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Safety, Liquidity, and the Natural Rate of Interest

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Abstract

Why are interest rates so low in the Unites States? We find that they are low mostly because the premium for safety and liquidity has increased since the late 1990s. We reach this conclusion using two complementary perspectives: a flexible time-series model of trends in Treasury and corporate yields, inflation, and long-term survey expectations, and a medium-scale DSGE model. We discuss the implications of this finding for the natural rate of interest.

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Interest rates have been persistently at or near historical lows in many advanced economies at least since the Great Recession. In the United States, short-term interest rates have only recently risen above their effective lower bound, while 10-year nominal Treasury bond yields have hovered around 2 percent since mid-2011. In comparison, 10-year yields averaged 6.7 percent in the 1990s and 4.5 percent in the first decade of the 2000s (in real terms, subtracting survey expectations of long run inflation, these numbers are 3.8 and 2.4 percent, respectively). The causes and macroeconomic consequences of this secular decline in interest rates have been widely discussed, even re-awakening the specter of secular stagnation, a chronic economic malaise characterized by low growth and low rates of return (e.g. Hansen, 1939; Summers, 2014). As shown by Kiley and Roberts (this issue), the decline in interest rates poses important challenges for monetary policy, but it also matters for fiscal policy and for our understanding of the nature of business cycles.

In this paper, we contribute to the debate on the extent of the secular decline in interest rates, and on its fundamental drivers, from two complementary perspectives. First, we estimate a flexible time-series model to extract the permanent component of the real interest rate from data on the nominal returns of bonds with various maturities and with different degrees of liquidity and safety, as well as data on inflation and long-run survey expectations of inflation and nominal rates. We also use this model to decompose the overall trend in interest rates into some of its fundamental drivers. Second, we estimate a medium-scale dynamic stochastic general equilibrium (DSGE) model that features nominal, real and financial frictions. This model provides a structural view of the underlying forces driving interest rates, which is complementary to that provided by the less restricted time-series model. Remarkably, the two models provide a very consistent view of the low frequency movements in the real interest rate, and of its underlying sources.

The common thread running through these two empirical exercises is that they both focus on recovering the properties of the *natural* rate of interest, or r_t^* for short. This concept was originally proposed by Wicksell (1898) and it has been formalized in the context of modern macroeconomics by Woodford (2003). We define r_t^* as the real return to an asset with the same safety/liquidity attributes as a 3-month US Treasury bill in a counterfactual economy without nominal rigidities. To the extent that these rigidities are the main source of the real effects of monetary policy, as they are in our DSGE model, the natural rate of interest is the counterfactual rate that would be observed "in the absence" of monetary policy. Therefore, it summarizes the real forces driving the movements in interest rates, abstracting from the influence of monetary policy decisions. We emphasize the safety/liquidity properties of r_t^*

because central banks' operational targets are generally returns on short-term liquid/safe assets. If r^* is meant to be a benchmark for monetary policy, it better be associated with the return of an asset that possesses such attributes.

Our main findings can be summarized as follows. First, the time-series and the DSGE models recover very similar estimates of the low-frequency component of the natural rate. According to both models, this trend was fairly stable around 2 to 2.5 percent from the early 1960s to the mid-1990s, reached a peak in the late 1990s, and has been declining steadily since then. We estimate its current level to be between 1 and 1.5 percent. Second, the main drivers of this decline are rising premia for the liquidity and safety of Treasury bonds, what Krishnamurthy and Vissing-Jorgensen (2012) refer to as the *convenience yield*. The rise in the convenience yield explains almost one percent of the decline in the natural rate, and is very precisely estimated. This finding adds to a growing body of recent evidence, which we discuss below, showing that Treasury bonds are valued not only for their pecuniary return, but also for their attributes of liquidity and safety. We arrive at this result by comparing the trends in yields on securities that are less liquid and less safe than Treasuries, such as Aaa and Baa corporate bonds, following Krishnamurthy and Vissing-Jorgensen (2012)'s empirical strategy: the returns on corporate bonds appear to have had much less of a secular decline than those on Treasuries, if at all. A version of the model that also includes data on consumption suggests that a persistent decline in its growth rate over the same period might account for most of the remaining trend decline in the real interest rate, although its exact contribution is statistically uncertain. Finally, our third finding is that, in the estimated DSGE model, safety and liquidity factors are also the main drivers of the low frequency movements in the natural rate. Moreover, they have important implications for its cyclical component, and for business cycles more broadly.

The paper's novel contribution is its identification of the convenience yield as a key driver of the trend in the natural rate of interest. To fix ideas on the relationship between the two, it is useful to start from the Euler equation for investing in a liquid, safe, short-term nominal government security, such as a 3-month US Treasury bill carrying a nominal return R_t:

$$1 = E_t \left[\frac{1 + R_t}{1 + \pi_{t+1}} (1 + CY_{t+1}) M_{t+1} \right], \tag{1}$$

where π_t is inflation and M_{t+1} is the stochastic discount factor, which in textbook formulations would be the marginal rate of substitution between consumption in two successive periods $\beta u'(c_{t+1})/u'(c_t)$. Expression (1) is a standard Euler equation, except for the presence of the convenience yield term $(1 + CY_{t+1})$. This is the premium associated with the special liquidity and safety characteristics of the Treasury security relative to assets with the same pecuniary payoff, but no such special attributes.[1] Therefore, an increase in the convenience yield depresses the safe real rate of return, for a given stochastic discount factor, since investors will be willing to accept a lower pecuniary return in exchange for the higher convenience. Similarly, in the counterfactual economy without nominal rigidities, an increase in the convenience yield will depress the natural rate of interest.[2] In the long run, this implies that trends in the convenience yield may drive trends in r_t^*. This is the main hypothesis we explore quantitatively in this paper.

Our two approaches to estimating r^* are related to the popular model of Laubach and Williams (2003) (henceforth LW). Their framework can be viewed both as a restricted version of our VAR, as well as a less tightly parametrized version of our DSGE model. As in our VAR, LW focus on the low frequency component of the natural rate, which they also model as an I(1) process. However, by assuming that r_t^* is a linear function of the growth rate of trend output, they impose more restrictions than in our VAR. The main drawback of their framework compared to a fully specified DSGE model is that the latter provides a more precise notion of the counterfactual that defines the natural rate, as detailed in Section III. Laubach and Williams (2016) update their earlier estimates of the natural rate. They find a more dramatic decline in r^* than the long-run rate identified by our VAR model during the Great Recession and in the years that followed it.[3] However, their estimate tracks relatively closely a shorter-term r^*, such as the 5-year forward natural rate implied by our DSGE model, since the early 1980s. We compare their estimated natural rate and the one resulting from our DSGE model in Section III.B.

[1] As Greenwood et al. (2015) put it, the recent literature "documents significant deviations from the predictions of standard asset pricing models — patterns that can be thought of as reflecting money-like convenience services — in the pricing of Treasury securities generally, and in the pricing of short-term T-bills more specifically." Krishnamurthy and Vissing-Jorgensen (2012) measure the historical convenience yield on Treasuries and show that it has been sizable, averaging 73 basis points per year. From a theoretical point of view, they model the convenience yield as arising from agents deriving direct utility from holding safe/liquid assets. In Kiyotaki and Moore (2012) the liquidity-related component of the convenience yield arises from so-called liquidity (or resaleability) constraints facing actors in financial markets: liquid assets are valued as they relax such constraints. In equation (1) we introduce the convenience yield following the specification in Kiyotaki and Moore (2012).

[2] Del Negro et al. (2017) discuss the impact on r_t^* of the liquidity shocks experienced after the Lehman crisis.

[3] Several other recent papers use unobserved component models to estimate a trend in the real interest rate, including Kiley (2015); Pescatori and Turunen (2015); Johannsen and Mertens (2016).

The extremely low levels of interest rates since the Great Recession have received a great deal of attention, and various explanations have been proposed. Laubach and Williams (2016) attribute a large fraction of the secular decline in the natural rate to a fall in the growth rate of trend output. Other authors, however, are more skeptical of such a tight connection. Looking at cross-country data starting in the 19th century, Hamilton et al. (2015) find only a tenuous link between r^* and output growth. For the U.S., this relationship can only go so far given that rates were high in the 1970s and 1980s when productivity growth was low, and they started declining in the 1990s, when productivity accelerated.

A second class of explanations for the low rates has focused on factors that can be expected to shift desired saving and investment.[4] The most prominent is arguably the ongoing demographic transition. For instance, Carvalho et al. (2016) and Gagnon et al. (2016) argue changes in the dependency ratio due to increased life expectancy and slower population growth can have potentially significant repercussions on aggregate saving, while Favero et al. (2016) argue that demographic factors help predict bond yields. Another factor contributing to higher desired saving and hence to lower interest rates is rising inequality, since richer households tend to save more out of marginal income. However, Auclert and Rognlie (2016) point out that, in general equilibrium, the fall in the interest rate tends to result in a boom in investment and output, which is clearly not a feature of the current environment. Increased uncertainty also has the potential to both increase precautionary saving and to depress investment through the channels emphasized by Bloom (2009). Moreover, the decline in the price of capital associated with rapid investment-specific technical change, by reducing the amount of saving needed to finance each unit of capital, might create an imbalance between desired saving and investment that would put downward pressure on the interest rate (e.g., Eichengreen, 2015).

A third class of explanations for the prevalence of low rates in the US and around the world since the financial crisis revolves around the idea of secular stagnation, which presumes a permanent aggregate demand deficiency, or equivalently an imbalance between desired saving and investment, which cannot be cleared by a sufficient fall in the real interest rate. Such a barrier to lower real rates can be connected most naturally to a binding zero lower bound, as in Eggertsson and Mehrotra (2014), where real rates are permanently pushed against this barrier by a deleveraging shock interacted with an overlapping generation structure.

[4] Rachel and Smith (2015) provide a comprehensive overview of this literature.

In contrast to all these explanations, our analysis emphasizes the role of spreads between Treasury and corporate bonds. We uncover a prominent role for low frequency movements in the convenience yield in accounting for the observed decline in real interest rates which was previously largely ignored in the literature on r^*.[5] Our findings are very much in line with the recent literature discussing the causes and the macroeconomic consequences of the shortage of safe assets (e.g., Bernanke et al., 2011; Caballero and Krishnamurthy, 2009; Caballero, 2010; Caballero and Farhi, 2014; Caballero et al., 2015; Gourinchas and Rey, 2016).[6] One implication of this shortage is that the yield of safe assets, relative to assets that are less safe, should have seen a secular decline, consistently with what we find.[7] Interestingly, Gourinchas and Rey (2016) reach very similar conclusions to ours using a very different approach based on the determinants of the consumption-to-wealth ratio.

This shortage of safe assets is of course related to the saving glut hypothesis first proposed by Bernanke (2005). According to this view, the current account imbalances that grew from the late 1990s to just before the Great Recession, and the globally low rates that accompanied them, were the result of a massive shift in desired saving in developing economies following the Asian crisis of 1997. This glut did not translate into a generic demand for assets, but into a specific one for safe (and liquid) assets. Bernanke et al. (2011) provide evidence that from 2003 to 2007 foreign investors acquired substantial amounts of U.S. Treasuries, Agency debt, and Agency-sponsored mortgage-backed securities. Greenwood et al., 2016 show that foreign holdings of money-like claims produce in the U.S. have risen sharply since the early 2000s. In Caballero (2010)'s words: "...there is a connection between the safe-assets imbalance and the more visible global imbalances: The latter were caused by the funding countries' demand for financial assets in excess of their ability to produce them (...), but this gap is particularly acute for safe assets since emerging markets have very limited institutional capability to

[5]Kiley (2015) includes a corporate spread as an exogenous variable in his analysis, since it helps to forecast output. He finds that this modification to the LW specification reduces the estimated movements in r^* around the Great Recession. Pescatori and Turunen (2015) find that proxies for the demand for safe assets help to explain some of the (cyclical) movements in their estimate of r^*, especially since the late 1990s.

[6]See Gorton (2016) for a definition of safe assets and for a broad discussion of their role in economics. Hall (2016) takes a related but slightly different perspective, as he emphasizes heterogeneity in beliefs and risk aversion, and how changes in the wealth distribution in favor of more risk averse/pessimistic investors can lead to a decline in the real rate on safe securities.

[7]Caballero and Farhi (2014) also show that the expected return on stocks is currently much higher than the yield of safe assets, consistently with their theory. Our empirical analysis is arguably more direct in that the safety premium is only one of the determinants of the stock market risk premium, while we are able to identify the convenience yield more sharply using spreads.

produce these assets."

While much of the macroeconomic literature mentioned above emphasizes safety, we also stress the role of liquidity. Liquidity has long played a prominent role in finance.[8] For instance, Fleckenstein et al. (2014) provide evidence of what they call the "TIPS-Treasury bond puzzle," that is, significant differences in prices between Treasury bonds of various maturities and inflation-swapped Treasury Inflation-Protected Securities (TIPS) of the same maturities.[9] Starting with Kiyotaki and Moore (2012), liquidity has also been incorporated in modern macroeconomic models to study its role for business cycles and the Great Recession.[10] We show that the liquidity convenience yield plays an important role in explaining why interest rates on liquid assets are currently low, and argue more broadly that for both secular trends and cyclical movements in interest rate liquidity plays a role that is as important as that of safety.[11]

The remainder of the paper proceeds as follows. Section I introduces the empirical model, a VAR with common trends, and Section II uses this framework to estimate trends in interest rates. Section III briefly describes the DSGE model and presents the results. Section IV concludes.

I A VAR with Common Trends

The model is given by the measurement equation

$$y_t = \Lambda \bar{y}_t + \tilde{y}_t, \tag{2}$$

where y_t is an $n \times 1$ vector of observables, $\bar{y}_t$ is a $q \times 1$ vector of trends, with $q \leq n$, $\Lambda(\lambda)$ is a $n \times q$ matrix of loadings which is restricted and depends on the vector of free parameters

[8]See Longstaff et al. (2004), Acharya and Pedersen (2005), Longstaff et al. (2005), Amihud et al. (2006), Garleanu and Pedersen (2011), Amihud et al. (2012), and Fleckenstein et al. (2014) among many others.

[9]Specifically, they find that the price of a Treasury bond and an inflation-swapped TIPS issue exactly replicating the cash flows of the Treasury bond can differ by more than \$20 per \$100 notional – a difference that, they argue, is orders of magnitude larger than the transaction costs of executing the arbitrage strategy.

[10]See for instance, Kurlat (2013), Bigio (2015), Ajello (forthcoming), Del Negro et al. (2017), Cui and Radde (2014), and Guerron-Quintana and Jinnai (2015).

[11]Our VAR and DSGE models treat safety and liquidity as essentially independent factors, which we try to distinguish empirically by looking at the returns on assets with different characteristics. However, safety and liquidity are clearly interrelated. For instance, in Kurlat (2013) market freezes (illiquidity) take place precisely because agents are uncertain about the safety of the assets in the market.

λ, and $\tilde{y}_t$ is an $n \times 1$ vector of stationary components. The rank of Λ, which is equal to q, determines the number of common trends, and the number of cointegrating relationships is therefore $n - q$. Both $\bar{y}_t$ and $\tilde{y}_t$ are latent and evolve according to a random walk

$$\bar{y}_t = \bar{y}_{t-1} + e_t \tag{3}$$

and a VAR

$$\Phi(L)\tilde{y}_t = \varepsilon_t, \tag{4}$$

respectively, where $\Phi(L) = I - \sum_{l=1}^{p} \Phi_l L^l$ and the Φ_l's are $n \times n$ matrices, and the $(q+n) \times 1$ vector of shocks $(e_t', \varepsilon_t')'$ is independently and identically distributed according to

$$\begin{pmatrix} e_t \\ \varepsilon_t \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0_q \\ 0_n \end{pmatrix}, \begin{pmatrix} \Sigma_e & 0 \\ 0 & \Sigma_\varepsilon \end{pmatrix} \right), \tag{5}$$

with the $\Sigma_.$'s being conforming positive definite matrices, and where $\mathcal{N}(.,.)$ denotes the multivariate Gaussian distribution. Equations (3) and (4) represent the transition equations in the state space model. The initial conditions $\bar{y}_0$ and $\tilde{y}_{0:-p+1} = (\tilde{y}_0', .., \tilde{y}_{-p+1}')'$ are distributed according to

$$\bar{y}_0 \sim \mathcal{N}(\underline{y}_0, \underline{V}_0), \;\; \tilde{y}_{0:-p+1} \sim \mathcal{N}(0, V(\Phi, \Sigma_\varepsilon)) \tag{6}$$

where $V(\Phi, \Sigma_\varepsilon)$ is the unconditional variance of $\tilde{y}_{0:-p+1}$ implied by (4).[12] Constants or deterministic trends can be easily accommodated into this framework. The procedure also straightforwardly accommodates missing observations.

The model above is essentially Villani (2009)'s VAR model, except that his deterministic trend is replaced by the stochastic trend (3). It also corresponds to the multivariate trend-cycle decomposition described in Stock and Watson (1988) (equation 2.4) with the important difference that the shocks affecting the trend and the cycle are orthogonal to one another (in Watson (1986)'s parlance, our model is an "independent trend/cycle decomposition"). In a nutshell, the model is a multivariate extension of a standard unobserved component model (e.g.,Watson (1986), Stock and Watson (2007), Kozicki and Tinsley (2012)). Recently, Crump et al. (2016) and Johannsen and Mertens (2016) have also estimated models that are very similar to ours.[13]

[12] We impose stationarity to the VAR (4), as discussed below, so that $V(\Phi, \Sigma_\varepsilon)$ is always well defined.

[13] Crump et al. (2016) estimate the parameters by maximizing the likelihood. Johannsen and Mertens (2016) use a Gibbs sampler, like we do, but impose that the elements of the matrix Λ are known. Johannsen and Mertens (2016)'s sophisticated model allows for stochastic volatility in the shocks distribution and for

The priors for the VAR coefficients $\Phi = (\Phi_1, \ldots, \Phi_p)'$ and the covariance matrices Σ_ε and Σ_e have standard form, namely

$$p(\varphi|\Sigma_\varepsilon) = \mathcal{N}(vec(\underline{\Phi}), \Sigma_\varepsilon \otimes \underline{\Omega})\mathcal{I}(\varphi), \; p(\Sigma_\varepsilon) = \mathcal{IW}(\kappa_\varepsilon, (\kappa_\varepsilon + n + 1)\underline{\Sigma_\varepsilon}),$$
$$p(\Sigma_e) = \mathcal{IW}(\kappa_e, (\kappa_e + q + 1)\underline{\Sigma_e}), \quad (7)$$

where $\varphi = vec(\Phi)$, $\mathcal{IW}(\kappa, (\kappa + n + 1)\underline{\Sigma})$ denotes the inverse Wishart distribution with mode $\underline{\Sigma}$ and κ degrees of freedom, and $\mathcal{I}(\varphi)$ is an indicator function which is equal to zero if the VAR is explosive (some of the roots of $\Phi(L)$ are less than one) and to one otherwise.[14] The prior for λ is given by $p(\lambda)$, the product of independent Beta, Gamma, or Gaussian distributions for each element of the vector λ (all the details, as well as the actual values used in the prior, are given below, when discussing the application).

The model (2) through (6) is a linear Gaussian state-space model. Therefore, it is straightforward to estimate efficiently in spite of the large size of the state space using modern simulation smoothing techniques (Carter and Kohn, 1994, or Durbin and Koopman, 2002). Section A in the Appendix describes the Gibbs sampler, which accommodates VARs of any size and with any estimated cointegrating relationship.

II Estimating and Decomposing the Trend in r_t

In this section, we estimate the trend in the return on safe and liquid assets r_t and analyze its determinants. We do so using the VAR discussed in Section I with data on nominal Treasury yields at different maturities, as well as inflation, inflation expectations, and measures of credit spreads associated with liquidity and safety. Under the generally accepted assumption that the gap between the observed real rate r_t and the natural rate r_t^* is stationary, we can learn about the trend in the latter, which we denote by $\bar{r}_t^*$, by conducting inference on $\bar{r}_t$. This is the strategy pursued in this section. As we will show in Section III, the trend in $\bar{r}_t$

explicit treatment of the zero lower bound on nominal rates. Our model can certainly be amended to accommodate the former, along the lines of Del Negro and Primiceri (2015), and in principle also the latter, following Johannsen and Mertens (2016)'s approach.

[14] The inverse-Wishart distribution with parameters κ and $(\kappa + m + 1)\underline{\Sigma}$ is given by

$$p(\Sigma; \kappa, (\kappa + m + 1)\underline{\Sigma}) = \frac{|(\kappa + m + 1)\underline{\Sigma}|^{\kappa/2}}{2^{m\kappa/2}\Gamma(\kappa/2)} |\Sigma|^{-(r+\kappa+1)/2} \exp\left(-\frac{\kappa + m + 1}{2} \operatorname{tr}(\Sigma^{-1}\underline{\Sigma})\right),$$

where m is the size of Σ. Under this parametrization $\underline{\Sigma}$ is the mode and κ are the degrees of freedom.

estimated using the VAR nearly coincides with the low frequency component of the natural rate of interest obtained from the DSGE model, corroborating this assumption.[15]

We start the exposition in Section II.A with a very simple specification that only includes data on nominal yields for Treasuries with short (3-month) and long (20-year) maturity, and on inflation and its expectations. This is the minimum amount of information needed to identify the trend in the real interest rate separately from that in inflation. We use both short and long-term bond yields because we are interested in a trend that is common across maturities, and because the long-term yield continues to provide information on that trend even over the years in which the short-term rate is constrained by the zero lower bound.[16] The trend in the real interest rate estimated in this simple model falls by about one-and-a-quarter percentage points from the late 1990s to the end of 2016. This estimated decline, displayed in Table 1, is very robust across specifications, and always significant.

Section II.B presents a richer model that also includes data on Baa and Aaa corporate bond yields. The spreads between these yields and those of Treasuries of comparable maturity allow us to identify trends in liquidity and safety, and hence in the overall convenience yield on Treasury yields. Our main finding is that these trends account for a large and statistically significant fraction of the trend decline in r_t—about 90 basis points. In Section II.C we also include data on consumption growth to verify the extent to which trends in this variable might account for some of the secular movements in the interest rate, as a textbook Euler equation would suggest. We find some evidence of a connection between the two trends, although this relationship is not sharply estimated.

Finally, Section II.D explores the robustness of the main results to several alternative specifications. The prominent role of the convenience yield in driving the real interest rate lower over the last two decades remains a robust finding across all these specifications.

[15] Although very common, the assumption of a stationary interest rate gap, or that monetary policy cannot affect the growth rate of the economy in the long-run, is not entirely uncontroversial. For instance, it is violated in models featuring endogenous growth with nominal rigidities (e.g. Benigno and Fornaro, 2016). Perhaps more importantly, equation (3) implies that trends evolve smoothly over time. Therefore, our approach cannot capture abrupt shifts from one long-run regime to another, as envisioned for example in the theory of Secular Stagnation (e.g., Summers, 2014; Eggertsson and Mehrotra, 2014).

[16] In principle, we could use many more maturities, but doing so would require taking a stance on the possible presence of different trends at different maturities, a task that is beyond the scope of this paper.

Table 1: Change in Trends, 1998Q1-2016Q4

	(1) Baseline	(2) Conv.Yield	(3) Liq.+Safe.	(4) Consumption
$\bar{r}_t$	-1.29**	-1.27**	-1.30**	-1.40**
	[-1.70, -0.85]	[-1.60, -0.92]	[-1.63 -0.95]	[-1.84, -0.92]
	(-2.07, -0.43)	(-1.91, -0.56)	(-1.95, -0.60)	(-2.23, -0.43)
$\bar{m}_t$		-0.34	-0.33	-0.61
		[-0.65, -0.02]	[-0.65, -0.01]	[-1.04, -0.15]
		(-0.96, 0.29)	(-0.95, 0.31)	(-1.45, 0.30)
$\bar{g}_t$				-0.56
				[-0.98, -0.13]
				(-1.37, 0.29)
$\bar{\beta}_t$				-0.04
				[-0.21, 0.12]
				(-0.37, 0.29)
$-\overline{cy}_t$		-0.93**	-0.97**	-0.78**
		[-1.14, -0.71]	[-1.18, -0.75]	[-0.99, -0.57]
		(-1.35, -0.49)	(-1.40, -0.53)	(-1.20, -0.36)
$-\overline{cy}^s_t$ (safety)			-0.45**	-0.33**
			[-0.60, -0.31]	[-0.47, -0.18]
			(-0.74, -0.16)	(-0.61, -0.04)
$-\overline{cy}^l_t$ (liquidity)			-0.52**	-0.45**
			[-0.65, -0.38]	[-0.58, -0.32]
			(-0.77, -0.24)	(-0.71, -0.19)
$\Delta\bar{c}_t$				-0.80
				[-1.38, -0.21]
				(-1.91, 0.39)

Note: The table shows the change in the trends for the different specifications described in Sections II.A (baseline model: column (1)), II.B (convenience yield model: column (2); safety and liquidity model: column (3)), and II.C (consumption: column (4)). For each trend, the table shows the posterior median, the 68 (square bracket) and 95 (round bracket) percent posterior coverage intervals. The $**$ symbol indicates that the decline is significant, in that the 95 percent coverage intervals do not include zero.

