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THE EFFECTS OF INVESTING IN EARLY EDUCATION ON ECONOMIC GROWTH*

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April 2006

Introduction

Economists have long believed that investments in education, or "human capital," are an important source of economic growth. Over the last 40 years output has grown about 3.5 percent a year and the productivity of labor has grown about 2.4 percent per year. The contribution of education to labor productivity growth is estimated to be between 13 and 30 percent of the total. Further, many people believe that investments in human capital will become even more important in the future as we become a post-industrial, knowledge-based economy, and they worry that we are giving insufficient policy attention to the development of an educated workforce.

Why might a more highly educated workforce increase economic growth? A more educated labor force is more mobile and adaptable, can learn new tasks and new skills more easily, and can use a wider range of technologies and sophisticated equipment (including newly emerging ones). It is also more autonomous and thus needs less supervision, and is more creative in thinking about how to improve the management of work. All of these attributes not only make a more highly skilled worker more productive than a less skilled one but also enable a work place that employs more educated workers to organize differently, manage differently, choose technologies and equipment

* The views expressed in this working paper are those of the authors and should not be attributed to the staff, officers or trustees of the Brookings Institution. This work was funded by the National Institute for Early Education Research with a grant from The Pew Charitable Trusts – Advancing Quality Pre-Kindergarten for All.

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differently, and adjust better to changes necessitated by competition, by technical advances, or by changes in consumer demand. What is true for the firm may also be true for the whole economy. Skills beget more skills and new ways of doing business, workers learn from one another, and firms adapt their technology and their use of capital to the skills of the available workforce. The benefits of having a more educated workforce accrue to everyone, not just to the organization where these individuals happen to work. Further, these kinds of indirect (or spillover) effects for the firm or the economy as a whole may be especially important in an increasingly competitive global marketplace. Imagine an economy lacking in people able to read directions, use a sophisticated copier or a computer, or understand prevailing norms of behavior. Even if a single organization in that economy were able to find or import such skills, other organizations would not be able to invest in certain kinds of equipment or certain kinds of businesses with any assurance that they could make the investment profitable. Beyond that, a more educated workforce may produce a less crime-ridden and healthier environment with better functioning civil institutions and all the benefits that flow to the business sector from that environment.

In this paper we develop a model that is flexible enough to allow a wide range of assumptions about the role of education in promoting economic growth. The model is particularly elaborate in its treatment of the breakdown of the population into different cohorts and in determining the amount of education people in different cohorts receive. This treatment allows us to develop a realistic estimate of the timing of the growth effects of a program that will take many years to have its full impact.

Theories of Growth: A Brief Review of the Literature and the Models Used in this Analysis

Past Work on Education and Growth

Since nearly its inception, the study of economic growth has focused on the importance of education. Robert Solow (1957, pp.312, 317) described growth of national income as resulting from three sources: increases in the stock of physical capital (machines and buildings that are used to produce goods and services), increases in the size of the labor force, and a residual representing all other factors. This residual contributed considerably more to per capita growth than the increase in the capital stock. Solow dubbed the residual “technical progress” and noted that increasing levels of education were one of the factors that contributed to its growth. Using the same basic approach as Solow, but taking explicit account of the role of education, Edward Denison (1985) estimated that between 1929 and 1982, increasing levels of education were the source of 16 percent of the growth of total potential output in nonresidential business (and 30 percent of the growth per person employed in that sector). A more recent study by Dale Jorgenson and Kevin Stiroh (2000) puts the contribution of education to economic growth at 8.7 percent of total growth over the recent period 1959 to 1998 and 13 percent of growth in output per worker.

In the expanded Solow framework education is treated as a separate factor of production. The "stock" of human capital is measured in a way similar to the stock of physical capital. A person year of education is valued at the cost of producing it and all the person years are added up to get the stock. Increases in the stock of human capital, or in any other specific factor of production, are assumed to produce less than proportional increases in output since the various factors must be combined to obtain an increase in output. Specifically, a one percent increase in the stock of human capital is assumed to cause somewhat less than a half a percent increase in national income. In the Solow framework the impact of a one percent change in the stock of a factor of production is equal to its share in national income. A one percentage point increase in the capital stock causes only about a third of a percentage point increase in output because capital's share of national income is only about a third. Labor gets most of the other two-thirds, and typical studies attribute two-thirds of that to human capital as opposed to raw labor. Thus increases in the number of workers, with no change in the total number of years of education, would have an impact smaller than an equal proportional increase in the stock of physical capital. But an equal proportional increase in the stock of human capital (person years of education) would have an effect twice that size. Some other theories of how education affects output suggest the gain could be even larger than this.

An alternative to treating human capital as a separate factor of production is to take account of the effects of education by assuming that it is not a separate factor of production but instead simply increases the productivity of labor. In this framework an economy with twice as much human capital per worker could produce the same amount with half as many workers. Hirofumi Uzawa (1965) was the first to propose a model of economic growth with human capital impacts of this sort. In his model a one percent increase in the amount of human capital per worker causes an increase in national income of about two-thirds of a percent. This is because a one percentage point increase in the stock of human capital per worker causes a full percent increase in the effective supply of labor, which produces an effect on output equal to labor's share of income.

But Uzawa's model may still understate the role of education in determining the level of output and growth. Both the Solow and Uzawa models assume that if we double the amount physical capital and human capital per worker we will at most double the amount of output and that each factor will get a fixed share of the increase equal to its average share of existing output. However, many economists have argued that doubling capital inputs more than doubles output and that this accounts for some of the unexplained residual in a Solow-type analysis of the sources of growth. When doubling inputs results in more than a doubling of output there are said to be "increasing returns to scale."[1]

[1] Increasing returns to scale cause a problem if they are thought to apply at the level of the firm. With even modest increasing returns to scale large firms have huge cost advantages over smaller rivals and can easily drive them out of business. In a world where increasing returns to scale were common at the firm level a single large firm would dominate in nearly all industries. However, if the advantages of increasing scale do not accrue to the firm, but to the economy as a whole (that is they are external to the firm) then this is not a problem. Thus growth models with increasing returns to scale typically assume that education has effects that are external to the firm. Many reasons have been proposed as to why a more educated workforce, in

Robert Lucas (1988), building on Paul Romer (1986, 1987), proposes a model of economic growth where the effects of human capital are greater than even the Uzawa model. Since the model has increasing returns to scale, increasing all factors of production proportionally can cause a greater than proportional increase in output. Thus the impact of factors need not equal their shares and add up to one. Instead their impact can add to a number greater than one and the impact of human capital can be greater than labor's share in output.

Is there any way to determine what model is the right one for analyzing the effects of human capital on growth for the United States? We do not have a satisfying answer to this question. In the mathematical appendix we show that the augmented Solow model, the Uzawa model, and the Lucas model can all be represented by the same equation with the only differences between them being the magnitude of the impact of years of education per worker on output. Unfortunately, there is no good way to estimate this parameter. We can look at the correlation of human capital growth and output growth, but we can't separate out the effects of human capital growth on output from output growth on human capital in a convincing way. However, there is another approach that can be taken to this question.

