

# Analyzing the Interplay Between Public-Pension Finances and Governmental Finances:

*Lessons from Linking an Economic Model to a Pension Fund Model*

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## Contents

Summary .....	3
Linking an economic scenario generator to a pension simulation model and tax revenue models .....	3
Examining compounding risks.....	4
Summary of results .....	5
Conclusions and lessons.....	8
Introduction .....	10
The economy, pension fund finances, and governmental finances.....	10
How we address these issues using linked models .....	11
How the models fit together .....	11
The economic scenario generator .....	14
Modeling cyclical tax variability for individual taxes .....	17
Why we did not model the property tax and how we might incorporate it in the future .....	20
Combining trend and cyclical growth .....	21
Constructing tax portfolios for stylized governments .....	23
Compounded risks from correlated business cycles, investment returns, and tax revenue .....	25
Illustration of a single simulation .....	26
Distribution of growth in real tax revenue.....	28
Simulating the finances of public pension plans.....	30
Evaluating the potential compounding of risks .....	31
Summary of simulation results .....	34
Conclusion.....	38
Technical Appendices .....	40
Linking an economic scenario generator, a pension simulation model, and cyclical tax revenue models.....	40
Alternative investment return approaches.....	94
Conclusion .....	97
Endnotes .....	99

## Summary

We and other researchers have examined the investment-related risks to public pension funds, to the governments that contribute to them, and to stakeholders in pension funds and governments, using stochastic simulation models of pension fund finances. These models simulate 1,000 or more potential future paths of pension fund finances, with each path having different investment returns drawn from an assumed distribution of investment returns. We analyze risks to governments by examining required contributions relative to projected government revenue.

These models generally use simple investment return assumptions. For example, our models have assumed that investment returns have an expected compound annual return of 7.5 percent and a standard deviation of 12 percent, are drawn from a normal distribution, and are independent from year to year. Our analysis of fiscal risks to governments has assumed that investment returns and governmental tax revenue are uncorrelated.

Research suggests that the real world differs from these assumptions, in some ways that mean the assumptions may understate risks, and in other ways that mean the assumptions may overstate risks. Investment returns may not be normally distributed and may not be independent over time. Perhaps more important, investment returns and tax revenue may be correlated: a poor economy may cause investment returns to fall short of expectations, and may also cause tax revenue to fall short. The resulting increase in required employer contributions may cause additional fiscal pressure if increases come when tax revenue is low.

### [Linking an economic scenario generator to a pension simulation model and tax revenue models](#)

We address these issues, focusing on the correlation between tax revenue and the economy, by building a small macroeconomic model that can generate internally consistent stochastic scenarios of growth in real gross domestic product (GDP) and returns from stock and bond investments.

We use outputs from the economic scenario generator to simulate anticipated tax revenue of stylized governments, based upon historical relationships between business cycles and tax revenue for different tax types, some of which are quite volatile, and others of which are more stable. We construct two stylized governments. The first is an income-tax-dominant government that has highly cyclical tax revenue that is even more cyclical in recessions in which financial markets fall. The second is a sales-tax-dominant government with tax revenue that is similar in cyclicity to the economy but less cyclical than the income-tax-dominant state. We compare these stylized governments to a baseline government in which tax revenue grows at a constant rate, with no cyclicity.

We use investment-return outputs of the economic scenario generator to drive a stochastic simulation model of pension fund finances that tracks pension fund financial status and calculates required employer contributions.

Because the economic scenario generator drives tax revenue and also drives pension fund finances, tax revenue will be correlated with investment returns in ways consistent with past relationships, which is a conceptual improvement compared to simpler models. We use these results to calculate fiscal pressure and risks to stylized governments and compare these risks to those resulting from simpler assumptions in which tax revenue is not cyclical.

### Examining compounding risks

We can examine the potential compounding of risks with the simulated tax revenue of stylized governments and simulated pension finances, with both simulations linked to the macroeconomic simulation. Economic growth and asset returns are jointly simulated by a regime-switching process that captures the historical pattern of the business cycle. The tax revenues of stylized governments are then calculated based on the simulated economic conditions and asset returns. The asset returns are fed in to our pension simulation model to calculate the required employer contributions, among other variables of interest.

It is assumed that the required employer contribution under the policy scenario “slow repayment of UAAL” and “7.5 percent discount rate” accounts for 5 percent of the total tax revenue of the sponsoring government in year-1. The total tax revenue calculated based on this assumption is also applied to the other three policy scenarios. The shares of employer contribution in the total tax revenue in year 1 under the other three policy scenarios are 7.2% (fast repayment of UAAL; 7.5 percent discount rate), 7.0 percent (slow repayment of UAAL; 6 percent discount rate), and 9.8 percent (fast repayment of UAAL; 6 percent discount rate). The sponsoring governments are assumed to pay full ADC in each year.

Tax revenues of the income-tax-dominant government and sales-tax-dominant government grow at the simulated growth rate. We also created a benchmark government for which the tax revenue grows at a constant rate that is equal the assumed long-term trend growth rate of GDP (1.9 percent). Employer contributions as a percentage of total tax revenue in each simulation are then calculated for all types of government and policy scenarios.

We examine two types of risks that the sponsoring governments of public pension plans may face:

1. The required employer contributions become very high relative to fiscal resources available to the sponsoring government, creating great fiscal pressure and potentially crowding out other public services. In our analysis of the simulation results, we measure this type of risk by the probability that employer contributions as a percentage of total tax revenue will become more than 5 percentage points higher than the level in year 1 at any time during the 30-year simulation period.

2. The required employer contributions rise sharply in a short period of time, creating difficulty in budget planning and short-term fiscal pressure. We measure this type of risk by the probability that employer contribution rises more than 3 percent of total tax revenue in a 2-year period at any time during the 30-year simulation period.

## Summary of results

### The risk of high employer contributions

Table 1 shows the risk of employer contributions rising by 5 percentage points or more relative to the starting point under our different models and funding policies. The three rows of the table show our three funding policies and the six columns show the size of the risk under our six models.

The first column presents the risk measure when tax revenue grows smoothly over time without any business cycle effects, and investment returns are normally distributed and independent over time, without any correlation to the economy. It is akin to the kinds of simulations we have done in past papers, and the risk measures are lower than measures for all of the models to the right. For example, there is a 6.8 percent chance that employer contributions will rise by more than 5 percentage points of tax revenue above their year 1 levels at some point during the 30-year simulation period – a smaller risk than in any column to the right.

The second column also assumes that tax revenue grows smoothly, but now investment returns are generated by the regime-switching simulation model that results in a heavier left tail in the return distribution than in the normal distribution, and returns are correlated with economic conditions rather than being independent over time. Thus, comparing this column to column 1 shows us the pure effect of linking investment returns to the economy, without considering tax revenue cyclicity, shedding light on the impact of introducing non-normality in investment returns. The risks of high employer contributions are only slightly higher than in the base case.

Column 3 now allows tax revenue to be exactly as cyclical as real GDP is – for example, if real GDP grows 2 percentage points more quickly than its trend, tax revenue will grow at the same rate. Investment returns are drawn from the normal distribution rather than being generated by our ESG. Thus, comparing column 3 to column 1 shows us the pure effect of introducing volatility in tax revenue that is not correlated with investment returns, without accounting for differences in tax structure. The risks are slightly greater than in the base case and not much different from those in column 2.

Column 4 allows tax revenue to be exactly as cyclical as real GDP and has investment returns generated by the model. Comparing this column to column 3 shows the risk introduced by having investment returns linked to the economy in addition to cyclical tax revenue. Comparing this column to column 2 shows the risk introduced by having cyclical revenue. In either comparison, the increase in risk is quite large. For example, under a 10-year open constant-dollar funding policy, the risk of a contribution increase that is at least 5 percentage points of

tax revenue is 17.8 percent, or more than a one in six chance of such an increase during the 30-year simulation period. Comparing column 4 to column 1 demonstrates that while the isolated effects of linking tax revenue or investment returns to the business cycle are modest, the compounding effect of correlated tax revenue and investment returns is quite large. For example, under the 15-year open constant dollar funding policy, the risk of high pension contribution is 2.7 times as great as in column 1 (17.2 percent compared to 6.4 percent), while for columns 2 and 3 the measures are only 1.4 and 1.5 times as great.

Columns 5 and 6 introduce our two stylized governments, with tax revenue cycles depending not just on real GDP cycles but also on tax structure. In both models, returns are generated by the economic scenario generator. The risks for the sales-tax-dominant government actually are slightly smaller than if revenue is exactly as cyclical as real GDP.<sup>1</sup> The risks for the income-tax-dominant state are much larger: the risk of a large employer contribution is 22.8 percent, compared to 16.5 percent for the sales-tax-dominant state, and compared to 6.8 percent for the base case where neither investment returns nor tax revenue are linked to the economy – the kinds of simulations that we have reported on in past papers.

Table 1 Risk of high pension contribution relative to tax revenue

**Risk of high employer contribution relative to tax revenue under alternative funding policies**

Amortization method for unfunded liability	Probability of employer contribution as a percentage of total tax revenue being more than 5 percentage points above the year-1 level at any time during the 30-year simulation period					
	Constant growth of total tax revenue (equal to trend GDP growth)		Cyclical growth of total tax revenue (equal to trend + cycle GDP growth)		Stylized governments	
	Normally distributed returns	Simulated returns	Normally distributed returns	Simulated returns	Sales-tax-dominant state	Income-tax-dominant state
					Simulated returns	Simulated returns
(1)	(2)	(3)	(4)	(5)	(6)	
10-year open constant dollar	6.8%	9.2%	9.5%	17.8%	16.5%	22.8%
15-year open constant dollar	6.4%	8.9%	9.3%	17.2%	16.6%	22.8%
30-year open constant percent of payroll	1.3%	2.0%	4.9%	11.1%	10.1%	18.0%

Note: In year 1, employer contribution as a percentage of total tax revenue under the three amortization methods are 8.67% (10-year open constant dollar), 7.25% (15-year open constant dollar), and 5% (30-year open constant percent of payroll).

Comparing risks as we move down the rows of Table 11 also is instructive, as it shows the interplay between funding policy and contribution risk. Risks of large employer contributions fall somewhat as the funding policy lengthens, particularly as we move from 15 years to 30 years and change from level dollar funding to level-percentage-of-pay funding. Because this risk measure compares every year in a 30-year simulation to the starting year, it is a measure of longer-term risks, and funding policies that smooth contributions have only limited ability to reduce this risk. They have much greater ability to reduce the risk of large employer contributions in a short period of time, as discussed below.

The risk of large contribution increases in a short period of time

Table 2 presents our short-term risk measure – the probability that employer contributions will rise by more than three percent of tax revenue in any two-year period during our 30-year simulation - for the same models and funding policies. The format and interpretation are the same as for Table 1.

As with our other risk measure, the risk increases as we move to the right in the table, but not as much as with our longer-term risk measure, the risk of a large contribution increase over the 30-year period. Furthermore, risks of sharp employer contribution increases diminish dramatically as we move down the rows of the table, lengthening the contribution smoothing period and liberalizing the policy, in the bottom row, by using level-percentage of pay funding: long smoothing periods and more-liberal funding policies reduce short-term employer contribution risks substantially.

Table 2 Risk of sharp increases in pension contributions relative to tax revenue

Risk of sharp increase in employer contribution relative to tax revenue under alternative funding policies

Amortization method for unfunded liability	Probability of employer contribution rising more than 3 percent of total tax revenue in any 2-year period during the 30-year simulation period					
	Constant growth of total tax revenue (equal to trend GDP growth)		Cyclical growth of total tax revenue (equal to trend + cycle GDP growth)		Stylized governments	
	Normally distributed returns	Simulated returns	Normally distributed returns	Simulated returns	Sales-tax-dominant state	Income-tax-dominant state
					Simulated returns	Simulated returns
(1)	(2)	(3)	(4)	(5)	(6)	
10-year open constant dollar	43.0%	44.1%	44.2%	48.7%	47.8%	54.7%
15-year open constant dollar	20.9%	23.7%	23.3%	31.4%	30.5%	37.8%
30-year open constant percent of payroll	0.5%	1.2%	1.1%	3.1%	2.5%	6.9%

Note: In year 1, employer contribution as a percentage of total tax revenue under the three amortization methods are 8.67% (10-year open constant dollar), 7.25% (15-year open constant dollar), and 5% (30-year open constant percent of payroll).

Contribution smoothing policies can reduce employer risks, but they do not make risks go away: they transfer them to the pension fund and its stakeholders, as we discuss in the next section.

Contribution-smoothing policies cannot make risks go away: They transfer risks from governments to pension plans

Lower risks for sponsoring governments under funding policies with stronger contribution-smoothing effects come at the expense of higher risk that the pension plan will become severely underfunded, as Table 3 demonstrates.

The first column of Table 3 ties to the fourth column of Table 2 to provide a useful point of reference, showing how the risk of sharp increases in employer contributions declines as the funding period lengthens and the policy liberalizes. The last column shows the risk that the plan’s funded ratio will fall below 40 percent, which we consider to be crisis territory, at some point during the 30-year simulation period. (We explain the intervening columns after the

table.) As we move down the rows, the risk of this crisis-level funding rises from 7.1 percent with 10-year constant-dollar funding policy to 30.9 percent, or nearly a one in three chance, with 30-year constant percent of payroll funding. (This latter risk is far greater than risks we estimated in earlier papers, when investment returns were not linked to economic conditions.) The risk protection that governments gain from stretched-out funding policies results in greater risk to pension plans.

*Table 3 Greater contribution smoothing leads to higher risk of pension underfunding*

	Risk of sharp increase in employer contribution relative to tax revenue*	Employer contribution as a % of tax revenue in year 1	Median Present value at year 1 of total employer contribution for year 1-15**	Median Present value at year 1 of total employer contribution for year 16-30**	Probability of low funded ratio***
<b>10-year open constant dollar</b>	48.7%	8.7%	1.32	0.64	7.1%
<b>15-year open constant dollar</b>	31.4%	7.3%	1.23	0.67	11.7%
<b>30-year open constant percent of payroll</b>	3.1%	5.0%	1.00	0.68	30.9%

Notes:

\* Probability of employer contribution rising more than 3 percent of total tax revenue in any 2-year period during the 30-year simulation period based on Model (4) (Cyclical growth of total tax revenue with simulated investment returns).

\*\* The present value at year 1 of total employer contribution in year 1-15 under the policy "30-year open constant percent of payroll" is standardized to 1. All other values are standardized accordingly. ☐

\*\*\* Probability of low funded ratio: the probability of funded ratio falling below 40% in any year during the 30 year simulation period.

## Conclusions and lessons

In this paper we examine how our assessment of risks to pension funds and their sponsoring governments changes when we take into consideration the correlation between economic conditions, investment returns, and governmental tax revenue. We do this by linking an economic scenario generator - a small stochastic macroeconomic model that simulates real GDP growth and investment returns – to models of tax revenue cyclical and a pension finance simulation model. Our paper builds upon other recent research that has examined risks to pension funds and governments under alternative economic scenarios.

The simulation results demonstrate that it is important to allow for correlation among economic conditions, investment returns, and tax revenue when examining the investment-related risks to public pension funds and the sponsoring governments. Contribution increases required after economic downturns are much larger, relative to tax revenue, when we allow tax revenue to be cyclical (varying with the economy) than under simpler assumptions of stable tax revenue growth.

Pension-related risks for governments can be further exacerbated by how state tax revenue structures respond to economic conditions. All else equal, income-tax-dominant states, with highly cyclical tax revenue, face higher pension-related risks than sales-tax-dominant states, which have less cyclical tax revenues.



The choice of funding policies for public pension funds also has a significant impact on the risks that sponsoring governments face. Funding policies that pay down unfunded liabilities slowly and have strong smoothing effects on contribution requirements can protect the sponsoring governments from the risks of high or sharp increase in pension contributions, at the expense of greatly increasing the risk of severe underfunding for the pension funds.

This paper shows how important it is to incorporate budgetary resources in pension fund risk analysis, and how that analysis can be deepened by modeling business cycles and investment returns together. Our work in this area is preliminary and can be extended and improved upon.

What do the insights from this work mean for larger efforts underway to encourage pension funds and governments to conduct stress testing and to report on risks to plan beneficiaries, legislators, citizens, and taxpayers? This paper highlights that these risks are even larger than commonly understood, and that these efforts are all the more important.

Stress testing and risk reporting are, we hope, the wave of the future. The recent trend has been in that direction. The Society of Actuaries called for stress testing in its 2014 Blue Ribbon Panel report. While relatively few plans and states conduct risk reporting as a matter of course, more have been doing so, with seven states now requiring it. The Actuarial Standards Board recently has encouraged greater reporting on risks with a recent actuarial standard of practice. More states and plans should follow this lead.

First steps that plans and governments are taking now are important, with simple measures that plans can readily calculate and the public can easily understand. Comparisons to economic and revenue resources assuming stable growth are an important part of those steps, as are stress testing under asset and economic shock scenarios. Integrated stochastic analysis of the impact of business cycles and the economy, such as we do here, can be adapted to plans and governments and can further deepen risk reporting and stress testing. It not need not be part of a first step, but it should not delay first steps. Understanding and assessing risks are necessary first steps toward managing risks.

## Introduction

Public pension underfunding represents the single-largest long-term threat to the finances of the state and local government sector, although it is a much bigger threat for some governments than for others. Assessing the risks is difficult because the magnitude and likelihood of pension underfunding depends upon investment returns, and the resulting governmental fiscal stress depends upon pension funding needs in relation to governmental fiscal resources. Investment returns and governmental resources both depend, in part, upon underlying economic conditions.

In this paper we link an “economic scenario generator” that generates stochastic future paths of gross domestic product, and investment returns for several asset classes, to (1) a pension fund simulation model, and (2) models of governmental tax revenue. We use these linked models to analyze how risks to governments and pension plans can be compounded because investment returns and tax revenue are driven by the same underlying economic conditions, building upon previous research on pension fund stress-testing.<sup>2</sup>

## The economy, pension fund finances, and governmental finances

We and others have modeled public pension fund finances stochastically, addressing how investment returns may affect pension fund finances and the finances of governments that contribute to plans. When these models link pension fund finances to government finances, they generally do so in ways that assume that governmental fiscal resources are independent of investment returns, and that fiscal resources grow smoothly rather than varying cyclically. In addition, these models often generate investment-return scenarios simply, assuming that portfolio returns are normally distributed and independent over time. (Recent research has improved upon this by examining shock scenarios in which both governmental revenue and investment returns are affected.<sup>3</sup>) Both sets of assumptions are imperfect.

First, governmental fiscal resources may be correlated with pension fund financial condition, because tax revenue and investment returns both are related to underlying economic conditions. For example, some recessions may cause sharp falloffs in governmental tax revenue and also cause investment shortfalls. This was true of both the 2001 and 2007 recessions, and some earlier recessions.<sup>4</sup> Increases in governmental contributions may be required when tax revenue available to pay contributions is falling (or falling short of expectation), causing more fiscal stress to governments than simple models suggest. Greater fiscal stress could make governments more likely to underpay requested contributions, heightening risks to pension fund finances.

Second, portfolio investment returns may be far more complex than models traditionally assume: Investment returns may be related to economic conditions rather than independent over time, they may be distributed non-normally, correlations among asset classes may change over time.<sup>5</sup> These real-world considerations could make risks to pension funds from investing in risky assets greater or less than simple models suggest.

An improved understanding of these issues could lead to plan designs and funding policies that are better suited to the finances of the governments that fund public pensions.

### How we address these issues using linked models

To examine these issues, we develop and link a small-scale macro-economic and investment-returns model to a pension fund simulation model and models of governmental tax revenue. This allows us to generate stochastic economic, investment-return, and fiscal scenarios in an integrated fashion. The model allows us to explore how economic scenarios can cause investment returns and governmental fiscal resources to vary together.

In future work, the model could be used to examine policy options that are best analyzed in a stochastic framework that integrates economic variables and asset returns. For example, policy options in which Cost of Living Adjustments (COLAs) or other contingent benefits depend upon plan funded status require such a framework. (The size of the COLA may depend upon inflation, while plan funded status will depend heavily on asset returns. Because inflation and asset returns may be correlated, an integrated model is valuable.)

The model we describe in this paper links investment returns to economic conditions. However, the model currently cannot produce other departures from traditional investment-return assumptions. For example, we cannot require that investment returns for a particular asset class follow a specific non-normal distribution. In future work, we plan to supplement the model with other methods for generating investment returns. We describe our work to date on this in an appendix.

### How the models fit together

We have built an integrated economic scenario generator that allows us to construct internally consistent simulations of real gross domestic product (GDP) and investment returns on several asset classes, far into the future, following paths over time that reflect parameters estimated from historical data or obtained from projections of future economic environments. By adding random errors to model results, we can simulate thousands of such paths.

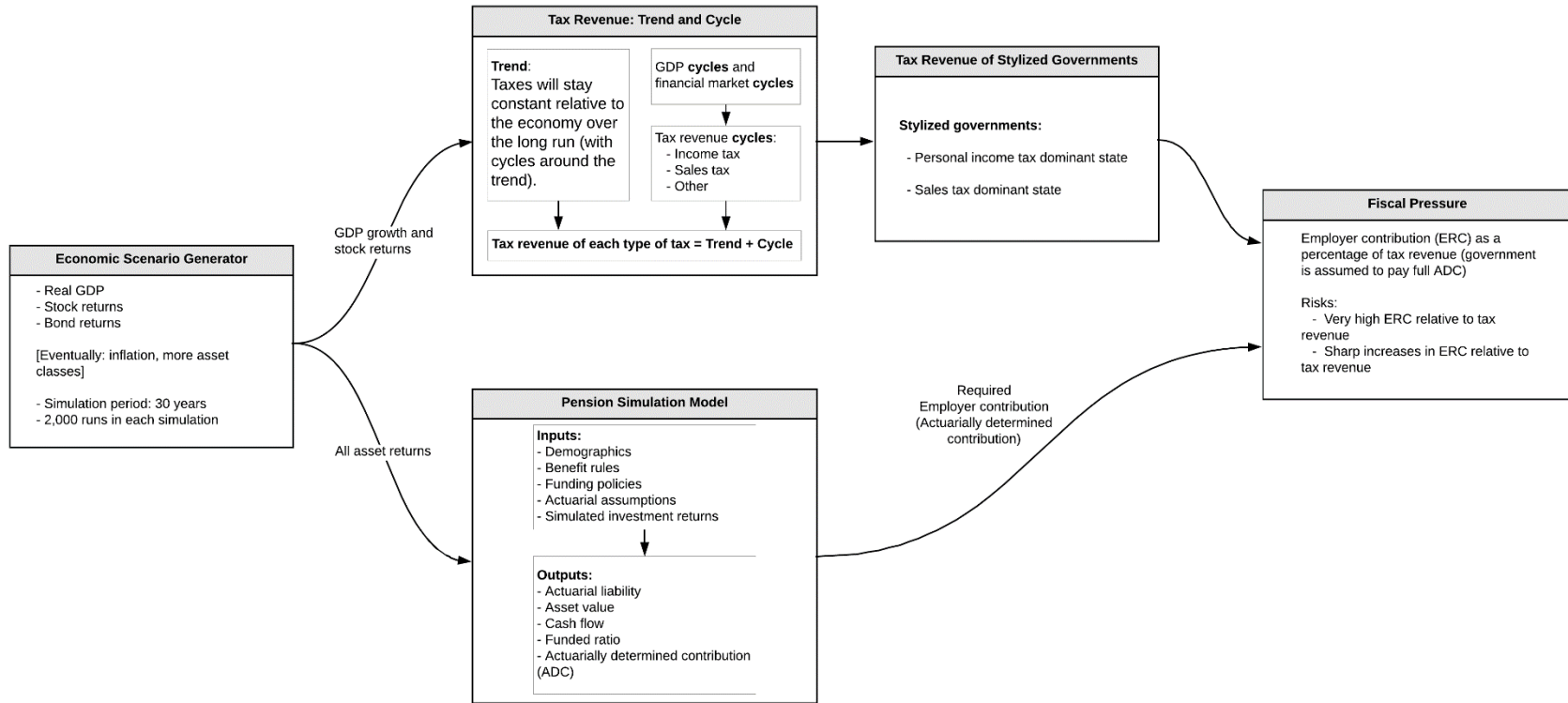
We use outputs from the economic scenario generator as inputs to:

1. A pension fund simulation model, which yields as outputs information on funded status of the plan and contribution requirements by governments.
2. Models of governmental tax revenue for income taxes, general sales taxes, selective sales taxes, and other taxes. The models allow revenue from these taxes to respond to economic cycles in ways that are consistent with historical patterns. We combine revenue from these taxes to create two stylized governments: an income-tax-dominant government and a sales-tax-dominant government. The economic scenarios affect the revenue resources of the stylized governments in different ways. For example, some economic scenarios will cause sharper revenue cycles in the income-tax-dominant government than in the sales-tax-dominant government.

We combine employer contribution requirements from the pension fund simulations and tax revenue of the stylized governments to analyze the stress that pension contributions may place upon the finances of governments. This considers the relationships among economic conditions, investment returns, pension fund finances, and governmental finances in a way that has not, to our knowledge, been done by other researchers.

Figure 1 summarizes how the models work together.

Figure 1 Model structure and linkage



### The economic scenario generator

We have constructed an economic scenario generator with three equations resulting in three outputs: growth in real GDP, total stock market returns, and total returns on long-term bonds. The model's equations are estimated from quarterly data, and they produce quarterly outputs that we convert to annual values. (See the appendix section, **Building an integrated economic scenario generator**, for details.)

The equation for real GDP growth allows the economy to cycle between two regimes, economic expansion and recession. The model estimates average GDP growth in each regime (faster average growth in expansion than recession), and the probabilities of switching from expansion to recession, or recession to expansion, from one quarter to the next. This approach is known as a Markov-switching model and is based upon a seminal paper by James Hamilton.<sup>6</sup> The specific equation we use models GDP growth as a random walk with drift, which does a good job of capturing the general historical pattern of expansions and recessions, as we discuss in the appendix. Because we are particularly interested in cyclical economic and tax revenue patterns, we chose this specification.

Stock returns are generally considered to be a random walk process with drift. Our equation for total stock market returns allows for two regimes - high-return-low-volatility periods, and low-return-high-volatility periods, based upon our examination of historical data. We implement this by modeling total stock returns as a random walk with a Markov-switching drift term and variance.

The economic scenario generator allows for correlation between stock returns and GDP growth by aligning their regimes. We estimated separate regime-switching models for GDP growth and stock return and found that the recession regimes of GDP growth are generally aligned with the low-return-high-volatility regimes of stock returns, especially in periods of severe economic downturn. When producing economic scenarios, the simulated regimes of GDP growth are also applied to stock returns, with the expansion regimes of GDP serving as the high-return-low-volatility regimes of stock return, and the recession regimes of GDP serving as the low-return-high-volatility regimes of stock return.

We have not modeled bond returns econometrically, because of their weak historical relationships to business cycles. Instead, when we produce economic scenarios, we construct stochastic bond returns that have correlations to stock returns that are consistent with historical correlations.

### Examining the performance of our economic scenario generator

To analyze how well these equations mimic historical patterns of expansion and recession, and of stock and bond returns, we conducted 2,000 simulations of the quarterly equations over 63 years (the period over which we estimated the equations). The model requires parameters such as the average growth rate in expansions, average decline in recessions, probability of switching from expansion to recession from one quarter to the next, or vice versa, expected bond returns

and standard deviation, correlation between stock and bond returns, and so on. For this evaluation purpose, we used parameters estimated from historical data. We then examine the extent to which the simulation results are consistent with historical patterns.

We summarize the results in Table 4, which compares historical values to the median simulation result, and shows where the historical value fits in the distribution of our simulations. (The appendix examines not just the median of simulation results, but also the distribution.) The table is organized into four blocks: economic regimes, GDP growth, stock returns, and bond returns, which we explain immediately after the table.

Table 4 Simulated values using historical parameters are quite consistent with historical data

**Summary statistics for historical and simulated data**

	<b>Historical value for 1953-2015 (63 years)</b>	<b>Median of the simulated distribution (2,000 simulations)</b>	<b>Historical value's percentile in the simulated distribution</b>
<b>Economic regimes</b>			
Number of recessions	10	10	52
Number of expansions	10	11	40
Average length of recession (quarters)	3.7	3.0	79
Average length of expansion (quarters)	20.2	19.4	57
<b>GDP growth</b>			
Mean	3.0%	3.1%	38
Standard Deviation	2.5%	2.2%	91
<b>Stock return</b>			
Mean	10.4%	10.9%	40
Standard Deviation	15.7%	17.2%	19
Kurtosis (Measure of heavy-tailedness)	0.13	0.10	52
<b>Bond return</b>			
Mean	6.6%	6.6%	49
Standard Deviation	9.6%	10.6%	14
Kurtosis (Measure of heavy-tailedness)	0.63	-0.16	88

Note: The kurtosis measure compares the "heavy-tailedness" of our simulated distributions to the normal distribution. Values greater than 1 mean our distribution has heavier tails than the normal distribution, and less than 1 mean the opposite.



The economic regimes block summarizes how well the GDP equation reproduces historical patterns of recession and contraction. It shows that there were 10 recessions over our 63-year historical period with average length of 3.7 quarters, while the median model run produces 10 recessions over 63 years, with an average duration of 3.0 quarters in the median simulation. Similarly, the historical period had 10 expansions with average length of 20.2 quarters, while the median simulation had 11 expansions with average length of 19.4 quarters. The last column of the table shows where the historical value fits in the simulated distribution: for example, the historical number of expansions, 10, was slightly below the simulated median of 11 recessions, and fell at the 40<sup>th</sup> percentile of our simulations (below the 50<sup>th</sup> percentile, of course).

The GDP-growth block shows the mean and standard deviation of GDP growth (quarterly growth at an annualized rate). The historical growth rate is slightly below our median simulation; the standard deviation, while only moderately higher than our median simulation, is at the 91<sup>st</sup> percentile of simulated values.

The second and third blocks of the table compare historical means and standard deviations for stock and bond returns to simulated values. They also include a measure of “heavy-tailedness” – the extent to which stock or bond returns have more extreme values than the normal distribution would suggest. This measure, called “kurtosis,” is positive when the simulated distribution is more heavy-tailed than the normal distribution, and negative if it is less so.

Overall, we think that simulations from the model, constructed using parameter values estimated from history, are quite consistent with historical values. The appendix evaluates the model in greater detail and examines alternative specifications that did not perform as well.

#### [Modeling cyclical tax variability for individual taxes](#)

We estimate the cyclical relationships between taxes and the economy for state personal income taxes, the state general sales tax, state selective sales taxes, and all other state government taxes as a group. We focus on these taxes because they play prominent roles in states’ tax portfolios. We also examine local government property taxes but do not model them, as we discuss further below. Table 5 shows the roles these taxes play in state and local government tax portfolios. (For further information, see *Composition of state tax revenue* in the appendix.)