II.A Extracting $\bar{r}_t$ from Nominal Treasury Yields and Inflation

Model Specification. Call $R_{\tau,t}$ the net yield on a nominal Treasury of maturity τ (with τ expressed in quarters). Following the VAR (2) of Section I, we decompose the term structure as the sum of a trend $\bar{R}_{\tau,t}$ and a stationary component $\tilde{R}_{\tau,t}$

$$R_{\tau,t} = \bar{R}_{\tau,t} + \tilde{R}_{\tau,t}. \tag{8}$$

Define r_t as the net real return on an asset that is as liquid and safe as a 3-month Treasury bill, and that therefore satisfies

$$E_t\left[(1+r_t)\left(1+CY_{t+1}\right)M_{t+1}\right] = 1 \tag{9}$$

where M_{t+1} is the stochastic discount factor. Assuming that the Fisher equation holds in the long run, we can decompose the trend in the nominal short-term rate as

$$\bar{R}_{1,t} = \bar{r}_t + \bar{\pi}_t,$$

where $\bar{r}_t$ and $\bar{\pi}_t$ are the trends in the real interest rate and in inflation, respectively. For a nominal 3-month bill ($\tau = 1$) we can therefore write equation (8) as

$$R_{1,t} = \bar{r}_t + \bar{\pi}_t + \tilde{R}_{1,t}. \tag{10}$$

From (10) we cannot separately disentangle movements in $\bar{r}_t$ and $\bar{\pi}_t$.[17] We address this problem by extracting the nominal trend $\bar{\pi}_t$ from inflation π_t (measured as log changes in the GDP deflator) and, whenever available, inflation expectations obtained from surveys π_t^e using an unobserved component model à la Stock and Watson (1999):

$$\begin{aligned} \pi_t &= \bar{\pi}_t + \tilde{\pi}_t, \\ \pi_t^e &= \bar{\pi}_t + \tilde{\pi}_t^e. \end{aligned} \tag{11}$$

In principle expressions (10) and (11) are enough to conduct inference on $\bar{r}_t$. However, we do not want to use short rates information for the zero lower bound (henceforth, ZLB) period given the concern that these may distort our inference on the trends. Therefore, we do not use data on $R_{1,t}$ after 2008Q3. Moreover, inference on trends can be made sharper by using two additional sources of information: long-maturity Treasury yields and forecasters' expectations of long-run averages of the short-term rate.

If the expectation hypothesis were correct, long-maturity Treasuries would indeed be the ideal observable for extracting trends, being simply averages of expected short-term rates. Of course, the expectation hypothesis does not hold, and movements in the term premium are

[17] Cieslak and Povala (2015) also allow for a persistent inflation component in an empirical model of nominal Treasury yields.

key drivers of yields. We model possible trends in the nominal term premium by including an exogenous component $\overline{tp}_t$. We use the yield on 20-year Treasuries as a measure of long-term yields and model it as

$$R_{80,t} = \bar{r}_t + \bar{\pi}_t + \overline{tp}_t + \tilde{R}_{80,t}, \tag{12}$$

where $\tilde{R}_{80,t}$ captures stationary movements in long term yields.[18] Recall that we allow for a correlation in the innovations to the trend, hence expressions (10) and (12) do not necessarily imply that trends in $\bar{r}_t$, $\bar{\pi}_t$, or $\overline{tp}_t$ are independent. However, since we impose a fairly strong prior that the correlation matrix is diagonal, Section II.D explores the possibility that trends in inflation might affect the term premium by introducing a term premium component that is proportional to trends in inflation $\gamma^{tp}\bar{\pi}_t$ with $\gamma^{tp} > 0$.

Finally, inspired by Crump et al. (2016), we also use forecasters' expectations of long-run averages of the short-term rate, which we call $R^e_{1,t}$, and model them as

$$R^e_{1,t} = \bar{r}_t + \bar{\pi}_t + \tilde{R}^e_{1,t}. \tag{13}$$

The system of equations (10) through (13) can be expressed as the VAR (2), where $y_t = \left(\pi_t, \pi^e_t, R_{1,t}, R_{80,t}, R^e_{1,t}\right)$ and $\bar{y}_t = \left(\bar{r}_t, \bar{\pi}_t, \overline{tp}_t\right)$ evolve according to (3), and the stationary components $\left(\tilde{\pi}_t, \tilde{\pi}^e_t, \tilde{R}_{1,t}, \tilde{R}_{80,t}, \tilde{R}^e_{1,t}\right)$ evolve according to (4). Note that we impose only two, arguably quite natural, cointegrating restrictions: one between inflation and inflation expectations, and another one between short-term interest rates and their expectations. We estimate this model using as observables annualized PCE inflation, long-run (10-year averages) PCE inflation expectations, the 3-month Treasury Bill rate, the long-run (10-year averages) expectations for the 3-month Treasury Bill rate, and the 20-year Treasury constant maturity rate.[19] With the exception of long-run expectations, all the data are available from

[18]Several papers (most recently Johannsen and Mertens, 2016) assume that the term premium is stationary. We have also considered a constant term premium and found the results to be robust. We use the 20-year yield because that is the natural counterpart in terms of maturity for the corporate bonds we will use in the next section (see Krishnamurthy and Vissing-Jorgensen, 2012). Results obtained using the 10-year yield are very similar.

[19]Annualized PCE inflation, the 3-month Treasury Bill rate and the 20-year Treasury constant maturity rate are available from FRED and their mnemonics are DPCERD3Q086SBEA, TB3MS, and GS20, respectively. The long-run PCE inflation expectations are obtained from the Survey of Professional Forecasters (henceforth, SPF) from 2007 onward, while from 1970 to 2006 we use the survey-based long-run (5- to 10-year-ahead) PCE inflation expectations series of the Federal Reserve Board of Governors FRB/US econometric model. This same dataset is employed by Clark and Doh (2014), and we are grateful to Todd Clark for making the data available. The long-run expectations for the 3-month Treasury Bill rate are also

1954Q1 to 2016Q4. We use the period 1954Q1-1959Q4 as presample and estimate the model over the sample 1960Q1-2016Q4. Because of the zero lower bound on interest rates, we treat the short-term rate as unobservable from 2008Q4 onward.

The prior for Σ_e, the variance covariance matrix of the innovations to the trends $\bar{y}_t$, is very conservative, in the sense of limiting the amount of variation that it attributes to the trends. The matrix $\underline{\Sigma_\varepsilon}$ is therefore diagonal with elements equal to 1/400, which imply that a priori the standard deviation of the expected change in the trend over one century is only one percentage point. For the trend in inflation, we use a higher, but still conservative, prior of 1/200 (one percentage point in fifty years).[20] In addition, these priors are quite tight, as we set $\kappa_e = 100$. As shown below, these conservative priors do not prevent us from finding trends where these are clearly present, such as in inflation or in the convenience yield. Moreover, the robustness section shows that with a looser prior we simply let $\bar{y}_t$ capture some higher frequency movements, with not much impact on the substantive results.

The prior for the VAR parameters describing the components $\tilde{y}_t$ is a standard Minnesota prior with the hyperparameter for the overall tightness equal to the commonly used value of .2 (see Giannone et al., 2015), except that of course the prior for the "own-lag" parameter is centered at zero rather than one, as we are describing stationary processes.[21] The initial conditions $\underline{y}_0$ for the trend components $\bar{y}_t$ are set at presample averages for inflation, the real rate, and the term spread (2, .5, and 1 for $\bar{\pi}_0$, $\bar{r}_0$, and $\overline{tp}_0$, respectively), with $\underline{V}_0$ being the identity matrix. Finally, the VAR uses five lags ($p = 5$).

Results. The left panel of Figure 1 shows the estimates of $\bar{r}_t$. The dashed black line shows the posterior median of $\bar{r}_t$ while the shaded areas show the 68 and 95 percent posterior coverage intervals (this convention applies to all latent variables shown below). $\bar{r}_t$ rises from the 1960s to the early 1980s, remains roughly constant until the late 1990s, and then begins

obtained from the SPF and are available once a year starting in 1992Q1. The 20-year Treasury constant maturity rate is not available from 1987Q1 to 1993Q3. For this period, following Haver Analytics, we use instead an average of the 10 and 30-year Treasury constant maturity rates (GS10 and GS30, respectively). We use quarterly averages for all variables that are available at higher frequency than quarterly.

[20]Results with a tighter prior of 1/400 for the variance of the inflation trend only change in that the trend in inflation does not rise as much as long-run inflation expectations in the mid-1970s, but are otherwise very similar to the ones shown here.

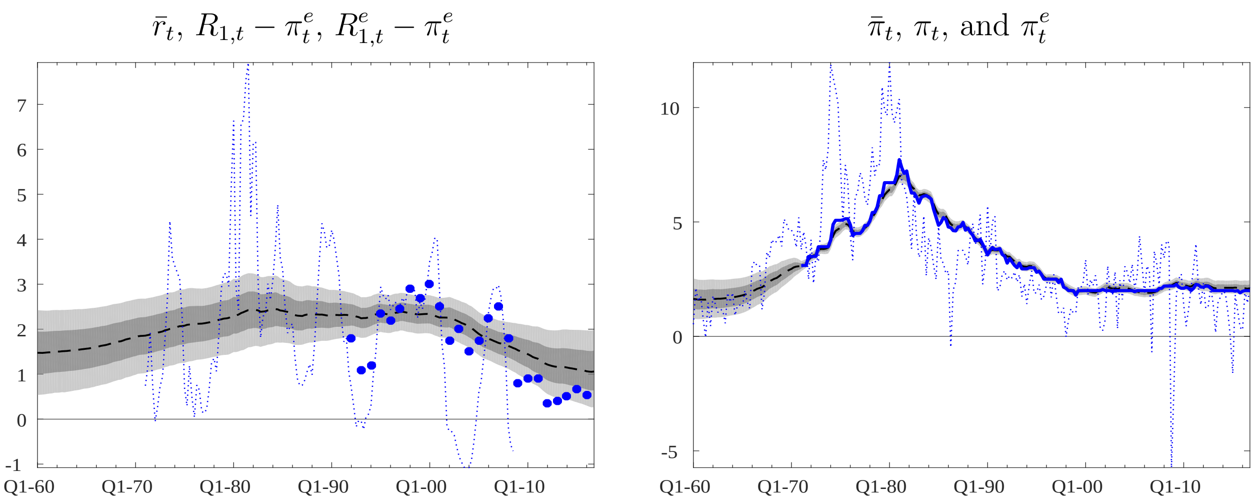
[21]Our prior for the variance Σ_ϵ is a very uninformative Inverse Wishart distribution centered at a diagonal matrix with unitary elements (except for inflation, for which the diagonal element is 2, and expectations, for which the variance is .5; these numbers reflect presample variances, except for expectations which are not available) with just enough degrees of freedom ($n + 2$) to have a well-defined prior mean. We do not use the "co-persistence" or "sum-of-coefficients" priors of Sims and Zha (1998).

to decline. This result is consistent with previous findings in the literature. In addition to LW, Bauer et al. (2012, 2014); Christensen and Rudebusch (2016), using a term structure model; Crump et al. (2016), using data on survey expectations; and Lubik et al. (2015), using a time-varying parameter VAR, also find that long-term forward rates have fallen substantially over the past twenty years. The median decline in $\bar{r}_t$ from 1998Q1 to 2016Q4 is about one-and-a-quarter percentage points, as shown in the first column of Table 1, from 2.36 to 1.06 percent. This decrease is significant, in that the 95 percent credible intervals range from -.43 to -2.07 percent. The left panel also shows the short-term rate $R_{1,t}$ and the long-run expectations for the short-term rate $R^e_{1,t}$, both expressed in deviations from long-run inflation expectations π^e_t so that trends in the real variables become more apparent.[22] $\bar{r}_t$ declines since the late 1990s along with the decline in long-term expectations for the short-term real rate $R^e_{1,t} - \pi^e_t$. Toward the end of the sample the trend remains above the data for $R^e_{1,t} - \pi^e_t$, which is arguably reasonable in light of the fact that these 10-year averages partly reflect cyclical movements – e.g., the slow renormalization of real rates in the aftermath of the crisis. It is also apparent from the figure that the use of long-run short-rate expectations helps in terms of the inference on the trend, as the bands for $\bar{r}_t$ get considerably narrower when these data become available (the bands become somewhat wider again in the ZLB period as we are not using data on the short-term rate during this period).

The right panel of Figure 1 shows the data, π_t (dotted blue line), and π^e_t (solid blue line), together with the trend $\bar{\pi}_t$. We find that $\bar{\pi}_t$ appears to capture well the trend in inflation and essentially coincides with long-run inflation expectations, whenever these are available, even though the model only imposes that π_t, and π^e_t share a common trend.

[22]The time series for $R_{1,t} - \pi^e_t$ begins in 1970 simply because long-run inflation expectations were not available before then. Figure A1 in the Appendix shows the estimated trends in the term premium together with the term spread $R_{80,t} - R_{1,t}$. Figure A2 shows all the data y_t used in the estimation together with $\Lambda\bar{y}_t$ and $\tilde{y}_t$, the non-stationary and stationary components, respectively. The figure shows that the model fits the trends in the data reasonably well, including those in the 20-year yield, in that the $\tilde{y}_t$'s do indeed look stationary. In the aftermath of the Great Recession, however, all of the stationary components are persistently negative, including those for inflation and long-run rates expectations. The model suggests that the Great Recession has had a persistently negative effect on the cyclical component of inflation and interest rates, possibly capturing headwinds to the recovery.

Figure 1: Trends and Observables, Baseline Model



Note: The left panel shows $R_{1,t} - \pi^e_t$ (dotted blue line), and $R^e_{1,t} - \pi^e_t$ (blue dots), together with the trend $\bar{r}_t$. The right panel shows π_t (dotted blue line), and π^e_t (solid blue line), together with the trend $\bar{\pi}_t$. For each trend, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

II.B Drivers of $\bar{r}_t$: The Role of the Convenience Yield

Trends in the Convenience Yield.

Model Specification. In this section we refine the approach outlined above with the goal of assessing the component of long-term movements in r due to changes in the convenience yield. In order to do that we bring into the analysis assets whose safety/liquidity attributes are not the same as those of nominal Treasuries.

The Euler equation (9) implies that trends in r_t are driven by trends in the convenience yield CY_t and in the stochastic discount factor M_t. In order to proceed we make the assumption that the covariance between CY_t and M_t is stationary and write:

$$\bar{r}_t = \overline{m}_t - \overline{cy}_t, \tag{14}$$

where $cy_t = log(1 + CY_t)$ and $m_t = -logM_t$. In addition, we assume that the trends $\overline{cy}_t$ and $\overline{m}_t$ evolve independently from one another according to equation (3) (although shocks to the trends are allowed to be correlated).

Using the above decomposition we can replace $\bar{r}_t$ with $\overline{m}_t - \overline{cy}_t$ in expressions (10), (12), and (13). Implicitly this amounts to assuming that in the long run all Treasuries, regardless of maturity, benefit in equal measure of the same safety and liquidity attributes as 3-month bills (an assumption we discuss below). This implies that data on $R_{1,t}$, $R_{80,t}$, or $R^e_{1,t}$ are of no use in disentangling $\overline{cy}_t$ from $\overline{m}_t$. In order to do that we need to consider assets who carry less of a convenience yield than Treasuries. Krishnamurthy and Vissing-Jorgensen (2012) use the spread between Baa corporate bonds and Treasuries to identify the convenience yield. We follow their lead and augment the set of observables with the yield of Baa corporate bonds, which we model as follows:

$$R_t^{Baa} = \overline{m}_t - \lambda_{cy}^{Baa}\overline{cy}_t + \bar{d}_t + \bar{\pi}_t + \overline{tp}_t + \tilde{R}_t^{Baa}, \tag{15}$$

where $0 \leq \lambda_{cy}^{Baa} < 1$, indicating that Baa corporate bonds are less liquid/safe than Treasuries, and where $\bar{d}_t$ reflects trends in the actual default probability of corporate bonds. We use the same term premium that we use in equivalent maturity Treasuries (following Krishnamurthy and Vissing-Jorgensen, 2012, we use 20-year Treasury yields as the reference), which means that we constrain the term premium to be the same, at least in the long run. In the reminder of this section we will ignore $\bar{d}_t$, on the grounds that there is clear secular trend in the average corporate default probability. In the robustness section we discuss the results of a model that explicitly accounts for $\bar{d}_t$, and show that our results are even stronger.

From equations (15) and (12) it follows that the trends in the spread between Baa corporate bonds yields and equivalent maturity Treasuries is given by

$$\bar{R}_t^{Baa} - \bar{R}_{80,t} = (1 - \lambda_{cy}^{Baa})\overline{cy}_t, \tag{16}$$

which implies that trends in the spread reflect trends in the convenience yield. Specifically, we will assume that $\lambda_{cy}^{Baa} = 0$, that is, that Baa corporates do not have any convenience yield whatsoever. Given the measured difference in trends $\bar{R}_t^{Baa} - \bar{R}_{80,t}$ between Baa corporate bonds yields and equivalent maturity Treasuries this assumption is the most conservative in terms of extracting $\overline{cy}_t$. We should also stress that our results focus on secular *changes* in the convenience yield, as opposed to its *level*. The level of the Baa/Treasury spread may be affected by factors other than safety and liquidity premiums (e.g., the average default probability of corporate bonds). The key identifying assumption we use is that secular changes in the spread primarily reflect secular changes in the convenience yield.

Equation (16) deserves additional comments. First, as explained very clearly in Krishnamurthy and Vissing-Jorgensen (2012) the spread $R_t^{Baa} - R_{80,t}$ captures not just the current value of the convenience yield, but rather the expected average convenience yield throughout the remaining maturity of the bond. But this is precisely what we need since we are after trends in the convenience yield. Second, we assumed that long-term Treasuries benefit of the same convenience yield as short-term Treasuries. In making this assumption, we are arguably underestimating the convenience yield on short-term Treasuries, which is what we are after. All Treasuries are equally safe, irrespective of their maturity, hence it is reasonable to assume that the component of the convenience yield deriving from safety applies evenly across maturities. As for the component associated with liquidity, Greenwood et al. (2015) provide some evidence that the liquidity premium is a decreasing function of maturity. They compute what they call *z-spreads*, which capture deviations in the pricing of Treasury Bills from the an extrapolation based on the rest of the yield curve, and argue that these *z-spreads*, which are sizable, "reflect a money-like premium on short-term T-bills, above and beyond the liquidity and safety premia embedded in longer term Treasury yields" (pg. 1687). In conclusion, for these reasons we think that our assumption that the convenience yields extracted from long-term Treasuries applies in the same measure to Treasury Bills is conservative; it is an assumption nonetheless, and one should bear that in mind in interpreting our results.

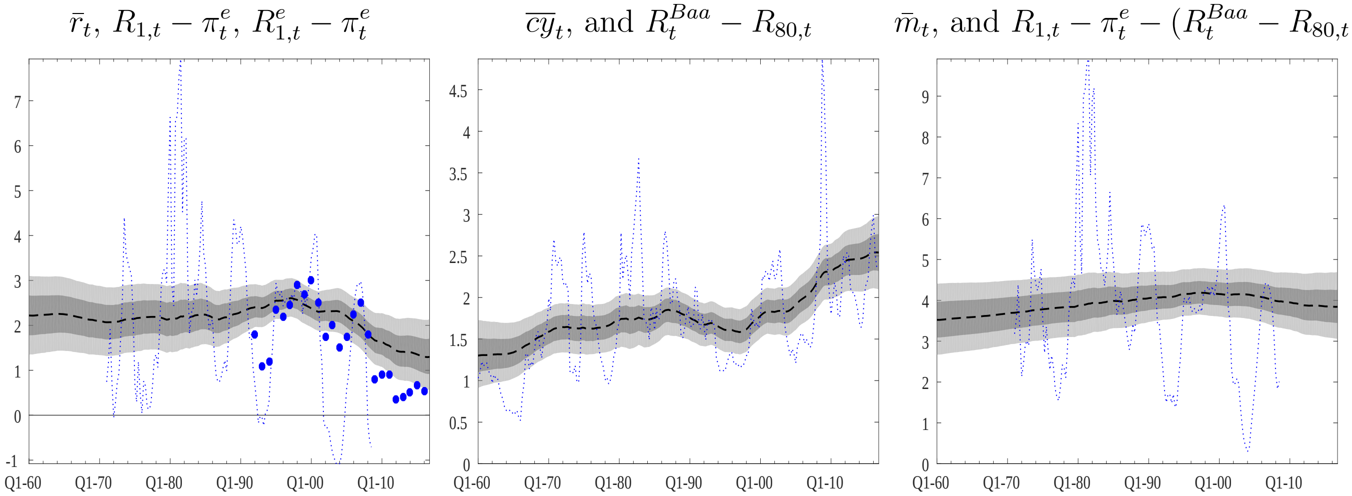
The system of equations given by (10)–(13) and (15) can be expressed as a VAR for $y_t = \left(\pi_t, \pi_t^e, R_{1,t}, R_{80,t}, R_{1,t}^e, R_t^{Baa}\right)$ with common trends $\bar{y}_t = \left(\bar{r}_t, \bar{\pi}_t, \overline{cy}_t, \overline{tp}_t\right)$.[23] We use exactly the same priors as described in Section II.A, except that since we decompose the trend $\bar{r}_t$ into two components, $\bar{m}_t$ and $\overline{cy}_t$, we center the corresponding diagonal value of $\underline{\Sigma_\varepsilon}$ to a number that is 1/2 the value chosen for $\bar{r}_t$ (we use 1/800 as opposed to 1/400).[24]

Results. The left panel of Figure 2 shows $\bar{r}_t$ together the short-term rate $R_{1,t}$ and the long-run expectations for the short-term rate $R_{1,t}^e$, both expressed in deviations from long-run inflation expectations π_t^e, similarly to the right panel of Figure 1. The time series of $\bar{r}_t$ is very similar to that shown in Figure 1, albeit not identical at the beginning of the sample (recall we are now using a larger cross section of yields to pin down $\bar{r}_t$). In terms of the question this paper addresses, the decline in $\bar{r}_t$ from the late 1990s to the present is 1.27 percentage points, the same as estimated before, as shown in the second column of Table 1. The other two panels show that much of this decline is attributable to an increase in the convenience yield, rather than to a fall in $\bar{m}_t$. The middle panel shows $\overline{cy}_t$, and the spread between Baa securities and comparable Treasuries $R_t^{Baa} - R_{80,t}$. This spread has a clear upward trend, especially starting right before the turn of the century, which is picked up by the estimate

[23]The Baa yield is available from FRED (mnemonic, Baa). As described in Krishnamurthy and Vissing-Jorgensen (2012, pg. 262) "The Moody's Baa index is constructed from a sample of long-maturity ($\geq$ 20 years) industrial and utility bonds (industrial only from 2002 onward)." This series is available throughout the whole sample, but ends in 2016Q3.

[24]The initial condition $\overline{cy}_0$ is set at 1 using presample averages for the Baa/Treasury spread, and correspondingly set $\overline{m}_0$ to 1.5 $(\bar{r}_0 + \overline{cy}_0)$. The variance of the initial conditions is 1, as is the case for all other trends.

Figure 2: Trends and Observables, Convenience Yield Model



Note: The left panel shows $R_{1,t} - \pi^e_t$ (dotted blue line), and $R^e_{1,t} - \pi^e_t$ (blue dots), together with the trend $\bar{r}_t$. The middle panel shows the Baa/Treasury spread $R^{Baa}_t - R_{80,t}$ (dotted blue line), together with the trend $\overline{cy}_t$. The right panel shows $R_{1,t} - \pi^e_t - (R^{Baa}_t - R_{80,t})$ (dotted blue line), together with the trend $\bar{m}_t$. For each trend, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

of $\overline{cy}_t$. Table 1 shows that the convenience yield increases by 92 basis points from 1998Q1 to 2016Q4, with 95 percent credible intervals ranging from 49 to 135 basis points. The right panel shows the "real rate" $R_{1,t} - \pi^e_t$ minus the spread $R^{Baa}_t - R_{80,t}$. It shows that there is a fall in $\bar{m}_t$ (the median decline is about 35 basis points) but is imprecisely estimated, as the upper bound of the 68 percent credible interval is essentially 0. We should stress once again that the reader should not focus on the level of $\bar{m}_t$ and $\overline{cy}_t$, but on their changes. Our statement is not "Were it not for the convenience yield from liquidity/safety, the secular components of real rates would be x percent" but rather "Much of the decline in rates over the past twenty years is due to the convenience yield." This is because the *level* of the spread $R^{Baa}_t - R_{80,t}$ is affected by factors — mostly the probability of default — other than the convenience yield.[25]

Another perspective on what we find is that the secular decline in real rates for unsafe/illiquid securities has been much less pronounced, if it has taken place at all, than that for safe/liquid securities. As discussed in the introduction, the trend increase in the safety/liquidity convenience yield since the late 1990s is very much in line with the narrative put forth by Caballero (2010) and the "safe assets" literature more broadly. The Asian crisis first resulted in excess supply of savings which, being institutional (that is, intermediated via

[25]Figure A3 in the Appendix shows the remaining estimated trends ($\bar{\pi}_t$ and $\overline{tp}_t$) along with the relevant data. Figure A4 shows all the data y_t used in the estimation together with $\Lambda\bar{y}_t$ and $\tilde{y}_t$, the non-stationary and stationary components, respectively.

central banks), was naturally directed toward safe and liquid assets. The NADSAQ crash further rendered safe assets more attractive. The housing boom and the related creation of allegedly safe securities partly met this increased demand, but this suddenly came to a halt with the housing crisis and the Great Recession, which resulted in an additional increased demand, and reduced supply, of safe and liquid assets.

