

DEBT SUSTAINABILITY IN A LOW INTEREST RATE WORLD

Neil R. Mehrotra

Brown University & Federal Reserve Bank of Minneapolis

ABSTRACT

Conditions of secular stagnation – low growth and low real interest rates – have counteracting effects on the cost of servicing the public debt. With sufficiently low interest rates relative to growth, governments can raise revenues by increasing the debt to GDP ratio. Mehrotra analyzes empirically and theoretically the tradeoffs involved with increased public debt. Using data from 1870 for advanced economies, interest rates on government debt are frequently less than GDP growth. However, despite current conditions of $r < g$, he finds a moderate probability of reversion to conditions with $r > g$ over a 5 or 10 year horizon and substantial variability in $r - g$. Using a 56 period quantitative lifecycle model calibrated to the US, the author shows that slower population growth worsens the cost of servicing the debt, while slower productivity growth improves this cost. Despite $r < g$, the level of public debt that minimizes the cost of servicing the debt is lower than current levels. JEL Classification: E43, E62, H68.

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1 Introduction

A decade after the onset of the Great Recession, interest rates and GDP growth remain historically low in the US. The same holds true for most advanced economies around the world. The secular stagnation hypothesis, most forcefully articulated by Summers (2013), argues that this era of low interest rates and modest growth may represent a “new normal.” In an environment of low interest rates and slow growth, monetary and fiscal policy face fresh challenges.

High deficits in the wake of the Great Recession and the subsequent uneven recovery in the US may worsen a fiscal outlook already troubled by rising health care expenditures and the looming retirement of the baby boom generation. However, despite slow growth, interest costs on the US public debt remain relatively low as real interest rates have fallen sharply.¹ Nevertheless, despite low rates, concerns remain that, given the large stock and short maturity of the US debt, a sharp rise in interest rates can quickly worsen debt dynamics and trigger the need for a fiscal consolidation.²

The cost of servicing the public debt depends on the gap between the real interest rate paid on government debt r and the growth rate of real GDP $g + n$ (real GDP growth is the sum of GDP per capita growth g and population growth n).³ In recent years in the US, this gap has been negative, implying that the cost of servicing the government debt is negative.⁴ To put it more simply, if $g = n = 0$ and $r < 0$, then investors are paying to lend the government and, holding these constant, an increase in the stock of debt would *increase* the real resources available to the government to either reduce taxes or increase expenditures. Were this a permanent state of affairs, the government could simply increase the public debt to the point that their negative interest payment fully financed any government spending. However, as emphasized in Ball, Elmendorf and Mankiw (1998), such a strategy is clearly risky. A sudden rise in interest rates relative to growth with a large stock of debt could quickly become quite costly to service. This is the basic tradeoff explored in this paper - the goal of this paper is to analyze empirically and theoretically how the cost of servicing the public debt changes in a low interest rate, low growth environment.

To be sure, the object $r - (g + n)$ is not a sufficient statistic for the optimal level of debt, and this paper does not attempt to address this issue. Rather, I adopt the perspective of a policymaker that narrowly wishes to minimize the direct tax burden for a given level of government expenditure and prefers to raise revenues via government debt, particularly when $r < g + n$. The implicit

¹See Elmendorf and Sheiner (2016) for an analysis of the implications of demographic trends and interest rates for the US public debt sustainability.

²See Hilscher, Raviv and Reis (2014) for a description of the maturity structure of the US public debt and the difficulty in inflating away the debt.

³I adopt the convention of separating growth into population growth and GDP per capita growth given the differing implications of changes in g and n for interest rate determination.

⁴Average interest rates on US marketable public debt are available from: <https://www.treasurydirect.gov/govt/rates/pd/avg/avg.htm>.

assumption is that policymakers must finance a fixed amount of public expenditures and perceive high economic efficiency costs or political costs to explicit taxation relative to debt financing. Policymakers preference for debt financing over taxation seems clear in light of recent political debates over fiscal policy.⁵

To assess the tradeoff between the level and volatility of servicing costs, I start by gathering some empirical evidence on historical episodes in which real GDP growth outstrips the real interest rate on government debt. Using a recently assembled macrofinancial database of 17 advanced economies, I find that cases in which the real interest rate is less than GDP growth and, hence, the cost of servicing the public debt is negative, are fairly common. Taking five year averages, for all countries, roughly half the observations carry a net negative fiscal cost, $r < (g + n)$. These episodes are not driven by the World Wars or the interwar years/Great Depression, and, in the case of the US, $r < g + n$ holds nearly 70% of the time in the postwar period. Put another way, with constant government expenditures of 20% of GDP and using the actual realizations of real interest rates and economic growth in the US since 1870, a government that targeted a debt to GDP ratio of 70% would have reduced taxation by 0.5% of GDP.

However, despite the frequency of these negative cost episodes, I also find a moderate probability of reversion to $r > g + n$ conditions over a 5 to 10-year horizon. I present the distribution of net fiscal cost over 5 or 10 year horizons conditioning on current negative fiscal costs. I also estimate probit regressions for the probability of positive fiscal cost over a 5 or 10 year horizon conditioning on the current net fiscal cost measure and observables like population growth and the debt to GDP ratio. These regressions show that the conditional probability of $r > g + n$ does exhibit some persistence but does not depend on the debt to GDP ratio. When using current values for these covariates in the US, I find close to a 50% probability that net fiscal cost $r - (g + n)$ turns positive in 5 to 10 years. At a debt to GDP ratio of 70% and given the historical variability in fiscal cost, the US would require significant austerity measures to stabilize the debt to GDP ratio. For instance, at the median level of fiscal cost conditional on $r > g + n$, the US would require austerity of 1.5% to 2% of GDP to stabilize the debt to GDP ratio - at the upper range of recent deficit reduction efforts.

Finally, I also consider the covariance between fiscal cost and economic growth, finding evidence that slower population growth and slower real GDP per capita growth are correlated with a higher fiscal cost. To the extent that higher levels of public debt are more valuable in periods of slow growth, the fiscal cost of servicing the debt rises in these periods. In other words, the fiscal burden of the public debt rises in periods of slow growth.

Overall, the empirical findings suggest that, on average, governments can raise real resources

⁵As I discuss further in the Section 4, in an environment with persistently negative $r - (g + n)$, the optimal level of debt will depend on a host of factors: costs of distortionary taxation, whether the economy suffers from oversaving, the manner in which fiscal policy redistributes across generations, etc.

by holding a high stock of government debt but face considerable uncertainty over future fiscal cost. Moreover, the fiscal burden of the debt worsens in slow growth periods, where, arguably greater fiscal stimulus is desired. Furthermore, while the current configuration of interest rates in the US relative to growth suggests benefits from a higher level of public debt, the historical record provides little assurance that current favorable conditions are likely to persist over the medium-run.

The empirical treatment so far treats fiscal cost as exogenous and does not consider the underlying factors driving interest rates and growth. Interest rates are determined endogenously, and theory offers multiple factors determine interest rates on government debt. I highlight the key factors determining interest rates in both the standard representative agent model and the canonical two-period OLG model. The representative agent model has a somewhat restrictive view of interest rate determination at odds with the data: it must be the case that $r > g + n$ and the interest rate typically only responds to changes in productivity growth g . That is, $dr/dn = 0$, and, with Ricardian equivalence, the real interest rate is invariant to the level of public debt. By contrast, in the OLG model, real interest rates depend on g, n and the public debt; OLG models also admit the possibility that $r < g + n$. These simplified models are useful in providing some intuition for the effects seen in the full lifecycle model.

To allow for richer interest rate determination and to make quantitative statements about the variability of fiscal costs, I extend the OLG model to a fully quantitative 56-period lifecycle model. This model allows me to consider how some of the candidate factors behind low interest rates (slow population growth, low rates of productivity growth, elevated risk/liquidity premia) quantitatively affect the real interest rate and the cost of servicing the debt. Using the quantitative model, I can consider how plausible ranges for population growth, productivity growth, and risk/liquidity premia affect the real interest rate and, hence, the cost of servicing the debt.

The lifecycle model includes several features to more realistically capture interest rate determination: households face a hump-shaped earnings profile, mortality risk prior to the terminal period, and accumulate physical capital and public debt. Intermediation frictions ensure that the return on risk-free debt falls below the rental rate of capital. Monopolistic competition among retailers provides a markup in steady state, which places a further wedge between the rental rate and the marginal product of capital. Crucially, while the model allows for $r < g + n$ where r is the risk free rate, the economy remains dynamically efficient and there is no oversaving of capital.

I calibrate the lifecycle model to match key moments of the US data, including interest rates, risk premia, labor share, and investment rates. The baseline calibration has $r < g + n$, a dynamically efficient level of capital, and a net negative fiscal cost of servicing the public debt of 0.6% of GDP. I show that, locally, a further slowdown in productivity growth will tend to *improve* the fiscal picture by lowering interest rates by more than the direct effect on growth. Since $dr/dg < 1$, $r - (g + n)$ becomes more negative, and slower productivity growth lowers the fiscal cost (equiv-

alently, increases the fiscal benefit) from a debt to GDP ratio of 70%. By contrast, a further slowdown in population growth *worsens* the fiscal outlook by lowering interest rates by less than the direct effect on growth. These conclusions are reversed for an acceleration in either productivity or population growth. It is worth noting that the current set of immigration and economic policies under consideration would, through the lens of this model, move rates in a way that worsens debt dynamics.

While population growth estimates are fairly certain, prospects for future productivity growth remain murky. I consider the model's implications for interest rates and the cost of debt service under optimistic (1990s levels) and pessimistic (Japanese/European levels) scenarios. Over a plausible range, further slowdown population growth over the next decade in the US impose a fairly small fiscal cost in terms of austerity required to keep the debt to GDP ratio constant. By contrast, an acceleration in productivity growth (which equals GDP per capita growth in the steady state) back to 1990 levels would imply a more sizable fiscal consolidation of 1% of GDP.

The quantitative model here can also be used to investigate how a decline in risk premia affects interest rates, investment, and debt sustainability. As I show, falling risk premia boost the investment to output ratios while raising the real interest rate. The overall sensitivity of interest rates to risk premia is fairly low, but the effect is to raise the cost of servicing the public debt since productivity growth and population growth are unaffected by risk premia. I investigate the fiscal implications of a return of risk premia (as proxied by the corporate Aaa spread over the 10-year Treasury rate) back to the average spread since 1980, finding only modest effects on the fiscal cost of servicing the debt.

Lastly, I use the quantitative model to investigate how changes in the level of the public debt itself affect interest rates and the cost of servicing the debt. In the baseline calibration, $r < g + n$, so, in principle, increases in the public debt may raise real resources for the government and lower the tax burden (holding government expenditures fixed). However, I find that the tax minimizing level of public debt is actually *lower* than the current 70% of GDP level of debt. This result stems from the elasticity of the real interest rate to changes in the level of debt; a reduction in debt reduces r relative to GDP growth, which further increases real resources raised by government despite the fall in the overall stock of debt. Taxes as a share of GDP are minimized for a level of debt at approximately 60% of GDP. Reduction in the debt to GDP ratio would also have the benefit of increasing the investment to GDP ratio in the model.

Overall, three lessons emerge from the model. First, the drivers of slow growth matter for determining the risk of higher or lower fiscal costs should growth slow. Second, on the narrow basis of maximizing resources raised from debt issuance, the steady state level of debt should be lower than the current 70% of GDP. Second, due to the presence of intermediation frictions and markups, the fiscal cost of servicing the debt may be negative, but increased debt accumulation still crowds out capital and reduces consumption per capita. Alternatively, if the economy

is dynamically inefficient, higher debt accumulation would raise consumption per capita, reducing capital accumulation. Given the relative stability of the average return on capital in the US documented in [Gomme, Ravikumar and Rupert \(2015\)](#), the former case would appear to be the empirically relevant one.⁶

The paper is laid out as follows. Section 2 presents basic statistics on the cost of servicing the public debt and evidence for the probability of reverting to positive fiscal cost over 5 and 10 year time horizons. Section 3 analytically characterizes the relationship of interest rates to productivity and population growth in the benchmark representative agent model and a simplified two period OLG model. Section 4 presents an overview of a 56-period OLG model, outlines the calibration strategy, and discusses the quantitative findings using the lifecycle model. Section 5 concludes.

