# ONLINE APPENDIX

**Appendix Table A1. Effect of using weights and truncation of the sample on basic moments for inflation forecasts and perceived inflation in New Zealand.**

| | Sample and weights | | | | | |
|---|---|---|---|---|---|---|
| **Survey Date** | Full sample | | Truncated sample with sample weights | | Full sample with sample weights | |
| | Mean | SD | Mean | SD | Mean | SD |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| | **Panel A: 1-year-ahead inflation forecasts, percentage points** | | | | | |
| **2013Q4** | 7.1 | 5.9 | 5.3 | 3.3 | 6.5 | 5.0 |
| **2014Q1** | 7.5 | 5.0 | 6.0 | 2.7 | 8.0 | 5.7 |
| **2014Q3** | 4.4 | 2.8 | 4.3 | 2.5 | 4.5 | 2.9 |
| **2014Q4** | 4.9 | 3.1 | 4.8 | 2.9 | 5.0 | 3.2 |
| | **Panel B: 5-to-10-year-ahead inflation forecast, percentage points** | | | | | |
| **2014Q3** | 3.5 | 2.6 | 3.6 | 2.5 | 3.7 | 2.8 |
| | **Panel C: 1-year inflation nowcasts/backcasts, percentage points** | | | | | |
| **2013Q4** | 5.3 | 4.2 | 4.3 | 3.5 | 5.1 | 4.6 |
| **2014Q1** | 5.9 | 4.3 | 5.8 | 3.4 | 6.9 | 5.4 |
| **2014Q3** | n.a. | n.a. | n.a. | n.a. | n.a. | n.a. |
| **2014Q4** | 4.3 | 2.5 | 4.1 | 2.3 | 4.1 | 2.5 |

Notes: The table presents inflation forecasts/nowcasts/backcasts of firms' managers. *Truncated sample* refers to when we exclude responses that are greater than 15 percentage points or less than -2 percentage points. *Full sample* includes all observations. Moments in columns (3)-(6) are constructed using sample weights. See the text and note to Table 1 for more details. Sample weights are based on firm counts. Results are similar if weights are based on employment counts.