Table 5 Personal income and general sales taxes are the most important state government taxes, and the property tax is the most important local tax

<b>State and local government tax shares in 2015</b>		
	<b>State government</b>	<b>Local government</b>
<b>Personal income tax</b>	36.9%	4.8%
<b>General sales tax</b>	31.4%	12.5%
<b>Selective sales taxes</b>	15.9%	4.8%
<b>Property tax</b>	1.7%	72.1%
<b>Other</b>	14.1%	5.8%
<b>Total</b>	100.0%	100.0%

Source: U.S. Census Bureau, 2015 Annual Survey of State and Local Government Finances

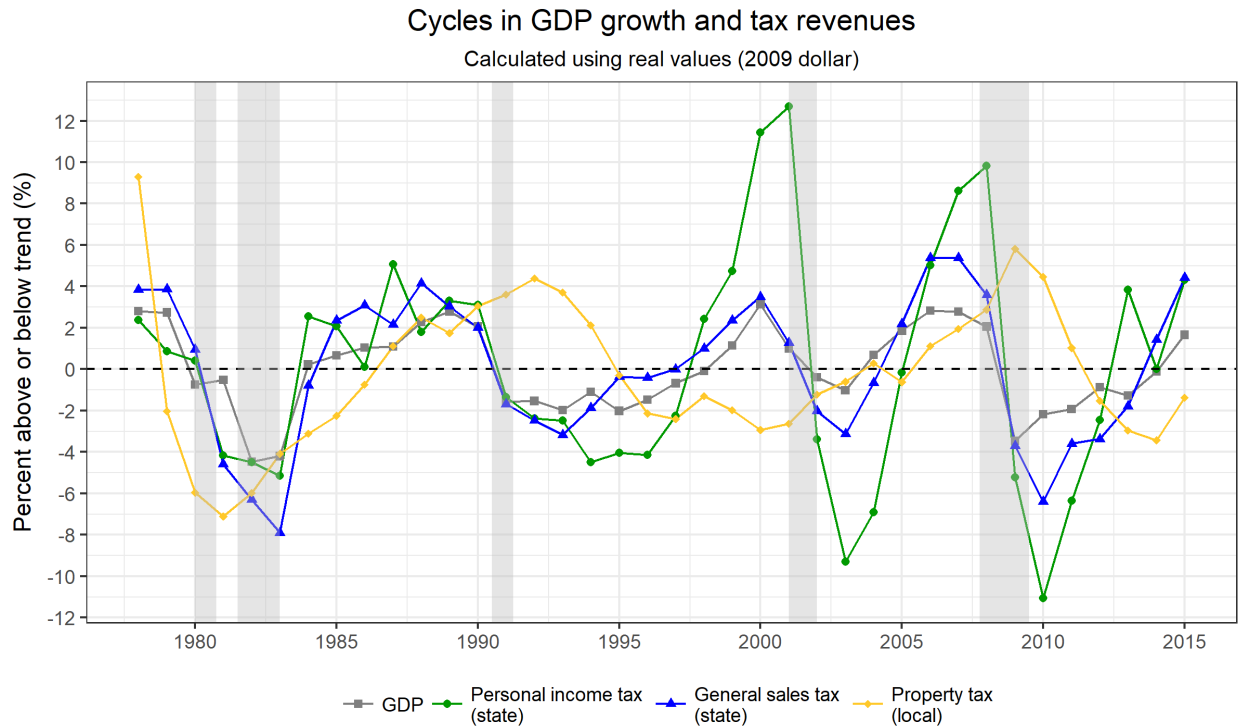
Individual taxes can exhibit different long-run behavior relative to the economy – for example, progressive income taxes generally will grow more quickly than the economy, while state sales tax bases have been declining relative to the economy.<sup>7</sup> In our analyses below, we assume that politicians will adjust tax bases and rates to maintain their shares of the economy over the long run, but that there will be cycles around this trend.<sup>8</sup> Our job is to construct reasonable estimates of these cycles, for different taxes, that relate tax revenue cycles to GDP and other outputs from our economic scenario generator.

To estimate relationships between the cyclical components of tax revenue and the economy, we use annual national data for 1977-2015 on tax revenue for the major tax types, obtained from the Urban Institute’s Data Query System (SLF-DQS).<sup>9,10</sup> We adjust each series for inflation using the GDP price index. We then remove the trend from each tax series, from real GDP, and from stock market returns using a decomposition approach known as the Hodrick–Prescott filter, which allows the trend to move slowly over time.<sup>11</sup> We subtract the trend of each series from its unadjusted value to obtain the cyclical component.<sup>12</sup>

The cyclical components of the three largest state government tax revenue sources (personal income taxes, state sales taxes, and local property taxes) are shown in Figure 2, along with the cyclical component of real GDP, each as a percentage of its trend. We make several observations: First, the income tax and sales tax have cycles that occur close in time to GDP cycles, but the cyclical pattern of the property tax, if there is one, is less clear.<sup>13</sup> Second, both

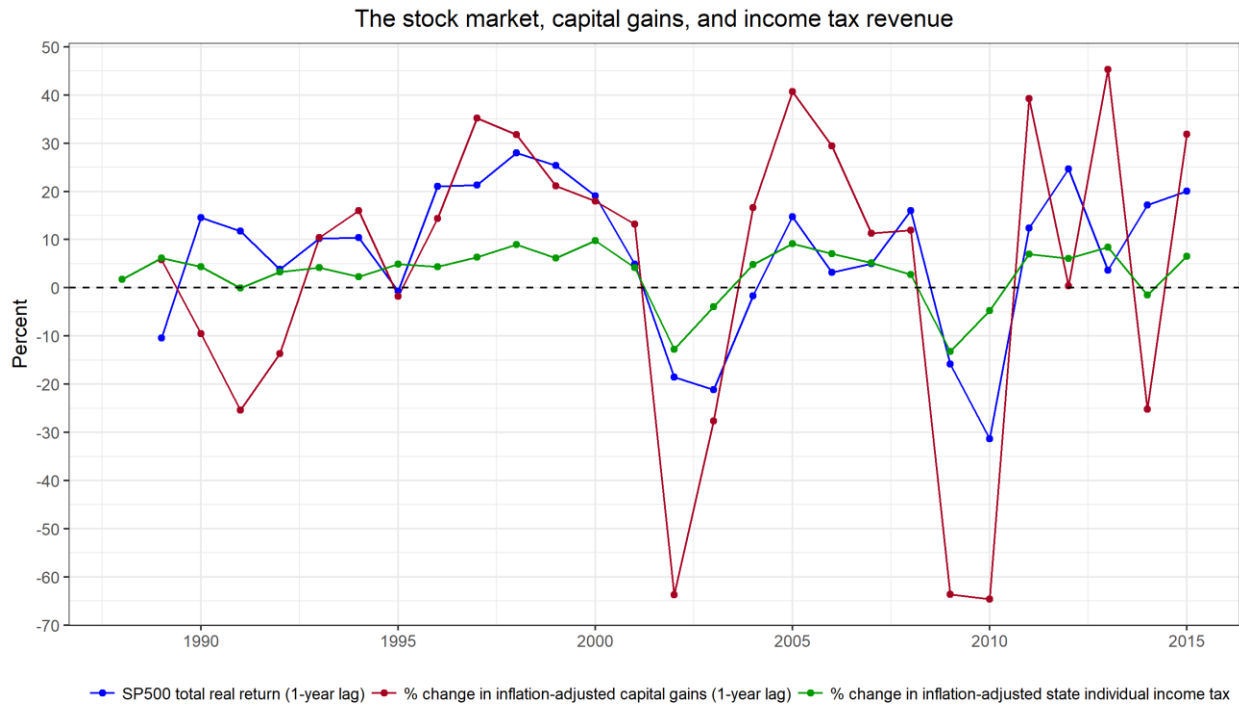
the income tax and the sales tax tend to have greater cyclical swings than GDP, relative to their trends. Third, the income tax cycles became far larger near the recessions of 2001 and 2007 than they had been near earlier recessions. We explore this further below.

Figure 2 Cycles in revenues of major taxes



The income tax presents interesting issues, because its cyclical behavior is highly dependent not just upon economic growth, but also upon how recessions and recoveries affect asset values and thereby capital gains from assets.<sup>14</sup> Figure 3 shows inflation-adjusted total stock market returns as measured by the S&P 500, inflation-adjusted capital gains, and inflation-adjusted state income tax revenue. The influence of stock market returns and values on capital gains is apparent, as is the impact of capital gains on income tax revenue. This figure shows that it is important to incorporate asset returns into a model of the cyclical behavior of income tax revenue.

Figure 3 Income tax revenue is influenced heavily by stock-market-driven capital gains



Based on these and other observations, we model the cyclical growth rate of real tax revenues as a function of the cyclical growth rate of real GDP and, in the case of the personal income tax, the cyclical growth rate of real stock market values.<sup>15</sup> In this approach, the coefficient of the percentage change in cyclical tax revenue relative to cyclical GDP growth can be interpreted as a cyclical elasticity.

#### Why we did not model the property tax and how we might incorporate it in the future

We do not include the property tax in our models, despite its importance to local governments, because we do not think its relationship to the economy can be modeled simply. In general, the property tax is more stable and resilient than other major taxes such as the personal income tax and the sales tax. This is especially important in recessionary periods. However, this conclusion also depends greatly on the institutions and rules affecting the property tax.

The economy has direct impacts on income and sales taxes: economic changes cause tax-base changes (e.g., a recession causes slower growth, or even declines, in wages), and the tax-base changes cause changes in tax collections (e.g., slower wage growth causes slower income tax withholding growth), generally with some lags.

Property tax mechanisms are more complicated: Economic changes may or may not lead to substantial changes in the market values of properties. This has varied from recession to recession. Often, property-value changes are not large (the 2007 recession was an exception).

Market-value changes may lead to changes in assessed values. How long that will take depends upon several factors including cycles for assessing property (which may be as long as 3 years or 5 years, or even longer), and both general and ad hoc governmental policies on how quickly changes in market values should be reflected in assessed values. For example, Proposition 8 in California required rapid adjustments in assessed values in the wake of the Great Recession.

Assessed value changes may lead to changes in taxable assessed value. However, these changes may not occur all at once. They may be phased in over several years. They may differ from one type of property to another, such as when governments place limits on whether or how rapidly taxable assessed values of homeowner properties may be adjusted. Some may not be adjusted until properties are sold.<sup>16</sup>

Finally, many governments calculate their property tax levies and then adjust tax rates to achieve those levies. But other governments are constrained by institutions that limit the size of the levy, or growth in the levy, or changes in tax rates, or tax rates as a percentage of property values. In the Great Recession, California, governments found themselves constrained by Proposition 13, which limited tax rates as a percentage of property value making it hard for most governments to maintain tax levies in the face of declining property market values and assessed values. But local governments in other states, such as New Jersey and New York, often were able to raise tax rates to offset the impact of declining assessed values. Examination of property tax revenue trends for individual local governments in these states shows very different revenue responses to the Great Recession.

Thus, institutions and rules that vary by state and even by specific local governments can influence how property tax revenue is affected by economic changes, and how quickly those changes affect property tax revenue. For these reasons, we have not modeled the property tax in this paper.

In future analyses, we may be able to incorporate the property tax by constructing several stylized property-tax-dominant governments that operate in different institutional environments. For example, one stylized government might have institutions that cause it to reassess properties slowly, and allow it to adjust tax rates to maintain the tax levy, while another might be required to reassess properties quickly and have limited ability to adjust tax rates.

### Combining trend and cyclical growth

In our simulation model, we obtain growth for a revenue source by adding its trend growth to its cyclical growth, where the cyclical component will depend upon GDP growth from our economic scenario generator and, in the case of the income tax, upon stock market returns. As noted earlier, we assume – lacking any basis for a better assumption - that politicians will adjust tax revenue periodically to keep revenue as a constant share of the economy, meaning that trend revenue will grow at the same rate as trend GDP. We adopt the Congressional Budget Office’s assumption that trend growth in GDP over the next 30 years is likely to approximate 1.9

percent annually, reflecting anticipated labor force growth, labor force participation, and productivity. Thus, we assume trend tax growth is 1.9 percent for each tax source.

Table 6 shows the trend growth rate for each tax and our assumptions, based upon econometric estimates, of cyclical revenue elasticities. The first row shows the trend growth rate. The second row shows the cyclical elasticity for the revenue source, relative to GDP growth. For example, the sales tax cyclical elasticity is 1.2. Thus, if GDP has a cyclical decline (relative to its trend) of 3 percent, the sales tax will have a cyclical decline of 3.6 percent relative to its trend. The income tax is more complicated because it also reflects a cyclical relationship to stock market returns.

Table 6 Tax revenue elasticities

Assumptions about trend and cyclical growth rates of tax revenues for simulation analysis					
Growth rates in inflation-adjusted tax revenue					
		Individual income tax	General sales tax	Selective sales tax	Other taxes
<b>Trend growth rate</b>		1.9%	1.9%	1.9%	1.9%
<b>Elasticity of cyclical change</b>	<b>Real GDP growth</b>	1.0	1.2	0.5	1.3
	<b>Real Stock returns</b>	0.2	-	-	-

An example may help to explain how we apply these parameters in our simulation. Table 7 illustrates this for a hypothetical expansion year in a hypothetical simulation, in which real GDP grows 1 percentage point above its trend growth and real stock returns are 2 percentage points above their trend return.<sup>17</sup>

The first row shows how we calculate growth for the individual income tax, which is the most-complex case. The first two columns show that we estimate the elasticity of the cyclical component of the real income tax to the cyclical component of real GDP is 1.0, and the elasticity of the cyclical component of the real income tax to the cyclical component of real stock returns is 0.2 (these results are drawn from Table 6.) The next two columns show the cyclical real GDP growth and cyclical real stock return values for our hypothetical year, as mentioned in the paragraph above.

The final column shows the calculation of the cyclical component of the income tax: (a) it is boosted by 1 percentage point because real GDP is growing 1 percentage point above trend, and the elasticity is 1.0; and it is boosted by another 0.4 percentage points because real stock

returns are 0.4 percentage points above trend and the elasticity relative to stock returns is 0.2. Adding the two calculated results, cyclical real income tax growth is 1.4 percentage points. Total real income tax growth is the sum of its trend revenue growth plus this cyclical component.<sup>18</sup>

The remaining rows of the table illustrate the calculation for the other taxes. This calculation is much simpler because these taxes generally are not correlated with stock returns and our models for these taxes do not incorporate stock returns.

Table 7 How we use model parameters to calculate tax revenue growth in the simulations

**An example of the calculation of cyclical tax revenue growth in the simulation**

	Elasticity with respect to		Hypothetical Cyclical GDP growth (c)	Hypothetical Cyclical Stock return (d)	Cyclical tax revenue growth (a)×(c) + (b)×(d)
	Cyclical GDP growth (a)	Cyclical stock return (b)			
<b>Individual income tax</b>	1.0	0.2	1%	2%	$1\% \times 1 + 2\% \times 0.2 = 1.4\%$
<b>General sales tax</b>	1.2	0	1%	2%	$1\% \times 1.2 = 1.2\%$
<b>Selective sales tax</b>	0.5	0	1%	2%	$1\% \times 0.5 = 0.5\%$
<b>Other taxes</b>	1.3	0	1%	2%	$1\% \times 1.3 = 1.3\%$

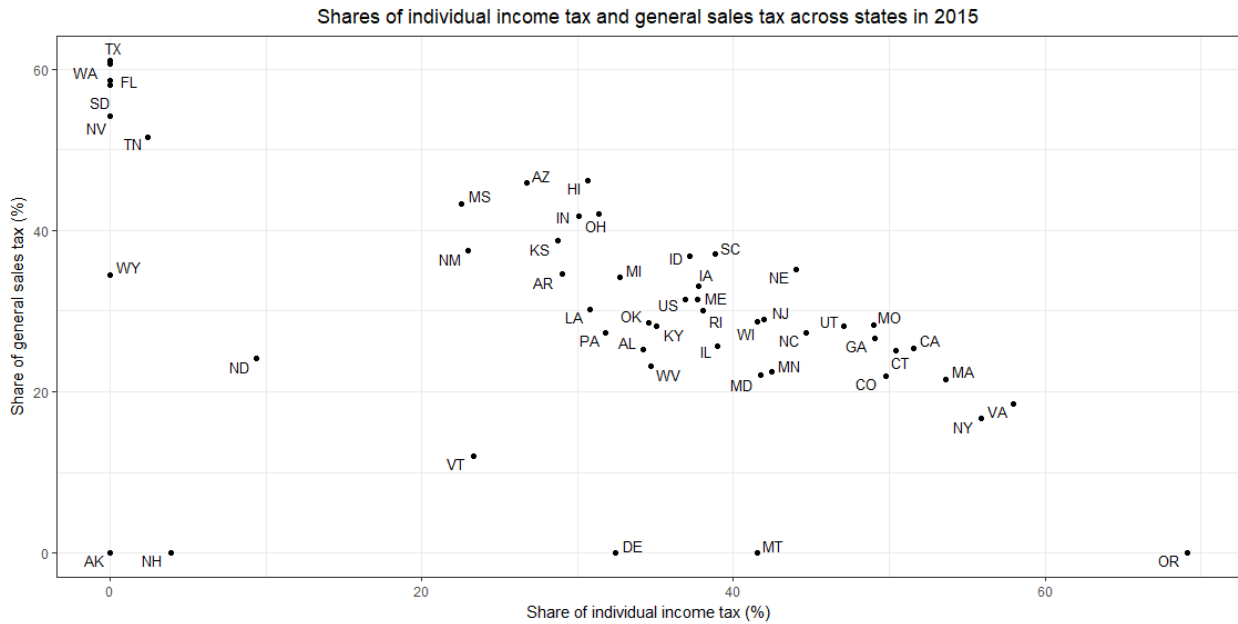
Notes:

1. All rates are inflation-adjusted values.
2. Cyclical growth of GDP and stock return are defined as the difference between total growth rate and trend growth rate.
3. For each tax category, total tax revenue growth is the sum of the cyclical growth calculated in the table and the assumed trend growth (1.9% in the simulation).

### Constructing tax portfolios for stylized governments

After we model the cyclical behavior of individual taxes, we construct tax portfolios for two stylized governments, consisting of a mix of income, sales, and other taxes. In Table 5 we showed that the income tax and the general sales tax are the two largest state government taxes, on average, and that the property tax dominates local government taxation. Before constructing stylized governments, it is useful to look at the distribution of the two largest state taxes, as shown in Figure 6.

Figure 4 Individual income tax and general sales tax shares of state tax revenue



States clustered in the upper left corner of Figure 6 rely heavily on the sales tax and have no income tax, including the large states of Florida, Texas, and Washington. These sales-tax-dominant states constitute an important group, especially given that the sales tax and the income tax have different cyclical behaviors, as discussed earlier. Most of the other states rely partly on the income tax and the sales tax. (Alaska and New Hampshire are the only two states with no general sales tax and no broad-based income tax.<sup>19</sup> Oregon is unique in relying extremely heavily on the income tax, and having no general sales tax.) While there are not other obvious groupings like the sales-tax dominant states, we think it makes sense to examine states that rely heavily on the income tax, with some sales tax reliance as well, as several large states fit this model, including California, Massachusetts, New York, and Virginia.

Based upon this analysis, we construct two stylized governments: a sales-tax-dominant state, relying on the general sales tax for 60 percent of its revenue, with no income tax, and an income-tax-dominant state, relying on the income tax for 55 percent of its tax revenue and using a mix of other taxes for the remainder.



Table 8 Tax structure of stylized state governments

<b>Tax revenue structure of stylized state governments</b>					
Each tax as a percentage of total tax revenue					
	<b>Personal income tax revenue</b>	<b>General sales tax revenue</b>	<b>Selective sales tax revenue</b>	<b>Other taxes</b>	<b>Total</b>
<b>Personal income tax dominant state</b>	55%	20%	10%	15%	100%
<b>Sales tax dominant state</b>	0%	60%	25%	15%	100%

Tax revenue in these stylized states will reflect the composition of the government’s tax structure as well as a business cycle and its character.

In our analyses below, we measure the burden of pension contributions as a percentage of tax revenue for these stylized governments. We also examine a “baseline” government in which tax revenue grows at the same rate as GDP, without any further cycle.

### Compounded risks from correlated business cycles, investment returns, and tax revenue

We use our model to examine the potential compounding of risks when investment returns and governmental tax revenue are driven by the same underlying economic conditions. Our key modeling assumptions are listed in Table 9.

Table 9 Key modeling assumptions

Simulation parameters based on forward-looking assumptions			
Parameter	Assumption on quarterly data (Annualized rates in parentheses except for the transition probabilities)		Target assumptions for annual data
	Expansion period	Recession period	
	4.8% (expansion to recession)	32% (recession to expansion)	
<b>GDP growth (real)</b>			Target expected geometric mean of annual GDP growth is 1.9%.
Transition probability			
Expected GDP growth	0.67% (2.7%)	-0.83% (-3.4%)	
Standard deviation of GDP growth	0.74% (1.48%)	0.74% (1.48%)	
<b>Stock return (nominal)</b>			Target expected geometric mean of annual stock return is 6.7%
Expected return	2.54% (10.6%)	-2.05% (-8.5%)	
Standard deviation	6.9% (13.8%)	11.9% (23.8%)	Target standard deviation of stock return is 17%
<b>Bond return (nominal)</b>			Target expected geometric mean of annual stock return is 3.6%
Expected return	0.9% (3.6%)	0.9% (3.6%)	
Standard deviation	2% (4%)	2% (4%)	Target standard deviation of annual bond return is 4%
Correlation between stock return and bond return	0.15		

Notes:

1. Adjustments are made to the quarterly parameters estimated with historical data to ensure the simulated results for annual data (converted from simulated quarterly data) are consistent with the target assumptions for annual data. The variables that have been adjusted and their historical-data based estimates (all for quarterly data) are expected GDP growth in expansion (0.95%), GDP growth in recession (-0.55%), expected stock return in expansion (3.2%), expected stock return in recession (-1.4%), expected bond return (1.6%), standard deviation of bond return (5.1%). (Historical bond returns are for long-term corporate bond)

3. The target annual GDP growth is obtained from the 30-year projection of potential growth GDP made by CBO (2017). The target assumptions on annual stock and bond returns are generally consistent with the capital market assumptions used in Mennis, et. al (2017).

We use our economic scenario generator to generate 2,000 simulations of 30 years of economic growth, using ESG outputs, in turn, to drive tax revenue models and a pension fund simulation model. Finally, we construct measures of fiscal pressure based upon required pension contributions and governmental tax revenue.

### Illustration of a single simulation

We first illustrate the simulation results using a single simulation (#2) selected from 2000 stochastic simulation runs.

Figure 5 presents simulated GDP growth and stock returns. The years marked by dashed vertical lines have at least one quarter in recession<sup>20</sup>. In this particular simulation, economic downturns are mostly associated with sharp declines in stock returns (the model may generate recessions without large drops in stock returns in other simulations).

Figure 5 Illustrating the relationship between simulated economic conditions and tax revenues using a single run of simulation

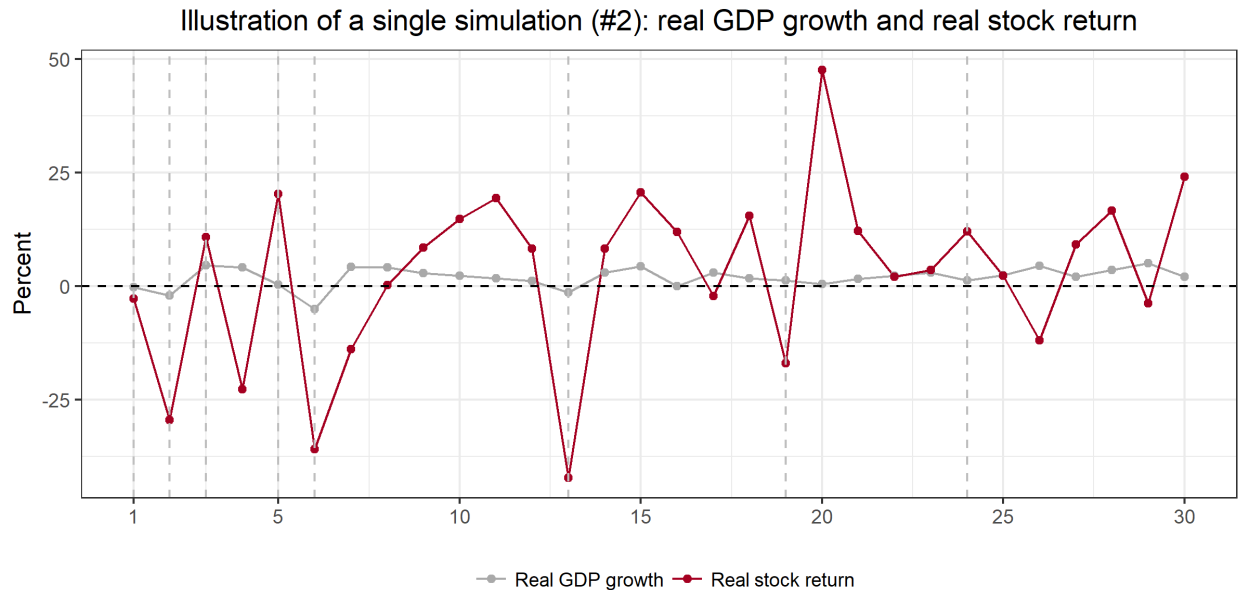
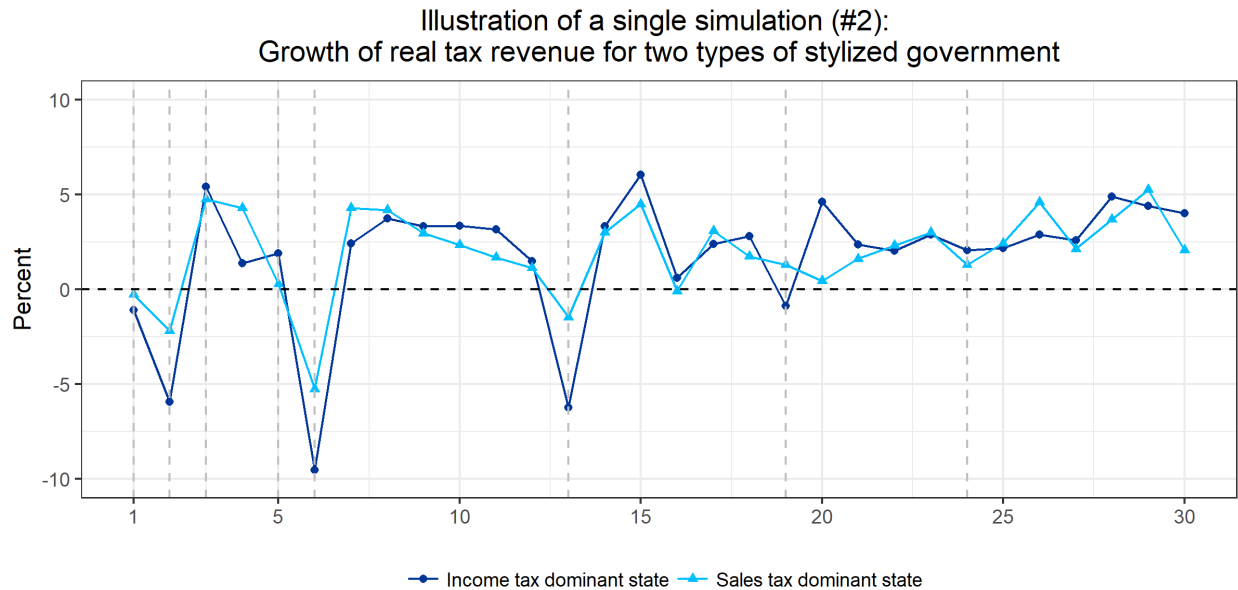


Figure 6 shows how tax revenues of the two types of stylized governments respond to the simulated economic conditions. The tax revenue of the income-tax-dominant state is generally more volatile than tax revenue of the sales-tax-dominant state and has larger declines during recession periods, consistent with historical patterns.

Figure 6 Simulated tax revenue growth is more volatile in the income-tax-dominant state than in the sales-tax-dominant state



### Distribution of growth in real tax revenue

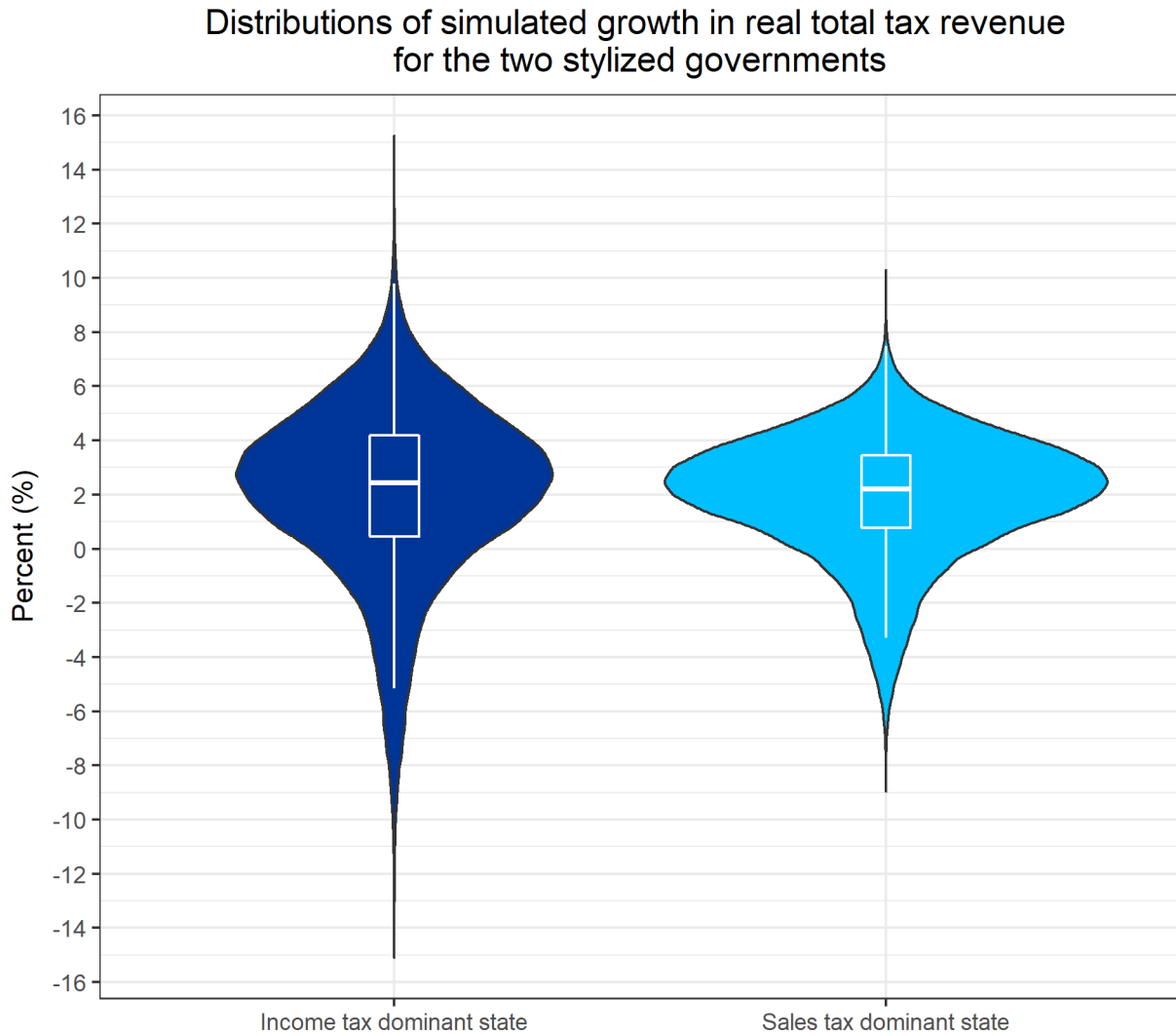
Figure 7 summarizes the distribution of total tax revenue growth of the two stylized governments across all 2,000 simulations and all years.

