FROM NIC TO TIC TO RAY:
CALCULATING TRUE LIFETIME COST OF CAPITAL
FOR MUNICIPAL BORROWERS

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Abstract: Cost of capital metrics for state/municipal government and not-for-profit borrowers have evolved over time from net interest cost (NIC) to true interest cost (TIC) to all-in TIC. However, each of these metrics is incomplete in that they all ignore the likelihood of refinancing given they are calculated using debt service to maturity. This is a significant shortcoming given the majority of fixed-rate, municipal bond issues are callable and issued with premium coupon rates that make future refinancing highly likely. This paper describes an improved lifetime cost of capital metric called Refunding Adjusted Yield (RAY). RAY incorporates refinancing probabilities utilizing the issuer’s own refinancing criteria in calculating cost of capital. RAY offers significant advantages in optimal bond structuring and is a more comprehensive and complete metric for use in financial policy decisions involving true capital cost.

Key words: municipal bonds, public financial management, net interest cost, true interest cost, refunding adjusted yield

1. Introduction

Like private sector businesses, state/municipal governments and not-for-profit entities sell debt instruments to fund their capital and operating budget activities. These instruments, often called municipal securities, finance critical infrastructure like roads, bridges, and airports as well as societal institutions like schools, hospitals, and universities.¹ Total capitalization of the U.S. municipal securities market is approximately $4 trillion, representing roughly 2% of the world’s financial assets.² Over the last ten years municipal borrowers have issued an average of $379.5 billion in long term fixed rate bonds per year.³ Despite the size and significance of the market, the primary cost of capital measures employed by municipal borrowers today are fundamentally incomplete in that they fail to account for the likelihood of refinancing callable bonds for interest cost savings. Such failure reduces financial management transparency and can lead to suboptimal capital market policy decisions by these borrowers. This paper describes an improved lifetime cost of capital metric called Refunding Adjusted Yield (RAY) which incorporates refinancing probabilities utilizing the issuer’s own refinancing criteria. RAY offers significant advantages in optimal bond structuring and is a more comprehensive and complete metric for

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¹ This paper will refer to state, municipal and not-for-profit issuers of municipal securities as “municipal borrowers”
² McKinsey Global Institute estimated the value of all debt and equity worldwide at $212 trillion in 2010.
³ The Bond Buyer, Statistical Supplement, 2016.
use in financial policy decisions that rely on cost of capital metrics.

2. Previous Research

Cost of capital refers to the cost of funds (usually equity or debt) required to finance an activity. For municipal borrowers this mainly entails the interest cost on their debt instruments since these entities generally do not sell equity. Prior to the 1970s, most municipal borrowers evaluated their cost of capital using the net interest cost (NIC) measure. NIC is calculated as the total amount of interest accrued in a bond issue less the amount of any premium or plus the amount of any discount divided by the product of the principal amount of the bonds maturing on each maturity date by the number of years from the issue date to their respective maturities. Beginning in the late 1960s and early 1970s, the NIC metric came under criticism as it did not take into account the time value of money. Hopewell and Kaufman (1974) evaluated the disadvantages of using NIC relative to true interest cost (TIC), a more internal rate of return-type metric that appropriately captures the time value of money. TIC is the rate that sets the present value of principal and interest payments equal to the net proceeds from the issue. If proceeds are further reduced by the costs of issuance at closing, this is called All-in TIC. TIC is formally defined in the following equation:

\[
\text{Net Bond Proceeds} = \sum_{i=1}^{n} \left( P_i + I_i \left( 1 + \frac{TIC}{2} \right)^{t_i} \right)
\]

where

- \( TIC \) = true interest cost
- \( i \) = scheduled payment dates for principal and/or interest
- \( P \) = principal payment at date \( i \)
- \( I \) = interest payment at date \( i \)
- \( t_i \) = number of 30/360 semi-annual periods from issue date to date \( i \)
- \( n \) = number of payment dates through final bond maturity

Subsequent research also criticized the use of NIC claiming it lead to flawed financial policy making. Braswell, Nosari and Sumners (1983) analyzed the use of net interest cost in evaluating which

\[4\] By convention, TIC and all-in TIC are calculated using semi-annual discounting and a 30/360 day count.
bond sale method, competitive or negotiated, results in the lowest borrowing costs. Their research used TIC rather than NIC to evaluate the sale method question as they claimed NIC was an inferior measure of the dependent variable in this line of research. More recent research continued to detail the benefits of TIC over NIC. For example, Benson (1999) estimated the cost to municipalities still using NIC instead of TIC in competitive bond sales.

