A Macroeconomic Perspective on Border Taxes

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1 Introduction

Tax policy that differentially treats domestically produced and foreign produced goods have long been a part of the arsenal of policy makers. These ‘border taxes’ can be explicit and take the form of import tariffs and export subsidies or more subtle in the form of value-added taxes (VAT) and payroll tax cuts. More recently discussions of corporate tax reform in the U.S. have included proposals (Auerbach, Devereux, Keen, and Vella (2017)) for a border adjustment tax (BAT) that disallows deductions of imported input costs from corporate revenue when computing taxable corporate profits, and excludes export revenue from taxation.

In some situations these policies have been used as a tool to stimulate economies. Famously, Keynes in 1931 proposed in the Macmillan Report to the British Parliament that a combination of an import tariff and an export subsidy be used to mimic the effects of an exchange rate devaluation while maintaining the gold pound parity. Farhi, Gopinath, and Itskhoki (2014) demonstrate the equivalence of the VAT-payroll tax swap policy to replicate the effects of a nominal exchange rate devaluation in economies with a fixed exchange rate. This analysis motivated the adoption of a fiscal devaluation in France in 2012.

In other circumstances, as in the current debate on BAT, the taxes are argued to be neutral, that is having no effect on real allocations, as flexible exchange rates adjust to undo any real effect of the border tax. This prediction of neutrality has its origins in a classic result in the field of international trade, called Lerner (1936) symmetry, and in its applications in Grossman (1980)

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and Feldstein and Krugman (1990). According to this result, when prices and wages are fully flexible and trade is balanced a combination of a uniform import tariff and an export subsidy of the same magnitude must be neutral, having no effect on imports, exports and other economic outcomes. This is because the tax leads to an increase in domestic wages relative to foreign wages (in a common currency), which in turn leaves unchanged the post-tax relative price of imported to domestically produced goods in all countries. That is, despite the higher tax on imports relative to domestically produced goods the lower relative wage of foreign products leaves the relative price of imported to domestic goods unchanged. Similarly on the export side, despite the export subsidy, the higher relative domestic wage, leaves unchanged the relative price of domestic goods in foreign markets.

The assumptions of flexible prices and balanced trade are unrealistic and so the question is under what circumstances do we retain neutrality when we depart from these assumptions. Barbiero, Farhi, Gopinath, and Itskhoki (2017) provide answers to this question, both qualitative and quantitative, for BAT, in a dynamic general equilibrium model. In this note, I illustrate the arguments using a simplified and static version of their model. The main takeaways are that for neutrality to hold all of the following five conditions need to be satisfied:

1. When prices/wages are sticky, if there is symmetry in the pass-through of exchange rates and taxes into prices faced by buyers in each market then neutrality is preserved. This symmetry is satisfied when prices are sticky in the producer’s currency or in the local currency. In the former case, with fully preset prices, the pass-through of either is 100% and consequently the exchange rate appreciation offsets taxes and there are no real effects. In the later case the pass-through is zero in either case and there are no real effects.

In reality though prices of traded goods are sticky in dollars regardless of origin and destination, as argued in Gopinath (2015), which leads to a break down of neutrality. In this case, with fully preset prices, the exchange rate appreciation has no pass-through into import prices faced by domestic households and firms while taxes have 100% pass-through. On the flip side the tax has no pass-through into export prices (in foreign currency) while the exchange rate has 100% pass-through. In this case, the exchange rate appreciation leads to a decline in imports and in exports and therefore a decline in overall trade in the short-run. These results hold more generally with staggered or state-contingent pricing.

One might question if sticky dollar pricing is a reasonable assumption in the face of large exchange rate changes when presumably more firms choose to adjust prices. This argument however fails to recognize that most exporters to the U.S. are also importers and therefore
have a significant fraction of costs that are stable in dollars. As the value added share of trade is much smaller than trade flows and with most trade invoiced and sticky in dollars even for trade with non-U.S. partners the scope to cut dollar prices is limited. Factors such as these explain why despite a substantial and rapid appreciation of the dollar by 15% between the third quarter of 2014 and third quarter of 2015 the pass-through into border prices remained low at around 35% (as opposed to a full pass-through of 100%).