The larger the sum of the impact coefficients of physical capital and human capital, the longer lasting will be the effect of an increase in investment on growth. If the sum of them is one or greater (as it is in the Uzawa and Lucas models) an increase in the rate of investment produces a permanent self-sustaining increase in the rate of growth. The reason is that new physical and human capital are produced in fixed proportions to national income with the investment rates[2] for the two types of capital determining the ratios of the value of new capital to output. Then an increase in either investment rate causes an initial increase in the rate of growth as output increases in response to the increase in capital. This increase in output will cause a further increase in the amount of physical and human capital that will cause yet another increase in output. If the sum of the impacts of physical and human capital on output are less than one then the second round increase in output will be less than proportional to the increase in human and physical capital induced by the first round increase in output. Thus the next round of increases in physical and human capital will be smaller than the last. As the process continues each round of increases will be smaller than the last until after a long time almost no effect remains. However, if the sum of the impacts is greater than or equal to one, the increases are self-perpetuating and a one-time increase in the investment rate for either physical or human capital will cause a permanent increase in the rate of growth of output. Thus long-lasting correlations between savings rates and the rate of growth are indicative of models where the impact of education and physical capital are large.

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particular, may have effects on productivity external to the firm. Some of these were discussed in the introduction.

[2] Or the savings rate since savings and investment are equal in nearly all growth models (including ours).

Models where the rate of growth depends on these investment rates or other economic variables are referred to as endogenous growth models. Growth is endogenous in the sense that it is determined by the values of the parameters of the system rather than being given by external factors such as the rate of scientific progress. If we can rule out endogenous growth models we can rule out the larger values of our human capital impact parameter. If we believe, on the other hand, that investments in education lead to self-sustaining increases in growth because they have effects that improve economy-wide efficiency, and the ability to invent or adopt new technologies, then we can rule out exogenous growth models and it that makes it more likely that a large value for the coefficient of human capital is appropriate.

There is a large empirical literature on exogenous versus endogenous growth models in which scholars have used some combination of historical and cross-national data in an attempt to determine what drives economic growth. As part of this literature, Paul Romer (1987) argued that the standard Solow growth model could not account for the high correlation between the growth of capital stocks and the growth of output, or for the sometimes negative correlation of labor force growth with rates of economic growth. He used these facts to argue for the endogenous growth model since they predict both outcomes. However, Gregory Mankiw, David Romer, and David Weil (1992) showed that most of the problems Paul Romer had pointed to went away when the Solow model was augmented to consider human capital as well as physical capital. The large role attributed to physical capital in Paul Romer's analysis was argued to be due to its correlation with growth of human capital. The negative effects of labor force growth were due to the fact that countries with rapid population growth happened to be countries with relatively little human capital growth. Mankiw et al. argued that the augmented Solow model was adequate to describe differences in output and rates of growth between countries. But in a more recent study, Ben Bernanke and Refet Gürkaynak (2001) argue that investment (or savings) rates are correlated with growth rates, even in the long run, in a way that suggests endogenous growth.

Thus we feel the jury is still out on the endogenous versus exogenous growth debate and analyze our model employing a range of assumptions about the importance of human capital. At one extreme we use the values implied by the augmented Solow model of Mankiw, Romer, and Weil. At the other we assume a value of the impact of human capital that allows the accumulation of physical and human capital to explain all economic growth in some periods[3] with no residual. This would be an extreme version of the Lucas-Romer model in which physical and human capital accumulation explains all growth in at least some years.

The Model in Words

The appendix to this paper provides a mathematical presentation of the model. Here we describe its workings in pictures and words. Figure 1 diagrams the flow of cause and effect from a policy change that impacts the level of educational attainment to the final outcome of increasing national income.

[3] We choose the value so that exogenous growth is close to zero at its minimum.

The first step in this chain of cause and effect (effect 1 in figure 1) is outside the model. To analyze a policy the user must determine what its impact will be on educational attainment. Specifically, the model takes as input the age of people affected by the policy, its starting year, and the magnitude of the effect of the policy in years of additional education. Given this input the model computes growth effects.

The model tracks the number of years of education attained by each cohort. The population projections for each cohort in 2004 are taken from the U.S. Census Bureau's Interim Population Projections, 2000-2050. We assume that each subsequent cohort increases in size by a set amount (the compound average population growth rate projected by the Census Bureau over the period) relative to the prior cohort.[4] Years of education for cohorts that have already entered the labor force are estimated from the March 2004 Supplement to the Current Population Survey. The model operates under the assumption that each member of a cohort is identical with regard to the duration of their schooling (their ultimate level of educational attainment). Members of a cohort enter the labor force once they have reached the age at which they complete their schooling. These individuals have an impact on output that rises with their level of educational attainment.

Take as an example a preschool program that covers half the population and causes people who attend the program to get an additional half year of education. The effect on the average years of schooling attained would be a quarter of a year for those in the first cohort to receive the program. We assume the program starts in the first year that the model simulates. For the first thirteen to fourteen years the program has no effect on growth as the first cohort of students who received the program move through grade school to high school and then into what would, in the absence of the preschool program, be their final year of post-secondary education. At this point two things happen. First, there is an increase in the number of years of education these students get. Since they are staying in school longer this reduces the size of the labor force and has a negative impact on output (effects 2 and 3 in figure 1). However, when these students graduate they are more productive because of their additional schooling and this has a positive impact on output (effects 4 and 5 in figure 1). As time goes by, and more students who have been in the preschool program graduate, the impact of the program on the size of the labor force remains roughly the same[5] but the impact on the productivity of the workforce grows as a larger and larger fraction of the population has the extra education. The effect continues to grow until the first cohort to receive the preschool education reaches retirement age. This is the direct effect of the program on output and growth and is displayed on the model output page as the contrast between the baseline simulation (the simulated growth path with no policy change) and the "static" model.

The direct effects are called static because they do not take into account the dynamic feedback effects that result when output changes in response to the policy. There

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[4] Assumptions regarding the population growth rate are adjustable from the model user input page.

[5] It grows slightly as each cohort is larger than the last.

are two types of dynamic effects. First, when there is an increase in output, savings and investment increase as well (effect 6 in figure 1). The model makes the relatively common assumption in growth theory that people save a fixed fraction of their income and that this savings becomes new investment in physical capital (effect 7 in figure 1) or human capital (effect 9 in figure 1). When the stock of physical capital increases, this further increases output (effect 8 in figure 1). We assume that the stock of physical capital depreciates at a constant rate so that an increasing level of investment is necessary to maintain a growing capital stock.

The model embodies a similar set of assumptions about human capital. In particular, the years of education obtained by each cohort are determined by the total resources expended in the production of education in each year. In the absence of a policy intervention, these resources are proportional to output in the year.[6] Thus an increase in output causes an increase in years of education (effect 9 in figure 1) and increases in the years of education feedback into output as described above (effects 2 through 5 in figure 1). The dynamic effects of physical and human capital accumulation go on year after year with the persistence of the effects on growth depending on the values of the coefficients on physical capital and human capital in the equation that determines output, otherwise known as the production function.[7]

Policy Simulations and Results

In order to estimate the potential impact of a preschool policy initiative on economic growth, we must first assume a reasonable baseline projection of economic growth that we expect to occur in the absence of any policy intervention. The model will

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[6] We further assume that these expenditures are proportional to the foregone earnings of students still in school. The ratio of foregone earnings to output in each year is assumed to equal a constant we call the educational savings rate. Foregone earnings are computed assuming that each year of schooling causes a fixed percentage increase in earnings that we refer to as the rate-of-return to schooling.