1.1 Related Literature

This paper builds on several strands of the literature including a recent emerging literature on low interest rates and secular stagnation and a mature literature thinking about the optimal level of the public debt. The secular stagnation hypothesis was resurrected by [Summers \(2013\)](#) and formalized using a OLG model with downward nominal wage rigidity in [Eggertsson and Mehrotra \(2014\)](#). Recent work by [Eggertsson, Mehrotra and Robbins \(2017\)](#) analyzes the sources of low real interest rates in a quantitative lifecycle model, which is used here to think about the cost of servicing the public debt.⁷ [Gordon \(2015\)](#) argues in favor of a supply-side, productivity driven view of low growth and interest rates. While [Eggertsson, Mehrotra and Robbins \(2017\)](#) emphasize factors like low population and productivity growth in accounting for low interest rates, [Caballero and Farhi \(2014\)](#) stress a shortage of safe assets and an elevated risk premium in accounting for low interest rates. The model in this paper incorporates both views to consider how the costs of servicing the debt respond to plausible variations in secular stagnation “risk factors.”

This paper is also related to a literature prominent in the late 1980s and 1990s concerning the sustainability of large US deficits and rising US debt. See, for example, [Auerbach \(1994\)](#) for a discussion of the large US deficits in the late 1980s and early 1990s and its implications for the medium term fiscal outlook. In contrast to the conventional wisdom, [Woodford \(1990\)](#) showed how high levels of debt may be welfare improving and may crowd-in capital in the presence of financial frictions, while stressing the empirical fact of low r relative to g for US government debt in the historical record. [Aiyagari and McGrattan \(1998\)](#) show in a quantitative model with uninsurable income risk that the level of public debt in the late 1990s may not be excessive relative to

⁶Work by [Abel et al. \(1989\)](#) argued that, as of the late 1980s, capital accumulation in the US was dynamically efficient. In more recent work, [Geerolf \(2013\)](#) argues that capital accumulation advanced economies may no longer be dynamically efficient.

⁷See [Gagnon, Johannsen and Lopez-Salido \(2016\)](#) and [Jones \(2016\)](#) for quantitative models of low interest rates due to demographic factors.

the optimal level. The model presented here offers a similar possibility - in the face of a depressed real interest rate, higher levels of public debt allow the government to raise real resources without resorting to taxation.

An extensive literature has studied optimal fiscal policy in representative agent models with distortionary taxation. This literature, starting with Barro (1979) and Lucas and Stokey (1983), emphasizes the importance of allowing the public debt to fluctuate to minimize the volatility of taxes; the level of debt is less crucial. Indeed, Barro (1979) shows that the public debt should follow a random walk. As noted earlier, this paper does not consider optimal fiscal policy and only examines the stationary equilibrium, but it does offer a possibility for why the level of public debt may matter in addition to its shock absorption role. A more recent literature considers optimal fiscal policy in models with heterogeneous agents and incomplete markets (see, for example, Bhandari et al. (2017)).

2 Empirical Evidence

In this section, I examine empirical evidence on whether interest rates on government debt typically exceed real GDP growth. The degree to which interest rates exceed GDP growth determines whether positive levels of public debt impose a net fiscal cost to the government. In addition to concerns that government debt may crowd out private investment, servicing the interest payments on the public debt may impose high costs for governments that are reticent to raise taxes for economic or political reasons.

Factors determining the cost of servicing the public debt can be easily seen by inspecting the government's flow budget constraint:

$$\begin{aligned} T_t + B_{t+1}^g &= G_t + (1 + r_t) B_t^g \\ \Rightarrow \tilde{\tau}_t + \tilde{b}_{t+1}^g \frac{A_{t+1}}{A_t} \frac{N_{t+1}}{N_t} &= \tilde{g}_t + (1 + r_t) \tilde{b}_t^g \end{aligned} \quad (1)$$

$$\Rightarrow \tilde{\tau} = \tilde{g} + ((1 + r) - (1 + g)(1 + n)) \tilde{b}_g \quad (2)$$

where T_t is real tax revenue (net of any transfers), G_t is real government expenditures, B_t^g is real government debt, and r_t is the effective real interest rate paid on government debt. For any variable X_t , $\tilde{x}_t = \frac{X_t}{A_t N_t}$ is the variable X_t detrended by output per capita A_t and population N_t . Along the balanced growth path, GDP will grow at $g + n$ (where $1 + g = \frac{A_t}{A_{t-1}}$ and $1 + n = \frac{N_t}{N_{t-1}}$) and the debt to GDP ratio will remain stable so long as debt grows at this rate. Equation (1) is obtained by dividing through using A_t and N_t , and equation (2) is the steady state obtained by dropping time subscripts.

The difference between the gross return on public debt $R = 1 + r$ and $(1 + g)(1 + n)$ represents the fiscal cost of servicing the public debt. Approximately, if $r < g + n$, public debt delivers a net

fiscal benefit - higher levels of public debt reduce the tax revenues needed to finance a given level of government spending. In this section, I provide some empirical evidence on the behavior of $r - (g + n)$ and the probability of switching between periods of positive and negative fiscal costs of the public debt.

2.1 Dataset

To analyze the behavior of the net fiscal cost measure $r - (g + n)$, I draw on the recently constructed historical macroeconomic dataset of [Jordà, Schularick and Taylor \(2016\)](#).⁸ This dataset provides macroeconomic and financial variables for 17 advanced economies including the US from 1870 to 2013. In particular, I utilize annual data on real GDP growth, inflation rates, population growth, long-term interest rates on government debt, and public debt to GDP ratios.

To compute measures of the cost of servicing the debt: $r - (g + n)$, I need a measure of the ex-ante real interest rate. I use a three-year moving average of inflation as a proxy for expected inflation in line with the approach in [Hamilton et al. \(2016\)](#). The real interest rate is then the nominal measure from [Jordà, Schularick and Taylor \(2016\)](#) less the three-year moving average of inflation. When using annual data, I drop extreme observations of $r - (g + n)$ above 10% and below -10%. By using a measure of long-term nominal rates, I am adopting a conservative measure of the net fiscal cost; short-term nominal rates are considerably lower and the effective interest rate on government debt will likely be lower than the long-term nominal rate.⁹ The resulting dataset is an unbalanced panel of 2107 observations; when limiting to non-missing observations of the debt to GDP ratio, the number falls slightly to 2002 observations.

Table 1 provides basic summary statistics for the real interest rate, the population growth rate and the real GDP per capita growth rate in the dataset. Values are shown for all countries and after limiting the sample to the US. For all countries, the median nominal long-term interest rate is 4.6% with a median inflation rate of 2.2%. For the US, both interest rates and inflation rates are slightly lower than the global median. Population growth is somewhat higher in the US, as is per capita real GDP growth. Debt to GDP ratios are, on average, slightly lower in the US.

It is worth emphasizing that current real interest rates (approximately zero in the US) are in the bottom quartile of the distribution of historical observations. Likewise, current population growth and real GDP growth are also in the bottom quartile. By contrast, the current values of the US public debt to GDP ratio (approximately 70% of GDP) are in the top quartile. Together, the current levels of population growth and real GDP growth along with high levels of debt to GDP may appear *prema facie* problematic for debt sustainability; however, low real interest rates have kept US debt servicing costs quite low.

⁸Their data set is available from <http://www.macrohstory.net/data>.

⁹[Jordà, Schularick and Taylor \(2016\)](#) provide a single long-term and short-term nominal rate. Interest rates for all maturities or an effective interest rate on public debt is not available.

Table 1: Summary statistics

Variable	17 Advanced Countries			United States		
	Median	25th perc.	75th perc.	Median	25th perc.	75th perc.
Long-term nominal interest rate	4.64	3.65	6.42	3.97	3.35	5.56
Inflation rate	2.19	0.11	4.61	1.84	0.00	3.55
Real interest rate	2.77	1.23	4.85	2.76	1.57	4.10
Real GDP per capita growth	2.05	0.25	3.84	1.91	-0.48	4.02
Population growth	0.78	0.44	1.16	1.39	0.98	1.91
Debt to GDP ratio	43.3	23.8	68.0	35.7	13.8	58.9
No. of observations	2107			131		

Real interest rate is the long-term nominal interest rate less a three-year moving average of inflation rates. All variables expressed as percentage points. Statistics based on data set after observations with net fiscal cost $> 10\%$ or less than -10% are dropped.

2.2 Median Servicing Cost

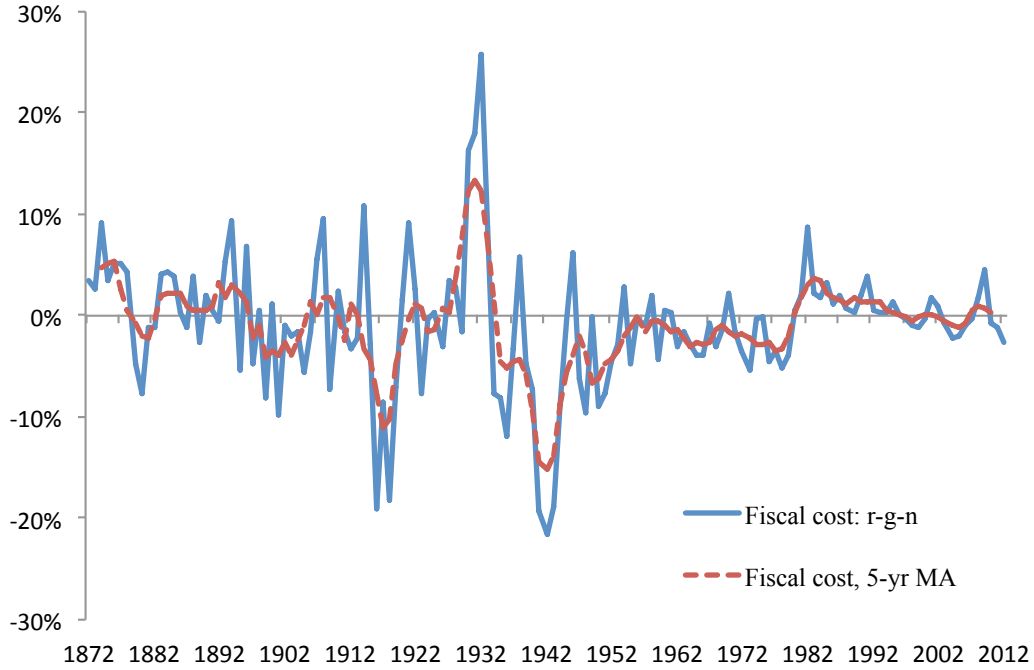
Figure 1 plots the fiscal cost measure for the US - long-term real interest rates less population growth and real GDP per capita growth. The solid line shows this measure for the US where the real interest rate is calculated using a three year moving average of inflation. The dashed red line is a five-year moving average of the solid line to smooth out fluctuations. The figure shows clearly that the cost of servicing the debt has frequently been negative historically and for a large part of the postwar period. Indeed, the period since 1980 is one of the few periods where real interest rates consistently exceeded real GDP growth. In the postwar period, the fiscal cost measure displays less volatility and greater persistence than the prewar or interwar periods.

Table 2 presents statistics on the fiscal cost of servicing the debt: $r - (g + n)$. I take averages over five year periods (non-overlapping) of $r - (g + n)$ for the US and 16 other advanced economies, presenting median values and ranges.¹⁰ As the table shows, over the full sample of advanced economies, the median value of net fiscal cost $r - (g + n)$ is nearly zero (8 basis points). That is, for any stationary level of the public debt, the cost of servicing that debt in terms of taxes in excess of government expenditures were minimal. In the US, that median value (-16 basis points) has been negative over the past 140 years. The finding that median values of $r - (g + n)$ are negative is not a function of historically extreme periods. Excluding the world wars and the interwar years (including the Great Depression) leaves the median value slightly higher for all countries and unaffected for the US. Limiting the sample to only the postwar period, net fiscal cost becomes *more* negative for both the all country sample and the US.