Trends in the Compensation for Safety and Liquidity.

Model Specification. Following Krishnamurthy and Vissing-Jorgensen (2012), we decompose the convenience yield $(1 + CY_t)$ into two parts, one due to liquidity $(1 + CY_t^l)$ and one to safety $(1 + CY_t^s)$, and write the Euler equation for a safe/liquid security as

$$E_t[(1 + r_t)(1 + CY_{t+1}^l)(1 + CY_{t+1}^s)M_{t+1}] = 1.$$

Under the assumption that the covariances between CY_t^l, CY_t^s, and M_t are stationary we obtain that:

$$\bar{r}_t = \overline{m}_t - \overline{cy}_t^l - \overline{cy}_t^s. \tag{17}$$

The distinction between liquidity and safety has two benefits. First, from an economic point of view, it allows us to disentangle the importance of the two components in explaining trends in r^*. In order to do so, of course, we have to be able to identify the two trends separately. Following once again Krishnamurthy and Vissing-Jorgensen (2012) we do so by bringing into the analysis the Aaa corporate yield, an index of securities that virtually never default, and hence carry as much of a safety discount as Treasuries, but are less liquid than Treasuries, and hence enjoy less of a liquidity premium. We therefore write:

$$R_t^{Aaa} = \overline{m}_t - \lambda_l^{Aaa}\overline{cy}_t^l - \overline{cy}_t^s + \bar{\pi}_t + \overline{tp}_t + \tilde{R}_t^{Baa}, \tag{18}$$
$$R_t^{Baa} = \overline{m}_t - \lambda_l^{Aaa}\overline{cy}_t^l - \lambda_s^{Baa}\overline{cy}_t^s + \bar{\pi}_t + \overline{tp}_t + \tilde{R}_t^{Baa}, \tag{19}$$

where $0 \leq \lambda_l^{Aaa} < 1$ and $0 \leq \lambda_s^{Baa} < 1$, indicating that both Aaa and Baa corporate bonds are less liquid than Treasuries (we assume that their degree of illiquidity is the same, hence $\lambda_l^{Baa} = \lambda_l^{Aaa}$), and that Baa corporate bonds are less safe than Treasuries. From equations (18), (19) and (12) it follows that

$$\bar{R}_t^{Aaa} - \bar{R}_{80,t} = (1 - \lambda_l^{Aaa})\overline{cy}_t^l,$$
$$\bar{R}_t^{Baa} - \bar{R}_t^{Aaa} = (1 - \lambda_s^{Baa})\overline{cy}_t^s.$$

As before, we will make the conservative assumptions that Baa bonds earn no safety and liquidity premium whatsoever, and that Aaa bonds are completely illiquid. These assumptions are conservative in the sense that they minimize time variation in the trends $\overline{cy}^l_t$ and $\overline{cy}^s_t$ given the observed trends in the spreads $\bar{R}^{Aaa}_t - \bar{R}_{80,t}$ and $\bar{R}^{Baa}_t - \bar{R}^{Aaa}_t$.

The system of equations given by (10)–(13) and (19)-(18) can be expressed as a VAR for $y_t = \left(\pi_t, \pi^e_t, R_{1,t}, R_{80,t}, R^e_{1,t}, R^{Aaa}_t, R^{Baa}_t\right)$ with common trends $\left(\bar{r}_t, \bar{\pi}_t, \overline{cy}^s_t, \overline{cy}^l_t, \overline{tp}_t\right)$.[26] We use exactly the same priors as described above, except that since we decompose the trend $\overline{cy}_t$ into two components, $\overline{cy}^s_t$ and $\overline{cy}^l_t$, we center the corresponding diagonal values of $\underline{\Sigma_\varepsilon}$ to a number that is 1/2 the value chosen for $\overline{cy}_t$ (we use 1/1600 as opposed to 1/800).[27] This obviously makes it harder to find a trend in these convenience yields.

Results. Figure 3 shows the trend $\bar{r}_t$ and its decomposition between trends in the convenience yield for safety and liquidity $\overline{cy}_t = \overline{cy}^l_t + \overline{cy}^s_t$ (we are actually plotting $-\overline{cy}_t$) and the stochastic discount factor $\bar{m}_t$. The estimates for $\bar{r}_t$ appear in all three panels, and the level of both $-\overline{cy}_t$ and $\bar{m}_t$ are normalized so that in 1998Q1 the three series coincide (at the median), so that the source of the post-1998 decline in $\bar{r}_t$ is more apparent. The estimates of $\bar{r}_t$ are virtually the same as those shown in Figures 2, and have $\bar{r}_t$ fall by 1.3 percentage points between 1998Q1 and 2016Q4 (see column 3 of Table 1). Again, this decline is precisely estimated. The middle panel shows that roughly one percentage point of this decline is attributable to an increase in the convenience yield. The converse of the convenience yield ($-\overline{cy}_t$) falls by one percent, and the decrease is very precisely estimated, with the 68 and 95 percent posterior coverage intervals ranging from -.75 to -1.18 percent and from -.53 to -1.40 percent, respectively. $\bar{m}_t$ also declines in the new century, by about 30 basis points, but its estimates are much more uncertain: the 68 percent intervals of the estimated fall in $\bar{m}_t$ range from -0.01 to -.65 percent.

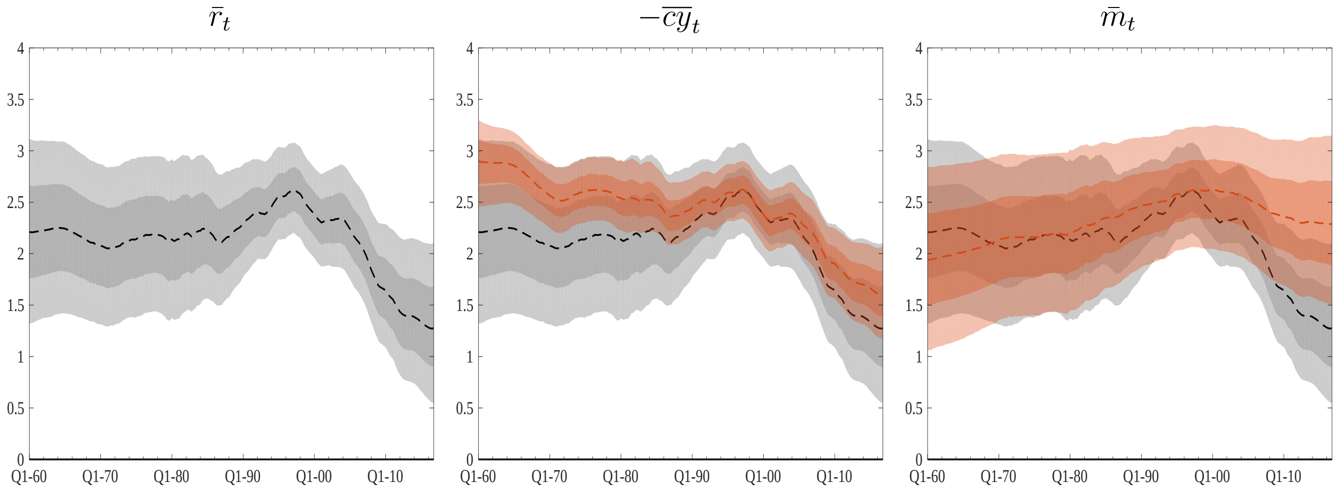
Figure 4 shows the estimated trends in the overall convenience yield $\overline{cy}_t$, and the convenience yields attributed to safety ($\overline{cy}^s_t$) and liquidity ($\overline{cy}^l_t$), along with the information that the model uses to extract these trends.[28] The left panel shows $\overline{cy}_t = \overline{cy}^s_t + \overline{cy}^l_t$, and the spread between Baa securities and Treasuries $R^{Baa}_t - R_{80,t}$. Again, in spite of the fact that

[26] The Aaa yield is also available from FRED (mnemonic, AAA) and has similar characteristics as the Baa index in terms of maturity. This series is available throughout the whole sample, but ends in 2016Q3.

[27] The initial conditions $\overline{cy}^s_0$ and $\overline{cy}^l_0$ are set at .75 and .25 using presample averages for the Baa/Aaa and the Aaa/Treasury spreads. The variance of the initial conditions is 1, as is the case for all other trends.

[28] Figure A5 in the Appendix shows the remaining estimated trends ($\bar{\pi}_t$, $\bar{r}_t$, $\bar{m}_t$, and $\overline{tp}_t$) along with the relevant data. Figure A7 shows all the data y_t used in the estimation together with $\Lambda\bar{y}_t$ and $\tilde{y}_t$, the nonstationary and stationary components, respectively. Figure A6 in the Appendix shows the prior and posterior distributions of the standard deviations of the shocks to the trend components—the diagonal elements of the matrix Σ_e.

Figure 3: $\bar{r}_t$, $\overline{cy}_t$, and $\bar{m}_t$



Note: The dashed black line shows the posterior median and the shaded gray areas show the 68 and 95 percent posterior coverage intervals for $\bar{r}_t$. The dashed orange line shows the posterior median and the shaded orange areas show the 68 and 95 percent posterior coverage intervals for $-\overline{cy}_t$ (middle panel) and $\bar{m}_t$ (right panel).

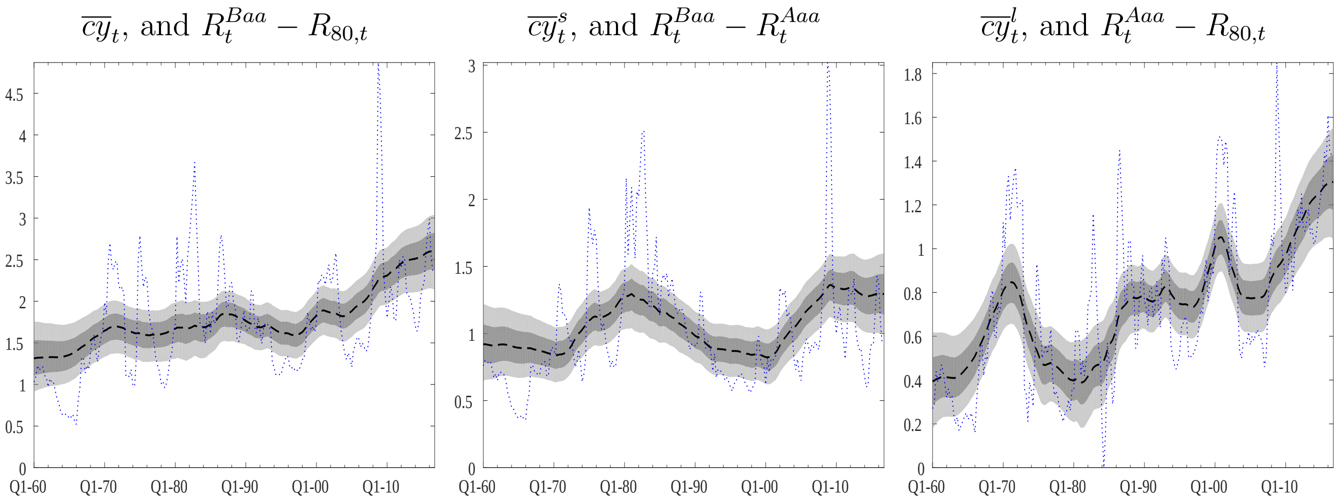
the trends $\overline{cy}_t^s$ and $\overline{cy}_t^l$ are now separately estimated, the inference for $\overline{cy}_t$ is broadly similar to that shown in Figure 2. The middle panel shows $\overline{cy}_t^s$ and the spread between Baa and Aaa bonds $R_t^{Baa} - R_t^{Aaa}$. The trend in this spread, according to the model, has less of a secular increase in the overall sample than the overall convenience yield. The trend in the safety premium increases in the 1970s, reaches a pick in the early eighties, declines progressively until the NASDAQ crash, and finally increases by a little less than 50 basis points until the end of the sample. The estimated increase in the safety convenience yield between 1998Q1 and 2016Q4 is 45 basis points, and is very significantly different from zero.

The right panel shows $\overline{cy}_t^l$, and the spread between Aaa securities and Treasuries $R_t^{Aaa} - R_{80,t}$. The trend $\overline{cy}_t^l$ has a more pronounced secular increase since the early 1980s.[29] From the perspective of the focus of the paper — the sources of the decline in real rates since the 1990s — the right panel shows an increase in $\overline{cy}_t^l$ by about 50 basis points since 1998 (see column 3 of Table 1).[30] Much of this increase occurred during and after the financial crisis. This is not surprising, because the liquidity shock in the aftermath of the Lehman

[29]While the transitory spikes in the convenience yield for liquidity are easily explained by financial events (e.g., the stock market crash of 1987, the burst of the 1990s stock market bubble and September 11, the Lehman crisis), this secular increase is for us not straightforward to explain, but we find it an interesting question for future research. One possibility is that it is related to the growth of the shadow banking system documented in Adrian and Shin (2009, 2010).

[30]Note that the high frequency spike in illiquidity occurred during the financial crisis does not seem to play an important role in the extraction of the trend; in other words, the increase in the compensation for liquidity appears to be mostly driven by the low frequency movements in the spreads.

Figure 4: Trends in Compensation for Safety and Liquidity, and Observables



Note: The left panel shows the Baa/Treasury spread $R_t^{Baa} - R_{80,t}$ (dotted blue line), together with the trend $\overline{cy}_t$. The middle panel shows the Baa/Aaa spread $R_t^{Baa} - R_t^{Aaa}$ (dotted blue line), together with the trend $\overline{cy}^s_t$. The right panel shows the Aaa/Treasury spread $R_t^{Aaa} - R_{80,t}$ (dotted blue line), together with the trend $\overline{cy}^l_t$. For each trend, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

crisis drastically curtailed the supply of liquid assets (as several asset classes became less liquid; see for instance Del Negro et al., 2017; Gorton and Metrick, 2012) and at the same time increased its demand. In addition, the regulatory changes after the crisis (see the liquidity requirements for financial institutions under Basel III; Basel Committee on Banking Supervision, 2013) also led to an increased demand for liquid assets, as well as a decline in the supply of liquid liabilities from the financial system. In conclusion, we find that the increase in the convenience yield since the late 1990s is roughly evenly split between compensation for safety and liquidity.

II.C The Role of Consumption

Model Specification. The VAR specifications that we have considered so far have all been agnostic on the fundamental determinants of the trends in the stochastic discount factor m_t. We chose this approach because there is no consensus in the literature on how to model this variable. Many asset pricing theories, however, connect the pricing kernel to some function of consumption growth. This list includes the consumption Euler equation that holds in the DSGE model of the next section. These theories, in fact, are the basis for the often discussed relationship between trends in rate of returns and in the growth rate of the economy (e.g. Laubach and Williams, 2003; Hamilton et al., 2015).

This section explores this relationship by including a measure of per capita consumption growth in the VAR. This model is an extended version of the baseline specification of Section II, in which $\bar{m}_t$ is decomposed into two factors. The first factor, denoted by $\bar{g}_t$, is common between the trends in m_t and in the growth rate of per capita consumption, which we call $\overline{\Delta c}_t$. Motivated by the fact that trends in m_t may in principle be driven by factors that are not associated with consumption, we also introduce a residual factor, $\bar{\beta}_t$, so that

$$\bar{m}_t = \bar{g}_t + \bar{\beta}_t. \tag{20}$$

In addition, we do not impose that $\bar{g}_t$ is the same as the trend in overall consumption growth, as would be the case in a textbook version of the Euler equation with log utility. Instead, we allow for another trend in consumption growth, or

$$\overline{\Delta c}_t = \bar{g}_t + \bar{\gamma}_t.$$

This specification admits the possibility that the relevant consumption pricing factor for interest rates is not aggregate consumption, but possibly a subset of consumption with a different trend from the aggregate. This would be the case, for instance, in a limited participation model in which only a subset of consumers have access to financial markets and the low frequency component of their consumption growth is different from that of non-participants (e.g., Vissing-Jorgensen, 2002). Given the steady growth in inequality over the last few decades, such a persistent divergence in the consumption prospects of richer asset holders and poorer households excluded from financial markets seems plausible.

In sum, we augment the system of equations given by (10)–(13) and (18)-(19) with an equation for consumption growth

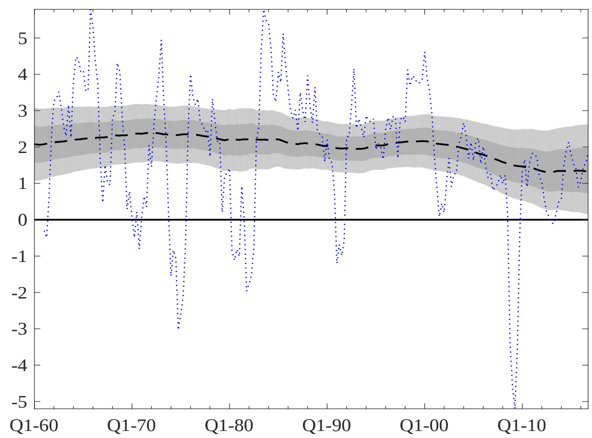
$$\Delta c_t = \bar{g}_t + \bar{\gamma}_t + \tilde{\Delta} c_t, \tag{21}$$

and set $\bar{m}_t = \bar{g}_t + \bar{\beta}_t$ in all the equations involving $\bar{m}_t$.[31] In terms of priors, we want to allow ample room for the trend in consumption growth $\bar{g}_t$ to account for the decline in $\bar{r}_t$. Therefore, we assume that its prior standard deviation is four times as large as that of $\overline{cy}_t^s$ and $\overline{cy}_t^l$, which implies a value of $1/400$ for the corresponding diagonal element of the matrix $\underline{\Sigma}_e$. We also assume the same prior for $\bar{\gamma}_t$, while the standard deviation of $\bar{\beta}_t$ is set to 1/8 of that of $\bar{g}_t$.[32] All other priors are the same as in the baseline model.

[31]We use the same measure of real per capita consumption as in the DSGE model, namely Personal Consumption Expenditures divided by the GDP deflator and by a smoothed version of population. See the DSGE data Appendix for more details. Consumption growth is quarterly annualized.

[32]The initial condition $\bar{\gamma}_0$ is calibrated by splitting in two the average growth rate of per capita consumption in the 1950s. The initial conditions $\bar{\gamma}_0$ and $\bar{\beta}_0$ are set to zero.

Figure 5: Consumption Growth and Its Trend



Note: The left panel shows Δc_t (4-quarter average; dotted blue line) and $\overline{\Delta c}_t = \bar{g}_t + \bar{\gamma}_t$. For the trend, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

Results. Figure 5 shows the 4-quarter average of the growth rate of per capita consumption together with its trend $\overline{\Delta c}_t = \bar{g}_t + \bar{\gamma}_t$. The figure shows that the estimated trend in consumption growth has fallen over the past twenty years. This decline has been notable, as shown in column 4 of Table 1. The median estimate is .80 percentage point, although it is imprecisely estimated. Table 1 also shows that the component attributable to $\bar{g}_t$, which is the part of the trend in consumption growth that affects the interest rate, is around 55 basis points at the posterior median, and it is also surrounded by significant uncertainty. Nonetheless, the estimated decline in $\bar{r}_t$ — 1.40 percent points — and the increase in the convenience yield — 0.78 percentage points — are close to the figures shown before, and are still precisely estimated.[33] In sum, the increase in the convenience yield still accounts for the majority of the overall secular trend decline in r_t.[34]

Results were very similar in a model in which we substituted the growth rate of consumption with that of labor productivity among the observables. The motivation for also

[33]Figure A8 in the Appendix shows the remaining estimated trends ($\bar{\pi}_t$, $\bar{r}_t$, $\bar{m}_t$, $\overline{tp}_t$, $\overline{cy}^s_t$, and $\overline{cy}^l_t$) along with the relevant data. Figure A9 shows all the data y_t used in the estimation together with $\Lambda\bar{y}_t$ and $\tilde{y}_t$, the non-stationary and stationary components, respectively.

[34]We also estimated a more restricted model with a common trend between aggregate consumption and the interest rate — that is, eliminating $\bar{\gamma}_t$. In that model, the trend in consumption moves much less, and the effects on $\bar{m}_t$ are smaller, suggesting that the restriction that all of the trend in consumption growth translates into secular changes in the discount factor is at odds with the data. Otherwise the results are quite similar to those just discussed. We also tried to estimate the intertemporal elasticity of substitution — that is, modifying (20) as $\bar{m}_t = \sigma^{-1}\bar{g}_t + \bar{\beta}_t$. This only resulted in more uncertain estimates of the decline in $\bar{m}_t$. This possibly reflects the well-know difficulties in pinning down the intertemporal elasticity of substitution.

experimenting with this specification comes from the neoclassical growth model, in which the interest rate, productivity growth and consumption growth are all tied together along the balanced growth path. Therefore, productivity growth provides an alternative source of information on the trend growth rate of the economy. The two trends might not coincide for several reasons, including persistent movements in the current account in an open economy, as well as trends in the labor force participation rate that drive a wedge between the growth rates of population (in the denominator of per capita consumption) and of hours worked (in the denominator of labor productivity). Both these phenomena have been observed in the United States since the 1990s and they are often mentioned as possible secular drivers of the decline in interest rates that has occurred over the same period. As shown in column 4 of Table A1 in the Appendix, the estimated trend decline in the real interest rate in this model is centered around 1.64 percentage points, the highest value of all the models we estimated. Of this decline, 89 basis points are accounted for by the increase in the convenience yield, and another 68 by the decline in the trend growth rate of productivity. As before, the former contribution is very tightly estimated, while the latter is quite uncertain.

In summary, the results of this augmented model corroborate our conclusion that the increase in the convenience yield has been a crucial factor in the secular decline of Treasury yields. In addition, the model suggests that the concomitant fall in the trend growth rate of economic activity — measured either in the form of consumption or of labor productivity — also played a relevant role, although this conclusion is subject to significant uncertainty.

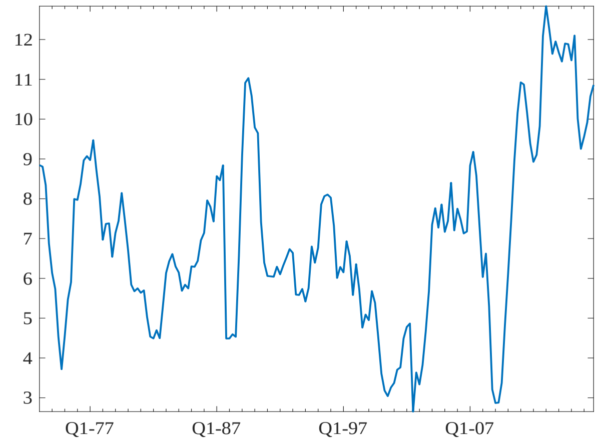
II.D Robustness

This section considers some variants to our benchmark specification — the model with convenience from both safety and liquidity.

Accounting for Trends in Corporate Default.
Figure 6 shows the median distance to default (DD) in the non-financial corporate sector used in Gilchrist and Zakrajsek (2012) — the higher DD the *lower* the default probability.[35] This measure has a clear upward trend since the late 1990s, implying that there is, if anything, a secular *decrease* in the default probability, which is why we did not include in the benchmark models. For completeness, here we estimate a model where we extract trends in the distance

[35]These are the data shown in Figure 2 of Gilchrist and Zakrajsek (2012). We are very grateful to Egon Zakrajsek for providing us with updated estimates.

Figure 6: Distance to Default



Note: The figure shows the median distance to default (DD) in the non-financial corporate sector used in Gilchrist and Zakrajsek (2012).

to default $\bar{D}_t$ from the data shown in Figure 6, and include them in the equation for the Baa yield, which becomes

$$R_t^{Baa} = \overline{m}_t - \gamma_l^d \bar{D}_t + \bar{\pi}_t + \overline{tp}_t + \tilde{R}_t^{Baa}, \tag{22}$$

where γ^d is estimated (we use an exponential distribution with mean 1/10 as the prior).[36] Table A1 in the Appendix shows that the estimated increase in the convenience yield is stronger than reported in Table 1, about 1.4 percentage points, and remains precisely estimated.