[7] The growth model could serve as the basis for a model of the long-term fiscal impact of a preschool program and also of its long-term total costs and benefits. The fiscal effects of the policy change could be derived from the cost assumptions the user enters, the growth effects the policy induces, and the average ratio of federal revenue to the GDP. The user would input the cost of the program per pupil to the federal government. The model would then compute the total cost in each year. The model would also calculate the growth in federal tax revenue in the policy simulation relative to the baseline and subtract the costs from the increased revenue in each year to obtain the total fiscal impact.

Total costs and benefits of the program could be computed from several user inputs and the effects of the policy on output and investment. First, the model could be set up to take user inputs of the dollar value of non-economic (or non-pecuniary) benefits to society for each program participant of each age. For example, one important benefit is decreased commission of crimes. These benefits are largest when program participants are in their late teens and early twenties, but much smaller to non-existent when participants are much younger or older. The model would keep track of the number of individuals in each cohort who have taken part in the program and multiply this times the value of benefits at each age to arrive at the total of non-economic benefits in each year. Total net benefits in each year would be computed as the sum of non-economic benefits and the additional consumption above baseline minus the foregone earnings of those still in school. The increase in consumption would be computed as the increase in output minus the increase in investment in physical capital plus educational expenditures. The discounted present value of the net benefits over the 75-year horizon of the model could then be computed. This work would take more time and resources but could be done as part of a second phase of this project.

then be capable of predicting the magnitude of the growth effect of a particular preschool policy initiative relative to size of the economy assumed in the baseline case. For this purpose, we adopt the 75-year projections of the Social Security Trustees (Social Security Board of Trustees, 2005). However, the model is also adaptable to a wide range of per-capita growth assumptions.

The Social Security Trustees estimate that the growth rate of output per worker beyond 2012 will be 1.60 percent per year. In order to calibrate our model we augment the production function scaling constant (A_t) in each year by the amount necessary to match the per-capita growth projections of the Social Security Trustees. Under our preferred set of values for each of the relevant model parameters, the compound average annual supplemental growth necessary to calibrate the model amounts to less than 0.60 percent per year.

The growth model is highly flexible in that it allows for user-selection of the values ascribed to an extensive set of variables, including the parameters that represent the rate of return to an additional year of education, the rate of return to an additional year of labor market experience, the depreciation rate of physical capital, the investment rate in physical capital, the investment rate in human capital, and the factor share ascribed to physical capital. To determine our preferred set of parameter values, we mainly used historical averages. In particular, we rely heavily upon data from the National Income and Product Accounts (Bureau of Economic Analysis) and the March Current Population Surveys (Bureau of Labor Statistics). Table 1 details the data sources used to select the preferred parameter values for the policy simulation described in the next subsection.

Policy Input

In recent years, parents, policymakers, and business leaders have expressed mounting interest in providing publicly funded, voluntary, universal preschool programs (see, e.g., Committee for Economic Development, 2002). While only Georgia and Oklahoma currently operate preschool programs that are universally accessible, New York, West Virginia, and Florida have each committed themselves to implementing universal programs (Karoly and Bigelow, 2005, pp.7-8). In this simulation, we analyze the growth effects expected from a high-quality, national, universal preschool program for three and four year-old children.

The initial effects of the policy are captured in a single value—the amount of additional years of education those who receive program services will ultimately obtain. Few evaluations of large-scale programs possess the methodological rigor necessary to draw conclusions about causation. Even fewer large-scale programs could be characterized as high quality. Indeed, only 20 of the 38 states that provide any public preschool require lead classroom teachers to hold a baccalaureate degree (Barnett, Hustedt, Robin, and Schulman, 2004). Instead, the best evidence flows from a set of small-scale experimental programs that featured random assignment and longitudinal evaluation of study participants (with low attrition). For this simulation, we have selected the Perry Preschool program, which operated in Ypsilanti, Michigan in the 1960s.

The Perry Preschool program provided low-income, at-risk, three- and four-year-old children with center-based care, two-and-a-half hours per day, five days per week, for thirty weeks each year. The center-based care was supplemented on a weekly basis with one-and-a-half hour home visits by the child's instructor. The Perry program was characterized by a high degree of instructor quality, as well as remarkably low student-teacher ratios. Study participants were selected on the basis of their low socioeconomic status (SES), with SES determined by parents' years of schooling, parents' occupational levels, and rooms per person in their households. The children in both the "program" and "no-program" groups were monitored on a periodic basis until the present-day, with study participants most recently surveyed at the age of forty (Schweinhart et. al, 2005).

At age twenty-seven, members of the program group were found to have levels of educational attainment 0.9 years greater than members of the no-program group. This difference was both statistically significant ($p < 0.016$) and economically substantial (Schweinhart, Barnes, and Weikart, 1993). We utilize this finding as the primary input for this policy simulation. It is worth noting, however, that this difference in educational attainment likely understates the productivity improvements benefiting program children, who also experienced gains in non-cognitive characteristics, including persistence and diligence. We conclude this because male participants experienced strong earnings gains, relative to men in the control group, despite having much smaller gains in education than women.

Projecting the effects of implementing a small scale program like Perry on a national level raises a host of complicated issues. First, a universal preschool program, if it is not compulsory, will not serve 100 percent of eligible children. Second, many of the children served by this new policy may already be enrolled in existing preschool programs, whereas most of children in the no-program group evaluated in the Perry study did not receive any early childhood education. The assumed impact of the program will need to be reduced for these children. Third, universal preschool programs will serve children that are less disadvantaged than the pool analyzed in the Perry Preschool experiment. It is not clear whether students with higher SES will experience comparable gains in educational attainment. Finally, we might also be concerned that preschool administrators may experience considerable difficulty in maintaining an equally high level of program quality in a program enrolling millions, rather than dozens, of children.

In order to address the first two concerns, we follow a procedure that is similar, to that employed by Lynn Karoly and James Bigelow in the recent RAND cost-benefit analysis of universal preschool in California. Based on experiences in Oklahoma and Georgia, Karoly and Bigelow assume that participation in a high-quality, voluntary, universal public preschool program would reach 70 percent of eligible children. Ten percent of children aged three and four would obtain preschool from the private market. The remaining 20 percent would not enroll in preschool of any kind.

Based on data from the October 2001 Current Population Survey, Karoly and Bigelow report that 52 percent of three- and four-year-olds are already enrolled in preschool of some kind. Of the 52 out of every 100 children who are enrolled in preschool, 26 are enrolled in public programs and 26 are enrolled in private programs. We assume that out of every 100 children, 70 will enroll in the proposed Perry-type program. These 70 children will be comprised of all of the children previously enrolled in public programs, roughly half of the children previously enrolled in private programs, and 28 children previously not enrolled in any preschool program.

We assume that children who would not have attended any preschool in the absence of this universal program now reap 100 percent of the benefits estimated for the Perry program (the full 0.9 year gain in educational attainment). We also assume that children who would have attended public preschool programs in the absence of this policy initiative will receive 50 percent of the policy effect, as the new initiative should be higher in quality than the average public program. Finally, we assume that children previously enrolled in private preschool programs receive no additional educational benefit.[8] Certainly it would be desirable to have concrete data on the myriad preschool programs in existence and their level of quality relative to Perry. However, in the absence of such detailed data, we believe these are reasonable assumptions.