Though the median value is negative, I nevertheless find substantial variability for the cost of servicing the debt. Table 2 also provides the interquartile range of $r - (g + n)$ for both the full sample and the US. The 75th percentile is roughly 1% in the US, while the 25th percentile is substantially negative. These percentiles display the same level shift; $r - (g + n)$ is lower at each

¹⁰Five-year periods with fiscal cost above 10% or below -10% are winsorized at these levels.

Figure 1: US fiscal cost: $r - (g + n)$



quartile than the corresponding figure for the all country sample. An interquartile range of four to five percentage points shows substantial variability in net fiscal cost.

Table 2 also shows the fraction of observations with a negative fiscal cost or a substantially negative fiscal cost (i.e. $r - (g + n) < -2\%$). In the all country sample, half of the observations are negative and between 20% to 30% of five-year periods feature a substantially negative value for the fiscal cost measure. In other words, in over half of the years, servicing the public debt raised real resources for governments to finance expenditures. In the case of the US, these percentages are somewhat higher than those for the global sample. Again, the percentage of years with negative fiscal cost for the public debt are not driven by the interwar years and the Great Depression, or the world wars. If anything, the postwar period has featured a greater percentage of years with $r < g + n$. Quite remarkably, 70% of five-year periods in the US and 55% of five-year periods across all advanced countries show negative net fiscal cost in the postwar period.

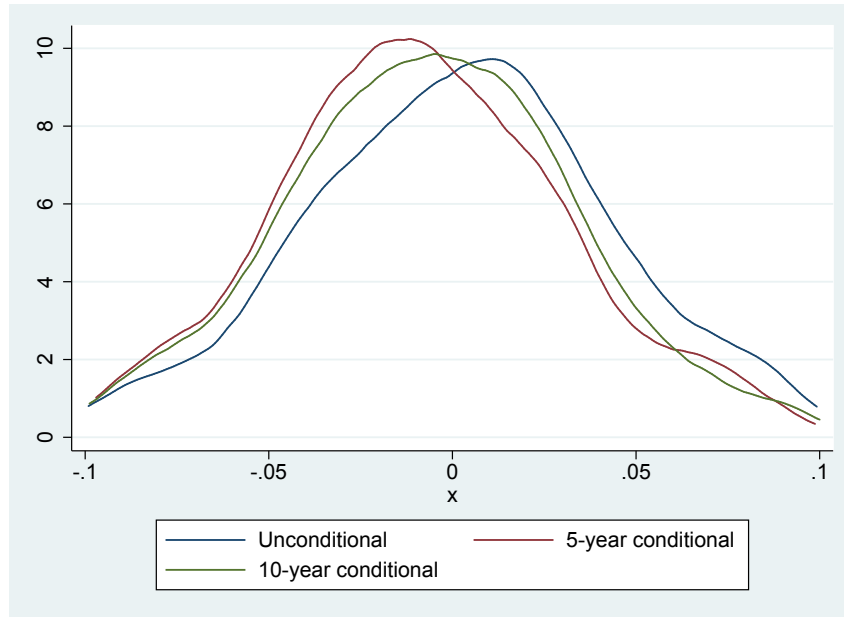
2.3 Reversion Risk

Given the current negative configuration of interest rates relative to GDP growth, what is the likelihood that interest rates will revert to a higher level so that $r > g + n$? Despite the high incidence of $r < g + n$ episodes, the probability of a reversion to higher fiscal cost over a five or ten year horizon turns out to be substantial. Conditional on $r < g + n$ in the current five-year period, there is a 30% probability that fiscal cost will turn positive in the subsequent five-year

Table 2: Moments of net fiscal cost measure

	17 Advanced Countries			United States		
	1870-2013	1870-1914 & 1946-2013	1946-2013	1870-2013	1870-1914 & 1946-2013	1946-2013
Net fiscal cost: $r - (g+n)$						
25th percentile	-2.64	-1.72	-2.74	-2.15	-1.72	-1.72
Median	0.08	0.51	-0.38	-0.16	-0.12	-1.35
75th percentile	2.28	2.23	1.55	1.09	1.42	0.57
Fraction < 0	49.3%	44.9%	54.3%	55.2%	54.5%	69.2%
Fraction < -2%	31.4%	18.3%	32.6%	31.0%	22.7%	23.1%
No. of observations	493	374	221	29	22	13

Real interest rate is the long-term nominal interest rate less a three-year moving average of inflation rates. Fraction < 0 is the percentage of years with negative net fiscal cost. Fraction < -2% is the percentage of years with net fiscal cost of less than -2%. Statistics based on data set after observations with net fiscal cost > 10% or less than -10% are winsorized at thresholds.

Figure 2: Empirical densities conditional on negative fiscal cost

period and a 38% probability that fiscal cost will turn positive over years 6-10; these conditional probabilities reflect all 17 countries in the dataset. Similar probabilities hold when just looking at the US; conditional on $r < g + n$ today, there is a 30% probability that $r > g + n$ in the subsequent period and a 46% probability two periods forward (6-10 years).

Figure 2 shows the entire conditional distribution of $r - (g + n)$. The five year forward conditional distribution is shifted left relative to the unconditional distribution indicating some degree of persistence. The same holds for the 10-year forward distribution. However, each conditional distribution displays a fairly wide range of outcomes; countries with currently negative fiscal cost face a moderate probability of reversion to higher rates in the near future. A high level of public debt would tend to exacerbate the consequences of a such a reversion.

To formalize the probability of a reversion to positive fiscal cost, I estimate a probit regression of the five-year and ten-year forward probability (using five-year averages) of a positive fiscal cost conditional on current fiscal cost and other controls. Of particular interest is whether the current debt to GDP ratio displays any correlation with the future probability of a reversion to a $r > g + n$ world. Specifically, I estimate the following specification on five-year averaged data:

$$P(fisc_{i,t+j} > 0 | X) = \Phi(c + \beta_f fisc_{i,t} + \beta_p popgrwth_{i,t} + \beta_d dgdp_{i,t} + \epsilon_{i,t}) \quad (3)$$

where $fisc_{i,t} = r_{i,t} - (g_{i,t} + n_{i,t})$ in country i and period t , $popgrwth_{i,t}$ is population growth in country i in period t , $dgdp_{i,t}$ is the public debt to GDP ratio in country i in period t , and $\Phi(\cdot)$ is the standard normal CDF. I estimate the regression model (3) using maximum likelihood methods. The regression results are shown in Table 3. In all regressions, standard errors are clustered at the country level.¹¹

The baseline specification is shown in columns (2) and (5) in Table 3. The coefficients on current fiscal cost $fisc_{i,t}$ and current population growth $popgrwth_{i,t}$ are statistically significant. The positive coefficient for β_f implies some degree of persistence in fiscal cost; a negative fiscal cost today lowers the probability of reversion in the next five year period and the subsequent five year period consistent with the distributions shown in Figure 2. The negative coefficient on β_p shows that higher current population growth lowers the probability of positive fiscal cost. Finally, the coefficient on the debt to GDP ratio β_d is insignificant across specifications. In the case of the 10-year forward probability, the point estimates go in the other direction; a higher debt to GDP ratio *lowers* the probability of future positive fiscal cost for the public debt. Columns (1) and (4) show the specifications setting $\beta_p = 0$

Columns (3) and (6) provide estimates when the sample is limited to postwar data. In these specifications, the coefficients rise markedly on current $r - (g + n)$ as do the regression R-squareds consistent with the higher degree of persistence exhibited by net fiscal cost in the postwar period (see Figure 1). However, population growth is no longer statistically significant in predicting future fiscal cost though the point estimates remain largely unchanged. The constant terms are largely stable and significant across specifications.

To interpret the probit regressions, it is useful to calculate the predicted probabilities at 1) at current US values and 2) under alternative optimistic and pessimistic scenarios. Current US values for long-term real interest rates are approximately 0.5% (given a 10-year rate of 2.3% and average inflation rate of 1.8%), population growth of 0.7%, and total factor productivity growth of 0.7%. With a debt to GDP ratio of 70%, the likelihood of reverting to $r > g + n$ is given in Table 4. Under the baseline specification for equation (3), the probability of $r > g + n$ is 48% and 47% under the

¹¹Given the time series component, standard errors may also be autocorrelated over time. Newey-West standard errors with 1 or 2 lags deliver standard errors comparable to country clustered standard error and coefficient significance is unchanged.

Table 3: Probit regressions of net fiscal cost

Variable	5-year forward: ($r > g+n$)			10-year forward ($r > g+n$)		
	(1)	(2)	(3)	(4)	(5)	(6)
Current value: $r - (g+n)$	12.931*** (1.116)	12.483*** (1.174)	25.869*** (3.430)	6.095*** (1.470)	5.707*** (1.421)	12.895*** (3.281)
Debt to GDP ratio	-0.049 (0.173)	-0.146 (0.183)	-0.142 (0.280)	-0.134 (0.189)	-0.212 (0.189)	-0.204 (0.315)
Population growth		-29.185** (11.762)	-24.618 (23.497)		-24.450** (10.973)	-28.148 (22.866)
Constant	0.062 (0.090)	0.363** (0.145)	0.373* (0.200)	0.046 (0.094)	0.300** (0.143)	0.483** (0.207)
McFadden pseudo R-squared	0.106	0.120	0.260	0.027	0.037	0.110
No. of observations	448	448	204	431	431	187

The dependent variable is a dummy variable that takes a value of 1 if the fiscal cost measure is positive (i.e. $r > g+n$) in next period (1-5 years forward) and in the subsequent period (6-10 years forward) respectively. Columns (1) and (4) do not include population growth; columns (3) and (6) limit the sample to the postwar period. Each column presents a separate regression. Standard errors are clustered at the country level. *** are coefficients significant at the 1% level, ** are coefficients significant at the 5% level, and * are coefficients significant at the 10% level.

five-year and 10-year horizons respectively. These probabilities come close to the unconditional probabilities shown in Table 2 for all countries. Effectively, low current population growth cancels out the effect of a slightly negative current value for $fisc_{i,t}$. Using only the postwar sample, the reversion probability is slightly lower at 45%, but the 10 year probability is higher. These differences are driven, in part, by the higher degree of persistence in the postwar period.

Population growth in the near-term is likely fixed (particularly without any liberalization of immigration policies), but if GDP per capita growth accelerates to 1.5%, the 5/10 year reversion probabilities drop to 44.0% and 45.4%; using the postwar sample, these declines are somewhat starker at 37.1% and 47.2% respectively. Conversely, if GDP per capita growth instead falls further to 0, the 5/10 year reversion probabilities rise to 51.5% and 48.8% respectively.

In each case, there remains a non-trivial probability of reverting to $r > g + n$ in the medium term. The striking feature of the likelihoods shown in Table 4 is that the reversion probabilities are not very sensitive to realistic swings in productivity or further increases in the debt to GDP ratio. This is understandable in lieu of the lower pseudo R-squareds, but nevertheless highlights a moderate risk of reversion to a positive fiscal cost environment.

It is worth noting that these findings are largely consistent with the 10-year projections from the Congressional Budget Office for interest rates, productivity, and population growth. The CBO projects the average nominal interest rate on public debt rising steadily from 2.0% in 2016 to 3.5% in 2027. At 2% inflation and 1.4% real GDP growth, net fiscal cost is $r - (g + n) = 0.1\%$. The CBO projects slightly higher real GDP growth of 1.9%, generating a value of -0.4% which is close to the postwar median value for the US.