This is known as a “violin graph” because of its shape. The box in the figure for each stylized government describes the middle 50 percent of outcomes – the horizontal line in middle of the box marks the median, the line at the top of the box marks the 75<sup>th</sup> percentile, and the line at the bottom marks the 25<sup>th</sup> percentile. Thus, we can see that the median tax revenue growth in the income-tax-dominant state was between 2 and 3 percent, growth at the 75<sup>th</sup> percentile was just over 4 percent, and growth the 25<sup>th</sup> percentile was just below 1 percent. We can see that the box for the income-tax-dominant state is larger than the box for the sales-tax-dominant state, indicating that the income-tax-dominant state had a greater range of growth rates than the sales tax state (its tax revenue was more volatile). The lines, or “whiskers” of the graph extend up and down from the box, indicating the 90<sup>th</sup> and 10<sup>th</sup> percentiles.

The mirror-image bulges on each graph show the distribution of outcomes, much like a bell curve turned sideways. (The left and right bulges, for a given stylized state, are symmetric, both showing the shape of the distribution.)

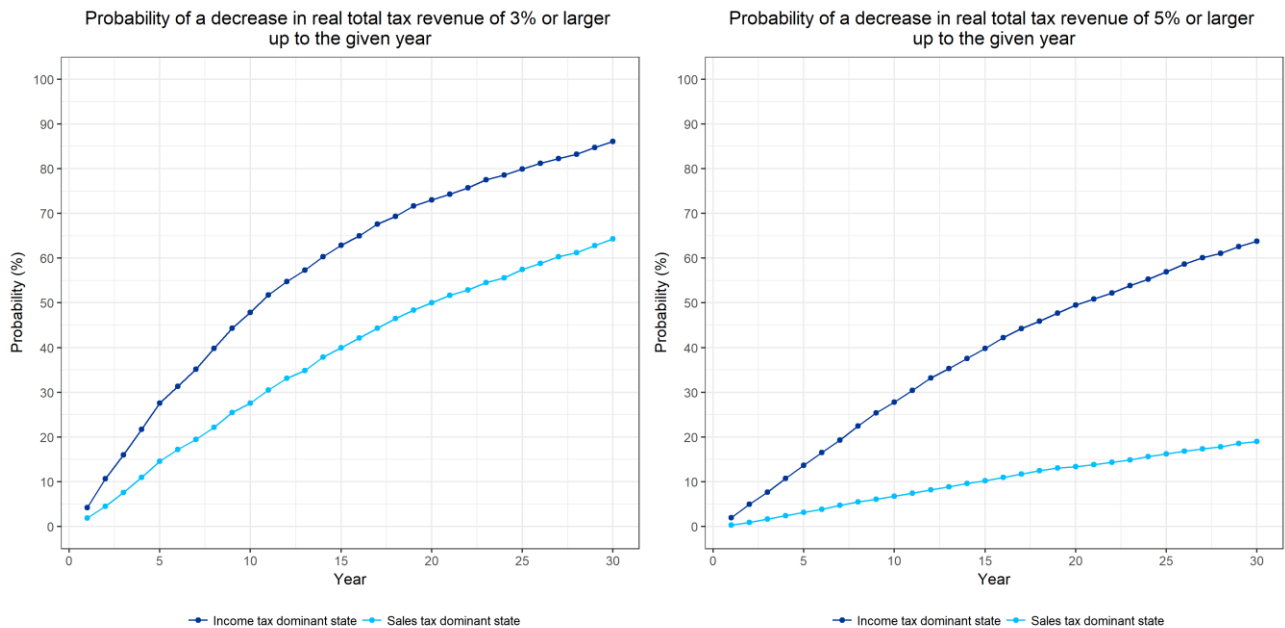
Tax revenue of the income-tax-dominant state is more volatile than tax revenue of the sales-tax-dominant state primarily because its revenue responds not only to GDP growth but also to capital gains driven in large part by stock market returns.

Figure 7 Simulated tax revenue is more volatile in the income-tax-dominant state than the sales-tax-dominant state



Next, we examine the likelihood of large declines in tax revenue for the two stylized governments in the 30-year simulation period. Figure 8 shows the probabilities of the stylized government having experienced a 3 percent or 5 percent drop of tax revenue in a single year up to a given simulation year. As expected, the income-tax-dominant state is much more likely to experience a large decline in tax revenue compared to the sales-tax-dominant state.

Figure 8 Probability of large tax revenue declines is greater in the income-tax-dominant state than the sales-tax-dominant state



### Simulating the finances of public pension plans

Separately, we have developed a simulation model that can be used to evaluate the implications of public pension plan investment risk.<sup>21</sup> The model calculates the actuarial liabilities, annual cash flows, funded status, and covered payroll of a public pension plan for future years based on the benefit rules, actuarial and economic assumptions, and demographic structure of the plan. Each year the model starts with beginning asset values and computes ending assets by subtracting benefits paid, adding employee and employer contributions (including any amortization), and adding investment income, which we calculate in the model. The model keeps track of these values and other variables of interest, such as the funded ratio and employer contributions as a percentage of payroll. It saves all results so that they can be analyzed after a simulation run in any way desired.

The model can be used to examine prototypical pension funds or can be used with data for actual pension funds. In the analysis that follows, we use a prototypical fund that resembles real-world pension plans in important ways. The key elements of the prototypical plan are described below.

- Plan characteristics.** It has a typical age distribution of workers and retirees, and benefits generally are calculated as 2.2 percentage points per year of service multiplied by the average of the final three years of salary, plus a 2 percent annual increase akin to a cost-of-living adjustment (COLA).<sup>22</sup> The age structure of the plan population is based on our analysis of data in the Public Plans Database, and is similar to the population of the Arizona Public Employees Retirement System, which we found to be fairly typical in

many ways. We assume that the plan has new hires each year sufficient to keep the number of active workers stable. The plan sponsor makes contributions each year. The plan starts off 75 percent funded with the actuarially liability calculated using a 7.5 percent discount rate.

- **Discount rates.** (assumed rate of return): 7.5 percent, a common assumption among public pension plans.
- **Funding policies.** Most pension funds adopt funding policies that can dampen the volatility in contribution caused by unexpected investment losses and gains and other deviations from their actuarial assumptions. The choice of funding policy also determines how fast the plan pays down its Unfunded Actuarial Accrued Liability (UAAL). We examine three funding policies with differing smoothing effects on contributions.
  - 1) 10-year level-dollar open amortization with 5-year asset smoothing. (Fastest repayment of UAAL; least contribution-smoothing effect)
  - 2) 15-year level-dollar open amortization with 5-year asset smoothing.
  - 3) 30-year level-percent open amortization with 5-year asset smoothing. (Slowest repayment of UAAL; strongest contribution-smoothing effect; common in plans with high UAAL)
- **Investment returns.** We examine two sets of stochastic investment returns:
  - 1) Returns generated by the regime-switching simulation model. As described earlier, the regime-switching model generates 2,000 simulations each with a 30-year series of equity returns and bond returns, linked to GDP growth. We construct a portfolio consisting of 70 percent of equities and 30 percent bonds.
  - 2) Returns drawn from normal distribution. The expected annual return and standard deviation are equal to those of the simulation-based 70/30 portfolio. 2,000 random returns are drawn from the normal distribution for each year in the simulation.

Given a funding policy and a set of investment returns, we run 2,000 simulations and compute the required employer contributions, the funded ratio, and other variables of interest of the prototypical pension plan. This allows us to gain insight into the fiscal stress that the pension plan creates for the stylized governments by comparing the simulated employer contributions against the simulated tax revenues.

### [Evaluating the potential compounding of risks](#)

By this point, we have tax revenues, asset returns, and pension finances that are all generated within a coherent simulation framework. Changes in tax revenues and investment returns, which in turn affect required pension contributions, are both driven by business cycles that are simulated by the regime-switching process of GDP growth between recessions and expansion periods.

With these simulation outputs of the linked models, we can examine the compounding of risks from correlated investment returns and tax revenues. We evaluate the pension-related risks for the two stylized governments - an income-tax-dominant state and a sales-tax-dominant state - and compare that with the risks under a baseline model that has no linkage between tax revenue and asset returns. This comparison will demonstrate how pension-related risks can be understated if the linkage is ignored.

### Using alternative model structures to decompose risks

We constructed alternative model structures to investigate how much of the increase in risk can be attributed to each of the following: (1) linking tax revenue to the economy, (2) linking asset returns to the economy, (3) the combined effects of linking tax revenue and asset returns to the economy, and (4) tax structures of the stylized governments, and the fact that income tax revenue is linked directly to GDP growth as well as to stock returns, potentially creating additional risk for the income-tax-dominant state.

The table below summarizes the six model structures we compare.

*Table 10 Simulation models for comparison*

Model	Description	Tax revenue linked to GDP?	Asset return linked to GDP?	Based on stylized government?	Tax revenue linked to asset return?
(1)	<b>Unlinked model:</b> - Constant growth of total tax revenue (equal to trend GDP growth). - Returns from normal distribution.	No	No	No	No
(2)	<b>Asset return linked only:</b> - Constant growth of total tax revenue (equal to trend + cycle GDP growth). - Returns from regime-switching simulation model.	No	Yes	No	No
(3)	<b>Tax revenue linked only:</b> - Cyclical growth of total tax revenue (equal to trend plus cycle GDP growth). - Returns from normal distribution.	Yes	No	No	No
(4)	<b>Both tax revenue and asset return linked:</b> - Cyclical growth of total tax revenue (equal to trend plus cycle GDP growth). - Returns from regime-switching simulation model.	Yes	Yes	No	No
(5)	<b>Stylized government: sales-tax-dominant state</b> - Cyclical growth of total tax revenue; estimated responsiveness to GDP growth. - Returns from regime-switching simulation model.	Yes	Yes	Yes	No
(6)	<b>Stylized government: income-tax-dominant state</b> - Cyclical growth of total tax revenue; estimated responsiveness to GDP growth and asset return. - Returns from regime-switching simulation model.	Yes	Yes	Yes	Yes

Model (1) serves as the baseline case in which neither tax revenue nor asset return are linked to the regime-switching process of GDP. It assumes that the total real tax revenue of the



government grows at a constant rate which is equal to the trend growth rate of GDP (1.9 percent) and investment returns are normally distributed.

Model (2) only links asset returns to GDP cycles and Model (3) only links tax revenue, so the simulation results of these two models reflect the isolated effects of linking only one variable to the business cycle.

Model (4) links both tax revenue and asset returns to GDP cycles and reflects the compounding effects of correlated tax revenue and asset returns. In Models (3) and (4), the growth rate of total tax revenue equals the total GDP growth rate (trend plus cycle).

Models (5) and (6) also link both tax revenue and asset returns to business cycles, but they differ from Model (4) in that they incorporate our analysis of how tax revenues respond to business cycles for the two stylized governments – the sales-tax-dominant state in Model (5), and the income-tax-dominant state in Model (6). A distinguishing feature of Model (6) is that the dominant tax category – personal income tax – is linked not only to GDP growth, but also to stock market returns.

#### Measures of potential fiscal pressure

Under each model structure, actuarially determined pension contributions under the three alternative funding policies described in the section. Simulating the finances of public pension plans are generated by our pension simulation model. We treat actuarially determined contributions as required employer contributions, although governments do not always pay them. Employer contributions as a percentage of the total tax revenue in the first simulation year are determined as follows:

- We assume that the required employer contribution under the policy “30-year open constant percent of payroll” accounts for 5 percent of the total tax revenue of the sponsoring government in year 1.
- The total tax revenue calculated based on this assumption is then applied to the other two policies.
- The shares of employer contribution in the total tax revenue in year 1 are 7.3% under the “15-year open constant dollar” policy and 8.7 percent under the “10-year open constant dollar” policy.

We examine two types of risks that the sponsoring governments of public pension plans may face:

1. The required employer contributions become very high relative to fiscal resources available to the sponsoring government, creating great fiscal pressure and potentially crowding out other public services. In our analysis of the simulation results, we measure this type of risk by the probability that employer contributions as a percentage of total tax revenue will become more than 5 percentage points higher than the level in year 1 at any time during the 30-year simulation period.

2. The required employer contributions rise sharply in a short period of time, creating difficulty in budget planning and short-term fiscal pressure. We measure this type of risk by the probability that employer contribution rises more than 3 percent of total tax revenue in a 2-year period at any time during the 30-year simulation period.

Our 3-percent- and 5-percent-of-tax-revenue thresholds for our two risk measures are arbitrary, but we think they are broadly indicative of tax-revenue changes that cause fiscal stress. They are based in part upon our experience analyzing and participating in state budget decision making over several decades. One way to think about this is to examine tax revenue shortfalls in recessions. In the first two years of the 1990, 2001, and 2007 recessions, tax revenue in the median state fell short by about 3.6 percent, 13 percent, and 14 percent, respectively.<sup>23</sup> From this perspective, our two-year risk measure involves less fiscal pressure than tax shortfalls in recent recessions, but given that pension contributions typically only are three to seven percent of state tax revenue, the risk measure will only be triggered if contributions increase very significantly. Our longer-term risk measure is larger than our short-term indicator, but will only be triggered if there is a sizable shift in the use of tax revenue over a long period of time.

Another way of thinking about the risk measures is to conduct sensitivity analysis, using different (but still arbitrary) thresholds, to see whether alternative measures lead to different conclusions about the impact of considering linkages between the economy, investment returns, and tax revenue. We discuss the results of sensitivity analysis in the next section; alternative thresholds do not change any of our fundamental conclusions.

### Summary of simulation results

#### The risk of high employer contributions

Table 11 shows the risk of employer contributions rising by 5 percentage points or more relative to the starting point under our different models and funding policies. The three rows of the table show our three funding policies and the six columns show the size of the risk under our six models.

The first column presents the risk measure when tax revenue grows smoothly over time without any business cycle effects, and investment returns are normally distributed and independent over time, without any correlation to the economy. It is akin to the kinds of simulations we have done in past papers, and the risk measures are lower than measures for all of the models to the right. For example, there is a 6.8 percent chance that employer contributions will rise by more than 5 percentage points of tax revenue above their year 1 levels at some point during the 30-year simulation period – a smaller risk than in any column to the right.

The second column also assumes that tax revenue grows smoothly, but now investment returns are generated by the regime-switching simulation model that results in a heavier left tail in the return distribution than in the normal distribution, and returns are correlated with economic

conditions rather than being independent over time. Thus, comparing this column to column 1 shows us the pure effect of linking investment returns to the economy, without considering tax revenue cyclicity, shedding light on the impact of introducing non-normality in investment returns. The risks of high employer contributions are only slightly higher than in the base case.

Column 3 now allows tax revenue to be exactly as cyclical as real GDP is – for example, if real GDP grows 2 percentage points more quickly than its trend, tax revenue will grow at the same rate. Investment returns are drawn from the normal distribution rather than being generated by our ESG. Thus, comparing column 3 to column 1 shows us the pure effect of introducing volatility in tax revenue that is not correlated with investment returns, without accounting for differences in tax structure. The risks are slightly greater than in the base case and not much different from those in column 2.

Column 4 allows tax revenue to be exactly as cyclical as real GDP and has investment returns generated by the model. Comparing this column to column 3 shows the risk introduced by having investment returns linked to the economy in addition to cyclical tax revenue. Comparing this column to column 2 shows the risk introduced by having cyclical revenue. In either comparison, the increase in risk is quite large. For example, under a 10-year open constant-dollar funding policy, the risk of a contribution increase that is at least 5 percentage points of tax revenue is 17.8 percent, or more than a one in six chance of such an increase during the 30-year simulation period. Comparing column 4 to column 1 demonstrates that while the isolated effects of linking tax revenue or investment returns to the business cycle are modest, the compounding effect of correlated tax revenue and investment returns is quite large. For example, under the 15-year open constant dollar funding policy, the risk of high pension contribution is 2.7 times as great much as in column 1 (17.2 percent compared to 6.4 percent), while for columns 2 and 3 the measures are only 1.4 and 1.5 times as great.

Columns 5 and 6 introduce our two stylized governments, with tax revenue cycles depending not just on real GDP cycles but also on tax structure. In both models, returns are generated by the economic scenario generator. The risks for the sales-tax-dominant government actually are slightly smaller than if revenue is exactly as cyclical as real GDP.<sup>24</sup> The risks for the income-tax-dominant state are much larger: the risk of a large employer contribution is 22.8 percent, compared to 16.5 percent for the sales-tax-dominant state, and compared to 6.8 percent for the basecase where neither investment returns nor tax revenue are linked to the economy – the kinds of simulations that we have reported on in past papers.

Table 11 Risk of high pension contribution relative to tax revenue

Risk of high employer contribution relative to tax revenue under alternative funding policies

Amortization method for unfunded liability	Probability of employer contribution as a percentage of total tax revenue being more than 5 percentage points above the year-1 level at any time during the 30-year simulation period					
	Constant growth of total tax revenue (equal to trend GDP growth)		Cyclical growth of total tax revenue (equal to trend + cycle GDP growth)		Stylized governments	
	Normally distributed returns	Simulated returns	Normally distributed returns	Simulated returns	Sales-tax-dominant state	Income-tax-dominant state
	(1)	(2)	(3)	(4)	(5)	(6)
10-year open constant dollar	6.8%	9.2%	9.5%	17.8%	16.5%	22.8%
15-year open constant dollar	6.4%	8.9%	9.3%	17.2%	16.6%	22.8%
30-year open constant percent of payroll	1.3%	2.0%	4.9%	11.1%	10.1%	18.0%

Note: In year 1, employer contribution as a percentage of total tax revenue under the three amortization methods are 8.67% (10-year open constant dollar), 7.25% (15-year open constant dollar), and 5% (30-year open constant percent of payroll).

Comparing risks as we move down the rows of Table 11 also is instructive, as it shows the interplay between funding policy and contribution risk. Risks of large employer contributions fall somewhat as the funding policy lengthens, particularly as we move from 15 years to 30 years and change from constant dollar funding to level-percentage-of-pay funding. Because this risk measure compares every year in a 30-year simulation to the starting year, it is a measure of longer-term risks, and funding policies that smooth contributions have only limited ability to reduce this risk. They have much greater ability to reduce the risk of large employer contributions in a short period of time, as discussed below.

To examine the sensitivity of our conclusions to our 5-percent-of-tax-revenue threshold, we constructed two additional tables that are in the same form as Table 11, but one was calculated with a 3 percent threshold and the other was calculated with a 7 percent threshold. While the numbers all differ from those calculated with the 5 percent threshold, the qualitative conclusions do not change: risks of high contributions are significantly greater when we consider the impact of correlations among GDP cycles, investment return cycles, and tax revenue cycles, and risks for the income-tax-dominant stylized government are considerably greater than for the sales-tax-dominant stylized government.

The risk of large contribution increases in a short period of time

Table 12 presents our short-term risk measure – the probability that employer contributions will rise by more than three percent of tax revenue in any two-year period during our 30-year simulation - for the same models and funding policies. The format and interpretation are the same as for Table 11.

As with our other risk measure, the risk increases as we move to the right in the table, but not as much as with our longer-term risk measure, the risk of a large contribution increase over the 30-year period. Furthermore, risks of sharp employer contribution increases diminish

dramatically as we move down the rows of the table, lengthening the contribution smoothing period and liberalizing the policy, in the bottom row, by using constant-percentage of pay funding: long smoothing periods and more-liberal funding policies reduce short-term employer contribution risks substantially. As with our first risk measure, we also conducted sensitivity analysis of our large-contribution-increases-in-a-short-period risk measure. We examined thresholds based upon 2 percent of tax revenue and 4 percent of tax revenue, as alternatives to our measure reported on here based upon 3 percent of tax revenue. As with the first risk measure, our fundamental conclusions do not change.

Table 12 Risk of sharp increases in pension contributions relative to tax revenue

**Risk of sharp increase in employer contribution relative to tax revenue under alternative funding policies**

Amortization method for unfunded liability	Probability of employer contribution rising more than 3 percent of total tax revenue in any 2-year period during the 30-year simulation period					
	Constant growth of total tax revenue (equal to trend GDP growth)		Cyclical growth of total tax revenue (equal to trend + cycle GDP growth)		Stylized governments	
	Normally distributed returns	Simulated returns	Normally distributed returns	Simulated returns	Sales-tax-dominant state	Income-tax-dominant state
					Simulated returns	Simulated returns
(1)	(2)	(3)	(4)	(5)	(6)	
10-year open constant dollar	43.0%	44.1%	44.2%	48.7%	47.8%	54.7%
15-year open constant dollar	20.9%	23.7%	23.3%	31.4%	30.5%	37.8%
30-year open constant percent of payroll	0.5%	1.2%	1.1%	3.1%	2.5%	6.9%

Note: In year 1, employer contribution as a percentage of total tax revenue under the three amortization methods are 8.67% (10-year open constant dollar), 7.25% (15-year open constant dollar), and 5% (30-year open constant percent of payroll).

Contribution smoothing policies can reduce employer risks, but they do not make risks go away: they transfer them to the pension fund and its stakeholders, as we discuss in the next section.

**Contribution-smoothing policies cannot make risks go away: They transfer risks from governments to pension plans**

Lower risks for sponsoring governments under funding policies with stronger contribution-smoothing effects come at the expense of higher risk that the pension plan will become severely underfunded, as Table 13 demonstrates.

The first column of Table 13 ties to the fourth column of Table 12 to provide a useful point of reference, showing how the risk of sharp increases in employer contributions declines as the funding period lengthens and the policy liberalizes. The last column shows the risk that the plan’s funded ratio will fall below 40 percent, which we consider to be crisis territory, at some point during the 30-year simulation period. (We will explain the intervening columns in a minute.) As we move down the rows, the risk of this crisis-level funding rises from 7.1 percent with 10-year constant-dollar funding policy to 30.9 percent, or nearly a one in three chance, with 30-year constant percent of payroll funding. (This latter risk is far greater than risks we estimated in earlier papers, when investment returns were not linked to economic conditions.)

The risk protection that governments gain from stretched-out funding policies results in greater risk to pension plans.

Table 13 Greater contribution smoothing leads to higher risk of pension underfunding

	Risk of sharp increase in employer contribution relative to tax revenue*	Employer contribution as a % of tax revenue in year 1	Median Present value at year 1 of total employer contribution for year 1-15**	Median Present value at year 1 of total employer contribution for year 16-30**	Probability of low funded ratio***
10-year open constant dollar	48.7%	8.7%	1.32	0.64	7.1%
15-year open constant dollar	31.4%	7.3%	1.23	0.67	11.7%
30-year open constant percent of payroll	3.1%	5.0%	1.00	0.68	30.9%

Notes:

\* Probability of employer contribution rising more than 3 percent of total tax revenue in any 2-year period during the 30-year simulation period based on Model (4) (Cyclical growth of total tax revenue with simulated investment returns).

\*\* The present value at year 1 of total employer contribution in year 1-15 under the policy "30-year open constant percent of payroll" is standardized to 1. All other values are standardized accordingly. ☐

\*\*\* Probability of low funded ratio: the probability of funded ratio falling below 40% in any year during the 30 year simulation period.

## Conclusion

In this paper we examine how our assessment of risks to pension funds and their sponsoring governments changes when we take into consideration the correlation between economic conditions, investment returns, and governmental tax revenue. We do this by linking an economic scenario generator - a small stochastic macroeconomic model that simulates real GDP growth and investment returns – to models of tax revenue cyclicity and a pension finance simulation model. Our paper builds upon other recent research that has examined risks to pension funds and governments under alternative economic scenarios.

The simulation results demonstrate that it is important to allow for correlation among economic conditions, investment returns, and tax revenue when examining the investment-related risks to public pension funds and the sponsoring governments. Contribution increases required after economic downturns are much larger, relative to tax revenue, when we allow tax revenue to be cyclical (varying with the economy) than under simpler assumptions of stable tax revenue growth.

Pension-related risks for governments can be further exacerbated by how state tax revenue structures respond to economic conditions. All else equal, income-tax-dominant states, with highly cyclical tax revenue, face higher pension-related risks than sales-tax-dominant states, which have less cyclical tax revenues.

The choice of funding policies for public pension funds also has a significant impact on the risks that sponsoring governments face. Funding policies that pay down unfunded liabilities slowly and have strong smoothing effects on contribution requirements can protect the sponsoring

governments from the risks of high or sharp increase in pension contributions, at the expense of greatly increasing the risk of severe underfunding for the pension funds.

This paper shows how important it is to incorporate budgetary resources in pension fund risk analysis, and how that analysis can be deepened by modeling business cycles and investment returns together. Our work in this area is preliminary and can be extended and improved upon.

What do insights from this work mean for larger efforts underway to encourage pension funds and governments to conduct stress testing and to report on risks to plan beneficiaries, legislators, citizens, and taxpayers? This paper highlights that these risks are even larger than commonly understood, and that these efforts are all the more important.

Stress testing and risk reporting are, we hope, the wave of the future. The recent trend has been in that direction. The Society of Actuaries called for stress testing in its 2014 Blue Ribbon Panel report. While relatively few plans and states conduct risk reporting as a matter of course, more have been doing so, with seven states now requiring it. The Actuarial Standards Board recently has encouraged greater reporting on risks with a recent actuarial standard of practice. More states and plans should follow this lead.

First steps that plans and governments are taking now are important, with simple measures that plans can readily calculate and the public can easily understand. Comparisons to economic and revenue resources assuming stable growth are an important part of those steps, as are stress testing under asset and economic shock scenarios. Integrated stochastic analysis of the impact of business cycles and the economy, such as we do here, can be adapted to plans and governments and can further deepen risk reporting and stress testing. It not need not be part of a first step, but it should not delay first steps. Understanding and assessing risks are necessary first steps toward managing risks.

## Technical Appendices

### Linking an economic scenario generator, a pension simulation model, and cyclical tax revenue models

#### Introduction and project goals

This project continues and extends the Rockefeller Institute's pension simulation modeling, by developing a small integrated model of the economy and investment returns that we link to a pension fund simulation model. This approach allows determination of economic variables, including governmental tax bases, and investment returns, in a joint cohesive framework.

This is important for several reasons. Perhaps the most important reason is that governmental tax bases are affected by some of the same economic phenomena that influence investment returns. Intuitively, many people believe that governments are called on to pay increased contributions at the worst time, after economic variables have caused not just poor investment performance, but also deterioration in tax bases. Current stochastic simulations do not take this relationship into account and may understate the likelihood that employer contributions will become large relative to governmental resources. When contributions become especially large governments may be less willing to make full actuarially determined contributions. Determining investment returns and economic variables jointly will make it possible to assess risks to pension funds and governments more realistically than otherwise.

Our model building and simulation has the following steps:

1. Build an integrated economic scenario generator that allows us to construct internally consistent simulations of real GDP and returns on several asset classes, far into the future, following paths over time that reflect parameters estimated from historical data. (Future versions of the model also will forecast and simulate inflation.) By incorporating random errors, we simulate thousands of such paths.
2. Use outputs from the economic scenario generator as inputs to:
  - a. A pension fund simulation model, which yields as outputs information on funded status of the plan and contribution requirements by governments.
  - b. A model of the revenue resources of a small set of stylized governments that illustrate the great diversity of tax bases that different governments rely on. These simulations will affect the revenue resources of different stylized governments in different ways. For example, some economic scenarios will affect the revenue of a stylized income-tax-dependent government very differently than they will affect the revenue of a sales-tax-dependent or property-tax-dependent stylized government.
3. Combine the pension fund simulations (2.a) and the revenue-resource simulations (2.b) to analyze the stress that pension contribution requirements may place upon the finances of governments, taking into account the potential relationships between

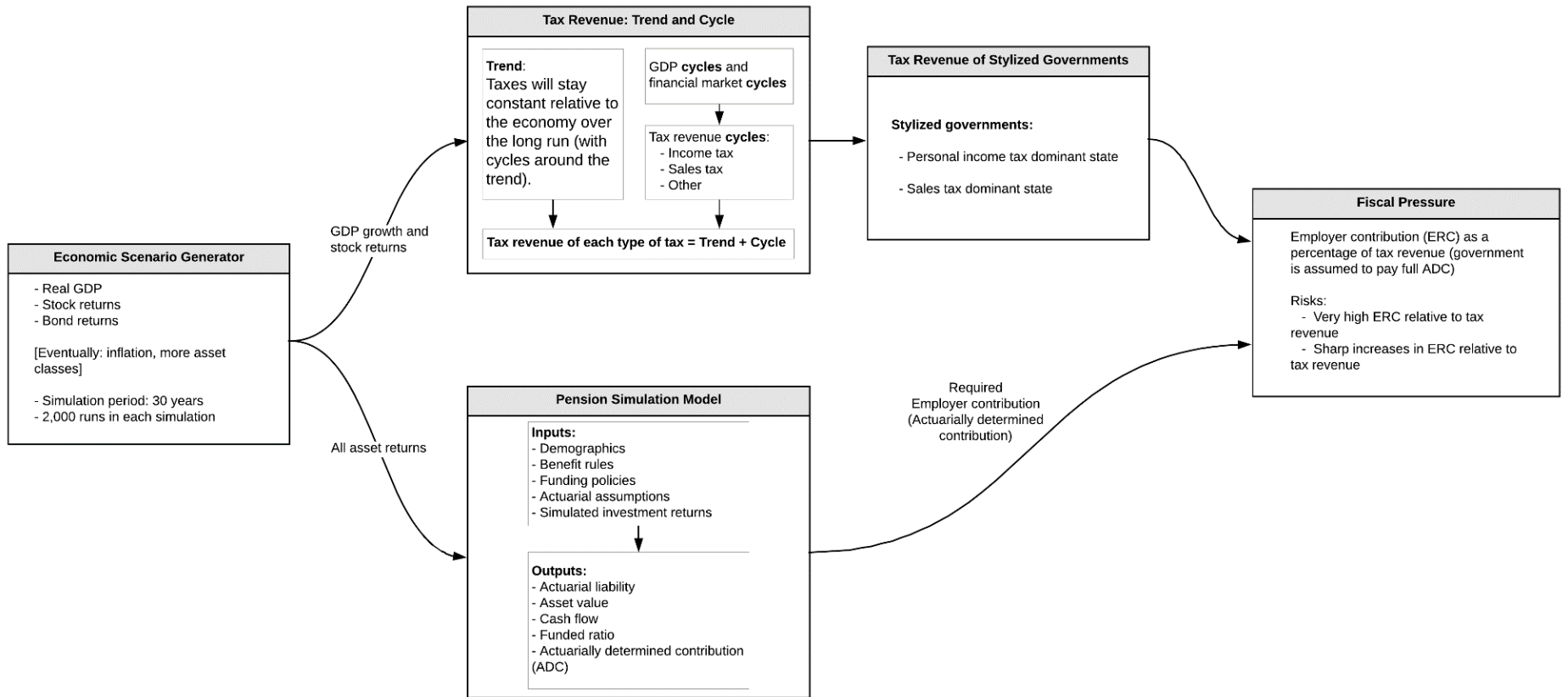


investment returns and tax revenue in a way that has not been done by other researchers.

