Appendix to Report "Patenting Prosperity: Invention and Economic Performance in the United States and its Metropolitan Areas"

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This methodological appendix is for readers interested in more detailed information about the data sources and statistical methods used in the report "Patenting Prosperity: Invention and Economic Performance in the United States and its Metropolitan Areas." This document, the main report, and companion web graphics are available at the Brookings Institution's website.

Main Patent Statistics

This section provides details on the Strumsky Patent Database and how it was used in the foregoing analysis. The address of the inventors of granted patents is used to allocate patents to metropolitan areas. Some patents have multiple inventors living in different metropolitan areas. The USPTO and other researchers use both whole and fractional counts. In the case of whole counts, a single patent is ascribed to a metropolitan area every time an inventor patents who lives in that area. For fractional counts, only the partial contribution of each researcher to a single patent is ascribed to a metropolitan area, so if there were three inventors living in three different areas, each metro area would get one third of a patent.

The analysis and summary statistics reported in the body of this report use the more intuitive whole count method. In practice, the rankings are nearly identical, and the findings of this report are not substantially affected.

In addition to counting and classifying patents by geography, this report also looks at the characteristics of patents in order to distinguish potentially higher value patents from those that are less valuable. The two main variables considered here are claims and citations. Other characteristics—including the average age of the patent codes, a patent's citations of previous work (prior art), and the number of inventors—were examined as well. Claims was the most robust predictor of productivity.

Patent claims define the invention and what aspects of it are legally enforceable. It is, in a sense, the intellectual property of the patent. The number of claims per patent is the measure used in this analysis. Generally, this is interpreted here to mean the breadth of a patent's intellectual property, but this is not necessarily the case. For a specific patent, the breadth will depend on the wording and interpretation of the claims, in addition to the number, but it is not clear how one could quantify the meaning of language in patent claims. It may be more helpful to think of claims as specific intellectual property attributes of legal value, since claims form the basis of intellectual property disputes.

Other patent and innovation statistics

In terms of presenting results and characterizing the technological trends in patenting, this report introduces its own simplified categorization, which aims to reduce the 500 USPTO major classes to a more manageable overview that roughly corresponds to industries as generally characterized in the North American Industrial Classification System (NAICS). In this modified system, each patent is grouped into a broad category and narrower subcategory.

Each patent has one or more inventors, and if the inventors work for a company, university, of government agency, the patent usually has what is called an assignee. The assignee is the entity assigned ownership rights to the patent, typically because the assignee hired the inventor to research and develop the patented invention. In order to characterize who is doing the inventing, the analysis distinguishes company assignees, from individual independent inventors, from universities and government assignees. Moreover, each patent must disclose if it received government funding from agencies like the National Institutes of Health or a Department of Defense agency, and those patents are also analyzed here.

The most prominent assignees are listed for each metro area on the Brookings website. To calculate these statistics, assignees are assigned to metro areas based on the inventor's residence. For example, if an IBM patent has two inventors, one living in Austin and one in Boston, then IBM receives one patent in both Austin and Boston.

National data on historic patent counts were obtained from the USPTO's website. Utility patents were added to design and plant patents, and foreign patents were subtracted from that sum to get total domestic patents.ⁱ Data on historical GDP and population were downloaded from the Measuring Worth project.ⁱⁱ R&D spending data came from the National Science Foundation's Science and Engineering Indicators reports.

Data on awards given to metropolitan area entrepreneurs through the Small Business Innovation Research (SBIR) and Small Business Technology Transfer (SBTT) were obtained from the SBIR website.ⁱⁱⁱ SBIR reports award data for each business establishment, including the zip code. Zip codes were matched to metropolitan areas using a zip-code to MSA code match from Moody's Analytics.

Source and Description of Economic Data

Time series data for metropolitan areas was obtained for each decennial year from 1980 to 2010. Value added per worker Metropolitan data on Gross Metropolitan Product (GMP) and total employment is from Moody's Analytics. GMP is in chained (or inflation-adjusted) 2005 dollars. The U.S. Census Bureau is the source of data for education and population.