What are the fiscal consequences if fiscal cost turns positive? In Panel B of Table 4, I provide

Table 4: Reversion likelihoods and fiscal cost

	Years: 1870-2013		Years: 1946-2013	
	5-year forward	10-year forward	5-year forward	10-year forward
Panel A: Likelihood of $r > g + n$				
Current values	48.0%	47.2%	45.2%	51.2%
Optimistic prod. (1.5%)	44.0%	45.4%	37.1%	47.2%
Pessimistic prod. (0%)	51.5%	48.8%	52.4%	54.9%
	$r - (g + n)$	Fiscal cost	$r - (g + n)$	Fiscal cost
Panel B: Fiscal consolidation				
Median $r > g + n$	2.24%	1.6%	1.68%	1.2%
75th percentile $r > g + n$	4.11%	2.9%	2.84%	2.0%

Reversion probabilities obtained as fitted values from regression specifications (2) - (3), and (5) - (6) with $r - g - n = 0.0054$, $g = 0.007$, $n = 0.007$, and debt to GDP ratio of 0.7. Optimistic and pessimistic scenarios consider alternatives with $g = 0.015$ and $g = 0$ respectively. Fiscal cost (% of GDP) is $r - g - n$ multiplied by debt to GDP ratio of 70%.

the median and 75th percentile of $r - (g + n)$ in both the overall sample and the postwar sample conditional on fiscal cost being positive. At a debt to GDP ratio of 70%, the fiscal cost of servicing the debt is nontrivial. At the median, debt service costs 1.2% to 1.6% of GDP, while at the 75th percentile, fiscal cost rises to 2% to 3% of GDP. By way of comparison, major deficit reduction legislation in the US such as the Omnibus Budget Reconciliation Act of 1990 and 1993 or the Budget Control Act of 2011 have typically raised revenues and reduced outlays by 0.5% to 1.5% of GDP.¹² Given current values of r and $g + n$, the public debt raises 0.6% of GDP in fiscal resources; thus a reversion would require austerity at the higher end of past deficit reduction.

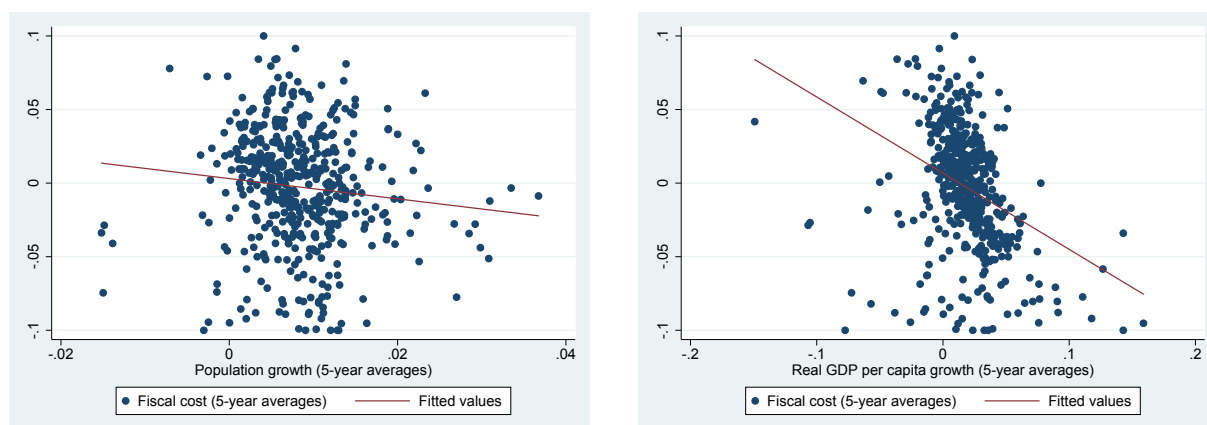
Overall, our findings show that periods of negative fiscal cost are not uncommon historically in the both the US and for advanced countries more generally. Moreover, advanced economies experience significant periods in which the cost of servicing debt is significantly negative. However, the probabilities of reversion over a relatively short time horizon (5 to 10 years) are significant and, with a large stock of public debt, the cost of servicing the public debt can quickly rise, potentially requiring a substantial fiscal tightening.

2.4 Growth and Fiscal Cost

While low levels of interest rates relative to GDP growth on average increase the attractiveness of public debt, policymakers may worry this mechanism of financing becomes more expensive precisely in the states of the world in which it may be most useful. The ability to increase the debt to GDP ratio for financing expenditures may be useful in cyclical or long-term downturns when tax revenues (as a share of GDP) falls or when additional fiscal stimulus is needed for demand-side fiscal stabilization or productivity enhancing government investments (i.e. infrastructure,

¹²See [Ruffing \(2011\)](#) for an analysis of past deficit reduction legislation.

Figure 3: Scatter plots of fiscal cost v. growth



education, basic research). Figure 3, shows how the cost of servicing the public debt varies with population growth and GDP per capita growth.

The left panel displays a scatter plot of $r - (g + n)$ against population growth - both five-year averages of all 17 advanced economies. The right panel displays the similar scatter plot against real GDP per capita growth. Both population growth and productivity growth negatively covary with the fiscal cost of servicing the debt; the negative relationship is weak for population growth, but stronger for real GDP per capita growth. Overall, in times of slow growth, the cost of servicing the public debt is typically higher. To the extent that governments wish to increase the debt to GDP ratio in slow growth period for reasons considered above, these plots suggest that the fiscal cost of servicing the debt rises in periods when debt is most useful. In other words, from a fiscal perspective public debt has poor hedging properties as it does not provide greater fiscal resources in slow growth periods.

3 Determinants of Real Interest Rates

Before turning to a realistic quantitative lifecycle model, it is helpful to characterize the relationship between the real interest rate, the public debt, population growth, and productivity growth in both the standard representative agent model and a simple two period OLG model in the spirit of Samuelson (1958) and Diamond (1965). This analysis helps provide a benchmark for the quantitative results that follow and illustrate how changes in population and productivity growth impact the fiscal cost of servicing the public debt.

3.1 Representative Agent Benchmark

Consider a representative household maximizing lifetime discounted utility subject to the economy's resource constraint:

$$\begin{aligned} \max_{\{c_t\}} \quad & \sum_{t=0}^{\infty} \beta^t N_t u(c_t) \\ F(K_t, A_t N_t) = & K_{t+1} - (1 - \delta) K_t + N_t C_t \end{aligned}$$

where N_t is the population at time t , which is growing at rate n . A unit measure of labor is supplied exogenously by each member of the household. Productivity is labor augmenting and growing at rate g . With log utility: $u(c) = \ln(c)$ and Cobb-Douglas production: $F(K, AN) = K^\alpha (AN)^{1-\alpha}$, equilibrium can be summarized by a resource constraint and an Euler equation for the representative household's utility maximizing consumption/saving decision.

With Cobb-Douglas production, the economy's resource constraint and Euler equation can be expressed in detrended per capita terms (i.e. for variable X_t , $\tilde{x}_t = \frac{X_t}{A_t N_t}$):

$$\tilde{k}_t^\alpha = \tilde{k}_{t+1} (1 + n) (1 + g) - (1 - \delta) \tilde{k}_t + \tilde{c}_t \quad (4)$$

$$1 = \frac{\beta}{1 + g} \frac{\tilde{c}_t}{\tilde{c}_{t+1}} \left(\alpha \tilde{k}^{\alpha-1} + 1 - \delta \right) \quad (5)$$

As can be seen from the resource constraint, consumption per capita along the balanced growth path will be maximized if the return to capital (MPK + undepreciated capital) equals the gross population growth rate times the gross productivity growth rate. To an approximation, consumption is still increasing with capital so long as $MPK - \delta > g + n$. As equation (5) shows, so long as population growth is such that household utility is well defined, this inequality always holds and the representative agent economy is always dynamically efficient.¹³

The representative agent model has strong implications for public debt. With a representative household and lump sum taxation, variations in the public debt and the economy's debt to GDP ratio are irrelevant for capital determination and the return on capital. If government debt provides a liquidity benefit, the real interest rate on government debt may be depressed relative to the marginal product of capital.¹⁴ However, even if the liquidity premium is large enough so that $r < g + n$, the tax benefits from higher levels of public debt remain irrelevant for real allocations due to Ricardian equivalence.¹⁵

¹³For household utility to be well-defined, it must be the case that $\beta(1 + n) < 1$. This condition together with (5) implies dynamic efficiency. See Acemoglu (2008) for an overview of dynamic efficiency.

¹⁴See Abel et al. (1989) for an illustration of how risk premia may depress the interest rate on riskless assets in an otherwise dynamically efficient economy.

¹⁵Optimal debt policy would follow a Friedman type rule with government debt sufficiently high to satiate households and eliminate the liquidity premium.

3.2 OLG Benchmark

In contrast to the representative agent model, capital accumulation in the benchmark OLG model may not be dynamically efficient and may be impacted by the level of public debt. A household lives two periods and makes consumption and saving decisions. Labor, normalized to unity, is supplied exogenously in the first period:

$$\begin{aligned} \max_{C_{1,t}, C_{2,t+1}} \quad & u(C_{1,t}) + v(B_{t+1}^g) + \beta u(C_{2,t+1}) \\ \text{s.t.} \quad & C_{1,t} + K_{t+1} + B_{t+1}^g = w_t - T_{1,t} \\ & C_{2,t+1} = (1 + r_t) B_{t+1}^g + (r_{t+1}^k + 1 - \delta) K_{t+1} - T_{2,t+1} \end{aligned}$$

where β is the rate of time preference, $T_{1,t}, T_{2,t+1}$ are taxes levied in each period, K_{t+1} are holdings of physical capital, and B_{t+1}^g are households' holdings of public debt. Public debt provides utility to households as a way of modeling demand for liquid or safe assets and introducing a liquidity premium.¹⁶

The government's budget constraint is similar to what was as shown in Section 2 but must now take into account population growth:

$$N_t T_{1,t} + N_{t-1} T_{2,t} + N_t B_{t+1}^g = (1 + r_t) N_{t-1} B_t^g$$

where N_t is the size of the cohort at time t and government expenditures are set to zero. The firm side of the economy is identical to the standard neoclassical model and is relegated to the appendix.

From the household problem and government budget constraint, equilibrium capital accumulation for the two-period OLG economy with log consumption and Cobb-Douglas production is given by the following equations:

$$1 = \frac{\beta}{1 + g} \frac{\hat{c}_{1,t}}{\hat{c}_{2,t+1}} \left(\alpha \tilde{k}^{\alpha-1} + 1 - \delta \right) \quad (6)$$

$$\hat{c}_{1,t} = (1 - \alpha) \tilde{k}_t^\alpha - \hat{\tau}_{1,t} - (1 + g) (1 + n) \left(\tilde{k}_{t+1} + \tilde{b}_{g,t+1} \right) \quad (7)$$

$$\hat{c}_{2,t+1} = \left(\alpha \tilde{k}^{\alpha-1} + 1 - \delta \right) (1 + n) \tilde{k}_{t+1} + (1 + r_{t+1}) (1 + n) \tilde{b}_{g,t+1} - \hat{\tau}_{2,t+1} \quad (8)$$

$$(1 + r_t) \tilde{b}_{g,t} = \tilde{b}_{g,t+1} (1 + g) (1 + n) + \hat{\tau}_{1,t} + \frac{\hat{\tau}_{2,t}}{1 + n} \quad (9)$$

where I have substituted capital and bond market clearing conditions (for any variable X_t , $\hat{x}_t = X_t/A_t$ and $\tilde{x}_t = X_t/A_t N_t$). The first equation (6) is the household's Euler equation, while equations (7) – (9) are the period and government budget constraints respectively.

¹⁶The quantitative model introduces an intermediation wedge instead of using money/bonds in the utility function.

