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OPTIMAL PENSION FUNDING STRATEGY AND PENSION INSURANCE REFORM

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ABSTRACT

I develop a model to improve our theoretical understanding of corporate pension policies and to forecast the behavioral impact of changes to pension policy. The model calculates optimal pension funding decisions and compares them to the actual decisions companies make. I find that the ability to save money in a tax sheltered account motivates companies to make higher than required contributions, and that the limits on deductible contributions discourage contributions above the maximum. I find limited evidence suggesting that companies are deterred from making additional contributions because it would reduce the value of their PBGC insurance. In a surprising result, I find no evidence that companies contribute additional money to reduce their variable rate premium.

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I. INTRODUCTION

Private employment-based defined benefit (DB) pension plans promise payments to retirees based on a formula. For example employees might receive a retirement benefit equal to one percent of their average salary over their tenure at a company for each year that they worked; this benefit would be paid each year that they remain alive in retirement. Based on rules that originated with the Employee Retirement Income Security Act (ERISA) of 1974, companies fund defined benefit pensions by contributing money to trust funds and investing those funds in a mix of investments. Funding rules require companies to contribute an amount equal to the present value of their benefit promises, with penalties for not complying. But sometimes those investments can perform less well than expected and pension funds become insufficient to cover promised benefits. In that situation, the rules require the plan sponsor to remedy the underfunding with additional contributions. If the sponsor goes out of business before remedying the underfunding, workers can lose promised benefits. To avoid these losses, the Pension Benefit Guaranty Corporation (PBGC) takes over the pension fund, pays the funded portion out of the plan assets, and covers the unfunded benefit up to guarantee limits with its own funds, which accumulated from the premiums paid by companies participating in the defined benefit system.

Private pension plans face serious long-term financial challenges. Despite the a trend towards defined contribution plans, private sector defined benefit plans still cover more than 40 million workers and retirees. In recent years, defined benefit plans have been chronically underfunded because of poor asset price performance, lower interest rates, and legislation relaxing the funding rules. The PBGC too is deeply underfunded because historically its premiums have not been adequate given the risks it is required to take.

This study compares actual company decisions about how well to fund their defined benefit pensions to optimal strategies from an infinite horizon model. It develops a Markov Decision Process model to determine optimal funding decisions, and estimates the relationships between those optimal decisions and actual observed decisions using maximum likelihood estimation. The goal of the model is to improve our theoretical understanding of corporate pension policies while also developing a practical tool to forecast how companies will react to policy changes aimed at improving the pension insurance system.

The effectiveness of any reform to the pension system in improving funding levels depends heavily on how it affects the strategic decisions that stakeholders regularly make. While the funding rules constrain choices available to plan sponsors, companies still have considerable flexibility in deciding how much to contribute and how to allocate their pension funds among investments. A wide literature suggests how companies might make those choices strategically to maximize their shareholder value. A separate literature focuses on measuring the cost of PBGC insurance, and how that cost might change in reaction to policy changes. Models focused on measuring the value of PBGC insurance assume that companies would make the minimum contributions regardless of whether a higher contribution might be optimal from their financial perspective. Meanwhile, corporate finance models of optimal pension funding decisions carefully model the basic incentives of pension plans but use simplified versions of the funding rules in limited two-period frameworks. While useful for gaining insights into the interplay of moral hazard and tax considerations in determining pension plan decisions, the highly simplified two-period framework makes these models unsuitable for the simulation of complicated policy changes over a longer horizon. The goal of the current study is to incorporate the incentive-based decisions of the corporate finance models into a more realistic model that can draw parameters from the empirical data. To do so I develop a Markov-Decision Process approach, where the condition of plans maps into a "state space" and the optimal decision for each plan is identified for any location in that state space.

I review the literature on the determinants of funding decisions starting with the insurance and tax effects identified in early studies, and other justifications such as actuarial bias, hedging of salary increases, and risk management factors. I then develop a Markov Decision Process model that can incorporate those factors in the optimal decision making of private pension plans. Under the Markov decision model companies consider both the impacts of these effects in the current period and the present value over the indefinite future. It identifies the optimal choice in each state, in terms of asset allocation and contribution levels, as the one that maximizes the present value net benefit of the pension plan taking as given the optimal choice in all other states.

The empirical model relates actual contributions to the optimal decisions from the Markov Decision Process model (MDP) following the approach described in Rust (1994) and Hansen & Sargent (1980). Under the empirical model companies are assumed to base their decisions on the present values associated with each possible action from the MDP and an unobservable error term. So while the theoretical model identifies the optimal decision as the one that maximizes the present value benefit of the pension plan, in the empirical model the optimal decision depends also on the value of an error term, and all actions have some probability of being optimal. The likelihood of any action being optimal and thus chosen increases with the present value associated with the MDP.

I find the parameters that are most consistent with actual decisions being made by pension plan sponsors. The empirical model adjusts the weights of each of the tax, insurance, premium and other effects to the levels that best predict actual decisions in the real world. The weights for each effect measure their importance in reality and thus can shed light on the theory of what determines pension plan decisions. This approach to investigating the importance of difference factors determining pension plan decisions improves upon the methods of previous empirical studies. The estimated parameters of the model can also be used to incorporate estimates of company reactions in the modeling of policy changes.

In the empirical analysis, I find that the ability to save money in a tax sheltered account motivates companies to make higher than required contributions. I find limited evidence suggesting that companies are deterred from making additional contributions because they would reduce the value of their PBGC insurance. But the effect is one tenth of the size predicted in theory. In an even more surprising result, I find no evidence that variable rate premiums induce companies to make higher than required contributions. Companies have no tendency to make higher contributions when their plan funding ratios are in the range where additional contributions can reduce their Variable Rate Premium (VRP) payment. I find this non-result even when controlling for sponsor creditworthiness. I suspect that the VRP rate is too low in the sample period to generate a measurable behavioral response. Now that the VRP rate is near three percent and rising, it still seems likely that companies will take measures to avoid it, and future research should revisit the estimation performed in this paper.

II. Review of the Literature

This paper will build on and combine aspects of past studies on corporate defined benefit pensions that fall (roughly speaking) in four categories. First, limited horizon (one or two-period) stylized models that apply corporate finance theory to pension funding decisions; second, empirical studies measuring the statistical relationship between corporate pension funding decisions or stock market valuations and plan sponsor characteristics and funding levels; third, models that use options pricing and simulation approaches to measure the cost and value of Pension Benefit Guaranty Corporation insurance; and fourth, infinite horizon models of optimal funding decisions developed by actuarial researchers. This study incorporates methods and approaches from each one of these strands in the literature to develop an infinite horizon model of pension funding decisions, which is grounded in corporate finance theory and empirical data and yields a measure of the value and cost of PBGC insurance. The model estimating the cost of PBGC insurance in this study, unlike previous models, considers how companies make funding decisions and how they might alter them in response to policy changes.

A. Theoretical models of pension asset allocation and contributions

In 1974, ERISA established the legal framework connecting pension liabilities and assets to the corporate balance sheet. Following its passage, several limited horizon models placed the pension plan in the context of corporate finance theory. In particular they related the optimal choice of pension funding levels and asset allocation to the capital structure theory that followed Modigliani and Miller (1958), and to the Black-Scholes Merton options pricing framework. The original theory of capital structure starts from the proposition that capital structure would be irrelevant to corporate valuation under certain conditions, and then explores the implications of real world deviations from those conditions. Similarly, early work on pension funding established conditions under which pension funding allocation would be irrelevant for company valuation and then turned to PBGC insurance, premiums, taxation and other factors as sources of deviations from that situation. More recent work incorporated new approaches in capital structure theory to the study of pension funding decisions.

The Insurance Effect: The Value of the PBGC Guarantee and Pension Funding Decisions

While cash wages supply workers with the most flexibility, for convenience and other reasons workers also enjoy receiving some of their compensation in benefits such as health care and pensions paid for and managed by their employer. Such fringe benefits enjoy tax advantages and lower group pricing than what individuals can achieve on their own. Workers care about the total value of their compensation and trade non-cash for cash benefits of equal value to them. The insurance of pension benefits is valuable to companies because it raises the value of its pension promises to workers and thus lowers the cash it needs to pay in salaries to attract and retain them.

Several models have studied how insurance impacts decision making through this channel. Sharpe (1976) considered a benchmark case where employees rationally assess the risk of their pension with full access to equity markets and without insurance. Under these conditions he found that there should be no advantage to any particular asset allocation, because employees could always compensate for changes in composition by adjusting their exposures outside the pension plan, and would demand higher cash wages that are exactly equal to the risk created by pension plan underfunding. Sharpe then considered how PBGC insurance reduces the risk facing employees, and concluded that it short-circuited the mechanism through which workers would discourage underfunding. He concluded that companies would be likely to pursue policies designed to maximize the value of PBGC insurance, which in general meant minimizing contributions and investing entirely in equity. Similarly Treynor (1977) argued that plan sponsors would have incentives to place risk on the beneficiaries by underfunding their plan as much as the law allowed and investing the pension fund in relatively volatile assets.

The Premium Effect: PBGC premiums and funding decisions

PBGC premiums offset the benefit of insurance for underfunded plans and can thus reduce or reverse the incentive to remain underfunded. The PBGC charges two types of premiums under law, the flat rate premiums, and the variable rate premium (VRP). The flat rate premium is a fixed amount per participant, while the VRP change is assessed on the plan's level of unfunded benefits, until recently at a rate of $9 per $1000 of unfunded benefits. The flat rate premium has encouraged companies to offer separated and retired employees lump sums to avoid counting them as participants, but does not vary with the funding ratio. The VRP is a more direct and powerful incentive to increase funding. As Love, et al. (2011) argue, PBGC insurance replaces the increment wages a worker would demand for bearing the risk with the PBGC's own insurance premium, partially so when the insurance guarantee is partial and fully in the case of a full guarantee. The authors report optimal funding levels and asset allocation under different parameter values, initial conditions, and PBGC insurance fees. Their results suggest that less creditworthy companies will tend to minimize funding levels while more creditworthy companies maximize them.

The Tax Shelter Effect

Black (1980) and Tepper (1981) argue that the tax consequences of pension funding strategy should drive companies towards full funding. Pension plans are tax shelters in the sense that income earned between the times the company contributes to the fund and when the benefit is paid to the worker is neither subject to corporate nor individual taxation. Pension fund contributions are tax deductible. Companies receive a tax deduction when they contribute to a pension plan, instead of the deduction they would have gotten when actually making a benefit payment funded by that contribution to the worker many years later. Because the two tax deductions are equal in present-value, the tax deduction on the contribution does not itself represent a benefit. But the exemption of investment income during the build-up period represents a benefit similar to that of a traditional IRA that the company can take maximum advantage of by remaining fully funded.

Tepper (1981) argues further that companies can maximize the value of the tax shelter by investing in bonds. The pension plan passes through the interest income from its assets to the shareholder as dividends or capital gains where it receives a preferential tax rate relative to the interest income at the individual level.

Tax Penalties: Non-Deductible Contributions and Reversion Taxes.

Statutory limits on tax deductible contributions limit their use as tax shelters. Above maximum contribution levels companies can no longer immediately deduct contributions from their income, although they can carry over the deduction to future years, resulting in a loss of present value.

A few studies suggested that companies faced penalties for excessive overfunding. Bulow & Scholes (1983) argue that workers have some claim on excess pension assets, through the bargaining power of their unions. Corporations therefore are prevented from fully taking advantage of the surplus assets. Corporate takeovers in the 1980s frequently led to the termination of overfunded pension plans for the benefit of the investors buying the corporations, and the potential detriment of workers who wished to access the excess pension assets. In response to concern over those workers, Congress enacted a set of taxes, called reversion taxes, in the late eighties, equal to fifty percent of the money extracted from overfunded pension plans. One study (Ippolito, 2002) argued that these reversion taxes, rather than encouraging the companies to maintain the overfunding in the pension plan for the benefit of the workers, made them reluctant to contribute any more the minimum required. He relates a fall in funding ratios in the 1990s to the advent of the taxes.

Convexity: The Cost of Contribution Volatility

There are both tax and other reasons why companies might wish to limit volatility in their pension contributions by either building a buffer stock of funding above minimum levels, or investing in mostly risk free assets. Black (1980) found that reducing the variability and risk of the pension fund increased the capacity of the company sponsoring the pension plan to fund itself with debt rather than equity, and receive the tax benefits of deducting its interest. In the trade-off theory of capital structure a company optimizes its mix of debt and equity at a level where the marginal tax benefit of an additional dollar of debt equals its marginal impact on the expected costs of bankruptcy. Investing the pension fund in debt lowers the risk of bankruptcy and thus shifts that optimal level of debt to a higher level, and allows the company to enjoy a greater benefit from its tax shield.

Companies facing volatility in their minimum funding requirements might miss out on profitable business opportunities, consistent with the concepts of Froot, et al. (1993), that companies face convex costs in raising capital when they face economic distress. Rauh (2009) suggests that companies with higher probabilities of bankruptcy might invest less money in equities because of this risk management motive, countering the impact of insurance.

Other Factors Impacting Asset Allocation

Love, et al. (2007) and Love, et al. (2011) supplied an additional reason for pension plans remaining fully funded and investing in debt, which derives from the inability of their workers to mitigate the risk of underfunding in ways available to the firm's other stakeholders. Love, et al. (2007) argued that companies face a greater penalty when placing risk on their workers than on bond holders. Underfunding, in their argument is a form of borrowing from workers, and exposes them to the risk of loss. Workers charge companies for that risk by demanding higher wages, in a competitive labor markets, just as a creditor charges a higher interest rate for the risk of loss on a bond. Workers are likely to require a higher premium for risk when lending to their companies, because their labor income is already at risk when their employer faces financial distress. While creditors can mitigate their exposure to the plan sponsor as part of a diversified pool, imperfections in capital markets prevent workers from avoiding a concentration of risk with their employer.

Other studies have proposed additional motives for investment in equity, beyond the insurance effect identified by Sharpe (1976) and Treynor (1977). Black (1989) related investment in equity to the impact of salary increases on pension liabilities, which are the present value of expected future benefit payments. Many benefit formulas relate pension payments to a worker's average or highest annual salary over their tenure at a company. In such formulas, pension benefits increase with the average salary each time the company gives employees a raise. Because wages and equity prices show some positive correlation over multiyear periods, equity serves as a potential hedge. Sundaresan & Zapatero (1997) and Lucas & Zeldes (2006) supported Black's position while Bodie (1990) criticized the notion that companies should base funding strategy on projection benefit obligation saying that there is no clear reason to think that companies will want or need to hedge an obligation that depends on their own decision to offer employees a raise. Gold (2005) argues that companies invest their pension plans largely in equity, despite the benefits of debt, because of something he calls actuarial bias. According to him the actuarial profession tends to embrace the false view that stocks are safe investments in the long run despite the objections financial economics raises to that proposition. The stocks for the long run view is that under the equity premium that has prevailed in the United States historically, and lognormal investment returns, the likelihood that a portfolio of stocks outperforms a portfolio of bonds approaches one over long horizons. This proposition however implies an arbitrage opportunity in financial markets that runs contrary to standard finance theory. So the stock for the long run idea is anathema to financial economists but not always to actuaries who frequently advise clients to take advantage of the long investment horizon of their pension plans.

Models attempting a synthesis of different viewpoints

Several studies including Harrison & Sharpe (2000) and Bicksler & Chen (1985) combined the insurance effect and tax effect into a unified model. Harrison & Sharpe (2000) found that depending on which effect was stronger, a company should pursue either full funding or investment in bonds, or minimal funding or investment in stocks. In general financially strong sponsors should pursue the full funding strategy and financially weak sponsors the minimal funding strategy. Bicksler & Chen (1985) found that in the presence of progressive and asymmetric corporate income taxes, the optimal funding strategies did not necessarily follow extreme cases, and could be of the mixed form that one observes in reality. Similarly Love, et al., (2011) report optimal funding levels and asset allocation under different parameter values, initial conditions, and PBGC insurance fees. Their results tend towards either extreme as in Harrison and Sharp (1983). Either a company should pursue a risk elimination strategy or a strategy that maximizes the net value of the PBGC insurance, with very few situations in between.