With regard to the third concern, the possibility of differential program effects on children from households with different levels of SES, we turn to evidence from Oklahoma's universal preschool program, which has recently been subjected to quasi-experimental evaluation by Gormley, Gayer, Phillips, and Dawson (2005). Gormley et al. exploit the strict age-eligibility cutoff in the Oklahoma program to examine children of approximately the same age who just made the cutoff and have finished a year of preschool (the treatment group) and those children who just missed the cutoff (the control group). Of course, no long-term analysis can be conducted with this research design, as every member of the control group is now slated to start the preschool program. Still, Gormley et al. find strong (and nearly comparable) gains across all income classes (as proxied by children's eligibility for free lunchs). These results indicate that children from both low and high income families may receive roughly comparable educational gains from participation in high-quality preschool programs. Similar findings have recently been reported by Steven Barnett and his colleagues at the National Institute for Early Education Research in their evaluation of preschool programs in Michigan, New Jersey, Oklahoma, South Carolina, and West Virginia (Barnett, Lamy, and Jung, 2005).

In light of this evidence, we assume that the effect of the preschool initiative on educational attainment is the same for children of all SES. Since we are already discounting the effects for children in private preschool who are primarily upper income we did not believe that it was necessary to reduce the effects further.[9] Moreover, we

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[8] Not everyone would agree that private programs are higher quality than public programs, but we assume they attract a relatively affluent population for whom preschool may not be as valuable as it is for their less advantaged peers. This has been a common assumption in the literature to date, but see the discussion of the Oklahoma program.

[9] The conclusion that the effect of preschool on educational attainment is roughly equivalent across SES groups does not necessarily bear on the issue of effect attenuation in other areas. Notably, rates of crime and teenage pregnancy are far less prevalent among high SES children. In these areas, we expect the positive effects of preschool to be smaller for high SES children.

expect that a significant segment of those children who enroll in the universal program could fairly be characterized as at-risk. Indeed, nearly 20 percent of children under age six live in families below the poverty line (DeNavas-Walt, Proctor, and Mills, 2004).

Under these assumptions, we calculate that the average increase in educational attainment for all children due to the preschool program will be 0.365 years. This is the value assumed in our preferred estimate of effects.

Results

As discussed previously, assuming different values for the impact of human capital per worker on output we can simulate: (1) an augmented (with human capital) Solow-Swann (exogenous) growth model (Mankiw, Romer, and Weil, 1992), (2) an endogenous growth model with constant returns to physical and human capital (Uzawa, 1965), and (3) an endogenous growth model with increasing returns to physical and human capital (Lucas, 1988).

Figure 2 displays the year-by-year predictions of the three models relative to the Social Security Trustees baseline over the full 75-year time horizon. The Mankiw, Romer, and Weil model serves as a lower bound for our estimates, with the Lucas model, featuring external effects from human capital investments, serving as the upper bound. The Uzawa model yields our preferred estimates.

The first effect of the policy initiative is to reduce the supply of labor when the first participants reach the age at which they would normally enter the labor force but instead extend the time they spend in school. This effect begins in 2025. However, when they enter the workforce, they are more productive due to the additional education and that has a positive effect on output. By 2046, all three models predict that the positive effects start to outweigh the negative effects. In the Lucas model, the effects turn positive as early as 2038. From here, the effects rapidly increase in magnitude, as additional treated cohorts enter the labor force and increased economic growth starts to result in positive dynamic feedbacks.

Table 2 displays estimates of the effect of the program on GDP for the 45-, 60-, and 75-year time horizon. The table also shows the effects on the level of human and physical capital relative to the baseline. Under the Mankiw, Romer, and Weil model, GDP increases by 1.34 percent, or 778 billion 2005 dollars in 2080. This equates to $2,943 per member of the labor force. Under the Uzawa model, GDP increases by 3.50 percent, or 2,034 billion 2005 dollars in 2080 ($7,699 per capita), and under the Lucas model GDP increases by 4.02 percent, or 2,341 billion 2005 dollars in 2080 ($8,859 per capita).

These findings are robust to a wide range of reasonable values for key parameters. While we conduct a full battery of sensitivity analysis, we find that the conclusions of the model are most heavily influenced by two key factors. First, assumptions regarding the program's take-up rate and the level of impact attenuation unsurprisingly play a pivotal role. However, even with a take-up rate of 50 percent, with benefits accruing exclusively to those children previously not enrolled in any preschool program, effects are still as high as 1.69 percent of GDP in 2080 under the preferred Uzawa model. GDP increases reach as high as 7.82 percent under the assumption of 90 percent program take-up, with full benefits accruing to all participants.

Likewise, results are also highly sensitive to the expected rate of return on education. It is possible that the individual return to education exceeds the social return (due to the sorting value of education). As such, we evaluate the preschool policy under assumptions of a dramatically lower rate of return to education. Even under the assumption of 5.3 percent rate of return to an additional year of education, the Uzawa model still yields a 0.30 percent increase in GDP in 2080.

Table 3 presents the results of our sensitivity analysis. The impact of the policy on GDP, the physical capital stock, and the stock of human capital, relative to baseline, are shown for the years 2050, 2065 and 2080 for a wide range of different parameter values. Figure 3 is a histogram showing the distribution of the results for 2080 from Table 3. In only one specification – that with the smallest value for the impact of human capital on individual productivity and the smallest coefficient on human capital in the production function – is the effect of the policy on GDP negative in 2080. It is worth noting that if this value for the return to education is correct, not only would investments in preschool contribute nothing to growth, but it would also be the case that we are probably hugely over investing in all types of education as the productivity of the more educated would be mostly the same is it is now even if they did not obtain the education that differentiates them from the rest of the population.

In all other cases impacts are positive. Also, with our most optimistic assumptions about take-up rates, program effects and the impact of human capital on output the GDP gain is greater than 15 percent in 2080.

Conclusion

The model predicts substantial gains in GDP, and the stocks of physical and human capital across a wide range of assumptions about the growth process of the economy. With our preferred assumptions, we predict an increase in GDP in 2080 of over two trillion 2005 dollars—an increase of about 3.5 percent. These effects compare favorably to many other programs that have been promoted as ways of increasing economic growth, particularly state and local subsidies to attract private business (Rolnick and Grunewald, 2003).

To put these gains in perspective consider that federal revenue is likely to increase by about 20 percent of the total increase in GDP or by about 400 billion (2005) dollars (.20 x $2 trillion). We estimate that in 2080 the cost of the program to the federal government, net of existing early childhood education and childcare expenditures, will be $59 billion for a net fiscal surplus of $341 billion using our preferred parameter values. Using those values the fiscal impact of the program becomes positive (costs less than new revenues) for the first time in 2050. Even using our lower bound estimates from the Mankiw, Romer, and Weil model the program pays for itself more than two times over by 2080.