2. Monetary policy should respond only to the output gap and CPI inflation, and not respond to the exchange rate, to maintain neutrality. Even if sticky prices satisfy the assumptions for neutrality if the monetary authority targets the exchange rate it will generate real effects. This is precisely why the same border taxes are proposed as a stimulative policy tool under pegged exchange rates, while being neutral under flexible exchange rates. Relatedly, if foreign monetary authorities attempt to mitigate the depreciation of their currencies, a reasonable assumption, it will also lead to a break down in neutrality.

3. When trade is not balanced neutrality continues to hold as long as all international assets and liabilities are in foreign currency. If however, some international holdings are in domestic currency then neutrality is no longer preserved. Since for the United States, foreign assets are mostly in foreign currency, while foreign liabilities are almost entirely in dollars, this generates a large one-time transfer to the rest of the world and a capital loss for the US of the order of magnitude of around 10% of US annual GDP. Barbiero, Farhi, Gopinath, and Itskhoki (2017) however show that because this transfer is a small fraction of U.S. wealth quantitatively the real impact is small.

4. The implementation of the border adjustment tax must take the form of a one-time permanent and unanticipated policy shift for it to be neutral. Otherwise, expectations of a border tax in the future will cause immediate exchange rate appreciations that impact portfolio choices of private agents and therefore will have real consequences. Similarly, neutrality fails to hold if the policy is expected to be reversed and therefore transitory, or if the other countries are expected to retaliate with their own policies in the future.

5. Neutrality requires that the border taxes be uniform and cover all goods and services. Service sectors such as tourism whose sales to foreigners take place within borders are not treated the same as exports that cross borders, which in turn affects neutrality.

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1 We make this calculation in Emmanuel Farhi, Gita Gopinath and Oleg Itskhoki “Trump’s Tax Plan and the Dollar.”
In the next section I sketch a simple model to analytically illustrate these arguments.

2 Benchmark Case for Neutrality

In this section I lay out a simple general equilibrium model with BAT to demonstrate neutrality. This is a static version of the model in Barbiero, Farhi, Gopinath, and Itskhoki (2017) with simplifying restrictions on functional forms. The world has two regions Home (H) and Foreign (F) and the border adjustment tax is implemented in H. I assume all foreign variables to be exogenous and unchanging, consistent with assuming that H is small relative to F. The results are unchanged if instead both regions are large. To simplify exposition I use specific functional forms, however, the predictions apply more generally as demonstrated in Barbiero, Farhi, Gopinath, and Itskhoki (2017). I focus here on BAT, but the results apply to other uniform border taxes listed in the introduction.

The model has four agents: ‘Bundlers’ that combine domestically produced and imported goods to produce a non-traded good that is used for household consumption and as an intermediate input for production; Households that consume the non-traded good and supply labor; Firms that produce a unique variety of traded good that is sold in H and F; Government that taxes and transfers.

I will start by assuming that prices and wages are fully flexible and then consider the case of sticky prices.

2.1 Bundlers

The non-traded consumption good (C) and intermediate good (X) are produced by competitive firms\(^2\) called ‘bundlers’ that combine domestic and imported goods using the technology,

\[ \mathcal{F} = \mathcal{F}_{HH}^{\theta} \mathcal{F}_{FH}^{\theta} \]

\[ \mathcal{F}_{HH} = \left( \int \omega \mathcal{F}_{HH}(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}} \]

where \( \mathcal{F}_{HH} \) is a CES bundle of domestic varieties \( \omega \).

\(^2\)The analysis remains unchanged if instead I assumed that the firms had monopoly power and charged mark-ups.
The price of the bundle $F$ is given by,

$$P = \frac{P_{HH}^{1-\theta} P_{FH}^\theta}{(1-\theta)^{1-\theta} \theta^\theta}$$  \hspace{1cm} (2)$$

$$P_{FH} = \frac{P_{FH}^* \mathcal{E}}{1-\iota \tau}$$  \hspace{1cm} (3)$$

where $P_{HH}$ is the price of the domestically produced good, $P_{FH}^*$ is the price of the imported good in $F$ currency, $\mathcal{E}$ is the exchange rate defined as $H$ currency per unit of $F$ currency so that an increase in $\mathcal{E}$ is a depreciation of $H$ currency. $\tau$ is the corporate tax rate and $\iota = 1$ with BAT and equals zero otherwise. This formulation captures the fact that under BAT imported goods are subject to taxation relative to domestically produced goods by the margin of the corporate tax rate $\tau$. The demand for domestic and imported goods are,

$$F_{HH} = (1-\theta) \frac{P}{P_{HH}} (C+X) \hspace{1cm} F_{FH} = \theta \frac{P}{P_{FH}} (C+X)$$  \hspace{1cm} (4)$$

$$F_{HH}(\omega) = \left( \frac{P_{HH}(\omega)}{P_{HH}} \right)^{-\sigma} F_{HH}$$