B. Empirical Studies of Pension Funding and Asset Allocation Decisions

Several empirical studies related pension contributions and asset allocation to the existing theory, for example by testing whether funding ratios might be negatively correlated with equity holdings. For example Friedman (1983) used data from 1977 to show that companies with lower profits and higher earnings volatility tended to minimize volatility in their pension plan by holding less equity, somewhat contrary to the notion of an insurance effect, and that firms with younger workers tended to invest more heavily in equity. Bodie, et al. (1985) used data from 1980 supply evidence supporting the view that tax considerations motivate companies to fully fund their plans with relatively safe assets. They show that more profitable companies, subject to higher tax rates, tend to have better funded pensions. The also show a link between funding level and relatively safe assets, with the riskiest firms keeping their pension plans underfunded and invested in relatively volatile assets, although they characterize the evidence supporting that link as weak.

Papke (1991) used data from 1981 to 1987 to show that despite the theoretical literature describing the optimality of extreme portfolio allocations, most pension plans in that era invested in a similar non-extreme mix of stocks and bonds. Coronado & Liang (2006) investigate whether the insurance effect lowers funding levels and raises the share of assets held in equity. They find evidence that riskier companies tend to contribute less, consistent with the insurance effect, but not that they funded more of their pensions in equity. According to their data, firms tend to congregate around an equity share of between 60 to 75 percent.

Rauh (2006) and Rauh (2009), using data from between 1990 and 2003 argue for the importance of risk management considerations in pension plan funding decisions. He supplies evidence that companies facing a high likelihood of bankruptcy are less rather than more likely to invest in risky assets most likely because they face greater costs than benefits from their volatility. Rauh (2006) uses a regression discontinuity approach to show that companies facing minimum funding requirements depress investment at the firm level. Companies facing volatility in their minimum funding requirements might miss out on profitable business opportunities, consistent with the concepts of (Froot, et al., 1993). Rauh (2009) shows that companies with higher probabilities of bankruptcy, if anything tend to invest less money in equities all else equal, suggesting that the risk management motive is stronger than the insurance effect. Franzoni (2009) seems to offer corroborating evidence in stock market valuations, showing that they drop more precipitously in response to a fall in pension assets for companies that are financially constrained. That additional drop in stock prices might reflect the secondary impact of pension assets on company value through the channel of underinvestment.

Rauh (2009) also shows that companies with a higher percent of active participants tend to invest more in equity, consistent with the notion in Black (1989) and Lucas & Zeldes (2006) that equity serves as a hedge against salary increases. The parameter estimates presented in Rauh (2009) do not suggest, however, that such hedging is the main motive for companies holding equity. A company whose participants are all retired is predicted to hold only a few percent less of its assets in equity than an identical company with only active participants.

Several studies examined the relationship between stock market prices and pension plan funding status. That relationship has significant implications for pension funding decisions, as managers should make decisions that raise their company's share price. According to Feldstein & Seligman (1981), stock prices properly reflected the future costs of unfunded liabilities, although perhaps in ways that were distorted by accounting conventions for pension liabilities. Bulow, et al. (1987) confirm those results with a wider set of regressions and controls.

However more recent studies suggest that stock prices offer companies weaker incentives to fully fund their pension plans and reduce their volatility. Coronado & Sharpe (2003) offer evidence for what they call the "opaque" model of corporate pensions where accounting conventions systematically mislead investors. They find large valuation errors based on stale accounting values that do not reflect the latest market data, for example that the stock price of plan sponsors did not fall as much as it should during the early years of the 2000-2001 market decline, when accounting values had not yet incorporated the fall in equity prices. Franzoni & Marin (2006) found that the stock price of underfunded plans tended to significantly underperform other companies in the years following a drop in pension assets, suggesting that the initial fall in stock was not sufficient to incorporate the impact of the fall in pension assets on the future profitability of the company.

C. Estimates of the Cost of PBGC Insurance

Several studies focused on measuring the cost of the pension system to its government insurer, the Pension Benefit Guaranty Corporation (PBGC). The cost is equivalent to a put option to sell the pension assets for the price of the plan's liabilities, but only in the event of company bankruptcy or serious financial distress. Marcus (1987) developed an options pricing formula for PBGC costs, assuming the PBGC would receive the assets and liabilities of plans after their bankruptcy. Pennacchi & Lewis (1994) improved his analysis, by incorporating the one-sided nature of the PBGC's exposure, where the PBGC would receive the assets and liabilities only when a plan was underfunded. Their model resulted in an analytical formula for the cost of PBGC insurance. Others used simulation based approaches starting with Estrella & Hirtle (1989).

Studies aimed at valuing PBGC insurance have assumed simple rules for pension contributions and asset allocation that companies would follow regardless of what might be optimal. For example Pennacchi & Lewis (1994) assumed that plans contribute the same fraction of pension fund liabilities, regardless of the plan condition or the pension funding rules. This very strong assumption offered no way to study how different funding rules would impact the PBGC's finances or plan funding decisions, let alone predict how plan sponsors might use the flexibility they have under the funding rule to contribute more than the minimum. Boyce & Ippolito (2002) sought to remedy this shortcoming by modeling the minimum funding rules in their complete detail, but assumed that companies would make the minimum contributions regardless of whether a higher contribution might be optimal from their financial perspective, representing a conservative assumption for projecting insurance claims. The PBGC uses the Boyce and Ippolito approach in its Pension Insurance Modeling System (PIMS) to project insurance claims. Assuming minimum contributions tends to overstate the amount of underfunding subject to the variable rate premium and thus VRP receipts. So the PBGC applies significant haircuts to its projections of underfunding to develop official projections of PBGC premiums. Congressional Budget Office (2005) presents a simplified version of the PIMS model that also assumes contributions equal the minimum required under the funding rules.

Studies of the cost of PBGC insurance have often highlighted the distinction between the expected value and market value of future of insurance claims and premiums. Expected values do not include the premium participants in financial markets earn for bearing risk, while market (or fair) values do. The Congressional Budget Office (2005) shows that the fair market value of PBGC net costs is approximately double their estimate of its expected value, and Boyce & Ippolito (2002) show the same ratio of market value to expected cost. Like in Falkenheim & Pennacchi (2003) study of deposit insurance, this model will distinguish between the actual probabilities, and "risk neutral" probabilities that are constructed to price claims at market value. Plan sponsors base their decisions in the Markov Decision Process Model on risk adjusted, or fair value, estimates of the impact of their funding strategy on future pension plan funding. Investors will judge the pension funding situation on a fair value basis, by definition, in determining the price at which they will buy and sell the company, and those investor decisions motivate managers to pay attention to fair value rather than a different measure. While I assume that fair value drives corporate decisions I do not mean to take a stand on the issue of whether or not it should be used for budgeting for agencies like the PBGC, in place of projections that rely on statistical expected values.

D. Actuarial Research

Studies by actuarial researchers have investigated optimal funding rules for pension plans by applying dynamic programming and related methods in infinite horizon models but base what is optimal on criteria that are less grounded in corporate finance theory than what I apply in this study. For example, Owadally & Haberman (2004) use optimal control to determine the funding approaches that minimize the variance of the funding ratio and contribution rate, finding that the spread method outperforms amortization. Under those same criteria the optimal proportion invested in risky assets declines as the funding ratio increases. Several commenters on the article criticized the criteria of minimizing those variances as not sufficiently grounded in corporate finance principles, a criticism I attempt to address in this study. The model developed in this study is similar to those in that it attempts to determine what is optimal in each possible state under an infinite horizon model using dynamic programming methods.

E. Summary of Literature Review and implications for my model

The Markov Decision Process (MDP) model developed in this study serves as a unifying framework for the four strands of literature. It predicts corporate behavior based on the motivations identified in the theoretical literature and estimates the weights companies give those motivations in actual decisions. Like the actuarial studies it applies optimal control theory to the question of how companies fund their pensions. But unlike those studies, it determines optimal decisions based on the motivations identified in the corporate finance literature. It incorporates insurance and tax effects into a matrix of rewards associated with each possible action at each location in the state space. Different tax effects are modeled individually as negative rewards in the case of tax penalties and positive rewards in the case of tax shelters and tax arbitrage effects.

Existing theoretical models simplify the environment impacting the company's pension funding decisions beyond the point where researchers could estimate their parameters with real world data. They model corporate funding decisions under illustrative policy regimes, if under any policy regimes at all. They predict broad associations between corporate characteristics and pension funding strategies. Empirical studies explore whether those broad associations exist in reality, but do not estimate parameters associated with theoretical models directly.

The goal of this study is to hit the sweet spot in terms of simplification. I develop a model that simplifies the environment impacting the company's decision enough to make it feasible to compute its optimal funding strategy. It does not oversimplify to the point where its predictions will lack real world significance that empirical estimation can directly test. Directly testing the validity of a theoretical model's predictions has some advantages of the approach that empirical studies have taken. While the theoretical literature does not always suggest monotonic relationships between firm and plan characteristics and plan funding strategy, the empirical analysis has generally tested for those relationships using linear regressions which assume monotonic relationships. Policy can clearly change the importance of different effects and in so doing change the predicted sign and shape of relationships between plan characteristics. Empirical analysis has generally not considered changes in policy regimes in testing for statistical associations.

Notwithstanding their limitations, the theoretical models and empirical studies generated some consistent findings about the relationship between firm characteristics and contribution levels, but much less conclusive findings about asset allocation. Theory suggests and the empirical studies have affirmed that less creditworthy sponsors would be less likely to contribute above the minimum and fully fund their plans, while financially healthy companies will be more likely to pursue full funding. The empirical data has supplied only limited evidence in support of the theoretical propositions about how companies should invest their pension funds. In general the theory supports more extreme and varied asset allocation than seem present in the data, where most plans fall in the 60 to 75 percent range for equity investments. Because past research has shown that most plans follow a convention in determining asset allocation, I focus my empirical estimate for now on how pension plan sponsors determine contributions.

III. Mathematical Structure of the Markov Decision Process Model

A. Overview

The key building blocks of a Markov Decision Process are the state space, the set of possible actions in each state, the transition probabilities between states as a function of actions, and the immediate rewards associated with each action in each state. Optimization in an MDP uses these building blocks to compute a value and optimal action in each state. Table 1 shows how I apply each of these concepts to the problem of optimal pension funding strategy.

Table 1: Implementation of Markov Decision Process Modeling in this Study

MDP Element	Implementation in this study
State Space	Two dimensional space: Pension Plan Funding Ratio X Company Financial Strength
Action Set	Two dimensional action space **Contributions**, subject to minimum funding requirements **Fraction of assets** allocated to high risk/return portfolio
Transition Probabilities	Bivariate Normal Distribution on funding ratio and company financial strength, conditional on contributions and asset allocation.

Rewards	Value of promised benefits minus Contributions minus Premiums plus Value of Tax Benefits
Results in:	
Values	Net Present Value of pension plan to company
Decisions	Contribution level and asset allocation that maximize Net Present Values

My representation of the state space and transition probabilities borrow heavily from models of PBGC insurance, including Lewis and Pennacchi (1999) and Congressional Budget Office (2005). The state space represents all of the possible situations the company could face with the funding ratio of the pension plan and a variable representing the failure risk of the sponsoring company. The variable representing failure risk is based on Merton's approach which treated corporate default as an option that could be related to the Black Scholes framework. Company default occurs when the value of its assets falls below a threshold related to its liabilities. I chose this measure of default risk because of its wide availability, consistency with earlier studies of PBGC insurance, and because it has been shown as a relatively timely, direct and accurate measure compared to account values (Hillegeist, et al., 2004) and credit ratings (Delianedis & Geske, 2003) and yield spreads (Delianedis & Geske, 2001). I measure the net worth with financial statement and stock price data.

B. The State Space

The company sponsors a pension plan with a log ratio of assets to liabilities (log funding ratio) of x, and its financial strength is captured by the variable y, where the default probability is given by the standard cumulative normal function $\Phi(-y)$. The model represents all possible values of x and y with a discrete set of points. I assume that the log funding ratio can equal any one of N_x values spaced k_x units apart between $\underline{X}$ and $\underline{X}+(N_x-1)k_x$. Let $\tilde{x}_k$ be the kth element in the set of possible log funding ratios described by $X=\{\underline{X},\underline{X}+k_x,\ldots,\underline{\underline{X}}+(N_x-1)k_x\}$. Similarly define N_y non-default levels of company financial strength, spaced between and $\underline{y}+(N_y-1)k_y$. Let $\tilde{y}_k$ be the kth element in the set Then the set of states $S=X\otimes Y$ contains all possible combinations of ratings and funding ratios, and has $N_S=N_yN_x$ elements.

C. The Action Set

I define c as the ratio of contributions to liabilities, and β as the fraction of assets ($0\le\beta\le 1$) that the company invests in a diversified portfolio of risky, return seeking assets. I assume that the company invests the remainder of its portfolio $1-\beta$ with an asset liability matching strategy. Under asset liability matching a pension plan invests in long term bonds whose values move inversely with interest rates to counter the sensitivity to interest rates in the value of the pension benefits that the company has promised. The contribution is subject to a minimum requirement that depends on the current state $\underline{c}(\theta)$, and is a function of a vector of policy parameters I will call (θ). That vector contains parameters governing the funding rules, including the funding target and speed at which the sponsor must remediate underfunding. I will discuss these parameters in more detail in later sections.

D. The Transition Matrix

The transition probabilities measure the likelihood of ending the year with any combination of funding ratio and net worth conditional on the initial state, and the "action," in other words on the contribution and asset allocation. The probability distribution of future log funding ratios and net worth is bivariate normal, with a positive correlation between funding ratios and company net worth, reflecting the tendency of individual companies to gain value when asset returns are systematically higher than expected. Contributions and asset allocation affect the transition probabilities associated with different funding ratios. Higher contributions shift the distribution of transition probabilities towards states with higher funding ratios. More risky asset allocation (higher β) spreads the transition probabilities across a wider range of funding ratios.

To risk adjust the estimates I use a common technique in the finance literature, applying "risk neutral" probability distributions where all assets on average earn a return equal to the risk free rate, rather than distributions that reflect the actual probabilities of different investment returns. This technique incorporates the notion that the risk adjustment on any asset's return should equal its risk premium, and thus the risk premium and adjustment cancel each other out.

I express all pension plan variables as a ratio to plan liabilities. Liabilities change from one period to the next because of the passage of time, changes in interest rates, benefit accruals, and benefit payouts. The passage of time increases the value of pension plan liabilities, which equal the discounted value of expected future benefit payments. As their payment draws one year closer they are discounted by one year less, and thus their present value grows at a rate equal to the discount rate. Pension plan liabilities are annuities whose value shifts in opposite direction from interest rates. As plans pay benefits, they extinguish liabilities, but they accrue new liabilities as workers earn new benefits under the plan formula.

I risk adjust bankruptcy probabilities using a risk neutral method as well, assuming that the firm's asset to liability ratio will evolve based on the risk neutral distribution, rather than on one that reflects actual probabilities. Under the risk neutral distribution company assets and liabilities also expect the risk-free rate of return. The ratio does not have the same upwards drift as in the actual probability distribution. Without the upward drift, probabilities of the company's asset to liability ratio falling below the failure threshold, which triggers default, are higher.

I assume that the company fails if the log net worth ratio falls below a threshold (assets fall below liabilities) at the end of the period. If the company survives, I assume that it adjusts its borrowing to move its net worth ratio towards a target level that depends on the volatility of its net worth ratio, consistent with Congressional Budget Office (2005). I apply targeting parameters from Collin-Dufresne & Goldstein

(2001). Consistent with the Falkenheim & Pennacchi (2003) model of bank capital targeting, I assume that the net worth target is a function of the volatility, such that companies with higher net asset volatility target a higher ratio of asset to liabilities y^*.