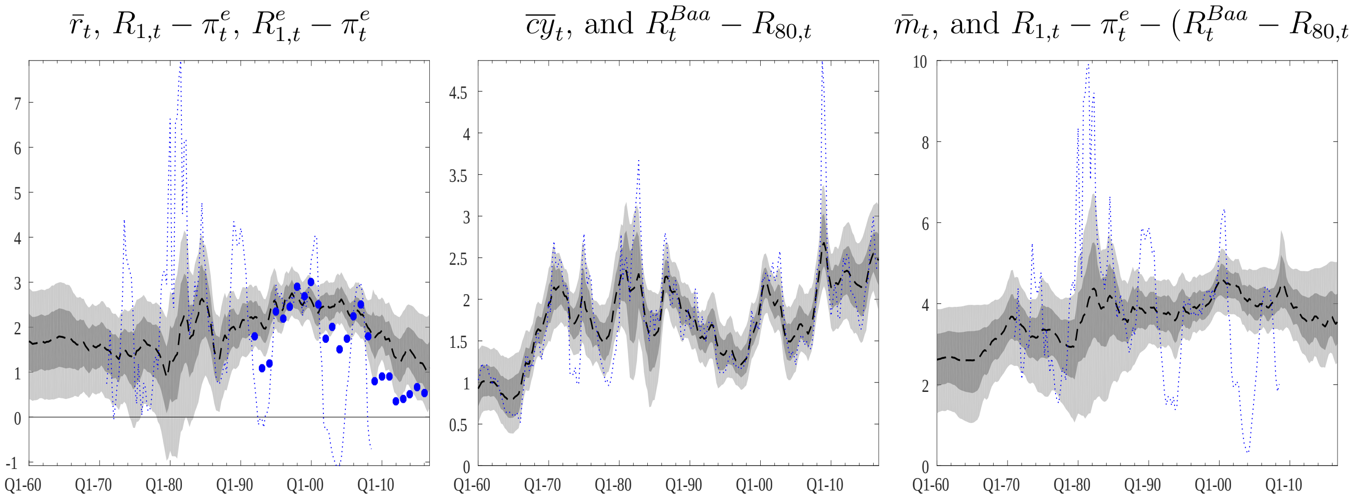
Loose Prior on the Trend.

Our main results is that trends in the convenience yield account for a large chunk of the decline in $\bar{r}_t$, while the effect of changes in the trend for the discount factor $\bar{m}_t$ is not as large, and is quite imprecisely estimated. One natural objection to our argument is that our prior on the standard deviation of the trends innovation is too conservative and too tight. If we had a looser prior, we may possibly find more evidence of a trend in $\bar{r}_t$ and consequently $\bar{m}_t$.

Figure 7 dispels this notion. This figure shows the outcome of reestimating the model of

[36]The prior mean for γ^d is loosely based on the results of the panel regressions reported in Gilchrist and Zakrajsek (2012), who estimate the effect of the distance to default on corporate spreads. The prior on the variance of the trend $\bar{D}_t$ (that is, the corresponding diagonal element of the matrix $\underline{\Sigma}_e$) is 1/400, which is the same prior we used for $\bar{r}_t$ in the first model. The exponential distribution with parameter $\underline{\gamma}^{-1}$ is $p(\gamma; \underline{\gamma}^{-1}) = \underline{\gamma}^{-1} \exp(-\underline{\gamma}^{-1}\gamma)\mathcal{I}\{\gamma \geq 0\}$, where $\mathcal{I}\{.\}$ is an indicator function.

Figure 7: Trends and Observables, Loose Prior on the Trend



Note: The left panel shows $R_{1,t} - \pi^e_t$ (dotted blue line), and $R^e_{1,t} - \pi^e_t$ (blue dots), together with the trend $\bar{r}_t$. The middle panel shows the Baa/Treasury spread $R^{Baa}_t - R_{80,t}$ (dotted blue line), together with the trend $\overline{cy}_t$. The right panel shows$R_{1,t} - \pi^e_t - (R^{Baa}_t - R_{80,t})$ (dotted blue line), together with the trend $\bar{m}_t$. For each trend, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

Section II.B loosening the prior on variance-covariance matrix as much as possible (we use 8 degrees of freedom, barely enough so that the prior has a well defined mean, as opposed to the 100 used in the baseline specification). The result is that the trend is no longer a trend in the sense that it also captures high frequency fluctuations in the observables — which is why we think the original tight prior is appropriate. But the broad contours of the results are the same: $\overline{cy}_t$ trends upward while $\bar{m}_t$ does not move much.[37]

Inflation Affecting the Nominal Term Premium.

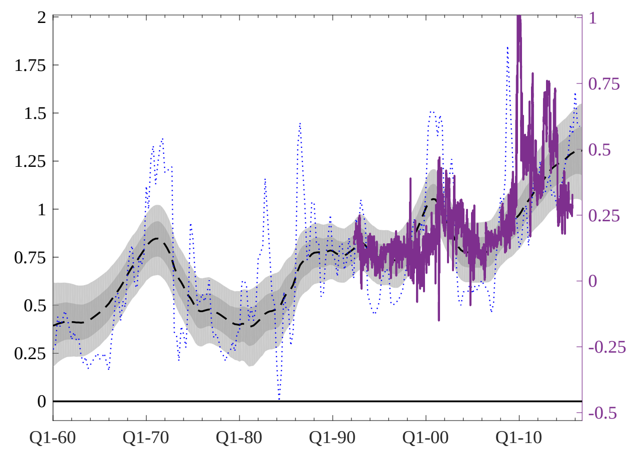
As anticipated, we also allow for the possibility that trends in inflation affect the nominal term premium — that is, we model the term premium as the sum of an exogenous component $\overline{tp}_t$ and a component $\gamma^{tp}\bar{\pi}_t$, where γ^{tp} is estimated (we use an exponential distribution with mean 1/10 as the prior). This specification is motivated by the work of Wright (2011), who found a positive correlation between the level of the nominal term premium and the volatility of inflation. Here, we use the level of inflation as a proxy for the latter. We therefore replace $\overline{tp}_t$ with $\overline{tp}_t + \gamma^{tp}\bar{\pi}_t$ in equations (12), (18), and (19). The results under this specification are nearly identical to those shown in Section II.B and therefore are relegated to the online appendix (Figure A11; Figure A10 shows the posterior distribution of γ^{tp}).

[37] We obtain similar results when we instead quadruple the variance of the trend innovations, but leave intact the number of degrees of freedom.

Callability.

Many corporate bonds are callable, while Treasuries are not (at least since 1985). One may wonder whether secular changes in the value of the call option drive secular changes in the spread. Fortunately, there are other spreads mainly reflecting liquidity other than the Aaa/Treasuries spread. The Refcorp/Treasury spread is one of them. This spread, according to Longstaff et al. (2004), is mostly (if not entirely) due to liquidity as Refcorp bonds are effectively guaranteed by the U.S. government, are subject to the same taxation and, importantly, to the best of our knowledge they are not callable.[38]

Figure 8: Trends in the Liquidity Convenience Yield and the Refcorp Treasury Spread



Note: The figure shows the estimated trend in the liquidity convenience yield $\overline{cy}^l_t$ (the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals; left axis), the Aaa/Treasury spread $R^{Aaa}_t - R_{80,t}$ (dotted blue line; left axis) and the Refcorp/Treasury spread (solid purple line; right axis). See footnote 38 for a description of how this spread is constructed.

Figure 8 plots the estimated trend in the liquidity convenience yield $\overline{cy}^l_t$ shown in the right panel of Figure 4 together with daily data on the Refcorp/Treasury spread collected by Del Negro et al. (2017) from 4/16/1991 to 9/06/2014. The figure shows that the trend in liquidity estimated using the Aaa/Treasury spread matches very well the trends in the Refcorp/Treasury spread, whenever this is available. In addition to suggesting that callability

[38] Refcorp bonds differ from most other agency bonds in that their principal is fully collateralized by Treasury bonds and full payment of coupons is guaranteed by the Treasury under the provisions of the Financial Institutions Reform, Recovery, and Enforcement Act of 1989. Longstaff et al. (2004) does not mention callability as a feature of these bonds. Lehman Brother's "Guide to Agency and Government-Related Securities" does not mention callability in reference to Refcorp bonds, while it discusses callability for other agency securities. As in Longstaff et al. (2004), we measure the spread by taking the differences between the constant maturity 10-year points on the Bloomberg fair value curves for Refcorp and Treasury zero-coupon bonds. The Bloomberg mnemonics are 'C091[X]Y Index' and 'C079[X]Y Index', respectively, where [X] represents the maturity.

is not the driving force behind secular movements in the Aaa/Treasury spread, Figure 8 provides important external validation to our analysis.[39]

III The Natural Rate of Interest in DSGE Models

Our analysis so far focused on long-run trends in r_t^* and on the factors that drive them. However, the natural rate of interest also fluctuates over the business cycle. This section presents estimates of r_t^* based on an empirical medium-scale Dynamic Stochastic General Equilibrium (DSGE) model that features nominal price and wage rigidities, as well as a host of real and financial frictions. Within this New Keynesian environment, we define r_t^* as the real interest rate that would prevail in equilibrium in the absence of sticky prices and wages, as discussed in the introduction.[40]

This particular notion of r_t^* is a useful tool in macroeconomic and monetary analysis for several related reasons. First, the natural rate does not depend on monetary policy. In the equilibrium without nominal rigidities, monetary policy is neutral, in the sense that it does not affect any real variable, including the real interest rate.[41] Therefore, the natural rate answers the question: what would the real interest rate be, "without" monetary policy?

Second, the gap between actual interest rates and their natural level is a more appropriate measure of the impetus (or restraint) imparted by monetary policy to aggregate demand than the level of the policy rate itself, as further discussed in Section III.B. In Wicksell's own words, "it is not a high or low rate of interest in the absolute sense which must be

[39] An alternative approach to addressing the issue of callability is the one taken by Gilchrist and Zakrajsek (2012) in the construction of the excess bond premium, who use a panel regression where they regress individual corporate spreads on individual measures of default probability as well as variables that likely capture the value of the call option. While this micro-level approach has several advantages relative to our aggregate approach — especially in terms of the exact computation of the spreads in terms of maturity and the removal of default probability — it has one important drawback from our perspective: in order to remove the call option, the spreads are regressed on the level of the interest rate, among other variables, thereby removing the very trends we are interested in.

[40] Neiss and Nelson (2003) were the first to evaluate the properties of the natural rate in a calibrated DSGE model. Edge et al. (2008), Justiniano and Primiceri (2010), Barsky et al. (2014), and Curdia et al. (2015) do so in estimated models.

[41] This statement holds in the model proposed below, but it might need to be qualified in other environments. Depending on the exact specification of the financial and real frictions, monetary policy might affect real variables even in the absence of nominal rigidities. However, these effects tend to be quantitatively limited in empirical models.

regarded as influencing the demand for raw materials, labour, and land or other productive resources, and so indirectly as determining the movement of prices. The causality factor is the current rate of interest on loans as compared to [the natural rate]."

This property of the natural rate does not imply that closing the interest rate gap is optimal. This is the case only in extremely simple models that do not feature a trade-off between real and nominal stabilization. In these models, closing the interest rate gap stabilizes the output gap, and at the same time inflation. In larger, more realistic DSGE models, this "divine coincidence" (Blanchard and Galí, 2007) between price stability and full employment does not hold. Nonetheless, a monetary policy strategy in which the real policy rate tracks the natural rate generally promotes stable inflation and economic activity even in those models, providing a more explicitly normative rationale for using estimates of the natural rate as an input in monetary policy making.[42]

The DSGE perspective on r_t^* described above is complementary to that explored in the first part of the paper. While the VAR only provides an estimate of the low frequency component of r_t^*, a fully specified DSGE model gives us the entire time-path of r_t^*. This is especially relevant in a policy context, This is especially relevant in a policy context, where estimates of r_t^* might be used to inform decisions on the appropriate level of the policy rate.

Of course, the flip side of this more comprehensive view of the movements in r_t^* provided by the DSGE approach is that inference is conditional on the exact structure of the DSGE model. As such, it is more likely to be affected by model misspecification than the VAR estimates. However, one of the key findings of our DSGE exercise is that its conclusions in terms of low frequency fluctuations in r_t^* agree with those of the reduced form exercise. The fact that the DSGE model can capture these low frequency movements is a surprising result in itself. Moreover, the two approaches also agree that the rise in the convenience yield is the source of this secular decline in r_t^*, as illustrated in Section III.B. Also, we find that the natural rate plunged to its historical lows during the Great Recession, making the lower bound on nominal interest rates bind, and hence severely impairing the ability of the Federal Reserve to stabilize the economy through its conventional policy tool, as we discuss further in Section III.B.[43]

[42] For instance, Justiniano et al. (2013) find that the there is a minimal trade-off between nominal and real stabilization in an estimated model similar to the one used here, approximating the divine coincidence that holds exactly in much simpler environments.

[43] We should note that studies based on a variety of empirical DSGE models tend to deliver a fairly consistent view of the business cycle fluctuations in r_t^*, even if their estimates do not coincide at any given point in time (see for instance Figure 1 in Yellen (2015)).

III.A DSGE model

The DSGE model considered here is a version of the FRBNY DSGE model described in Del Negro et al. (2015). It builds on the model of Christiano et al. (2005) and Smets and Wouters (2007), and expands it with various features, most notably financial frictions similarly to Bernanke et al. (1999b) and Christiano et al. (2014). At the core of the model lies a frictionless neoclassical structure in which monetary policy has no effects. This neoclassical core is augmented with frictions such as stickiness of nominal prices and wages (i.e., nominal frictions), various real frictions (such as adjustment costs of capital) and financial frictions that interfere with the flow of funds from savers to borrowers. In addition, the model includes several structural shocks which are the ultimate causes of economic fluctuations, such as shocks to productivity, the marginal efficiency of investment (e.g., Greenwood et al. (1998), Justiniano et al. (2010)), and price and wage markup shocks. We also allow for shocks to liquidity, safety, credit premia in line with the empirical model of Section II, as well as anticipated policy shocks as in Laseen and Svensson (2011) to account for the zero lower bound on nominal interest rates and forward guidance in monetary policy. The equilibrium conditions are approximated around the non-stochastic steady state, and we express all variables in (log) deviations from that steady state. A complete model specification is detailed in Appendix B. Here, we focus the discussion on the parts of the model most closely related to the natural rate of interest and its drivers.

We include two types of wedges between the Treasury rate and the rate at which corporations finance their investment.[44] The first wedge arises from financial frictions à la Bernanke et al. (1999a), which we model building on the work of Christiano et al. (2003), De Graeve (2008), and Christiano et al. (2014).[45] The second wedge arises exogenously and captures the convenience yield — the fact that investors prefer to hold Treasuries over alternative assets.[46] These wedges affect the spread between corporate bond yields and Treasury yields

[44]Wu and Zhang (2016) also model the spread between private and government bonds in the context of a New Keynesian model.

[45]We assume that banks collect deposits from households and lend to entrepreneurs who use these funds as well as their own wealth to acquire physical capital, which is rented to intermediate goods producers. Entrepreneurs are subject to idiosyncratic disturbances that affect their ability to manage capital. Their revenue may thus turn out to be too low to pay back the loans received by the banks. The banks therefore protect themselves against default risk by pooling all loans and charging a spread over the deposit rate.

[46]We model the convenience yield as a simple transaction cost/subsidy, following Smets and Wouters (2007). Fisher (2015), following Krishnamurthy and Vissing-Jorgensen (2012), models it by including Treasury bonds in households' preference. Our transaction "subsidy" can be recast as a linear utility benefit

according to the (linearized) equation

$$\mathbb{E}_t\left[\tilde{R}^k_{t+1} - R_t\right] = cy_t + \zeta_{sp,b}\left(q^k_t + \bar{k}_t - n_t\right) + \tilde{\sigma}_{\omega,t}, \tag{23}$$

where $\mathbb{E}_t\tilde{R}^k_{t+1}$ is the entrepreneurs' expected (short-term) return on capital, R_t is the 3-month Treasury bill rate, cy_t is the wedge arising from the convenience yield, the term $q^k_t + \bar{k}_t - n_t$ captures the entrepreneurs' leverage (i.e. the value of capital $q^k_t + \bar{k}_t$ relative to net worth n_t), and $\tilde{\sigma}_{\omega,t}$ consists of Christiano et al. (2014)'s "risk shocks" — that is, mean-preserving changes in the cross-sectional dispersion of entrepreneurial ability.

Two comments are in order regarding the convenience yield. First, it is assumed to be exogenous, unlike in the models of Kurlat (2013), Bigio (2015), or Del Negro et al. (2017). Although this is a theoretical limitation of our approach, it is functional to our empirical aim to use the DSGE model to map the effects of changes in cy_t on the macroeconomy, and on r^*_t in particular. Second, unlike in the papers just mentioned, in our model's counterfactual economy without nominal rigidities changes in cy_t affect the real rate on Treasuries, but have no effect on allocations. This "neutrality" of convenience yield shocks in the flexible price economy implies that in the actual economy, monetary policy could in principle completely isolate the economy from fluctuations in cy_t by adjusting the policy rate appropriately. This is a strong assumption, but one with some support: Del Negro et al. (2017) find that indeed monetary policy can undo most of these effects.

We assume that cy_t contains both a liquidity component cy^l_t and a safety component cy^s_t

$$cy_t = cy^l_t + cy^s_t, \tag{24}$$

which we identify through the same spreads used in Section II.B—the spreads between Aaa and Baa corporate bonds and 20-year Treasuries. The former, which we assume mainly reflects liquidity, is modeled as

$$\text{Aaa} - \text{20-year Treasury Spread} = cy^l_* + \mathbb{E}_t\left[\frac{1}{80}\sum_{j=0}^{79} cy^l_{t+j}\right] + e^{Aaa}_t,$$

while the latter, which reflects both liquidity and safety, as well as the actual probability of default, is modeled as

$$\text{Baa} - \text{20-year Treasury Spread} = cy^l_* + cy^s_* + SP_* + \mathbb{E}_t\left[\frac{1}{80}\sum_{j=0}^{79} \tilde{R}^k_{t+j+1} - R_{t+j}\right] + e^{Baa}_t,$$

from holding Treasuries, as in Anzoategui et al. (2015).

where the terms $\mathbb{E}_t\left[\tilde{R}^k_{t+j+1} - R_{t+j}\right]$ include all the components included in expression (23).

These equations highlight that measured spreads are between long-term yields. Therefore, in the model, they capture expectations of future convenience yields (and other sources of spreads) over the maturity of the bonds. We set the steady-state premia for liquidity and safety, cy^l_* and cy^s_*, to the values found in Krishnamurthy and Vissing-Jorgensen (2012) (46 and 27 basis points, respectively), and let e^{Aaa}_t and e^{Baa}_t capture measurement errors or other possible discrepancies between data and the model-implied concepts.[47]

Finally, in parallel with Section II.B, we allow for both (near) permanent and transitory components in safety and liquidity. The (near) permanent components have an an autocorrelation fixed at .99, with the same tight prior on the standard deviation of the shocks used in their VAR counterpart. The permanent components capture secular movements in safety and liquidity similar to those highlighted in the VAR, while the transitory components capture shocks such as those that hit the economy after the Lehman crisis.[48]

We conclude the model's description by returning to the Euler equation mentioned in the introduction. In the DSGE model, it takes the log-linearized form

$$\begin{aligned} c_t = &-\frac{1-\bar{h}}{\sigma_c(1+\bar{h})}\left(R_t - \mathbb{E}_t[\pi_{t+1}] + cy_t\right) + \frac{\bar{h}}{1+\bar{h}}\left(c_{t-1} - z_t\right) \\ &+ \frac{1}{1+\bar{h}}\mathbb{E}_t\left[c_{t+1} + z_{t+1}\right] + \frac{(\sigma_c - 1)}{\sigma_c(1+\bar{h})}\frac{w_* L_*}{c_*}\left(L_t - \mathbb{E}_t[L_{t+1}]\right), \end{aligned} \tag{25}$$

where c_t is consumption, L_t denotes hours worked (utility is non-separable in consumption and leisure), π_t is inflation, and z_t is productivity growth.[49] As in equation (1), this expression contains the convenience yield: as households' preferences for holding Treasuries increases and cy_t rises, the real rate should drop, holding everything else constant.

The model is estimated via Bayesian methods using numerous time series over the 1960Q1-2016Q3 period. In addition to spreads, these time series are real output growth (including both GDP and GDI measures), consumption growth, investment growth, real wage growth, hours worked, inflation (measured by both core PCE and GDP deflators), the

[47] We fix cy^l_* and cy^s_* as they are not clearly identified from the initial conditions on the exogenous processes. We also estimated versions of the model where we estimate the coefficients cy^l_* and cy^s_*, and where we do not have measurement error for the spreads, and found the results to be very similar to those shown below.

[48] The transitory component of the safety convenience yield might capture some of the changes in credit market sentiment emphasized by López-Salido et al. (2016).

[49] The parameter σ_c captures the degree of relative risk aversion while $\bar{h} \equiv he^{-\gamma}$ depends on the degree of habit persistence in consumption, h, and steady-state growth, γ.

federal funds rate, the ten-year Treasury yield, and Fernald's measure of TFP (constructed as in Basu et al., 2006).[50] We should stress that, differently from Smets and Wouters (2007), we allow for both stationary productivity shocks and very persistent shocks to the growth rate of productivity, in the attempt of capturing secular trends in latter. Finally, we also use survey-based long-run inflation expectations to capture information about the public's perception of the Federal Reserve's inflation objective, market data on expectations of future federal funds rates up to six quarters ahead to incorporate the effects of forward guidance on the policy rate. Appendix B provides more details on data construction and on the prior and posterior distributions for all parameters.

III.B DSGE Estimates of r^*

Long-term forecasts of the natural rate.
We start our discussion of the DSGE estimates of the natural rate by focusing on its persistent component, since this is the dimension in which the VAR and DSGE approaches are most directly comparable. Remarkably, the two models provide a very similar characterization of this component of interest rates, both in terms of their time-series behavior, as well as with regard to their fundamental drivers. This consistency between the two models is all the more notable given their very different characteristics in terms of their theoretical assumptions as well as of the data used in their estimation.

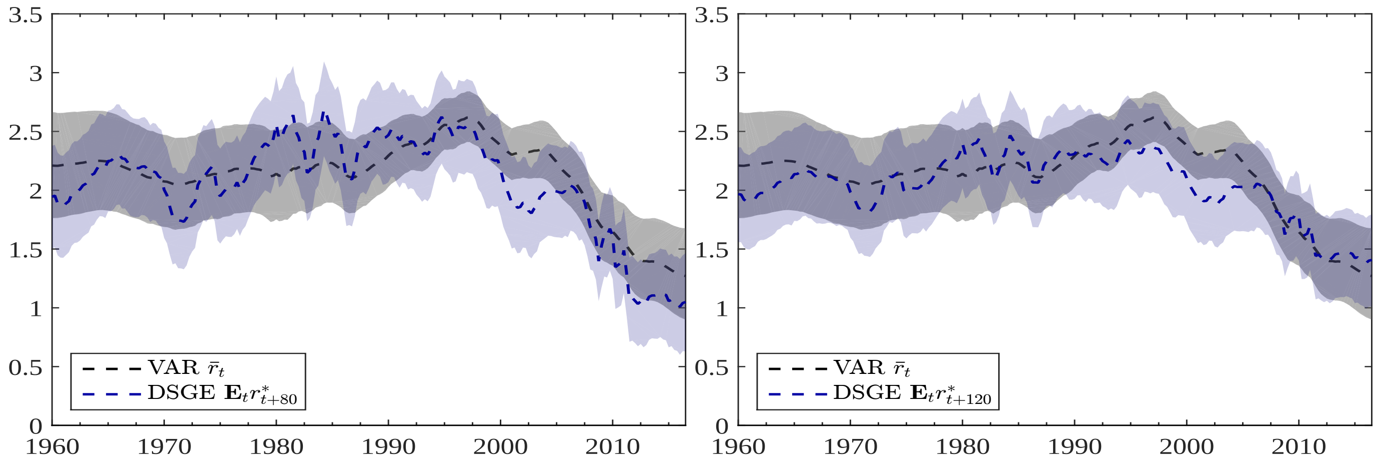
As a way of isolating persistent movements in real rates, Figure 9 compares forecasts of the short-term natural rate at the 20- and 30-year horizons both for the DSGE model and the VAR. We refer to these forecasts as (implied) forward rates. The key result highlighted by this graph is that forecasts of the real interest rate at long horizons behave very similarly in the two models.[51] This is true whether we use forecasts of either natural or actual rates

[50] We assume that some of the observables equal the model implied value plus an AR(1) exogenous process that can be thought of capturing either measurement error or some other unmodeled source of discrepancy between the model and the data as in Boivin and Giannoni (2006). For instance, these processes capture discrepancies between the noisy measures of output (real GDP and real GDI) and the corresponding model concept, and the gaps between the measures of inflation (based on the core PCE deflator and the GDP deflator) and inflation in the model. Instead, for the 10-year Treasury bond yield, such a process represents fluctuations in bond yields above and beyond those due to expectations of future short-term rates, such as movements in the term premium.