\subsection*{2.2 Households}

Households consume $(C)$ and supply labor $(N)$ and they maximize utility, $U(C, N) = \log C - N$ subject to a budget constraint

$$PC + \mathcal{E} B^* = WN + \Pi + T$$  \hspace{1cm} (5)$$

where $B^*$ is the inherited net foreign debt in foreign currency, $\Pi$ is after tax corporate profits, $W$ is the wage rate, and $T$ are transfers from the government. From the optimality of the labor-leisure decision we have,

$$- \frac{U_N}{U_C} = W \frac{P}{P} \rightarrow PC = W$$  \hspace{1cm} (6)$$

In a static environment there are no other decisions to be made.

\subsection*{2.3 Firms}

Firms produce a unique variety of good $\omega$ that they sell domestically and export. The production function is given by,

$$Y(\omega) = AL(\omega)^{1-\alpha} X(\omega)^\alpha$$  \hspace{1cm} (7)$$
where $L$ is labor, $X$ is the intermediate input, and $A$ is productivity. The firm’s profits are given by,

$$
\Pi(\omega) = (1 - \tau) \left( P_{HH}(\omega)Y_{HH}(\omega) + P_{HF}(\omega)Y_{HF}(\omega) - MC \cdot Y(\omega) \right) + \iota\tau P_{HF}(\omega)Y_{HF}(\omega) \tag{8}
$$

where $\tau$ is the corporate profit tax rate, $P_{HH}(\omega)$ and $Y_{HH}(\omega)$ are the price the firm sets and the quantity it sells in the $H$ market (to $H$ bundlers). Similarly $P_{HF}(\omega)$ and $Y_{HF}(\omega)$ are the price and quantity for the $F$ market. $MC$ is the (constant) marginal cost of production and $Y(\omega) = Y_{HH}(\omega) + Y_{HF}(\omega)$.

$$
MC = \frac{1}{\alpha^\alpha (1 - \alpha)^{1-\alpha}} \frac{W^{1-\alpha} P^\alpha}{A}
$$

With BAT, $\iota = 1$, and consequently export revenues are not taxed. This is the second margin on which the BAT works. The optimality conditions for hiring labor and intermediate inputs satisfy,

$$
(1 - \alpha) \frac{Y}{L} = \frac{W}{MC} \quad \alpha \frac{Y}{X} = \frac{P}{MC}
$$

Market clearing will imply that $F_{HH} = Y_{HH}$.

### 2.4 Government

From the government’s budget constraint we have,

$$
T = \frac{\tau}{1 - \tau} \Pi - \frac{\iota \tau}{1 - \tau} \left[ P_{HF}Y_{HF} - \mathcal{E}P_{FH}^*Y_{FH} \right] \tag{9}
$$

Combining the government’s and households budget constraint we have the condition,

$$
\mathcal{E}B^* = P_{HF}Y_{HF} - \mathcal{E}P_{FH}^*Y_{FH}
$$

that is the debt needs to be repaid by running a trade surplus.

**Proposition 1** When prices and wages are flexible, the equilibrium with BAT ($\iota = 1$) has the same real allocation as the equilibrium without BAT ($\iota = 0$), but with a higher relative domestic wage and more appreciated real exchange rate. That is,

$$
\frac{\mathcal{E}W^*}{W'} = (1 - \tau) \frac{\mathcal{E}W^*}{W} \quad \frac{\mathcal{E}P^*}{P'} = (1 - \tau) \frac{\mathcal{E}P^*}{P}
$$
where the primes refer to the BAT equilibrium.