**Appendix Table A2. Distribution of firm sizes (firm count) in the survey data vis-à-vis census data.**

| Employment size | Finance, Insurance, and Business services | | | | Manufacturing | | | | Other services | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | survey | | | census | survey | | | census | survey | | | census |
| | raw | weight A | weight B | | raw | weight A | weight B | | raw | weight A | weight B | |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) |
| Panel A: Share in the total number of firms | | | | | | | | | | | | |
| 6-9 | 0.27 | 0.42 | 0.43 | 0.43 | 0.24 | 0.33 | 0.34 | 0.33 | 0.29 | 0.44 | 0.44 | 0.43 |
| 10-19 | 0.31 | 0.34 | 0.34 | 0.33 | 0.36 | 0.33 | 0.35 | 0.33 | 0.32 | 0.35 | 0.35 | 0.34 |
| 20-49 | 0.27 | 0.17 | 0.16 | 0.16 | 0.23 | 0.22 | 0.19 | 0.22 | 0.26 | 0.16 | 0.15 | 0.16 |
| 50-99 | 0.13 | 0.04 | 0.04 | 0.04 | 0.14 | 0.07 | 0.07 | 0.07 | 0.12 | 0.04 | 0.05 | 0.05 |
| 100+ | 0.02 | 0.03 | 0.03 | 0.04 | 0.03 | 0.05 | 0.06 | 0.05 | 0.01 | 0.01 | 0.02 | 0.03 |
| Panel B: Share in the total employment | | | | | | | | | | | | |
| 6-9 | 0.07 | 0.13 | 0.13 | 0.13 | 0.06 | 0.08 | 0.08 | 0.08 | 0.09 | 0.18 | 0.18 | 0.15 |
| 10-19 | 0.15 | 0.20 | 0.20 | 0.18 | 0.15 | 0.14 | 0.14 | 0.14 | 0.18 | 0.27 | 0.26 | 0.22 |
| 20-49 | 0.31 | 0.23 | 0.22 | 0.20 | 0.25 | 0.24 | 0.21 | 0.20 | 0.36 | 0.29 | 0.26 | 0.24 |
| 50-99 | 0.31 | 0.11 | 0.12 | 0.12 | 0.31 | 0.17 | 0.17 | 0.16 | 0.32 | 0.16 | 0.17 | 0.15 |
| 100+ | 0.17 | 0.32 | 0.33 | 0.38 | 0.23 | 0.37 | 0.40 | 0.43 | 0.05 | 0.09 | 0.13 | 0.24 |
| Panel C: Average firm size | | | | | | | | | | | | |
| 6-9 | 7.4 | 7.4 | 7.4 | 7.3 | 7.7 | 7.8 | 7.8 | 7.3 | 7.4 | 7.4 | 7.4 | 7.2 |
| 10-19 | 13.8 | 13.9 | 13.9 | 13.3 | 13.7 | 13.8 | 13.7 | 13.4 | 13.9 | 13.8 | 13.8 | 13.3 |
| 20-49 | 32.4 | 32.3 | 32.3 | 29.7 | 36.1 | 36.5 | 36.3 | 29.7 | 33.0 | 32.7 | 32.6 | 29.2 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 50-99 | 68.5 | 68.3 | 68.2 | 68.4 | 73.6 | 74.0 | 73.6 | 69.8 | 63.9 | 63.9 | 63.3 | 68.6 |
| 100+ | 247.8 | 246.1 | 241.5 | 260.7 | 220.9 | 243.9 | 241.2 | 252.1 | 128.4 | 119.1 | 118.1 | 191.9 |

Notes: Columns (1), (5),and (9) report distributions in the survey without weights. Columns (4), (8), and (12) report distributions in the population (Census). Columns (2), (3), (6), (7), (10), and (11) reports distribution after applying weights to match the distribution in the population. Sample “weight A” is based on firm count. Sample “weight B” is based on firm employment.

## Proofs

### Proof of Proposition 1.

Suppose expectations are ideally $\epsilon$-anchored at time $t$ for horizon $\tau$, then $\forall i \in [0,1]$ we have

$$\pi^i_{t+\tau|t} = \int_{\pi^*-\epsilon}^{\pi^*+\epsilon} x dF^i_{t+\tau|t}(x) \leq \pi^* + \epsilon$$

Similarly we can show that $\pi^i_{t+\tau|t} \geq \pi^* - \epsilon$. Hence,

$$\left(\pi^i_{t+\tau|t} - \pi^*\right)^2 < \epsilon^2 \Rightarrow \int_0^1 \left(\pi^i_{t+\tau|t} - \pi^*\right)^2 di < \epsilon^2$$

$$\Rightarrow \int_0^1 \left(\pi^i_{t+\tau|t} - \int_0^1 \pi^i_{t+\tau|t} di + \int_0^1 \pi^i_{t+\tau|t} di - \pi^*\right)^2 di < \epsilon^2$$

$$\Rightarrow \int_0^1 \left(\pi^i_{t+\tau|t} - \int_0^1 \pi^i_{t+\tau|t} di\right)^2 di + \int_0^1 \left(\int_0^1 \pi^i_{t+\tau|t} di - \pi^*\right)^2 di < \epsilon^2$$

$$\Rightarrow bias^2_{t+\tau|t} + \int_0^1 \left(\pi^i_{t+\tau|t} - \int_0^1 \pi^i_{t+\tau|t} di\right)^2 di < \epsilon^2$$

Thus, $bias_{t+\tau|t} < \epsilon$. Q.E.D.