Combining these equations and considering a stationary equilibrium, I obtain the following expression that determines the capital stock in this economy:

$$\frac{\beta}{1+\beta} \left((1-\alpha) \tilde{k}^\alpha - \hat{\tau}_1 \right) = (1+g)(1+n) \left(\tilde{k} - \frac{\hat{\tau}_2}{1+n} + \tilde{b}_g \gamma \right) \quad (10)$$

$$\Rightarrow \frac{\beta}{1+\beta} (1-\alpha) \tilde{k}^\alpha = (1+g)(1+n) \left(\tilde{k} + \tilde{b}_g \gamma \right) \quad (11)$$

where equation (11) follows under the assumption that the government follows a tax rule that precisely cancels out the effect of taxes on household's saving decision. The parameter γ takes values between $\frac{1}{1+\beta}$ and 1, and depends on the degree of liquidity premium attached to the public debt. When the liquidity premium is zero, the return on public debt equals the return on physical capital and $\gamma = 1$. When $\tilde{b}_g = 0$, equation (11) reduces to the standard expression for capital in the canonical two period OLG model as summarized in [Acemoglu \(2008\)](#).

Under the assumption that households are satiated in public debt at all times (or equivalently, no liquidity benefit of public debt), the equilibrium level of the capital stock can be represent by means of a Solow diagram. Figure 4 below shows the comparative statics of an increase in public debt and a reduction in productivity growth on the capital stock. The curved blue line represents the left hand side of equation (11), while the grey straight line represents the left-hand side of equation (11). The intersection points give the level of the capital stock in the stationary equilibrium. Depending on parameter values, the equilibrium capital stock may exceed the dynamically efficient level of capital.

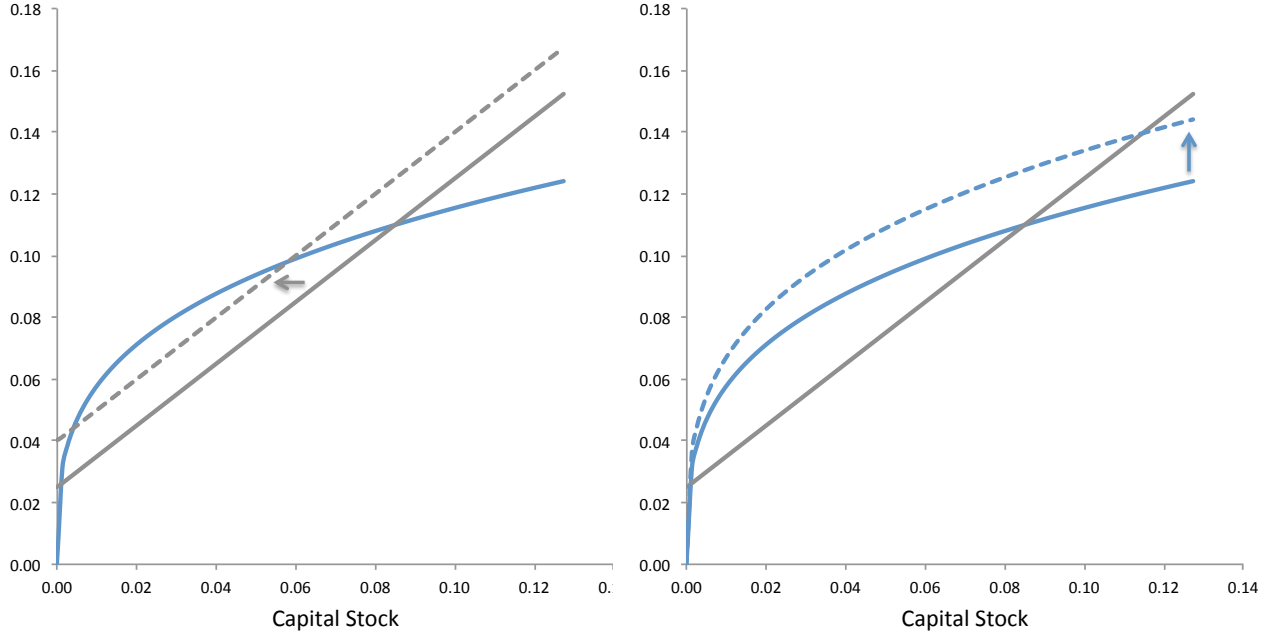
As can be seen in Figure 4, a higher level of public debt lowers the capital stock due to crowding out while lower productivity growth (or population growth) shifts the blue segment outward raising the capital stock. The rental rate (and hence the real interest rate) move inversely with the capital stock; higher public debt raises rates, while lower productivity and/or population growth lowers the real interest rate.¹⁷

In addition to the degree to which the real interest rate is depressed relative to GDP growth, a related consideration is how sensitive are interest rates locally to changes in the population growth and productivity. Let $R = \alpha \tilde{k}^{\alpha-1} + 1 - \delta$ (or equivalently the gross real interest rate). It is straightforward to show that $dR/d(1+g) = \frac{R}{1+g}$ and $dR/d(1+n) = 0$ for the representative agent model. Given that $R > (1+g) \times (1+n)$ in steady state in the representative agent model, it must be the case that $dR/d(1+g) > (1+n)$.

For the representative agent economy, since $R > (1+g) \times (1+n)$, positive public debt carries a net fiscal cost. Since $dR/d(1+g) > (1+n)$, lower productivity growth reduces the interest rate by more than the fall in the growth rate; therefore, locally, lower productivity growth *lowers* the

¹⁷This simple model features multiple steady states. I focus on the behavior of the high capital stock steady state in response of changes in parameters. The low capital stock steady state represents a poverty trap and is locally unstable. The presence of multiple steady states may be due to the particular fiscal rule assumed here.

Figure 4: OLG model - comparative statics



net fiscal cost of the public debt. Symmetrically, higher productivity growth raises the interest rate by more than the increase in the growth rate raising the net fiscal cost of the public debt. Lower population growth, since it has no effect on interest rates, will raise the net fiscal cost of financing the public debt.

The same calculation can be conducted for the OLG economy. As the proposition below summarizes, it can be shown that the interest elasticity to population and productivity growth have simple expressions. The fact that interest rates now respond to population growth contrasts with the infinite horizon case.¹⁸ Furthermore, in a dynamically inefficient world where $R < (1 + g) \times (1 + n)$, public debt carries a negative net fiscal cost. These elasticities imply that slower productivity (population) growth *raises* the net fiscal cost of the public debt (or, in other words, lower the net fiscal benefit). Effectively, the risk of slower growth worsens, locally, the sustainability of the public debt. Given that the local response of interest rates to changes in productivity and population growth depends on the type of model, it remains a quantitative question of whether the lifecycle model behaves more like the OLG model or more like the representative agent model. I turn to that question next.

Proposition 1. *If $\delta = 1$, $\tilde{b}_g = 0$, then $dR/d(1 + g) = \frac{R}{1+g}$ and $dR/d(1 + n) = \frac{R}{1+n}$. Furthermore, in a dynamically inefficient economy with $R < (1 + g) \times (1 + n)$, the net fiscal cost of servicing the public debt*

¹⁸The fact that population growth does not affect interest rate determination in the representative agent model is *not* generic. Under alternative assumptions over preferences, population growth may affect the interest rate. However, the benchmark discussed here is fairly standard.

in the stationary equilibrium is decreasing in both population growth and productivity growth.

Proof. Under the assumptions listed, from equation (11), the expression for the gross real interest rate R and the elasticity of R to the gross growth rate of productivity and the gross population growth rate is given by:

$$\begin{aligned}\frac{\beta}{1+\beta} \frac{1-\alpha}{\alpha} R &= (1+g)(1+n) \\ \frac{dR}{d(1+g)} &= (1+n) \frac{1+\beta}{\beta} \frac{\alpha}{1-\alpha} = \frac{R}{1+g} \\ \frac{dR}{d(1+g)} &= (1+g) \frac{1+\beta}{\beta} \frac{\alpha}{1-\alpha} = \frac{R}{1+n}\end{aligned}$$

If parameters α and β are such that $R < (1+g) \times (1+n)$, then the elasticity of the cost of servicing the public debt $R - (1+g)(1+n)$ to productivity growth (resp. population growth) must be:

$$\begin{aligned}\frac{dR}{d(1+g)} - (1+n) &= \frac{R}{1+g} - (1+n) \\ \Rightarrow \frac{d}{d(1+g)} \{R - (1+g)(1+n)\} &< 0\end{aligned}$$

Therefore, the cost of servicing the public debt is falling in both productivity and population growth as required. \square

4 Quantitative Lifecycle Model

4.1 Model Summary

In this section, I analyze a 56-period quantitative lifecycle following closely on Eggertsson, Mehrotra and Robbins (2017) and in the spirit of Auerbach and Kotlikoff (1987) and Ríos-Rull (1996). The quantitative model will be used to endogenize the real interest rate and examine how changes in population or productivity growth or the intermediation wedge impact debt sustainability and the sensitivity of the cost of financing the public debt under realistic ranges for these forcing variables. For brevity, I only outline the key elements of the model here while leaving the presentation of the full model to the appendix.

The model consists of 56 generations of households that supply labor inelastically, accumulate physical capital and claims on the government (public debt), face mortality risk each period, pay taxes, and passively collect profits from firms and financial intermediaries. Households are modeled as entering the working period of life at age 25 and, conditional on survival, dying with certainty at age 81. These households face an exogenous effective labor productivity profile that is increasing over time to match the lifecycle profile of earnings. Households save for retirement and earn no labor income from age 65 to age 81. Prior to the terminal period, household have

access to one-period annuity markets ensuring that the assets of those households that die prior to the terminal period are redistributed to the survivors. Firms hire capital and labor and have some degree of market power, setting prices at a constant markup to marginal cost.

To capture the fact that returns on business capital typically exceed returns on government debt due to 1) risk premia, 2) liquidity premia, 3) real intermediation costs, and 4) regulatory capital requirements in the banking system, I introduce an intermediation wedge. A fraction ω of the return to physical capital goes to intermediaries that, in turn, return these proceeds to households.¹⁹ If a fraction ω of the return on capital is diverted to intermediary profits, the real return received by households is given by:

$$1 + r_t = r_t^k (1 - \omega) + 1 - \delta \quad (12)$$

The fiscal authority issues public debt and raises tax revenues by taxing labor income to pay interest on previously issued public debt and to finance government purchases. The government's aggregate budget constraint is given below:

$$B_{t+1}^g + \tau_t w_t \sum_{j=0}^J N_{j,t} h c_j = G_t + B_t^g (1 + r_t)$$

where $h c_j$ is the effective human capital of households of age j and $N_{j,t}$ is the population of cohort j . Taxes are levied proportional to labor income, but labor income taxes here are non-distortionary since labor supply is exogenous.

4.2 Calibration Strategy

I calibrate the quantitative model outlined in Section A to fit current US economic conditions and use this quantitative model to assess how interest rates and fiscal cost are affected by changes in population growth, productivity growth, and changes in the intermediation wedge. The model can also be used to assess how interest rates and the net fiscal cost of servicing the public debt change with the level of public debt. Finally, the quantitatively model can be used to assess the degree of crowding out as the public debt increases.

The calibration strategy I follow chooses a subset of parameters directly from moments in the data, sets a smaller set of parameters from the literature, and chooses a final set of parameters to match a set of targeted moments. Panel A of Table 5 shows the parameters directly taken from the data. Age specific mortality rates $\{p_{j,t}\}_{j=0}^J$ are taken from mortality tables from the Centers for Disease Control (CDC). I assume households enter at age 25 with zero initial level of assets and face mortality risk throughout their life. The population growth rate is the average US population growth rate for the past decade taken from Census Bureau estimates. Total factor productivity

¹⁹See [Curdia and Woodford \(2010\)](#) and [Mehrotra and Sergeyev \(2016\)](#) for a discussion of this modeling strategy.