[51] Comin and Gertler (2006) refer to fluctuations over these horizons as medium-term cycles, in contrast to business cycles that take place at frequencies of 2 to 8 years. Given our focus on interest rates, we stick to the "long" horizon characterization, since this is the adjective most commonly used to define the yield on

Figure 9: Forward Natural Rates $\mathbb{E}_t r^*_{t+h}$ in the DSGE vs $\bar{r}_t$

20 years ($h = 80$) 30 years ($h = 120$)

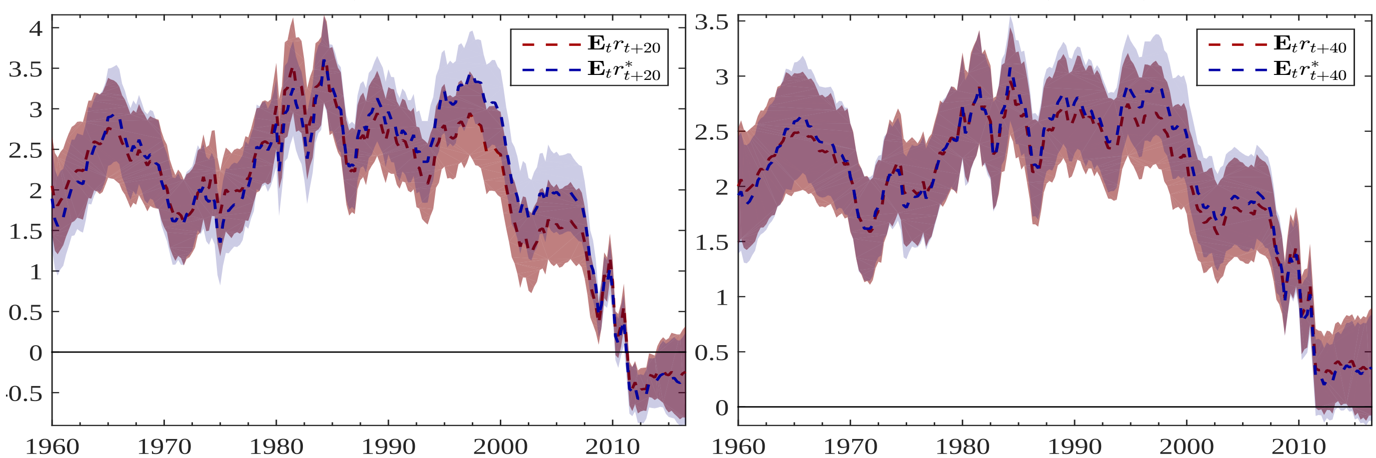


Note: The dashed black lines show the posterior medians and the shaded gray areas show the 68 percent posterior coverage intervals for the VAR estimates of $\bar{r}_t$. The dashed blue lines show the posterior median and the shaded blue areas show the 68 percent posterior coverage intervals for the DSGE estimates of $E_t r^*_{t+h}$.

in the DSGE, since the two are essentially identical starting at horizons of around 10 years. This similarity is illustrated in Figure 10, which compares 10- and 5-year forecasts of the two rates implied by the DSGE model. Actual and natural rates are quite close at the shorter horizon, but they do diverge at times by as much as about 50 basis points. However, this distance shrinks to just a few basis points at the 10 year horizon.

Figure 10: Forward Natural Rates $\mathbb{E}_t r^*_{t+h}$ and Forward Actual Rates $\mathbb{E}_t r_{t+h}$

5 years ($h = 20$) 10 years ($h = 40$)



Note: The dashed blue lines show the posterior median and the shaded blue areas show the 68 percent posterior coverage intervals for the DSGE estimates of $\mathbb{E}_t r^*_{t+h}$. The dashed red lines show the posterior median and the shaded red areas show the 68 percent posterior coverage intervals for the DSGE estimates of $\mathbb{E}_t r_{t+h}$.

As we discussed in Section II, the VAR provides useful information on the persistent

bonds with maturities in this range.

component of the natural rate of interest only if the gap between the actual and the natural economy is less persistent than the natural variables themselves. The convergence between projections of actual and natural rates in the DSGE model, at horizons at which both rates are still expected to display significant variation, is an important piece of evidence in favor of the key assumption needed for the VAR to be informative on the natural rate.

The fact that the DSGE model projects meaningful fluctuations in the interest rate at very long horizons, and even more that these fluctuations resemble those identified by the VAR, is a surprising finding. The (transformed) DSGE model is stationary around its steady state. Therefore, its infinite horizon forecasts of the interest rate are constant, unlike those of the VAR that are affected by its permanent shocks.[52] Yet, we find that the estimated model describes the trend in the real interest rate as well as the VAR, even if it has no power at exactly zero frequency.[53]

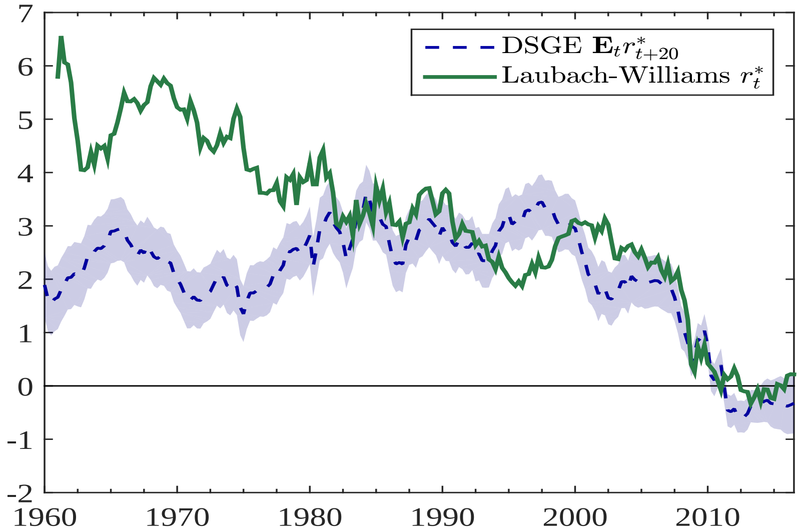
The flexibility of our estimated DSGE model as a tool to characterize the persistent component of real interest rates allows us to address an open question in the literature, namely how to integrate "longer-run" estimates of the natural rate, such as those provided by Laubach and Williams (2016) and our VAR, and "shorter-run" estimates derived from DSGE models. The presumption so far has been that the two types of estimates are mostly complementary, since longer-run approaches focus exclusively on permanent movements in the natural rate, while shorter-run approaches assume that the natural rate is stationary (see for instance the discussion on this point in Laubach and Williams (2016), especially Section 6). In contrast, our results suggest that DSGE models can provide a more comprehensive view of the fluctuations in the natural rate across frequencies than has generally been assumed until now, encompassing both "longer-run" and "shorter-run" measures of the natural rate.

As a further illustration of this point, Figure 11 shows that LW's estimates of the natural rate co-move quite closely with the 5-year forward natural rate derived from our DSGE model, at least starting in the early 1980s. This similarity with a relatively short-horizon

[52] Along the model's balanced growth path, the log levels of output, consumption and investment share a unit root that they inherit from productivity. As a result, their (log) ratios are stationary, and so are all the other variables, including interest rates.

[53] The ability of a stationary DSGE model to approximate the low frequency behavior implied by the VAR is related to the approach of Stock and Watson (1998) and Stock and Watson (2007). They characterize the persistent component of what look like stationary variables, such as GDP growth and inflation, through unit root processes with a "small" variance. Our results suggest a similarly blurred line between the stationary vs unit root characterization of interest rates provided by the DSGE and the VAR.

Figure 11: Comparison between 5-Year Forward Natural Rate $\mathbb{E}_t r^*_{t+20}$ in the DSGE and Laubach-Williams Estimate of r^*_t

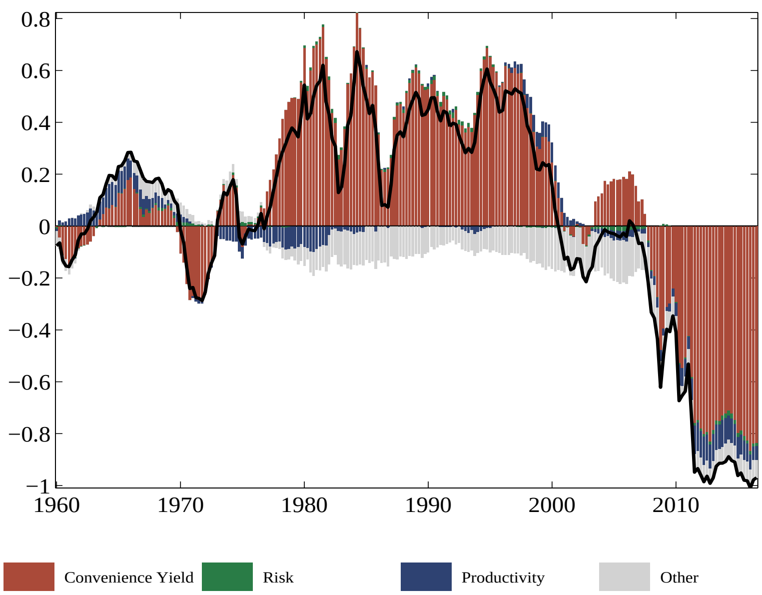


Note: The figure shows the 5-year forward real natural rate (i.e. the quarterly real natural rate expected to prevail 5 years in the future) from the DSGE model (blue dashed line), and Laubach and Williams' one-sided estimate of r^* (green line). The blue shaded area represents the 68 percent posterior coverage interval for the former.

forward rate suggests that LW's model includes a fair amount of transitory variation in its estimates of the natural rate, even if it follows an I(1) process. This result therefore confirms the blurry line between short- and long-run estimates of the natural rate. Before the early 1980s, though, the two models disagree strongly. LW estimate real natural rates to be as high as 6% in the 1960s, while the DSGE model sees them fluctuating around levels very similar to those that prevail in the subsequent decades. This stationarity of the DSGE estimates in the first twenty years of the sample is quite consistent with the trend from the VAR, which is essentially flat through the 1990s.

The consistency of the long-horizon forecasts of the real interest rate implied by the VAR and DSGE models strengthens our substantive conclusions, especially given the significant differences between the two empirical approaches. Next, we show that the two models also agree qualitatively on the main sources of persistent fluctuations in the natural rate. This result is illustrated in Figure 12, which shows the 20-year forward real natural rate in deviations from its steady state (solid black line) and the combined contribution of liquidity and safety shocks (in red) — the very shocks driving the Aaa and Baa corporate spreads. Liquidity and safety, which together represent the convenience yield, account for almost all the low frequency movements in the natural rate. This result is consistent with that obtained from the VAR as it attributes a large portion of the persistent movements in the natural rate to shifts in the convenience yield (see Figure 4). Instead, productivity shocks (in blue) are responsible for a much smaller fraction of the fluctuations, and risk shocks $\tilde{\sigma}_{\omega,t}$ (in green) have barely a noticeable impact on the 20-year forward natural rate. The remaining shocks (in grey) include aggregate demand shocks, which capture exogenous movements in government expenditures and foreign demand, as well as shocks to the marginal efficiency of investment.

Figure 12: 20-Year Forward Natural Rate and its Drivers



Note: The figure shows the 20-year forward real natural rate (i.e. the quarterly real natural rate expected to prevail 20 years into the future) implied by the DSGE model (black solid line) in deviations from the steady state. It also shows the contribution of shocks to the convenience yield (in red), risk shocks $\tilde{\sigma}_{\omega,t}$ (in green), productivity shocks (in blue), and other shocks (in grey) in causing fluctuations in the 20-year forward natural rate.

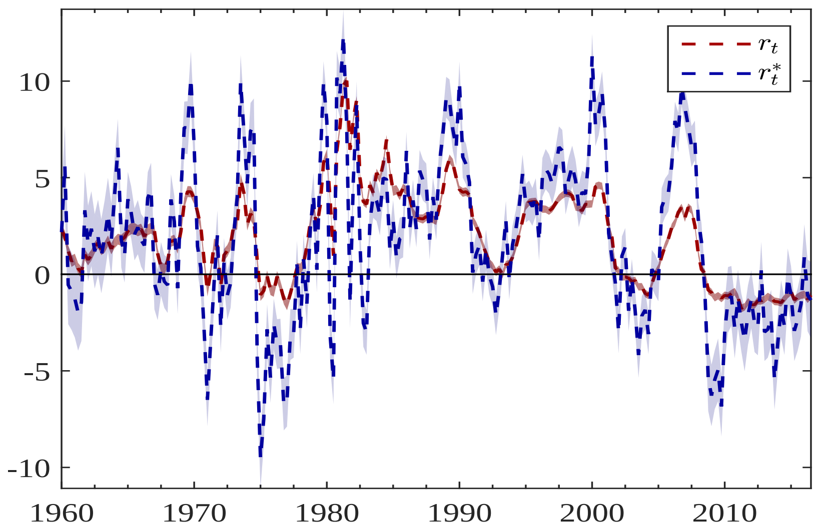
This section established that the view of persistent fluctuations in the natural rate of interest provided by the DSGE model is surprisingly consistent with that gleaned through the lens of the VAR. With this reassuring consistency in hand, we proceed with the derivation of the entire time path of the natural rate, exploiting the full structure of the DSGE framework.

Short-term r_t^*.

As mentioned above, the DSGE model provides estimates of the short-term natural rate. Figure 13 shows the estimate of r_t^* implied by the DSGE model, along with the real federal funds rate (measured as the nominal federal funds rate minus the model-based expected inflation), from 1960 through 2016. Several observations stand out. First, the estimate of r_t^* moves considerably over time. This is at odds with the assumptions commonly made of either a constant or slow-moving r_t^*, but is common among estimates of r_t^* based on

DSGE models.[54] Second, r_t^* displays a clear cyclical pattern: it tends to be high and rising during booms, while it declines quite abruptly in recessions. This decline in r_t^* is especially pronounced during the Great Recession, when the model sees it fall into negative territory. The estimate of r_t^* remains persistently low through the first phase of the recovery, but is estimated to have increased somewhat at the end of the sample. Third, the estimate of r_t^* displays fairly pronounced high frequency variation. These quarter-to-quarter gyrations reflect the short-run nature of the natural rate, which moves in reaction to many of the shocks that buffet the economy.

Figure 13: Short-term r_t^* and r_t



Note: The figure shows the estimated real natural rate of interest (blue dashed line) and the model-implied short-term real interest rate (red line). The red and blue shaded areas represent the respective 68 percent posterior coverage intervals.

Comparing the natural rate to the actual real interest rate in Figure 13 makes clear that monetary policy was constrained by the zero lower bound when the natural rate fell to sharply negative values in the wake of the Great Recession. From 2009 to 2014, the natural rate hovered below the real federal funds rate, leading the FOMC to engage in unconventional monetary policy (i.e., large-scale asset purchases and forward guidance) to mitigate these effects. By the end of our sample though, the natural rate recovers, surpassing again the short-term real interest rate.

Fluctuations in r_t^* are driven by real and financial factors, but not by monetary factors, since in the model monetary policy has no effect in the absence of price and wage rigidities. To understand some of the drivers of the natural rate, consider the consumption Euler

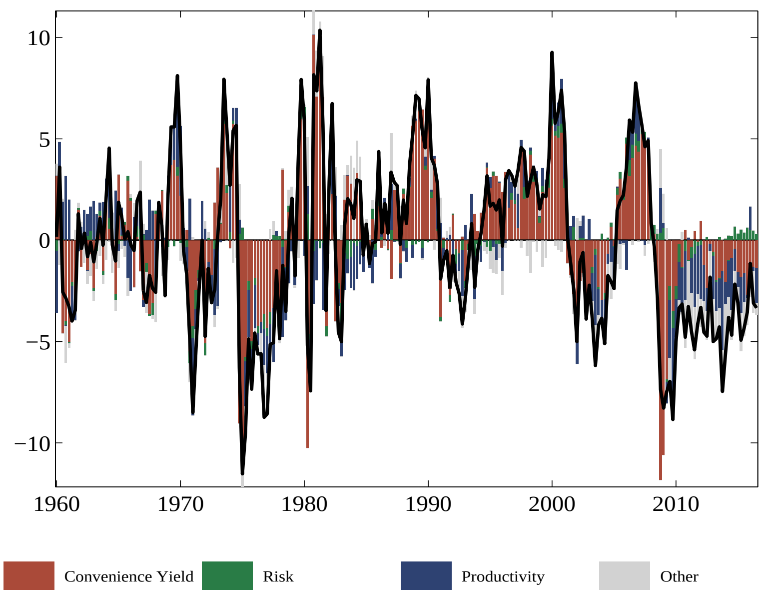
[54] See, e.g., Justiniano and Primiceri (2010), Barsky et al. (2014), and Curdia et al. (2015).

equation (25). The same equation holds also in the counterfactual economy in which prices and wages are fully flexible. Solving that equation for r_t^*, we obtain

$$r_t^* = -cy_t + \frac{\sigma_c}{1-\bar{h}}\left(\mathbb{E}_t\left[c_{t+1}^* - c_t^* + z_{t+1}\right] - \bar{h}\left(c_t^* - c_{t-1}^* + z_t\right)\right) - \frac{(\sigma_c - 1)}{(1-\bar{h})}\frac{w_* L_*}{c_*}\mathbb{E}_t[L_{t+1}^* - L_t^*], \tag{26}$$

where c_t^* and L_t^* denote respectively the level of consumption and hours worked in the flexible price economy. This expression reveals that r_t^* falls one-for-one with any increase in the convenience yield cy_t. The natural rate also increases with higher expected consumption growth or lower growth in labor supply in the flexible-price economy.

Figure 14: Short-term r_t^* and its Drivers



Note: The figure shows the real natural rate implied by the DSGE model (black solid line) in deviations from the steady state. It also shows the contribution of shocks to the convenience yield (in red), risk shocks $\tilde{\sigma}_{\omega,t}$ (in green), productivity shocks (in blue), and other shocks (in grey) in causing fluctuations in the natural rate.

Figure 14 decomposes the r_t^* estimate shown in Figure 13 in terms of the shocks that account for its movements. As before, the red bars capture exogenous fluctuations in convenience yields cy_t; green bars display fluctuations due to risk shocks $\tilde{\sigma}_{\omega,t}$, which increase the cost of external finance for firms, reducing the demand for investment. Productivity shocks (in blue) also depress desired consumption and investment, and lower the natural rate. The remaining shocks are in gray.

We draw two main lessons from Figure 14. First, r_t^* plunged during the recent financial crisis and the recession that followed due to an unusual combination of severe financial, risk, and productivity shocks. Second, among these negative contributions, shocks to convenience yields and negative productivity shocks have had particularly pronounced effects.

IV Conclusion

We estimated the natural rate of interest and its fundamental drivers using two very different methodologies. The first one is a flexible multivariate unobserved component model estimated using data on Treasury and corporate bond yields of various maturities, inflation, and survey expectations, which we used to make inference on slow-moving trends in the natural rate. The second is a medium-scale DSGE model with nominal and financial frictions, estimated using the same data on yields, along with a large set of other macroeconomic variables, whose tighter structure allows us to recover the entire time path of the natural rate.

The two approaches yield remarkably consistent results. First, they both isolate a slow-moving trend in the real interest rate that is fairly flat between 2 and 2.5 percent until the late 1990s, when it starts declining towards a recent trough at around 1 percent. Second, they both attribute most of this decline to an increase in the convenience yield on Treasuries, which they identify as a low-frequency component in the spreads between corporate and Treasury bonds with the same maturity, but different characteristics in terms of liquidity and safety. In addition, the DSGE model sees these factors as also playing an important role in the movements of the natural rate at business cycle frequencies. Finally, the DSGE model suggests that the short-term interest rate was severely constrained by the effective lower bound on nominal interest rates starting in late 2008, when the natural rate plunged well into negative territory.

Going forward, both our models suggest that the natural rate of interest will likely remain low due to its depressed secular component. Yet, this conclusion is subject to significant uncertainty, since sudden changes in expectations, regulation, market structure, investors' degree of risk aversion, or in their perceptions of the safety and liquidity attributes of U.S. Treasuries could all be sources of shocks to this trend. Although we have identified a rise in the measured convenience yield as a key driver of the secular decline in the natural rate of interest, we have not investigated the underlying sources of these changes in the premia commanded by liquid and safe assets. This is something we leave for future research.

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Online Appendix for
"Safety, Liquidity, and the Natural Rate of Interest"

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A Gibbs Sampler for VARs with Common Trends

Let use the notation $x_{i:j}$ to denote the sequence $\{x_i, .., x_j\}$ for a generic variable x_t. The Gibbs sampler is structured according to the following blocks:

1. $\bar{y}_{0:T}, \tilde{y}_{-p+1:T}, \lambda|\varphi, \Sigma_\varepsilon, \Sigma_e, y_{1:T}$
   (a) $\lambda|\varphi, \Sigma_\varepsilon, \Sigma_e, y_{1:T}$
   (b) $\bar{y}_{0:T}, \tilde{y}_{-p+1:T}|\lambda, \varphi, \Sigma_\varepsilon, \Sigma_e, y_{1:T}$
2. $\varphi, \Sigma_\varepsilon, \Sigma_e|\bar{y}_{0:T}, \tilde{y}_{-p+1:T}, \lambda, y_{1:T}$
   (a) $\Sigma_\varepsilon, \Sigma_e|\bar{y}_{0:T}, \tilde{y}_{-p+1:T}, \lambda, y_{1:T}$
   (b) $\varphi|\Sigma_\varepsilon, \Sigma_e, \bar{y}_{0:T}, \tilde{y}_{-p+1:T}, \lambda, y_{1:T}$

Details of each step follow:

1. $\bar{y}_{0:T}, \tilde{y}_{-p+1:T}, \lambda|\varphi, \Sigma_\varepsilon, \Sigma_e, y_{1:T}$
   This is given by the product of the marginal posterior distribution of λ (conditional on the other parameters) times the distribution of $\bar{y}_{0:T}, \tilde{y}_{-p+1:T}$ conditional on λ (and the other parameters).
   (a) $\lambda|\varphi, \Sigma_\varepsilon, \Sigma_e, y_{1:T}$
   The marginal posterior distribution of λ (conditional on the other parameters) is given by
   $$p(\lambda|\varphi, \Sigma_\varepsilon, \Sigma_e, y_{1:T}) \propto L(y_{1:T}|\lambda, \varphi, \Sigma_\varepsilon, \Sigma_e)p(\lambda),$$
   where $L(y_{1:T}|\lambda, \varphi, \Sigma_\varepsilon, \Sigma_e)$ is the likelihood obtained from the Kalman filter applied to the state space system (2) through (6). $p(\lambda|\varphi, \Sigma_\varepsilon, \Sigma_e, y_{1:T})$ does not have a known form so we will use a Metropolis Hastings step.

(b) $\bar{y}_{0:T}, \tilde{y}_{-p+1:T}|\lambda, \varphi, \Sigma_\varepsilon, \Sigma_e, y_{1:T}$
Given λ and the other parameters of the state space model we can use Durbin and Koopman (2002)'s simulation smoother to obtain draws for the latent states $\bar{y}_{0:T}$ and $\tilde{y}_{-p+1:T}$. Note that in addition to $\bar{y}_{1:T}$ and $\tilde{y}_{1:T}$ we also need to draw the initial conditions $\bar{y}_0$ and $\tilde{y}_{-p+1:0}$ in order to estimate the parameters of (4) and (3) in the next Gibbs sampler step.

Note that missing observations do not present any difficulty in terms of carrying out this step: if the vector y_{t_0} has some missing elements, the corresponding rows of the observation equation (2) are simply deleted for $t = t_0$.