In this first working paper emerging from our project, we have not attempted to determine the net benefits from the additional growth caused by this policy initiative. However, such estimates, along with other extensions of the model, are feasible.[10]

In an ideal world without any increasing returns to scale, the net economic benefits of additional growth caused by additional investment would be exactly offset by the cost of the additional resources devoted to production. However, we do not live in such an ideal world. Additional net benefits could be had by increasing the amount of education people get if we are under-investing in education for some fraction of our population now. This is more likely to be the case to the extent that spillover (or external) effects of education are important and to the extent that individuals fail, for various reasons (lack of finances, short-sightedness), to make investments that payoff over the longer run. Indeed, James Heckman has suggested that at current levels of support, the United States substantially under-invests in early childhood education (Heckman and Masterov, 2004). Further, we must emphasize that any net benefits from these growth effects are all in addition to the well-documented net social benefits of early education programs (Belfield, Nores, and Barnett, 2005).

Because most of these benefits are long term while the costs of mounting the programs are immediate, the political system tends to be biased against making such investments. But any business that operated in this way would likely fail to succeed. A similarly dim prospect may be in store for a country that fails to take advantage of such solid investment opportunities.

[10] See footnote 7.

APPENDIX: THE MATHEMATICAL SPECIFICATION OF THE MODEL

The model used for the analysis in this paper has two main parts: the equation that describes how the inputs to the production process are combined to get output (the production function), and the equations that describe the evolution of the inputs. In this model there is particular attention paid to the accumulation of human capital which requires a more elaborate set of equations to describe the evolution of the population and the acquisition of human capital. By modeling the gestation lags in the creation of new human capital the model is able to track the fiscal effects of investments over time.

The Production Function

The core of the model is the production function. Ours is a modification of the standard Cobb-Douglas production function which has the property that a percentage increase in one of the inputs always causes the same percentage increase in output (constant elasticity). That increase is equal to the coefficient of the input. Our production function is specified with three inputs: physical capital, human capital services per worker-hour, and hours of labor.

$$(1)\quad Y_t = A_t K_t^{\alpha} (H_t L_t)^{1-\alpha} H_t^{\gamma}$$

$$where \qquad H_t = \frac{\sum_{i=18}^{65} f_{i,t} N_{i,t} (1 - F_{i,t}) \ell}{\sum_{i=18}^{65} N_{i,t} (1 - F_{i,t}) \ell},$$

$$L_t = \sum_{i=18}^{65} N_{i,t} (1 - F_{i,t}) \ell ,$$

$$and \qquad f_{i,t} = e^{b * E_{i,t} + z_1 * \varepsilon_{t,i} + z_2 * (\varepsilon_{t,i})^2 + z_3 * (\varepsilon_{t,i})^3}$$

Here Y_t denotes output, or GDP, in period *t*. The variable K_t is the stock of physical capital in year t, H_t is the average flow of human capital services per worker hour, and L_t is the total hours of labor provided. The variable A_t is a normalizing constant. The changes in A_t reflect exogenous technical change.

The average flow of human capital services per hour of labor provided is computed as the average of the flow value of the services of those at age *i* in time *t* ($f_{i,t}$) weighted by the number of hours of labor provided by people of age *i* in year *t*. The hours provided by workers of a particular age is determined as the number of workers ($N_{i,t}$) times the fraction of the year that they are not in school (1-$F_{i,t}$) times the average number of hours worked per worker (l). The variable L_t is thus the total number of labor hours.

The flow value of human capital services per hour of labor is derived by estimating a regression of the log of hourly wages on a constant, years of education (E), years of work experience (ε), experience squared, and experienced cubed. The values b, and z_1 to z_3 are the coefficients of the variables in that regression.

Equation (1) is a very flexible functional form. With γ set equal to α-1 the function yields the standard Cobb-Douglas production function without human capital. With γ larger than α-1 but less than zero the function is the extended Cobb-Douglas with human capital as in Mankiw, Romer and Weil (1992). When γ is set equal to zero the equation yields Uzawa's (1965) production function. Finally, for values of γ greater than zero the production function exhibits increasing returns to scale in human and physical capital and is equivalent to that used in Lucas (1988).

Population

We model only the output produced by domestically born workers. Thus our cohorts do not grow with age. We also assume that all workers remain in the labor force until they retire at 65. The youngest cohort represented in our model is 6 years old thus

$$(2) \quad N_{i,t} = N_{i-1,t-i} \quad for\ all\ t, and\ \ i = 7\ to\ 65\ .$$

The size of each new cohort is given by the equation

$$(3) \quad N_{6,t} = (1 + g_t) N_{6,t-1}\ .$$

The growth rate g_t is set equal to a constant user-determined parameter. The initial values of $N_{i,t}$ are taken from census projections for 2005.

Capital Accumulation

We assume that a constant fraction of GDP, S_K, is devoted to the production of physical capital. We value the stock of physical capital in terms of the cost of producing it so that

$$(4) \quad K_t = (1 - \delta_K) K_{t-1} + S_K Y_{t-1}\ ,$$

where δ_K is the rate of depreciation for physical capital.

Our treatment of human capital is very different from this and very different from the standard treatment in abstract growth modeling. Rather than represent human capital as a stock, like physical capital, we instead model the accumulation of human capital by keeping track of the number of years of education attained by people of each age. We do this to properly model the gestation lags in the generation of new human capital due to a change in educational policy. We assume that all members of each cohort get the same amount of education. We model the education of the cohort of age i at time t as

$$(5)\quad E_{i,t} = E_{i-1,t-1} + F_{i,t} \qquad for\ \ i = 7\ \ to\ \ 65\ ,$$

where $F_{i,t}$ is the fraction of the year that cohort i spends in school in year t. That fraction is determined by the equation

$$(6)\quad F_{i,t} = Max\ (Min\ (1, X_t - i + P_{i,t}), 0)\ .$$

The value X_t+$P_{i,t}$ is the age at which people in cohort i leave school in year t. For those who have had the preschool treatment, $P_{i,t}$ is set equal to p which is a parameter entered by the user. For older cohorts it is set equal to zero. Thus the fraction of the year each cohort spends in school is 0 for those older than X_t+$P_{i,t}$, X_t+$P_{i,t}$–i for the cohort for which this value is between zero and 1, and 1 for younger cohorts.

The value of X_t in each year is determined by a relationship like the savings equation for physical capital in that we assume that, in the absence of the preschool policy, the value of new human capital will be proportional to GDP in that year. We further assume that the total cost of new human capital is proportional to the forgone earnings of those obtaining the additional schooling. While the model operates in units of discrete time, we utilize the following continuous time approximation for obtaining X_t in each time period:

$$X_t = \frac{\ln\left[\dfrac{S_H Y_{t-1}(b-g)}{(2000\mu)\, e^{c+d-6b}} + e^{(b-g)\,\lambda}\right]}{b-g}\ ,$$

where S_H is the savings rate for human capital, λ is the earliest age at which we consider output to be lost by having children in school, μ is the ratio of total compensation to labor earnings, c is the constant in the log wage equation, and the other terms are as previously defined. A typescript of the derivation of this approximation is available from the authors.