The detailed proof is provided in the appendix. Basically a BAT is associated with an increase in relative wages at home, that is an increase in \( \frac{W}{W^*} \).

\[
\frac{W'}{\varepsilon W^*} = \frac{W'}{P'} = \frac{1}{1 - \tau \varepsilon W^*}
\]

where the last equality follows from the fact that real wages \((W/P)\) are unchanged across the two equilibria with the same level of consumption, which follows from the labor-leisure decision Eq. (6). The increased demand for \( H \) goods following BAT leads to an increase in relative wages at home that perfectly offsets the tax advantage. The demand at \( H \) and \( F \) therefore remain unchanged as post-tax relative prices in \( H \) and \( F \) remain unchanged, that is,

\[
\frac{P'_{HH}}{P'} = \mu \left( \frac{W'}{P'} \right)^{1-\alpha} = \mu \left( \frac{W}{P} \right)^{1-\alpha} = \frac{P_{HH}}{P}
\]

\[
\frac{P'_{HF}}{P^*} = \frac{(1 - \tau)\mu \left( \frac{W'}{P'} \right)^{1-\alpha}}{\varepsilon P^*} = \frac{\mu \left( \frac{W}{P} \right)^{1-\alpha}}{\varepsilon P} = \frac{P^*_{HF}}{P^*}
\]

where \( \mu = \frac{\sigma}{\sigma - 1} \) is the constant mark-up.\(^3\) Consequently, \( Y_{HH} \) and \( Y_{HF} \) remain unchanged. Relatedly, the terms of trade which measures the relative price of imports to exports at the border, remain unchanged.

\[
TOT = \frac{P^*_{HF}}{P_{HF}^*} = \frac{P^*}{(1 - \tau)\mu(W'/P')^{1-\alpha}\varepsilon'/P'} = \frac{P^*}{\mu(W/P)^{1-\alpha}\varepsilon/P} = \frac{P^*}{P^*_{HF}}
\]

From the consolidated budget constraint we have,

\[
C' + \frac{\varepsilon'}{1 - \ell \tau} B^* = \frac{W'}{P'} N + \frac{\Pi'/P'}{1 - \tau}
\]

(10)

With real wages and real after-tax profits unchanged\(^4\) the real appreciation offsets the BAT leaving the real allocation unchanged.

If we continue to assume flexible prices, and introduce money into the model and pick a particular nominal equilibrium where the monetary authority targets a constant \( CPI \), that is \( P = 1 \), the implication in this case is for the nominal exchange rate to appreciate one-to-one

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\(^3\)Because of symmetry across firms I drop the \( \omega \) reference.

\(^4\)This can be demonstrated using equation Eq. (8) and the relation that \( \frac{P'_{HF}}{P} = (1 - \tau) \frac{P'_{EF}}{P} \).
with the tax.

**Proposition 2** If prices are flexible and the monetary authority targets a fixed $P = 1$ then, $\frac{e'}{e} = \frac{1}{1-\tau}$, that is the nominal exchange rate does all the work.

In the event that the U.S. corporate tax rate is reduced to 20%, the scenario with BAT involves a U.S. dollar appreciation of 25% as compared to one without BAT. This 25% number has been reported frequently in discussions of the BAT.

# 3 Departures from the Benchmark

## 3.1 Sticky Prices and Wages

In the previous section I assume flexible prices. In this section I demonstrate that when the exchange rate changes by the same magnitude as the BAT, that is $\frac{e'}{e} = (1 - \tau)$, two form’s of price stickiness, namely producer currency pricing and local currency pricing maintain neutrality, while a third, dollar currency pricing, which arguably is a more realistic description of price stickiness in international trade, leads to a break down of neutrality.

### 3.1.1 Producer currency pricing

The Mundell-Fleming benchmark assumes that prices are sticky in the producer’s currency and the law of one price holds, that is, $P'_{HH} = \bar{P}_{HH}$ and $P'_{HF} = (1 - \tau)\bar{P}_{HH}$. In this case the nominal exchange rate appreciation is sufficient to mimic the flexible price equilibrium.

$$TOT' = \frac{e'P'_{FH}}{(1 - \tau)\bar{P}_{HH}} = \frac{e'P_{FH}}{\bar{P}_{HH}} = TOT$$

The price of imports in $H$ and exports to $F$ given by,

$$P'_{FH} = \frac{\bar{P}_{FH}e'}{1 - \tau} \quad \quad P'_{HF} = \frac{(1 - \tau)\bar{P}_{HH}}{e'}$$

are unchanged as the exchange rate appreciation fully offsets the higher tax on imports, and while the subsidy lowers the home currency price at which $H$ sells to $F$ the exchange rate depreciation of $F$ currency raises its price in foreign currency with a complete offset. There is therefore no change in demand for $H$ goods from either region. Importantly, this result follows from the symmetry in pass-through (100%) of the exchange rate and the BAT into buyers prices ($P'_{FH}$ and $P'_{HF}$).
3.1.2 Local Currency Pricing

