00	-0.07	0.00	0.00	0.00
Employment-pop.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.51

Table 5: The table reports the target moments in each of the two subsamples 1984-2000 and 2001-2016, as well as the change between samples, and the contribution of each parameter to each change in moment, for the model estimated with IES=1. See text for details.

	β	μ	p	δ	α	g_P	g_Z	g_Q	$\overline{N}$
Gross profitability	1.76	2.76	-1.52	0.68	0.00	-0.00	-1.17	-1.63	-0.00
Capital share	0.00	4.13	0.00	0.00	-0.03	0.00	0.00	0.00	0.00
Risk-free rate	1.14	0.00	-3.11	0.00	-0.00	-0.00	-0.76	-0.41	0.00
Price-dividend ratio	-35.66	0.00	27.75	0.00	0.04	-2.20	11.47	6.38	0.00
Investment-capital	0.00	-0.00	-0.00	0.47	-0.00	-0.07	-0.39	-0.88	-0.00
Growth of TFP	0.00	-0.14	0.00	0.00	-0.00	-0.00	-0.26	0.06	0.00
Growth of invt. price	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.64	0.00
Growth population	0.00	0.00	0.00	0.00	0.00	-0.07	0.00	0.00	0.00
Employment-pop.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.51

Table 6: The table reports the target moments in each of the two subsamples 1984-2000 and 2001-2016, as well as the change between samples, and the contribution of each parameter to each change in moment, for the model estimated with IES=0.5. See text for details.

	Baseline			IES=1			IES=0.5		
	1984-'00	2001-'16	Diff.	1984-'00	2001-'16	Diff.	1984-'00	2001-'16	Diff.
A. MPK-RF spread									
Total spread	11.22	15.24	4.02	11.22	15.24	4.02	11.22	15.24	4.02
- Depreciation	4.55	4.37	-0.18	4.55	4.37	-0.18	4.55	4.37	-0.18
- Market power	3.39	5.55	2.17	3.39	5.55	2.17	3.39	5.55	2.17
- Risk premium	3.15	5.23	2.08	3.15	5.23	2.08	3.15	5.23	2.08
B. Rate of returns									
Equity return	5.85	4.90	-0.96	5.85	4.90	-0.96	5.85	4.90	-0.96
Equity premium	3.07	5.25	2.18	3.07	5.25	2.18	3.07	5.25	2.18
Risk-free rate	2.79	-0.35	-3.14	2.79	-0.35	-3.14	2.79	-0.35	-3.14
C. Valuation ratios									
Price-dividend	42.34	50.11	7.78	42.34	50.11	7.78	42.34	50.11	7.78
Price-earnings	17.85	25.79	7.94	17.85	25.79	7.94	17.85	25.79	7.94
Tobin's Q	2.50	3.84	1.34	2.50	3.84	1.34	2.50	3.84	1.34
D. Income shares									
Share Labor	70.11	66.01	-4.10	70.11	66.01	-4.10	70.11	66.01	-4.10
Share Capital	22.59	21.24	-1.35	22.59	21.24	-1.35	22.59	21.24	-1.35
Share Profit	7.30	12.76	5.46	7.30	12.76	5.46	7.30	12.76	5.46
E. Macroeconomy									
K/Y	2.13	2.28	0.15	2.13	2.28	0.15	2.13	2.28	0.15
I/Y	17.28	16.50	-0.78	17.28	16.50	-0.78	17.28	16.50	-0.78
Detrend Y (% chg)	–	–	-0.30	–	–	-0.30	–	–	-0.30
Detrend I (% chg)	–	–	-4.95	–	–	-4.95	–	–	-4.95

Table 7: The table reports the target moments in each of the two subsamples 1984-2000 and 2001-2016, as well as the change between samples, and the contribution of each parameter to each change in moment, for the model estimated with IES=0.5. See text for details.

In terms of matching this model to data, we will consider as "tangible" all capital except intellectual property products (IPP), that is, tangible is the sum of residential, equipment and structures. We will assume, similar to section 6.5 of the paper, that measured IPP investment is a fraction λ of true intangible investment:

$$X_{U,t}^{obs} = \lambda X_{U,t},$$

and hence along the balanced growth path we also have $K_{U,t}^{obs} = \lambda K_{U,t}$. The same points made in section 6.5 about the mismeasurement of GDP, profits, and the labor share apply. We estimate this model given a fixed λ, and find the same parameters as the baseline model, plus α_U, δ_U, and the growth rate of Q_U, using similar moments as the baseline model. Here mismeasurement rises over time not because λ is changing but because intangibles are growing faster than other types of capital. Specifically, we use as target moments the growth rates of investment prices in both tangible and intangible capital, the ratio of measured profits to tangible capital and the ratio of profits to intangible capital, and finally the ratio of tangible investment to tangible capital, and of intangible investment to intangible capital.

Table 8 presents the estimated parameters for different values of λ, and table 9 presents the model implications. fFirst, note that the estimated α_U is small with no mismeasurement, corresponding to the share of IPP capital in total capital: α_U is estimated to rise from 3.4% to 4.8%. The depreciation rate of intangible investment is quite high, over 20%, consistent with the usual estimates. This high depreciation is precisely the reason why the share of IPP in the capital stock is small, despite a fairly large share in investment (about 25% lately). Finally, there is progress in the technology to make IPP, but it is slower than for equipment.