Table 1:

Sensitivity Analysis Bounds

Parameter	Parameter Meaning	Preferred Value	Other values checked (1)	(2)	Notes
β	Log-wage coefficient	0.113	0.083	0.053	Preferred value derived from regressions done with 2001 March CPS. We expect the result to be upward biased (because of ability bias, education's value as a sorting mechanism, etc). Therefore, the sensitivity analysis only reviews parameter values less than the preferred value.
δ_K	Depreciation rate of physical capital	4.57%	6.57%	3.57%	Preferred value represents average from 1995-2004 (Calculated from Bureau of Economic Analysis NIPA Tables)
S_K	Savings rate on physical capital	15.90%	16.41%	14.48%	Preferred value calculated from Table 1.5.5 of the Bureau of Economic Analysis's National Income and Product Accounts and represents the average percentage of private domestic fixed investment from 1995-2004. Iterations 1 and 2 represent +/- 1 Standard Deviation of the average of the 11 5-year periods from 1950-2004.
α	Physical Capital Coefficient	0.347	0.323	0.389	Preferred value equals the 40 year average share of national income not allocated to labor earnings. Iterations (1) and (2) represent the minimum and maximum physical capital factor shares over the past 40 years. All figures are calculated from the National Income and Product Accounts (Table 1.12).
g	Average Population Growth Rate of 18 Year-Olds	0.53%	0.28%	0.78%	Preferred value is the average of Census Projections and reflects domestic population growth (i.e., no immigration).
p	Average increase in educational attainment as a result of policy initiative	0.3650	0.1774	0.8100	Preferred value (0.9 year increase) is taken from "Significant Benefits: The High/Scope Perry Preschool Study Through Age 27" Schweinhart, Barnes, & Weikart (1993), pp. 55, 57. Lower bound assumes a 50% take-up rate with benefits accruing only to those children who would not have otherwise attended preschool. Upper bound assumes a 90% take-up rate with benefits accruing in full (100% of Perry Preschool effect) to all children. Methodology based on RAND study by Lynn Karoly and James Bigelow (March 2005).

Table 2: Key Results at 2050, 2065, and 2080
(ALL INCREASES MEASURED RELATIVE TO BASELINE)

Mankiw, Romer, Weil, 1992

	% Increase in Gross Domestic Product	% Increase in Physical Capital	% Increase in Human Capital	Dynamic Prediction – Baseline (2005 dollars)
2050	0.20%	-0.12%	2.36%	$62 billion
2065	0.92%	0.34%	4.22%	$391 billion
2080	1.34%	0.87%	5.18%	$778 billion

Uzawa, 1965

	% Increase in Gross Domestic Product	% Increase in Physical Capital	% Increase in Human Capital	Dynamic Prediction – Baseline (2005 dollars)
2050	0.88%	0.18%	2.49%	$270 billion
2065	2.34%	1.12%	4.72%	$988 billion
2080	3.50%	2.31%	6.37%	$2,034 billion

Lucas, 1988

	% Increase in Gross Domestic Product	% Increase in Physical Capital	% Increase in Human Capital	Dynamic Prediction – Baseline (2005 dollars)
2050	1.02%	0.24%	2.52%	$314 billion
2065	2.65%	1.30%	4.83%	$1,123 billion
2080	4.02%	4.02%	6.64%	$2,340 billion

Table 3: Sensitivity Analysis

		Mankiw, Romer, & Weil (1992)			Uzawa (1965)			Lucas (1988)		
		$\gamma = -.25$			$\gamma = 0$			$\gamma = 0.05$		
		%Δ from Baseline			%Δ from Baseline			%Δ from Baseline		
	Year	**GDP**	**Physical Capital**	**Human Capital**	**GDP**	**Physical Capital**	**Human Capital**	**GDP**	**Physical Capital**	**Human Capital**
Preferred Values	**2050**	0.20%	-0.12%	2.36%	0.88%	0.18%	2.49%	1.02%	0.24%	2.52%
	2065	0.92%	0.34%	4.22%	2.34%	1.12%	4.72%	2.65%	1.30%	4.83%
	2080	**1.34%**	0.87%	5.18%	**3.50%**	2.31%	6.37%	**4.02%**	2.65%	6.64%
Log-Wage Coefficient										
0.083	**2050**	-0.14%	-0.27%	1.54%	0.27%	-0.10%	1.61%	0.36%	-0.06%	1.63%
	2065	0.29%	-0.04%	2.84%	1.14%	0.44%	3.11%	1.34%	0.54%	3.17%
	2080	**0.48%**	0.26%	3.44%	**1.73%**	1.12%	4.12%	**2.03%**	1.32%	4.27%
0.053	**2050**	-0.44%	-0.38%	0.77%	-0.25%	-0.30%	0.80%	-0.21%	-0.28%	0.80%
	2065	-0.23%	-0.34%	1.64%	0.17%	-0.11%	1.75%	0.26%	-0.06%	1.77%
	2080	**-0.24%**	-0.26%	1.89%	**0.30%**	0.14%	2.16%	**0.42%**	0.23%	2.23%
δ_K										
6.57%	**2050**	0.20%	-0.11%	2.35%	0.90%	0.24%	2.49%	1.04%	0.32%	2.52%
	2065	0.95%	0.44%	4.22%	2.41%	1.35%	4.73%	2.74%	1.55%	4.84%
	2080	**1.39%**	1.01%	5.20%	**3.63%**	2.66%	6.43%	**4.18%**	3.04%	6.71%
3.57%	**2050**	0.20%	-0.12%	2.36%	0.87%	0.15%	2.49%	1.01%	0.20%	2.52%
	2065	0.91%	0.29%	4.23%	2.29%	1.00%	4.72%	2.60%	1.15%	4.82%
	2080	**1.31%**	0.78%	5.17%	**3.42%**	2.10%	6.34%	**3.93%**	2.41%	6.60%

Table 3: Sensitivity Analysis (Continued)

		Mankiw, Romer, & Weil (1992)			Uzawa (1965)			Lucas (1988)		
		γ = -.25			γ = 0			γ = 0.05		
		%Δ from Baseline			%Δ from Baseline			%Δ from Baseline		
	Year	**GDP**	**Physical Capital**	**Human Capital**	**GDP**	**Physical Capital**	**Human Capital**	**GDP**	**Physical Capital**	**Human Capital**
S_K										
16.41%	**2050**	0.20%	-0.12%	2.36%	0.88%	0.18%	2.49%	1.02%	0.24%	2.52%
	2065	0.92%	0.34%	4.22%	2.34%	1.12%	4.72%	2.65%	1.30%	4.83%
	2080	**1.34%**	0.87%	5.18%	**3.50%**	2.31%	6.37%	**4.02%**	2.65%	6.64%
14.48%	**2050**	0.20%	-0.12%	2.36%	0.88%	0.18%	2.49%	1.02%	0.24%	2.52%
	2065	0.92%	0.34%	4.23%	2.33%	1.12%	4.72%	2.65%	1.29%	4.83%
	2080	**1.34%**	0.86%	5.18%	**3.50%**	2.31%	6.37%	**4.02%**	2.65%	6.64%
α_P										
0.323	**2050**	0.23%	-0.11%	2.36%	0.90%	0.19%	2.49%	1.04%	0.25%	2.52%
	2065	0.99%	0.37%	4.24%	2.39%	1.15%	4.74%	2.70%	1.32%	4.84%
	2080	**1.43%**	0.93%	5.24%	**3.57%**	2.36%	6.41%	**4.09%**	2.70%	6.68%
0.389	**2050**	0.14%	-0.13%	2.36%	0.83%	0.17%	2.49%	0.98%	0.23%	2.52%
	2065	0.80%	0.27%	4.19%	2.24%	1.07%	4.70%	2.56%	1.25%	4.81%
	2080	**1.17%**	0.75%	5.08%	**3.36%**	2.22%	6.29%	**3.90%**	2.56%	6.57%
λ										
12	**2050**	0.26%	-0.09%	2.44%	0.98%	0.22%	2.57%	1.13%	0.29%	2.60%
	2065	0.96%	0.37%	4.16%	2.35%	1.17%	4.58%	2.66%	1.34%	4.67%
	2080	**1.35%**	0.90%	5.01%	**3.42%**	2.32%	6.02%	**3.91%**	2.64%	6.24%
18	**2050**	0.34%	-0.04%	2.44%	1.08%	0.31%	2.50%	1.23%	0.38%	2.51%
	2065	1.06%	0.45%	3.98%	2.37%	1.25%	4.15%	2.64%	1.41%	4.19%
	2080	**1.39%**	0.98%	4.49%	**3.09%**	2.25%	4.89%	**3.46%**	2.52%	4.98%