4. Examine policy options that are best analyzed in a stochastic framework, ideally including real economic variables as well as asset returns. For example, policy options in which COLAs or other contingent benefits depend upon plan funded status require such a framework.

The major components of the model are summarized in Figure 9. The combined macroeconomic and stochastic investment model enables us to analyze important public policy issues such as stress-testing for pension funds<sup>25</sup>, and addressing or managing the cyclical relationship between pension fund finances and governmental finances. It also can be used to examine and develop optimal employer-employee risk sharing arrangements, and to examine other policy issues such as the "California rule," under which, in some states, governments cannot reduce the rate or manner in which current workers earn future benefits.

Figure 9 Structure of linked models



## Building an integrated economic scenario generator

Economists and actuaries often build small economic models that include investment returns by asset classes, and one or more other variables such as inflation, or real GDP, or other economic variables. They use these models to generate multiple future economic scenarios – perhaps thousands of scenarios – by varying the random errors in the model. In this context, the models often are called Economic Scenario Generators (ESGs). Analysts use them for risk analysis or management – for example, to examine bank capital adequacy or, in our case, to examine risks to pension funds and their sponsors.

### *Literature review*

#### Patterns of historical asset returns

##### Time-varying patterns of asset returns

It is well documented in the literature that the expected return and volatility of major asset classes exhibit time-varying patterns, which can be usually described by stochastic regime-switching processes. Regime-switching models assume that the behavior of asset returns randomly switches between several “states” in which returns have distinct return-risk profiles.

Hardy (2001) modeled the US and Canadian large-cap stock returns as a two-regime process: one with high expected return and low volatility and the other with low expected return and high volatility. They found that the regime-switching model can capture the observed extreme returns and clustered high volatility periods better than the conventional normal models and other more sophisticated time-series models.

Guidolin and Timmermann (2006) also found that a two-regime process can drive large and small firms’ stock returns, while a three-regime model is chosen for bond returns, with the states only weakly correlated the regimes of stocks, showing that stock and bond returns are governed by very different regime-switching patterns and more complex models are needed to describe the joint distribution of stock and bond returns.

Other examples of using regime-switching models to examine stock returns include Ryden et al. (1998), Turner et al. (1989), and Whitlaw (2001).

##### Time-varying correlation across asset classes

Correlation of investment returns across asset classes, or lack thereof, is the source of the benefits of portfolio diversification. There is evidence that correlations across asset classes are not time-invariant, implying that an asset allocation assuming constant correlation may fail to provide the anticipated benefits of diversification during certain time periods.

Sheikh and Qiao (2009) examined, among other topics, the correlation breakdown in joint asset class returns and found that the returns of almost all major asset classes become more correlated during periods with high market volatility, which means the “diversification may not materialize precisely when an investor needs it the most”.

Jammazi et al. (2015) examined the time-varying patterns of the dependences between stock and long-term government bond returns for a wide range of developed economies. They observed that the stock-bond association was positive during 1990s and the became negative from the early 2000s, which supports the presence of flight-to-quality effects during the two recessions in 2000s.

Guidolin and Timmermann (2006) examined the complexity of modeling the joint process of stock and bond returns and found that a four-regime model is required to capture the time-variation in the mean, variance and correlation between stock and long-term bond returns.

#### *Non-normality of asset returns*

Simulation modelers often assume that investment returns are normally distributed, and independent over time, so that current returns are not dependent upon past returns. We have made these assumptions in much of our work. Real-world investment returns can differ from these assumptions in several important ways.

#### *Fat tails*

Investment returns are said to have “fat tails” if the probabilities of particularly good outcomes or of particularly bad outcomes are greater than what we would expect if returns were distributed normally. If the investment returns have fat left tails, then the probability of particularly bad outcomes is greater than if returns were distributed normally. If investment returns have fat right tails, then the probability of particularly good outcomes is greater than if returns were distributed normally.

Sheikh and Qiao (2009) show that the historical monthly returns of all major asset classes exhibit fat left tails compared to the normal distribution. Their work also suggests that fat tails are far less prominent for annual returns than for returns at higher frequencies, such as monthly and quarterly.

#### *Mean-reversion*

Mean reversion in asset returns occurs if periods of high returns or low returns are followed by an eventual reversion to longer-term norms. There has been a great deal of academic research into the question of mean reversion, and the results are mixed. Much of the work is specific to stock market returns, although our concern must be broader: The sometimes-argued presumption that pension funds will eventually get their returns typically pertains to portfolios as a whole, not just stocks.

Two early, frequently cited papers by Poterba and Summers (1988) and by Fama and French (1988) concluded that there was evidence of long-term mean reversion in stock market returns between 1926 and 1985, generally for period lengths of three-five years. Recent research generally concludes that either there is no evidence for long-term mean reversion, or that the evidence is mixed and has been limited to specific markets such as United States equities (Jorion, 2003), or that mean reversion is more than offset by other factors (Dimson et al. 2013, Pastor and Stambaugh, 2012).

### How time-varying characteristics of asset returns are related to business cycles

To evaluate how fluctuations in investment returns would translate into pension funding pressures on sponsoring governments, it is key to understand how the regime-switching behaviors of asset returns are related to the boom-and-bust cycles of the real economy, which in turn affects the fiscal conditions of governments that sponsor pension plans.

Hamilton and Lin (1996) investigate the joint behavior of monthly stock return volatility and growth in industrial production using bi-variate models and conclude that the fluctuations in stock return volatility are primarily driven by economic recessions.

Guidolin and Timmermann (2006) construct a four-regime model to describe the joint behavior of stock and long-term bond returns and observe that the high volatility regimes occur around NBER recession periods.

Andersen et al. (2004) found that correlation between stock and bond returns switches sign in expansion versus recession.<sup>26</sup>

Other studies that have identified the correlation between asset return regimes and business cycles include Vervurt (2016), Nyberg (2012), and, Dzikevicius & Vetrov (2012).

### Two broad approaches to models of GDP and investment returns: Cascading models, and interdependent models

Economic scenario generators can be used to produce plausible stochastic future paths of economic and financial variables that are relevant to the finances of public pension funds and the sponsoring government. The key economic variables include GDP growth, short-term and long-term interest rates and returns of major asset classes including equity, bond, and real estate. (Future versions of our model also will forecast and simulate inflation.) Such a model should be (1) capable of capturing the long-term joint distribution of relevant economic and financial variables, and (2) as parsimonious as possible so that it is practical to estimate based on moderate amounts of historical data.

We examined two commonly used approaches to constructing economic scenario generators: cascading models where each equation is independent of the others, and interdependent models where equations are not independent.

#### Cascading models: The Wilkie model and its variants

One approach to ESG models has a cascade-structure in which economic and financial variables are driven by a set of core variables through a series of time series regression equations. The model proceeds sequentially: for example, this period's inflation may drive next period's stock returns and next period's bond yields, but stock and bond returns do not influence each other directly.

Wilkie (1986) developed the first comprehensive economic scenario generator of this type in which inflation serves as the core variable and drives the dynamics of stock returns and long-term bond yield. The Wilkie model was then extended to incorporate more variables including short-term bond yields, real estate prices, and wage inflation (Wilkie 1995). Among many economic scenario generation models following Wilkie's approach, Hibert et al. (2001) and Ahlgrim et al. (2005) specify the dynamics of equity return as a regime-switching Markov process, which captures the time-varying characteristics of equity returns.

#### Markov-switching vector autoregressive model (MS-VAR)

We also consider an economic scenario generator driven by the Markov-switching autoregression model (MS-VAR) (Harris, 1999), which simultaneously models the dynamics and interdependence of all variables of interest. Sherris and Zhang (2009) developed an economic scenario generator based on a MS-VAR model of 11 variables with CPI and unemployment rates being selected to be regime-switching and calibrated it to quarterly data of Australia. (See Moore and Pederson, 2013.)

#### Comparison of the two approaches

Each of the two types of economic scenario generation models - a cascade-structure model and an MS-VAR model – has advantages and disadvantages, and there is little research that compares the performance of these models as engines for an economic scenario generator. The cascade-structure model relies on economic and financial theory to a greater extent and has a more parsimonious structure compared with the MS-VAR model, which makes the cascade-structure model easier to estimate and interpret. Moreover, because many parameters in the model have explicit economic interpretations, it is easier to generate meaningful alternative simulations by altering model parameters for specific variables of interest. However, the potential misspecification of the theory-based interdependence in the model may compromise its ability to capture the joint-distributions of variables in historical data. By contrast, the MS-VAR model is generally data-driven and takes a more flexible structure, which gives it better potential to capture the joint distribution of key variables and their time-varying characteristics. But the MS-VAR model is more difficult to estimate and interpret.

Existing models of these types mostly focus on inflation and interest rates. For our purpose, we also need to incorporate real economic activity, which is associated with governmental tax bases and affects tax revenue of sponsoring governments of public pension funds. (Moody's Analytics has developed an economic scenario generator based on Hardy et al. (2001) that has GDP growth as one of the output variables. But the methodology of incorporating GDP is not described in the original paper. Moore and Pederson (2013) modeled economic regimes only; we also need to model and simulate the path of economic growth.)

#### *Historical patterns in data that we will model*

In the sections that follow, we describe historical patterns in key variables of interest: GDP growth, and returns on stocks and bonds. The quarterly United States GDP series from 1953Q1 to 2017Q4 is obtained from the Federal Reserve Bank of St. Louis (FRED). The total return

indices of U.S. stocks, long-term government bonds and long-term corporate bonds from 1926 to 2015 are obtained from the SBBI Yearbook 2016. The stock total return, which is based on the S&P Composite Index, consists of capital appreciation return and dividend income return.<sup>27</sup> The long-term government bond return index and long-term corporate bond return index are calculated based on bonds with terms of approximately 20 years. The bond total returns include both interest income and capital appreciation due to changes in yields. Monthly total return indices of stocks, government bonds, and corporate bonds are provided in the SBBI Yearbook, which are converted to quarterly indices by using March, June, October, and December values, and to annual indices by using June values. The common sample period for GDP and asset return series of 1953Q1 to 2015Q4 is used for modeling.

#### Analysis of individual variables

Table 14 presents descriptive statistics for annual and quarterly GDP growth and asset returns. The table includes the mean and standard deviation, and two measures of the shape of the distribution, skewness and excess kurtosis.

Skewness is a measure of symmetry. If it is zero, the distribution is symmetric. If it is negative, the distribution has a long tail to the left, and if it is positive, the distribution has a long tail to the right (the greater the magnitude of the measure, the more the distribution tails to left or right).

Kurtosis measures the heaviness of the tail of a distribution – the greater the value, the heavier the tail. The value in our table is adjusted so that it is the excess over the kurtosis of the standard normal distribution. Distributions with excess kurtosis greater than zero have fatter tails than the standard normal distribution, and distributions with excess kurtosis less than zero have lighter tails than the standard normal distribution. Kurtosis can be heavily influenced by a few extreme observations.

The excess kurtosis measure in Table 1 provides evidence for fat tails for all quarterly variables, while fat tails are less prominent for annual data.

Table 14 Summary statistics of key variables

**Summary statistics of GDP growth, stock return and bond return**

	<b>N</b>	<b>Mean</b>	<b>Standard Deviation</b>	<b>Skewness</b>	<b>Kurtosis</b>
<b>Annual</b>					
GDP growth	62	3.0%	2.5%	-0.32	0.39
Stock return	62	10.4%	15.7%	-0.38	0.13
Long-term Treasury bond return	62	6.3%	9.9%	0.58	0.33
Long-term corporate bond	62	6.6%	9.6%	0.58	0.63
<b>Quarterly</b>					
GDP growth	251	0.7%	0.9%	-0.39	1.43
Stock return	251	2.5%	7.9%	-0.92	1.76
Long-term Treasury bond return	251	1.6%	5.1%	0.66	2.12
Long-term corporate bond	251	1.6%	4.7%	0.52	2.99

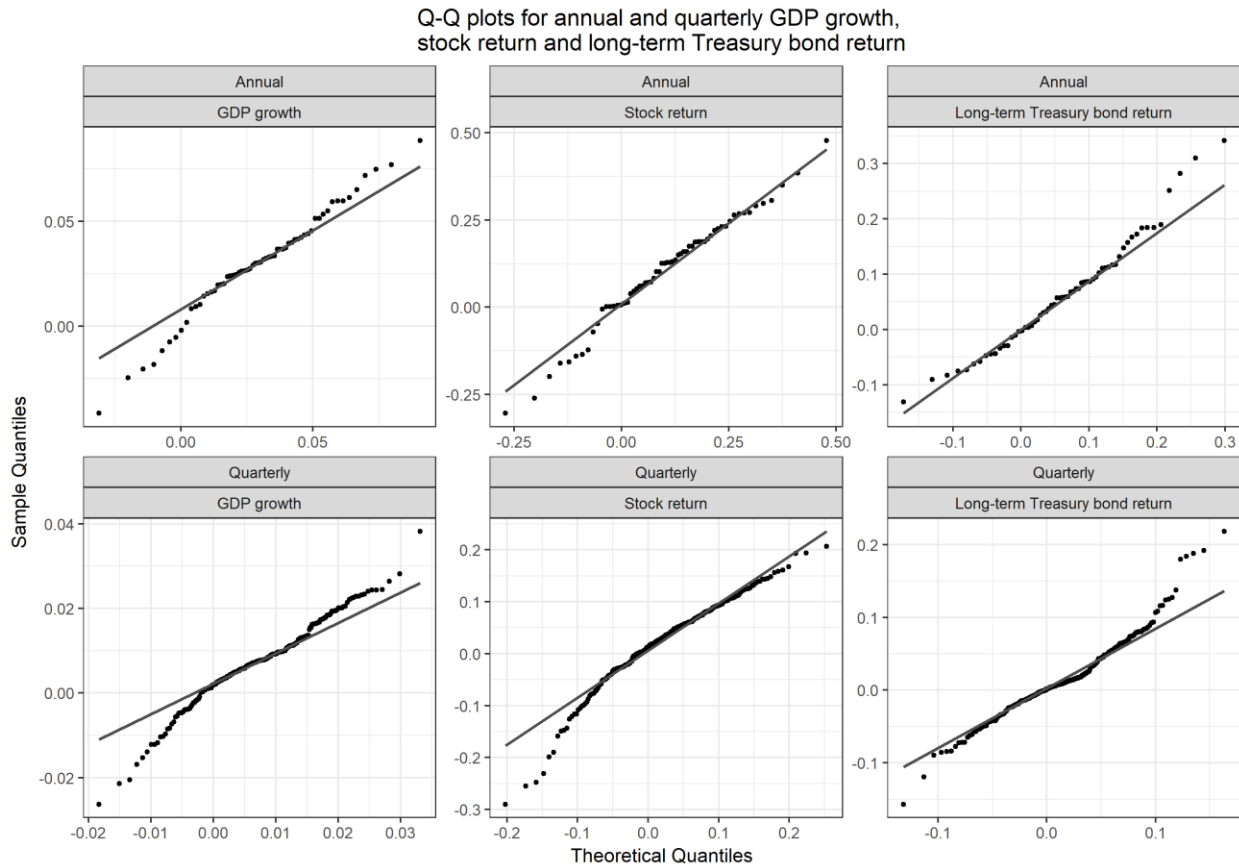
Figure 10 shows special plots that compare the empirical distributions of these variables against the normal distribution, known as “Q-Q plots”. These plots show on the horizontal axis the percentiles of observations that we would expect to see if the variable was normally distributed. On the vertical axis, they show the percentiles that we observe in our data. The data points would locate very close to the 45° line if the data were normally distributed.)

While the issue of fat tails is frequently raised in discussions of modeling stock returns, we saw in Table 14 the kurtosis of quarterly and annual stock total returns is relatively low. However, fat lower tails are visually prominent in the Q-Q plots of stock returns, especially for quarterly stock returns. In other words, we have observed more frequent large negative stock returns than we would expect if stock returns were normally distributed. This suggests it could be important to capture this non-normality when modeling pension fund returns, which has not generally been done by pension simulation modelers.

The distribution of GDP growth shows fat tails in both lower and higher ends. Long-term bond total return exhibits fat tails only in the upper end of its distribution, implying that extreme positive bond returns occur more frequently than a normal distribution would suggest.



Figure 10 The historical distributions of GDP growth and stock returns show fat tails, which are more prominent in quarterly data than in annual data.



### How variables move together over time

Next, we examine the historical paths of stock and bond returns and their relationship with economic growth and business cycles. Figure 2 shows 12-quarter moving average return and standard deviation for stock total return (upper panel) and long-term government bond return (lower panel), along with moving average GDP growth. The shaded areas indicate recession periods defined by National Bureau of Economic Research (NBER).

Figure 2(a) shows that stock total return has experienced sharp declines and elevated volatility in most severe economic downturns after 1970. In contrast, the pattern of long-term government bond returns is not aligned with economic cycles in a consistent manner. Although the volatility of government bond returns surged abruptly during the economic downturns in the early 1980s and in the Great Recession of 2007 to 2009, in other periods government bond returns and volatility show very little correlation with business cycles.

Figure 11 The patterns of stock returns and volatility usually synchronizes with business cycles, while the patterns of bond returns generally do not.

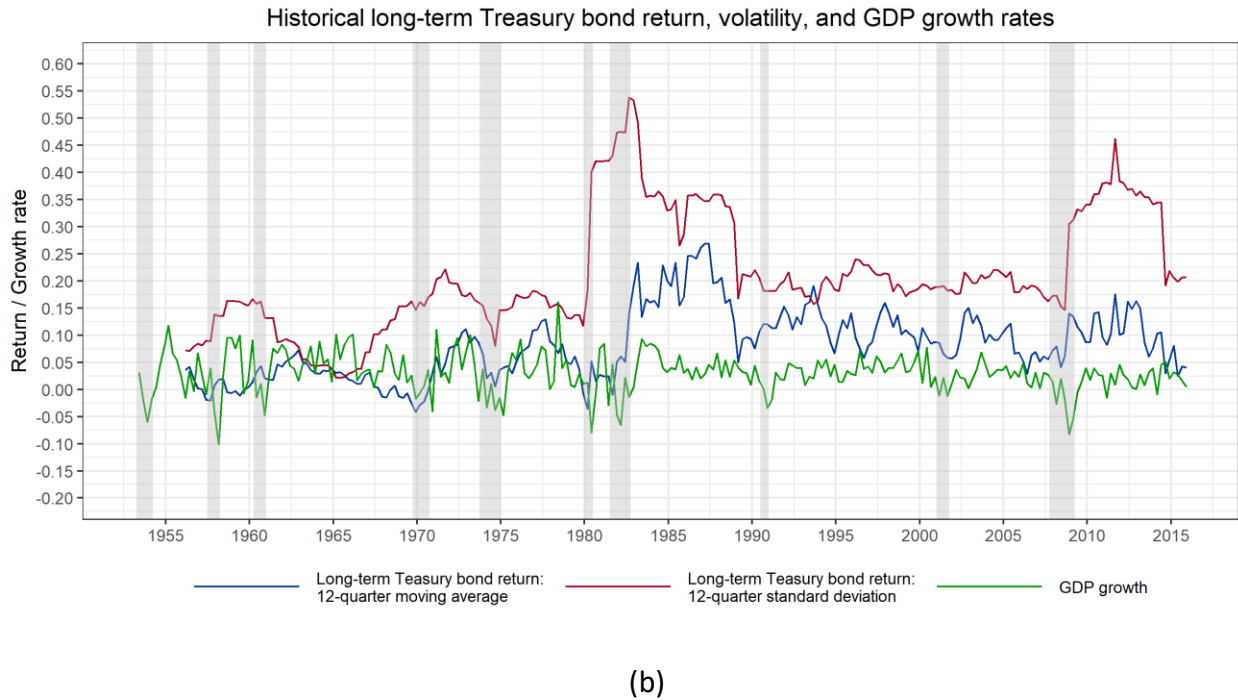
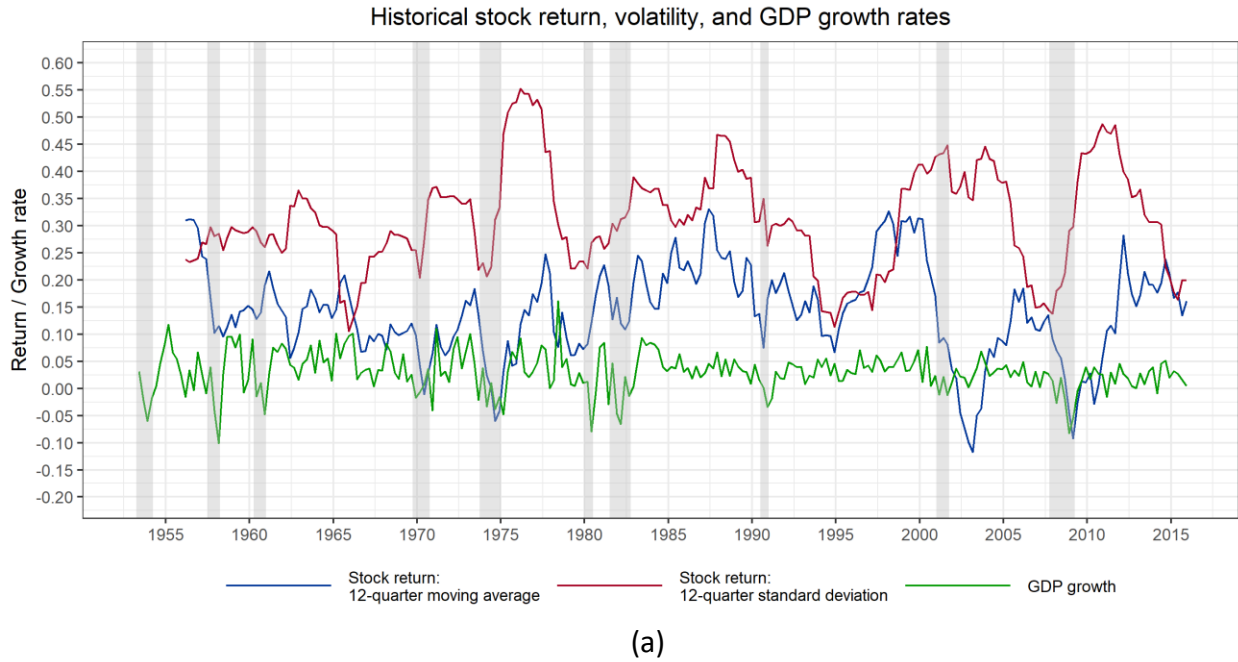
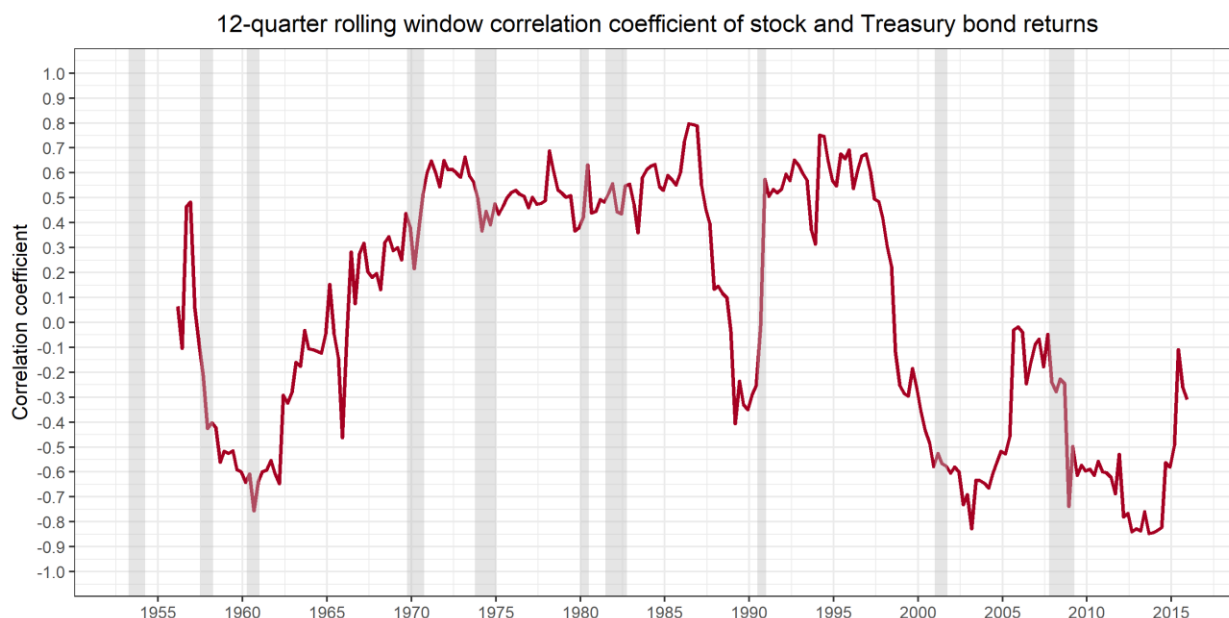


Figure 12 shows that the correlation between stock returns and long-term government bond returns varies widely over time. The stock-bond correlation started negative in 1950s and early 1960s, then become mostly positive from 1965 through the end of 1990s before turning negative again after 2000. This time-varying pattern of stock-bond correlation is consistent with

the results of Jammazi et al. (2015), who employed more sophisticated statistical models to examine the time-varying dependences of stock and bond returns in developed economies.

Figure 12 The correlation between stock returns and long-term government bond returns varies widely over time



### Modeling GDP growth and asset returns

In this section, we describe the individual equations in our macroeconomic model (i.e., economic scenario generator). We have two main equations in the current version of the model: an equation for real GDP growth, and an equation for nominal stock returns. We have not modeled bond returns econometrically, because of their minimal historical relationship to business cycles. Instead, when we produce economic scenarios, we construct stochastic bond returns that are consistent with their historical correlations with stock returns.

### Modeling GDP growth

In Hamilton's (1989) seminal paper that introduced Markov-switching models, the US GNP series was modeled as an autoregressive model of order 4 with the mean switching between two regimes (economic expansion and recession). The model can be described as follows:

$$y_t = \mu_{S_t} + \phi_1(y_{t-1} - \mu_{S_{t-1}}) + \phi_2(y_{t-2} - \mu_{S_{t-2}}) + \phi_3(y_{t-3} - \mu_{S_{t-3}}) + \phi_4(y_{t-4} - \mu_{S_{t-4}}) + \varepsilon_t$$

where  $y_t$  is the quarterly growth rate of GNP,  $\mu_{S_t}$  denotes the mean of the process in time period  $t$  with  $S_t$  being the regime the process belongs to in  $t$ . The innovation term  $\varepsilon_t$  follows the normal distribution and is independent across time periods. In each time period, the regime transitions following the matrix of transition probabilities,

$$\begin{bmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{bmatrix}$$

where  $p_{ij}$  is the probability of transitioning from regime  $i$  to regime  $j$ .

We first examine the regime-switching behavior of the US GDP series by applying the regime-switching autoregression model described above to the quarterly GDP growth series from 1953q2 to 2015q4. The first row of Table 15 shows the estimated model parameters along with the expected durations of expansion (regime 1) and recession (regime 2) implied by the estimated transition probabilities. The estimated mean quarterly GDP growth rates are 0.88 percent in expansion regime and -1.15 percent in recession regime. There are two noteworthy issues with the estimation results. First, the expected duration of recession regimes implied by the transition probabilities is 1.44 quarters, which is much shorter than the average duration of recession of 3.7 quarters based on the historical data on the same sample period. Second, the estimated parameters of the third and the fourth autoregression terms are not statistically significant.

To explore whether there are model specifications that can better capture the historical patterns of economic expansion and recession, we fit two alternative model specifications:

- Model (2): Markov-switching AR(2) model. Markov-switching autoregression model with order 2 (obtained by removing the insignificantly autoregression terms from the Hamilton (1989) model);
- Model (3): Random walk with a Markov-switching drift term (obtained by removing all autoregression terms from the Hamilton (1989) model), resulting in the model:

$$y_t = \mu_{S_t} + \varepsilon_t; \quad \varepsilon_t \sim N(0, \sigma^2)$$

The estimation results of these two alternative models are presented in the second and third row of Table 15 respectively. Model (2) gives estimates of regime-specific means, variance, and expected duration of the recession regime that are very similar to those for the Hamilton model. Model (3) produces expected durations of recession and expansion regimes that are close to the historical data, and a slightly higher estimate for the mean growth rate of the recession regime.