Industrial productivity was measured indirectly. The goal was to isolate the effects of industrial concentration on productivity from the effects of patenting and other variables. As a result, average output per worker was calculated for every 2-digit NAICS sector using data from Moody's Analytics for the relevant decennial years. Then for each metro area in each decennial year, a weighted average of industrial productivity was calculated using Moody's metro area data. This calculation multiplied U.S. productivity in each sector by the share of total metro area employment working in that sector.^{iv} This captures for all sector-wide characteristics that are relevant to productivity, such as commodity prices, profitability, and skill level, and adjusting the results for this variable puts metro areas on an even playing field when comparing productivity.

To distinguish industries with high propensities to patent versus those that do not, this analysis defined the tech-sector according to the Moody's Analytics special aggregate for "high tech." Using this variable as a control, helps to isolate the importance of patents, as opposed to just having tech-sector workers in high-patenting industries. These industries consist of 17 distinct 4-digit NAICS that Moody's deems especially important to the tech sector. They largely overlap with the manufacturing industries listed by the USPTO as patent-intensive industries, which include Computer and Electronics Manufacturing, Chemical Manufacturing, Machinery Manufacturing, Electrical Equipment, Appliance, and Component Manufacturing, and Miscellaneous Manufacturing, which includes medical equipment.^V The Moody's list add a number of service industries to this list including software publishing, telecommunications and other information providers, as well as computer systems design, scientific research and development services, and medical labs.

Historic data on housing prices were gathered for each county from 1980 to 2000 using data from the University of Minnesota's National Historic Geographic Information System and aggregated to metropolitan areas using a crosswalk from Moody's Analytics.^{vi} Contemporary 2010 metropolitan area housing data was obtained from the 2010 American Community Survey.

Data on metropolitan area unemployment rates were obtained from the Bureau of Labor Statistics (BLS). The user should note that for many New England metro areas the metropolitan area boundaries used by the BLS and the Census Bureau are not identical.

The report also presents metropolitan data on high-quality research programs in science. The data for this is from the National Research Council and described later in the report. Programs were considered "top-ranked," if they scored in the 90th percentile on the two major summary measures of academic quality reported by the NRC, using data from 2005-2006.

In an attempt to explain patent growth trends by metropolitan area, the analysis also includes a measure of the "patent class effect" on metropolitan patenting. To calculate this, the number of patents per USPTO class per year was calculated for each metropolitan area. Then the US share of patents developed by inventors in each metropolitan area by class was calculated for 1980-1984, using the yearly average. This 5-year "market share" was multiplied by the change in patenting from 1980-2011 (using the year patents were granted rather than applied for to avoid an artificial dip in 2011) and summed across patent classes to get a single metropolitan level measure for each year. This summed product equaled predicted patent class growth. This was multiplied by the average number of patents from 1980-1984 to measure predicted patents for 2007-2011, which could then be used as an independent variable in a regression to estimate actual 2007-2011 patents.^{vii}

Econometric Analysis

A primary goal of this paper is to report findings on how patents affect economic growth. Economists have long noted that capital and labor produce economic output. These fundamental inputs to production are qualified by technology and human capital (i.e. skills and education). The growth of these factors is ultimately what propels innovation and higher living standards. Living standards are measured as value added per worker, or productivity.

The production model used in this analysis follows this tradition. The quality of technology in a given metropolitan area, as it relates to productivity, is measured by three factors: scale (or population size), industrial contribution to output, and patents. Population fosters specialization, which should increase productivity. Industrial orientation is the broadest possible measure of the technologies and work arrangements used in the metropolitan areas. Patents, on the other hand, are a more specific measure of technological quality, in that they pertain to a smaller number of mostly goods-producing and information service industries, and allow for distinctions within industries, since companies that patent more are arguably more competitive.