The risk-adjusted probability of transitioning between the initial state (x_t, y_t) to (x_{t+1}, y_{t+1}), given c_t and β_t, equals a double integral of the bivariate normal distribution:

$$p(x_{t+1}, y_{t+1} \mid x_t, y_t, c_t, \beta_t) = \int_{x_{t+1}-k_x}^{x_{t+1}+k_x} \int_{y_{t+1}-k_y}^{y_{t+1}+k_y} \phi(x_{t+1}, y_{t+1} \mid \mu(x_t, y_t, c_t), \Sigma(\beta_t))\, dy\, dx \qquad (1.)$$

Where ϕ is the bivariate normal probability distribution function with mean μ and variance Σ. Appendix A derives the following formulas for those parameters.

$$\mu(x_t, y_t, c_t) = \begin{pmatrix} \dfrac{x_t + c_t - \upsilon}{1 + a - \upsilon} \\ (1-\alpha)y_t + \alpha y^* \end{pmatrix}, \ \Sigma(\beta_t) = \begin{pmatrix} \beta_t^2 \sigma_E^{\ 2} & \rho\beta_t(1-\alpha)\sigma_E \\ \rho\beta_t(1-\alpha)\sigma_E & (1-\alpha)^2 \end{pmatrix} \qquad (2.)$$

Where:

- a is the rate of new benefits accruing to participants, as a ratio to liabilities,
- υ is the payout rate of pension benefits to retirees as a ratio to liabilities,
- α is the targeting speed for the sponsor's asset to liability ratio,.
- y^* is the target distance to default.
- σ_S is the volatility of the difference in value between an equity portfolio and the pension liabilities backed by it, and
- ρ is the correlation between the equity portfolio return and the change in asset to liability ratio of the plan sponsor, representing the fraction of the plan sponsor's return that are systematic.

I estimate these probabilities using MATLAB quadrature functions, which apply the numerical integration techniques described in Genz (2004). Plans in each state have a certain probability of terminating, with the sponsors bankruptcy in each year, based on the default risk level associated with their state. The termination probability is given by the cumulative normal function $\Phi(-y_t)$.

The transition matrix will contain the probabilities as calculated in (1.) from each possible initial state to each possible final non-default state:

$$P(c,\beta) = \begin{pmatrix} p(x_{t+1} = \tilde{x}_1, y_{t+1} = \tilde{y}_1 \mid x_t = \tilde{x}_1, y_t = \tilde{y}_1, c_t, \beta_t) & . & p(x_{t+1} = \tilde{x}_N, y_{t+1} = \tilde{y}_N \mid x_t = \tilde{x}_1, y_t = \tilde{y}_1, c_t, \beta_t) \\ . & . & . \\ p(x_{t+1} = \tilde{x}_1, y_{t+1} = \tilde{y}_1 \mid x_t = \tilde{x}_N, y_t = \tilde{y}_N, c_t, \beta_t) & & p(x_{t+1} = \tilde{x}_N, y_{t+1} = \tilde{y}_N \mid x_t = \tilde{x}_N, y_t = \tilde{y}_N, c_t, \beta_t) \end{pmatrix}$$

(3.)

The sum of elements in each row of $P(c,\beta)$ will equal the likelihood that the plan sponsor does not default and thus that the plan does not terminate, and that probability equals $1-\Phi(-y_t)$. The transition matrix is a function of the vectors c and β containing the choice of contribution and asset allocation in each state. Under a Markov Process the t period transition matrix is equal to $P(c,\beta)^t$.

E. REWARDS

The term "reward" in the context of a Markov Decision Process refers to the immediate benefits an actor (in this case a company sponsoring a pension plan) gets while taking an action in a given state. The immediate benefits for a plan sponsor, from its pension plan, equal the lower cash wages workers will accept in exchange for their pension benefits, plus the net tax benefits associated with the plan minus the cost of the plan in terms of contributions, premiums and tax penalties. The value of the pension plan to the company is the expected present value of these immediate rewards over an infinite horizon.

Define $r(x,y,c,\beta)$ as a function for immediate rewards to company stakeholders in each state associated with each action in the state (x,y), and $R(c,\beta)$ as the vector of rewards in each state given the vectors c and β of containing the choice of contribution and asset allocation in all state.

$$\begin{aligned}
r(x,y,c,\beta) &= && \text{Total Rewards} = \\
a-(1-\omega)E[u_f \mid x,y,c,\beta] & && \text{Benefit accruals net of unguaranteed benefit losses} + \\
+\tau_E x\eta & && \text{Benefit of Tax Shelter} - \\
-c & && \text{contributions} - \\
-\zeta[c-a]_+^2 & && \text{cost of contribution convexity} \\
-\pi[x-\pi^*] & && \text{variable rate premiums} + \\
-\tau_{ND}[c-\bar{c}]_+ & && \text{tax losses on non-deductible contributions} + \\
(1-\tau_o)E[o_f \mid x,y,c,\beta] & && \text{Reversion of overfunding in default} + \\
(1-\tau_R)E[v \mid x,y,c,\beta] & && \text{Reversion of overfunding outside of default}
\end{aligned} \qquad (4.)$$

Net Benefits to Participants: The first term $a-(1-\omega)E[u \mid x,y,c,\beta]$ represents the net benefits flowing to participants in the pension plan, which should equal the amount of cash wages they are willing to forgo in exchange for the pension benefits. The net benefits equal the rate of accrual of new benefits a minus the losses of unguaranteed benefits suffered in cases of default: $(1-\omega)E[u \mid x,y,c,\beta]$. $E[u \mid x,y,c,\beta]$ represents the expected value of underfunding at failure, and is equal to the probability of the company failing times the conditional expectation of underfunding in the event of failure, and ω is the fraction of liabilities that is guaranteed.

Benefits of the Tax Shelter: The company benefits from the tax shelter covering the earnings of the pension fund by an amount equal to the forgone taxes on earnings in the fund. Given a rate of earnings of η and a tax rate on earnings of τ_E, the benefit of the tax shelter under a funding ratio of x will equal $\tau_E x\eta$.

Contributions and Convexity: The total cost of contributions is assumed to be convex. It includes the term c and a second term representing the cost of contribution volatility $\zeta[c-a]_+^2$. The second term is a quadratic function of the squared excess of contributions over accruals. A plan that always contributes an amount equal to benefit accruals, and applies an asset-liability matching strategy $(\beta = 0)$ to avoid the possibility of losses and the need for additional contributions would always have a zero value of this term. Plans with more volatile contributions would have on average a positive value for this term, with its convexity making it larger in expectation, the higher the volatility of contributions for any positive value of ζ.

Variable Rate Premiums: The term $\pi[\pi^* - x]$ represents the variable rate premium the plans sponsor pays given a premium target of π^* and a premium rate of π. The variable $\pi^* - x$ represents the amount of underfunding subject to the tax.

Reversions: The plan can return money to the sponsor company when the pension plan is terminated or sufficiently funded. I assume that reversions will occur either when better than expected asset returns push the funding ratio over the maximum level in the state space, $\overline{\overline{x}}$ or if the plan undergoes a distress termination in the case of its sponsor going into default with an overfunded pension plan. $(1-\tau_o)E[o \mid x, y, c, \beta]$ represents the value of reversions of overfunding in the case of sponsor default, where τ_o is the effective tax rate applied to reversions in that situation, and $E[o \mid x, y, c, \beta]$ is the expected level of overfunding in default. The term $E[v \mid x, y, c, \beta]$ represents the expected value of reversions net of taxes in non-default states, where τ_R is the effective tax rate applied to reversions applied to companies outside of default.

Under these formulas, the expectations, $E[u \mid x, y, c, \beta]$, $E[o \mid x, y, c, \beta]$, and $E[v \mid x, y, c, \beta]$ are double integrals on $\phi(x_1, y_1 \mid \mu, \Sigma)$, the bivariate distribution of x and y:

$$E[u \mid x, y, c, \beta] = \int_{-\infty}^{0}\int_{-\infty}^{0}[1-e^{x_1}]_+ \phi(\mu(x,y,c), \Sigma(\beta))dxdy \quad (5.)$$

$$E[o \mid x, y, c, \beta] = \int_{-\infty}^{0}\int_{0}^{\infty}[e^{x_1}-1]_+ \phi(\mu(x,y,c), \Sigma(\beta))dxdy \quad (6.)$$

$$E[v \mid x, y, c, \beta] = \int_{0}^{\infty}\int_{\overline{\overline{x}}}^{\infty}\left[e^{x_1}-e^{\overline{\overline{x}}}\right]_+ \phi(\mu(x,y,c), \Sigma(\beta))dxdy \quad (7.)$$

Like with the probabilities in the transition matrix, I solve these using the MATLAB numerical integration functions, which apply the quadrature methods described in Shampine (2008).

The vector form of equation (4) is as follows:

$$R(c,\beta) = a - (1-\omega)u + \tau_E x\eta - c - \zeta[c-a]_+^{\,2} - \pi[x-\pi^*]_+ + (1-\tau_o)o(1-\tau_V)v \quad (8.)$$

F. The Present Value of the Pension Plan to the Sponsor

Define D as the discount factor and G as the annual growth rate in pension plan liabilities. Values for D and G are derived in Appendix A. a vector of present value of the pension plan to the company, as a function of the actions the company takes in each state will equal:

$$V(c,\beta) = \sum_{t=0}^{\infty} D^t G^t P(c,\beta)^t R(c,\beta) \quad (9.)$$

Applying the formula for the sum of an infinite series of matrixes, with I as the identity matrix, this formula becomes:

$$V(c,\beta) = \left(I - DGP(c,\beta)\right)^{-1} R(c,\beta) \quad (10.)$$

G. Decomposing the Present Value into the Effects Identified in the Theoretical Literature

A separate formula equates current funding levels and inflows to the pension fund to outflows on a present value basis:

$$x + \sum_{t=0}^{\infty} D^t G^t P^t c = 1 + \sum_{t=0}^{\infty} D^t G^t P^t (a + o + v) - \sum_{t=0}^{\infty} D^t G^t P^t u \quad (11.)$$

This formula states that the current funding ratio x plus the expected present value of future contributions is equal to current benefit promises plus the expected present value of future benefit promises and reversions minus the present value of benefit promises on which the plan will default. The one represents the value of current benefit promises, which by definition are equal to one when expressed as a ratio to plan liabilities.

Combining equations (8), (9), and (11), we obtain the following expression for the value of the pension plan as the sum of different elements related to the theoretical literature:

$$V = (x-1) + \left(I - DGP(c,\beta)\right)^{-1} * \\ \left\{r_{insurance} - r_{premiums} + r_{taxshelter} - r_{nondeductible} - r_{reversiontax} - r_{convexity}\right\} \quad (12.)$$

where:

$$r_{insurance} = \omega u$$
$$r_{premiums} = \pi[\pi^* - x]_+$$
$$r_{taxshelter} = \tau_E x\eta$$
$$r_{reversiontax} = \tau_o o + \tau_V v$$
$$r_{nondeductible} = \tau_{ND}[c - \bar{c}]_+$$
$$r_{convexity} = \zeta[c-a]_+^{\,2}$$

This formula expresses the value of the pension plan as equal to its net funding level $(x-1)$ plus the sum of different net present values related to insurance, tax, and other motivations. The component of the value equal to the net funding level reflects the fundamental value of the assets in the plan –that if the plan sponsor contributes a dollar to its pension plan, the pension fund will have one dollar's more net value as a result. The other components show how the funding level of the plan has additional value (potentially) because of the benefits of underpriced insurance and tax shelters. If there were no additional values from these components then plan sponsors would have no incentive one way or another to make higher or lower contributions. In that case, a dollar spent would generate exactly a dollar's value, and plan sponsors would be indifferent to their own level of contributions. But the tax and insurance factors can generate incentives to increase or decrease funding. A company maximizes insurance rewards by minimizing underfunding, but benefits from lower premiums and higher value of its tax shelter by contributing more. The optimal contribution will depend on the sum of these effects.

H. Optimal Actions in Each State

The value under the optimal actions are defined as the vector $V^* = \left(I - DGP(c^*, \beta^*)\right)^{-1} R(c^*, \beta^*)$. Each element $v^*(x, y)$ of the vector V^* represents the present value of the pension plan under the optimal pension funding strategy action in the state (x, y).

In any given state a company chooses the contribution level, and asset allocation that maximize the net present value of their pension subject to constraints imposed by the funding rules. One can specify the solution to this problem in terms of a response to the following optimization problem.

$$\{c^*, \beta^*\} = \underset{c,\beta}{\arg\max} \left(I - DGP(c,\beta)\right)^{-1} \mathrm{R}(\mathrm{c},\beta)\ s.t.\ c_t > \underline{c}(\theta_t) \quad 0 \le \beta \le 1 \qquad (13.)$$

The optimal contribution level and asset allocation in any state depend on the optimal contribution and asset allocation in all others and the optimal actions in all states can be solved simultaneously using dynamic programming. For the solution this study applies the INRA toolbox as described in (Chadès, et al., 2014), which solves equation (13.) using value iteration.

I. The policy parameters and how they have changed with legislation

Table 2 contains information on the parameters representing the funding rules and tax regimes in the rewards vector that I use to generate estimates of optimal contributions and asset allocation in each state, under the theoretical model. The parameters ω, π, and π^* relate to the PBGC guarantee and funding rules. On average the PBGC guarantee covers about nine tenths of unfunded benefits. The variable rate premium rate historically was 0.009, meaning that companies paid 9 dollars in premiums for every 1000 dollars of underfunding relate to the premium funding target. The threshold at which underfunding was assessed has changed with statute, but generally has vested liabilities valued with a different discount rate than in the funding rules. The discount rate for liabilities under the premium system has generally been lower, and less smoothed than the ones used in the funding rules. On a conceptual level the premium system liability was more closely approximating the termination liability (also called economic liability in this paper) than the funding rule definition of liability. Similarly the minimum and maximum tax deductible contributions vary by policy regime, and the policy regimes are described in Appendix D. Recent legislation generally has lowered the minimum contributions, and raised the maximum tax deductible contributions.

Pension reversions, where a fund returns overfunding to the sponsor, has been subject to a fifty percent excise tax throughout the policy regime. However that tax is lowered to twenty percent in cases where a company is undergoing bankruptcy, and the IRS does not have high priority for this tax in bankruptcy. I assume that it will recoup half of the twenty percent, consistent approximately with the average recovery on unsecured debt. So I assume 0.5 as the rate of tax on non-failure reversions, and 0.1 as the effective rate of tax on overfunding in default. To calculate the value of the tax shelter, my assumed tax rate on investment earnings is 0.25, approximately equal to the average individual tax rate and the effective tax rate on corporate income. The annual benefit of the tax shelter is equal to the tax rate times the risk-adjusted rate of return. I use the ten-year Treasury rate to proxy for that rate of return. For the present illustration I use the current rate of around two percent. A company making a non-deductible contribution will need to pay corporate or individual tax equal to that same rate but be able to recoup that tax by making a deduction later, through a carry over. I assume that the loss of present value with a non-deductible contribution is then 0.02, which is eight percent of the corporate tax rate of 0.25.

Table 2: Parameter Choices for an Illustration of the Optimization Model

Parameter	Definition	Assumed Value in Base Case
ω	The fraction of benefits with a PBGC guarantee	0.9
π	The Variable Rate Premium Rate	0.009
π^*	The Variable Rate Premium Target	Varies based on vested liabilities and statutory discount rate
τ_E	The effective tax rate on investment earnings	0.25
τ_O	The effective tax rate on reversions of overfunding of plans terminated in bankruptcy	0.1
τ_V	The effective tax rate on non-bankruptcy reversions	0.5
$\overline{c}$	The maximum non-deductible contribution	Varies by funding ratio and policy regime
$\underline{c_z}$	The minimum contribution	Varies by funding ratio and policy regime
τ_{ND}	The present value tax loss associated with contributions above the maximum tax-deductible limit	0.02

J. Optimal Contributions

Figure 1 shows the optimal contributions as a function of default risk and variable rate premium rates. The first panel shows how they vary with different levels of default probabilities under a common variable rate premium rate and the second panel shows different levels of variable rate premium for a company with a constant expected default rate. It shows that the lower default risk sponsor finds it optimal to make higher contributions than the higher default risk sponsor. That results from the higher degree of weight that the lower default risk sponsor places on the future benefits of higher funding. The increase in premium weights induces the lower default sponsor to make higher contributions across the range of funding ratios.