Though TIC (or all-in TIC) is now used predominately by municipal borrowers for calculating cost of capital, it has been criticized in recent years for not being consistent or complete. Simonsen and Robbins (2001; 2002) note that some municipal borrowers calculate TIC to the dated date, while others calculate it to the delivery date with associated offset of accrued interest. Further, they point out TIC fails to incorporate the effect of other funds associated with borrowings such as capitalized interest or debt service reserves. For these reasons they conclude that standard TIC calculations understate true borrowing cost. They introduce the concept of internal financing rate (IFR) which reflects payments on these funds, all issuance costs, and is calculated to the delivery date by definition. For these reasons, Simonsen and Robbins claim IFR is a more comprehensive measure in calculating the true cost of capital compared to TIC. However, many municipal security offerings do not include capitalized interest or debt service reserve funds (i.e., most general obligation bonds) so the benefit of IFR is limited to securities that include these funds.

3. Limitation of Previous Cost of Capital Measures

The evolution of cost of capital measures from NIC to TIC to IFR has certainly improved the accuracy and comprehensiveness of municipal bond borrowing cost calculations. However, all these previous measures fail to take into account a significant aspect of most municipal securities offerings: the

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5 We note that IFR appears to omit other costs of borrowing in the public markets such as trustee fees, meeting ongoing disclosure requirements, and tax compliance costs. If these estimates are identical across structures being compared, the relative attractiveness will not change. In that case, these costs may be safely omitted.
6 This paper focuses on the limitations of TIC rather than IFR since TIC is much more commonly used by municipal borrowers. However, our criticisms of TIC can just as easily be extended to IFR since neither measure takes into account future refinancing impacts.
ability to call bonds early through refinancing which can result in lower lifetime interest costs. Specifically, the majority of municipal fixed-rate bond issues have optional redemption features that give the issuer the right to redeem the bonds at a specified price, usually par. Currently, these callable bonds tend to be issued with a premium coupon where the bond yield is calculated to the call date. Despite this fact, the capital cost metrics in the existing literature and predominantly used in practice ignore call features, incorporating principal and interest calculations to maturity only. Note that the net proceeds raised by the borrower are clearly impacted by the existence of this call feature: the left side of the TIC equation (i.e., net bond proceeds) is calculated assuming certain bonds are priced to their call date. However, the right side of the equation shows cash flows to maturity only and ignores the likelihood that the bonds may be refinanced to achieve nominal interest cost reduction.

This cognitive and calculated disconnect is significant as the call feature can be worth 5% or more of originally issued par relative to the bond’s non-callable equivalent, particularly in light of the premium coupons commonly issued today. This disconnect in TIC, by definition, results in an overstatement of municipal borrowers’ true expected lifetime cost of capital because debt service used in the TIC calculation is assumed to run to maturity, even for callable bonds. The financial policy implications of this overstatement are far reaching and generally include 1) reduced transparency in bond borrowings that can be misleading to elected officials, rating agencies, investors and the public, 2) flawed decision-making in choosing optimal bond structures and in the timing and amount of future debt issues, and 3) inappropriate competitive bid awards. A fuller exposition of these policy implications is discussed later in this paper.

4. A Better Cost of Capital Measure: Refunding Adjusted Yield (RAY)

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7 See (Landoni, 2014) for optimal couponing by municipal issuers and optimal trading behavior by municipal bond investors. Also, see MSRB rules G-12(c)(v)(l) and G-15(a)(v)(l) defining when a municipal bond’s yield is yield-to-worst.