### Proof of Proposition 2.

Notice that similar to proof of Proposition 1 we can show that as expectations are strongly $\epsilon$-anchored,

$$\left(\pi^i_{t+\tau|t} - \int_0^1 \pi^i_{t+\tau|t} di\right)^2 < \epsilon^2, \forall i \in [0,1] \Rightarrow \int_0^1 \left(\pi^i_{t+\tau|t} - \int_0^1 \pi^i_{t+\tau|t} di\right)^2 di < \epsilon^2 \Rightarrow sd_{t+\tau|t} < \epsilon.$$

Note that we cannot say anything about the bias relative to the central bank's target here as being strongly anchored does not imply anything about expectations being close to that of the central bank's. Q.E.D.

**Proof of Lemma 1.** Suppose inflation expectations are ideally $\frac{\epsilon}{2}$-anchored, then from proof of Proposition 1 recall that

$$\pi_{t+\tau|t}^{i} < \pi^{*} + \frac{\epsilon}{2}, \forall i \in [0,1] \Rightarrow \int_{0}^{1} \pi_{t+\tau|t}^{i} di < \pi^{*} + \frac{\epsilon}{2} \Rightarrow \int_{0}^{1} \pi_{t+\tau|t}^{i} di - \epsilon < \pi^{*} - \frac{\epsilon}{2}$$

Now, since $F_{t+\tau|t}^{i}\left(\pi^{*} - \frac{\epsilon}{2}\right) = 0, \forall i \in [0,1]$ and any CDF is weakly increasing, we must have

$$F_{t+\tau|t}^{i}\left(\int_{0}^{1} \pi_{t+\tau|t}^{i} di - \epsilon\right) = 0, \forall i \in [0,1].$$

Similarly,

$$\pi_{t+\tau|t}^{i} > \pi^{*} - \frac{\epsilon}{2}, \forall i \in [0,1] \Rightarrow \int_{0}^{1} \pi_{t+\tau|t}^{i} di > \pi^{*} - \frac{\epsilon}{2} \Rightarrow \int_{0}^{1} \pi_{t+\tau|t}^{i} di + \epsilon > \pi^{*} + \frac{\epsilon}{2}.$$

Again, since $F_{t+\tau|t}^{i}\left(\pi^{*} + \frac{\epsilon}{2}\right) = 1, \forall i \in [0,1]$, and any CDF is weakly increasing, we must have

$$F_{t+\tau|t}^{i}\left(\int_{0}^{1} \pi_{t+\tau|t}^{i} di + \epsilon\right) = 1, \forall i \in [0,1].$$

Thus,

$$F_{t+\tau|t}^{i}\left(\int_{0}^{1} \pi_{t+\tau|t}^{i} di + \epsilon\right) - F_{t+\tau|t}^{i}\left(\int_{0}^{1} \pi_{t+\tau|t}^{i} di - \epsilon\right) = 1, \forall i \in [0,1].$$

which implies that expectations are strongly $\epsilon$-anchored. Notice that the reverse does not need to be true as being strongly anchored does not imply anything about expectations being close to the forecast of the central bank. Q.E.D.

**Proof of Proposition 3.**

Notice that

$$\begin{aligned} E_{t}^{i}\left\{\left(\pi_{t+\tau} - \pi_{t+\tau|t}^{i}\right)^{2}\right\} &= \int_{\pi_{t+\tau|t}^{i}-\epsilon}^{\pi_{t+\tau|t}^{i}+\epsilon} \left(x - \pi_{t+\tau|t}^{i}\right)^{2} dF_{t+\tau|t}^{i}(x) \\ &< \epsilon^{2} \int_{\pi_{t+\tau|t}^{i}-\epsilon}^{\pi_{t+\tau|t}^{i}+\epsilon} dF_{t+\tau|t}^{i}(x) \\ &= \epsilon^{2}. \end{aligned}$$

Q.E.D.