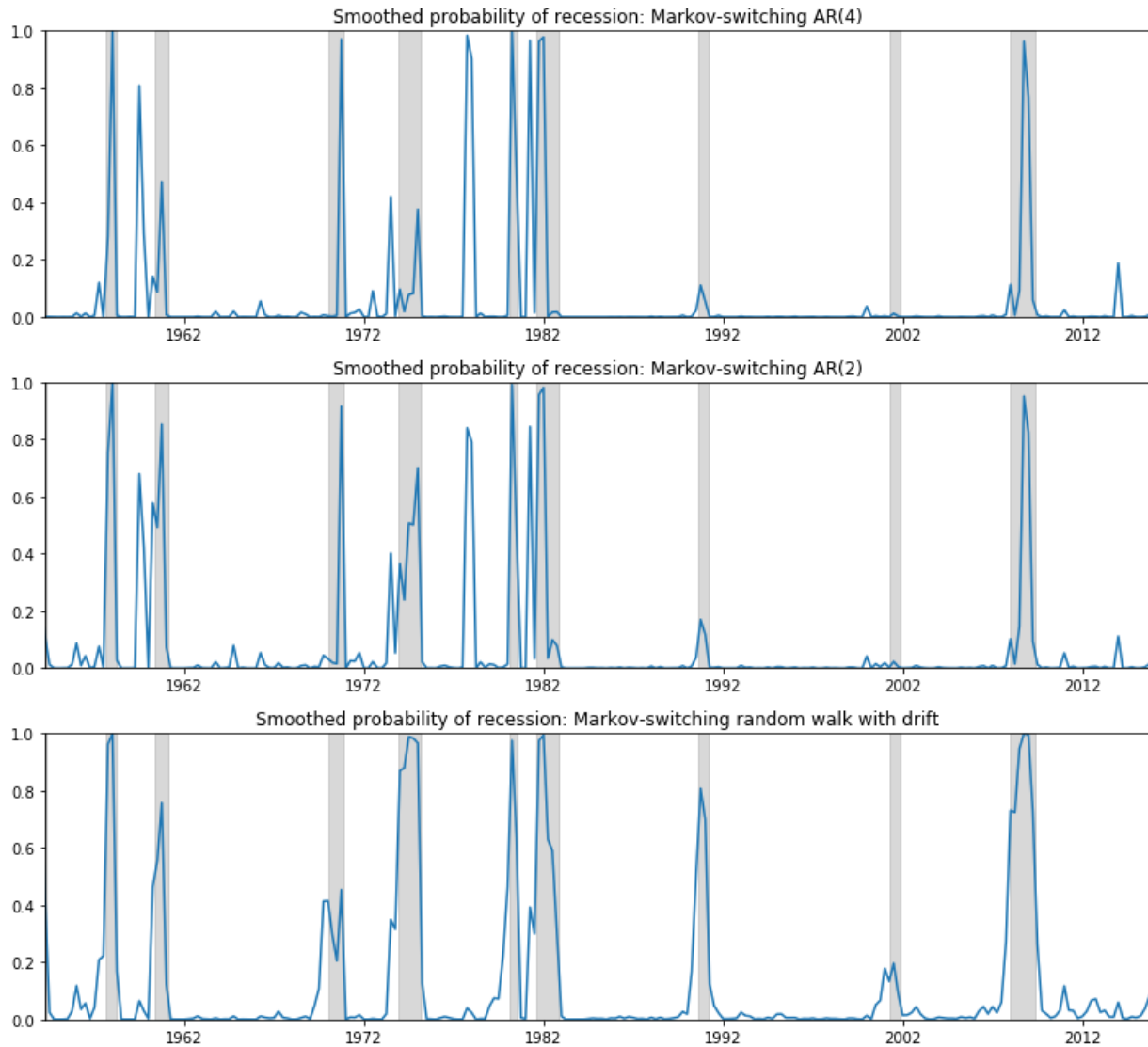
Theoretically, the Markov-switching models with autoregressive terms can be more suitable for describing quarterly GDP series because it allows for smoother transition between regimes and mean-reversion within regimes. By contrast, the random walk with Markov-switching drift term assumes abrupt changes across regimes and no mean reversion within regimes. Although the Markov-switching autoregression models have the theoretical merits above, they tend to greatly underestimate the expected length of recession periods (see Table 15), while the estimates by the model of random walk with Markov-switching drift are more consistent with historical data. Because capturing the general historical pattern of economic regimes is more important for our final goal, in this study we use Model (3) for simulation. We plan to run sensitivity analysis to see the impact of introducing AR terms into the model.

Table 15 The estimation results of the GDP growth model of random walk with a Markov-switching drift term are more consistent with the historical durations of recessions and expansions.

**Estimation results of Markov-switching models for quarterly GDP growth**

Models	Mean in Expansion	Mean in Recession	Standard deviation of error term	Probability of transition		Expected duration of regimes (Quarter)	
				Expansion to recession	Recession to expansion	Expansion	Recession
<b>(1) Markov-switching AR(4)</b>	0.0088 (0.001)	-0.0115 (0.003)	0.0068 (0.0022)	0.043	0.0695	23.23	1.44
<b>(2) Markov-switching AR(2)</b>	0.0089 (0.001)	-0.0097 (0.003)	0.0069 (0.0024)	0.049	0.576	20.5	1.74
<b>(3) Markov-switching Random walk with drift</b>	0.0094 (0.001)	-0.0055 (0.003)	0.0074 (0.0024)	0.048	0.319	20.7	3.13
<b>Historical Data</b>						20.2	3.7

Figure 13 The estimation results of the GDP growth model of random walk with a Markov-switching drift term are more consistent with the historical durations of recessions and expansions.



### Modeling stock returns

Stock returns are generally considered to be a random walk process with drift. Research has shown that stock returns exhibit regime-switching behavior, and the stock-return regimes can be correlated with the business cycle. Following Hardy (2001), we model stock total returns as a random walk with a Markov-switching drift term and variance.

$$y_t = \mu_{s_t} + \varepsilon_t; \quad \varepsilon_t \sim N(0, \sigma_{s_t}^2)$$

The two-state model has identified two regimes: a high-return-low-volatility regime, and a low-return-high-volatility regime. We examine the extent to which stock-return regimes are aligned

with regimes of GDP growth in Table 16. The first row shows estimated model parameters for the regime-specific mean stock return and standard deviation. We also examine the transition probabilities and implied average duration of regimes in the last four columns of Table 16. The second row of the table shows the average stock return and standard deviation in expansion and recession periods defined by NBER.

The probability of stock returns entering a low-return-high-volatility regime is about twice as high as the probability of GDP growth entering a recession regime suggested by the Model (3) of GDP. The expected duration of the low-return regime (3.36 quarters) is very close to the average duration of economic recessions (3.7 quarters), while the expected duration of the high-return regime is about half the length of the average duration of GDP expansion.

The model results show that economic recessions are almost always accompanied by periods of low return and high volatility in stock market. While the low-return and high-volatility regime of stock return may occasionally occur without an economic downturn.

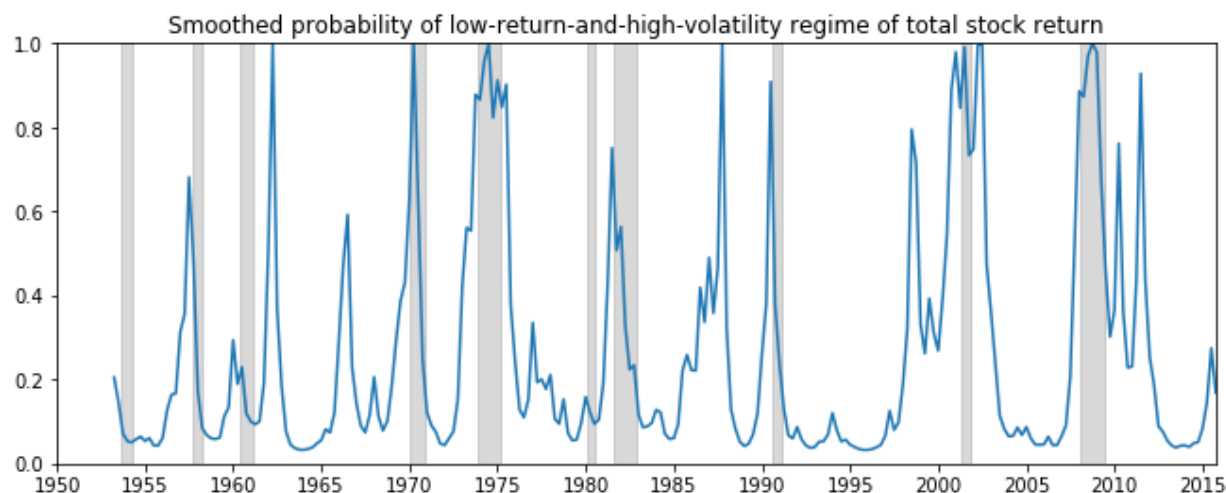
*Table 16 Expected returns and volatility of the 2 regimes detected by the Markov-switching model of stock*

**Estimation results of Markov-switching models for quarterly stock returns**

Models	Mean		Standard deviation of error term		Probability of transition		Expected duration of regimes (Quarter)	
	Regime 1 (High return and low volatility)	Regime 2 (Low return and high volatility)	Regime 1 (High return and low volatility)	Regime 2 (Low return and high volatility)	Regime 1 to Regime 2	Regime 2 to Regime 1	Regime 1 (High return and low volatility)	Regime 2 (Low return and high volatility)
Markov-switching Random Walk with drift	0.041 (0.006)	-0.018 (0.03)	0.055 (0.032)	0.113 (0.055)	0.106	0.298	9.47	3.36
Calculation based on NBER expansion/recession definition	0.032	-0.014	0.069	0.119	-	-	-	-

We first examine how the smoothed probabilities of low-return-high-volatility regime are aligned with NBER recession and expansion periods (Figure 14). There are 13 peaks in the probability of low-return regime, 7 of them are approximately aligned with the recession periods, and most of the other 6 peaks occur around recessions periods (exceptions are peaks in 1966 and 1987, where there were no economic downturn).

Figure 14 Economic recessions are almost always accompanied by periods of low return and high volatility in stock market, while the low-return and high-volatility regime of stock return may occasionally occur without an economic downturn



### Modeling bond returns

We have not modeled bond returns econometrically, because of their weak historical relationships to business cycles. Instead, when we produce economic scenarios, we construct stochastic bond returns that have correlations to stock returns that are consistent with historical correlations.

Preliminary analysis results show that the historical patterns in bond returns are generally not related to business cycles; and the time-varying pattern of the stock-bond dependences shows little correlation with business cycles. There can be modest serial correlation in long-term government bond returns. The unconditional distribution of bond return is quite close to normal distribution except for the upper end of the distribution. There are occasional extreme positive values in long-term bond returns.

### Simulation of GDP growth and asset returns

#### Simulation approach

- 1. GDP (quarterly, core variable):
  - Use Model (3) Random walk with a Markov-switching drift term.
  - Key parameters
    - Transition matrix that determines regime-switching behavior
    - Regime-dependent mean quarterly GDP growth rates
    - Standard deviation.
  - Outputs: 2,000 simulated series of quarterly GDP growth along with the information on which regime each simulated data points belongs to.
- 2. Total stock returns
  - Assume the 2 regimes of stock returns are perfectly aligned with the expansion and recession regimes of GDP growth.



- Parameters: Regime specific mean stock return and standard deviation are calculated based on the NBER recession and expansion periods.
- Outputs: 2,000 simulated series of stock total return.
- 3. Total bond returns
  - Modeled and simulated as a separate random walk process with drift. The innovation terms of bond returns and stock returns are drawn from a bivariate normal distribution. The covariance between the innovation term of stock and innovation term of bond is calibrated so that the correlation between the simulated annual stock returns and annual bond returns is approximately equal to the preset target values.
- 4. Converting quarterly growth rates and returns to annual rates and returns.
- 5. Return of Portfolio
  - Construct a hypothetical stock-bond portfolio.

Because the ultimate purpose of our simulation model is to shed light on the public-pension-related risks the sponsoring governments may face in the future, it would not be appropriate to set the simulation parameters directly based on the model parameters estimated with historical data because the historical trends and patterns do not necessarily extend to the future. For example, the CBO (2017) projects that the average annual growth rate of potential GDP in the next 30 years will be 1.9 percent, which is much lower than the historical average GDP growth rate of about 3 percent. Therefore, forward-looking assumptions for the economic variables should be incorporated into the simulation parameters. To do so, we made adjustments to the estimated model parameters to ensure that the simulation results are generally consistent with a set of forward-looking assumptions constructed based our review of external studies. The target annual GDP growth is obtained from the 30-year projection of potential growth GDP made by CBO (2017). The target assumptions on annual stock and bond returns are generally consistent with the capital market assumptions used in Mennis, et. al (2017). The forward-looking simulation parameters are provided in Table 17, the unadjusted historical-data based parameters that are different from the forward-looking parameters are provided in the footnote of the table.

Table 17 Model parameters used in the simulation

Simulation parameters based on forward-looking assumptions			
Parameter	Assumption on quarterly data (Annualized rates in parentheses except for the transition probabilities)		Target assumptions for annual data
	Expansion period	Recession period	
	4.8% (expansion to recession)	32% (recession to expansion)	
<b>GDP growth (real)</b>	Expected GDP growth	0.67% (2.7%)	Target expected geometric mean of annual GDP growth is 1.9%.
	Standard deviation of GDP growth	0.74% (1.48%)	
<b>Stock return (nominal)</b>	Expected return	2.54% (10.6%)	Target expected geometric mean of annual stock return is 6.7%
	Standard deviation	6.9% (13.8%)	
<b>Bond return (nominal)</b>	Expected return	0.9% (3.6%)	Target expected geometric mean of annual stock return is 3.6%
	Standard deviation	2% (4%)	
	Correlation between stock return and bond return	0.15	

Notes:

- Adjustments are made to the quarterly parameters estimated with historical data to ensure the simulated results for annual data (converted from simulated quarterly data) are consistent with the target assumptions for annual data. The variables that have been adjusted and their historical-data based estimates (all for quarterly data) are expected GDP growth in expansion (0.95%), GDP growth in recession (-0.55%), expected stock return in expansion (3.2%), expected stock return in recession (-1.4%), expected bond return (1.6%), standard deviation of bond return (5.1%). (Historical bond returns are for long-term corporate bond)
- 
- The target annual GDP growth is obtained from the 30-year projection of potential growth GDP made by CBO (2017). The target assumptions on annual stock and bond returns are generally consistent with the capital market assumptions used in Mennis, et. al (2017).

### Analysis of simulation results

We produced simulations with both historical-data based parameters and forward-looking-assumptions based parameters. The latter will be used in the section *Using the linked model to analyze risks and fiscal stresses* to evaluate the potential pension-related risks the sponsoring governments may face in the future. In this section, we examine the simulation results generated from the historical-data based parameters and see to what extent the simulation model can produce data that are consistent with the historical pattern.

Because historical-data based parameters are estimated a sample period of 63 years (1953 to 2015), the simulation horizon in each of the 2,000 simulations are also set to 63 years (252

quarters). Summary statistics, such as mean, standard deviation, and skewness, of the simulated variables are calculated for each of the 2,000 simulations. To examine the plausibility of the simulation results, we use a method that has similar logic with statistical tests: we compare the simulated distributions of the summary statistics and the statistics calculated based on historical data, and ask the question that if our simulation model was the true data-generating process in history, then how likely the actual historical data we observed today would be generated? We will also do graphical comparison of the distributions of historical and simulated variables.

### Simulated GDP growth rates and regimes

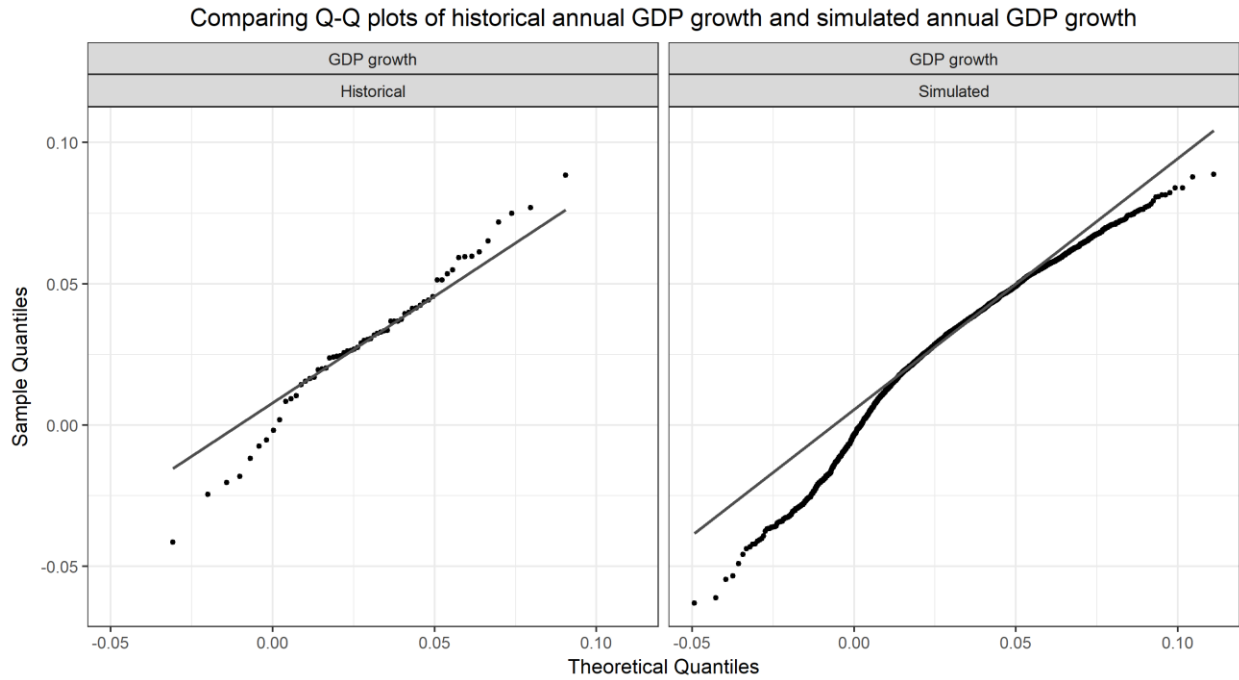
Table 18 compares the statistics of historical GDP growth rates with the distributions (characterized by median, 5th, 25th, 75th and 95th percentiles) of these statistics across the 2,000 simulations. The historical mean growth rate is very close to the median of the simulated mean growth. The historical standard deviation, average length of recession and expansion fall on the higher end of the simulated distribution but still cannot be considered as extreme values.

*Table 18 Simulated distribution of GDP growth and durations of recessions and expansions are generally consistent with historical data*

Summary statistics for historical and simulated GDP growth							
	Historical value for 1953-2015 (63 years)	Distribution of simulation results (2,000 simulations; 63 years in each simulation)					Historical value's percentile in the simulated distribution
		5th percentile	25th percentile	Median	75th percentile	95th percentile	
<b>Regime</b>							
Number of recession	9	6	9	10	12	15	0.38
Average length of recession (quarter)	3.7	1.9	2.5	3.0	3.6	4.6	0.79
Average length of expansion (quarter)	20.2	12.7	16.1	19.4	23.8	33.0	0.57
<b>GDP growth</b>							
Mean	3.0%	2.5%	2.9%	3.1%	3.3%	3.6%	0.38
Standard Deviation	2.5%	1.7%	2.0%	2.2%	2.4%	2.6%	0.91
Skewness	-0.33	-1.20	-0.89	-0.68	-0.48	-0.14	0.88
Kurtosis	0.39	-0.48	-0.01	0.47	1.05	2.27	0.46

Figure 15 compares the Q-Q plots of the historical and simulated annual GDP growth. Simulated GDP growth rates have similar fat left tail as the historical data, but has a thinner right tail.

Figure 15 Simulated GDP growth rates have similar fat left tail as the historical data but thinner right tail



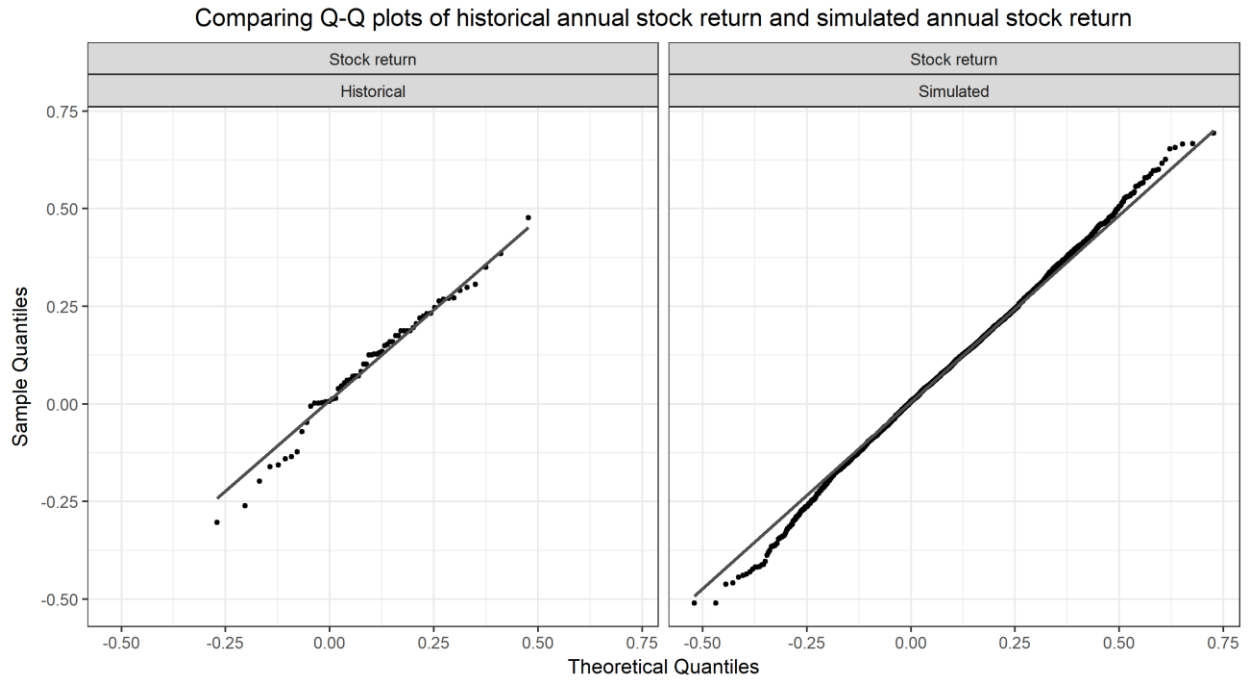
### Historical and simulated distribution of stock returns

Figure 16 compares the Q-Q plots of the historical and simulated annual stock returns. Simulated distribution of annual return has fat left tails for values less than -25%.

Table 19 Comparing historical and simulated stock returns

Summary statistics for historical and simulated stock return							
	Historical value for 1953-2015 (63 years)	Distribution of simulation results (2,000 simulations; 63 years in each simulation)					Historical value's percentile in the simulated distribution
		5th percentile	25th percentile	Median	75th percentile	95th percentile	
Mean	10.4%	7.1%	9.4%	10.9%	12.5%	14.6%	0.40
Standard Deviation	15.7%	14.4%	16.0%	17.2%	18.5%	20.4%	0.19
Skewness	-0.39	-0.56	-0.21	0.02	0.25	0.62	0.11
Kurtosis	0.13	-0.65	-0.28	0.10	0.61	1.61	0.52

Figure 16 Simulated distribution of annual stock returns have fatter tails than historical distribution



#### Historical and simulated distribution of bond returns

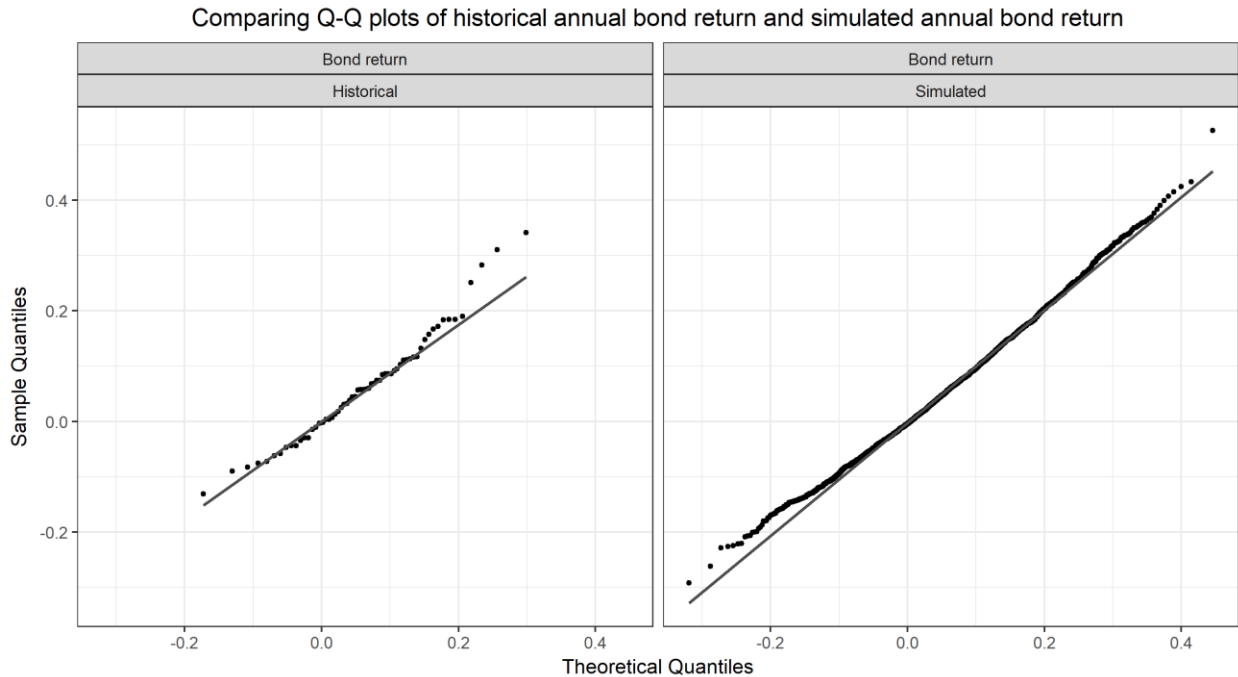
Figure 17 compares the Q-Q plots of the historical and simulated annual stock returns. Since bond returns are simulated using a simple normal distribution, it does not capture the occasional extreme positive bond returns (4 out of 61 data points) in history. Excluding the extreme values, the distribution of the historical bond returns is quite consistent with normal distribution.

Table 20 Comparing historical and simulated bond returns

**Summary statistics for historical and simulated bond return**

	Historical value for 1953-2015 (63 years)	Distribution of simulation results (2,000 simulations; 63 years in each simulation)					Historical value's percentile in the simulated distribution
		5th percentile	25th percentile	Median	75th percentile	95th percentile	
Mean	6.6%	4.4%	5.7%	6.6%	7.5%	8.8%	0.49
Standard Deviation	9.6%	9.1%	10.0%	10.6%	11.4%	12.4%	0.14
Skewness	0.60	-0.27	0.00	0.19	0.39	0.70	0.91
Kurtosis	0.63	-0.77	-0.46	-0.16	0.22	1.10	0.88

Figure 17 The bond returns simulated by normal distribution does not capture the extreme positive bond returns in history



### Summary of simulation results and implications

The simulated distribution of GDP growth is generally consistent with historical data except that the simulated distribution shows a thinner tail in the upper end (less likely to have very high growth rates than the historical data shows).

The regime-switching simulation approach is capable of capturing the dependences between low-return-high-volatility periods of stock returns and severe economic downturns, while the overall distribution of the simulated annual stock returns may have fatter tails than the historical distribution.

The simulation approach with synchronized regimes of stock returns and GDP growth, which will be further linked to tax revenues and pension contributions, will allow us to examine the risk of sharp rises in pension contributions when the sponsoring governments are under fiscal pressure due to economic downturns. The simulated stock-return distributions with fatter tails than normal distributions will allow us to evaluate the impact of more frequent large negative returns on pension funding and the fiscal pressure on sponsoring governments.

## Using the macroeconomic model to drive revenue resources of stylized governments

### *Goals and overall approach*

While investment returns can be related to the economy, governmental tax revenue also is related to the economy (and can be related directly to returns on assets as well). In recent recessions, investment returns have been driven down sharply and tax revenue has declined or weakened as well. Thus, pension contributions may increase when or shortly after tax revenue weakens, making it all the harder for governments to pay higher contributions.

In this project, we use the economic growth and investment-return outputs of the macroeconomic model to forecast tax revenue of stylized governments, which allows us to explore the extent to which risks to pension plans and governments may change when we take the endogeneity of tax revenue into account.

Our overall goal is to capture how resources available to governments to pay contributions may change over relevant portions of the business cycle. In addition, because different governments rely on different taxes, and different taxes respond differently to the business cycle, we want to construct our measure of resources from a portfolio of individual taxes that may change from one kind of government to another.

As we describe below, we plan to construct revenue forecasts for governments as follows:

1. We will construct a portfolio of taxes for three to four stylized governments, consisting of a mix of income, sales, property, and other taxes. For example, one stylized government might be an income-tax-dominant government that relies heavily on the income tax for 50-60 percent of its revenue and uses a mix of other taxes for the remainder, similar in some ways to New York or California. Another stylized government might rely exclusively on the property tax, as many local governments do.
2. Individual taxes in the portfolio will remain a constant long-run share of the economy.
3. Each tax will have a cycle around this trend that reflects how it tends to respond to economic changes. For example, income taxes will be highly cyclical, and will increase more or fall even more sharply when asset values rise or fall significantly. By contrast, the “other” category (which includes excise taxes) will be far more stable. Because asset values play a role, particularly for the income tax, revenue cycles will vary depending upon whether asset values change sharply during a recession or recovery.
4. Thus, governmental resources will reflect the business cycle and its character, and the composition of the government’s tax structure.

### *Summary measure of governmental resources to consider – own-source revenue, or taxes?*

Over the long run state governments can adjust their revenue structures to raise revenue to pay for services that taxpayers are willing to support, and they can allow local governments to do the same. Thus, over the long run, the economy’s capacity to generate revenue is the major constraint on resources. In the short run, political systems do not adapt so easily. Some governments have constitutional constraints on revenue sources they may use, and

constitutions do not change quickly. Some local governments have constraints on revenue they can raise, imposed by higher levels of government. Thus, in the short run, we are especially interested in the revenue that may be raised from existing tax structures.

But which measure of revenue? We are interested in revenue that governments can raise from their own resources, rather than counting on revenue from higher levels of government. The broadest such measure typically is called “own-source revenue,” which includes taxes, fees, and other miscellaneous revenue that a government can impose. Another commonly used summary measure of revenue is just total tax revenue, not including fees and other miscellaneous revenue.

In concept, own-source revenue seems like the preferred measure. However, data limitations often mean that it includes revenue we might not consider available to politicians to increase to help pay for pension contributions. For example, it often includes tuition revenue, for which there is an associated service, and it may also include patient fees for medical services.

Because of these complications, in this draft we focus on tax revenue, although we acknowledge that own-source revenue is preferable conceptually.<sup>28</sup>

#### *Specific taxes to examine and their trends and cycles*

Based on our analysis of Census Bureau data on the composition of governmental taxes, we think it makes sense to divide taxes into the following categories:<sup>29</sup>

- Personal income taxes
- General sales taxes
- Real property taxes
- Other taxes (the remainder – including excise taxes, corporate income taxes, estate taxes, and other miscellaneous taxes)

We use this breakdown because it allows us to capture the great variation in tax portfolios that governments have, and the great variation in how different taxes respond to changes in the economy.

Different taxes respond differently to changes in the economy. As we noted above, income taxes and excise taxes did have very different long-run behavior. They and other taxes also have very different cyclical behavior. It is hard to observe this behavior precisely because we have very little “pure” data on tax revenue: most of our observed data reflect actual tax collections, which reflect the frequent changes that policymakers make to tax bases and rates. Thus, year-to-year changes reflect economic trends, legislative changes, and a variety of other factors.<sup>30</sup>

#### *Trends*

The first question is what revenue responses to changes in the economy do we want to capture in our long-term simulations? Some taxes tend to grow more quickly than the economy and some taxes tend to grow more slowly. For example, many states have progressive income taxes



that impose higher marginal rates on higher levels of income. Income tax revenue will grow more quickly than the underlying income grows, because average incomes tend to rise over time due to productivity growth and inflation, and this pushes people into higher tax brackets.<sup>31</sup> Some other taxes, especially excise taxes imposed upon the quantity of goods sold, do not capture growth in the prices of these commodities and therefore their revenue tends to fall relative to the economy over time, except when politicians raise rates.