Table 5: Parameters taken from the data and related literature

<i>Panel A: Data</i>	<i>Symbol</i>	<i>Value</i>	<i>Source</i>
Mortality profile	$s_{j,t}$		US mortality tables, CDC
Income profile	hc_j		Gourinchas and Parker (2002)
Population growth rate	n	0.70%	US Census Bureau
Productivity growth	g	0.70%	Fernald (2012)
Government spending (% of GDP)	$\frac{G}{Y}$	19.2%	BEA
Public debt (% of GDP)	$\frac{b_g}{Y}$	70%	CBO
<i>Panel B: Related literature</i>	<i>Symbol</i>	<i>Value</i>	
Elasticity of intertemporal substitution	ρ	0.5	
Depreciation rate	δ	8%	
<i>Panel C: Matching targets</i>	<i>Symbol</i>	<i>Value</i>	<i>Target</i>
Rate of time preference	β	1.0042	Real US 10-year rate
Intermediation wedge	ω	0.1733	Corporate Aaa spread
Retailer elasticity of substitution	θ	4.6174	Labor share
Capital share parameter	α	0.2341	Investment to GDP ratio

growth is taken from estimates of utilization adjusted TFP by Fernald (2012). The wage profile hc_j is chosen to match the earnings profile estimated in the data by Gourinchas and Parker (2002).

Government purchases are set at 19.2% of GDP based on 10-year averages in the National Income and Product Accounts from the Bureau of Economic Analysis (BEA). The debt to GDP ratio is set at 70% of GDP based on federal debt held by the public (that is, netting that which is owed to the Social Security Trust Fund) as reported by the CBO. The tax rate on labor income adjusts so that the debt to GDP ratio is kept stable in the stationary equilibrium (i.e. equation (26) holds in the stationary equilibrium).

Panel B of Table 5 provides values for the parameters that are directly set based on the literature. In the baseline calibration, I choose an elasticity of intertemporal substitution of 0.5 - at the midpoint between Gourinchas and Parker (2002) and Ríos-Rull (1996). As documented in Eggertsson, Mehrotra and Robbins (2017), estimates in the literature typically range from 0.25 to 1. The depreciation rate is set at 8% - towards the lower end of depreciation rates applied to productive capital like equipment and software. I set the depreciation rate on the low side in order to take into account the low rate of depreciation for residential and nonresidential structures and ensure a more realistic capital to labor ratio.

Finally, Panel C of Table 5 provides values for the four parameters that are chosen to ensure that the model matches certain targets. Specifically, we chose the rate of time preference β , the intermediation wedge ω , the retailer's elasticity of intertemporal substitution θ , and the capital share α to match the following targets: the real interest rate on 10-year US government debt, the premium on Aaa corporate debt over the 10-year rate, the labor share, and the investment to output ratio. Though there is not a one to one correspondence between these parameters, the parameters are listed in the same order as the targets that they most influence.

The real interest rate target is 0.54% corresponding to the 2012-2017 average of the 10-year Treasury rate (2.13%) less the 2012-2017 average of core PCE inflation (1.60%). The labor share is set at 60% corresponding with the post 2000 average in Karabarbounis and Neiman (2014). The investment to output ratio is set at 16.7% to match the post 2000 average of nominal investment divided by nominal GDP in the National Income and Product Accounts. Finally, the risk premium given by the equation below is targeted at 1.79% to match the average Aaa spread from 2012-2017:

$$premium = r_k - \delta - r$$

Under this calibration, in the stationary equilibrium, the capital to output ratio is 1.77 which is slightly lower than the economywide capital to output ratio as reported in the BEA fixed asset tables (about 2.25) using post 2000 averages.²⁰ The rental rate on capital (less the depreciation rate) exceeds the real interest rate due to the intermediation wedge (premium plus the real interest

²⁰By contrast, restricting the capital to output ratio to the ratio of capital in the business sector to business income implies a ratio of 1.43. The model capital ratio is somewhat closer to the business capital to income ratio.

rate gives a net rental rate of 2.34%). However, the presence of pure profits due to monopolistic competition places a further wedge between the marginal product of capital (net of depreciation) and the rental rate. In our model, $MPK - \delta = 5.19\%$; the return on capital (net of depreciation) is therefore considerably higher than the real GDP growth rate. Therefore, the stationary equilibrium capital stock is dynamically efficient.

4.3 Quantitative Findings

This quantitative model allows one to ask how the fiscal cost of servicing the debt changes with key determinants of the real interest rate: population growth, productivity growth and the intermediation wedge. The secular stagnation literature (for example, [Eggertsson and Mehrotra \(2014\)](#) and [Caballero and Farhi \(2014\)](#)) have emphasized these factors as driving low interest rates; the quantitative model utilized here allows me to consider variations in each of these elements and their impact on debt sustainability.

4.3.1 Productivity and Population Growth

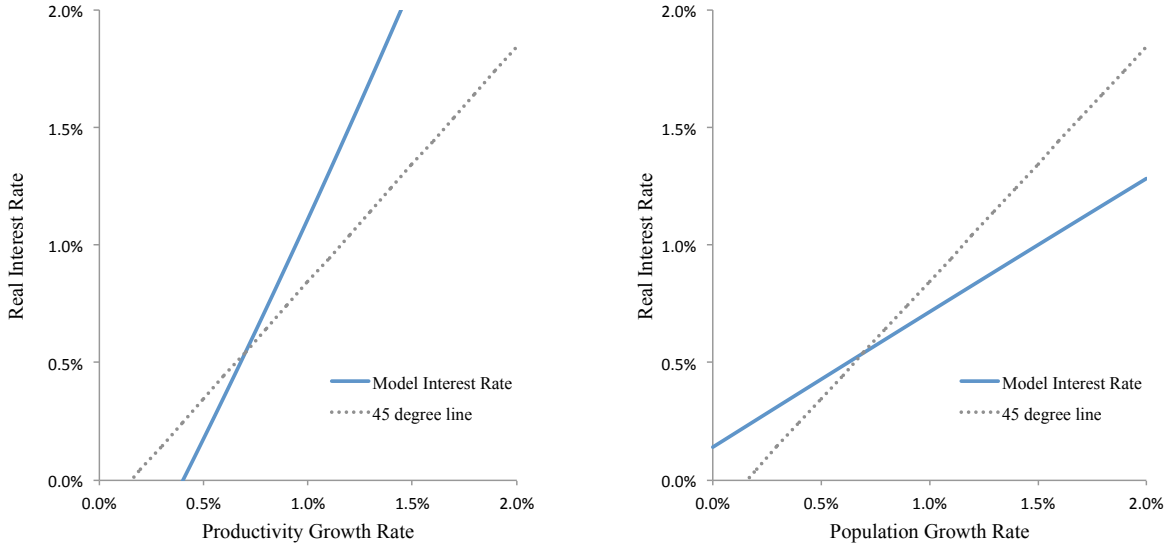
As shown in the Section 3, the representative agent and OLG benchmarks carry differing implications for the elasticity of the real interest rate to productivity and population growth. Figure 5 shows how the real interest rate changes with productivity and population growth respectively. I allow these parameters to vary between 0% and 2%. Forecasts for productivity growth are inherently uncertain as productivity has exhibited substantial and unpredicted variation in the postwar period. By contrast, population growth is far more stable and likely to remain close to its current level of 0.7% over the next decade.

The left panel of Figure 5 shows how the real interest rate changes with productivity growth. The 45 degree line going through the baseline level of r and g is also shown. In the representative agent model with an elasticity of substitution of 0.5, a 10 basis point increase in growth causes a 20 basis point increase in the real interest rate. The calibrated lifecycle model delivers a similar elasticity, with the real interest rate moving more than one for one with productivity growth. Approximately, the slope of the real interest rate line is about 1.8 in the vicinity of the steady state; the lifecycle model slightly attenuates the elasticity of the real interest rate to productivity. The right panel of Figure 5 shows variations in the real interest rate with population growth, along with a 45 degree line through the baseline level of r and n . In contrast to the representative agent model, $dr/dn \neq 0$, with a slope of approximately 0.6.

As in the canonical OLG model, lower growth implies a higher K/Y ratio. The investment to output ratio is related to the capital to output ratio by a simple expression:

$$\frac{I}{Y} = (\delta + g + n) \frac{K}{Y} \quad (13)$$

Figure 5: Lifecycle model - comparative statics



Changes in productivity and population growth have competing effects on investment rates as can be seen in equation (13). Slower growth has a direct effect reducing the investment ratio, but has an indirect effect through capital deepening. Lower real interest rates translate (imperfectly) into lower rental rates on capital, thereby raising the capital to output ratio. In the case of slower productivity growth, the capital deepening effect dominates with investment rates rising as TFP growth falls. By contrast, slower population growth has a weaker effect on rental rates and, therefore, the direct effect dominates; falling population growth lowers investment rates.²¹

The elasticity of the real interest rate to productivity and population growth carry implications for debt sustainability through effects on the net fiscal cost of servicing the debt. In the baseline calibration, a real interest rate of 0.54% and a debt to GDP ratio of 70% implies a negative fiscal cost of 0.6% of GDP or, in other words, taxes as a share of GDP are 18.6% while government purchases are 19.2% of GDP. As Figure 5 reveals, further productivity and population slowdown carry opposite implications for debt sustainability.

Locally, a slowdown in productivity growth reduces the real interest rate *more* than its direct effect on growth. Thus, net fiscal costs become more negative and debt sustainability improves since $dr/dg > 1$. By contrast, a slowdown in population growth worsens net fiscal costs. The direct effect of slower population growth outstrips the reduction in the interest rate on government debt since $dr/dn < 1$. Conclusions are reversed with higher productivity and population growth. The former worsens net fiscal costs as real rates rise sharply relative to growth, while the latter

²¹It should be noted that the passthrough from interest rates to rental rates is not one for one. Holding the intermediation wedge constant does not imply a constant risk premia. In the case of both productivity and population growth, risk premia expand as interest rates rise.

generates further benefits as the growth response outstrips the rate response.

Where do the balance of risks lie for productivity and population growth? Current productivity growth in the US appears quite low relative to postwar averages, though commensurate to productivity growth rates in other advanced economies. Our model suggests that higher productivity growth would worsen debt service costs. Holding constant population growth, a rise in productivity growth above 1.6% would ensure a net cost of servicing the public debt (i.e taxes would now have to exceed government purchases to service the debt). A rise to between 1.5% and 2.0% would represent a return to late 1990s/early 2000s productivity growth rates; given the uncertainty over the causes of the slowdown in the past decade, such an acceleration seems within the realm of possibility.

At a debt to GDP ratio of 70%, the implied rise in interest rates would require a nontrivial fiscal tightening of 0.5 to 1 percentage point of GDP to keep the debt to GDP ratio stable. By contrast, a collapse in productivity growth to 0% - commensurate to the slowdown experienced in the mid 2000s, would generate additional resources of 0.5 percentage points of GDP.²²

The degree of uncertainty facing population growth is considerably narrower than the uncertainty over productivity growth. A fall in population growth to 0.5% would require a tightening of less than 0.1 percentage points of GDP to keep the debt to GDP ratio stable. A rise in population growth to 1990 levels of 1.1% would allow for additional fiscal resources of 0.1 to 0.15 percentage points of GDP. Absent a drastic liberalization of immigration policies, social trends along with delays in household formation associated with educational requirements and low wage growth appear to likely to further slowdown US population growth. Over the relevant range of uncertainty, the fiscal implications of changes in population growth are muted.

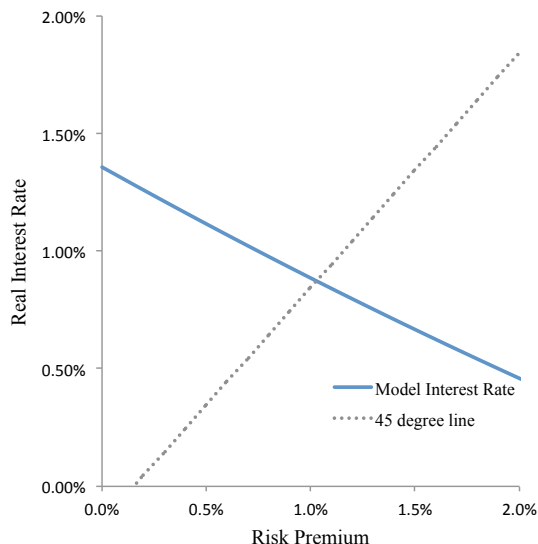
4.3.2 Risk/liquidity Premia

While much of the literature on low interest rates has focused on slow productivity and population growth, [Caballero and Farhi \(2014\)](#) and [Andolfatto and Williamson \(2015\)](#) emphasize a shortage of safe assets that has increased the spread between returns on safe assets like government debt and risky assets like corporate debt and equity. By varying the intermediation wedge, I can examine how interest rates and the fiscal cost of servicing the debt vary with changes in risk premia.