Similar to the simple analysis with mismeasurement of section 6.5, we find that (i) the model without mismeasurement behaves quite similarly to the baseline model; (ii) higher mismeasurement has no effect on most parameters except μ, α_T, and α_U. Specifically, more mismeasurement leads to lower estimated markups, lower α_T, and higher α_U. Here too, rising intangibles reduce the role of the markup story while preserving the risk story.

	$\lambda = 1$			$\lambda = 2/3$			$\lambda = 1/2$			$\lambda = 1/4$		
	1984-00	2001-16	Diff.	1984-00	2001-16	Diff.	1984-00	2001-16	Diff.	1984-00	2001-16	Diff.
β	0.961	0.973	0.012	0.961	0.973	0.012	0.961	0.973	0.012	0.961	0.973	0.012
μ	1.078	1.141	0.063	1.075	1.136	0.060	1.073	1.131	0.058	1.063	1.114	0.051
p	0.034	0.062	0.028	0.034	0.062	0.028	0.034	0.062	0.028	0.034	0.062	0.028
δ_T	1.792	2.585	0.794	1.792	2.585	0.794	1.792	2.585	0.794	1.792	2.585	0.794
α_T	0.210	0.199	-0.011	0.207	0.195	-0.012	0.203	0.190	-0.013	0.191	0.174	-0.017
g_P	1.171	1.101	-0.069	1.171	1.101	-0.069	1.171	1.101	-0.069	1.171	1.101	-0.069
g_Z	0.994	0.715	-0.280	0.984	0.684	-0.300	0.973	0.652	-0.321	0.919	0.509	-0.410
g_{QT}	1.781	0.809	-0.972	1.781	0.809	-0.972	1.781	0.809	-0.972	1.781	0.809	-0.972
α_U	0.034	0.048	0.014	0.050	0.070	0.020	0.065	0.091	0.026	0.123	0.167	0.044
δ_U	22.875	23.797	0.922	22.875	23.797	0.922	22.875	23.797	0.922	22.875	23.797	0.922
g_{QU}	1.710	2.150	0.440	1.710	2.150	0.440	1.710	2.150	0.440	1.710	2.150	0.440
$\overline{N}$	0.623	0.608	-0.015	0.623	0.608	-0.015	0.623	0.608	-0.015	0.623	0.608	-0.015

Table 8: The table reports the estimated parameters in the model with intangibles, for each of the two subsamples 1984-2000 and 2001-2016, as well as the change between samples.

	$\lambda = 1$			$\lambda = 2/3$			$\lambda = 1/2$			$\lambda = 1/4$		
	1984-00	2001-16	Diff.	1984-00	2001-16	Diff.	1984-00	2001-16	Diff.	1984-00	2001-16	Diff.
A. Spread MPK-RF												
Spread	11.95	16.19	4.24	11.95	16.19	4.24	11.95	16.19	4.24	11.95	16.19	4.24
B. Rates of Returns												
Equity return	5.86	4.73	-1.13	5.86	4.73	-1.13	5.86	4.73	-1.13	5.86	4.73	-1.13
Equity premium	3.07	5.08	2.01	3.07	5.08	2.01	3.07	5.08	2.01	3.07	5.08	2.01
Risk-free rate	2.79	-0.35	-3.14	2.79	-0.35	-3.14	2.79	-0.35	-3.14	2.79	-0.35	-3.14
C. Valuation ratios												
Price-dividend	42.34	50.11	7.78	42.34	50.11	7.78	42.34	50.11	7.78	42.34	50.11	7.78
Price-earnings	17.76	25.13	7.37	17.76	25.13	7.37	17.76	25.13	7.37	17.76	25.13	7.37
Tobin's Q	2.49	3.74	1.25	2.49	3.74	1.25	2.49	3.74	1.25	2.49	3.74	1.25
D. Income Distribution												
Labor	70.11	66.01	-4.10	69.12	64.75	-4.37	68.15	63.53	-4.62	64.53	59.09	-5.44
Tangible cap.	19.52	17.48	-2.04	19.24	17.14	-2.10	18.97	16.82	-2.15	17.96	15.64	-2.32
Intangible cap.	3.14	4.19	1.05	4.64	6.16	1.52	6.10	8.06	1.96	11.55	14.99	3.44
Profits	7.24	12.33	5.09	7.01	11.95	4.95	6.79	11.59	4.81	5.96	10.27	4.32
E. Macroeconomic variables (detrended, % change)												
K/Y	2.13	2.28	0.15	2.13	2.28	0.15	2.13	2.28	0.15	2.13	2.28	0.15
I/Y	14.47	13.04	-1.43	14.47	13.04	-1.43	14.47	13.04	-1.43	14.47	13.04	-1.43
Y	–	–	-4.36	–	–	-5.16	–	–	-5.67	–	–	-6.09
I	–	–	-6.73	–	–	-7.52	–	–	-8.03	–	–	-8.45

Table 9: The table reports some moments of interest calculated in the model with intangible capital, for different values of the mismeasurement parameters, using the estimated parameter values for each of the two subsamples 1984-2000 and 2001-2016, as well as the change between samples.