**Proof of Lemma 2.**

Recall from proof of Lemma 1 that if expectations are ideally $\frac{\epsilon}{2}$-anchored, then

$$\pi^i_{t+\tau|t} < \pi^* + \frac{\epsilon}{2}, \forall i \in [0,1] \Rightarrow \pi^i_{t+\tau|t} - \epsilon < \pi^* - \frac{\epsilon}{2}$$

Now, since $F^i_{t+\tau|t}\left(\pi^* - \frac{\epsilon}{2}\right) = 0, \forall i \in [0,1]$ and any CDF is weakly increasing, we must have

$$F^i_{t+\tau|t}\left(\pi^i_{t+\tau|t} - \epsilon\right) = 0, \forall i \in [0,1].$$

Similarly,

$$\pi^i_{t+\tau|t} > \pi^* - \frac{\epsilon}{2}, \forall i \in [0,1] \Rightarrow \pi^i_{t+\tau|t} + \epsilon > \pi^* + \frac{\epsilon}{2}.$$

Again, since $F^i_{t+\tau|t}\left(\pi^* + \frac{\epsilon}{2}\right) = 1, \forall i \in [0,1]$, and any CDF is weakly increasing, we must have

$$F^i_{t+\tau|t}\left(\pi^i_{t+\tau|t} + \epsilon\right) = 1, \forall i \in [0,1].$$

Thus,

$$F^i_{t+\tau|t}\left(\pi^i_{t+\tau|t} + \epsilon\right) - F^i_{t+\tau|t}\left(\pi^i_{t+\tau|t} - \epsilon\right) = 1, \forall i \in [0,1].$$

Q.E.D.

The argument is identical to proving that if expectations are strongly $\frac{\epsilon}{2}$-anchored then they are also weakly $\epsilon$-anchored.

**Proof of Proposition 4.**

For this proof we are going to use the identity that for any given random variable $X$ with CDF $F(.)$,

$$E\{X\} = \int_0^{+\infty} \big(1 - F(x)\big)dx - \int_{-\infty}^0 F(x)dx.$$

Now, let $FR_{t+\tau|t}^i \equiv E_t^i\{\pi_{t+\tau}\} - E_{t-1}^i\{\pi_{t+\tau-1}\}$, and notice that

$$\begin{aligned}
\left|FR_{t+\tau|t}^i\right| &= \Big| \int_0^{+\infty} \left(1 - F_{t+\tau|t}^i(x)\right) dx - \int_{-\infty}^0 F_{t+\tau-1|t-1}^i(x)dx \\
&\quad - \int_0^{+\infty} \left(1 - F_{t+\tau|t}^i(x)\right) dx + \int_{-\infty}^0 F_{t+\tau-1|t-1}^i(x)dx \Big| \\
&= \left| \int_R \left(F_{t+\tau|t}^i(x) - F_{t+\tau-1|t-1}^i(x)\right) dx \right| \\
&\leq \int_R \left|F_{t+\tau|t}^i(x) - F_{t+\tau-1|t-1}^i(x)\right| dx \\
&< \epsilon.
\end{aligned}$$

Q.E.D.

**Proof of Lemma 3.**

Notice that if inflation expectations for horizon $\tau$ are ideally $\frac{\epsilon}{2}$-anchored at $t$ and $t-1$, then

$$\int_R \left|F^i_{t+\tau|t}(x) - F^i_{t+\tau-1|t-1}(x)\right| dx = \int_{\pi^*-\frac{\epsilon}{2}}^{\pi^*+\frac{\epsilon}{2}} \left|F^i_{t+\tau|t}(x) - F^i_{t+\tau-1|t-1}(x)\right| dx < \int_{\pi^*-\frac{\epsilon}{2}}^{\pi^*+\frac{\epsilon}{2}} 1.\,dx = \epsilon$$

meaning that expectations are consistently $\epsilon$-anchored.

Now, for the second part of the lemma, suppose inflation expectations for horizon $\tau$ are strongly $\frac{\epsilon}{2}$-anchored at $t$ and $t-1$. Let for this part of proof

$$a \equiv \min\{\overline{\pi_{t+\tau|t}}, \overline{\pi_{t+\tau-1|t-1}}\}, b \equiv \max\{\overline{\pi_{t+\tau|t}}, \overline{\pi_{t+\tau-1|t-1}}\}$$

Notice that $b - a = \overline{FR_{t+\tau|t}} = \delta$.