8 Though no formal research exists, an idea named “TIC+” also uses cash flows to maturity but proposes increasing the net borrower proceeds by some estimate of call value to lower the capital cost calculation.

9 5% estimate of call value worth is based on author’s calculations which are available upon request.
The problematic assumption of TIC that debt service will be paid to maturity is addressed by a concept we call refunding adjusted yield (RAY). RAY aims to incorporate the possibility that a municipal borrower will refinance a new municipal securities offering sometime in the future. In the parlance of the municipal securities market, these refinancings are known as “refundings”, a term we will use in this paper. In order to calculate RAY, we first must realistically model municipal refundings. Municipal refundings (and therefore callable municipal bonds) are complicated in part because their complete analysis involves not only different points on the yield curve (“tenors”) but even entirely different markets. As such there is some debate as to the appropriate type of model. In this paper we use a real-world market model that offers the ability to capture multiple tenors from different markets simultaneously (Deguillaume, Rebonato, and Pogudin, 2013). The model by construction perfectly captures the historic covariance of each modeled tenor, both intra and inter-market. We create callable AAA, AA, A, BBB and state and local government securities (SLGS) escrow markets across 3 month and 1, 3, 5, 10, 15, 20, and 30 year tenors. Data is derived from Apr 5, 1987 through Apr 5, 2012. The starting and ending horizon yield curves for the borrower’s bond yields are shown in Figure 1.

[FIGURE 1]

In modeling refunding, we use the following assumptions:

1. Refunding bonds: assumed to be matched maturity par bonds; refunding bonds with maturities greater than 10 years are callable in 10 years at par
2. Refunding policy: 5% present value savings; refunding criteria are tested quarterly and a simulated refunding occurs on the same date criteria are satisfied.
3. Advance refundings: only tax-exempt advance refundings are calculated.

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10 Such debate is beyond the scope of this paper. However, we feel the use of standard, lognormal bond option pricing models like Black, Derman, Toy (1990) or Black-Karasinski models are inappropriate for two primary reasons. First, the municipal market is not arbitrage free. Second, the purpose of the analysis from the issuer’s perspective is one of performance and risk management, not relative pricing in a no-arbitrage setting. For more details in the tax-exempt market see Orr & de la Nuez (2013). For a more general discussion see Nawalkha and Rebonato (2011).

11 Although this is a 40 factor model as is, it can be extended to include interest rate swap curves, other fixed-income markets, currencies, and even investment returns. The covariance matrix for all simulated market elements is preserved and we are limited only by memory and computational resources.

12 For market model details see Deguillaume, Rebonato, and Pogudin, (2013) and Orr and de la Nuez (2014).

13 A more realistic assumption would be to introduce a 30 day lag between the time when criteria are satisfied and when refunding hypothetically occurs, though this has no effect on relative results.
4. Cash flow savings: interest cost reduction is taken in equal annual amounts starting from the simulated refunding date through the maturity of the refunded bond. Savings in short periods is pro-rated.

5. Costs of issuance: assumed one percent costs of issuance throughout.

6. Escrow cost: calculated using the yield to the call date from the escrow yield curve from the same simulated environment.

7. Present value savings: the difference between the then market value of the refunding bond, incorporating escrow and issuance costs, and the value of the refunded bond to maturity.

Based on the above assumptions, Figure 2 analyzes the interest costs on a hypothetical $10 million par bond with a 5% coupon rate maturing in 20 years callable in 10 years. One nuance in looking at true issuer capital cost is that the refunding bonds themselves may be callable, giving the issuer the ability to effectively refinance the original debt multiple times over the life of the project. We call an initial refunding of bonds originally issued to fund a project a “first generation” refunding. A refunding of the first generation refunding bonds is a “second generation” refunding. Figure 2 illustrates the difference in annual interest cost to maturity versus expected interest costs taking into account first and second generation refundings assuming a 5% present value savings policy as the refunding trigger. The black lines in Figure 2 show the difference between interest cost to maturity (solid) and expected interest expense (dashed). Note that the expected interest expense falls as first generation refundings lead to cash flow benefit. Close to year 10 we begin to see second generation refundings of the callable refunding bonds issued in the first generation. Probability of first generation refunding approaches 90% by the maturity of the bond (solid blue line, right vertical axis) while the probability of second generation refunding, tax-exempt refunding of the callable refunding bond, peaks at roughly 34% (dashed blue line, right axis). By the end of the ten years interest expense has dropped by over 10% to under $445,000. Armed with new expected cash flows, we can calculate RAY and compare this to traditional yields to call and maturity.