Figure 1: Optimal Contribution as a Ratio to Liabilities

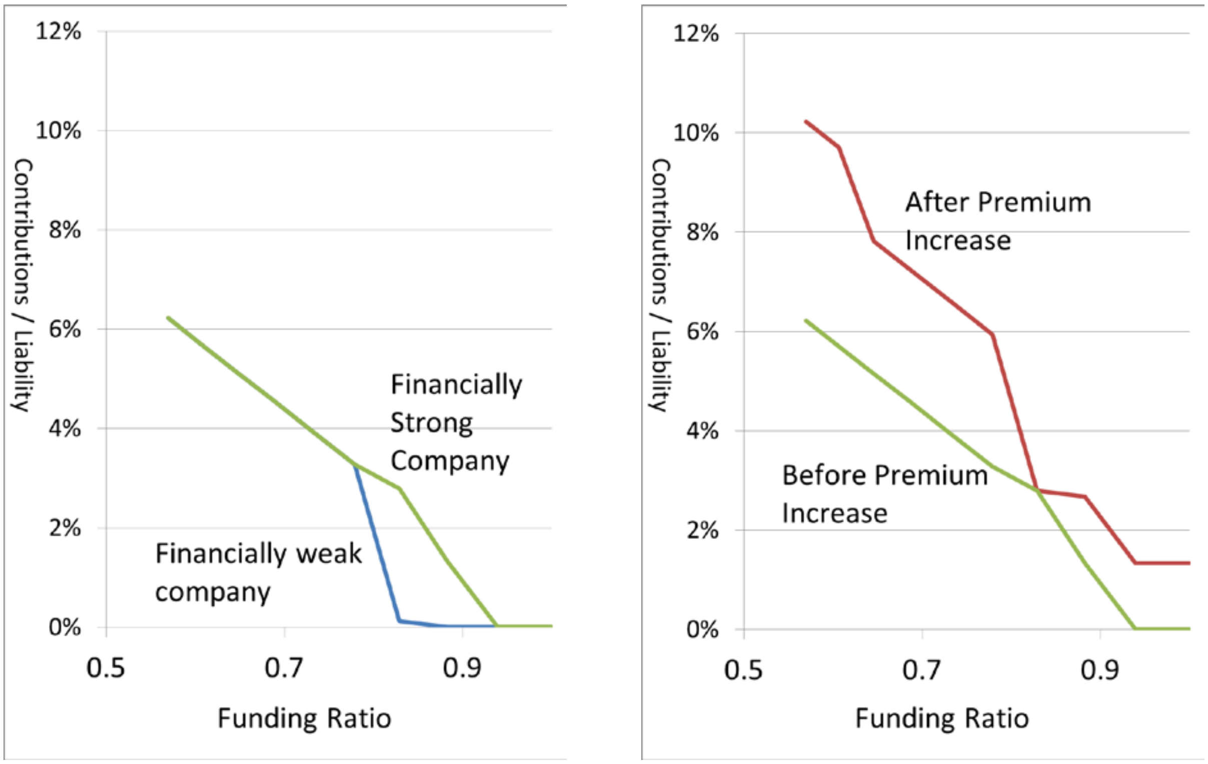
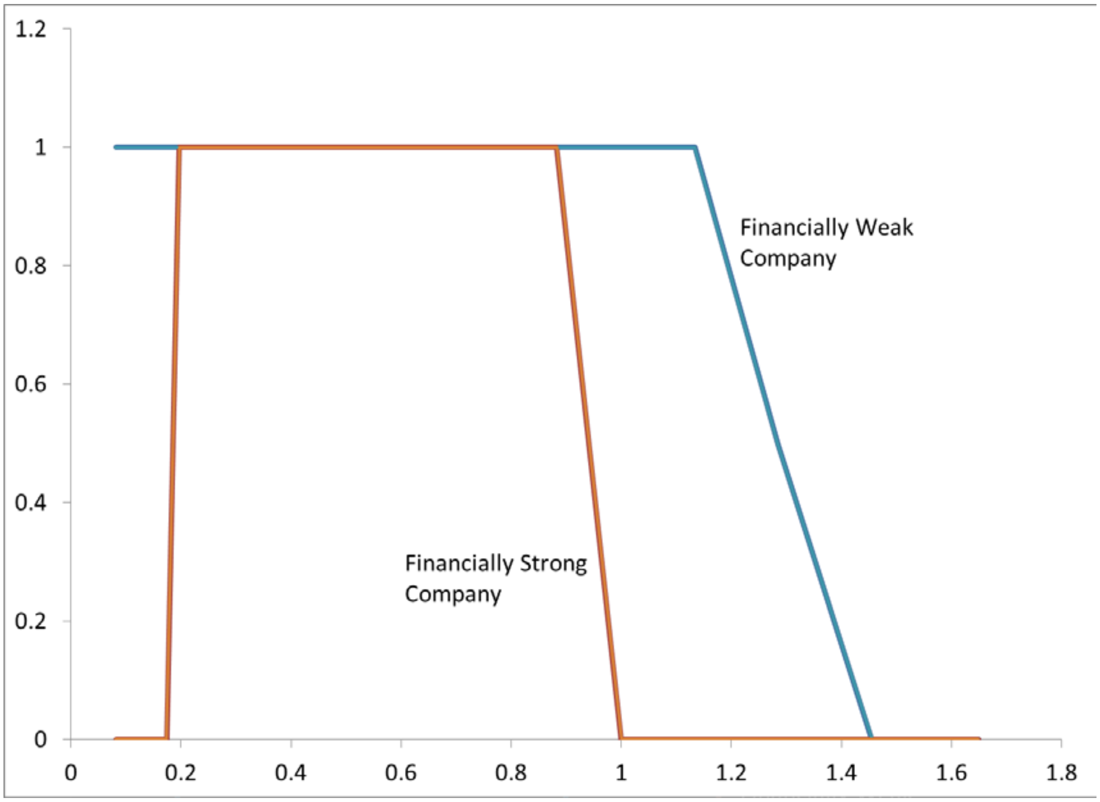


Figure 2 shows optimal asset allocation as a function of creditworthiness and the asset to liability ratio of the plan. The optimal allocation for most underfunded plans under the model is entirely in a return-seeking portfolio, because that maximizes the value of its insurance. For overfunded plans, the possibility of the reversion tax leads the optimal allocation to be in bonds, more so for relatively creditworthy plans where the opposing insurance effect is less potent.

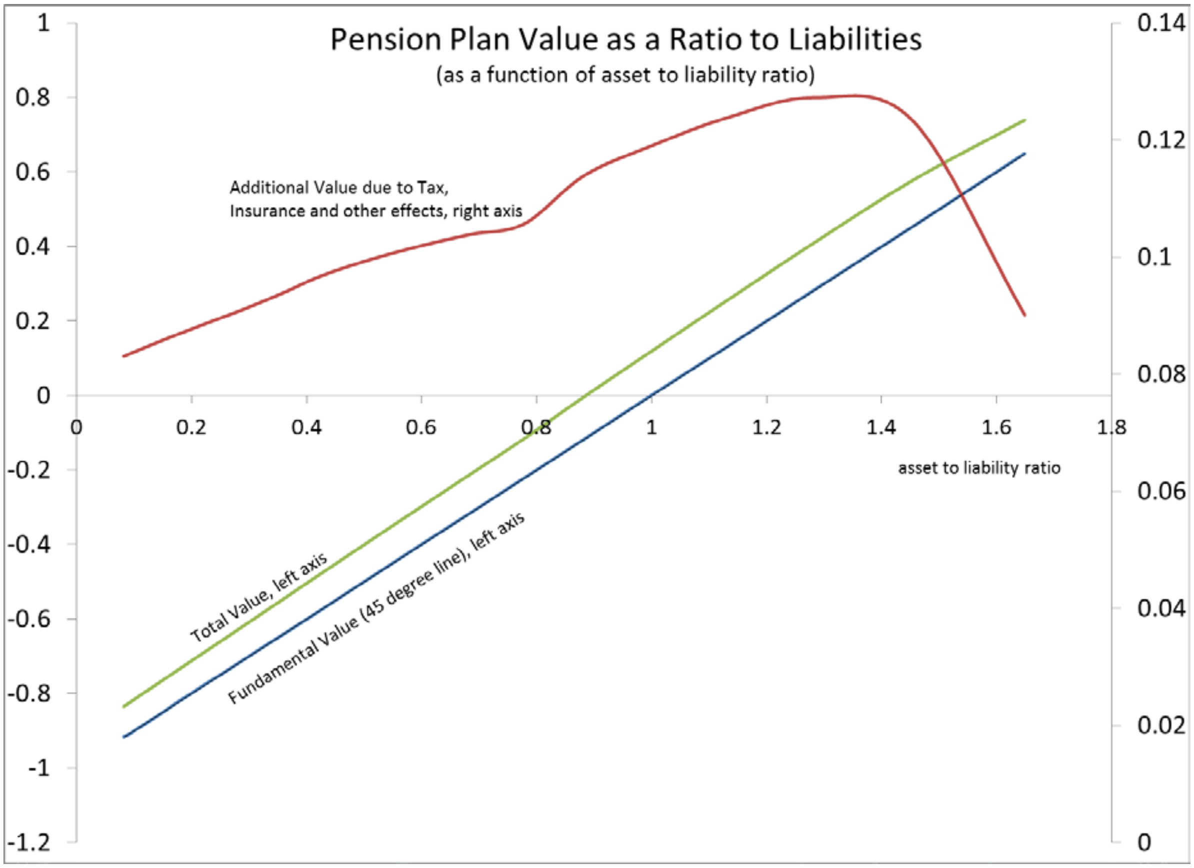
Figure 2: Optimal Allocation to Risky Portfolio by Funding Ratio for Financially Strong and Weak Companies



K. THE PRESENT VALUE ASSOCIATED WITH INSURANCE AND TAX EFFECTS

As noted in equation (12.), the value of the pension plan is the sum of a component equal to the value of net assets in the plan (assets minus liabilities) plus a component related to the tax, insurance, premium and other effects. Figure 3 shows this decomposition for a company with default risk consistent with an investment grade rating. The curve representing the net value of the assets is linear with a slope of one. It forms a 45% angle. That slope of one represents the notion that a dollars investment in a pension plan buys a dollar's worth of assets. Another line representing the total value of the pension plan lies above and roughly parallel to that 45-degree line. The total value is above the 45 degree line because of the additional value created by the underpriced insurance and tax benefits of the pension plan. The distance between the two lines represents the additional value of those benefits, and I also represent that additional value with a third line graphed against the right axis. It is a zoomed in measure of the distance between the first two lines. The additional value rises steadily and drops off before the level at which reversions are assumed to be mandatory.

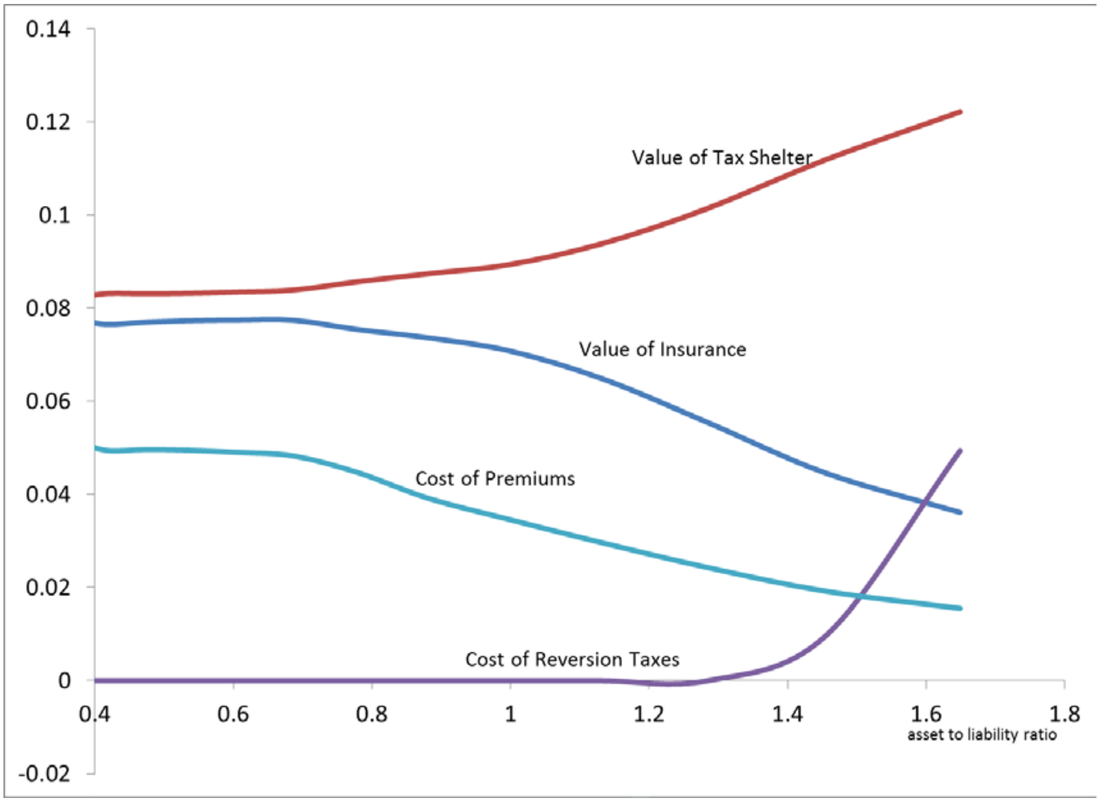
Figure 3: The fundamental and additional value of a pension plan



As long as the additional value is increasing in the asset-to-liability ratio, deductible contributions above minimum levels can make sense for the company. An additional dollar's contribution yields a dollar of fundamental value and a positive level of additional value over this range. A non-deductible contribution however might not make sense if the tax loss associated with non-deductibility exceeds the increase in value generated by the additional contribution. Also, convex costs of financing might offset the additional value of making additional contributions.