2. $\varphi, \Sigma_\varepsilon, \Sigma_e|\bar{y}_{0:T}, \tilde{y}_{-p+1:T}, \lambda, y_{1:T}$
This step is straightforward because for given $\bar{y}_{0:T}$ and $\tilde{y}_{-p+1:T}$ equations (3) and (4) are standard VARs where in case of (3) we actually know the autoregressive matrices. The posterior distribution of Σ_e is given by

$$p(\Sigma_e|\bar{y}_{0:T}) = \mathcal{IW}(\underline{\Sigma}_e + \hat{S}_e, \kappa_e + T)$$

where $\hat{S}_e = \sum_{t=1}^{T}(\bar{y}_t - \bar{y}_{t-1})(\bar{y}_t - \bar{y}_{t-1})'$. The posterior distribution of φ and Σ_ε is given by

$$\begin{aligned} p(\Sigma_\varepsilon|\tilde{y}_{0:T}) &= \mathcal{IW}(\underline{\Sigma}_\varepsilon + \hat{S}_\varepsilon, \kappa_\varepsilon + T), \\ p(\varphi|\Sigma_\varepsilon, \tilde{y}_{0:T}) &= \mathcal{N}\left(vec(\hat{\Phi}), \Sigma_\varepsilon \otimes \left(\sum_{t=1}^{T} \tilde{x}_t\tilde{x}_t' + \underline{\Omega}^{-1}\right)^{-1}\right), \end{aligned}$$

where $\tilde{x}_t = (\tilde{y}_{t-1}', .., \tilde{y}_{t-p}')'$ collects the VAR regressors,

$$\hat{\Phi} = \left(\sum_{t=1}^{T} \tilde{x}_t\tilde{x}_t' + \underline{\Omega}^{-1}\right)^{-1} \left(\sum_{t=1}^{T} \tilde{x}_t\tilde{y}_t' + \underline{\Omega}^{-1}\underline{\Phi}\right), \; \hat{S}_\varepsilon = \sum_{t=1}^{T} \hat{\varepsilon}_t\hat{\varepsilon}_t' + (\hat{\Phi} - \underline{\Phi})'\underline{\Omega}^{-1}(\hat{\Phi} - \underline{\Phi}),$$

and $\hat{\varepsilon}_t = \tilde{y}_t - \hat{\Phi}'\tilde{x}_t$ are the VAR residuals.

We use 100,000 draws and discard the first 50,000.

B DSGE model

This section describes the model specification, the data used, how they relate to the model concepts, and the priors distributions assumed for estimation.

The model economy is populated by eight classes of agents: 1) a continuum of households, who consume and supply differentiated labor; 2) competitive labor aggregators that combine labor supplied by individual households; 3) competitive final good-producing firms that aggregate the intermediate goods into a final product; 4) a continuum of monopolistically competitive intermediate good producing firms; 5) competitive capital producers that convert final goods into capital; 6) a continuum of entrepreneurs who purchase capital using both internal and borrowed funds and rent it to intermediate good producing firms; 7) a representative bank collecting deposits from the households and lending funds to the entrepreneurs; and finally 8) a government, composed of a monetary authority that sets short-term interest rates and a fiscal authority that sets public spending and collects taxes. We solve each agent's problem, and derive the resulting equilibrium conditions, which we approximate around the non-stochastic steady state. Since the derivation follows closely the literature (e.g., Christiano et al. (2005)), we describe here the log-linearized conditions.

Growth in the economy is driven by technological progress, $Z_t^* = e^{\frac{1}{1-\alpha}\tilde{z}_t} Z_t^p e^{\gamma t}$, which is assumed to include a deterministic trend ($e^{\gamma t}$), a stochastic trend (Z_t^p), and a stationary component ($\tilde{z}_t$), where α is the income share of capital (after paying mark-ups and fixed costs in production). Trending variables are divided by Z_t^* to express the model's equilibrium conditions in terms of the stationary variables. In what follows, all variables are expressed in log deviations from their steady state, and steady-state values are denoted by $*$-subscripts.

The stationary component of productivity $\tilde{z}_t$ and the growth rate of the stochastic trend $z_t^p = log(Z_t^p / Z_{t-1}^p)$ are assumed to follow AR(1) processes:

$$\tilde{z}_t = \rho_z \tilde{z}_{t-1} + \sigma_z \varepsilon_{z,t},\ \varepsilon_{z,t} \sim N(0,1). \tag{A-1}$$

$$z_t^p = \rho_{z^p} z_{t-1}^p + \sigma_{z^p} \epsilon_{z^p,t},\ \epsilon_{z^p,t} \sim N(0,1). \tag{A-2}$$

The growth rate of technology evolves thus according to

$$z_t \equiv log(Z_t^*/Z_{t-1}^*) - \gamma = \frac{1}{1-\alpha}(\rho_z - 1)\tilde{z}_{t-1} + \frac{1}{1-\alpha}\sigma_z \epsilon_{z,t} + z_t^p, \tag{A-3}$$

where γ is the steady-state growth rate of the economy.

The *optimal allocation of consumption* satisfies the following Euler equation:

$$c_t = -\frac{1-\bar{h}}{\sigma_c(1+\bar{h})}\left(R_t - I\!E_t[\pi_{t+1}] + cy_t\right) + \frac{\bar{h}}{1+\bar{h}}\left(c_{t-1} - z_t\right) + \frac{1}{1+\bar{h}} I\!E_t\left[c_{t+1} + z_{t+1}\right] + \frac{(\sigma_c - 1)}{\sigma_c(1+\bar{h})}\frac{w_* L_*}{c_*}\left(L_t - I\!E_t[L_{t+1}]\right), \quad \text{(A-4)}$$

where c_t is consumption, L_t denotes hours worked, R_t is the nominal interest rate, and π_t is inflation. The parameter σ_c captures the degree of relative risk aversion while $\bar{h} \equiv he^{-\gamma}$ depends on the degree of habit persistence in consumption, h, and steady-state growth. This equation includes hours worked because utility is non-separable in consumption and leisure.

The convenience yield cy_t contains both a liquidity component cy_t^l and a safety component cy_t^l

$$cy_t = cy_t^l + cy_t^l, \quad \text{(A-5)}$$

where we let each premium be given by the sum of two AR(1) processes, one that captures transitory fluctuations, and one that captures highly persistent movements.

The *optimal investment decision* satisfies the following relationship between the level of investment i_t, measured in terms of consumption goods, and the value of capital in terms of consumption q_t^k:

$$i_t = \frac{q_t^k}{S'' e^{2\gamma}(1+\bar{\beta})} + \frac{1}{1+\bar{\beta}}\left(i_{t-1} - z_t\right) + \frac{\bar{\beta}}{1+\bar{\beta}} I\!E_t\left[i_{t+1} + z_{t+1}\right] + \mu_t. \quad \text{(A-6)}$$

This relationship shows that investment is affected by investment adjustment costs (S'' is the second derivative of the adjustment cost function) and by an exogenous process μ_t, which we call "marginal efficiency of investment", that alters the rate of transformation between consumption and installed capital (see Greenwood et al. (1998)). The shock μ_t follows an AR(1) process with parameters ρ_μ and σ_μ. The parameter $\bar{\beta} \equiv \beta e^{(1-\sigma_c)\gamma}$ depends on the intertemporal discount rate in the household utility function, β, on the degree of relative risk aversion σ_c, and on the steady-state growth rate γ.

The *capital stock*, $\bar{k}_t$, which we refer to as "installed capital", evolves as

$$\bar{k}_t = \left(1 - \frac{i_*}{\bar{k}_*}\right)\left(\bar{k}_{t-1} - z_t\right) + \frac{i_*}{\bar{k}_*} i_t + \frac{i_*}{\bar{k}_*} S'' e^{2\gamma}(1+\bar{\beta})\mu_t, \quad \text{(A-7)}$$

where $i_*/\bar{k}_*$ is the steady state investment to capital ratio. Capital is subject to variable capacity utilization u_t; *effective capital* rented out to firms, k_t, is related to $\bar{k}_t$ by:

$$k_t = u_t - z_t + \bar{k}_{t-1}. \quad \text{(A-8)}$$

The optimality condition determining the *rate of capital utilization* is given by

$$\frac{1-\psi}{\psi} r_t^k = u_t, \tag{A-9}$$

where r_t^k is the rental rate of capital and ψ captures the utilization costs in terms of foregone consumption.

Real marginal costs for firms are given by

$$mc_t \; = \; w_t + \alpha L_t - \alpha k_t, \tag{A-10}$$

where w_t is the real wage. From the optimality conditions of goods producers it follows that all firms have the same *capital-labor ratio*:

$$k_t \; = \; w_t - r_t^k + L_t. \tag{A-11}$$

We include financial frictions in the model, building on the work of Bernanke et al. (1999), Christiano et al. (2003), De Graeve (2008), and Christiano et al. (2014). We assume that banks collect deposits from households and lend to entrepreneurs who use these funds as well as their own wealth to acquire physical capital, which is rented to intermediate goods producers. Entrepreneurs are subject to idiosyncratic disturbances that affect their ability to manage capital. Their revenue may thus turn out to be too low to pay back the loans received by the banks. The banks therefore protect themselves against default risk by pooling all loans and charging a spread over the deposit rate. This spread may vary as a function of entrepreneurs' leverage and riskiness.

The *realized return on capital* is given by

$$\tilde{R}_t^k - \pi_t = \frac{r_*^k}{r_*^k + (1-\delta)} r_t^k + \frac{(1-\delta)}{r_*^k + (1-\delta)} q_t^k - q_{t-1}^k, \tag{A-12}$$

where $\tilde{R}_t^k$ is the gross nominal return on capital for entrepreneurs, r_*^k is the steady state value of the rental rate of capital r_t^k, and δ is the depreciation rate.

The *excess return on capital* (the spread between the expected return on capital and the riskless rate) can be expressed as a function of the convenience yield cy_t, the entrepreneurs' leverage (i.e. the ratio of the value of capital to net worth), and "risk shocks" $\tilde{\sigma}_{\omega,t}$ capturing mean-preserving changes in the cross-sectional dispersion of ability across entrepreneurs (see Christiano et al. (2014)):

$$E_t\left[\tilde{R}_{t+1}^k - R_t\right] = cy_t + \zeta_{sp,b}\left(q_t^k + \bar{k}_t - n_t\right) + \tilde{\sigma}_{\omega,t}, \tag{A-13}$$

where n_t is entrepreneurs' net worth, $\zeta_{sp,b}$ is the elasticity of the credit spread to the entrepreneurs' leverage $(q^k_t + \bar{k}_t - n_t)$. $\tilde{\sigma}_{\omega,t}$ follows an AR(1) process with parameters ρ_{σ_ω} and σ_{σ_ω}. *Entrepreneurs' net worth* n_t evolves in turn according to

$$\begin{aligned} n_t &= \zeta_{n,\tilde{R}^k}\left(\tilde{R}^k_t - \pi_t\right) - \zeta_{n,R}\left(R_{t-1} - \pi_t + cy_{t-1}\right) + \zeta_{n,qK}\left(q^k_{t-1} + \bar{k}_{t-1}\right) + \zeta_{n,n} n_{t-1} \\ &\quad -\gamma_* \frac{v_*}{n_*} z_t - \frac{\zeta_{n,\sigma_\omega}}{\zeta_{sp,\sigma_\omega}} \tilde{\sigma}_{\omega,t-1}, \end{aligned} \tag{A-14}$$

where the ζ's denote elasticities, that depend among others on the entrepreneurs' steady-state default probability $F(\bar{\omega})$, where γ_* is the fraction of entrepreneurs that survive and continue operating for another period, and where v_* is the entrepreneurs' real equity divided by Z^*_t, in steady state.

The *production function* is

$$y_t = \Phi_p\left(\alpha k_t + (1-\alpha) L_t\right), \tag{A-15}$$

where $\Phi_p = 1 + \Phi/y_*$, and Φ measures the size of fixed costs in production. The *resource constraint* is:

$$y_t = g_* g_t + \frac{c_*}{y_*} c_t + \frac{i_*}{y_*} i_t + \frac{r^k_* k_*}{y_*} u_t. \tag{A-16}$$

where $g_t = log(\frac{G_t}{Z^*_t y_* g_*})$ and $g_* = 1 - \frac{c_* + i_*}{y_*}$. *Government spending* g_t is assumed to follow the exogenous process:

$$g_t = \rho_g g_{t-1} + \sigma_g \varepsilon_{g,t} + \eta_{gz} \sigma_z \varepsilon_{z,t}.$$

Optimal decisions for price and wage setting deliver the *price and wage Phillips curves*, which are respectively:

$$\pi_t = \kappa\, mc_t + \frac{\iota_p}{1+\iota_p \bar{\beta}} \pi_{t-1} + \frac{\bar{\beta}}{1+\iota_p\bar{\beta}} I\!E_t[\pi_{t+1}] + \lambda_{f,t}, \tag{A-17}$$

and

$$\begin{aligned} w_t = {} & \frac{(1-\zeta_w\bar{\beta})(1-\zeta_w)}{(1+\bar{\beta})\zeta_w((\lambda_w - 1)\epsilon_w + 1)}\left(w^h_t - w_t\right) - \frac{1+\iota_w\bar{\beta}}{1+\bar{\beta}}\pi_t + \frac{1}{1+\bar{\beta}}\left(w_{t-1} - z_t + \iota_w \pi_{t-1}\right) \\ & + \frac{\bar{\beta}}{1+\bar{\beta}} I\!E_t\left[w_{t+1} + z_{t+1} + \pi_{t+1}\right] + \lambda_{w,t}, \end{aligned} \tag{A-18}$$

where $\kappa = \frac{(1-\zeta_p\bar{\beta})(1-\zeta_p)}{(1+\iota_p\bar{\beta})\zeta_p((\Phi_p - 1)\epsilon_p + 1)}$, the parameters ζ_p, ι_p, and ϵ_p are the Calvo parameter, the degree of indexation, and the curvature parameter in the Kimball aggregator for prices, and ζ_w, ι_w, and ϵ_w are the corresponding parameters for wages. w_t^h measures the household's marginal rate of substitution between consumption and labor, and is given by:

$$w_t^h = \frac{1}{1-\bar{h}}\left(c_t - \bar{h}c_{t-1} + \bar{h}z_t\right) + \nu_l L_t, \tag{A-19}$$

where ν_l characterizes the curvature of the disutility of labor (and would equal the inverse of the Frisch elasticity in the absence of wage rigidities). The mark-ups $\lambda_{f,t}$ and $\lambda_{w,t}$ follow the exogenous ARMA(1,1) processes:

$$\lambda_{f,t} = \rho_{\lambda_f}\lambda_{f,t-1} + \sigma_{\lambda_f}\varepsilon_{\lambda_f,t} - \eta_{\lambda_f}\sigma_{\lambda_f}\varepsilon_{\lambda_f,t-1},$$

and

$$\lambda_{w,t} = \rho_{\lambda_w}\lambda_{w,t-1} + \sigma_{\lambda_w}\varepsilon_{\lambda_w,t} - \eta_{\lambda_w}\sigma_{\lambda_w}\varepsilon_{\lambda_w,t-1}.$$

Finally, the monetary authority follows a *policy feedback rule*:

$$\begin{aligned} R_t &= \rho_R R_{t-1} + (1-\rho_R)\left(\psi_1(\pi_t - \pi_t^*) + \psi_2(y_t - y_t^*)\right) \\ &\quad + \psi_3\left((y_t - y_t^*) - (y_{t-1} - y_{t-1}^*)\right) + r_t^m. \end{aligned} \tag{A-20}$$

where π_t^* is a time-varying inflation target, y_t^* is a measure of the "full-employment level of output," and r_t^m captures exogenous departures from the policy rule.

The time-varying inflation target π_t^* is meant to capture the rise and fall of inflation and interest rates in the estimation sample.[1] As in Aruoba and Schorfheide (2008) and Del Negro and Eusepi (2011), we use data on long-run inflation expectations in the estimation of the model. This allows us to pin down the target inflation rate to the extent that long-run inflation expectations contain information about the central bank's objective. The time-varying *inflation target* evolves according to

$$\pi_t^* = \rho_{\pi^*}\pi_{t-1}^* + \sigma_{\pi^*}\epsilon_{\pi^*,t}, \tag{A-21}$$

where $0 < \rho_{\pi^*} < 1$ and $\epsilon_{\pi^*,t}$ is an iid shock. We model π_t^* as a stationary process, although our prior for ρ_{π^*} will force this process to be highly persistent.

The "full-employment level of output" y_t^* represents the level of output that would obtain if prices and wages were fully flexible and if there were no markup shocks. This variable along with the natural rate of interest r_t^* are obtained by solving the model without nominal

[1] The assumption that the inflation target moves exogenously is of course a simplification. A more realistic model would for instance relate movements in trend inflation to the evolution of the policy makers' understanding of the output-inflation trade-of, as in Sargent (1999) or Primiceri (2006).

rigidities and markup shocks (that is, equations (A-4) through (A-19) with $\zeta_p = \zeta_w = 0$, and $\lambda_{f,t} = \lambda_{w,t} = 0$).

The exogenous component of the policy rule r_t^m evolves according to the following process

$$r_t^m = \rho_{r^m} r_{t-1}^m + \epsilon_t^R + \sum_{k=1}^{K} \epsilon_{k,t-k}^R, \tag{A-22}$$

where ϵ_t^R is the usual contemporaneous policy shock, and $\epsilon_{k,t-k}^R$ is a policy shock that is known to agents at time $t-k$, but affects the policy rule k periods later, that is, at time t. We assume that $\epsilon_{k,t-k}^R \sim N(0, \sigma_{k,r}^2)$, *i.i.d.* As argued in Laseen and Svensson (2011), such anticipated policy shocks allow us to capture the effects of the zero lower bound on nominal interest rates, as well as the effects of forward guidance in monetary policy.

B.1 State Space Representation and Data

We use the method in Sims (2002) to solve the system of log-linear approximate equilibrium conditions and obtain the transition equation, which summarizes the evolution of the vector of state variables s_t:

$$s_t = \mathcal{T}(\theta) s_{t-1} + \mathcal{R}(\theta)\epsilon_t. \tag{A-23}$$

where θ is a vector collecting all the DSGE model parameters and ϵ_t is a vector of all structural shocks. The state-space representation of our model is composed of the transition equation (A-23), and a system of measurement equations:

$$Y_t = \mathcal{D}(\theta) + \mathcal{Z}(\theta) s_t, \tag{A-24}$$

mapping the states into the observable variables Y_t, which we describe in detail next.

The estimation of the model is based on data on real output growth (including both GDP and GDI measures), consumption growth, investment growth, real wage growth, hours worked, inflation (measured by core PCE and GDP deflators), short- and long- term interest rates, 10-year inflation expectations, market expectations for the federal funds rate up to 6 quarters ahead, Aaa and Baa credit spreads, and total factor productivity. Measurement equations (A-24) relate these observables to the model variables as follows:

$$
\begin{aligned}
\text{GDP growth} &= 100\gamma + (y_t - y_{t-1} + z_t) + e_t^{gdp} - e_{t-1}^{gdp} \\
\text{GDI growth} &= 100\gamma + (y_t - y_{t-1} + z_t) + e_t^{gdi} - e_{t-1}^{gdi} \\
\text{Consumption growth} &= 100\gamma + (c_t - c_{t-1} + z_t) \\
\text{Investment growth} &= 100\gamma + (i_t - i_{t-1} + z_t) \\
\text{Real Wage growth} &= 100\gamma + (w_t - w_{t-1} + z_t) \\
\text{Hours} &= \bar{L} + L_t \\
\text{Core PCE Inflation} &= \pi_* + \pi_t + e_t^{pce} \\
\text{GDP Deflator Inflation} &= \pi_* + \delta_{gdpdef} + \gamma_{gdpdef} * \pi_t + e_t^{gdpdef} \\
\text{FFR} &= R_* + R_t \\
\text{FFR}^e_{t,t+j} &= R_* + \mathbb{E}_t\left[R_{t+j}\right],\ j = 1, ..., 6 \\
\text{10y Nominal Bond Yield} &= R_* + \mathbb{E}_t\left[\frac{1}{40}\sum_{j=0}^{39} R_{t+j}\right] + e_t^{10y} \\
\text{10y Infl. Expectations} &= \pi_* + \mathbb{E}_t\left[\frac{1}{40}\sum_{j=0}^{39} \pi_{t+j}\right] \\
\text{Aaa - 20-year Treasury Spread} &= cy_*^l + \mathbb{E}_t\left[\frac{1}{80}\sum_{j=0}^{79} cy_{t+j}^l\right] + e_t^{Aaa} \\
\text{Baa - 20-year Treasury Spread} &= cy_*^l + cy_*^s + SP_* + \mathbb{E}_t\frac{1}{80}\sum_{j=0}^{79}\left[\tilde{R}_{t+j+1}^k - R_{t+j}\right] + e_t^{Baa} \\
\text{TFP growth, demeaned} &= z_t + \frac{\alpha}{1-\alpha}(u_t - u_{t-1}) + e_t^{tfp}.
\end{aligned}
\tag{A-25}
$$

All variables are measured in percent. The terms π_* and R_* measure respectively the net steady-state inflation rate and short-term nominal interest rate, expressed in percentage terms, and $\bar{L}$ captures the mean of hours (this variable is measured as an index). We assume that some of the variables are measured with "error," that is, the observed value equals the model implied value plus an AR(1) exogenous process e_t^* that can be thought of either measurement errors or some other unmodeled source of discrepancy between the model and the data, as in Boivin and Giannoni (2006). For instance, the terms e_t^{gdp} and e_t^{gdi} capture measurement error of total output.[2] Alternatively, or the long-term nominal interest rate,

[2] We introduce correlation in the measurement errors for GDP and GDI, which evolve as follows:

$$
\begin{aligned}
e_t^{gdp} &= \rho_{gdp} \cdot e_{t-1}^{gdp} + \sigma_{gdp}\epsilon_t^{gdp},\ \epsilon_t^{gdp} \sim i.i.d.N(0,1) \\
e_t^{gdi} &= \rho_{gdi} \cdot e_{t-1}^{gdi} + \varrho_{gdp} \cdot \sigma_{gdp}\epsilon_t^{gdp} + \sigma_{gdi}\epsilon_t^{gdi},\ \epsilon_t^{gdi} \sim i.i.d.N(0,1).
\end{aligned}
$$

The measurement errors for GDP and GDI are thus stationary in *levels*, and enter the observation equation in first differences (e.g. $e_t^{gdp} - e_{t-1}^{gdp}$ and $e_t^{gdp} - e_{t-1}^{gdp}$). GDP and GDI are also cointegrated as they are driven the term e_t^{10y} captures fluctuations in term premia not captured by the model.

The 10-year inflation expectations contain information about low-frequency movements of inflation and are obtained from the Blue Chip Economic Indicators survey and the Survey of Professional Forecasters.

B.2 Inference, Prior and Posterior Parameter Estimates

We estimate the model using Bayesian techniques. This requires the specification of a prior distribution for the model parameters. For most parameters common with Smets and Wouters (2007), we use the same marginal prior distributions. As an exception, we favor a looser prior than Smets and Wouters (2007) for the quarterly steady state inflation rate π_*; it is centered at 0.75% and has a standard deviation of 0.4%. Regarding the financial frictions, we specify priors for the parameters SP_*, $\zeta_{sp,b}$, ρ_{σ_ω}, and σ_{σ_ω}, while we fix the parameters corresponding to the steady state default probability and the survival rate of entrepreneurs, respectively. In turn, these parameters imply values for the parameters of (A-14). Information on the priors and posterior mean is provided in Table A2.

B.3 Data Construction

Data on real GDP (GDPC), the GDP deflator (GDPDEF), core PCE inflation (PCEPILFE), nominal personal consumption expenditures (PCEC), and nominal fixed private investment (FPI) are produced at a quarterly frequency by the Bureau of Economic Analysis, and are included in the National Income and Product Accounts (NIPA). Average weekly hours of production and nonsupervisory employees for total private industries (AWHNONAG), civilian employment (CE16OV), and the civilian non-institutional population (CNP16OV) are produced by the Bureau of Labor Statistics (BLS) at a monthly frequency. The first of these series is obtained from the Establishment Survey, and the remaining from the Household Survey. Both surveys are released in the BLS Employment Situation Summary. Since our models are estimated on quarterly data, we take averages of the monthly data. Compensation per hour for the non-farm business sector (COMPNFB) is obtained from the Labor Productvity and Costs release, and produced by the BLS at a quarterly frequency. The data are transformed following Smets and Wouters (2007), with the exception of the civilian population data, which are filtered using the Hodrick-Prescott filter to remove jumps around

by a comment stochastic trend.

census dates. The federal funds rate is obtained from the Federal Reserve Board's H.15 release at a business day frequency. We take quarterly averages of the annualized daily data and divide by four. Let Δ denote the temporal difference operator. Then:

$$\begin{aligned}
\text{Output growth} &= 100 * \Delta LN((GDPC)/CNP16OV) \\
\text{Consumption growth} &= 100 * \Delta LN((PCEC/GDPDEF)/CNP16OV) \\
\text{Investment growth} &= 100 * \Delta LN((FPI/GDPDEF)/CNP16OV) \\
\text{Real wage growth} &= 100 * \Delta LN(COMPNFB/GDPDEF) \\
\text{Hours worked} &= 100 * LN((AWHNONAG * CE16OV/100)/CNP16OV) \\
\text{GDP Deflator Inflation} &= 100 * \Delta LN(GDPDEF) \\
\text{Core PCE Inflation} &= 100 * \Delta LN(PCEPILFE) \\
\text{FFR} &= (1/4) * FEDERAL\ FUNDS\ RATE
\end{aligned}$$

Long-run inflation expectations are obtained from the Blue Chip Economic Indicators survey and the Survey of Professional Forecasters available from the FRB Philadelphia's Real-Time Data Research Center. Long-run inflation expectations (average CPI inflation over the next 10 years) are available from 1991Q4 onward. Prior to 1991Q4, we use the 10-year expectations data from the Blue Chip survey to construct a long time series that begins in 1979Q4. Since the Blue Chip survey reports long-run inflation expectations only twice a year, we treat these expectations in the remaining quarters as missing observations and adjust the measurement equation of the Kalman filter accordingly. Long-run inflation expectations $\pi_t^{O,40}$ are therefore measured as

$$\text{10y Infl Exp} = (\text{10-year average CPI inflation forecast} - 0.50)/4.$$

where 0.50 is the average difference between CPI and GDP annualized inflation from the beginning of the sample to 1992. We divide by 4 to express the data in quarterly terms.