Politicians and taxpayers generally do not want tax revenue to grow on autopilot over the long run, continually rising (or falling) relative to the economy. Thus, over the long run politicians often cut income taxes and raise sin taxes, partially offsetting their underlying trends.

There have been episodes in the history of United States where governments have raised or lowered the overall level of taxation substantially, such as when state and local government taxes were increased to help pay the escalating education costs of baby boomers entering school. However, these are political decisions reflecting taxpayer and political preferences, and are not easy to predict.

For purposes of our analysis, we assume that taxes will stay constant relative to the economy over the long run, but that there will be cycles around this trend.<sup>32</sup> Our job is to construct reasonable estimates of these cycles, for different taxes.

Figure 18 Trends in major tax categories relative to GDP

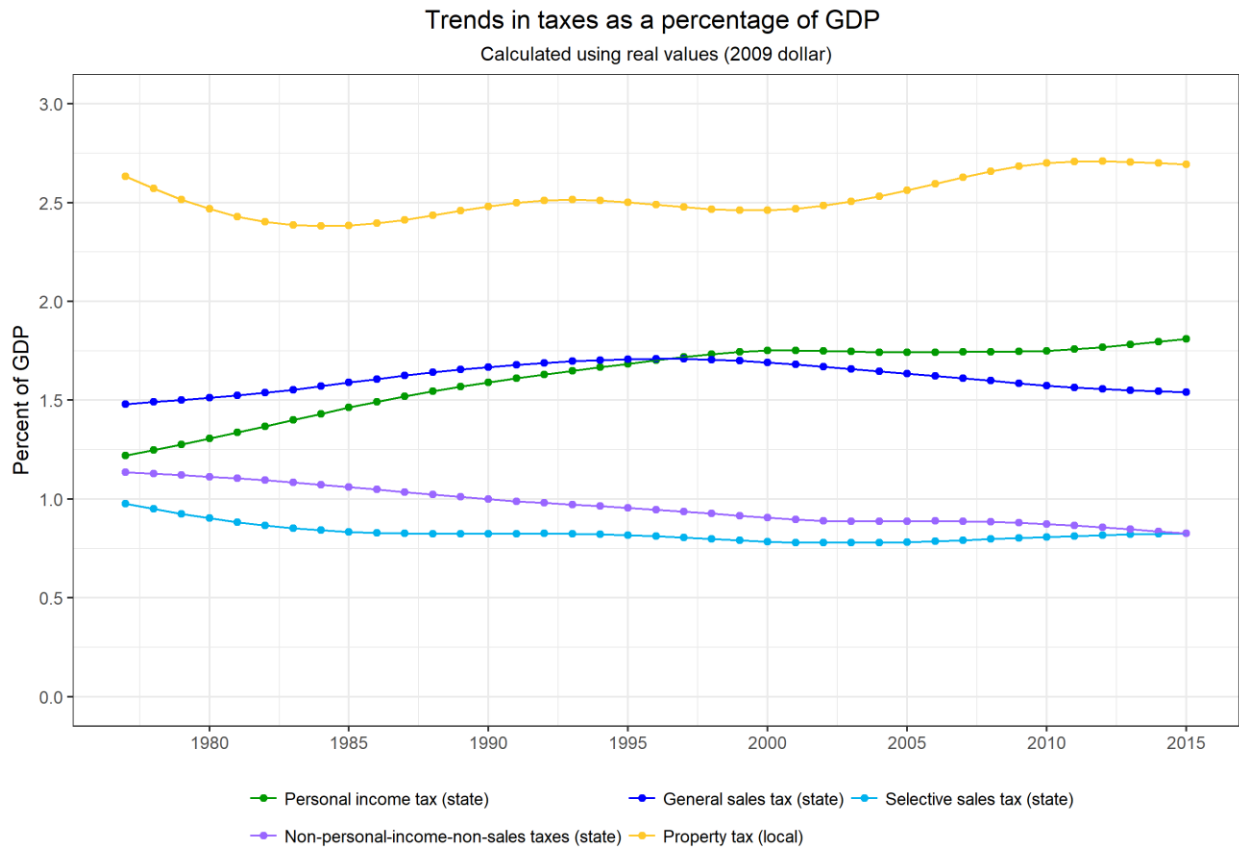
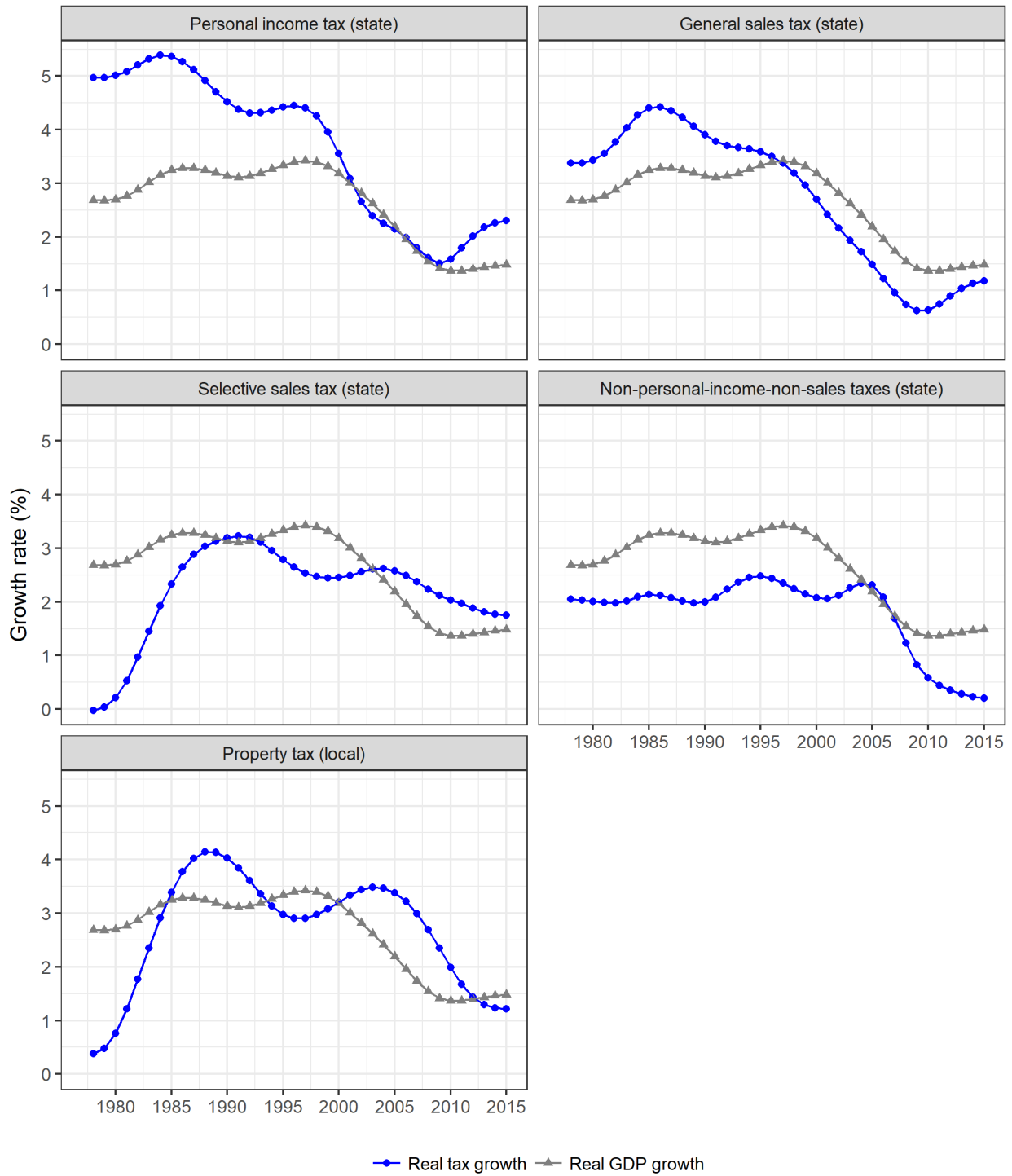


Figure 19 Comparing trend growth rate of GDP with trend growth rates of major tax categories

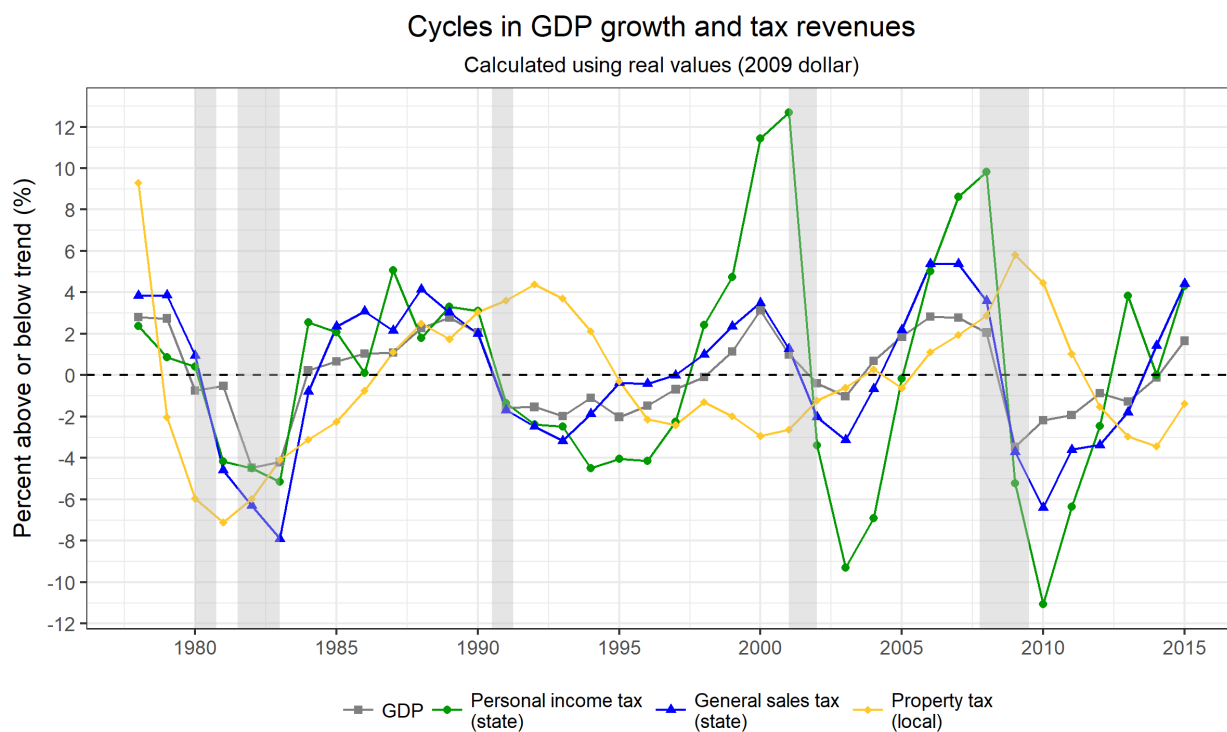
### Trend real growth of GDP and tax revenues

Real growth calculated based on 2009 dollar



## Cycles

Figure 20 Cycles in revenues of major taxes



Perhaps the most notable conclusion is that in recent decades the personal income tax has become far more cyclical than in the past, and far more cyclical than other taxes.<sup>33</sup> This heightened cyclicity has been driven in large part by changes in the nature of recessions and recoveries: income from financial markets, especially from capital gains on the sale of stock and other assets, have played a prominent role in income subject to tax and in resulting tax revenue. Thus, the nature of an economic recession or recovery, and not just its depth and duration, is important. A recession accompanied by sharp declines in asset values, such as either of the last two recessions, has had a far more detrimental impact on income tax revenue than have earlier recessions. We capture this in an approximate fashion in our analysis.

**General sales taxes:** Over the long run, the sales tax has been declining relative to the economy, for several reasons.<sup>34</sup> The cyclical behavior of these taxes is highly related to consumption of commodities that are taxed. Sales taxes almost always tax durable goods, the consumption of which tends to be quite cyclical (keeping people often postpone the purchase of a car during a recession), and many other goods and services that can be curtailed when times get hard, such as restaurant meals. Most states exempt many necessities from the general sales tax, including prescription drugs, and, in some states food for consumption at home. As a consequence, the

sales tax tends to be more cyclical than total consumption albeit less cyclical than personal income taxes, but all of this can vary depending upon the specific tax base that a state uses.

Property taxes: It is difficult to define and measure the cyclical behavior of property taxes for several reasons. First, tax rates tend to be adjusted every year by policymakers in a way that is not done with income and sales taxes. Second, changes in market values of property may take years to be reflected in legally assessed values of property due to lags inherent in the administrative and political aspects of tax assessment. Finally, idiosyncratic rules in individual governments will affect the assessments of individual properties and the ability of governments to raise or lower the aggregate levy. All this makes it very hard to observe over history how property taxes might change (absent legislative action) in response to changes in the economy. There have been some instances of significant changes in property taxes in cases where values changed dramatically on a sustained basis, such as during the Texas oil bust of the 1980s, and the real estate bust associated with the recent recession.<sup>35</sup> However, most episodes in history suggest that the property tax is very stable relative to the economy, except when policymakers make explicit choices about raising or lowering the tax. This varies greatly across places, depending upon tax limitations that are in place.

Other taxes: Governments use a broad array of other taxes that include excise taxes on specific commodities such as cigarettes, alcohol, and motor fuel, and corporate income taxes, estate taxes, and miscellaneous other taxes. Excise and selective sales taxes for many governments make up a large share of this category. Taxes on addictive and habit-forming products such as cigarettes and alcohol tend to be relatively stable even in recessions. Corporate income taxes are extremely volatile but rarely are important to governmental revenue. Many other taxes do not have clear cyclical properties.

#### Summary of stylized behavior of individual taxes

The table below summarizes our conclusions about the long run and cyclical behavior of the four major taxes we discussed, based upon our review of relevant research and our own analysis of data. As noted, we focus in this project on constructing the cyclical behavior taxes, and we assume that policymakers adjust rates and bases to keep the taxes a constant share of the economy over the long run.

*Table 21 Long run and cyclical behavior of major taxes*

<b>Tax</b>	<b>Long run behavior</b>	<b>Cyclical behavior</b>
Personal income tax	Grows faster than the economy due to progressivity	Much sharper swings than the economy (GDP), primarily because of sharp swings in income from financial assets, and how that income is taxed. Increased cyclicity, last few decades.

Tax	Long run behavior	Cyclical behavior
General sales tax	Grows more slowly than the economy due to changes in consumption and how different components are taxed, and due to difficulty of collecting taxes on e-commerce. See e.g., Mikesell	Somewhat sharper than the economy, primarily because sales taxes exempt many necessities and tax many goods and services that are easily postpone able (cars, restaurant meals, etc.)
Property taxes	Unclear.	Almost impossible to observe because in most places rates are set administratively (rate = desired levy / assessed value of property). Some places face severe limits on the rate or growth in the levy (e.g., California, Prop 13), others do not. If rates were held constant, then revenue would have same cyclical as the base (real estate values), allowing lags for assessment response. Property values these have been relatively stable or highly volatile depending on the specific kind of recession.
Other taxes	Often exhibit longer term decline relative to economy. Many of these taxes are quantity-based (e.g., cigarette tax cents per pack) and do not keep up with inflation-driven nominal GDP. Many follow real consumption of the associated products, and that consumption may be declining.	Generally, not very cyclical (e.g., cigarette taxes) or moderately cyclical (motor fuel taxes).

*Constructing the trend and cyclical behavior of tax revenues*

Trends in specific taxes

For purposes of our analysis, we assume that taxes will stay constant relative to the economy over the long run, but that there will be cycles around this trend. We assume the trend growth

rates of all tax categories are equal to the trend growth rate of GDP, which is 1.9 percent (in real term).

### Personal income tax revenue

In addition to GDP growth, realized capital gains is the ideal variable to include as an independent variable in a model of personal income tax revenue. Realized capital gains are not directly modeled in our macro-economic model. However, stock returns and realized capital gains have tended to move in tandem, especially during the recent two crises, although stock returns are less volatile than realized capital gains. Because of this relationship, we use stock returns as a proxy for realized capital gains in the regression analysis.

Figure 21 Income tax revenue is influenced heavily by stock-market-driven capital gains

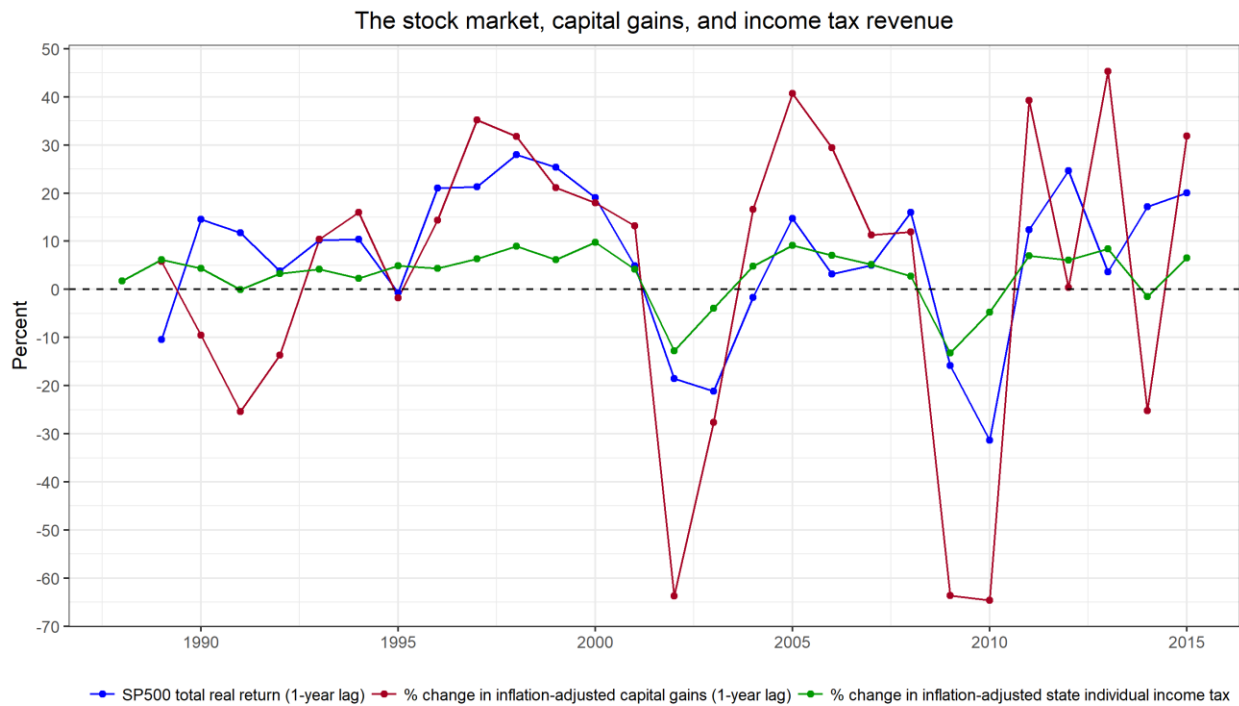


Table 22 shows the regression analysis results of the national aggregate of state personal income tax (PIT), all growth rates below are referred to as the cyclical components of the annual real growth rates.

When only GDP growth is included in the regression, the result shows that a 1 percent of GDP growth percent is associated with a 1.42 percent growth in state PIT. We also run a regression that also include an interaction term between GDP growth and a dummy variable for the time period after 1997. [add an endnote for the choice of shifting point] The estimated parameter of the interaction term is significant. The adjusted R-squared is 0.28<sup>36</sup>.

Including lagged realized capital gains greatly improve the goodness of fit of the model, increasing the adjusted R-squared from around 0.3 to over 0.8. [See models (3), (4), (5)] Parameters on lagged realized capital gains are highly significant in all three model specifications. (upper panel of the Table 22). The results also show that the state PIT become more responsive to realized capital gains after 1997: the estimated elasticity of total state PIT with respect to capital gains increases from 0.06 before 1997 to over 0.15 after 1999. With the realized capital gains included in the model, the elasticity of state GDP with respect to real economy growth decreases to below unity, and the interaction term in regression (4) for a shift of the GDP elasticity after 1999 is not significant.

Next we look at the model results when stock returns are used as a proxy for realized capital gains. [See models (3'), (4'), (5')]. While the parameters on lagged stock return are significant for the post-1997 period (0.19 in model (4') and 0.23 in model (5')), they are not for the pre-1999 period. The estimated elasticities of state PIT with respect to GDP in these models are close to 1 in model (3') and (5'). The values of adjusted R-squared for model (4') and (5') are around 0.6, which are about 0.3 lower than the values for their counterparts with realized capital gains as regressors.

Results of model (3') and (5') will be used as the basis for constructing simulation parameters.



Table 22 Regression results for individual income tax

**Growth of total state individual income tax; with GDP and realized capital gain tax as regressors  
Cyclical components; 1978-2015**

	Dependent variable: cyclical portion of state individual income tax growth									
	(1)		(2)		(3)		(4)		(5)	
	Estimate	p-value	Estimate	p-value	Estimate	p-value	Estimate	p-value	Estimate	p-value
GDP only										
GDP with a break point in 1997										
GDP and realized capital gains										
GDP and realized capital gains; both with a breaking point in 1997										
GDP and realized capital gains; capital gains with a break point in 1997										
GDP growth	<u>1.42</u>	0.001	<u>0.70</u>	0.145	<u>0.91</u>	0.000	<u>0.63</u>	0.011	<u>0.79</u>	0.000
GDP growth x after 1997			<u>1.73</u>	0.022			<u>0.40</u>	0.292		
Realized Capital Gains (1-year lag)					<u>0.12</u>	0.000	<u>0.06</u>	0.001	<u>0.06</u>	0.001
Realized Capital Gains (1-year lag) x after 1997							<u>0.09</u>	0.000	<u>0.16</u>	0.000
intercept	<u>0.00</u>	0.985	<u>0.00</u>	0.939	<u>0.00</u>	0.991	<u>0.00</u>	0.894	<u>0.00</u>	0.963
n		38		38		38		38		38
R-squared		0.28		0.38		0.79		0.88		0.88

**Growth of total state individual income tax; with GDP and stock returns as regressors  
Cyclical components; 1978-2015**

	Dependent variable: cyclical portion of state individual income tax growth					
	(3')		(4')		(5')	
	Estimate	p-value	Estimate	p-value	Estimate	p-value
GDP and stock return						
GDP and stock return; both with a breaking point in 1997						
GDP and stock return stock return with break point in 1997						
GDP growth	<u>0.99</u>	0.011	<u>0.57</u>	0.200	<u>0.97</u>	0.006
GDP growth x after 1999			<u>0.92</u>	0.172		
Stock return (1-year lag)	<u>0.17</u>	0.001	<u>0.06</u>	0.294	<u>0.04</u>	0.445
Stock return (1-year lag) x after 1999			<u>0.19</u>	0.024	<u>0.23</u>	0.004
intercept	<u>0.00</u>	0.895	<u>0.00</u>	0.853	<u>0.00</u>	0.917
n		38		38		38
R-squared		0.49		0.63		0.60

Note: Estimated parameters significant at 5% level are underscored.

Sales tax revenue

Table 23 shows the regression results of sales taxes. All growth rates below are referred to as the cyclical components of the annual real growth rates.

With the GDP growth as the only independent variable, the estimated elasticity of real general sales tax growth with real GDP growth is about 1.2. Note that this elasticity is lower than the

estimated GDP elasticity of PIT when GDP growth is the only independent variable (1.42), but higher than the estimated GDP elasticity of PIT when realized capital gains or stock returns is included (about 1). The R-squared is about 0.62.

In regression (2) and (2'), dummy variables for the last two recessions are added to the regression to capture the large response of sales tax to these recessions which may not be consistent with other time periods. The estimated GDP elasticities are 0.9 for the sample period 1985-2015 (regression (2)) and 0.7 for the sample period 1995-2015 (regression (2')), which are much lower than the estimated elasticity in (1). (2) and (2') show that there are 1.5-4 percent annual drops during the last two recessions that cannot be explained by the long-run GDP elasticity of general sales tax.

The estimated elasticity of select sales tax with respect to GDP is much lower than the elasticity of general sales tax and not statistically significant. Moreover, only a very small portion of the overall variation in the real growth of select sales tax can be explained by the real GDP growth (R-squared is only about 10 percent).

(1) is the preferred model for constructing simulation parameters because the estimated elasticity is more reasonable.

Figure 22 Cycles in sales taxes

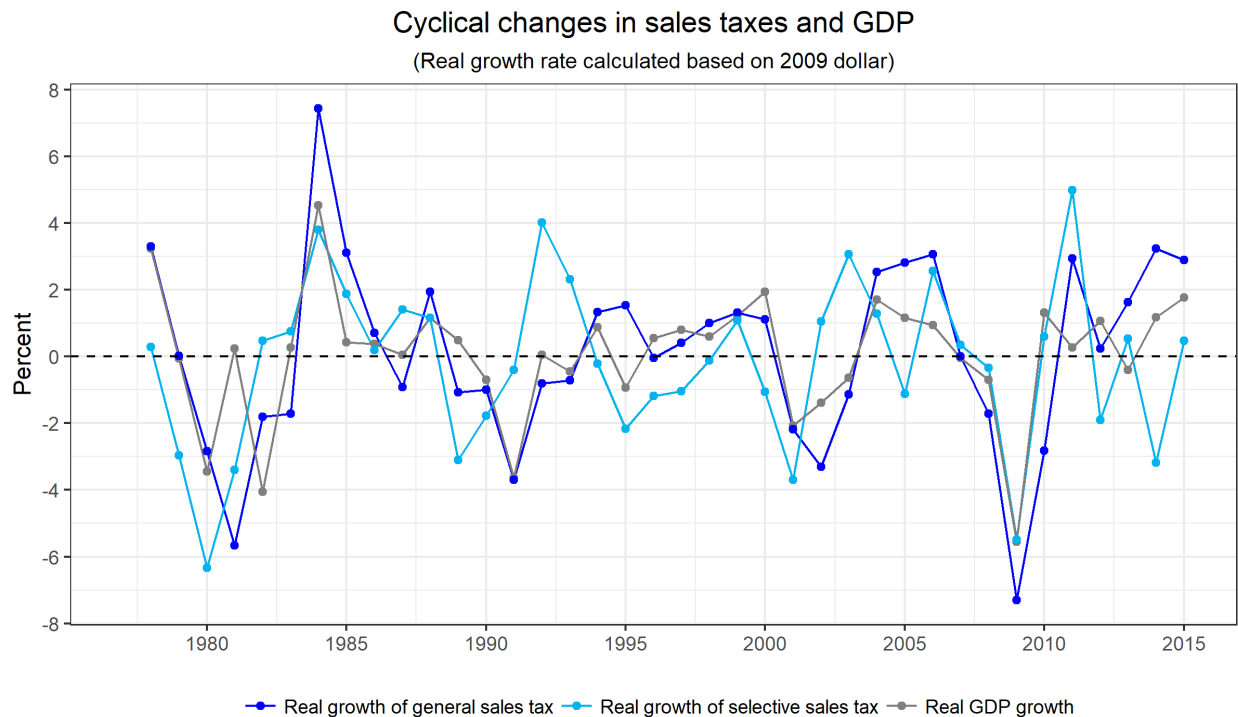


Table 23 Regression results for sales taxes

<b>Growth of general sales tax</b>						
<b>Cyclical component</b>						
	<b>(1)</b>		<b>(2)</b>		<b>(2')</b>	
	<b>GDP only (1985-2015)</b>		<b>GDP and dummies for the two recent recessions (1985-2015)</b>		<b>GDP and dummies for the two recent recessions (1995-2015)</b>	
	<b>Estimate</b>	<b>p-value</b>	<b>Estimate</b>	<b>p-value</b>	<b>Estimate</b>	<b>p-value</b>
<b>GDP growth</b>	<u>1.20</u>	0.000	<u>0.906</u>	0.000	<u>0.703</u>	0.004
<b>Recession 2001-03</b>			-0.015	0.081	<u>-0.025</u>	0.033
<b>Recession 2008-10</b>			<u>-0.030</u>	0.002	<u>-0.038</u>	0.001
<b>intercept</b>	0.00	0.689	0.006	0.045	<u>0.010</u>	0.012
<b>n</b>	31		32		21	
<b>R-squared</b>	0.62		0.74		0.80	

<b>Growth of selective sales tax</b>						
<b>Cyclical component</b>						
	<b>(1)</b>		<b>(2)</b>		<b>(2')</b>	
	<b>GDP only (1985-2015)</b>		<b>GDP and dummies for the two recent recessions (1985-2015)</b>		<b>GDP and dummies for the two recent recessions (1995-2015)</b>	
	<b>Estimate</b>	<b>p-value</b>	<b>Estimate</b>	<b>p-value</b>	<b>Estimate</b>	<b>p-value</b>
<b>GDP growth</b>	0.50	0.054	0.484	0.110	0.779	0.051
<b>Recession 2001-03</b>			0.008	0.582	0.019	0.265
<b>Recession 2008-10</b>			-0.009	0.527	0.002	0.913
<b>intercept</b>	0.00	0.968	0.000	0.998	-0.006	0.328
<b>n</b>	31		32		21	
<b>R-squared</b>	0.12		0.15		0.26	

Notes:

1. Estimated parameters significant at 5% level are underscored.

Other taxes (other than personal income and general sales taxes)

State non-personal-income-non-sales tax revenue includes corporate income tax, property tax received by state, and various other taxes. These taxes account for about 15 percent of total

state tax revenue in 2015. Table 24 shows the regression results of non-personal-income-non-sales tax revenue taxes.

If GDP growth is the only independent variable (model (1)), the estimated elasticity is about 1.3. For model (2), the interaction term between real GDP growth and the dummy variable for post-2000 period is significant. The result shows that the GDP elasticity of non-personal-income-non-sales taxes after 2000 is about three times as much as that before 2000. (0.6 before 2000 and 1.8 after 2000). This should be largely attributable to large decreases in corporate income tax during the two recent recessions.

For model (3), the interaction term between real GDP growth and the dummy variable for the recent two recessions is significant. For model (4), the dummy variables for the recent two recessions are significant.

Since the share of non-personal-income-non-sales in total state tax revenue is low, we choose to use the simplest model specification (model (1)) as the basis for the simulation parameters.