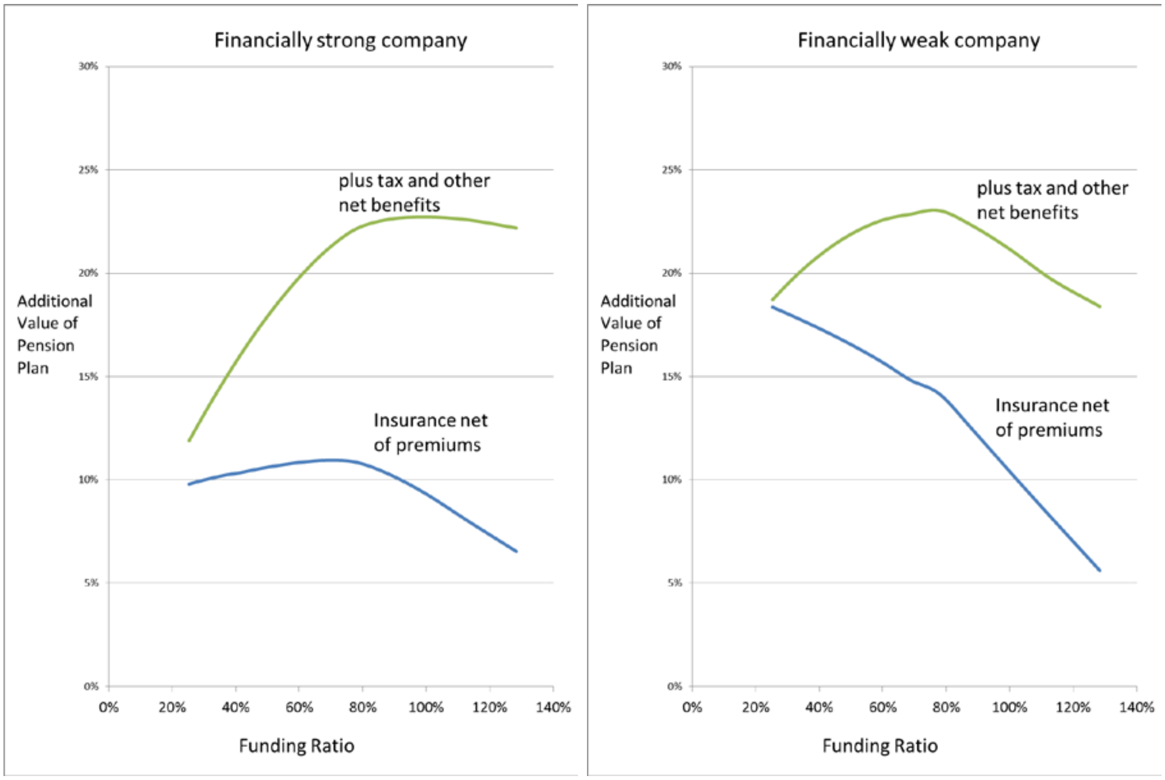
Figure 4 shows the individual tax and insurance effects in additional default for a company of medium default risk. The present value of premiums and insurance decline with the current funding ratio because higher funding ratio mean lower near term exposures of the pension insurance system. The graph shows that the present value of insurance exceeds that of the premiums at all funding ratios. The value of the tax shelter goes up with the current funding ratio as more fully funded plans are more able to take advantage of it. The cost of reversion taxes increases abruptly as the company approaches the 1.6 funding ratio threshold where reversions are assumed to be mandatory in the model.

Figure 4: Present Value of Tax Shelter, Insurance, Premiums as a Function of Funding Ratio



Figures 5 (a) and (b) show how each of these additional present values relate to the plan's current funding level and default risk. The left panel shows the present values of each of these rewards as a function of funding ratio for a company whose default risk is equivalent to an investment grade company. Figure 2b shows the same present values for a company with junk-bond level default expectations. For each company the insurance value decreases as the funding level increases. It starts out larger and declines more steeply for the junk-rated company. The value of the tax shelter is larger for the investment grade company, both because it is more likely to survive to enjoy many years of benefit from the tax shelter, and because its optimal funding strategy is to make higher contributions and to be higher funded.

Figure 5 (a), the additional value of tax and insurance benefits as a percent of pension plan liabilities by funding ratio for a low default risk company, and (b) the same values for a high default risk company.



IV. Empirical Estimation of the Factors Impacting Funding Decisions

The previous section developed formulas for optimal funding decisions and pension fund values in each possible state, conditional on policy parameters. Actual funding decisions in each state may deviate from these optimal funding decisions for a variety of reasons. The simplifications of the modeling may miss important factors in pension funding decisions. Plan sponsors might have idiosyncratic reasons for preferring a funding strategy over another in ways that the model cannot capture. However, the goal of the model is to supply an estimate of funding decisions that centers around actual funding decisions, with as little error as possible.

Under the theoretical model optimal contributions are a non-linear function of policy and other parameters. The non-linearities relate to the binding constraint of minimum contributions, where one quarter of observations reside, and the tendency for contributions levels to bunch at the tax deductible maximum. Under these circumstances a linear regression of excess contributions on policy, company and plan variables would be badly specified. Instead I opt for an approach that follows Rust (1994) and

Hansen & Sargent's (1980) recommendations on how to estimate parameters for a Markov Decision Process model.

I relate data on actual pension funding decisions to theoretically optimal decisions by introducing error terms into the equation identifying the optimal strategy. By including the error term, I create a probabilistic prediction. Instead of identifying a single strategy as optimal, it creates a probability distribution governing the optimal decision. The theoretical model's optimal decision has the maximum probability density in this distribution.

I estimate parameters that weight factors outlined in the theory section of the literature review and equation (11). Under that equation the value of the pension plan is the sum of the net funding level and a set of values related to other factors, including PBGC insurance, PBGC premiums, the tax shelter, penalties for exceeding maximum tax deductible contributions, and contribution volatility. The weighted version of this function is as follows:

$$V_W = (x-1) + (I - DGP(c,\beta))^{-1} * \{w_I r_{insurance} - w_\pi r_{premiums} + w_{TS} r_{taxshelter} - w_{ND} r_{nondeductible} - w_V r_{reversiontax} - w_\sigma r_{convexity}\} \quad (14.)$$

The weighted version of the rewards function in equation (7) is as follows:

$$R_W(c,\beta) = a - u - c + v + o + w_I \omega u - w_\pi \pi[x - \pi^*]_+ + w_{TS}\tau_E x\eta + w_v(\tau_o o - \tau_V v) - w_\sigma[c-a]_+^2 \quad (15.)$$

Under the theoretical model I presented in the last section, the weights on the insurance, premium, tax shelter and reversion taxes should all equal one if companies are behaving optimally and my model properly specifies the benefits and costs of their funding decisions. Finding weights other than one for those parameters could imply misspecification of the model, estimation errors such as mismeasurement of the financial strength of companies, or suboptimal decision making on the part of the companies such inattention on the part of companies to the factors outlined in the theory section. The weights on the gross tax loss associated with non-deductible contributions should be between zero and one because it represents the loss of present value companies face when they make a contribution over the tax deductible maximum and carry forward that deduction to future years. (In the last section I guessed its value at 0.1 but estimate it here). The weight on contribution volatility is something I expect to be greater than zero to reflect the costs associated with contribution volatility but I treat its estimation as an empirical matter.

Based on the weights the optimal actions ($c_w{}^*, \beta_w{}^*$) and values ($V_W{}^*$) are functions of the weights as defined in the following:

$$\{c_W *, \beta_W *\} = \arg\max_{c,\beta} (I - DGP(c,\beta))^{-1} \mathrm{R_W}(\mathrm{c},\beta)\ s.t.\ c_t > \underline{c}_i(\theta) \quad 0 \le \beta \le 1 \quad (16.)$$

$$V_W{}^* = (I - DGP(c_w{}^*, \beta_w{}^*))^{-1} R_W(c_w{}^*, \beta_w{}^*). \quad (17.)$$

Equation (17) can be decomposed into a current period reward and future period values as follows. That decomposition allows me to apply the method suggested by Rust for estimation of MDP, where companies calculate the value of the pension plan in future periods as if there were going to be no error term beyond the current year. Companies estimate the value associated with different possible states in the next period based on what this paper calls the theoretically optimal decisions.

$$\sum_{t=0}^{\infty} D^t G^t P(c,\beta^*)^t R(c^*,\beta^*) = R(\mathrm{c},\beta) + DGP(c^*,\beta^*)V^* \qquad (18.)$$

The value of the pension fund, given any choice of c^*, β^* is equal to the rewards in the current period plus the present value of rewards in all future periods $DGP(c^*,\beta^*)V^*$.

The version of this matrix formula for an individual company i , at time t , is the following:

$$v_t^i\left(x_t^i, y_t^i, c, \beta \mid \theta_t\right) = r * \left(x_t^i, y_t^i, c, \beta \mid \theta_t\right) + DG\sum_{j=1}^{N} p\left(x_j, y_j \mid x_t^i, y_t^i, c, \beta\right) v^*\left(x_j, y_j \mid \theta_t\right) \qquad (19.)$$

The values at each discrete point in the state space from the theoretical model enter this equation as the $v^*\left(x_j, y_j \mid \theta_t\right)$ terms.

The marginal net benefit of another dollar of contribution depends on the first derivative of the value given in equation (15), which is the following:

$$\frac{\partial v_t^i\left(x_t^i, y_t^i, c_t^i, \beta \mid \theta_t\right)}{\partial c_t^i} = \frac{\partial r\left(x_t^i, y_t^i, c_t^i, \beta \mid \theta_t\right)}{\partial c_t^i} + DG\sum_{j=1}^{N} \frac{\partial p\left(x_j, y_j \mid x_t^i, y_t^i, c_t^i, \beta\right)}{\partial c_t^i} v^*\left(x_j, y_j \mid \theta_t\right)$$
$$+ \kappa^i + \lambda_1 b_t^i + \lambda_2 S_t^i + \lambda_3 W_t^i + \varepsilon_t^i$$

(20.)

In this equation I introduce the error term ε_t^i to represent the notion that idiosyncratic factors might give companies additional benefits or costs associated with additional contributions. I estimate a fixed effect for each company. The variables b_t^i, S_t^i, and W_t^i represent the credit balance as a ratio to liabilities, the log of the size of the plan measured in dollars of liabilities, and the log ratio of the plan liabilities to company liabilities, respectively. I expect all of these variables might influence the marginal benefit of additional contributions. Higher credit balances might lower the marginal benefit of additional contributions because of diminishing benefits to higher credit balances in terms of the flexibility they give a plan. Size could either lower or raise the benefits of higher contributions depending on the impact of unobservable characteristics correlated with size.

The derivative should equal zero if the company made a contribution above the minimum but below the tax deductible maximum, because that zero marginal benefit would represent a situation where a dollar lower or higher contribution would not make the company better off. The derivative should be non-positive for a company making the minimum contribution. The situation at the tax deductible maximum is more complicated and discussed in Appendix C.

Those conditions on the derivative imply restrictions or unique values of the error term ε_t^i, which I assume to be a normally distributed independent variable. To determine what those restrictions or values are given a set of parameters, I calculate the values $v^*(x_j, y_j \mid \theta_t)$ which are fixed across observations under a policy regime. Then I calculate the derivatives $\frac{\partial r(x_t^i, y_t^i, c_t^i, \beta \mid \theta_t)}{\partial c_t^i}$ and $\frac{\partial p(x_j, y_j \mid x_t^i, y_t^i, c_t^i, \beta)}{\partial c_t^i}$ which vary by individual based on their state and action. To calculate the derivatives of the transition probabilities, $\frac{\partial p(x_j, y_j \mid x_t^i, y_t^i, c_t^i, \beta)}{\partial c_t^i}$, I apply equation (1). The derivatives of the rewards function are $\frac{\partial r(x_t^i, y_t^i, c_t^i, \beta \mid \theta_t)}{\partial c_t^i}$ are based on equation (15) and are as follows:

$$\frac{\partial r(x_t^i, y_t^i, c_t^i, \beta \mid \theta_t)}{\partial c_t^i} = -1 - \frac{\delta u_t^i}{\partial c_t^i} + \frac{\delta v_t^i}{\partial c_t^i} + \frac{\delta o_t^i}{\partial c_t^i} + w_I \omega \frac{\delta u_t^i}{\partial c_t^i} - w_\pi \pi * 1[x < \pi^*] + w_v \left(\tau_o \frac{\delta o_t^i}{\partial c_t^i} + \tau_V \frac{\delta v_t^i}{\partial c_t^i} \right) - 2 w_\sigma [c - a]_+ \qquad (21.)$$

While the theoretical model uses discretization and simplifying assumptions to make computation feasible, the empirical model applies those methods only to periods in the future and not to the current period. I assume that companies calculate the current year impact of their decisions exactly and then use the values of the theoretical model to measure the impact of their decisions on the expected future value of their pension plan. I model their set of choices as a continuous choice space, and calculate the current year rewards associated with those choices using a non-simplified version of the funding rules and premium schedule.

I assume that the error term is drawn from a normal distribution with mean zero and standard deviation σ_ε. Define c_t^i as the observed levels of c and β for company i at time t.

$$L_i(\Omega \mid c_t^i, \beta_t^i, x_t^i, y_t^i, \theta_t) = \Pr \left\{ \begin{array}{l} \mathrm{c}_\mathrm{t}^\mathrm{i} = \arg\max_c v_t^i(x_t^i, y_t^i, c, \beta \mid \theta_t, \Omega) \\ s.t. \quad c \geq \underline{c}_i(\theta), \quad 0 \leq \beta \leq 1 \end{array} \right\} \qquad (22.)$$

The likelihood under the parameters of the observation $\mathrm{c}_\mathrm{t}^\mathrm{i}$ equal the probability that it is a solution to this optimization problem, under the probability distribution governing the value of the error term. The observation c_t^i does not always imply a unique value of ε_t^i under equation (19) because of the kinks and discontinuities in the rewards functions, and potentially binding constraints. There are kinks in the rewards function where contributions reach a level where VRP liability goes to zero and at the point of maximum tax deductibility. There is also a potential discontinuity in the premium function in the period prior to the passage of the Pension Protection Act, where companies at or above the Full Funding Limit were exempted from paying the VRP. The minimum contribution can be a binding constraint on c_t^i. The corner solutions associated with those kinks create ranges of possible values of ε_t^i that would be consistent with optimal behavior. When c_t^i and β_t^i are neither situated at binding constraints, nor at kinks in the rewards functions then the first order conditions of the optimization problem yield a unique value of ε_t^i. Appendix C derives the likelihood function in detail covering all of these cases.

The parameters of the system are estimated through the following maximum likelihood equation:

$$\hat{\Omega} = \arg\max_{\Omega} \sum_{\forall i,t} \log L_i\left(\Omega \mid c_t^i, \beta_t^i, x_t^i, y_t^i, \theta_t\right) \quad (23.)$$

Table 3 Parameters in the Estimated Set Ω.

Parameter	Description
w_I	The weight companies give the value of insurance pension funding decisions.
w_π	The weight companies give PBGC premiums pension funding decisions.
w_{TS}	The weight companies give the value of tax free build up in pension funding decisions.
w_{ND}	The weight companies give the penalty for non-deductible contributions in pension funding decisions.
w_σ	The weight companies give to convex costs of capital in pension funding decisions.
κ^i	A fixed effect by individual plan sponsor
λ_1	The coefficient on the credit balance as a ratio to liabilities
λ_2	The coefficient on the log liabilities (size) of the pension plan
λ_3	The coefficient on the log ratio of liabilities of the pension plan to the liabilities of the company (relative size of the pension plan to the sponsor)
σ_ε	Standard deviation of ε_t^i

V. DATA AND MEASUREMENT

The data come from four sources: the annual form 5500 filings for private pension plans, data from the premium filings that plans submit to the PBGC, Compustat, and the Center for Research on Security Prices (CRSP). I use the form 5500 data to measure the actual and required contributions as a percent of liabilities as well as the maximum deductible contribution, and the statutory asset to liability ratio for each plan. I use the premium filings, in combination with the form 5500 data, to estimate the threshold asset to liability ratio below which companies pay the variable rate premium. The PBGC's Policy Research and Development office supplied the form 5500 and premium filing data.

The premium filing data is available for some companies but not others and is more informative in some years than others. In the years since passage of the Pension Protection Act in my sample, 2009 to 2012, the premium filing data contains the funding target under the premium rules. That target allows me to pinpoint exactly the level of assets at which companies can avoid paying the variable rate premium, for companies covered by the premium filing data. For the companies where premium filings are not available in those years, I impute the premium target using the average ratio between the premium target and the vested liability funding rule target of companies where the information is available. For the years before implementation of the PPA I revalue the vested liabilities of each plan, based on the difference between the discount rate the company uses in the funding rules and the discount rate required in the premium calculation. I assume that the modified duration of benefits is 12 years in making that adjustment, such that a one percent lower discount rate leads to an approximately 12 percent higher funding target, because this is approximately the average duration of DB plans.

I linked the Compustat and CRSP data using the Wharton Research Data Service tools. They supplied the data used to develop the distance to default measure including financial statement liability, market capitalization and equity price volatility. The distance to default measures the likelihood of default in reference to the Black-Sholes-Merton model, with characterizes equity holders as holding a put option on bond holders, and default as the exercise of that option. The distance to default is the log ratio of assets to liabilities minus a threshold level at which default takes place, divided by the standard deviation of that ratio. It measures the number of standard deviations from the mean a negative shock to the company's net worth would need to be to throw the company into default. For example a company with a distance of default equal to two would default under a two-standard deviation or higher negative shock, which under the normal distribution would have a 0.0228 likelihood of default. Appendix B describes how I calculate the distance to default using standard methods from previous studies measuring credit risk.