[FIGURE 2]

14 A natural extension would be to include taxable advance refunding bonds for those bonds ineligible for tax-exempt advance refunding.
Table 1 summarizes the various capital cost metrics. RAY1 is the refunding adjusted yield incorporating only first generation refunding savings. RAY includes both first and second generation refundings and, as expected, is a lower rate than RAY1, in this case by 11 basis points. Yield to maturity assumes that the debt service is paid through maturity.\(^{15}\) Yield to call is the yield assuming debt service to the call date at which point the bond will be called for redemption. RAY is 11 basis points higher than yield to call and approximately 43 basis points lower than yield to maturity. Yield to maturity will always be an upper bound on RAY per bond, assuming the same target value for the yield.\(^{16}\) Table 1 also shows the expected present value savings (as a percentage of refunded bond par) both for first generation refunding (EPV1) and total present value savings (as a percentage of refunded par) from both refundings (EPV). Given a 5% refunding policy threshold, EPV1 is approximately 5.2%. When the savings from second generation refunding is included, EPV savings increases another 40% to 7.29%. These expected present value savings estimates clearly illustrate that refunding savings should not be ignored when measuring expected lifetime costs of capital.

**[TABLE 1]**

### 5. How Municipal Borrower Refunding Behavior Affects RAY

Since a municipal borrower’s call provisions are often either practically or legally fixed, the refunding decision embeds the borrower’s risk preference. This risk preference relates to the timing in which a municipal borrower thinks it is most advantageous to refinance its debt. The risk preference is manifest in its debt policy when it describes the parameters acceptable to refinance debt. As described earlier, a common practice by municipal borrowers is to execute refundings when the present value savings exceed some threshold (e.g., 5% present value savings). This risk preference drives a refunding tendency that will impact the overall debt service an issuer expects to pay on bonds both individually and

\(^{15}\) TIC is essentially the aggregate yield to maturity of all the individual bond maturities of a bond issue. We use yield to maturity instead of TIC in this section of the paper since we are only looking at one bond maturity not a bond issue.

\(^{16}\) Though rare, there may be cases however where RAY is actually lower than yield to call. With sufficiently low borrower and high escrow yield expectations, simulated refundings will lead to a RAY lower than yield to call.
a debt portfolio. We quantify this relationship in Figure 3 for first generation refundings for two bonds, a 4% and a 5% coupon bond both with 20-year maturities but callable in 10 years, at different present value savings refunding thresholds ranging from 1% to 12%.

[FIGURE 3]

The horizontal axis in Figure 3 represents different refunding policies used to simulate refundings that ultimately lead to adjusting debt service for each bond. The yield to maturity for the 5% and 4% bond (dotted green and blue line respectively) are invariant to the change in present value savings policy, and in all cases above their respective RAY1s. Starting with a low 1% present value savings threshold, RAY1 falls as the threshold increases for the 4% and 5% bond (solid blue and green lines). This occurs because, as we move to the right in the chart, the lower probability of refunding is more than offset by the improved cash flow savings when refunding occurs. However, this relationship has its limits. A minimum is reached for both bonds indicating that the aforementioned tradeoff begins to tilt more towards the fact that refundings occur too infrequently to compensate for the higher savings threshold. At this point, RAY1 begins to rise.