We measure *Spread* as the annualized Moody's Seasoned Baa Corporate Bond Yield spread over the 10-Year Treasury Note Yield at Constant Maturity. Both series are available from the Federal Reserve Board's H.15 release. Like the federal funds rate, the spread data are also averaged over each quarter and measured at a quarterly frequency. This leads to:

$$\text{Spread} = (1/4) * (Baa\ Corporate\ -\ 10\ year\ Treasury).$$

Similarly,

$$\text{10y Bond yield} = (1/4) * (10\ year\ Treasury).$$

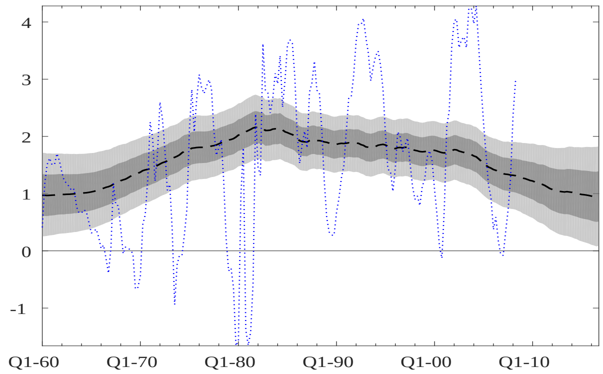
Lastly, TFP growth is measured using John Fernald's TFP growth series, unadjusted for changes in utilization. That series is demeaned, divided by 4 to express it in quarterly growth rates, and divided by Fernald's estimate of $(1-\alpha)$ to convert it in labor augmenting terms:

TFP growth, demeaned $= (1/4)*$(Fernald's TFP growth, unadjusted, demeaned) $/(1-\alpha)$.

C Additional Tables and Figures – VARs (Section II)

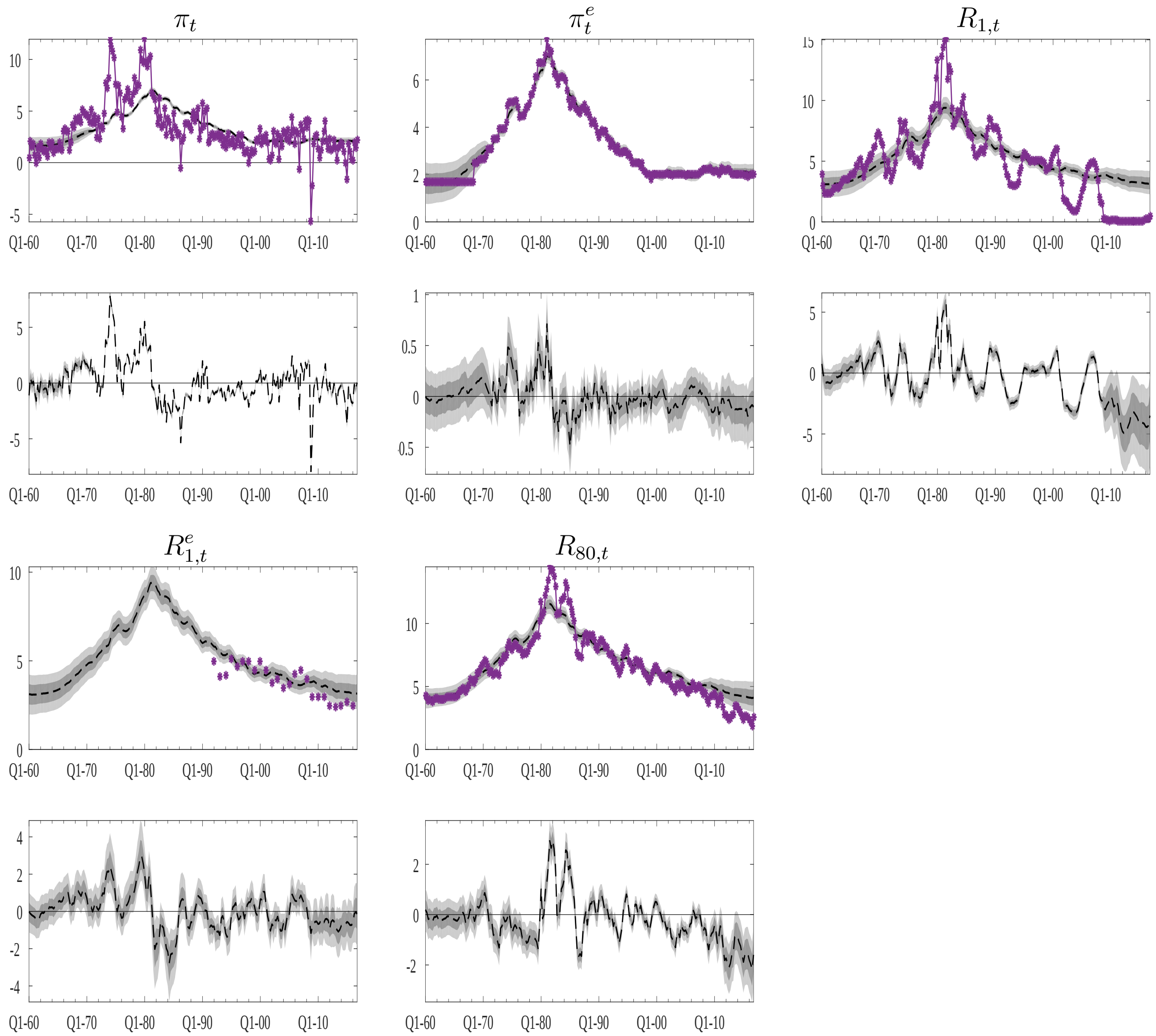
Figure A1: Other Trends and Observables, Baseline Model

$\overline{tp}_t$ and $R_{80,t} - R_{1,t}$



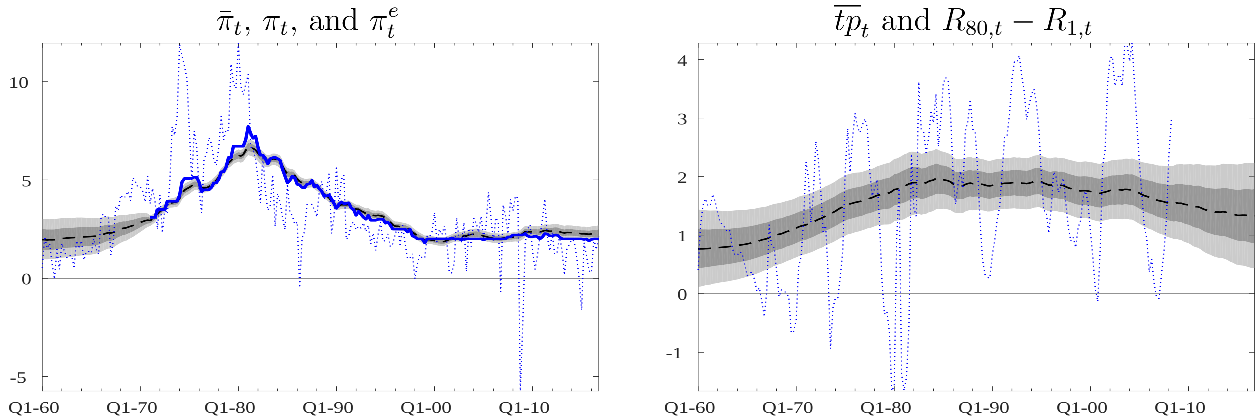
Note: The figure shows $R_{80,t} - R_{1,t}$ (dotted blue line) together with the trend $\overline{tp}_t$. For the trend, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

Figure A2: y_t, $\Lambda\bar{y}_t$, and $\tilde{y}_t$; Baseline Model



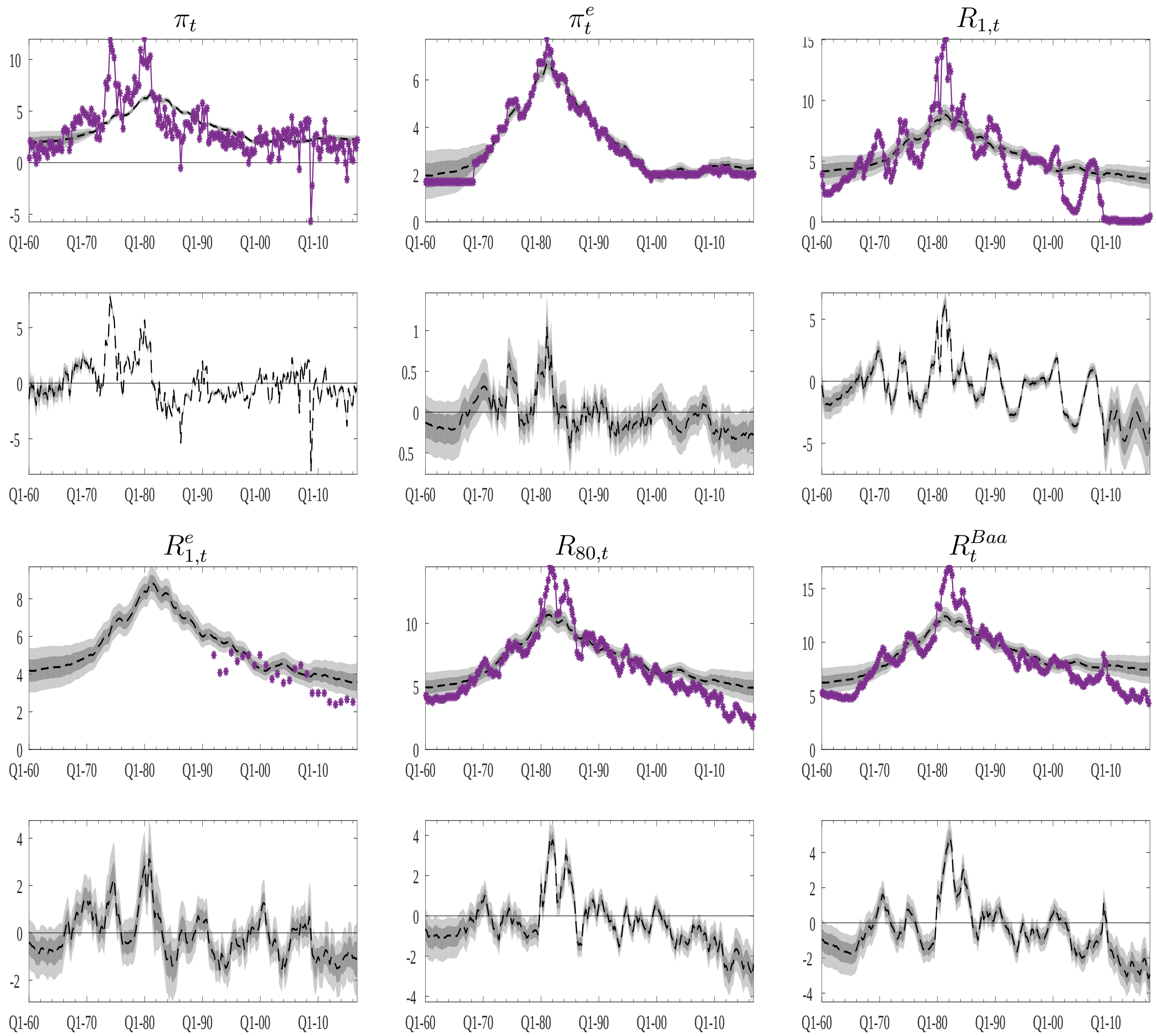
Note: For each variable the top panel shows the data y_t and the trend component $\Lambda\bar{y}_t$, and the bottom panel shows the stationary component $\tilde{y}_t$. For each latent variable, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

Figure A3: Other Trends and Observables, Convenience Yield Model



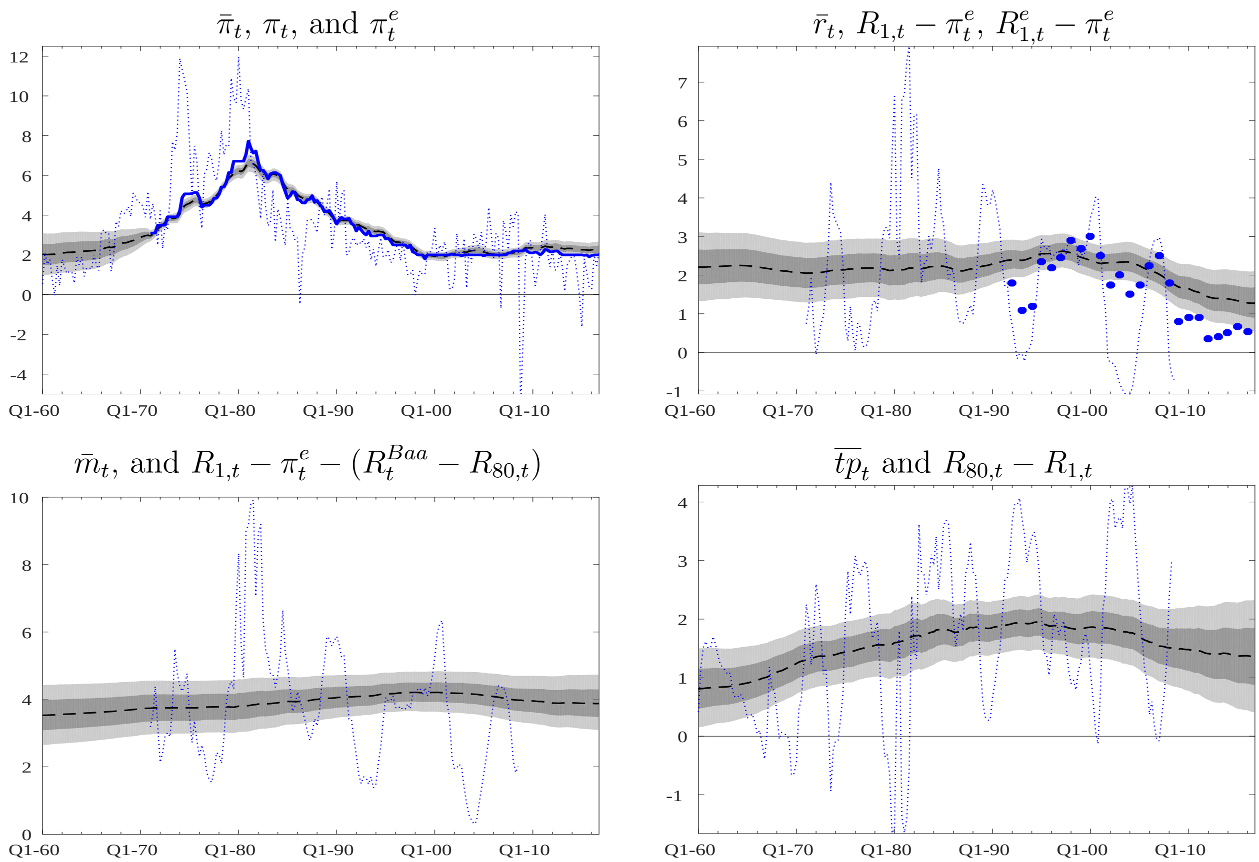
Note: The left panel shows π_t (dotted blue line), and π_t^e (solid blue line), together with the trend $\bar{\pi}_t$. The right panel shows $R_{80,t} - R_{1,t}$ (dotted blue line) together with the trend $\overline{tp}_t$. For the trend, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals. For each trend, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

Figure A4: y_t, $\Lambda\bar{y}_t$, and $\tilde{y}_t$; Convenience Yield Model



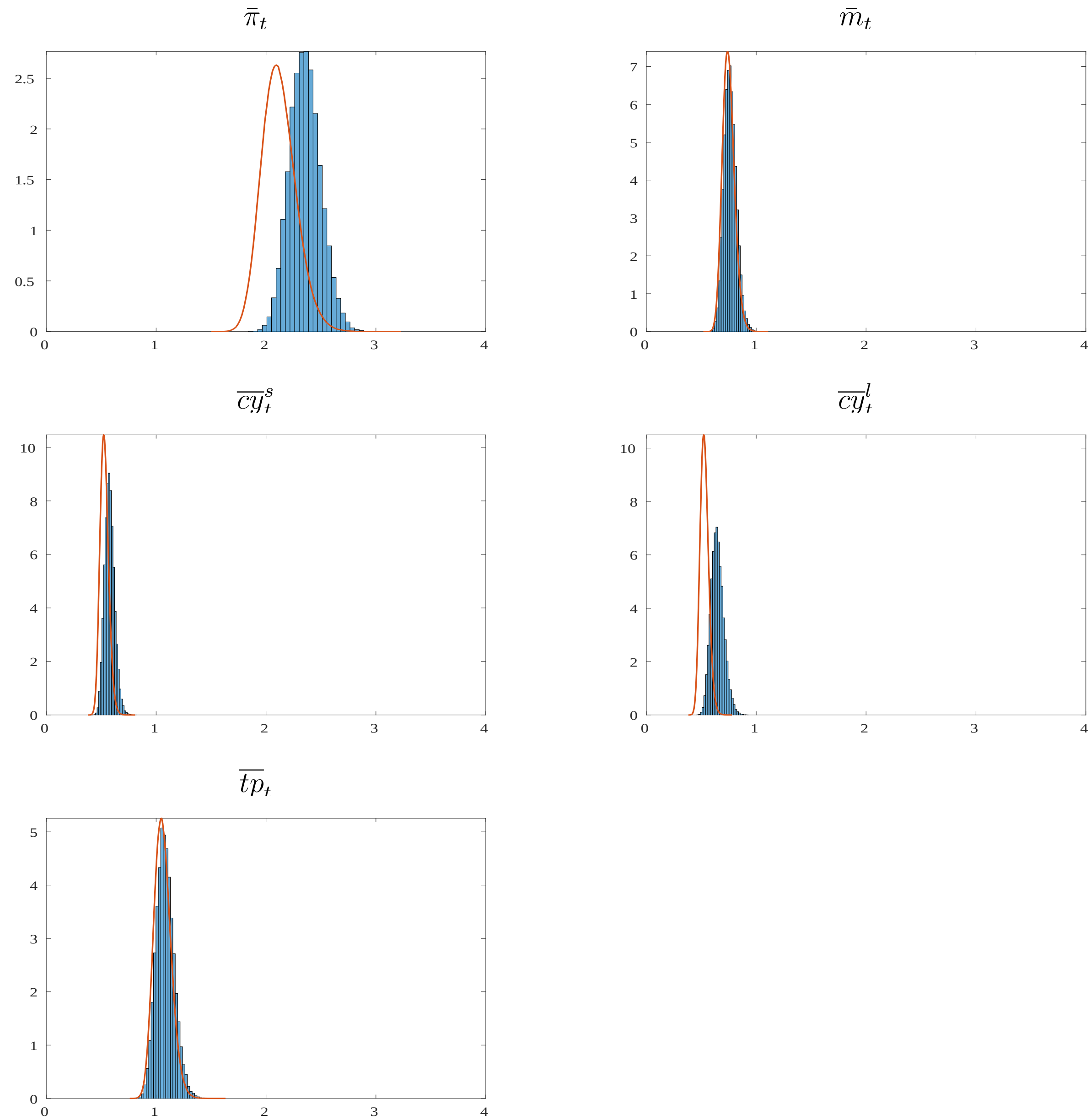
Note: For each variable the top panel shows the data y_t and the trend component $\Lambda\bar{y}_t$, and the bottom panel shows the stationary component $\tilde{y}_t$. For each latent variable, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

Figure A5: Other Trends and Observables, Safety and Liquidity Model



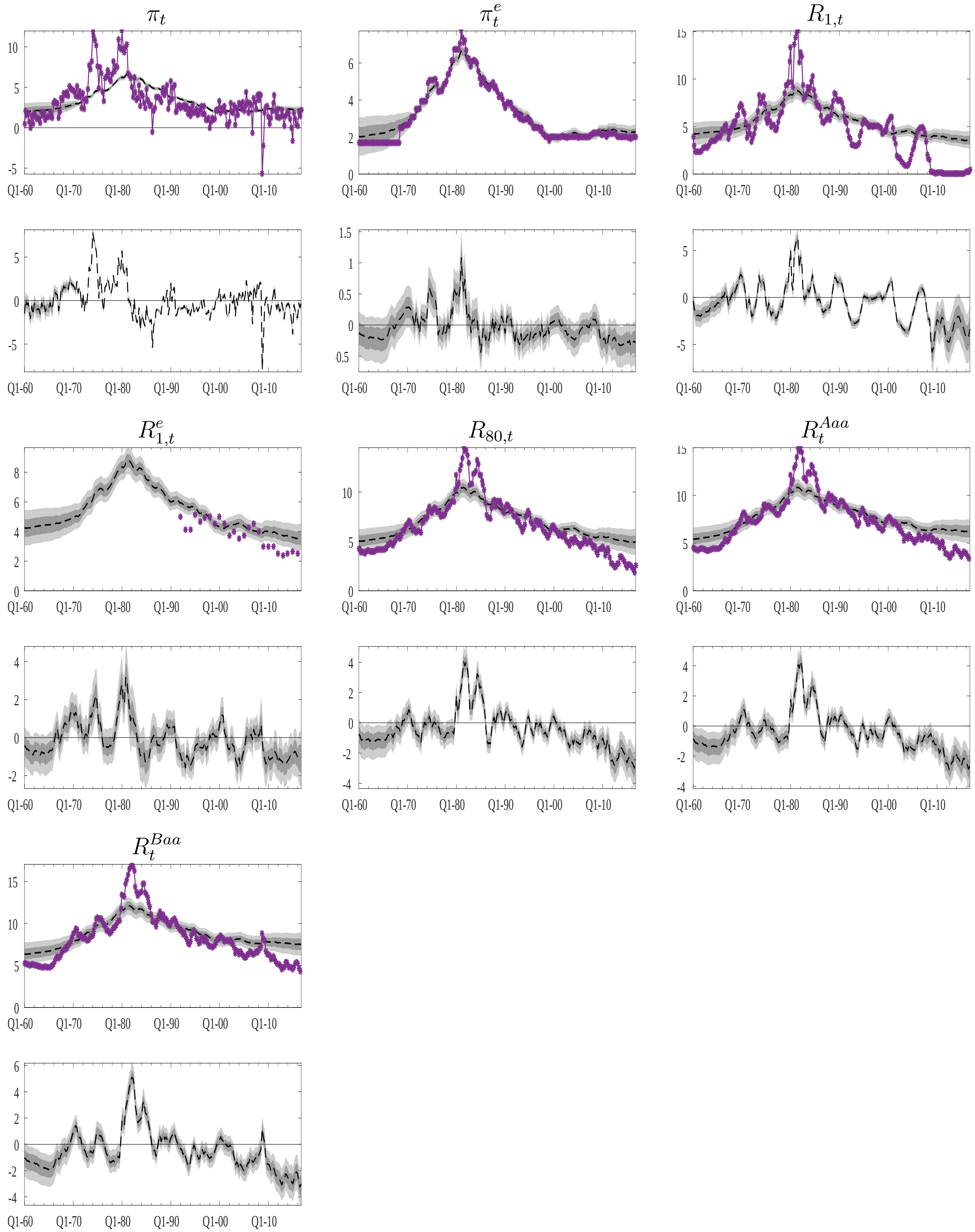
Note: The top left panel shows π_t (dotted blue line), and π^e_t (solid blue line), together with the trend $\bar{\pi}_t$. The top right panel shows $R_{1,t} - \pi^e_t$ (dotted blue line), and $R^e_{1,t} - \pi^e_t$ (blue dots), together with the trend $\bar{r}_t$. The bottom left panel shows $R_{1,t} - \pi^e_t - (R^{Baa}_t - R_{80,t})$ (dotted blue line), together with the trend $\bar{m}_t$. The bottom right panel shows $R_{80,t} - R_{1,t}$ (dotted blue line) together with the trend $\overline{tp}_t$. For each trend, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

Figure A6: Prior and Posterior Distributions of the Standard Deviations of the Shocks to the Trend Components



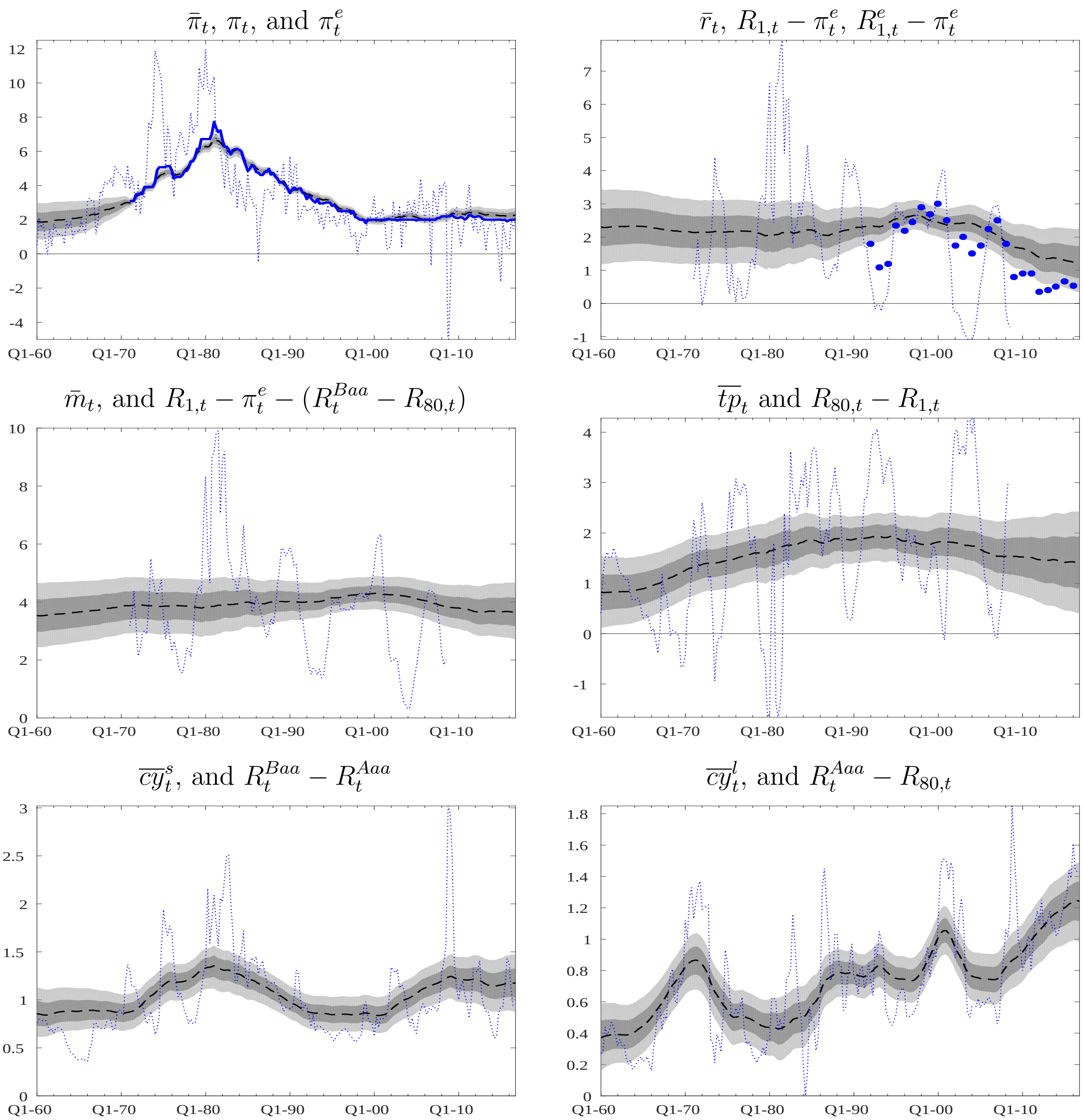
Note: The panels show the prior (solid red line) and posterior (histogram) distributions of the standard deviations of the shocks to the trend components – the diagonal elements of the matrix Σ_e. The units are expressed in terms of multiples of 1% per century, that is, $\sqrt{1/400}$.