I used two sources matching the Compustat and CRSP identification numbers to the employer identification numbers in the form 5500 and premium filings. The first was the table of ID matches from Lewis and Pennacchi (1999), and the second was the premium filings from the PBGC. The premium filings contained a field for the employer's CUSIP, which could be matched to the identification number used in CRSP, called the "PERMNO" using tools in the Wharton Research Data Service. The result of the matching process yielded approximately two thousand CRSP PERMNO, identifying publicly-traded entities, matched to approximately twice as many EINs, linked to separate companies owned by those entities. Each EIN could be associated with more than one pension plan, as it is common for large employers to have separate plans for different classes of employees (e.g. airline mechanics and flight attendants).

I excluded observations with extremely high or low funding ratios and contributions which do not fall in the range where I'd expect the incentives created by funding rules to have much relevance, and other observations with discrepancies based on the formulas linking different entries in the form 5500. Table 4 describes the dataset that resulted from this merging and data quality control.

Table 4: Number of Observations, unique publicly-traded holding company (PERMNO), and unique employers (EIN)

Year	Observations	Unique PERMNO	Unique EINs
1993	1884	901	1003
1994	1982	943	1066
1995	1418	748	832
1996	1732	870	984
1997	1741	931	1054

1998	1975	959	1115
1999	1506	740	867
2000	1453	752	891
2001	1689	828	1025
2002	1844	903	1139
2003	1870	872	1133
2004	1820	862	1131
2005	1815	856	1121
2006	1739	839	1084
2007	1580	787	993
2008		Data not available	
2009	1515	785	966
2010	1459	777	951
2011	1416	758	916
2012	1347	740	891
All Years	31785	1816	2313

There were 1816 unique PERMNO identification numbers, each representing a publicly-traded entity. In each year about half of the total number of PERMNO were observed in the data set. Frequently more than one employer corresponded to a PERMNO, as a publicly-traded holding company might own more than one employer. And many employers had more than one plan associated with them. As a result each publicly-traded holding company on average was associated with just under two pension plans per year.

Table 5: Summary Statistics for Key Variables

	Minimum	Maximum	Median	Mean	Standard Deviation
Distance to Default (y)	1.00	8.00	3.37	3.41	1.71
Funding Ratio	0.20	1.50	0.90	0.93	0.22
Required Contribution / Liabilities	0.00	0.77	0.03	0.04	0.06
Actual Contribution / Liabilities	0.00	0.56	0.03	0.06	0.08
Premium Threshold / Liabilities	0.11	2.09	1.16	1.16	0.14
Maximum Tax Deductible Contribution / Liabilities	0.00	1.10	0.03	0.07	0.10
Starting Credit Balance / Liabilities	0.00	1.00	0.13	0.17	0.17

The results of the estimation are presented in Table 6. It shows the estimated weights associated with each of the factors determining optimal contributions.

Table 6: Estimated Parameters

	Estimated Coefficient	Standard Error	T-Statistic	Prob. |t|>0
Weight on Insurance- Applied to estimated insurance payments	0.097	0.000	557.0	0.000
Weight on Premium - Applied to estimated premium payments	-2.506	0.003	-909.6	0.000
Weight on Tax Shelter Applied assuming effective tax rate of 0.25 and using average ten year Treasury rate for each time period	3.493	0.003	1050.9	0.000
Weight on Tax Deductibility Applied assuming effective tax rate of 0.25	0.137	0.001	153.6	0.000
Weight on Convexity –applied to quadratic term in equation (12.)	-0.016	0.001	-18.2	0.000
Sigma	0.033	0.000	218.9	0.000
Coefficient on Credit Balance	-0.072	0.002	-34.2	0.000
Coefficient on Log Liabilities of Pension Plan	-0.005	0.000	-82.4	0.000
Coefficient on Log Ratio of Pension Plan Liabilities to Company Liabilities	0.003	0.000	23.9	0.000

The weight on "tax shelter" is positive and significant, suggesting that the main motivation for making excess contributions is the benefit of tax shelter. The coefficient on the insurance is positive and significant but nowhere near one. That suggests that less creditworthy companies refrain from making contributions, but not to the extent predicted by the theory. The coefficient on premiums is highly negative and significant. This result is the most surprising because it suggests that companies in the funding ratio range that is subject to the VRP do not seem to make additional contributions to reduce their VRP liability –on the contrary they seem to refrain from making contributions that similar companies not subject to the variable rate premium would make in the same situation. It is possible that the VRP rate of 0.009 is too low to enter their considerations in practice to a measurable degree even though it is theoretically large enough to matter. The very high coefficient on tax shelter and negative coefficient on premiums suggests the presence of some omitted variable or problem in the specification. Companies are deterred from making above the maximum tax deductible level of contributions. The coefficient on convexity is negative and statistically significant, but close to zero. That suggests that companies are not concerned with variation in their contribution levels. On the contrary they seem to slightly prefer making lumpy contributions to smooth ones.

The three additional variables do not impact the weights very much, which are almost identical without their inclusion in the study. The coefficient on credit balances is negative as expected, suggesting that plans with large credit balances all else equal are likely to make lower contributions. Larger pension plans tend to make lower contributions, all else equal. However when pension plans liabilities are more significant in proportion to their sponsor's overall finances, sponsors tend to make higher contributions.

APPENDIX A: CALCULATION OF THE PROBABILITIES IN THE TRANSITION MATRIX

This Appendix derives formulas for the transition probabilities, discount and growth rates, which are functions of parameters governing asset returns and pension benefits. It then supplies the values of those parameters used in the study, and the sources for those values.

L. NOTATION

The pension plan fund has assets equal to A_t^P and liabilities equal to L_t^P at time t. The ratio of assets to liabilities at time t is defined as $x_t \stackrel{\text{def}}{=} \frac{A_t^P}{L_t^P}$. The plan sponsor has company assets and liabilities equal to A_t^S and L_t^S. The log ratio of assets to liabilities for the plan sponsor is $\tilde{y}_t \stackrel{\text{def}}{=} \log\frac{A_t^S}{L_t^S}$ .The pension plan pays out a fraction of its liabilities equal to υ over the year to retirees, and workers earn new benefits whose present value is equal to a fraction a of its liabilities. c is the ratio of contributions to liabilities. The liabilities will grow at the rate $G \equiv (1+a-\upsilon)(1+\eta)$, where η is the expected rate of return on assets and liabilities under the risk neutral distribution, where the expected return on all assets is equal to the risk free rate. The discount rate, D is $\frac{1}{1+\eta}$, and thus GD, representing the growth factor of the present value of the pension plan liabilities, equals $(1+a-\upsilon)$.

M. SEQUENCE OF EVENTS

The following sequence of events governs the transition probabilities:

1. New benefits accrue to active workers, retirees receive benefit payments, and the plan sponsor makes contributions.
2. Plan and sponsor asset and liabilities experience stochastic returns.
3. Sponsors with assets to liability ratios below the default threshold go bankrupt, closing their pension plans and potentially turning them over the PBGC. Plans with asset to liability ratios above the default threshold raise or lower their company liabilities to move their asset to liability ratio towards a target value over a period of years.

N. BENEFIT ACCRUALS, PAYOUTS, AND CONTRIBUTIONS

After the accruals, payouts and contributions assets will equal $A_{t+}^P = A_t^P + (c-\upsilon)L_t^P$, and liabilities will equal $L_{t+}^P = (1+a-\upsilon)L_t^P$.

The new ratio of asset to liabilities x_{t+} will be equal to the following:

$$x_{t+} = \frac{A_{t+}^P}{L_{t+}^P} \tag{24.}$$

which translates into:

$$x_{t+} = \frac{x_t + c - \upsilon}{1 + a - \upsilon} \tag{25.}$$

O. Stochastic Returns to Assets and Liabilities

The distribution of asset returns determines how much plan assets are likely to grow in value, while the distribution of interest rate changes determines how much plan liabilities might grow or shrink as a function of changing discount rates. Asset returns are governed by the following difference equations:

$$\log\left(A_{t+1}^P / A_{t+}^P\right) = \eta + \beta\sigma_E z_t^E + (1-\beta)\sigma_L z_t^L \tag{26.}$$

$$\log\left(L_{t+1}^P / L_{t+}^P\right) = \eta + \sigma_L z_t^L \tag{27.}$$

$$\log\left(A_{t+1}^S / A_t^S\right) = \eta + \sigma_Y z_t^Y \tag{28.}$$

$$\log\left(L_{t+}^S / L_t^S\right) = \eta \tag{29.}$$

The z terms are standard normal variables that represent the stochastic components of a diversified equity fund, pension plan liabilities, and the sponsor's corporate assets. The parameters σ_E, σ_L, and σ_Y represent the standard deviation of returns of the diversified equity fund, pension plan liabilities, and plan sponsor assets, respectively. The plan has two investment options: the diversified equity fund in which it invests a fraction β of its portfolio and a bond fund in which it invests the remainder. The bond fund has the same duration as the pension plan liabilities, and therefore is subject to the same stochastic component of the return $\sigma_L z_t^L$. The covariances of the stochastic components are as follows: $Cov(z^E, z^L) = \rho_{EL}$, $Cov(z^E, z^S) = \rho_{EY}$, $Cov(z^L, z^S) = \rho_{LY}$.

The plan funding ratio x_{t+1} after the stochastic asset price changes has the following relationship to x_{t+}, the plan funding ratio before it:

$$\log\frac{x_{t+1}}{x_{t+}} = \log\left(A_{t+1}^P / L_{t+1}^P\right) - \log\left(A_{t+}^P / L_{t+}^P\right) = \log\left(A_{t+1}^P / A_{t+}^P\right) - \log\left(L_{t+1}^P / L_{t+}^P\right) \tag{30.}$$

Combining this equation with (26.) yields an expression for the stochastic component of the new funding ratio x_{t+1}:

$$\log\frac{x_{t+1}}{x_{t+}} = \beta\left[\sigma_E z_t^E - \sigma_L z_t^L\right] \tag{31.}$$

The stochastic component in this equation is equal to β times the return on the portion of the portfolio invested in equity net of the return on a bond portfolio with a duration equal to the benefit payments that represent the plan's liabilities. The expected value of that component is zero because z terms are standard normal variables and thus are mean zero. So the expected value of $\log x_{t+1}$ is the same as $\log x_{t+}$. It equals $\frac{x_t + c - \upsilon}{1 + a - \upsilon}$.

The standard deviation of the funding ratio x_{t+1} is then a function of the parameters and the asset allocation variable

$$\sigma_x = \beta\sqrt{\sigma_E{}^2 + \sigma_L{}^2 - 2\rho_{EL}\sigma_E\sigma_L} \,. \tag{32.}$$

Defining $\sigma_{E-L} \equiv \sqrt{\sigma_E{}^2 + \sigma_L{}^2 - 2\rho_{EL}\sigma_E\sigma_L}$, note that $\sigma_x = \beta\sigma_{E-L}$. The standard deviation of the log funding ratio is directly proportional to the fraction of the asset in the diversified return-seeking portfolio. Note that a plan that practices complete duration matching $\beta = 0$ would have a funding ratio not subject to stochastic changes, where $\sigma_x = 0$.

P. BANKRUPTCY AND COMPANY LEVERAGE TARGETING

Define the variable y as the scaled log ratio of sponsor company assets to liabilities, where $y = \frac{\tilde{y}}{\sigma_Y}$, Let y_{t+} represent the log asset to liability ratio of the plan sponsor after the stochastic return to plan assets but before the plan sponsor undertakes any adjustments to its liabilities to return its asset-to-liability ratio towards the target value. It is given by the following formula:

$$y_{t+} = \log\left(A^S_{t+1} / L^S_{t+}\right) \tag{33.}$$

In combination with formulas (28.) and (29.), this yields:

$$y_{t+} = y_t + z^y_t \tag{34.}$$

The standard deviation of y_{t+} is one, based on the standard normal stochastic component. Thus the initial value y_t represents the number of standard deviations the plan is from the default threshold. The company defaults if the log asset to liability ratio of the company falls below zero. So the probability of default is $\Phi(-y_t)$ where Φ is the standard cumulative normal function.

If the sponsor does not go bankrupt, where $y_{t+} > 0$, then it adjusts its log asset to liability ratio towards a target value of y^* at a speed α, yielding a value of y_{t+1} equal to the following:

$$y_{t+1} = (1-\alpha)\mathrm{y}_{\mathrm{t+}} + \alpha y^* \tag{35.}$$

Combining this equation with (34.) yields:

$$y_{t+1} = (1-\alpha)\mathrm{y}_\mathrm{t} + (1-\alpha)\mathrm{z}_\mathrm{t}^\mathrm{y} + \alpha y^* \qquad (36.)$$

The expected value of y_{t+1} conditional on y_t, is thus $(1-\alpha)\mathrm{y}_\mathrm{t} + \alpha y^*$, and the standard deviation $(1-\alpha)$. The conditional covariance of x_{t+1} and y_{t+1} is:

$$\begin{aligned} \mathrm{cov}(x_{t+1}, y_{t+1} \mid x_t, y_t) &= \mathrm{cov}\left(\beta\left[\sigma_E z_t^E - \sigma_L z_t^L\right], (1-\alpha)\mathrm{z}_\mathrm{t}^\mathrm{y}\right) \\ &= (\sigma_E \rho_{EY} - \sigma_L \rho_{LY})(1-\alpha)\beta \qquad (37.) \end{aligned}$$

Define $\rho = \dfrac{(\sigma_E \rho_{EY} - \sigma_L \rho_{LY})}{\sigma_{E-L}}$, so that $\rho\sigma_{E-L} = (\sigma_E \rho_{EY} - \sigma_L \rho_{LY})(1-\alpha)\beta$. That supplies the last element for the expected value and covariance matrix for x_{t+1} and y_{t+1}:

$$\mu = \begin{pmatrix} \dfrac{x_t + c - \upsilon}{1 + a - \upsilon} \\ (1-\alpha)y_t + \alpha y^* \end{pmatrix}, \ \Sigma = \begin{pmatrix} \beta^2 \sigma_{E-L}{}^2 & \rho\beta(1-\alpha)\sigma_{E-L} \\ \rho\beta(1-\alpha)\sigma_{E-L} & (1-\alpha)^2 \end{pmatrix} \qquad (38.)$$

Q. VALUES USED IN THE STUDIES

Expression (38.) contains two variables characterizing the initial state of the pension plan, (x_t and y_t), two capturing the action taken by the plan sponsor (c and β), and several parameters. Table A-1 supplies the values for those parameters, along with their definitions and sources for the values.

Table A-1: Parameters of the Transition Matrix

Parameter	Definition	Value	Source
a	The ratio of new benefits accruals to existing pension liabilities	0.047	Department of Labor, Bureau of Economic Analysis
υ	The ratio of pension benefit payouts to existing pension liabilities	0.085	Department of Labor
σ_{E-L}	The standard deviation of the asset to liability ratio for pension benefits funded with a diversified portfolio of equity.	0.209	Ibbotson
ρ	The correlation of a diversified equity portfolio with the plan sponsor’s net worth.	0.265	Goyal and Santa Clara
α	The fraction of the gap towards the target asset to liability ratio remedied in each year	0.18	Collin Dufresne and Goldstein
y^*	The target asset to liability ratio divided by the annual volatility of that ratio	3.278	Collin Dufresne and Goldstein

To estimate the accrual rate a, the ratio of new benefit accruals to existing pension liabilities I relied on the Bureau of Economic Analysis estimate prepared for the National Accounts (Rassier, 2014), and divided them by the liabilities measures found in the PBGC Data Book. The average rate for the 1992-2012 period was 0.047. The Department of Labor Pension plan bulletin supplied aggregate levels of pension benefit payments, which I divided by the liability measures in the PBGC Data Book to yield an estimated value for υ, the ratio of pension benefit payouts to liabilities. Over the 1992-2012 period those payments averaged 0.085 percent of liabilities. The values of the accrual rate and the payout rate lead GD, representing the growth factor of the present value of the pension plan liabilities, to equal 0.962.