The other extreme is where prices buyers face in the destination market are preset in the destination market’s currency. That is $P'_{FH} = \bar{P}_{FH}$ and $P'_{{*}H} = \bar{P}_{{*}H}$ are sticky. In this case neither the exchange rate nor the BAT have any effect on the prices buyers face and therefore their demand. Similarly, there is no change in the terms of trade,

$$TOT' = \frac{(1 - \tau)\bar{P}_{FH}}{\mathcal{E}'\bar{P}_{{*}H}} = \frac{\bar{P}_{FH}}{\mathcal{E}'\bar{P}_{{*}H}} = TOT$$

where the numerator is the price $F$ sellers receive and the denominator is the price $H$ sellers receive. Again here there is symmetry in pass-through of the exchange rate and the BAT into export and import prices of buyers, with both being zero in this case.

3.1.3 Dollar Currency Pricing

As described in Gopinath (2015), and Casas, Díez, Gopinath, and Gourinchas (2017), the pricing of traded goods in world trade is dominated by dollar pricing. Gopinath and Rigobon (2008) and Gopinath, Itskhoki, and Rigobon (2010) report for the U.S. that around 94% of U.S. imports and 97% of U.S. exports are priced and sticky in dollars for durations of 10-12 months. Further, even conditional on a price change the pass-through into dollar prices is low.

In this more realistic case dollar prices at the border are sticky. The net of tax price faced by buyers at $H$ and $F$ are then,

$$P'_{FH} = \frac{\bar{P}^b_{FH}}{(1 - \tau)}$$
$$P'_{{*}H} = \frac{\bar{P}^b_{{*}H}}{\mathcal{E}}$$

$$\frac{P'_{FH}}{P'} = \frac{1}{(1 - \theta)1 - \theta(\mathcal{E})}$$
$$\frac{P'_{{*}H}}{P_{*}} = \frac{\bar{P}^b_{{*}H}}{\mathcal{E}P_{*}}$$

where the super-script $b$ stands for border and I have used the relation $P = \frac{P_{{*}H}^1 - \thetaP_H^1}{(1 - \theta)1 - \theta\mathcal{E}}$ and $P_{HH} = \bar{P}_{HH}$. In this case there is asymmetry in the pass-through of exchange rates and taxes into demand relevant prices. The exchange rate appreciation does not effect the border price of imports in dollars and consequently the net of tax prices rise leading to a shift in demand away from imports. In the case of exports, the border price in dollars does not change with the tax rate and consequently the appreciation of the dollar makes the foreign currency price of exports rise leading to a drop in demand for U.S. exports. Exchange rate appreciations cannot undo the tax change leading to
a drop in imports and exports. As shown in Barbiero, Farhi, Gopinath, and Itskhoki (2017) the negative impact on overall trade (the sum of exports and imports) is large, while on the trade balance is small.

### 3.2 Saving/Borrowing

The derivation in Section 2 was for a static environment without borrowing and lending. Neutrality is preserved as long as all assets and liabilities are in foreign currency and the BAT implementation is one-time and unanticipated. To illustrate this, consider the case when only foreign currency bonds that pay a gross interest rate of $R^*_t$ are traded internationally. From the Euler equation we have,

$$\beta R^*_t \frac{E_t}{E_{t+1}} \frac{C_t}{C_{t+1}} \frac{P_t}{P_{t+1}} = 1$$

When the BAT is one time and unanticipated the real exchange rate appreciates permanently, that is $\frac{E_t}{P_t} = \frac{E_{t+1}}{P_{t+1}}$, and there is no impact on $C$ through the Euler equation. If on the other hand there are predictable changes in the real exchange rate because of expectations of BAT or because of gradual adjustment in rates or reversals, then neutrality does not hold as the savings/borrowing decisions of $H$ agents are altered.

A second departure from neutrality is when $H$ trades in $H$ currency financial instruments. Suppose that $H$ has debt in $H$ currency, similar to the case of the U.S. whose liabilities are overwhelmingly in dollar bonds. In this case the consolidated budget constraint in Eq. (10) is,

$$C' + \frac{\xi'}{1-\tau'} B^* + \frac{1}{1-\tau'} B = \frac{W'}{P'} N + \frac{\Pi'/P'}{1-\tau}$$

Even with flexible prices the BAT leads to an increase in transfers to $F$ equivalent to the dollar appreciation which leads to a breakdown in neutrality. Since for the United States, the foreign assets are mostly in foreign currency, while foreign liabilities are almost entirely in dollars, this generates a one-time transfer to the rest of the world and a capital loss for the US of the order of magnitude of around 13% of US annual GDP.