Figure A7: y_t, $\Lambda\bar{y}_t$, and $\tilde{y}_t$; Safety and Liquidity Model



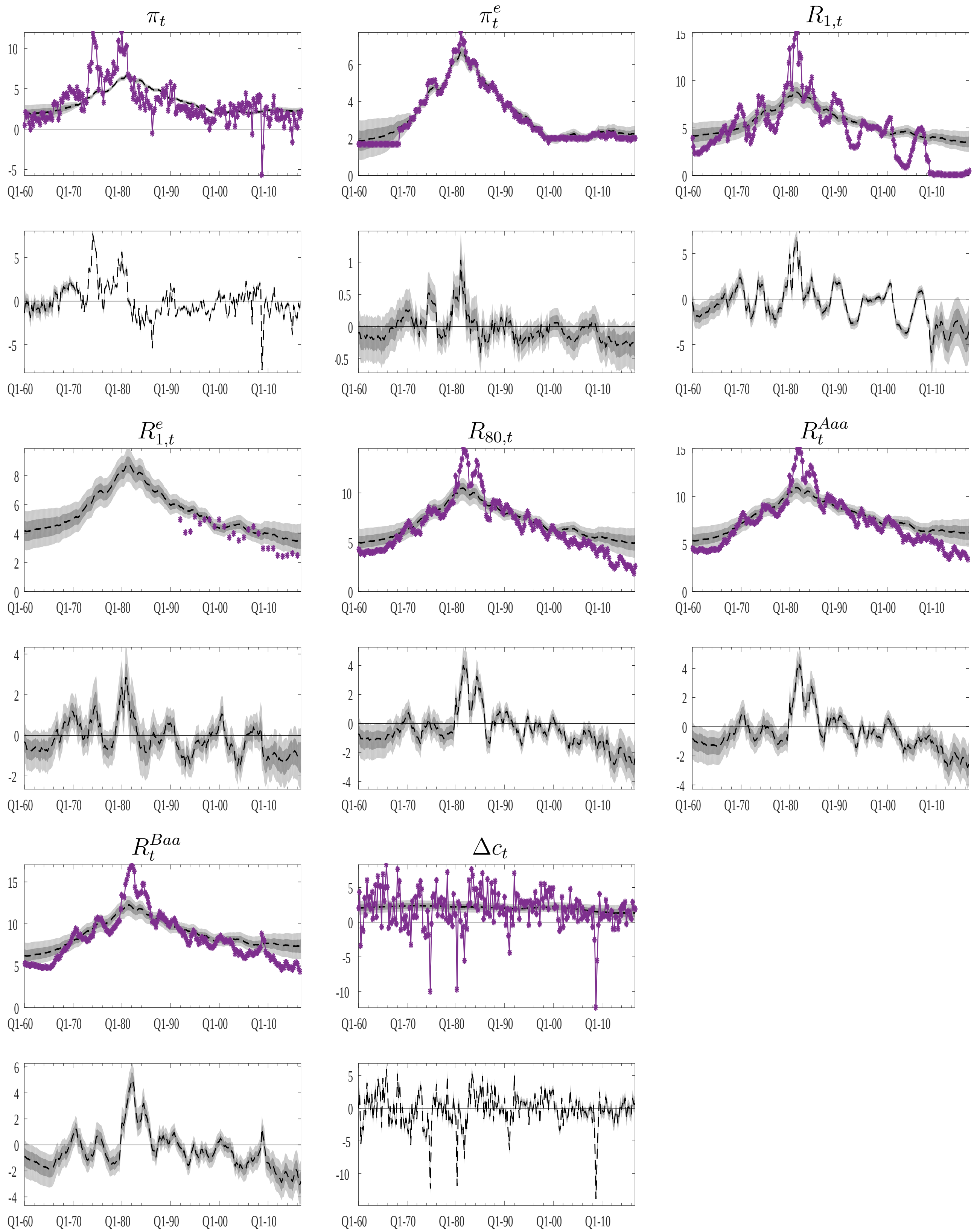
Note: For each variable the top panel shows the data y_t and the trend component $\Lambda\bar{y}_t$, and the bottom panel shows the stationary component $\tilde{y}_t$. For each latent variable, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

Figure A8: Other Trends and Observables, Consumption Growth Model



Note: The top left panel shows π_t (dotted blue line), and π_t^e (solid blue line), together with the trend $\bar{\pi}_t$. The top right panel shows $R_{1,t} - \pi_t^e$ (dotted blue line), and $R_{1,t}^e - \pi_t^e$ (blue dots), together with the trend $\bar{r}_t$. The middle left panel shows $R_{1,t} - \pi_t^e - (R_t^{Baa} - R_{80,t})$ (dotted blue line), together with the trend $\bar{m}_t$. The middle right panel shows $R_{80,t} - R_{1,t}$ (dotted blue line) together with the trend $\overline{tp}_t$. The bottom left panel shows the Baa/Aaa spread $R_t^{Baa} - R_t^{Aaa}$ (dotted blue line), together with the trend $\overline{cy}_t^s$. The bottom right panel shows the Aaa/Treasury spread $R_t^{Aaa} - R_{80,t}$ (dotted blue line), together with the trend $\overline{cy}_t^l$. For each trend, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

Figure A9: y_t, $\Lambda\bar{y}_t$, and $\tilde{y}_t$; Consumption Growth Model



Note: For each variable the top panel shows the data y_t and the trend component $\Lambda\bar{y}_t$, and the bottom panel shows the stationary component $\tilde{y}_t$. For each latent variable, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

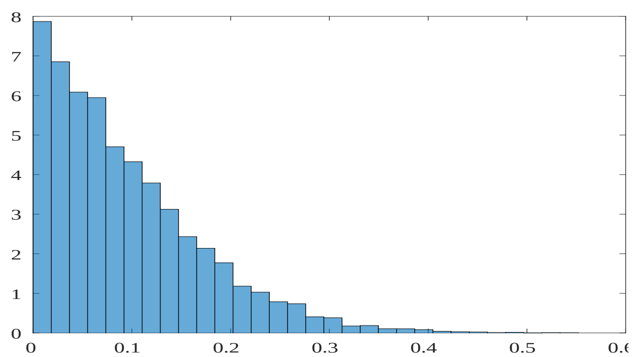
D Robustness – VAR (Section II)

Table A1: Change in Trends, 1998Q1-2016Q4 – Robustness

	(1) Default	(2) Loose Prior	(3) Inflation Term Spread	(4) Productivity
$\bar{r}_t$	-1.46**	-1.56**	-1.27**	-1.61**
	[-1.80, -1.11]	[-2.06, -0.98]	[-1.61, -0.92]	[-2.00, -1.19]
	(-2.12, -0.76)	(-2.51, -0.34)	(-1.92, -0.57)	(-2.36, -0.75)
$\bar{m}_t$	-0.08	-0.38	-0.32	-0.76
	[-0.40, 0.24]	[-0.91, 0.24]	[-0.64, -0.00]	[-1.17, -0.35]
	(-0.70, 0.56)	(-1.40, 0.91)	(-0.95, 0.32)	(-1.55, 0.08)
$\bar{g}_t$				-0.72
				[-1.10, -0.32]
				(-1.47, 0.09)
$\bar{\beta}_t$				-0.05
				[-0.21, 0.12]
				(-0.38, 0.29)
$-\overline{cy}_t$	-1.38**	-1.18**	-0.95**	-0.84**
	[-1.58, -1.17]	[-1.48, -0.88]	[-1.16, -0.73]	[-1.05, -0.63]
	(-1.78, -0.97)	(-1.78, -0.56)	(-1.37, -0.51)	(-1.27, -0.41)
$-\overline{cy}^s_t$	-0.69**	-0.46**	-0.44**	-0.39**
(safety)	[-0.83, -0.54]	[-0.65, -0.28]	[-0.59, -0.29]	[-0.54, -0.24]
	(-0.97, -0.40)	(-0.85, -0.09)	(-0.73, -0.15)	(-0.68, -0.10)
$-\overline{cy}^l_t$	-0.69**	-0.72**	-0.51**	-0.45**
(liquidity)	[-0.82, -0.55]	[-0.89, -0.54]	[-0.64, -0.37]	[-0.58, -0.32]
	(-0.95, -0.42)	(-1.06, -0.36)	(-0.76, -0.24)	(-0.71, -0.19)
$\Delta\overline{Prod}_t$				-1.06
				[-1.57, -0.54]
				(-2.05, 0.01)

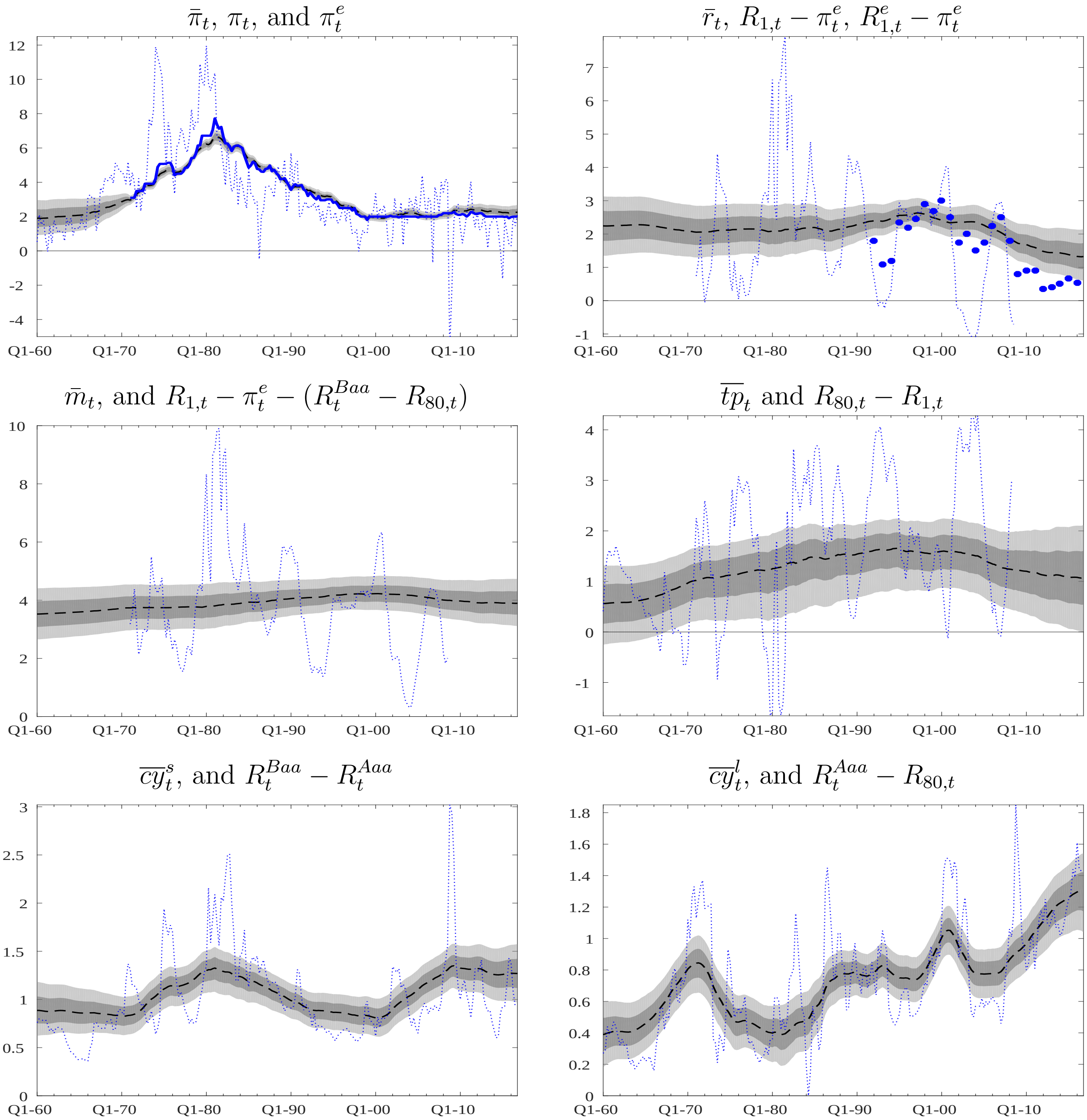
Note: The table shows the change in the trends for the different specifications described in section II.D, the model with default (column (1)), loose prior (column (2)), inflation trends in the term spread (column (3)), and labor productivity (column (4)). For each trend, the table shows the posterior median, the 68 (square bracket) and 95 (round bracket) percent posterior coverage intervals. The ** symbol indicates that the decline is significant, in that the 95 percent coverage intervals do not include zero.

Figure A10: Posterior Distribution of γ^{tp} – Model with Inflation Affecting the Nominal Term Premium



Note: The figure shows the posterior distribution of γ^{tp}. The prior is an expoential with mean .10.

Figure A11: Trends and Observables, Inflation Affecting the Nominal Term Premiuml



Note: The top left panel shows π_t (dotted blue line), and π_t^e (solid blue line), together with the trend $\bar{\pi}_t$. The top right panel shows $R_{1,t} - \pi_t^e$ (dotted blue line), and $R_{1,t}^e - \pi_t^e$ (blue dots), together with the trend $\bar{r}_t$. The middle left panel shows $R_{1,t} - \pi_t^e - (R_t^{Baa} - R_{80,t})$ (dotted blue line), together with the trend $\bar{m}_t$. The middle right panel shows $R_{80,t} - R_{1,t}$ (dotted blue line) together with the trend $\overline{tp}_t$. The bottom left panel shows the Baa/Aaa spread $R_t^{Baa} - R_t^{Aaa}$ (dotted blue line), together with the trend $\overline{cy}_t^s$. The bottom right panel shows the Aaa/Treasury spread $R_t^{Aaa} - R_{80,t}$ (dotted blue line), together with the trend $\overline{cy}_t^l$. For each trend, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

E Additional Table – DSGE (Section III)

Table A2: DSGE Parameter Estimates

		Prior		Posterior		
Parameter	Type	Mean	SD	Mean	90.0% Lower Band	90.0% Upper Band
			Steady State			
100γ	N	0.400	0.100	0.381	0.313	0.450
α	N	0.300	0.050	0.177	0.151	0.201
$100(\beta^{-1}-1)$	G	0.250	0.100	0.201	0.100	0.301
σ_c	N	1.500	0.370	1.097	0.922	1.282
h	B	0.700	0.100	0.503	0.436	0.574
ν_l	N	2.000	0.750	2.614	1.808	3.419
δ	-	0.025	0.000	0.025	0.025	0.025
Φ	-	1.000	0.000	1.000	1.000	1.000
S''	N	4.000	1.500	1.556	1.153	2.037
ψ	B	0.500	0.150	0.558	0.408	0.712
$\bar{L}$	N	-45.000	5.000	-47.415	-48.910	-45.793
λ_w	-	1.500	0.000	1.500	1.500	1.500
π_*	-	0.500	0.000	0.500	0.500	0.500
g_*	-	0.180	0.000	0.180	0.180	0.180
			Nominal Rigidities			
ζ_p	B	0.500	0.100	0.945	0.931	0.960
ζ_w	B	0.500	0.100	0.906	0.867	0.948
ι_p	B	0.500	0.150	0.237	0.098	0.365
ι_w	B	0.500	0.150	0.583	0.360	0.801
ε_p	-	10.000	0.000	10.000	10.000	10.000
ε_w	-	10.000	0.000	10.000	10.000	10.000
			Policy			
ψ_1	N	1.500	0.250	2.294	1.973	2.620
ψ_2	N	0.120	0.050	0.206	0.148	0.265
ψ_3	N	0.120	0.050	0.317	0.270	0.363
ρ_R	B	0.750	0.100	0.848	0.809	0.888
ρ_{rm}	B	0.500	0.200	0.229	0.139	0.318
			Financial Frictions			
$F(\bar{\omega})$	-	0.030	0.000	0.030	0.030	0.030
γ_*	-	0.990	0.000	0.990	0.990	0.990
$\zeta_{sp,b}$	B	0.050	0.005	0.047	0.039	0.055
SP_*	G	1.000	0.100	1.039	0.878	1.189
cy_*^l	-	0.117	0.000	0.117	0.117	0.117
cy_*^s	-	0.065	0.000	0.065	0.065	0.065
			Exogenous Processes			
ρ_g	B	0.500	0.200	0.983	0.971	0.996
ρ_μ	B	0.500	0.200	0.975	0.955	0.995
ρ_{z^p}	B	0.500	0.200	0.898	0.764	0.974
ρ_z	B	0.500	0.200	0.942	0.910	0.976
$\rho_{cy^p,l}$	-	0.990	0.000	0.990	0.990	0.990

Table A2: DSGE Parameter Estimates

	Prior			Posterior		
Parameter	Type	Mean	SD	Mean	90.0% Lower Band	90.0% Upper Band
$\rho_{\tilde{c}y,l}$	B	0.500	0.200	0.639	0.357	0.931
$\rho_{cy^p,s}$	-	0.990	0.000	0.990	0.990	0.990
$\rho_{\tilde{c}y,s}$	B	0.500	0.200	0.649	0.425	0.910
ρ_{σ_ω}	B	0.750	0.150	0.628	0.488	0.873
ρ_{π^*}	-	0.990	0.000	0.990	0.990	0.990
ρ_{λ_f}	B	0.500	0.200	0.791	0.677	0.911
ρ_{λ_w}	B	0.500	0.200	0.497	0.190	0.858
η_{λ_f}	B	0.500	0.200	0.630	0.429	0.848
η_{λ_w}	B	0.500	0.200	0.499	0.217	0.816
η_{gz}	B	0.500	0.200	0.434	0.132	0.722
σ_g	IG	0.100	2.000	2.234	2.039	2.418
σ_μ	IG	0.100	2.000	0.477	0.305	0.640
σ_{z^p}	IG	0.100	2.000	0.095	0.047	0.158
σ_z	IG	0.100	2.000	0.526	0.472	0.581
$\sigma_{cy^p,l}$	IG	0.013	100.000	0.013	0.012	0.015
$\sigma_{\tilde{c}y,l}$	IG	0.100	2.000	0.092	0.040	0.150
$\sigma_{cy^p,s}$	IG	0.013	100.000	0.011	0.010	0.013
$\sigma_{\tilde{c}y,s}$	IG	0.100	2.000	0.124	0.057	0.178
σ_{σ_ω}	IG	0.050	4.000	0.556	0.178	0.776
σ_{π^*}	IG	0.030	6.000	0.054	0.044	0.064
σ_{λ_f}	IG	0.100	2.000	0.079	0.062	0.096
σ_{λ_w}	IG	0.100	2.000	0.379	0.333	0.429
σ_{rm}	IG	0.100	2.000	0.229	0.206	0.251
$\sigma_{1,r}$	IG	0.200	4.000	0.096	0.075	0.117
$\sigma_{2,r}$	IG	0.200	4.000	0.087	0.068	0.106
$\sigma_{3,r}$	IG	0.200	4.000	0.089	0.069	0.109
$\sigma_{4,r}$	IG	0.200	4.000	0.086	0.067	0.103
$\sigma_{5,r}$	IG	0.200	4.000	0.088	0.068	0.108
$\sigma_{6,r}$	IG	0.200	4.000	0.094	0.071	0.115
$\sigma_{7,r}$	IG	0.200	4.000	0.236	0.111	0.375
$\sigma_{8,r}$	IG	0.200	4.000	0.241	0.108	0.386
$\sigma_{9,r}$	IG	0.200	4.000	0.242	0.111	0.374
$\sigma_{10,r}$	IG	0.200	4.000	0.248	0.107	0.399
$\sigma_{11,r}$	IG	0.200	4.000	0.247	0.107	0.400
$\sigma_{12,r}$	IG	0.200	4.000	0.245	0.108	0.390
				Measurement		
δ_{gdpdef}	N	0.000	2.000	-0.003	-0.049	0.043
γ_{gdpdef}	N	1.000	2.000	1.036	0.965	1.108
ρ_{gdp}	N	0.000	0.200	0.077	-0.137	0.293
ρ_{gdi}	N	0.000	0.200	0.946	0.909	0.986
ϱ_{gdp}	N	0.000	0.400	-0.138	-0.774	0.466
ρ_{gdpdef}	B	0.500	0.200	0.499	0.367	0.636
ρ_{pce}	B	0.500	0.200	0.269	0.064	0.467
ρ_{Aaa}	B	0.500	0.100	0.621	0.484	0.754
ρ_{Baa}	B	0.500	0.100	0.810	0.736	0.892

Table A2: DSGE Parameter Estimates

Parameter	Prior Type	Prior Mean	Prior SD	Posterior Mean	Posterior 90.0% Lower Band	Posterior 90.0% Upper Band
ρ_{10y}	B	0.500	0.200	0.959	0.935	0.984
ρ_{tfp}	B	0.500	0.200	0.175	0.073	0.270
σ_{gdp}	IG	0.100	2.000	0.255	0.213	0.297
σ_{gdi}	IG	0.100	2.000	0.306	0.271	0.340
σ_{gdpdef}	IG	0.100	2.000	0.164	0.146	0.181
σ_{pce}	IG	0.100	2.000	0.101	0.083	0.119
σ_{Aaa}	IG	0.100	2.000	0.024	0.021	0.028
σ_{Baa}	IG	0.100	2.000	0.049	0.042	0.055
σ_{10y}	IG	0.750	2.000	0.124	0.113	0.135
σ_{tfp}	IG	0.100	2.000	0.749	0.678	0.820

Note: T N, B and G stand, respectively, for Normal, Beta and Gamma distributions. For Inverse Gamma (IG) distributions, we report the coefficients τ and ν instead of the prior mean and SD.

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