I use historical stock market data to measure σ_{E-L} and rely on a study that parses idiosyncratic and systematic risk in equity for ρ. The change in the asset to liability ratio of a portfolio invested in equity funded using long term debt will equal the difference in returns between the equity portfolio and the debt portfolio. Using data from Ibbotson the standard deviation of a portfolio the difference of the return on the S&P 500 and the return on 10-year Treasury bonds, over the 1928-2014 period is 0.209. I'd get similar results using a period that reaches less far back into history.

The parameter ρ represents the correlation of an individual company's net worth with that of the overall market, and is equal to the fraction of volatility that relates to systematic rather than idiosyncratic risk. To estimate ρ, I rely on Goyal & Santa-Clara (2003) who estimate a monthly standard deviation of 0.0436 for the return of a diversified portfolio, and 0.1646 for that of an individual stock. The ratio of the two values, 0.265, is used for the parameter ρ.

I use parameters from Collin-Dufresne & Goldstein (2001) for α and y^*. Under the risk neutral the measure the variable they use to measure distance to default, called ℓ in their model, reverts towards a value of 0.6556, at a rate of 0.18 per year. They estimate a standard deviation of 0.2, which leads to a ratio of 0.6556 / 0.2 of 3.278. (see section IIA).

APPENDIX B: ESTIMATION OF THE DISTANCE TO DEFAULT

Under the model the market value log asset to liability ratio, $\tilde{y}_t$, and its volatility, σ_Y, summarize the current health of the company and how rapidly that health is likely to change. The distance to default is based on the ratio of these two variables. I use stock market data to estimate the current asset liability ratio and its volatility.

Define s_t as the ratio of the stock market capitalization of a company to its liabilities. Under an options pricing model of equity value, as proposed by Merton (1974) the market capitalization is equal to value of an option to purchase the company's assets with an exercise price equal to the company's liabilities. Mathematically this relationship is the following:

$$s_t = \exp(\tilde{y}_t) N(d_{1t}) - N(d_{2t}) \qquad (39.)$$

where

$$d_{1t} = \left(\tilde{y}_t + \frac{1}{2} \sigma_Y{}^2 \right) / \sigma_Y, \; d_{2t} = d_{1t} - \sigma_Y$$

The volatility of the asset to liability ratio, σ_Y also figures in this equation, and is not directly observable.

The volatility of equity is given by a second equation.

$$\sigma_{S\,t} = \frac{\exp(y_t) N(d_{1_t})}{s_t} \sigma_Y \qquad (40.)$$

It is a function of the asset to liability ratio y_t, declining as the asset to liability ratio goes up, or leverage goes down. So while I assume that the volatility of the asset to liability ratio σ_Y is constant for a given company, and the volatility of equity changes continuously along with the variable y_t. Equation (40.) gives σ_{S_t} as a function of $\tilde{y}_t$, s_t, and σ_Y. The value of $\tilde{y}_t$ is an implicit function of σ_Y and s_t, under equation (39.) which I will call $\tilde{Y}(\sigma_Y, s_t)$. So σ_{S_t} can be expressed as a function of just σ_Y and s_t, which I will define as $S(\sigma_Y, s_t)$.

I use CRSP data to estimate the volatility of equity in each quarter, using daily observations of the stock market return for each company. Let $\hat{\sigma}^D_{S\,t}$ be the standard deviation of daily return for the stock in a quarter, based on the roughly 63 trading days in that quarter. Then the estimate $\hat{\sigma}_{S\,t}$ of the annualized volatility is $\hat{\sigma}_{S_t} = \sqrt{252}\hat{\sigma}^D_{S\,t}$, based on 252 trading days in a year. The estimate $\hat{\sigma}_{S\,t}$ measures $\sigma_{S\,t}$ with some error. The ratio of the two has the following Chi-Square distribution:

$$63\left(\frac{\hat{\sigma}_{St}}{\sigma_{St}}\right)^2 \sim \chi^2(63) \tag{41.}$$

The ratio of the observed sum of square deviations of daily returns from the mean, over 63 daily observations in the quarter, to the expected sum of squares is Chi-Squared with 63 degrees of freedom. So I will refer to the likelihood of an observed value of $\hat{\sigma}_{St}$ given that σ_{St} is the true value as:

$$L(\hat{\sigma}_{St} \mid \sigma_{St}) = \chi^2\left(63\left(\frac{\hat{\sigma}_{St}}{\sigma_{St}}\right)^2, 63\right). \tag{42.}$$

The approach taken will be to find the value of σ_Y and thus y for each company that maximize this likelihood, while also considering the implications of the difference equations (28.) and (29.) which govern movements in y. Under those difference equations, the value of y should tend towards $y*$, making values near $y*$ more likely to occur all else equal than those distant from it. Specifically y follows a normally distributed mean-reverting process. If I apply recursion to the formula $\mathrm{y_t} = (1-\alpha)\mathrm{y_{t-1}} + (1-\alpha)\mathrm{z_t^y} + \alpha y^*$, I obtain $\mathrm{y_t} = (1-\alpha)^{\mathrm{j}}\mathrm{y_{t-j}} + \sum_{k=0}^{j}(1-\alpha)^{\mathrm{k}}\left[\mathrm{z_{t-k}^y} + \alpha y^*\right]$ for all j. The limit as j goes to infinity this simplifies to: $\mathrm{y_t} = y^* + \sum_{k=0}^{j}(1-\alpha)^{\mathrm{k}}\mathrm{z_t^y}$. The unconditional variance is thus the following:

$$\begin{aligned}\mathrm{Var}(\mathrm{y_t}) &= Var\left(y^* + \sum_{k=0}^{j}(1-\alpha)^{\mathrm{k}}\mathrm{z_{t-k}^y}\right) = \\ \left(\sum_{k=0}^{j}(1-\alpha)^{\mathrm{k}}\right)^2 &= \frac{1}{1-(1-\alpha)^2}\end{aligned} \tag{43.}$$

With $\alpha = 0.18$, this leads to a standard deviation of 1.747, around the mean value of 3.35 I obtained from the Collin-Dufresne and Goldstein study. Those parameters imply a prior distribution of y_t that is $N(3.35, 1.75)$. The covariance of two value of y_t of the same company at different time is the following function of how far apart in time those observations are:

$$\mathrm{cov}(y_{t_1}, y_{t_2}) = \mathrm{cov}\left(\sum_{k=0}^{j}(1-\alpha)^{\mathrm{k}}\mathrm{z_{t_1-k}^y}, \sum_{k=0}^{j}(1-\alpha)^{\mathrm{k}}\mathrm{z_{t_2-k}^y}\right) = \frac{(1-\alpha)^{2|t_1-t_2|}}{1-(1-\alpha)^2} \tag{44.}$$

Let y^i be the set of values of y_t associated with a company i, and $\sum_Y^i$ be the covariance matrix associated with those observations based on formulas (43.) and (44.). Then the likelihood of the data s^i and $\hat{\sigma}_S^i$, given a value of $\sigma^i{}_Y$ is as follows:

$$L\left(s^i, \hat{\sigma}_s^i \mid \sigma^i{}_Y\right) = \sum \phi\left(\tilde{Y}\left(\sigma_Y, s^i\right), \mu_Y, \Sigma_Y^i\right) \chi^2\left(63\left(\frac{\hat{\sigma}_S^i}{S\left(\sigma_Y, s_t\right)}\right)^2, 63\right) \tag{45.}$$

After obtaining an estimate of $\sigma^i{}_Y$, by finding the value that maximizes this likelihood function, I use it to calculate the value of y_t.

APPENDIX C: THE LIKELIHOOD FUNCTION

The empirical model solves for parameters that maximize the likelihood of the actual contributions. The plan sponsor receives a net benefit for each dollar of contributions to the pension plan, equal to an estimated net benefit under the model and an error term. The parameters that maximize the likelihood of the data minimize that error term.

A sponsor's actual contributions imply a unique value to the error term in cases where they are above the minimum level and not at a kink in the rewards function. Observations where a company made the minimum contribution or a contribution at a kink in the rewards function are consistent with a range of values of the error term. There are two points at which kinks in the rewards function take place. First at the Variable Rate Premium Target, contributions abruptly stop earning the reward of lower VRP rates. Second at the maximum tax deductible contribution, additional contributions become subject to a delay in the tax deductibility of contributions. Finally there is a special case in the pre-2008 data only where reaching the full funding limitation can cause a discontinuous drop to zero in variable rate premiums, which also corresponds to a range of values of the error term.

The term $v\left(C_t^i \mid \varepsilon_t^i\right)$ refers to the value of the company at contribution level C_t^i given the value of the error term, and its derivative $v'\left(C_t^i \mid \varepsilon_t^i\right)$ is part of the first order conditions that determine the optimal. The likelihood function gives the probabilities associated with the values of ε_t^i that are solutions to the equations generated by the optimization problem.

1) INTERIOR SOLUTION

For a contribution not situated at a kink in the function $v\left(C_t^i\right)$, the first order condition suggests that $v'\left(C_t^i \mid \varepsilon_t^i\right) = 0$.

2) MINIMUM CONTRIBUTION

At the minimum contribution the marginal value of an additional dollar of contribution is either zero or less. In the case where the marginal value of an additional contribution is less, the plan sponsor would benefit from making a lower than minimum contribution but is constrained by the funding rules. Mathematically it is true that $v'\left(C_t^i \mid \varepsilon_t^i\right) \le 0$.

Define $\overline{\varepsilon_t^i}$ as $v'\left(C_t^i \mid \overline{\varepsilon_t^i}\right) = 0$. Then $\varepsilon_t^i \le \overline{\varepsilon_t^i}$

R. KINKS

If the contribution is at a kink, or non-differentiable part of the function $v\left(C_t^i\right)$then $v'\left(C_t^i\right)$ does not exist, but I can place conditions on the limits of $v'\left(C\right)$ as C approaches C_t^i, under which the company would not benefit from increasing or decreasing its contribution. Namely the marginal net benefit of a dollar of contribution would be greater than equal to zero before the kink, and less than or equal to zero after the kink, meaning:

$$\lim_{C \to C_t^{i-}} v'\left(C \mid \varepsilon_t^i\right) \geq 0, \ \lim_{C \to C_t^{i+}} v'\left(C \mid \varepsilon_t^i\right) \leq 0$$

These conditions imply that the error term will fall within a range.

$$\underline{\varepsilon_t^i} \leq \varepsilon_t^i \leq \overline{\varepsilon_t^i} \text{ , where } \lim_{C \to C_t^{i-}} v'\left(C \mid \underline{\varepsilon_t^i}\right) = 0 \text{ and } \lim_{C \to C_t^{i+}} v'\left(C \mid \overline{\varepsilon_t^i}\right) = 0$$

3) DISCONTINUOUS JUMP AT THE FULL FUNDING LIMITATION

If a contribution is situated right above a discontinuous increase in $v(C)$, due to the fact that all variable rate premiums are extinguished at the meeting of the full funding limitation, I can say that $\lim_{C \to C_t^{i+}} v'(C) \leq 0$, that additional contributions beyond the full funding limitation do not bring a net benefit. Also the value at the full funding limitation $v\left(C_t^i\right)$ is greater than the local maximum for all levels of contributions below the jump in value. Suppose that the jump in rewards is equal to Π.

Let $C^*, \underline{\varepsilon_t^i}$ be defined by $v'\left(C^* \mid \underline{\varepsilon_t^i}\right) = 0$, and $v\left(C^* \mid \underline{\varepsilon_t^i}\right) = v\left(C_t^i \mid \underline{\varepsilon_t^i}\right)$. The contribution level C^* represents the contribution level below the full funding limit where the value of the function $v\left(C_t^i\right)$ is at a maximum. And $\underline{\varepsilon_t^i}$ is the level of the error term at which the value at that local maximum is exactly equal to the value at the FFL.

I will use the Taylor Series approximation to help solve for these two unknowns, in order to obtain $\underline{\varepsilon_t^i}$.

The first order Taylor series expansion of the first derivative, along with the first order condition imply:

$$v'\left(C^* \mid \underline{\varepsilon_t^i}\right) \approx v'\left(C_t^i \mid \underline{\varepsilon_t^i}\right) + v''\left(C_t^i \mid \underline{\varepsilon_t^i}\right)\left(C^* - C_t^i\right) = 0$$

$$C^* - C_t^i = -\frac{v'\left(C_t^i \mid \underline{\varepsilon_t^i}\right)}{v''\left(C_t^i \mid \underline{\varepsilon_t^i}\right)} \qquad (46.)$$

The second order expansion of the Taylor series on the value implies:

$$v\left(C^* \mid \underline{\varepsilon}_t^i\right) \approx v\left(C_t^i \mid \underline{\varepsilon}_t^i\right) - \Pi + \left(C^* - C_t^i\right)v'\left(C_t^i \mid \underline{\varepsilon}_t^i\right) + \frac{1}{2}\left(C^* - C_t^i\right)^2 v''\left(C_t^i \mid \underline{\varepsilon}_t^i\right) = v\left(C_t^i \mid \underline{\varepsilon}_t^i\right)$$
$$\Pi = \left(C^* - C_t^i\right)v'\left(C_t^i \mid \underline{\varepsilon}_t^i\right) + \frac{1}{2}\left(C^* - C_t^i\right)^2 v''\left(C_t^i \mid \underline{\varepsilon}_t^i\right) \tag{47.}$$

Combining (46.) and (47.) yields

$$\Pi = -\frac{v'\left(C_t^i \mid \underline{\varepsilon}_t^i\right)}{v''\left(C_t^i \mid \underline{\varepsilon}_t^i\right)}v'\left(C_t^i \mid \underline{\varepsilon}_t^i\right) + \frac{1}{2}\frac{v'\left(C_t^i \mid \underline{\varepsilon}_t^i\right)^2}{v''\left(C_t^i \mid \underline{\varepsilon}_t^i\right)}$$
$$\Pi v''\left(C_t^i \mid \underline{D}_t^i\right) = -v'\left(C_t^i \mid \underline{\varepsilon}_t^i\right)^2 + \frac{1}{2}v'\left(C_t^i \mid \underline{\varepsilon}_t^i\right)^2 \tag{48.}$$

So $\overline{\varepsilon_t^i}$ is the solution to the equation:

$$\Pi v''\left(C_t^i \mid \underline{\varepsilon}_t^i\right) + \frac{1}{2}v'\left(C_t^i \mid \underline{\varepsilon}_t^i\right)^2 = 0$$

There will in most cases be two solutions. If so, use the one that corresponds to the smallest positive value of $\frac{v'\left(C_t^i \mid \underline{\varepsilon}_t^i\right)}{v''\left(C_t^i \mid \underline{\varepsilon}_t^i\right)}$. If there are no real solutions that yield positive values of $\frac{v'\left(C_t^i \mid \underline{\varepsilon}_t^i\right)}{v''\left(C_t^i \mid \underline{\varepsilon}_t^i\right)}$ that are lower than $C_t^i - \underline{C}_t^i$, then $\underline{\varepsilon}_t^i \leq \varepsilon_t^i \leq \overline{\varepsilon_t^i}$ can be simplified to $\varepsilon_t^i \leq \overline{\varepsilon_t^i}$, because there is no level of ε_t^i below which it makes sense to contribute less than what is necessary to match the full funding limit.

APPENDIX D: REPRESENTING POLICY REGIMES IN THE MODEL

1) SIMPLIFICATION OF FUNDING RULES

The funding rules are complicated and this study will represent them in a significantly simplified form, to be compatible with the Markov Decision Process framework. The simplification process boils funding rule regimes down to two basic parameters: The target funding ratio and the remediation rate. The target funding ratio is the ratio of pension fund assets to pension fund liabilities at which the funding rules regard the plan as fully funded and require the sponsor to contribute only for newly accruing benefits. The remediation rate is the speed at which companies need to remedy gaps between the actual funding ratio and target funding ratio with additional contributions, beyond the cost of newly accruing benefits. It measures what fraction of that gap, on average, companies need to remediate in each year.