Now observe that

$$\int_R \left|F^i_{t+\tau|t}(x) - F^i_{t+\tau-1|t-1}(x)\right| dx = \int_{a-\frac{\epsilon}{2}}^{b+\frac{\epsilon}{2}} \left|F^i_{t+\tau|t}(x) - F^i_{t+\tau-1|t-1}(x)\right| dx$$

$$< \int_{a-\frac{\epsilon}{2}}^{b+\frac{\epsilon}{2}} 1.\,dx = b - a + \epsilon = \delta + \epsilon$$

Meaning that expectations are consistently $(\epsilon + \delta)$-anchored. Q.E.D.

**Proof of Proposition 5.**

The regression coefficient is given by

$$\beta_\tau = \frac{\int_0^1 (\pi^i_{t+\tau|t} - \overline{\pi_{t+\tau|t}})(\pi^i_{t+1|t} - \overline{\pi_{t+1|t}})di}{\int_0^1 \left(\pi^i_{t+1|t} - \overline{\pi_{t+1|t}}\right)^2 di}$$

Now notice that for $\tau \geq T$, since expectations are strongly $\epsilon_\tau$-anchored, we have

$$\begin{aligned}
\int_0^1 \left(\pi^i_{t+\tau|t} - \overline{\pi_{t+\tau|t}}\right)\left(\pi^i_{t+1|t} - \overline{\pi_{t+1|t}}\right)di &= \int_{\left\{i:\pi^i_{t+1|t} > \overline{\pi_{t+1|t}}\right\}} \left(\pi^i_{t+\tau|t} - \overline{\pi_{t+\tau|t}}\right)\left(\pi^i_{t+1|t} - \overline{\pi_{t+1|t}}\right)di \\
&+ \int_{\left\{i:\pi^i_{t+1|t} < \overline{\pi_{t+1|t}}\right\}} \left(\pi^i_{t+\tau|t} - \overline{\pi_{t+\tau|t}}\right)\left(\pi^i_{t+1|t} - \overline{\pi_{t+1|t}}\right)di \\
&< \epsilon_\tau \int_{\left\{i:\pi^i_{t+1|t} > \overline{\pi_{t+1|t}}\right\}} \left(\pi^i_{t+1|t} - \overline{\pi_{t+1|t}}\right)di \\
&- \epsilon_\tau \int_{\left\{i:\pi^i_{t+1|t} < \overline{\pi_{t+1|t}}\right\}} \left(\pi^i_{t+1|t} - \overline{\pi_{t+1|t}}\right)di \\
&< \epsilon_\tau \int_0^1 \left|\pi^i_{t+1|t} - \overline{\pi_{t+1|t}}\right| di
\end{aligned}$$

Similarly, we can show that

$$\int_0^1 \left(\pi^i_{t+\tau|t} - \overline{\pi_{t+\tau|t}}\right)\left(\pi^i_{t+1|t} - \overline{\pi_{t+1|t}}\right)di > -\epsilon_\tau \int_0^1 \left|\pi^i_{t+1|t} - \overline{\pi_{t+1|t}}\right| di$$

Thus,

$$|\beta_\tau| < \epsilon_\tau \frac{\int_0^1 \left|\pi^i_{t+1|t} - \overline{\pi_{t+1|t}}\right| di}{\int_0^1 \left(\pi^i_{t+1|t} - \overline{\pi_{t+1|t}}\right)^2 di}$$

So, as long as $\int_0^1 \left(\pi^i_{t+1|t} - \overline{\pi_{t+1|t}}\right)^2 di \neq 0$, $\beta_\tau \to 0$ as $\epsilon_\tau \to 0$. Q.E.D.