Figure 23 Cycles in other taxes



Table 24 Regression results for other taxes

Non-personal-income-non-sales taxes Cyclical component; 1978-2015								
	(1)		(2)		(3)		(4)	
	GDP only		GDP with a break point in 2000		GDP and interaction with the two recent recessions		GDP and dummies for the two recent recessions	
	Estimate	p-value	Estimate	p-value	Estimate	p-value	Estimate	p-value
GDP growth	<u>1.30</u>	0.000	0.62	0.111	<u>0.70</u>	0.061	<u>0.95</u>	0.006
GDP growth x after 2000			<u>1.80</u>	0.006				
GDP growth x recent 2 recessions					<u>2.04</u>	0.005		
recession 2001-2002							<u>-0.06</u>	0.023
recession 2008-2010							<u>-0.05</u>	0.032
intercept	0.00	0.881	0.00	0.982	0.00	0.505	0.01	0.304
n	38		38		38		38	
Adjusted R-squared	0.30		0.44		0.45		0.45	

Note: Estimated parameters significant at 5% level are underscored.

#### Model parameters for simulation

In our simulation model, we obtain growth for a revenue source by adding its trend growth to its cyclical growth, where the cyclical component will depend upon GDP growth from our economic scenario generator and, in the case of the income tax, upon stock market returns. As noted earlier, we assume – lacking any basis for a better assumption - that politicians will adjust tax revenue periodically to keep revenue as a constant share of the economy, meaning that trend revenue will grow at the same rate as trend GDP. We adopt the Congressional Budget Office’s assumption that trend growth in GDP over the next 30 years is likely to approximate 1.9 percent annually, reflecting anticipated labor force growth, labor force participation, and productivity. Thus, we assume trend tax growth is 1.9 percent for each tax source.

Table 25 shows the trend growth rate for each tax and our assumptions, based upon econometric estimates, of cyclical revenue elasticities. The first row shows the trend growth rate. The second row shows the cyclical elasticity for the revenue source, relative to GDP growth. For example, the sales tax cyclical elasticity is 1.2. Thus, if GDP has a cyclical decline (relative to its trend) of 3 percent, the sales tax will have a cyclical decline of 3.6 percent relative to its trend. The income tax is more complicated because it also reflects a cyclical relationship to stock market returns.

Table 25 Tax revenue elasticities

Assumptions about trend and cyclical growth rates of tax revenues for simulation analysis					
Growth rates in inflation-adjusted tax revenue					
		Individual income tax	General sales tax	Selective sales tax	Other taxes
<b>Trend growth rate</b>		1.9%	1.9%	1.9%	1.9%
<b>Elasticity of cyclical change</b>	<b>Real GDP growth</b>	1.0	1.2	0.5	1.3
	<b>Real Stock returns</b>	0.2	-	-	-

An example may help to explain how we apply these parameters in our simulation. Table 26 illustrates this for a hypothetical expansion year in a hypothetical simulation, in which real GDP grows 1 percentage point above its trend growth and real stock returns are 2 percentage points above their trend return.<sup>37</sup>

The first row shows how we calculate growth for the individual income tax, which is the most-complex case. The first two columns show that we estimate the elasticity of the cyclical component of the real income tax to the cyclical component of real GDP is 1.0, and the elasticity of the cyclical component of the real income tax to the cyclical component of real stock returns is 0.2 (these results are drawn from Table 25.) The next two columns show the cyclical real GDP growth and cyclical real stock return values for our hypothetical year, as mentioned in the paragraph above.

The final column shows the calculation of the cyclical component of the income tax: (a) it is boosted by 1 percentage point because real GDP is growing 1 percentage point above trend, and the elasticity is 1.0; and it is boosted by another 0.4 percentage points because real stock returns are 0.4 percentage points above trend and the elasticity relative to stock returns is 0.2. Adding the two calculated results, cyclical real income tax growth is 1.4 percentage points. Total real income tax growth is the sum of its trend revenue growth plus this cyclical component.<sup>38</sup>

The remaining rows of the table illustrate the calculation for the other taxes. This calculation is much simpler because these taxes generally are not correlated with stock returns and our models for these taxes do not incorporate stock returns.

Table 26 How we use model parameters to calculate tax revenue growth in the simulations

**An example of the calculation of cyclical tax revenue growth in the simulation**

	Elasticity with respect to		Hypothetical Cyclical GDP growth (c)	Hypothetical Cyclical Stock return (d)	Cyclical tax revenue growth (a)×(c) + (b)×(d)
	Cyclical GDP growth (a)	Cyclical stock return (b)			
<b>Individual income tax</b>	1.0	0.2	1%	2%	$1\% \times 1 + 2\% \times 0.2 = 1.4\%$
<b>General sales tax</b>	1.2	0	1%	2%	$1\% \times 1.2 = 1.2\%$
<b>Selective sales tax</b>	0.5	0	1%	2%	$1\% \times 0.5 = 0.5\%$
<b>Other taxes</b>	1.3	0	1%	2%	$1\% \times 1.3 = 1.3\%$

Notes:

1. All rates are inflation-adjusted values.
2. Cyclical growth of GDP and stock return are defined as the difference between total growth rate and trend growth rate.
3. For each tax category, total tax revenue growth is the sum of the cyclical growth calculated in the table and the assumed trend growth (1.9% in the simulation).

### Constructing stylized governments with specific mixes of taxes

#### Composition of state tax revenue

After we model the cyclical behavior of individual taxes, we construct tax portfolios for two stylized governments, consisting of a mix of income, sales, and other taxes. The income tax and the general sales tax are the two largest state government taxes, on average, and that the property tax dominates local government taxation. Before constructing stylized governments, it is useful to look at the distribution of the two largest state taxes, as shown in Figure 24.

Figure 24 Individual income tax and general sales tax shares of state tax revenue



States clustered in the upper left corner of Figure 24 rely heavily on the sales tax and have no income tax, including the large states of Florida, Texas, and Washington. These sales-tax-dominant states constitute an important group, especially given that the sales tax and the income tax have different cyclical behaviors, as discussed earlier. Most of the other states rely partly on the income tax and the sales tax. (Alaska and New Hampshire are the only two states with no general sales tax and no broad-based income tax.<sup>39</sup> Oregon is unique in relying extremely heavily on the income tax, and having no general sales tax.) While there are not other obvious groupings like the sales-tax dominant states, we think it makes sense to examine states that rely heavily on the income tax, with some sales tax reliance as well, as several large states fit this model, including California, Massachusetts, New York, and Virginia.



Table 27 Tax revenue structure of income-tax-dominant states and sales-tax-dominant states

**Structure of tax revenue of personal tax dominant states and sales tax dominant states  
in 2015**

	<b>Total tax revenue (million)</b>	<b>Personal Income tax</b>	<b>General sales tax</b>	<b>Select sales tax</b>	<b>Other taxes</b>
US	\$911,043	36.9%	31.4%	15.9%	15.8%
<b>Top 5 personal income tax dominant states</b>					
OR	\$10,575	69.1%	0.0%	14.2%	16.7%
VA	\$20,537	58.0%	18.5%	13.2%	10.4%
NY	\$78,205	55.9%	16.8%	13.8%	13.5%
MA	\$27,012	53.6%	21.5%	9.1%	15.7%
CA	\$151,173	51.6%	25.4%	9.2%	13.8%
<b>Top 5 sales tax dominant states</b>					
TX	\$55,086	0.0%	61.1%	25.4%	13.5%
WA	\$20,644	0.0%	60.6%	18.0%	21.3%
FL	\$37,218	0.0%	58.6%	22.9%	18.5%
SD	\$1,674	0.0%	58.0%	24.0%	18.0%
NV	\$7,533	0.0%	54.2%	26.0%	19.8%

[Constructing stylized governments](#)

After we model the cyclical behavior of individual taxes, we construct tax portfolios for two stylized governments, consisting of a mix of income, sales, and other taxes. One stylized government is income-tax-dominant, relying on the income tax for 55 percent of its tax revenue and using a mix of other taxes for the remainder, similar to Oregon, Virginia, or New York. A second stylized government is sales-tax dominant, relying on the general sales tax for 60 percent of its revenue, with no income tax, similar to Texas, Washington, or Florida.

Table 8 summarizes the tax portfolios for our two stylized governments.

Table 28 Tax structure of stylized state governments

<b>Tax revenue structure of stylized state governments</b>					
Each tax as a percentage of total tax revenue					
	<b>Personal income tax revenue</b>	<b>General sales tax revenue</b>	<b>Selective sales tax revenue</b>	<b>Other taxes</b>	<b>Total</b>
<b>Personal income tax dominant state</b>	55%	20%	10%	15%	100%
<b>Sales tax dominant state</b>	0%	60%	25%	15%	100%

The net effect of our approach is that governmental resources will reflect the business cycle and its character, and the composition of the government’s tax structure.

In our analyses below, we measure the burden of pension contributions as a percentage of tax revenue for these stylized governments. We also examine a “baseline” government in which tax revenue grows at the same rate as trend GDP, without any further cycle.

*Using the linked model to analyze risks and fiscal stresses*

We can examine the potential compounding of risks with the simulated tax revenue of stylized governments and simulated pension finances, with both simulations are linked to the macroeconomic simulation. Economic growth and asset returns are jointly simulated by a regime-switching process that capture the historical pattern of the business cycle. The tax revenues of stylized governments are then calculated based on the simulated economic conditions and asset returns. The asset returns are fed in to our pension simulation model to calculate the required employer contributions, among other variables of interest.

*Illustration of a single simulation*

We first illustrate the simulation results using a single simulation (#2) selected from 2000 stochastic simulation runs.

Figure 25 presents simulated GDP growth and stock returns. The years marked by dashed vertical lines have at least one quarter in recession<sup>40</sup>. In this particular simulation, economic downturns are mostly associated with sharp declines in stock returns (the model may generate recessions without large drops in stock returns in other simulations).

Figure 26 shows how tax revenues of the two types of stylized governments respond to the simulated economic conditions. The tax revenue of the income-tax-dominant state is generally more volatile than tax revenue of the sales-tax-dominant state and has larger declines during recession periods, which are consistent with the historical pattern.

Figure 25 Illustrating simulated economic conditions using a single run of simulation

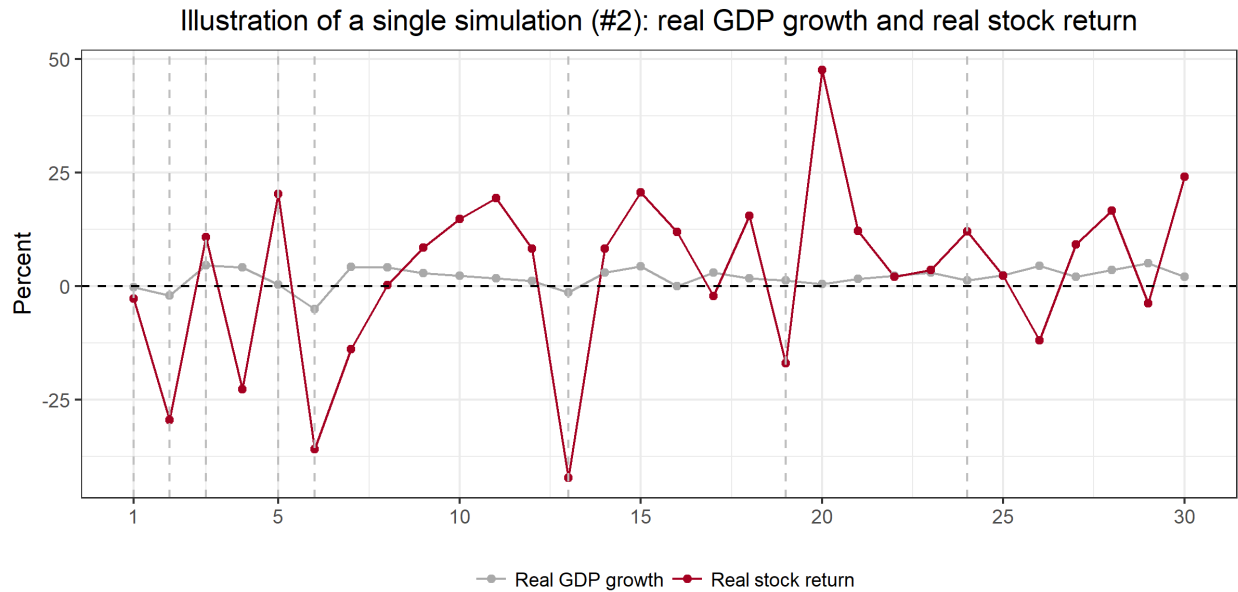
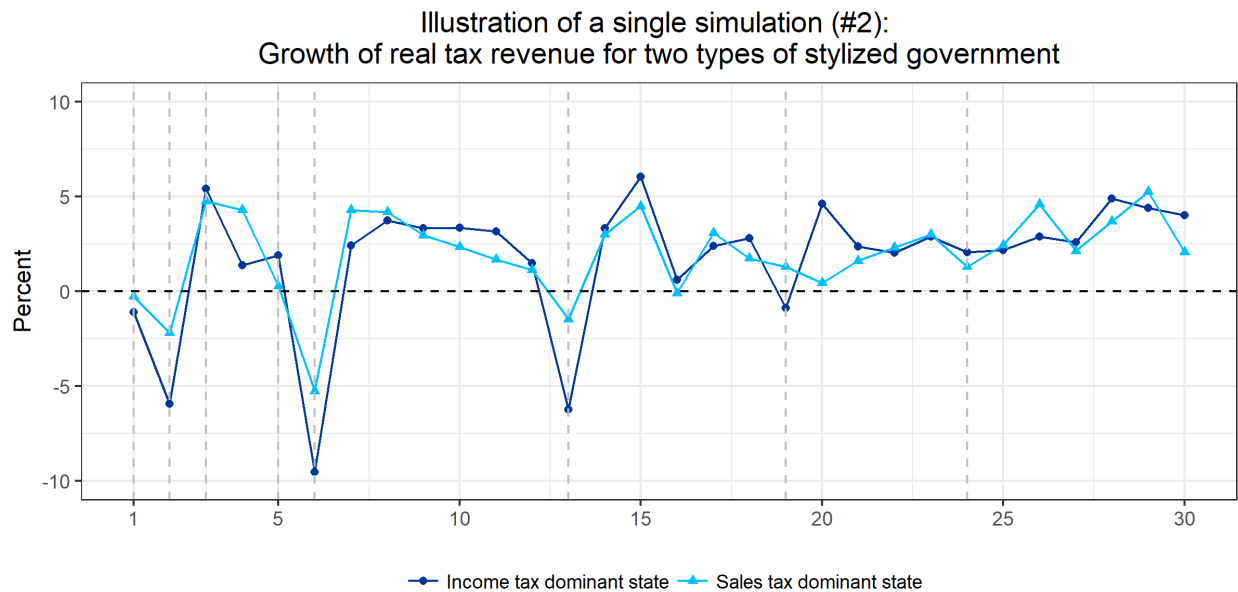


Figure 26 Illustrating the simulated tax revenue growth of the two stylized governments using a single run of simulation



*Distribution of real tax growth*

Figure 27 summarizes the distribution of total tax revenue growth of the two stylized governments across all 2,000 simulations and all years.

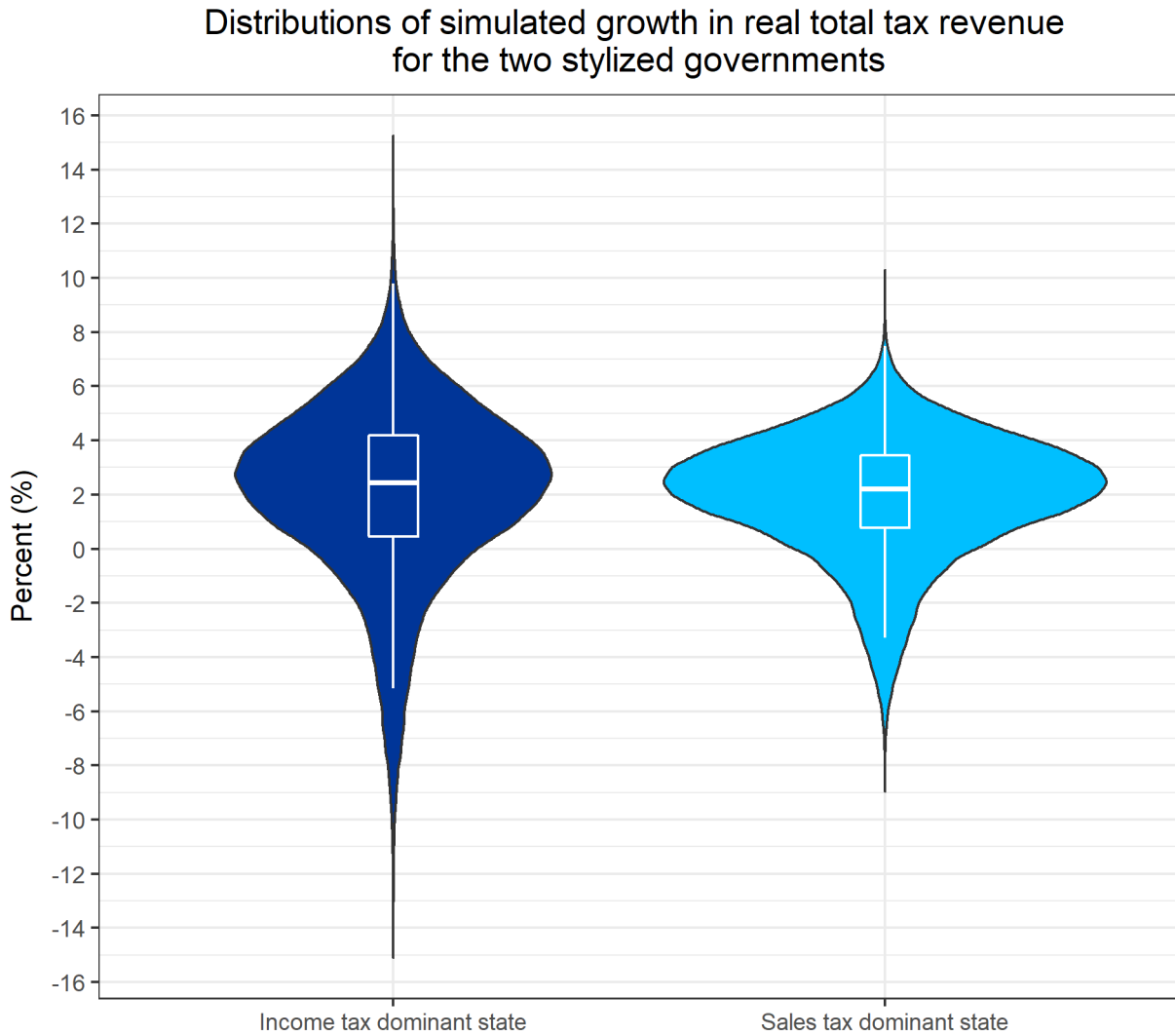
This is known as a “violin graph” because of its shape. The box in the figure for each stylized government describes the middle 50 percent of outcomes – the horizontal line in middle of the box marks the median, the line at the top of the box marks the 75<sup>th</sup> percentile, and the line at

the bottom marks the 25<sup>th</sup> percentile. Thus, we can see that the median tax revenue growth in the income-tax-dominant state was between 2 and 3 percent, growth at the 75<sup>th</sup> percentile was just over 4 percent, and growth the 25<sup>th</sup> percentile was just below 1 percent. We can see that the box for the income-tax-dominant state is larger than the box for the sales-tax-dominant state, indicating that the income-tax-dominant state had a greater range of growth rates than the sales tax state (its tax revenue was more volatile). The lines, or “whiskers” of the graph extend up and down from the box, indicating the 90<sup>th</sup> and 10<sup>th</sup> percentiles.

The mirror-image bulges on each graph show the distribution of outcomes, much like a bell curve turned sideways. (The left and right bulges, for a given stylized state, are symmetric, both showing the shape of the distribution.)

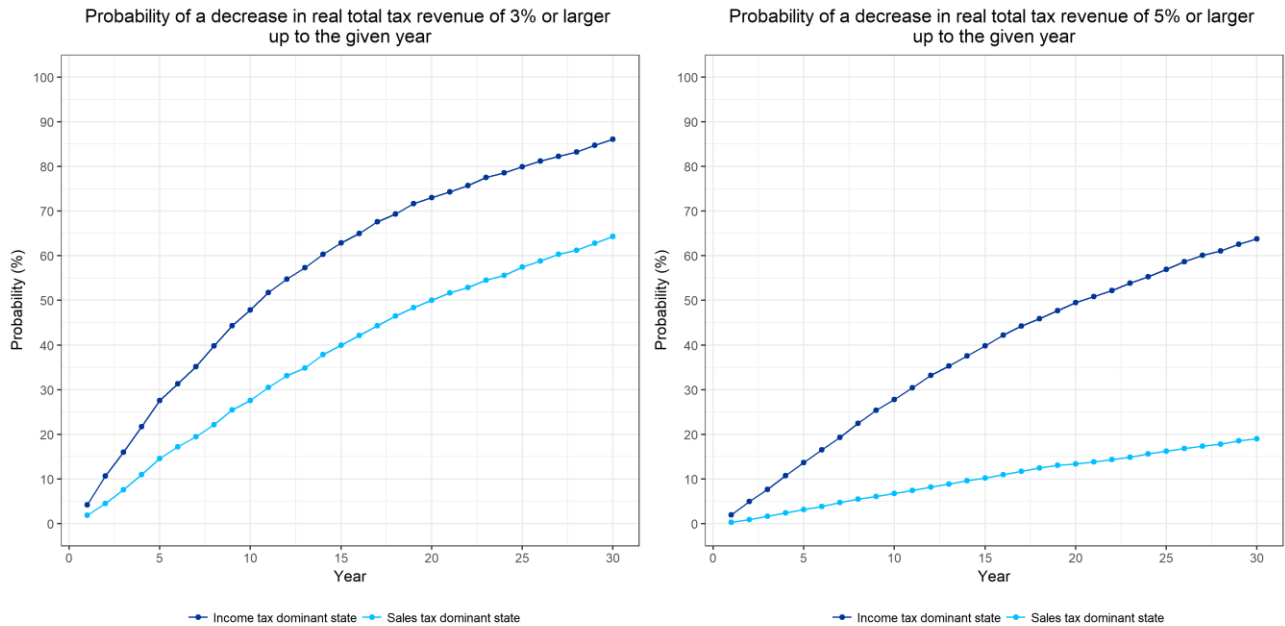
Tax revenue of the income-tax-dominant state is more volatile than tax revenue of the sales-tax-dominant state primarily because its revenue responds not only to GDP growth but also to capital gains driven in large part by stock market returns.

Figure 27 Simulated tax revenue is more volatile in the income-tax-dominant state than the sales-tax-dominant state



Next, we examine the likelihood of large declines in tax revenue for the two stylized governments in the 30-year simulation period. Figure 8 shows the probabilities of the stylized government having experienced a 3 percent or 5 percent drop of tax revenue in a single year up to a given simulation year. As expected, the income tax dominant state is much more likely to experience large decline in tax revenue compared to the sales tax dominant state.

Figure 28 The income tax dominant state is much more likely to experience large decline in tax revenue compared to the sales tax dominant state



### Simulating the finances of public pension plans

Separately, we have developed a simulation model that can be used to evaluate the implications of public pension plan investment risk.<sup>41</sup> The model calculates the actuarial liabilities, annual cash flows, funded status, and covered payroll of a public pension plan for future years based on the benefit rules, actuarial and economic assumptions, and demographic structure of the plan. Each year the model starts with beginning asset values and computes ending assets by subtracting benefits paid, adding employee and employer contributions (including any amortization), and adding investment income, which we calculate in the model. The model keeps track of these values and other variables of interest, such as the funded ratio and employer contributions as a percentage of payroll. It saves all results so that they can be analyzed after a simulation run in any way desired.

The model can be used to examine prototypical pension funds or can be used with data for actual pension funds. In the analysis that follows, we use a prototypical fund that resembles real-world pension plans in important ways. The key elements of the prototypical plan are described below.

- Plan characteristics.** It has a typical age distribution of workers and retirees, and benefits generally are calculated as 2.2 percentage points per year of service multiplied by the average of the final three years of salary, plus a 2 percent annual cost-of-living adjustment (COLA).<sup>42</sup> The age structure of the plan population is based on our analysis of data in the Public Plans Database, and is similar to the population of the Arizona

Public Employees Retirement System, which we found to be fairly typical in many ways. We assume that the plan has new hires each year sufficient to keep the number of active workers stable. The plan sponsor makes contributions each year. The plan starts off 75 percent funded with the actuarially liability calculated using a 7.5 percent discount rate.

- **Discount rates** (assumed rate of return). 7.5 percent, a common assumption among public pension plans.
- **Funding policies.** Most pension funds adopt funding policies that can dampen the volatility in contribution caused by unexpected investment losses and gains and other deviation from their actuarial assumptions. The choice of funding policy also determines how fast the plan pays down its Unfunded Actuarial Accrued Liability (UAAL). We examine three funding policies with differing smoothing effects on contributions.
  - 1) 10-year level-dollar closed amortization with 5-year asset smoothing. (Fastest repayment of UAAL; least contribution-smoothing effect)
  - 2) 15-year level-dollar closed amortization with 5-year asset smoothing.
  - 3) 30-year level-percent closed amortization with 5-year asset smoothing. (Slowest repayment of UAAL; strongest contribution-smoothing effect; common in plans with high UAAL)
- **Investment returns.** We examine two sets of stochastic investment returns:
  - 3) Returns generated by the regime-switching simulation model. As described earlier, the regime-switching model generates 2,000 simulations each with a 30-year series of equity returns and bond returns, linked to GDP growth. We construct a portfolio consisting of 70 percent of equities and 30 percent bonds.
  - 4) Returns drawn from normal distribution. The expected annual return and standard deviation are equal to those of the simulation-based 70/30 portfolio. 2,000 random returns are drawn from the normal distribution for each year in the simulation.

Given a funding policy and a set of investment returns, we run 2,000 simulations and compute the required employer contributions, the funded ratio, and other variables of interest of the prototypical pension plan. Then we can gain insight into the fiscal stress that the pension plan creates for the stylized governments by comparing the simulated employer contributions against the simulated tax revenues.

*Evaluating the potential compounding of risks from correlated investment returns and tax revenues.*

By this point, we have tax revenues, asset returns, and pension finances that are all generated within a coherent simulation framework. Changes in tax revenues and investment returns, which in turn affect required pension contributions, are both driven by business cycles that are

simulated by the regime-switching process of GDP growth between recessions and expansion periods.

With these simulation outputs of the linked models, we can examine the compounding of risks from correlated investment returns and tax revenues. We evaluate the pension-related risks for the two stylized governments - income-tax-dominant state and sales-tax-dominant state, and compare that with the risks under a baseline model that has no linkage between tax revenue and asset returns. This comparison will demonstrate how the pension-related risks would be understated if the linkage is ignored.

We constructed alternative model structures to investigate how much of the increase in risk can be attributed to each of the following: (1) linking tax revenue to the economy, (2) linking asset returns to the economy, (3) the combined effects of linking tax revenue and asset returns to the economy, and (4) tax structures of the stylized governments, and the fact that income tax revenue is linked directly to GDP growth as well as to stock returns, potentially creating additional risk for the income-tax-dominant state.

The table below summarizes the six model structures we compare.

*Table 29 Simulation models for comparison*

Model	Description	Tax revenue linked to GDP?	Asset return linked to GDP?	Based on stylized government?	Tax revenue linked to asset return?
(1)	<b>Unlinked model:</b> - Constant growth of total tax revenue (equal to trend GDP growth). - Returns from normal distribution.	No	No	No	No
(2)	<b>Asset return linked only:</b> - Constant growth of total tax revenue (equal to trend + cycle GDP growth). - Returns from regime-switching simulation model.	No	Yes	No	No
(3)	<b>Tax revenue linked only:</b> - Cyclical growth of total tax revenue (equal to trend plus cycle GDP growth). - Returns from normal distribution.	Yes	No	No	No
(4)	<b>Both tax revenue and asset return linked:</b> - Cyclical growth of total tax revenue (equal to trend plus cycle GDP growth). - Returns from regime-switching simulation model.	Yes	Yes	No	No
(5)	<b>Stylized government: sales-tax-dominant state</b> - Cyclical growth of total tax revenue; estimated responsiveness to GDP growth. - Returns from regime-switching simulation model.	Yes	Yes	Yes	No
(6)	<b>Stylized government: income-tax-dominant state</b> - Cyclical growth of total tax revenue; estimated responsiveness to GDP growth and asset return. - Returns from regime-switching simulation model.	Yes	Yes	Yes	Yes

Model (1) serves as the baseline case in which neither tax revenue nor asset return are linked to the regime-switching process of GDP. It assumes that the total real tax revenue of the



government grows at a constant rate which is equal to the trend growth rate of GDP (1.9 percent) and investment returns are normally distributed.

Model (2) only links asset returns to GDP cycles and Model (3) only links tax revenue, so the simulation results of these two models reflect the isolated effects of linking only one variable to the business cycle.

Model (4) links both tax revenue and asset returns to GDP cycles and reflects the compounding effects of correlated tax revenue and asset returns. In Models (3) and (4), the growth rate of total tax revenue equals the total GDP growth rate (trend plus cycle).

Models (5) and (6) also link both tax revenue and asset returns to business cycles, but they differ from Model (4) in that they incorporate our analysis of how tax revenues respond to business cycles for the two stylized governments – the sales-tax-dominant state in Model (5), and the income-tax-dominant state in Model (6). A distinguishing feature of Model (6) is that the dominant tax category – personal income tax – is linked not only to GDP growth, but also to stock market returns.

Under each model structure, required pension contributions under the three alternative funding policies described in section *Simulating the finances of public pension plans* are generated by our pension simulation model. The required employer contribution as a percentage of the total tax revenue in the first simulation year are determined as follows:

- We assume that the required employer contribution under the policy “30-year open constant percent of payroll” accounts for 5 percent of the total tax revenue of the sponsoring government in year-1.
- The total tax revenue calculated based on this assumption is then applied to the other two policies.  
The shares of employer contribution in the total tax revenue in year 1 are 7.3% under the “15-year open constant dollar” policy and 8.7 percent under the “10-year open constant dollar” policy.

We examine two types of risks that the sponsoring governments of public pension plans may face:

1. The required employer contributions become very high relative to fiscal resources available to the sponsoring government, creating great fiscal pressure and potentially crowding out other public services. In our analysis of the simulation results, we measure this type of risk by the probability that employer contributions as a percentage of total tax revenue will become more than 5 percentage points higher than the level in year 1 at any time during the 30-year simulation period.
2. The required employer contributions rise sharply in a short period of time, creating difficulty in budget planning and short-term fiscal pressure. We measure this type of risk

by the probability that employer contribution rises more than 3 percent of total tax revenue in a 2-year period at any time during the 30-year simulation period.