Figure 4 extends the previous analysis in Figure 3 to include the second generation refunding as well. As shown in the figure, the RAYs have become much closer between for the 4% and 5% coupon bonds as the second generation refundings for the 5% coupon bond has a greater impact on RAY than those for the 4% bond. This is intuitive as refundings of the 5% bond are more likely to be themselves callable, and hence available for future expected debt service reduction. Most importantly for understanding an issuer’s expected cost of capital, the RAYs for the 4% and 5% bond are 0.22% and 0.44% lower than their respective yields to maturity. Again, this illustrates the overstatement of capital cost using yield to maturity compared to a metric such as RAY that incorporates future refinancings.

[FIGURE 4]

6. NIC vs. TIC vs RAY – A Real World Example
Looking at a real-world example we analyzed the $387,025,000 State of Wisconsin’s General Obligation Bonds of 2015, Series C, issued in September 2015. Pricing and maturity details for this issue are shown in Table 2. This bond series has a twenty-year final maturity (2036) and first call date on May 1, 2024 for the bonds maturing between 2025 and 2036. At approximately 8.5 years to the first optional redemption date, this issue has a shorter call feature than the standard, 10-year call accompanying most fixed-rate municipal bond issues. Principal and interest payments (i.e. debt service) to maturity are shown in the bars in Figure 5.

The traditional NIC, TIC and all-in TIC calculations for this issue are 3.666%, 3.318% and 3.404% respectively. Using the AA simulation and the same refunding assumptions as those in the prior section, Table 3 shows the all-in RAY (assuming first and second generation refundings) for these bonds as 3.04% or 0.36% lower than all-in TIC. Assuming just a first generation refunding, the all-in RAY is 3.12% or 0.28% lower than all-in TIC. This calculation uses the same target value as all-in TIC but the principal and interest payments of the callable bonds reflect first or first and second generation refunding activity using the State’s own refunding criteria. The State of Wisconsin’s actual refunding criteria sets the present value savings threshold at 3% with sensitivity analysis on present value savings assuming interest rates decline in the future (see Note 2 in Table 3). Figure 5 shows aggregate debt service to maturity in the bars, and refunding adjusted (or expected) debt service in the dashed lines assuming first generation or first and second generation refundings. Note that starting from date of issue, simulated refunding activity occurs which gradually increases savings and decreasing expected debt service. However, as callable bonds begin to mature after the call date the amount of effective cash flows savings begins to decline and adjusted debt service moves back closer to debt service to maturity.
Table 3 evidences the robustness of the RAY analysis by altering the bond refunding criteria. The table shows refunding adjusted statistics for this bond issue using five different refunding policies. Similar to the 4% and 5% coupon case, we note that the RAY is higher at the high and low present value savings thresholds and for the same reasons discussed above. But more important are the differences between these numbers and the all-in TIC of 3.40%. RAY is between 30 and 36 basis points lower than all-in TIC under all five refunding criteria. From a dollar budgetary perspective, TIC overstates lifetime capital cost for the state of Wisconsin on a present value basis by over $14.75 million, or nearly 4% of issue par.

A couple other statistical features of RAY are worthy of note as shown in Table 3. First, the aggregate probability of callable bonds being refunded goes downward as the present value savings threshold increases, as one would expect. However, the present value savings generally increase as the refunding threshold becomes more stringent (i.e., higher) which provides empirical support for issuer’s adopting more stringent refunding policies. Second, the refunding adjusted weighted average life of the bonds is materially lower than the weighted average life to maturity which empirically shows how more traditional “to-maturity” bond statistics like TIC and weighted average life overstate an issuer’s debt burden both in terms of total interest costs and term to maturity.

7. Policy Implications

There are several policy implications that emanate from our claim that RAY is a superior estimate of lifetime cost of capital for municipal borrowers compared to TIC. The use of TIC rather than RAY leads to flawed financial policy in several ways. First, an overstated cost of capital measure can lead to incorrect project selection in the capital budgeting process. Since project selection is often based on net present value analysis, an overstated discount rate will lead to a lower net present value, all else equal. This could lead to some “borderline” projects not selected even though they provide a positive net present value. Second, an inappropriate cost of capital measure will bias bond structure decisions towards alternatives with less refinancing flexibility. For example, using a capital cost measure that does not take
into account the likelihood of future refinancing may result in a municipal borrower selecting coupon rates that ultimately result in higher interest costs over time. Third, overstated cost of capital measures inherently bias the bond structure decision towards the use of variable rate debt over fixed rate debt. Since TIC generally overstates lifetime interest costs, variable rate debt looks artificially more attractive given the historical interest cost benefit of variable rate compared to fixed rate debt.