Two issues arise in calculating the remediation rate. The first issue is how to represent the actual system of multiple overlapping amortization schedules as a single rate. The second relates to smoothing of interest rates and asset values in the calculation of total assets and liabilities. Under current rules most companies amortize “experience losses” arising in one year over seven years. Experience gains or losses represents the difference in asset values net of liabilities at the end of the year relative to a level that was consistent with the expectation that both assets and liabilities grow at the assumed rates. They result from both changes in the discount rate, which impact estimated liabilities and differences between the rate of return on assets and the expected rate. When a company experiences a loss, it starts a schedule of seven annual payments that are calculated to exactly offset the loss on a present value basis. If asset prices move in the wrong direction the following year, the company schedules a second set of annual payments which almost entirely overlap with the first. A full representation of this system in the model would base required contributions on the funding ratios from seven years. However the first order Markov process I use in this study can base required contributions on the state of the pension plan and its sponsor in the current year alone. I use results from Dufresne (1988) which show that for any amortization period there is a remediation rate that leads to the same average level of funding shortfall. I use formulas aligned with his analysis to translate amortization periods into remediation rates.

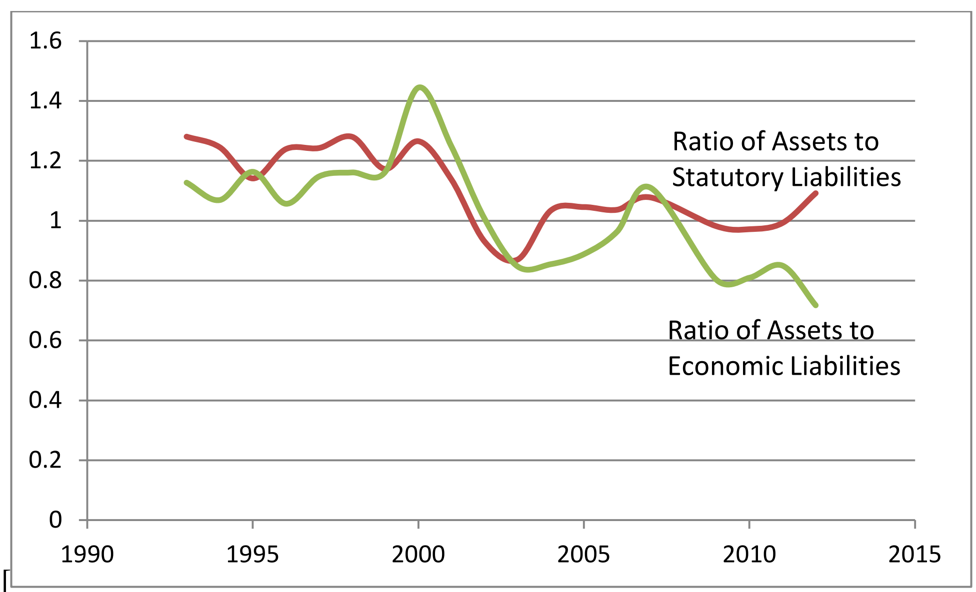
Smoothing of discount rates and asset values impacts the remediation rate. When companies use smoothed discount rates they delay fully incorporating changes in asset values into the calculation of required contributions. If the discount rate and asset values are smoothed over four years, that effectively delays full remediation by four years. So to obtain the remediation rate I first add the smoothing period to the amortization period, and then I transform the sum of those two periods into an annual remediation rate using the formulas I develop in Appendix E.

The calculation of pension liabilities depends on the discount rate, mortality rate and other assumptions, all of which have an impact on required contributions through their effect on the estimate of pension liability. The stringency of funding rules depends in part on the nominal target levels (for example whether the target is 100 vs 90 percent of statutory liabilities) and in part on requirements on the assumptions. For example if laws permitting the use outdated mortality rates reduce estimated liabilities by five percent, that has the same effect on required contributions as a five percent reduction in the target funding ratio. Higher discount rates have the same directional impact as higher mortality rates. They reduce estimated liabilities and thereby reduce the funding target.

This study will represent the funding target ratio as the product of two values: (1) the statutory target funding ratio and (2) the ratio of statutory liabilities to economic liabilities. The statutory target funding ratio was ninety percent of current liabilities before the passage of the Pension Protection Act and then transitioned to one hundred percent follows its passage. To obtain an estimate of the ratio of statutory to economic liabilities, I estimate an average ratio for each year and apply it to all companies. I estimate the average ratio for each year by comparing total liabilities in the PBGC data book, which are calculating based on a method designed to replicate current group annuity prices, to the estimated liabilities in the form 5500 data where companies report liabilities under the statutory definitions.

Figure D-1 shows the funding ratio over the 1993-2012 period measured two ways. Assets are shown in proportion to statutory liabilities and economic liabilities. Note that the statutory funding ratio has less variation and remains high throughout, while the economic funding ratio is more variable in the face of asset price volatility in the late nineties and 2000s, and ultimately declines significantly because of low asset returns and declining interest rates. That decline is blunted and even reversed in the statutory funding ratio because of the measure in MAP-21 to replace the discount rate established in the Pension Protection Act with one based on a twenty-five year historical average.

Figure D-1: Statutory and Economic Funding Ratios



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2) Legislative Changes during the Study Period

The data used for this study comes from the period spanning 1993 to 2012, which saw major legislation affecting the rules governing pension funding and premiums, as well as some significant changes to the tax code. Two pieces of legislation made significant modifications to the pension system, the Retirement

Protection Act of 1994, and the Pension Protection Act of 2006, and a few other pieces of legislation had significant impacts on some of the key parameters in the funding rules and premiums formulas. At the end of the period covered by the data, in 2012, the Moving Ahead for Progress in the 21st Century (MAP-21) significantly changed the discount rate and premium structure for the PBGC, giving companies a high degree of flexibility to reduce their required contributions but also charging them significantly higher premiums when they do so.

Box D.1 summarizes the legislative developments over the 1990 to 2013 period, and Table D.1 shows the estimated funding target ratio and assumed remediation rate I estimate for the model in each year. The remediation rate stayed remarkably constant over the period reflecting the offsetting changes in the pension law, as an increase in the amortization rate in the PPA offset a decrease in the smoothing period. In addition the Pension Protection Act, while widely seen as having made the funding rules more stringent, actually had a roughly null impact on funding targets because of offsetting provisions, raising the target after a transition period from 90 to 100% of liabilities but raising the discount used to calculate those liabilities.

BOX D-1: LEGISLATIVE DEVELOPMENTS IMPACTING THE SINGLE EMPLOYER PENSION SYSTEM 1993-2012

1994 — Retirement Protection Act (RPA 94). PL 103-465

- Eliminated VRP cap
- Requires the use of uniform mortality tables
- Limits the range of interest rates used to determine the current liability to between 90 and 100 percent of the weighted average on 30-year treasuries over the previous 4 years
- Reduces length of amortization period

2002 — Job Creation and Worker Assistance Act of 2002. PL 107-147

- Increases interest rate used to calculate liabilities from 85 percent of the 30-year treasury rate to 100 percent

2004 — Pension Funding Equity Act of 2004. PL 108-218

- Temporarily allows plans to use interest rates on high-quality bonds, instead of 30-year treasuries, to discount future liabilities

2005 — Deficit Reduction Act (DRA 2005). PL 109-171

- Increased flat-rate premium from $19 to $30 per person
- Established automatic indexing of flat-rate premium
- Established cap on VRP for plans sponsor by very small employers
- New premium of $1,250 per plan participant, per year, for 3 years, on plans that are terminated on an involuntary or distressed termination basis

2006 — Pension Protection Act of 2006 (PPA 2006). PL 109-280

- Introduces new funding requirements – requires plan funding to be equal to 100% of plan's liabilities, with unfunded liabilities being amortized over seven years.
    - Phased in at (92% in 2008, 94% in 2009, 96% in 2010, and 100% in 2011 and later.
- Changed basis for determining VRP – beginning in 2008, requires plans to use high-grade corporate bond interest rates, instead of 30-year treasuries, to discount pension obligations.
- Narrows the range for actuarial valuations to between 90% and 110% of fair market value, and reduces smoothing period to two years.
- Eliminated FFL exception

2007 — Worker, Retiree, and Employer Recovery Act of 2008. PL 110-458

- Waives required minimum distribution rules for calendar year 2009
-
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BOX D-1 (CONT'D): LEGISLATIVE DEVELOPMENTS IMPACTING THE SINGLE EMPLOYER PENSION SYSTEM 1993-2012

- Extends the transition provision, allowing a plan's funding shortfall for the year to be determined by only the applicable percentage of the funding target, rather than the full target. Sets percentages at 92 for 2008, 94 percent for 2009, and 96 for 2010.

2010 — Preservation of Access to Care for Medicare Beneficiaries and Pension Relief Act of 2010. PL 111-192

- Allows plan sponsors to elect to extend amortization period, offering 2 alternatives:
    - 2 plus 7 plan: for the first two years, the amortization payments required are equal to the interest on the amortization base for the plan year of election. The payments for the next 7 years are equal to the amount necessary to amortize the remaining balance.
    - 15-year plan: allows plan sponsors to elect to amortize over a 15 year period.

2012 — Moving Ahead for Progress in the 21st Century Act (MAP21). PL 112-141

- Increased flat-rate premium to $42 for 2012 and $49 for 2013
- Increased VRP rate from $9 per $1,000 UVB to $13 (indexed) for 2014 and $18 (indexed) for 2015
- Established cap on VRP based on participant count ($400 per P, indexed after 2012)
- Revises the specified percentage ranges for determining whether a segment rate must be adjusted:
    - 90 to 110 percent for plan years beginning 2012
    - 85 percent to 115 percent for 2013
    - 80 percent to 120 percent for 2014
    - 75 percent to 125 percent for 2015
    - 70 percent to 130 percent for 2016 or later

Table D-1: Parameters of Funding Rule and Remediation Regimes

	Premium Target / Economic Liabilities	Premium Trigger/ Economic Liabilities	Funding Target/ Economic Liabilities	Remediation rate	Tax Deductible Limit/ Economic Liabilities
1993-2001	1.16	0.85	0.85	0.15	0.85
2002-2003	1.08	0.85	0.85	0.15	0.85
2004-2007	0.96	0.85	0.85	0.15	0.85
2009-2011	0.82	0.82	0.79	0.15	1.03
2012	0.76	0.76	0.66	0.15	0.85

Under the model the value of the tax shelter depends not only on the tax rate on investment income but also on the nominal risk-adjusted rates of return on investment. I use the 10-year Treasury rate as a proxy for the rate of return on investment, taking the average within each of the five regimes. As can be seen in Table D-2, nominal rates of return have decreased over the study period, suggesting that the motivation to fully fund up plans to gain the benefits of a tax shelter have dwindled.

Table D-2: Investment Returns in each of the regimes

	Investment Return (proxy is 10-year Treasury rate)
1993-2001	6.03
2002-2003	4.31
2004-2007	4.50
2009-2011	2.81
2012	1.80

Variable rate premiums (VRP) equal the funding gap relative to the VRP target times a factor which for most of the period under study was equal to 0.009, but lately has been increased. The VRP target is different from the funding rules target. It includes only vested liabilities and depends on a different discount rate. Before the PPA became effective the VRP discount rate was 0.85 times the rate used to assess liabilities in the funding rules. All else equal this lower discount rate would lead to a higher target. However vested liabilities are less than or equal to total liabilities. The rules prior to the passage of the Pension Protection Act exempted companies from paying the variable rate premium if they had assets at the target in the funding rules, under something called the Full Funding Level exemption, even if the VRP formula indicated they faced a liability. The PPA eliminated the FFL exemption and changed the discount rate applied to variable rate premiums to a spot rate, while leaving the discount rate under the funding rules as a smoothed rate. Under the new rules companies could have a higher VRP target when the spot rate was lower than the smoothed rate, something which has been often been the case recently under falling interest rates. The MAP-21 Act significantly increased smoothing and thus the tendency for pension plans at full funding under the funding rules to face a VRP liability.

The most significant change to the tax system governing pension plans during the study period was the flexibility the PPA introduced for companies to make tax deductible contributions to plans that were already fully funded under the statutory measures. The PPA allowed companies to make tax deductible contributions up to a level that represented a buffer above full funding, whereas before any contributions over the full funding limit were not tax deductible. The current system of reversion taxes was in place by 1990 and has not changed much since then. However changes in the individual tax code potentially strengthened the tax arbitrage effects identified in the literature. Equity income began to receive a significantly more preferential rate starting with the Taxpayer Relief Act of 1997, and the Jobs and Growth Tax Relief Reconciliation Act of 2003 (JGTRRA). Those pieces of legislation increased the differential between the tax rate applied to interest and equity income, which under the propositions laid out in Tepper (1981) should have increased the incentive to invest pension funds in fixed income.

Legislation increased premiums several times over the course of the period under study. Many of the premium increases focused on the flat rate per participant premium, which would not vary depending on the funding decisions made by plan sponsors, and thus not produce incentives that impact funding strategy. Some legislation strengthened the variable rate premium. First the Pension Protection Act removed the full funding exemption from the variable rate premium, and subsequent legislation raised the rate from nine tenths of a percent to over two percent, and indexed the rate so that it would increase with inflation.

3) CONVERTING AMORTIZATION PERIODS INTO A REMEDIATION RATE

While the amortization method bases funding requirements on the evolution of asset prices over a several year period, the funding formula in the Markov Chain model as I have specified can only consider the current state of the plan. So exact modeling of the amortization method is not possible. Instead using results from Dufresne (1988) I observe that for any amortization funding rule, one can find a spread method funding rule that will generate the same distribution of funding shortfalls and therefore will lead to the same projection of claims, contributions, and premiums. Under the spread method of funding, companies remedy a fixed fraction of underfunding in each year.

Under the amortization funding rule, in any year the gap between the funding ratio and its target equal the following function of past year losses:

$$\tau - x(t) = \beta\sigma_x \sum_{w=1}^{W} \frac{W-w}{W} z_{s-a}(t-w) \qquad (49.)$$

where W is the amortization period and $z_{s-a}(t-w)$ is the experience loss or gain w years in the past. The variance of the total shortfall will be a linear function of the variances of the annual stochastic changes in funding ratio σ_X^2:

$$Var(\tau - x) = \beta^2 \sigma_x{}^2 \sum_{w=0}^{W-1} \left(\frac{W-w}{W} \right)^2 \qquad (50.)$$

Now consider a spread method rule where plans must remedy a fraction θ_1 of the difference between their funding ratio and the target. The portion of the shortfall w years back that remains unamortized will then equal $(1-\theta)^w$ and the total shortfall over all past years will equal:

$$\tau - x = \beta\sigma_x \sum_{w=0}^{\infty} (1-\theta_1)^w z_{s-a}(t-w) \qquad (51.)$$

Applying a formula for the sum of an infinite series I find that the variance of that total shortfall will equal:

$$Var(\tau - x) = \frac{\beta^2 \sigma_x{}^2}{1-(1-\theta)^2} \qquad (52.)$$

To approximate W-year amortization, I can set θ to a level where the variance given in equation (50.) is equal to the variance given in equation (52.). Applying formulas for the sum of square series one obtains the following relationship between θ and the amortization period W:

$$\theta = 1 - \sqrt{1 - \frac{6W}{(W+1)(2W+1)}} \qquad (53.)$$

Table D.3 gives the resulting values of θ that represent a spread method funding parameter equivalent to each amortization period.

Table D.3: Amortization and Equivalent Spread Method Parameters

Amortization Period	θ (equivalent spread method parameters)
1	1
2	0.552786
3	0.402386
5	0.261451
7	0.193774
10	0.139617

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