Our 3-percent- and 5-percent-of-tax-revenue thresholds for our two risk measures are arbitrary, but we think they are broadly indicative of tax-revenue changes that cause fiscal stress. They are based in part upon our experience analyzing and participating in state budget decision making over several decades. One way to think about this is to examine tax revenue shortfalls in recessions. In the first two years of the 1990, 2001, and 2007 recessions, tax revenue in the median state fell short by about 3.6 percent, 13 percent, and 14 percent, respectively.<sup>43</sup> From this perspective, our two-year risk measure involves less fiscal pressure than tax shortfalls in recent recessions, but given that pension contributions typically only are three to seven percent of state tax revenue, the risk measure will only be triggered if contributions increase very significantly. Our longer-term risk measure is larger than our short-term indicator, but will only be triggered if there is a sizable shift in the use of tax revenue over a long period of time.

Another way of thinking about the risk measures is to conduct sensitivity analysis, using different (but still arbitrary) thresholds, to see whether alternative measures lead to different conclusions about the impact of considering linkages between the economy, investment returns, and tax revenue. We discuss the results of sensitivity analysis in a not in the section that describes our results.

Table 30 shows the risk of employer contributions rising by 5 percentage points or more relative to the starting point under our different models and funding policies. The three rows of the table show our three funding policies and the six columns show the size of the risk under our six models.

The first column presents the risk measure when tax revenue grows smoothly over time without any business cycle effects, and investment returns are normally distributed and independent over time, without any correlation to the economy. It is akin to the kinds of simulations we have done in past papers, and the risk measures are lower than measures for all of the models to the right. For example, there is a 6.8 percent chance that employer contributions will rise by more than 5 percentage points of tax revenue above their year 1 levels at some point during the 30-year simulation period – a smaller risk than in any column to the right.

The second column also assumes that tax revenue grows smoothly, but now investment returns are generated by the regime-switching simulation model that results in a heavier left tail in the return distribution than in the normal distribution, and returns are correlated with economic conditions rather than being independent over time. Thus, comparing this column to column 1 shows us the pure effect of linking investment returns to the economy, without considering tax revenue cyclicity, shedding light on the impact of introducing non-normality in investment returns. The risks of high employer contributions are only slightly higher than in the base case.

Column 3 now allows tax revenue to be exactly as cyclical as real GDP is – for example, if real GDP grows 2 percentage points more quickly than its trend, tax revenue will grow at the same rate. Investment returns are drawn from the normal distribution rather than being generated by our ESG. Thus, comparing column 3 to column 1 shows us the pure effect of introducing volatility in tax revenue that is not correlated with investment returns, without accounting for differences in tax structure. The risks are slightly greater than in the base case and not much different from those in column 2.

Column 4 allows tax revenue to be exactly as cyclical as real GDP and has investment returns generated by the model. Comparing this column to column 3 shows the risk introduced by having investment returns linked to the economy in addition to cyclical tax revenue. Comparing this column to column 2 shows the risk introduced by having cyclical revenue. In either comparison, the increase in risk is quite large. For example, under a 10-year open constant-dollar funding policy, the risk of a contribution increase that is at least 5 percentage points of tax revenue is 17.8 percent, or more than a one in six chance of such an increase during the 30-year simulation period. Comparing column 4 to column 1 demonstrates that while the isolated effects of linking tax revenue or investment returns to the business cycle are modest, the compounding effect of correlated tax revenue and investment returns is quite large. For example, under the 15-year open constant dollar funding policy, the risk of high pension contribution is 2.7 times as great much as in column 1 (17.2 percent compared to 6.4 percent), while for columns 2 and 3 the measures are only 1.4 and 1.5 times as great.

Columns 5 and 6 introduce our two stylized governments, with tax revenue cycles depending not just on real GDP cycles but also on tax structure. In both models, returns are generated by the economic scenario generator. The risks for the sales-tax-dominant government actually are slightly smaller than if revenue is exactly as cyclical as real GDP.<sup>44</sup> The risks for the income-tax-dominant state are much larger: the risk of a large employer contribution is 22.8 percent, compared to 16.5 percent for the sales-tax-dominant state, and compared to 6.8 percent for the basecase where neither investment returns nor tax revenue are linked to the economy – the kinds of simulations that we have reported on in past papers.

Table 30 Risk of high pension contribution relative to tax revenue

**Risk of high employer contribution relative to tax revenue under alternative funding policies**

Amortization method for unfunded liability	Probability of employer contribution as a percentage of total tax revenue being more than 5 percentage points above the year-1 level at any time during the 30-year simulation period					
	Constant growth of total tax revenue (equal to trend GDP growth)		Cyclical growth of total tax revenue (equal to trend + cycle GDP growth)		Stylized governments	
	Normally distributed returns	Simulated returns	Normally distributed returns	Simulated returns	Sales-tax-dominant state	Income-tax-dominant state
					Simulated returns	Simulated returns
(1)	(2)	(3)	(4)	(5)	(6)	
10-year open constant dollar	6.8%	9.2%	9.5%	17.8%	16.5%	22.8%
15-year open constant dollar	6.4%	8.9%	9.3%	17.2%	16.6%	22.8%
30-year open constant percent of payroll	1.3%	2.0%	4.9%	11.1%	10.1%	18.0%

Note: In year 1, employer contribution as a percentage of total tax revenue under the three amortization methods are 8.67% (10-year open constant dollar), 7.25% (15-year open constant dollar), and 5% (30-year open constant percent of payroll).

Comparing risks as we move down the rows of Table 30 also is instructive, as it shows the interplay between funding policy and contribution risk. Risks of large employer contributions fall somewhat as the funding policy lengthens, particularly as we move from 15 years to 30 years and change from constant dollar funding to level-percentage-of-pay funding. Because this risk measure compares every year in a 30-year simulation to the starting year, it is a measure of longer-term risks, and funding policies that smooth contributions have only limited ability to reduce this risk. They have much greater ability to reduce the risk of large employer contributions in a short period of time, as discussed below.

**The risk of large contribution increases in a short period of time**

Table 31 presents our short-term risk measure – the probability that employer contributions will rise by more than three percent of tax revenue in any two-year period during our 30-year simulation - for the same models and funding policies. The format and interpretation are the same as for Table 30.

As with our other risk measure, the risk increases as we move to the right in the table, but not as much as with our longer-term risk measure, the risk of a large contribution increase over the 30-year period. Furthermore, risks of sharp employer contribution increases diminish dramatically as we move down the rows of the table, lengthening the contribution smoothing period and liberalizing the policy, in the bottom row, by using constant-percentage of pay funding: long smoothing periods and more-liberal funding policies reduce short-term employer contribution risks substantially.

Table 31 Risk of sharp increases in pension contributions relative to tax revenue

**Risk of sharp increase in employer contribution relative to tax revenue under alternative funding policies**

Amortization method for unfunded liability	Probability of employer contribution rising more than 3 percent of total tax revenue in any 2-year period during the 30-year simulation period					
	Constant growth of total tax revenue (equal to trend GDP growth)		Cyclical growth of total tax revenue (equal to trend + cycle GDP growth)		Stylized governments	
	Normally distributed returns	Simulated returns	Normally distributed returns	Simulated returns	Sales-tax-dominant state	Income-tax-dominant state
	(1)	(2)	(3)	(4)	(5)	(6)
10-year open constant dollar	43.0%	44.1%	44.2%	48.7%	47.8%	54.7%
15-year open constant dollar	20.9%	23.7%	23.3%	31.4%	30.5%	37.8%
30-year open constant percent of payroll	0.5%	1.2%	1.1%	3.1%	2.5%	6.9%

Note: In year 1, employer contribution as a percentage of total tax revenue under the three amortization methods are 8.67% (10-year open constant dollar), 7.25% (15-year open constant dollar), and 5% (30-year open constant percent of payroll).

Contribution smoothing policies can reduce employer risks, but they do not make risks go away: they transfer them to the pension fund and its stakeholders, as we discuss in the next section.

**Contribution-smoothing policies cannot make risks go away: They transfer risks from governments to pension plans**

Lower risks for sponsoring governments under funding policies with stronger contribution-smoothing effects come at the expense of higher risk that the pension plan will become severely underfunded, as Table 32 demonstrates.

The first column of Table 32 ties to the fourth column of Table 30 to provide a useful point of reference, showing how the risk of sharp increases in employer contributions declines as the funding period lengthens and the policy liberalizes. The last column shows the risk that the plan’s funded ratio will fall below 40 percent, which we consider to be crisis territory, at some point during the 30-year simulation period. (We will explain the intervening columns in a minute.) As we move down the rows, the risk of this crisis-level funding rises from 7.1 percent with 10-year constant-dollar funding policy to 30.9 percent, or nearly a one in three chance, with 30-year constant percent of payroll funding. (This latter risk is far greater than risks we estimated in earlier papers, when investment returns were not linked to economic conditions.) The risk protection that governments gain from stretched-out funding policies results in greater risk to pension plans.

Table 32 Greater contribution smoothing leads to higher risk of pension underfunding

	Risk of sharp increase in employer contribution relative to tax revenue*	Employer contribution as a % of tax revenue in year 1	Median Present value at year 1 of total employer contribution for year 1-15**	Median Present value at year 1 of total employer contribution for year 16-30**	Probability of low funded ratio***
<b>10-year open constant dollar</b>	48.7%	8.7%	1.32	0.64	7.1%
<b>15-year open constant dollar</b>	31.4%	7.3%	1.23	0.67	11.7%
<b>30-year open constant percent of payroll</b>	3.1%	5.0%	1.00	0.68	30.9%

Notes:

\* Probability of employer contribution rising more than 3 percent of total tax revenue in any 2-year period during the 30-year simulation period based on Model (4) (Cyclical growth of total tax revenue with simulated investment returns).

\*\* The present value at year 1 of total employer contribution in year 1-15 under the policy "30-year open constant percent of payroll" is standardized to 1. All other values are standardized accordingly. □

\*\*\* Probability of low funded ratio: the probability of funded ratio falling below 40% in any year during the 30 year simulation period.

### Summary of simulation results

The simulation results demonstrate that it is important to allow for the correlation between tax revenue, economic conditions, and investment returns when examining the investment-related risks to public pension funds and the sponsoring governments. Contribution increases required after economic downturns are much larger, relative to tax revenue, when we allow tax revenue to be cyclical (varying with the economy) than when it is under simpler assumptions of stable tax revenue growth.

The pension-related risks for governments are also affected by the responsiveness of the governments' fiscal resources to economic conditions. Income-tax-dominant states, the tax revenues for which is more cyclical, would bear higher pension-related risks compared with sales-tax-dominant states, which have less cyclical tax revenues.

The choice of funding policies for public pension funds also has a significant impact on the risks the sponsoring governments face. Funding policies that pay down unfunded liabilities slowly and have strong smoothing effects on contribution requirements can protect the sponsoring governments from the risks of high or sharp increase in pension contributions, at the expense of greatly increasing the risk of severe underfunding for the pension funds.

### Alternative investment return approaches

#### Preliminary discussion of alternative investment return scenarios

Our macroeconomic model generates investment returns that are internally consistent with our estimates of growth in real GDP. This is valuable, but there are other important ways to generate investment returns that may deviate from the traditional simple normal independent-over-time estimates of returns. We intend to use those approaches to provide us with a more-robust look at the potential impact of alternative investment return assumptions.

For example, two ways of generating what are known as “fat tails” (return distributions that have greater likelihood of extremely good or extremely bad returns, compared with the normal distribution) are (1) “bootstrapping” from historical returns (repeatedly drawing returns from historical data), and (2) drawing returns from statistical distributions that have fatter tails than the normal distribution by definition, such as Student’s t-distribution. We use these methods to show how stock market returns constructed in this fashion differ from the normal distribution. At this point, this analysis is preliminary and illustrative – we have not yet incorporated these returns into our pension simulation model.

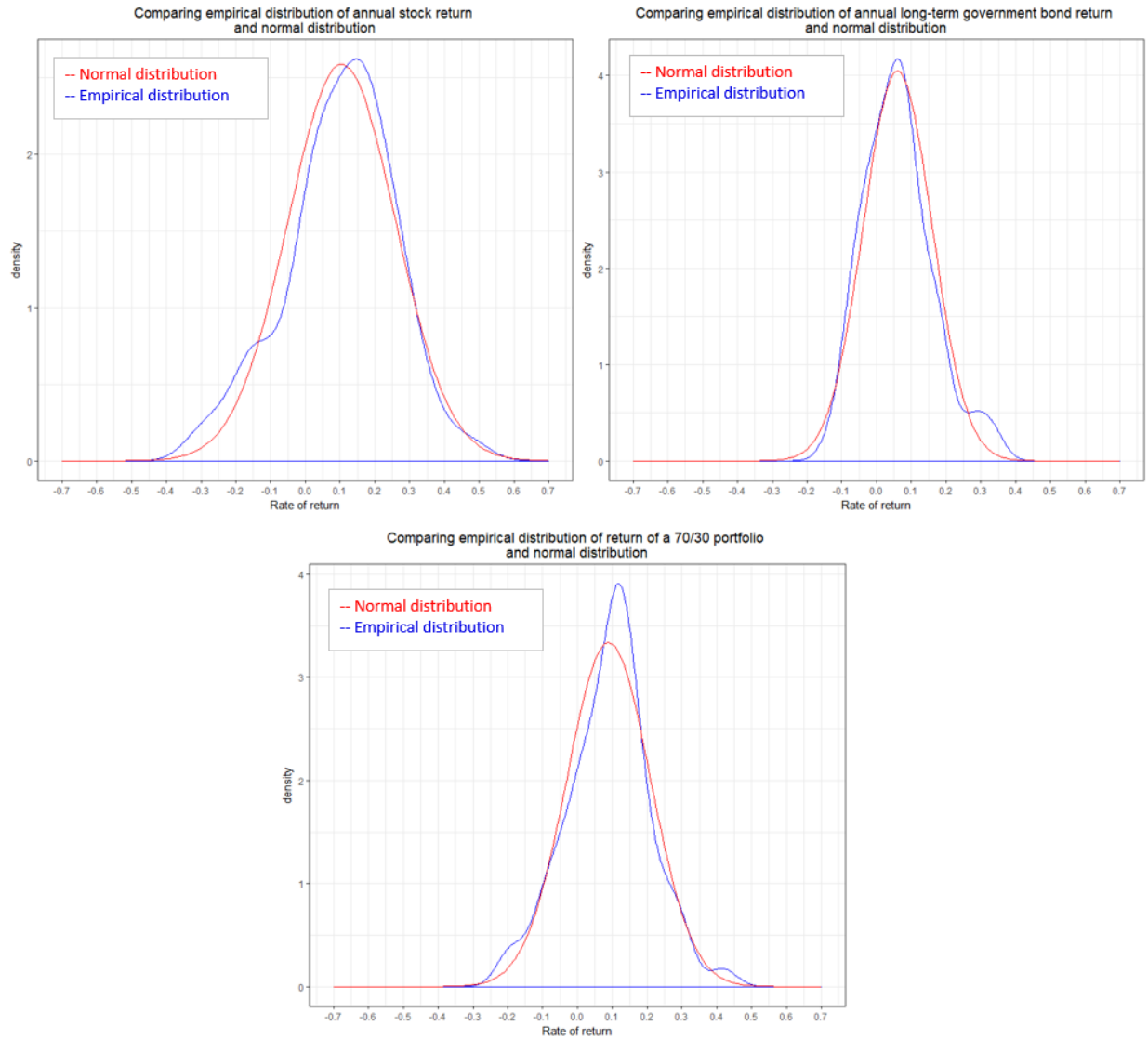
#### Departures from simple investment-return assumptions

We first examine how the empirical distributions of asset return differ from the commonly used normal distribution. Figure 29 compares the empirical distributions of annual returns based on historical data from 1955 to 2015, to otherwise similar normal distributions. It does this for three portfolios: a portfolio consisting solely of large-cap stocks, one consisting solely of long-term government bonds, and a portfolio consisting of 70 percent large-cap stocks and 30 percent government bonds. In each comparison, the mean and standard deviation of the normal distribution are the same as those of the asset return distribution it is compared against.

The upper-left panel of Figure 29 shows the fat lower tail of annual stock returns. The normal distribution underestimates the likelihood of losses of 15 percent or larger. The distribution of long-term government bond returns exhibits a slightly thinner lower tail compared to the normal distribution, and its upper tail is considerably heavier than the normal distribution for returns in the range of 30 percent to 40 percent. (This may be caused by sharp changes in bond yields in certain historical periods. We are going to examine whether these are special cases.

The distribution of the 70/30 portfolio exhibits fatter tails than the normal distribution in both the upper and lower ends.

Figure 29 “Fat tails” in empirical distribution of asset return



We constructed asset returns using the “bootstrap” method based on historical data from 1955 to 2015). Table 33 shows the probabilities of large negative returns under the bootstrapped distribution and the normal distribution with the same mean and standard deviation. For stock returns, the bootstrapped distribution is more than 3 times as likely as the normal distribution to generate a loss of 30 percent or larger (that is, the 1.48 percent probability is more than 3 times as large as the 0.42 percent probability). The bootstrapped distribution of stock returns is about 1.4 times as likely to generate a loss of 20 percent or larger as is the normal distribution.



Table 33 Comparing normal asset returns and bootstrapped asset returns

	Probability of return lower than			
	-40%	-30%	-20%	-10%
<b>Stock; Normal distribution</b>	0.10%	0.42%	2.20%	9.06%
<b>Stock; Bootstrapped distribution</b>	0.00%	1.48%	3.02%	12.26%
<b>70/30 portfolio; Normal distribution</b>	0.00%	0.06%	0.70%	4.76%
<b>70/30 portfolio; Bootstrapped distribution</b>	0.00%	0.00%	0.00%	6.30%

In a later version of this paper, we will examine further the deviations of bootstrapped asset returns from normally distributed asset returns and use our pension simulation model to compare their implications for the evaluation of pension finance risks.

### Conclusion

In this paper we examine how our assessment of risks to pension funds and their sponsoring governments changes when we take into consideration the correlation between economic conditions, investment returns, and governmental tax revenue. We do this by linking an economic scenario generator - a small stochastic macroeconomic model that simulates real GDP growth and investment returns – to models of tax revenue cyclicity and a pension finance simulation model. Our paper builds upon other recent research that has examined risks to pension funds and governments under alternative economic scenarios.

The simulation results demonstrate that it is important to allow for correlation among economic conditions, investment returns, and tax revenue when examining the investment-related risks to public pension funds and the sponsoring governments. Contribution increases required after economic downturns are much larger, relative to tax revenue, when we allow tax revenue to be cyclical (varying with the economy) than under simpler assumptions of stable tax revenue growth.

Pension-related risks for governments can be further exacerbated by how state tax revenue structures respond to economic conditions. All else equal, income-tax-dominant states, with highly cyclical tax revenue, face higher pension-related risks than sales-tax-dominant states, which have less cyclical tax revenues.

The choice of funding policies for public pension funds also has a significant impact on the risks that sponsoring governments face. Funding policies that pay down unfunded liabilities slowly and have strong smoothing effects on contribution requirements can protect the sponsoring governments from the risks of high or sharp increase in pension contributions, at the expense of greatly increasing the risk of severe underfunding for the pension funds.

This paper shows how important it is to incorporate budgetary resources in pension fund risk analysis, and how that analysis can be deepened by modeling business cycles and investment returns together. Our work in this area is preliminary and can be extended and improved upon.

## Endnotes

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<sup>1</sup> One reason the sales-tax-dominant state may have lower risk than the case where revenue grows at the same rate as GDP is that in simulations with many recessions, its general sales tax share of total tax revenue actually falls because, with a general sales-tax elasticity greater than one, it falls faster than GDP and the sales-tax-dominant state's tax revenue increasingly is made up of more-stable revenue sources.

<sup>2</sup> See, for example, Greg Mennis, Susan Banta, and David Draine, "Assessing the Risk of Fiscal Distress for Public Pensions: State Stress Test Analysis," M-RCBG Associate Working Paper Series (Harvard Kennedy School Mossavar-Rahmani Center for Business and Government, May 2018), [https://www.hks.harvard.edu/sites/default/files/centers/mrcbg/files/AWP\\_92\\_final.pdf](https://www.hks.harvard.edu/sites/default/files/centers/mrcbg/files/AWP_92_final.pdf).

<sup>3</sup> See, for example, Greg Mennis, Susan Banta, and David Draine., Andrew G. Biggs, "The Public Pension Quadrilemma: The Intersection of Investment Risk and Contribution Risk," *The Journal of Retirement*, Summer 2014.

<sup>4</sup> For example, in the 2007 recession, aggregate inflation-adjusted state government tax revenue fell more than 10 percent from peak to trough, based on the authors' analysis of quarterly state tax revenue from the U.S. Bureau of the Census, and the S&P 500 index fell by 40 percent from the quarter in which the recession started until the trough quarter.

<sup>5</sup> For an excellent discussion of these issues, see Abdullah Sheikh and Hongtao Qiao, "Non-Normality of Market Returns: A Framework for Asset Allocation Decision-Making" (J.P. Morgan Asset Management, 2009).

<sup>6</sup> James D. Hamilton, "A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle," *Econometrica* 57, no. 2 (1989): 357–84, <https://doi.org/10.2307/1912559>.

<sup>7</sup> In the case of income taxes, typical progressive tax rate structures and fixed exemptions and deductions mean that tax revenue generally rises faster than income over the long run, because productivity growth drives real incomes upward, and inflation drives nominal income further upward. Inflation-indexing, where used, only partially offsets this. For discussion of long-run relationships between the sales tax and the economy, see John L. Mikesell, "The Disappearing Retail Sales Tax," *STATE TAX NOTES*, March 5, 2012.

<sup>8</sup> This long-run assumption is similar to the assumption in the recent Pew analysis that own-source revenue would grow at the same rate as the economy. However, Pew did not impose a cycle around this trend. Greg Mennis, Susan Banta, and David Draine, "Assessing the Risk of Fiscal Distress for Public Pensions: State Stress Test Analysis," May 2018.

<sup>9</sup> <http://slfdqs.taxpolicycenter.org/index.cfm>

<sup>10</sup> These data are actual collections as reported to the U.S. Bureau of the Census. Thus, they include changes in revenue resulting from economic changes, and changes in revenue resulting from policy changes, administrative changes, and other technical factors. It would be preferable to have tax revenue data that are purged of policy changes and other similar changes, but no such data are available on a high-quality comprehensive basis, in a reproducible form. Thus, we use actual collections. We believe, based on our general knowledge of the field and on exploratory data analysis that these data allow us to capture the essence of the relationships we care about, although they are noisier than we like.

<sup>11</sup> Although the Hodrick-Prescott filter has suffered some criticism (see James D Hamilton, 2017, "Why You Should Never Use the Hodrick-Prescott Filter", Working Paper), it is a widely adopted approach to detrending time series. For our purpose, we think the Hodrick-Prescott filter does a good job decomposing the variables of interest into trend and cycle components

<sup>12</sup> The non-trend component includes any remaining noise in the data, as well as the cyclical component.

<sup>13</sup> The sharp dip in the property tax in the late 1970s and early 1980s probably is not related primarily to the business cycle, but rather likely is related to the property tax revolt of that era.

<sup>14</sup> The relationship between income taxes and the economy, or between any tax and the economy, is for more complex than we describe here. Our models are abstractions, intended to capture the most important elements of the relationship.

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<sup>15</sup> Income tax revenue is more closely related to capital gains income than it is to stock market values, which are only one factor influencing capital gains. However, our economic scenario generator produces forecasts of stock market growth, not capital gains growth, and so we use the former in our income tax model.

<sup>16</sup> For a good analysis of how institutional factors affected property tax in New York City, see Ana Champeny, “Stabilizing Revenue Collection During the Downturn: How Assessment Phase Ins and Caps Affect the City’s Property Tax,” Fiscal Brief (New York City Independent Budget Office, February 2011), <http://www.ibo.nyc.ny.us/iboreports/taxstability2102011.pdf>.

<sup>17</sup> In our simulations, we use long-run trend assumptions of 1.9 percent for real GDP and 4.7 percent for real stock returns. Thus, in this example, total real GDP growth in the hypothetical year would be 2.9 percent (1 percent cycle plus 1.9 percent trend) and total real stock returns would be 6.7 percent (2 percent cycle plus 4.7 percent trend).

<sup>18</sup> Because we posit, lacking better information, that politicians will strive to keep tax revenue a relatively constant share of the economy over the long run, we posit that trend revenue growth for each tax will be the same as trend real GDP growth, or 1.9 percent. Thus – not shown in the table – total real income tax growth in this hypothetical year, which is the sum of trend growth plus cyclical growth, would be 3.3 percent.

<sup>19</sup> Each state relies partly on selective versions of one or more of these taxes, but they are not broad based.

<sup>20</sup> In our macroeconomic simulation model, recessions are generated by a Markov regime-switching process with quarterly frequency. The model is specified such that the expected GDP growth rates in recession periods and expansion periods and the expected durations of recession and expansion periods are consistent with historical data. The characteristics of the simulated recessions may not be consistent with the definitions of recession that are sometimes found in other articles or studies, such as “two consecutive quarters of GDP decline”. In this report, the simulated quarterly growth rates of GDP are converted to annual rates and then, combined with the GDP elasticities of taxes, are used to produce annual growth rates of tax revenue.

<sup>21</sup> For example, see Donald Boyd and Yimeng Yin, “Investment Risk-Taking by Public Pension Plans: Potential Consequences for Pension Funds, State and Local Governments, and Stakeholders in Government,” Conference Paper (Rockefeller Institute of Government, July 2017).

<sup>22</sup> Technically, this is not a cost-of-living adjustment, but an automatic escalator. The benefit is assumed to increase annually regardless of what happens to the cost of living. Many plans have such an arrangement.

<sup>23</sup> Based upon authors’ analysis of revenue forecast data reported in National Governors’ Association, Fall Fiscal Survey of the States, various years.

<sup>24</sup> One reason the sales-tax-dominant state may have lower risk than the case where revenue grows at the same rate as GDP is that in simulations with many recessions, its general sales tax share of total tax revenue actually falls because, with a general sales-tax elasticity greater than one, it falls faster than GDP and the sales-tax-dominant state’s tax revenue increasingly is made up of more-stable revenue sources.

<sup>25</sup> We will do this in a manner that takes into consideration and builds upon the work described in Greg Mennis, Susan Banta, and David Draine. “Assessing the Risk of Fiscal Distress for Public Pensions: State Stress Test Analysis.” Harvard Kennedy School of Government, 2017.

<sup>26</sup> See Guidolin and Timmermann (2006).

<sup>27</sup> Daily reinvestment of dividends is assumed.

<sup>28</sup> In a later draft we will provide data on the relative size of own-source revenue and taxes. A recent paper used own-source revenue as its measure of resources: Greg Mennis, Susan Banta, and David Draine, “Assessing the Risk of Fiscal Distress for Public Pensions: State Stress Test Analysis,” accessed January 5, 2018, <https://www.hks.harvard.edu/sites/default/files/centers/mrcbg/files/pension/Mennis%20et%20al%2C%20Pew%20Submission%2010.18.17.pdf>.

<sup>29</sup> more details to come in later draft.

<sup>30</sup> In an unrelated project, fiscal analysts at the Pew Charitable Trusts have constructed a data set for state taxes that attempted to remove the impact of policy changes. These data may prove useful in a later stage of this project.

<sup>31</sup> Inflation-driven income growth can be compensated for by indexing a tax system fully to inflation but that is not the practice among states. And even then, productivity-driven income growth would still drive real incomes upward, causing revenue to rise more quickly than incomes.

<sup>32</sup> This long-run assumption is similar to the assumption in the recent Pew analysis that own-source revenue would grow at the same rate as the economy. However, Pew did not impose a cycle around this trend. Greg Mennis,

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Susan Banta, and David Draine, “Assessing the Risk of Fiscal Distress for Public Pensions: State Stress Test Analysis.”

<sup>33</sup> For example, see Leslie McGranahan and Richard H. Mattoon, “State Tax Revenues over the Business Cycle: Patterns and Policy Responses,” Essays on Issues, Chicago Fed Letter (The Federal Reserve Bank of Chicago, June 2012), [http://www.chicagofed.org/digital\\_assets/publications/chicago\\_fed\\_letter/2012/cfljune2012\\_299.pdf](http://www.chicagofed.org/digital_assets/publications/chicago_fed_letter/2012/cfljune2012_299.pdf).

<sup>34</sup> John L. Mikesell, “The Disappearing Retail Sales Tax.”

<sup>35</sup> For good analysis see: Byron F. Lutz, “The Connection Between House Price Appreciation and Property Tax Revenues,” Working Paper, Finance and Economics Discussion Series (Federal Reserve Board, September 12, 2008), <http://www.c.federalreserve.gov/pubs/feds/2008/200848/200848pap.pdf>.

<sup>36</sup> R-squared is a measure of model fit that shows how much of the variation in the dependent variable can be explained by the independent variables included in the model. The adjusted R-squared imposes a penalty for adding additional independent variables.

<sup>37</sup> In our simulations, we use long-run trend assumptions of 1.9 percent for real GDP and 4.7 percent for real stock returns. Thus, in this example, total real GDP growth in the hypothetical year would be 2.9 percent (1 percent cycle plus 1.9 percent trend) and total real stock returns would be 6.7 percent (2 percent cycle plus 4.7 percent trend).

<sup>38</sup> Because we posit, lacking better information, that politicians will strive to keep tax revenue a relatively constant share of the economy over the long run, we posit that trend revenue growth for each tax will be the same as trend real GDP growth, or 1.9 percent. Thus – not shown in the table – total real income tax growth in this hypothetical year, which is the sum of trend growth plus cyclical growth, would be 3.3 percent.

<sup>39</sup> Each state relies partly on selective versions of one or more of these taxes, but they are not broad based.

<sup>40</sup> In our macroeconomic simulation model, recessions are generated by a Markov regime-switching process with quarterly frequency. The model is specified such that the expected GDP growth rates in recession periods and expansion periods and the expected durations of recession and expansion periods are consistent with historical data. The characteristics of the simulated recessions may not be consistent with the definitions of recession that are sometimes found in other articles or studies, such as “two consecutive quarters of GDP decline”. In this report, the simulated quarterly growth rates of GDP are converted to annual rates and then, combined with the GDP elasticities of taxes, are used to produce annual growth rates of tax revenue.

<sup>41</sup> For example, see Donald Boyd and Yimeng Yin, “Investment Risk-Taking by Public Pension Plans: Potential Consequences for Pension Funds, State and Local Governments, and Stakeholders in Government,” Conference Paper (Rockefeller Institute of Government, July 2017).

<sup>42</sup> Technically, this is not a cost-of-living adjustment, but an automatic escalator. The benefit is assumed to increase annually regardless of what happens to the cost of living. Many plans have such an arrangement.

<sup>43</sup> Based upon authors’ analysis of revenue forecast data reported in National Governors’ Association, Fall Fiscal Survey of the States, various years.

<sup>44</sup> One reason the sales-tax-dominant state may have lower risk than the case where revenue grows at the same rate as GDP is that in simulations with many recessions, its general sales tax share of total tax revenue actually falls because, with a general sales-tax elasticity greater than one, it falls faster than GDP and the sales-tax-dominant state’s tax revenue increasingly is made up of more-stable revenue sources.