The fourth policy implication involves the competitive bid process. By not taking into account the likelihood of refinancing, competitive bids with reduced call flexibility will be advantaged even though bids with greater call flexibility will likely result in lower interest costs over time assuming future refinancing(s). Fifth, municipal borrowers will understate debt capacity given the overstatement of interest costs. The understatement of debt capacity will make the pay-as-you-go financing approach more attractive than the pay-as-you-use approach, which may not be optimal. Finally, using overstated cost of capital measures like TIC reduces the financial management transparency of municipal borrowers. Citizens will overstate the future debt burden of their jurisdictions based on debt service as a percentage of revenues or expenditures metrics. Such overstatement can also compromise municipal borrowers’ credit ratings as the rating agencies will rely on debt service disclosures by these governments in assessing their financial condition.

8. Conclusion

In this paper we show that there is significant value in using an alternative quantification of debt service costs of callable fixed rate bonds to TIC. We introduce a new measure called RAY that improves on TIC as a cost of capital measure because of its inclusion of future refinancing in its calculation. Through a calculation of expected debt service based upon a real-world market model and the issuer’s own refunding criteria, we conclude that RAY provides a more accurate estimate of lifetime project financing cost. This is an important component of many essential financial decisions for tax-exempt borrowers.

However, since RAY relies on probability analysis in calculating cost of capital, RAY will likely
never represent the actual cost of capital of a bond issue, just as TIC likely will not. Thus, this paper’s analyses may raise more questions than it answers as it relates to the actual use of RAY. For bond structuring purposes, should issuers use debt service to maturity or refunding adjusted debt service? How should RAY be incorporated in new issue bond structuring or refunded bond selection or both? This question also applies to debt capacity and feasibility analyses. Since RAY, as a mean, is essentially a first moment of an entire distribution of possible financing costs, should higher moments be explored? RAY volatility, a downside RAY or perhaps a “95% RAY-at-Risk,” similar to Value-at-Risk so frequently used in the context of risk management? These questions all get at the overarching question of when is it more important to be approximately correct rather than precisely wrong. In our case this question is very germane to the valuation of the call features embedded in callable municipal bonds in calculating cost of capital. Future research should build on the basic model presented in this paper to address these important operational questions.

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17 See forthcoming research, *Municipal Bond Structuring: Minimizing Lifetime Expected Borrowing Cost from Intuitive Analytics*

18 Full credit to George Box’s famous quote which we have paraphrased here.
9. References


10. Tables and Figures

Figure 1. 10 Year Callable AA Yield Simulation

![10Y Callable AA Yield Simulation](image)

Figure 2. Refunding Adjusted Interest vs. To Maturity Interest / Refunding Probabilities

![Refunding Adj vs To Maturity Interest / Refunding Probabilities](image)
Table 1. Refunding Statistics

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<th>Coupon Rate</th>
<th>Maturity</th>
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Figure 3. Cost of Capital Comparison between TIC and RAY, 4% and 5% Coupons, 1st Generation Refunding Only
Figure 4. Cost of Capital Comparison between TIC and RAY, 4% and 5% Coupons, 1st and 2nd Generation Refundings
Table 2. Bond Pricing