### 3.3 Monetary Policy

The real consequences of BAT depend crucially on the stance of monetary policy. Assumptions that support neutrality generate zero CPI inflation and a zero output gap. Therefore as long as the monetary authority only targets CPI inflation and the output gap neutrality is preserved.
With interest rates unchanged a *one-time and permanent* exchange appreciation is consistent with uncovered interest parity.

If on the other hand monetary policy targets the exchange rate then we no longer have neutrality. This is indeed the case of a fiscal devaluation wherein border taxes can stimulate the economy in a fixed exchange rate regime.

In the case when neutrality breaks down the prediction for the exchange rate is less straightforward. Barbiero, Farhi, Gopinath, and Itskhoki (2017) demonstrate that the extent of appreciation depends on trade openness and the relative magnitude of price and wage stickiness in non-linear ways. For parameters calibrated to the U.S. Barbiero, Farhi, Gopinath, and Itskhoki (2017) find that even when there is dollar currency pricing and $H$ currency international assets that lead to departures from neutrality the nominal exchange rate change is quantitatively close to $(1 - \tau)$.

Lastly, I comment briefly on the implications of the BAT for fiscal revenues.

### 3.4 Fiscal revenues

When BAT is neutral it is associated with an undistortive (*lump-sum*) transfer from the US private sector to the government budget in proportion with the trade deficit.

\[
T' - T = \frac{\tau}{1 - \tau} \left[ P'_{HF} Y_{HF} - P^*_{FH} Y_{FH} \right] = \tau \left[ P_{HF} Y_{HF} - P^*_{FH} Y_{FH} \right]
\]

The fiscal revenues are positive in periods of trade deficits, and negative in periods of trade surpluses. If as in the case of the U.S., the country has a negative net foreign assets position then it must imply that the present discounted value of transfers to the government will be negative, because to preserve long-run solvency the present discounted value of trade surpluses must be positive.
References


4 Appendix

Proof of Proposition: I list here the system of equations and variables, where all $H$ variables are scaled by the price level, that its $\bar{X} = X/P$.

$$C = \bar{W}$$
$$\bar{P}_{HH} = \mu \bar{W}^{1-\alpha}$$
$$\bar{P}_{HF} = (1 - \tau) \mu \bar{W}^{1-\alpha}$$
$$\bar{P}_{FH} = \frac{P_{FH} \bar{E}}{1 - \tau}$$
$$\bar{P}_{HH}^{1-\theta} \bar{P}_{FH}^{\theta} = 1$$
$$Y_{HH} = (1 - \theta) \frac{(C + X)}{\bar{P}_{HH}}$$
$$Y_{FH} = \theta \frac{(C + X)}{\bar{P}_{FH}}$$
$$Y_{HF} = \theta \frac{\bar{E}}{\bar{P}_{FH}} P^*(C^* + X^*)$$
$$(1 - \alpha) \frac{Y}{N} = \frac{\bar{W}}{\bar{MC}}$$
$$\alpha \frac{Y}{X} = \frac{1}{\bar{MC}}$$
$$\bar{MC} = \frac{1}{\alpha \alpha(1 - \alpha)^{1-\alpha}} \bar{W}^{1-\alpha}$$
$$\bar{\Pi} = (1 - \tau) \left( \bar{P}_{HH} Y_{HH} + \bar{P}_{HF} Y_{HF} - \bar{MC} \cdot Y \right) + \tau \bar{P}_{HF} Y_{HF}$$
$$C + \frac{\bar{E}}{1 - \tau} B^* = \bar{W} N + \frac{\bar{\Pi}}{1 - \tau}$$
$$Y = Y_{HH} + Y_{HF}$$

This is a system of fourteen equations in fourteen unknowns $\{C, \bar{W}, \bar{P}_{HH}, \bar{P}_{HF}, \bar{P}_{FH}, Y_{HH}, X, Y_{FH}, Y_{HF}, Y, N, \bar{E}, \bar{MC}, \bar{\Pi}\}$. The proof follows simply from recognizing that the real allocations are identical in the case with and without BAT as long as $\mathcal{E}'/\mathcal{E} = (1 - \tau)$. 

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