Table 3: Sensitivity Analysis (Continued)

		Mankiw, Romer, & Weil (1992)			Uzawa (1965)			Lucas (1988)		
		γ = -.25			γ = 0			γ = 0.05		
		%Δ from Baseline			%Δ from Baseline			%Δ from Baseline		
	Year	**GDP**	**Physical Capital**	**Human Capital**	**GDP**	**Physical Capital**	**Human Capital**	**GDP**	**Physical Capital**	**Human Capital**
g										
0.28%	**2050**				0.94%	0.19%	2.51%			
	2065				2.37%	1.13%	4.65%			
	2080				**3.50%**	2.31%	6.18%			
0.78%	**2050**				0.86%	0.15%	2.55%			
	2065				2.26%	1.09%	4.74%			
	2080				**3.35%**	2.28%	6.35%			
p										
0.177	**2050**	0.10%	-0.06%	1.15%	0.43%	0.09%	1.21%	0.50%	0.12%	1.23%
(50% take-up)	**2065**	0.45%	0.16%	2.04%	1.14%	0.55%	2.28%	1.29%	0.63%	2.33%
	2080	**0.65%**	0.42%	2.48%	**1.69%**	1.12%	3.05%	**1.95%**	1.28%	3.18%
0.810	**2050**	0.43%	-0.28%	5.28%	1.92%	0.35%	5.58%	2.24%	0.48%	5.65%
(90% take-up)	**2065**	2.01%	0.72%	9.53%	5.18%	2.45%	10.70%	5.91%	2.83%	10.96%
	2080	**2.91%**	1.90%	11.70%	**7.82%**	5.15%	14.58%	**9.03%**	5.92%	15.23%