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<td>22,795,000</td>
<td>5.00</td>
<td>2.870</td>
<td>3.634</td>
<td>116.168</td>
<td>26,480,495.60</td>
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<tr>
<td>5/1/32</td>
<td>23,990,000</td>
<td>5.00</td>
<td>2.910</td>
<td>3.714</td>
<td>115.837</td>
<td>27,789,296.30</td>
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<tr>
<td>5/1/33</td>
<td>25,245,000</td>
<td>5.00</td>
<td>2.950</td>
<td>3.786</td>
<td>115.507</td>
<td>29,159,742.15</td>
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<tr>
<td>5/1/34</td>
<td>26,570,000</td>
<td>5.00</td>
<td>2.990</td>
<td>3.850</td>
<td>115.178</td>
<td>30,602,794.60</td>
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<tr>
<td>5/1/35</td>
<td>27,950,000</td>
<td>5.00</td>
<td>3.020</td>
<td>3.903</td>
<td>114.932</td>
<td>32,123,494.00</td>
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<tr>
<td>5/1/36</td>
<td>29,420,000</td>
<td>5.00</td>
<td>3.060</td>
<td>3.957</td>
<td>114.605</td>
<td>33,716,791.00</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>387,025,000</strong></td>
<td></td>
<td><strong>3.060</strong></td>
<td><strong>3.957</strong></td>
<td><strong>114.605</strong></td>
<td><strong>448,395,714.50</strong></td>
</tr>
</tbody>
</table>

**Figure 5.** Debt Service to Maturity versus Expected (Refunding Adjusted) Debt Service

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- **Principal**: Amount paid towards the original loan amount.
- **Interest**: Amount paid towards the interest on the loan.
- **Expected DS, 1st Generation**: Expected debt service for the first generation of bonds.
- **Expected DS, 2nd Generation**: Expected debt service for the second generation of bonds.
### Table 3. Bond Summary Statistics

<table>
<thead>
<tr>
<th>Refunding Policy (PV savings as percent of refunded bonds Par, except “State”)</th>
<th>State²</th>
<th>3%</th>
<th>5%</th>
<th>7%</th>
<th>9%</th>
</tr>
</thead>
<tbody>
<tr>
<td>All-in-TIC¹</td>
<td>3.40%</td>
<td>3.40%</td>
<td>3.40%</td>
<td>3.40%</td>
<td>3.40%</td>
</tr>
<tr>
<td>RAY¹</td>
<td>3.04%</td>
<td>3.10%</td>
<td>3.07%</td>
<td>3.08%</td>
<td>3.10%</td>
</tr>
<tr>
<td>RAY¹²</td>
<td>3.12%</td>
<td>3.18%</td>
<td>3.14%</td>
<td>3.12%</td>
<td>3.12%</td>
</tr>
<tr>
<td>Average Refunded</td>
<td>225,193,867</td>
<td>229,265,966</td>
<td>201,419,337</td>
<td>171,134,329</td>
<td>140,064,332</td>
</tr>
<tr>
<td>Probability Callable Bonds Refunded</td>
<td>82.46%</td>
<td>83.95%</td>
<td>73.75%</td>
<td>62.66%</td>
<td>51.28%</td>
</tr>
<tr>
<td>Average PV Savings, $</td>
<td>16,481,056</td>
<td>13,631,850</td>
<td>14,759,124</td>
<td>14,811,828</td>
<td>14,007,432</td>
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<tr>
<td>Average PV Savings, %</td>
<td>6.46%</td>
<td>5.35%</td>
<td>5.79%</td>
<td>6.28%</td>
<td>5.94%</td>
</tr>
<tr>
<td>Average Time to Refunding</td>
<td>4.18</td>
<td>3.2</td>
<td>4.3</td>
<td>5.0</td>
<td>5.7</td>
</tr>
<tr>
<td>Weighted Average Life (WAL)</td>
<td>12.6</td>
<td>12.6</td>
<td>12.6</td>
<td>12.6</td>
<td>12.6</td>
</tr>
<tr>
<td>Refunding Adjusted WAL</td>
<td>8.25</td>
<td>8.1</td>
<td>8.5</td>
<td>9.0</td>
<td>9.6</td>
</tr>
</tbody>
</table>

¹ Assumes a cost of issuance of 1% of par.
² State of Wisconsin actual refunding policy: 3% PV savings, 50% Opportunity Cost Index (OCI) sensitivity and 90% negative arbitrage / PV savings.