Figure 6 shows how real interest rates vary with changes in risk premia induced by variations in the intermediation wedge parameter ω . As the intermediation wedge falls, real interest rates rise, thereby raising the fiscal cost of servicing the debt. Since productivity and population growth are unchanged, a rise in interest rates from a fall in risk premia is unambiguously negative for servicing the public debt. While rates rise as risk premia contract, rental rates on capital falls with

²²It is worth noting that, in this case, long-term real rates would be significantly negative; the ability to attain this negative rate given an inflation target of 2% remains an open question.

Figure 6: Variations in Risk Premia



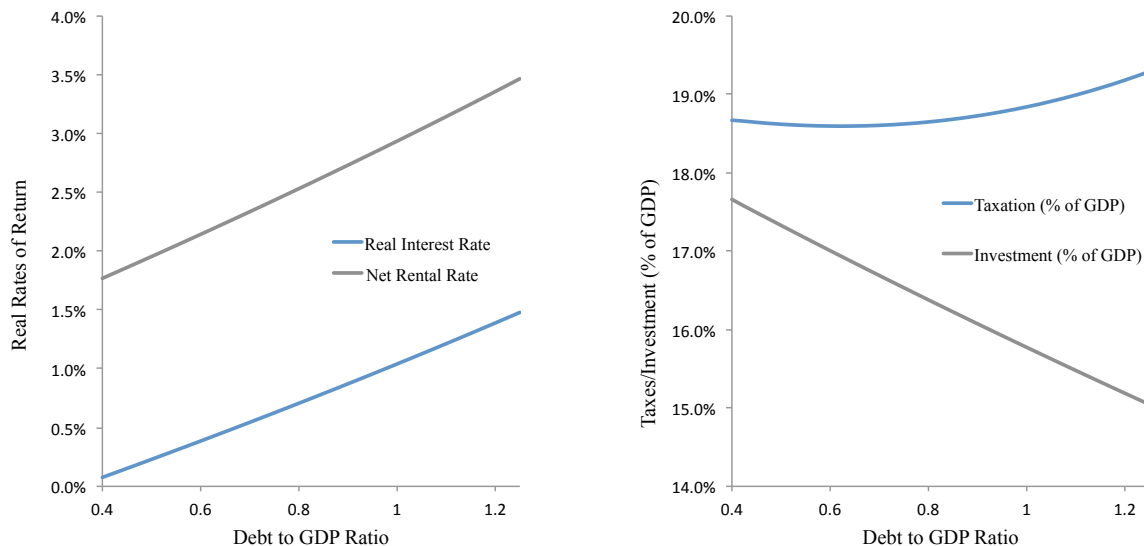
the risk premia. So despite the rise in risk free rate, the capital to output ratio and the investment to output ratio rise. Though the analysis here is restricted to comparison of stationary equilibria, it stands to reason that a fall in rental rates would lead to a transitory boom in investment rates along the transition path that could reduce the debt to GDP ratio in the transition.

What is the realistic variation in the intermediation wedge and risk premium? The baseline calibration sets the intermediation wedge to target a premium of 1.79% based on the average spread between Aaa corporate debt and 10-year Treasury rates. Over the period 1980 to 2016, this spread has averaged 1.24% showing some evidence of increased spreads in the post-Great Recession period.²³ If spreads revert to this historical average, the lifecycle model predicts a 30 basis point increase in the real interest rate and a rise in the investment rate from 16.7% to 17.3%. This rise in the real interest rate translates into a 0.2 percentage point of GDP loss in fiscal resources. A small fiscal consolidation would be required to maintain a stable debt to GDP ratio.

The finding that reductions in risk/liquidity premia raise the real rate is clearly conditional on the manner in which intermediation frictions are modeled and the role of the public debt. As discussed in the previous section, liquidity premia can be modeled as a utility benefit from holdings of government debt - analogous to the common assumption of money in the utility function. In this class of models, liquidity premia *only* affect the real interest rate while leaving the rental rate on capital (in steady state) constant. Hence, a reduction in the liquidity premium has no effect on real quantities like capital, investment, or output. [Abel et al. \(1989\)](#) show in the context of a simple Lucas style asset pricing model that elevated risk premia would only affect the safe rate, leaving the rental rate unchanged. The model here differs somewhat, but further research is needed on

²³At the trough of the Great Recession, this spread was closer to 2.5% to 3%.

Figure 7: Effect of changes in public debt



the precise sources of elevated premia in the Great Recession and their overall contribution to low interest rates.

4.3.3 Increases in Public Debt

In the baseline calibration with $r < g + n$, governments can raise additional real resources by increasing the public debt. However, a policy of increasing the public debt raises real interest rate closing the gap between r and $g + n$. Depending on the elasticity of the real interest rate to the public debt, increases in the public debt may raise little real resources and, at some point, may begin to impose a net cost. Secondly, an increase in the public debt, even if it raises fiscal resources, can crowd out private investment. Here, we consider quantitatively these effects.

The left panel of Figure 7 shows the response of real interest rates as the public debt varies from 40% of GDP to 125% of GDP - approximately the historical range of the US public debt over the 20th century. As the figure illustrates, real interest rates are increasing in the level of the public debt. However, the response of interest rates is quite low; over the entire range, real interest rates vary from 8 basis points to 150 basis points. In the vicinity of a 70% debt to GDP ratio, real interest rates increase by roughly 16 basis points for every 10 percentage point increase in the debt to GDP ratio. Figure 7 also shows how the net rental rate varies with the public debt; the slope is somewhat steeper, reflected in an increase in risk premia as the public debt rises.

The right panel of Figure 7 shows how changes in the public debt affect taxes and investment. Unsurprisingly, higher levels of public debt lower investment rates, and the effect of public debt on investment is fairly strong; a 10 percentage point increase in the public debt lowers the investment to GDP ratio by approximately 30 basis points. What is more surprising is that higher levels of

public debt do not reduce the government's tax burden.

As the blue line shows, the tax share is minimized for a public debt ratio of roughly 60% of GDP. Given current level of US debt, the effect of increases in the public debt on real interest rates (and the net cost of servicing the debt) outstrip the benefits from a higher stock of debt. At about 120% of GDP, real interest rates are high enough that servicing the public debt no longer carries any fiscal benefits. At current levels, *reductions* in the public debt, at least locally, provide a further fiscal benefit by reducing interest costs even at the expense of a lower stock of debt. However, over a fairly large range, the fiscal benefits from varying the debt stock are minimal; at the margin, favorable changes in $r - g - n$ are offset by unfavorable changes in $\frac{b_g}{Y}$. Quite apart from concerns about a reversion in rates towards higher historic levels, the quantitative model considered here suggests that austerity may be beneficial for both reasons of minimizing the net cost of servicing the public debt and increasing the productive capital stock.

4.4 Optimal Level of Debt

As noted earlier, I have so far restricted my analysis to empirical and quantitative assessments of a relatively narrow question: the fiscal cost of servicing the public debt. While the gap between the real interest rate and the growth rate of the economy is a standard metric for debt sustainability, this metric is not a sufficient statistic for the optimal level of debt. In the environment considered here, a full quantitative analysis of the optimal level of debt would depend on a host of considerations: level and type of distortionary taxes, the type of financial frictions faced by households and firms, the manner in which taxes redistribute income across households, the degree of crowding out (including whether the level of capital accumulation is dynamically efficient), and the way in which the government values the utility of current versus future generations.

While a full analysis of these factors is beyond the scope of this paper, it is worth highlighting how some of these factors may qualitatively impact the question of the optimal level of debt. The benefits from a higher public debt in a negative $r - (g + n)$ economy can be separated into steady state (or level) benefits versus more classic hedging/insurance benefits. In a stationary equilibrium, with $r < g + n$, a higher level of public debt may allow the government to reduce distortionary taxation holding constant the level of government expenditure. Lower distortionary taxes may boost factor supply (raising output) or raise welfare by reducing distortions in labor/leisure choices. To the extent that $r < g + n$ due to capital overaccumulation, a higher level of government debt reduces the capital stock, raising consumption per capita. Alternatively, to the extent that $r < g + n$ primarily due to liquidity considerations, a Friedman type rule would apply where the government should fully satiate households' desire for liquidity; this policy would not crowd out the capital stock. A higher provision of safe assets could also carry benefits for the banking system/financial intermediation (see, for example [Holmström and Tirole \(1998\)](#) and [Caballero](#)

and Farhi (2014)) or the functioning of the international financial system (see Caballero, Farhi and Gourinchas (2008) and Eggertsson et al. (2016)).

Classical treatments of optimal fiscal policy have typically focused on the importance of tax smoothing and allowing the level of public debt to vary with government expenditure shocks. In an environment with $r < g + n$ on average, the optimal level of debt would have to tradeoff gains from increasing the public debt in periods of low r relative to growth against the risks of raising taxes or allowing the public debt to drift higher in periods of high r relative to growth. Moreover, in a low r world, a higher average level of public debt could carry the benefit of mitigating the zero lower bound problem. While the higher level of public debt would increase crowding out, monetary policy would presumably have greater freedom to operate to stabilize the business cycle while keeping average inflation rates low. An analysis of the optimal level of debt in a stochastic setting would likely balance these insurance/hedging benefits against costs of crowding out or excessive volatility in tax rates.

Finally, political economy considerations may encourage government to favor higher levels of public debt in a low r world. In the US, raising taxes in recent decades has proved politically costly with deficit financing providing the path of least resistance in increasing expenditures or reducing taxes. The political salience of high levels of public debt appears to coincide with periods of high real interest rates (relative to growth). For example, debt and deficit reduction were key political issues in the late 1980s and early 1990s. Politicians with short time horizons or a high aversion to raising taxes and cutting expenditures may face less pressure to keep the public debt low in periods of $r < g + n$.

5 Conclusion

In this paper, I consider how the cost of servicing the public debt changes in a world of low interest rates. I offer empirical evidence on the frequency with which $r < g + n$ and the possibility of reversion to a higher interest rate regime. I build a quantitative lifecycle model to consider how interest rates, investment, and the cost of servicing the public debt vary with hypothesized drivers of low interest rates and changes in the level of public debt.

On the empirical side, I find that, among advanced economies, real interest rates on government debt frequently fall below the growth rate of real GDP, implying that the public debt provides real resources that governments can use to finance government expenditures. Periods of $r < g + n$ are not driven by historical outliers such as World War I or World War II or the interwar/Great Depression years. I also find that the US displays a *higher* propensity for periods of low rates of return on government debt relative to growth. Nevertheless, I find a moderate probability of reversion to conditions where $r > g + n$ over a 5 or 10 year horizon, meaning that a policy of building up a large stock of public debt may be fiscally unwise. A return to median levels of fiscal

cost would imply a fiscal consolidation of 0.5% of GDP, but the implied fiscal consolidation could easily rise to 1.5% to 2% of GDP - at the upper end of past deficit reduction efforts.

I analyze a 56 period quantitative lifecycle model, calibrating the model to match US interest rates, bond premia, the US labor share, and the US investment to GDP ratio. A simplified two period version is used to show how the real interest rate varies with the public debt and how the behavior of interest rates may differ fundamentally in this class of models relative to the standard representative agent framework. Using the quantitative model, I find that the risk of further slowdowns in productivity or population growth carry differing implications for debt sustainability. Lower productivity growth will actually increase the fiscal benefits of public debt, while slower population growth has the opposite effect. By contrast, a reduction in risk premia will raise interest rates and increase the cost of servicing the debt.