Figure 1: Flow Diagram

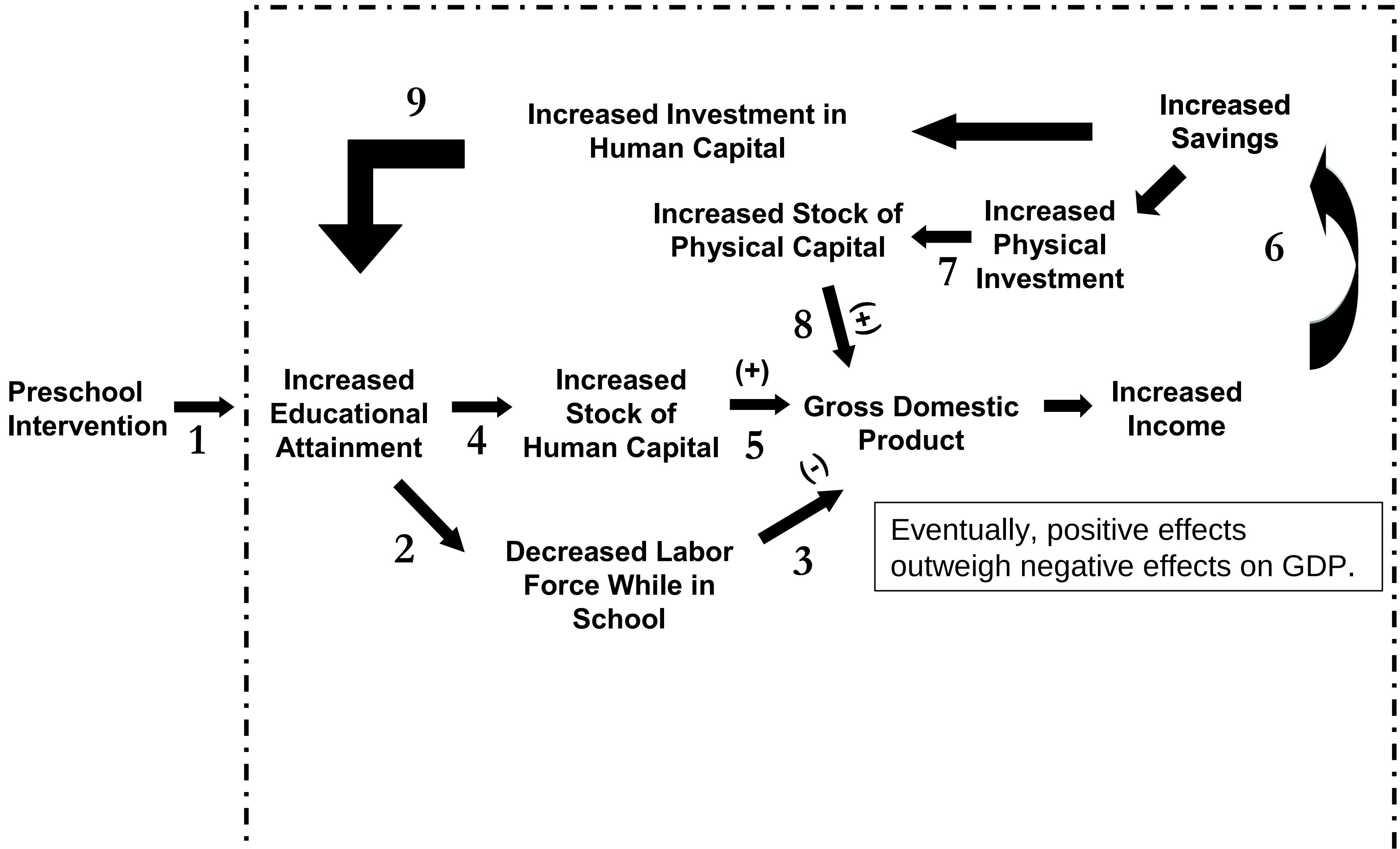


Figure 2: Comparative Predictions of Three Major Models

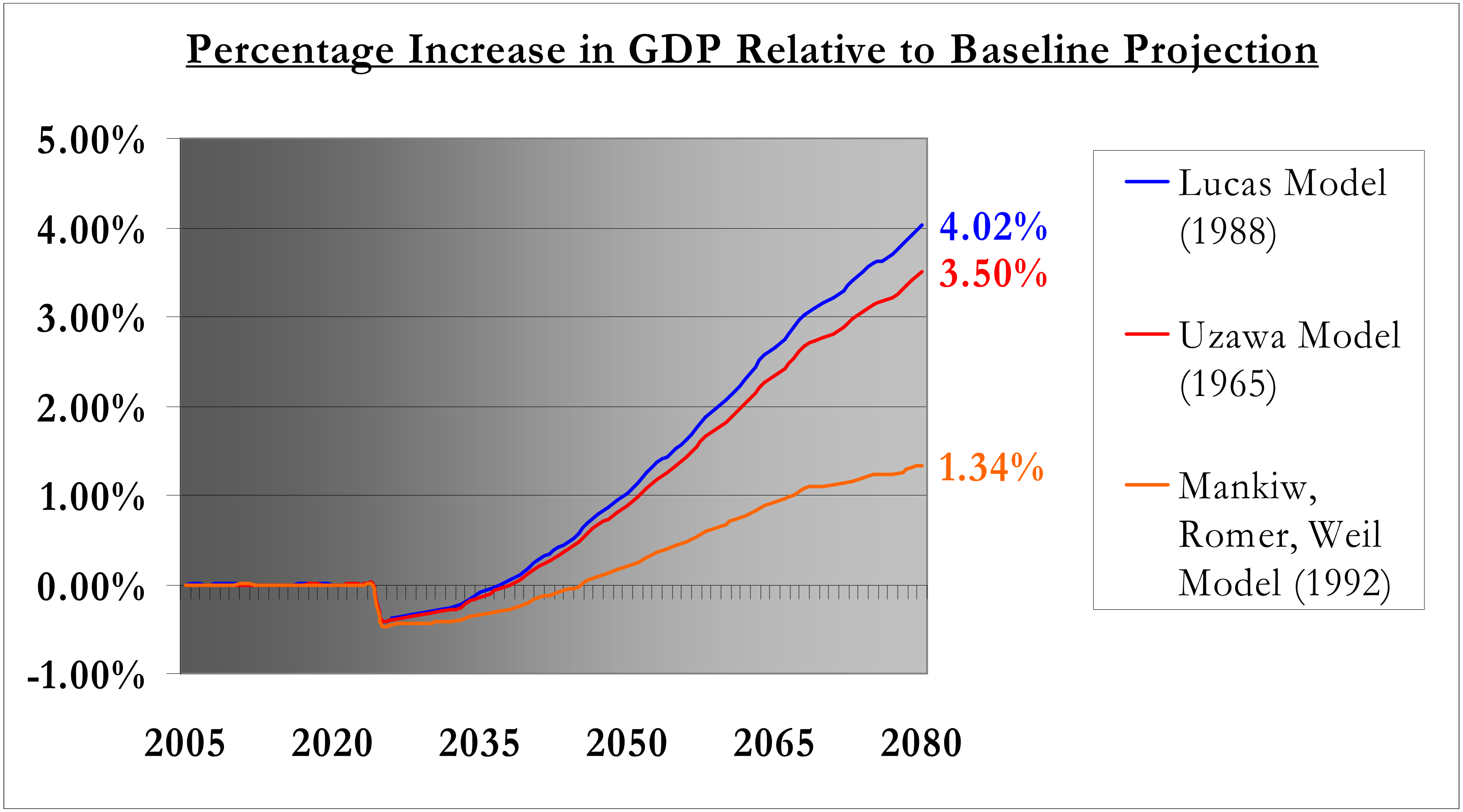
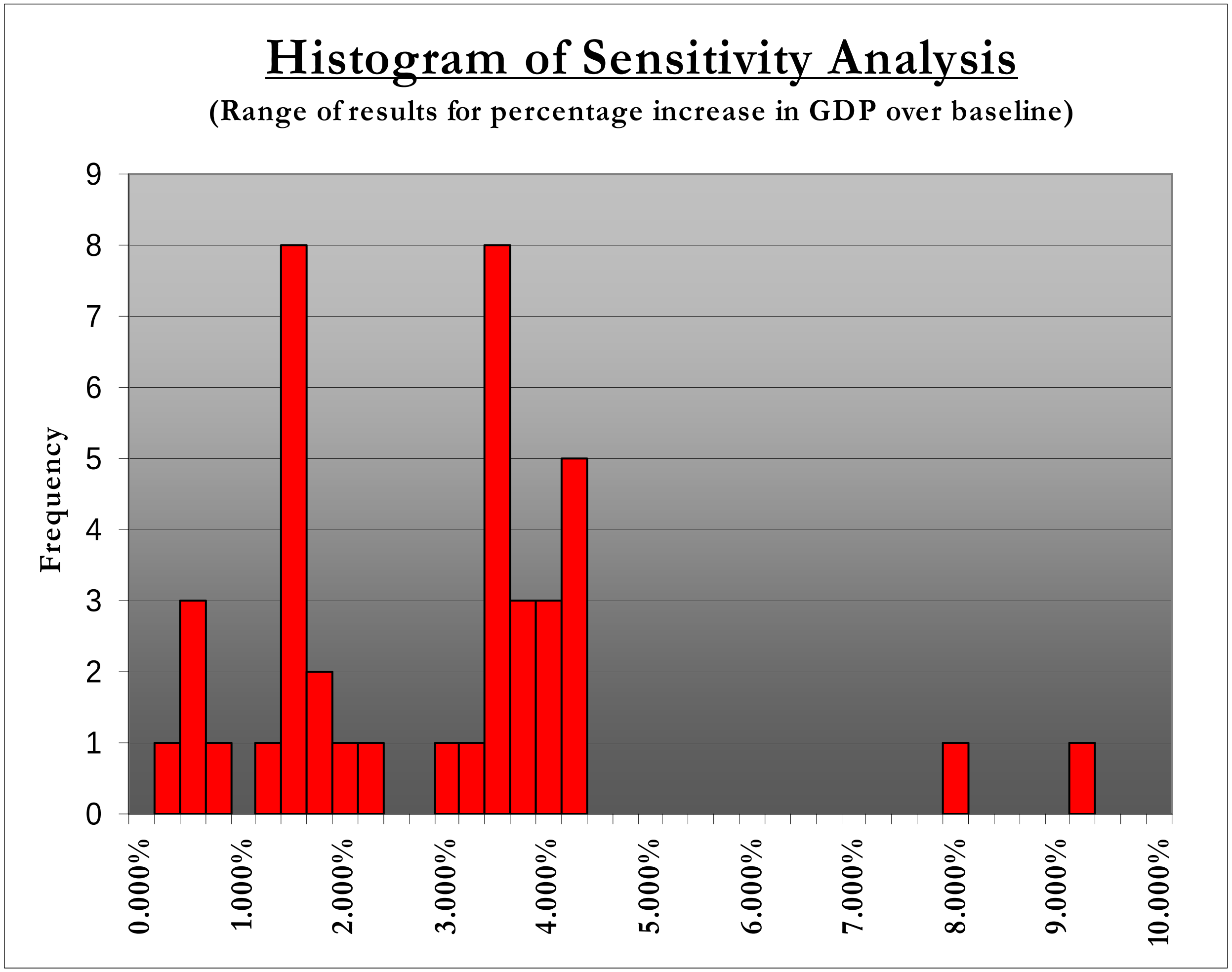


Figure 3: Distribution of Results from Sensitivity Analysis



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