The quantitative model can be used to assess the benefits and costs of increasing the level of public debt. I show that, at the current debt to GDP ratio of 70%, increases in the public debt do not offer further fiscal benefits even though $r < g + n$. Given current rates, a higher level of debt increases interest rates strongly enough to counter the benefits of a higher stock of debt. Moreover, increases in the public debt increase the fiscal consolidation needed if future conditions revert to $r > g + n$ while further crowding out investment in productive capital.

Though the lifecycle model used here is fairly standard, the model could be enriched in several directions to better match the overall capital to output ratio, including the fact that a substantial fraction of the capital stock is in housing. The model could also be extended to include the major government transfer programs - Social Security and Medicare/Medicaid - that represent important drivers of the long-term US fiscal imbalance. Finally, it is likely worth modeling in greater detail the liquidity and intermediation roles of public debt, which has important implications for the response of interest rates to changes in the public debt and the optimal level of debt.

The analysis here has so far abstracted from business cycle considerations. Of particular relevance in a low interest rate world is the possibility of more frequent future zero lower bound episodes assuming the Fed does not increase average inflation rates (see [Williams \(2016\)](#) for a discussion). If ZLB episodes are more frequent in the future, fiscal policy may take on greater importance in responding to future recessions. Governments must then tradeoff any fiscal benefits that come from having a high level of public debt on average given $r < g + n$ against the benefits of entering a recession with fiscal space for cyclical increases in the debt to GDP ratio to support greater fiscal stimulus. I leave this extension for future research.

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A Lifecycle Model

In this section, I describe a 56-period quantitative lifecycle following closely on Eggertsson, Mehrotra and Robbins (2017) and in the spirit of Auerbach and Kotlikoff (1987) and Ríos-Rull (1996). The quantitative model can be used to inform the tradeoff between the net fiscal benefits of the public debt in a low interest rate world and increased crowding out. I can also consider the implications for debt sustainability of a further slowdown in population or productivity growth or, alternatively, a reversion to earlier postwar growth rates.

A.1 Demographics and Labor Supply

Households are born at age 0 and live at most to age J . They have an exogenous survival probability $p_{j,t+1}$ which gives the probability of a household of age $j \in \{0, \dots, J\}$ surviving from t to $t + 1$. By assumption, $p_{J,t+1} = 0$ for all t . Let $N_{j,t}$ denote the population of age j at time t .

The total population at any point in time and the law of motion for the population is given by the following equations. I assume an exogenous birth rate n_{t+1} defined in terms of the total population at any point in time:

$$\begin{aligned} N_t &= \sum_{j=0}^J N_{j,t} \\ N_{j+1,t+1} &= p_{j,t+1} N_{j,t} \text{ for } j \in \{0, J\} \\ N_{0,t+1} &= n_{t+1} N_t \end{aligned}$$

Households supply labor inelastically over their lifetime. Effective labor supply changes over a household's lifetime, and the profile of effective labor is given by $\{hc_j\}_{j=0}^J$ so that wage income for a household of age j in time t is $w_t hc_j$. Without loss of generality, retirement can be modeled as an age j where effective labor supply falls to zero. Total labor supply by all households is given below:

$$L_t^s = \sum_{j=0}^J N_{j,t} hc_j$$

A.2 Firms

There exist a continuum of final goods firms i of measure one that costlessly differentiate an intermediate good and resell to the household. These firms are monopolistically competitive, set prices, and face a demand curve that takes the following form:

$$y_t(i) = Y_t \left(\frac{p_t(i)}{P_t} \right)^{-\theta}$$

where θ determines the degree of a firm's market power. An increase in θ decreases a firm's market power and lowers equilibrium markups.

A final good firm chooses real prices $\frac{p_t(i)}{P_t}$ and $y_t(i)$ to maximize real profits subject to the following constraints:

$$\begin{aligned} & \max \frac{p_t(i)}{P_t} y_t(i) - \frac{p_t^{int}}{P_t} y_t(i) \\ & \text{subject to } y_t(i) = Y_t \left(\frac{p_t(i)}{P_t} \right)^{-\theta} \end{aligned}$$

where $\frac{p_t^{int}}{P_t}$ is the price of the intermediate good taken as given by the firm.

The optimality condition for the real price of the firm's good is a constant markup over the price of the intermediate good:

$$\frac{p_t(i)}{P_t} = \frac{\theta}{\theta - 1} \frac{p_t^{int}}{P_t} \quad (14)$$

The nominal price index is given by the following expression and implies the following expression for the price of intermediate goods:

$$P_t = \left(\int p_t(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}} \quad (15)$$

$$\Rightarrow \frac{p_t^{int}}{P_t} = \frac{\theta - 1}{\theta} \quad (16)$$

There exists a perfectly competitive intermediate goods sector that sells their production to the final goods sector at real price $\frac{p_t^{int}}{P_t}$. These firms operate a Cobb-Douglas production function, hire labor, and rent capital.

The representative intermediate good firm maximizes real profits given the following production function:

$$\Pi_t^{int} = \max \frac{p_t^{int}}{P_t} Y_t - w_t L_t - r_t^k K_t \quad (17)$$

$$Y_t = K_t^\alpha (A_t L_t)^{1-\alpha} \quad (18)$$

The first order conditions that determine labor and capital demand are standard and are given below:

$$w_t = \frac{p_t^{int}}{P_t} (1 - \alpha) \frac{Y_t}{L_t} \quad (19)$$

$$r_t^k = \frac{p_t^{int}}{P_t} \alpha \frac{Y_t}{K_t} \quad (20)$$

A.3 Households

Households choose consumption $c_{j,t}$ and next period assets $a_{j+1,t+1}$ to maximize their lifetime utility. Assets consist of both public debt issued by the fiscal authority and physical assets. Assets

from households that die before the terminal period are redistributed to surviving households next period via a one-period annuity market. Households face a tax levied on labor income. The relevant state variable for households are the assets with which they enter each period $a_{j,t+j}$. The household's problem is given below:

$$U_t = \max_{c_{j,t+j}, a_{j+1,t+1}} \sum_{j=0}^J \beta^j s_{t,t+j} u(c_{j,t+j})$$

subject to $c_{j,t+j} + a_{j+1,t+j+1} + b_{j+1,t+j+1}^g = (1 - \tau_t) w_{t+j} h c_j + \Pi_{j,t+j} +$

$$(1 + r_{t+j}) \left(a_{j,t+j} + \frac{1 - p_{j-1,t+j}}{p_{j-1,t+j}} a_{j,t+j} \right) \quad (21)$$

$$a_{0,t} = 0 \quad (22)$$

where $s_{t,t+j}$ is the probability of surviving to age j :

$$s_{t,t+j} = \prod_{l=0}^{j-1} p_{t+l,t+l+1}$$

The household's consumption/saving decision satisfies a standard Euler equation:

$$u_c(c_{j,t+j}) = \beta u_c(c_{j+1,t+j+1}) (1 + r_{t+j}) \text{ for } j \in \{0, \dots, J\} \quad (23)$$

A.4 Intermediaries

Empirically, rates of return on real capital typically exceed rates of return on government debt owing to, among other things, 1) risk premia, 2) liquidity premia, 3) real intermediation costs, and 4) regulatory capital requirements in the banking system. To capture the fact that returns on capital exceed that of government debt, I introduce an intermediation wedge. I assume that a fraction ω of the return to physical capital goes to intermediaries that, in turn, return these proceeds to households.²⁴ If a fraction ω of the return on capital is diverted to intermediary profits, the real return received by households is given by:

$$1 + r_t = r_t^k (1 - \omega) + 1 - \delta \quad (24)$$

Total distributed profits to the household from monopolistically competitive firms and financial intermediaries must equal the total profit share:

$$\frac{Y_t}{\theta} + \omega r_t^k K_t = \sum_{j=0}^J N_{j,t} \Pi_{j,t} \quad (25)$$

We assume that profits are distributed proportionally to human capital (and hence labor income).

²⁴See [Curdia and Woodford \(2010\)](#) and [Mehrotra and Sergeyev \(2016\)](#) for a discussion of this modeling strategy.

A.5 Fiscal Authority

The fiscal authority issues public debt and raises tax revenue to pay interest on previously issued public debt and to finance government purchases. The government's aggregate budget constraint is given below:

$$B_{t+1}^g + \tau_t w_t \sum_{j=0}^J N_{j,t} h c_j = G_t + B_t^g (1 + r_t) \quad (26)$$

A.6 Competitive Equilibrium

A competitive equilibrium is a set of household allocations: $\{\{c_{j,t}, a_{j,t+1}, \Pi_{j,t}\}_{j=0}^J\}_{t=0}^\infty$, a set of aggregate quantities $\{Y_t, K_t, L_t\}_{t=0}^\infty$, a set of prices $\{w_t, r_t^k, r_t, \frac{p_t^{int}}{P_t}\}_{t=0}^\infty$, a fiscal policy $\{G_t, B_{t+1}^g, \tau_t\}_{t=0}^\infty$, and a set of exogenous processes $\{\{N_{j,t}, p_{j,t+1}\}_{j=0}^J\}_{t=0}^\infty$ that jointly satisfy:

1. Household Euler equations (J equations): (23)
2. Household budget constraints ($J + 1$ equations): (21)
3. No arbitrage condition: (24)
4. Profit clearing: (25)
5. Government budget constraint: (26)
6. Production function: (18)
7. Optimal factor demand: (19) – (20)
8. Markup condition: (14)
9. Market-clearing conditions:

$$L_t^s = L_t$$

$$K_{t+1} + B_{t+1}^g = \sum_{j=0}^J N_{j,t} a_{j+1,t+1}$$

Proposition A.1. Assume that $N_t/N_{t-1} = 1 + n$ and $A_t/A_{t-1} = 1 + g$. Assume that the ratios G_t/Y_t and $B_{g,t}/Y_t$ are held constant. There exists a stationary balanced growth path equilibrium with a constant rental rate of capital and constant real interest rate. Aggregate allocations grow at the rate $g + n$ while per capita consumption/saving allocations grow at the rate g .

Proof. To show the existence of a balanced growth path, I show that the rental rate of capital and the real interest rate remain constant in terms of stationary (detrended) aggregate and household

quantities. For any variable X_t , let $\tilde{x}_t = \frac{X_t}{A_t N_t}$ and let $\hat{x}_t = X_t A_t$. Then, equilibrium conditions can be expressed in terms of stationary quantities (time independent) and a constant rental rate of capital and real interest rate.

$$\tilde{y} = \tilde{k}^\alpha \tilde{l}^{1-\alpha} \quad (27)$$

$$\tilde{l} = \sum_{j=0}^J n_j h c_j \quad (28)$$

$$\tilde{k} + \tilde{b}_g = \frac{1}{1+n} \sum_{j=0}^J n_j \hat{a}_{j+1} \quad (29)$$

$$r_k = \frac{\theta-1}{\theta} \alpha \frac{\tilde{y}}{\tilde{k}} \quad (30)$$

$$\hat{w} = \frac{\theta-1}{\theta} (1-\alpha) \frac{\tilde{y}}{\tilde{l}} \quad (31)$$

$$r = r^k (1-\omega) - \delta \quad (32)$$

$$\left(\frac{\hat{c}_j}{\hat{c}_{j+1}} \right)^{-\sigma} = \beta (1+r) (1+g)^{-\sigma} \quad \text{for } j \in \{0, \dots, J-1\} \quad (33)$$

$$\hat{c}_j + \hat{a}_{j+1} (1+g) - (1+r) \left(\hat{a}_j + \frac{1-p_{j-1}}{p_{j-1}} \hat{a}_j \right) = (1-\tau) (\hat{w} h c_j + \hat{\pi}_j) \quad \text{for } j \in \{0, \dots, J\} \quad (34)$$

$$\sum_{j=0}^J n_j \hat{\pi}_j = \frac{\tilde{y}}{\theta} \quad (35)$$

$$\tilde{b}_g (1+g) (1+n) + \tau \sum_{j=0}^J n_j (\hat{w} h c_j + \hat{\pi}_j) = \tilde{g} + \tilde{b}_g (1+r) \quad (36)$$

□