To measure human capital, as it relates to metropolitan productivity, two factors are considered. The first is the bachelor's degree attainment rate of the population aged 25 and older. In this framework, more educated workers bring a higher level of skill to the workplace. In actual fact, they are more likely to be working and earn higher wages; so, by definition, they are more productive. The second measure is the average value of patents invented by people living in the metropolitan area. This speaks to the value and productivity of inventors. The variables used to measure the average value of patents are discussed above.

The analysis also adjusts for unchanging metropolitan characteristics (or fixed effects in statistical parlance). This is done by controlling for each metro area's average productivity,

given the other factors. In practice, this controls for things like weather, state laws, and history that do not change over the time of analysis—1980 to 2010. Time trends are also used as a control variable. In doing so, factors that affect all metros—like recessions, oil prices, fiscal and monetary policies—are accounted for.

Finally, the results of statistical modeling are translated into more readily meaningful findings by showing the average marginal effect of each main variable, evaluated at the mean, including its upper and lower bound within a confidence interval of 95 percent. This provides a rough sense of the size and importance of the effects of the variables (e.g. patenting, or educational attainment) on productivity growth. The variables were standardized to have mean zero and a standard deviation of one to facilitate accurate comparisons.

Results of Econometric Analysis

Appendix Table 1, shown below, reports the results of a regression of the log of productivity on the characteristics of metropolitan areas ten years previous. The log difference of two observations separated by time is the growth rate. Since the ten-year lag of productivity is included in the right hand size of the regressions, this equation effectively tests the growth of productivity, conditional on initial characteristics. A working technical paper is available on-line with more detail.^{viii}

One concern from academic reviewers is that these results are biased by endogeneity or omitted variables. In other words, anticipated productivity growth could lead to investments in R&D and patenting. The 10-year lag of patents in the econometric model makes this bias unlikely but it cannot be ruled out entirely. As for omitted variables, there is never any guarantee that all relevant variables are accounted for, but it is not obvious what else they would be. For example, one reviewer was concerned that differences in the cost of living bias the measure of productivity. As robustness check, we included data on average housing costs and average rental costs and entered them separately (in two different regressions) on the right-hand-side in logs, lagged 10 years, to be consistent with the other variables. Neither variable was significant in predicting future productivity, and more importantly, patent claims remained highly significant. We conclude that the results are consistent with the interpretation that patents cause growth and believe that is the most likely theoretical explanation, but we concede that this cannot be proven definitely in our model.

Appendix Table 1. Panel Regression of Productivity	on Patents for Metropolitan Areas, 1980-2010
	Ln MSA Productivity

Linnertroducting				
1	2	3	4	
	-			
0.0229*	0.0745***	0.0107		
(0.0118)	(0.0276)	(0.0123)		
		<u> 1 2 </u>	<u>1</u> <u>2</u> <u>3</u> <u>-</u> 0.0229* 0.0745*** 0.0107	

Log number of patent claims, lagged 10 years		0.0941***		0.0350***
		(0.0242)		(0.0103)
Average claims per patent, lagged 10 years			0.0146***	
			(0.00437)	
Log of GDP/Worker, lagged 10 years	0.196***	0.194***	0.193***	0.196***
	(0.0280)	(0.0278)	(0.0278)	(0.0279)
Log of Predicted Productivity based on				
industries	0.101***	0.103***	0.103***	0.102***
	(0.00867)	(0.00860)	(0.00862)	(0.00862)
Log of population, lagged 10 years	0.0725***	0.0732***	0.0770***	0.0636***
	(0.0246)	(0.0244)	(0.0245)	(0.0242)
Bachelor's degree attainment rate, lagged 10				
years	0.0206*	0.0244**	0.0198*	0.0188*
	(0.0109)	(0.0109)	(0.0108)	(0.0107)
Log of Tech-sector Employment, lagged 10	0.0200*	0.0200*	0.0220	0.0221
years	0.0269*	0.0266*	0.0239	0.0231
	(0.0151)	(0.0149)	(0.0150)	(0.0149)
Year 2000	0.00523	0.0171	0.0155	0.0114
	(0.0101)	(0.0105)	(0.0105)	(0.0103)
Year 1990	- 0.0619***	-0.0381**	-0.0444**	- 0.0512***
	(0.0170)	(0.0179)	(0.0177)	(0.0173)
Constant	3.467***	3.465***	3.470***	3.460***
	(0.118)	(0.117)	(0.117)	(0.118)
Observations	1,082	1,082	1,082	1,082
Adjusted R-squared	0.930	0.932	0.931	0.931

Standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1. Source: Brookings analysis of Strumsky Patent Database, Moody's Analytics, U.S. Census Bureau. All columns include metropolitan area fixed effects and decennial year effects. All variables shown are standardized to have mean zero, except lagged productivity and the decennial year binary variables.

Appendix Table 2 below reports the results of a regression analysis used to investigate the relationship between metropolitan area patenting and metropolitan area unemployment rates (columns 1-2) and employment growth (columns 3-4). The results show a strong negative relationship between patent growth and unemployment, and a positive relationship between patenting and job growth. However, that positive relationship becomes statistically insignificant when controlling for the change in human capital, which has a strong positive effect on employment growth. The education variable is very highly correlated with patents, and it is likely that patenting causes higher educational attainment by attracting them to the region.^{ix}

Appendix Table 2. Relationship between Patent Growth, Unemployment and Job Growth				
	Average Annual Growth			
Average unemployment	Rate in Employment,			
rate, 1990-2010	1980-2010			

	1		2	3
Annual growth rate of patents, 1990-2010	-22.13***	-11.86***		
	(4.317)	(4.173)		
Annual growth rate of population, 1990-2010	89.32***	83.85***		
	(24.94)	(23.25)		
Change in bachelor's degree attainment rate, 1990-2010	-23.72***	-16.32***		
	(4.808)	(4.548)		
Growth in housing price index, 1990-2010	-0.516	-0.237		
	(0.427)	(0.395)		
Share of workforce with bachelor's degree or higher in 1990		-13.12***		
		(1.817)		
Growth in tech-sector employment, 1990-2010		-0.864		
		(4.946)		
Predicted growth in productivity, 1990-2010		-85.43***		
Appual growth rate of patents 1080 2010		(26.28)	0.0215**	-0.00643
Annual growth rate of patents, 1980-2010			(0.0215**	-0.00643 (0.00937)
Annual growth rate of population, 1980-2010			(0.00878) 0.856***	(0.00937) 0.811***
Annual growth rate of population, 1980-2010			(0.0313)	(0.0303)
Predicted growth in productivity, 1980-2010			-0.249***	-0.186***
realized growthin productivity, 1900 2010			(0.0548)	(0.0521)
Change in bachelor's degree attainment rate, 1980-2010			(0.0340)	0.0292***
				(0.00700)
Growth in tech-sector employment, 1980-2010				0.0534***
				(0.0130)
Constant	7.562***	10.46***	0.00651***	0.00330***
	(0.525)	(0.700)	(0.000867)	(0.00103)
	included	included	included	included
Observations	317	317	356	356
Adjusted R-squared	0.457	0.551	0.866	0.882

Standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1. Source: Brookings analysis of data from Strumsky Patent Database, Bureau of Labor Statistics, Census Bureau, Moody's Analytics, and Federal Housing Finance Agency. Home price data and unemployment data begin in 1990.

The relationship between patenting and IPOs was explored in the text above. The main conclusion—that patents are predictive of private companies going public and higher valuations thereof—is supported by the regression results shown in Appendix Table 3. Higher patenting rates between 1995 and 2000 predicts higher value IPOs and more of them per capita from 2000 to 2006. That correlation remains significant controlling for tech-sector employment, education, population, and GDP per worker.

	Value of IPOs in millions	IPOs per capita
	1	2
Patent per capita, 1996-2000	2.359***	2.356***
	(0.273)	(0.270)

Tech-sector Share of Employment, 2000	0.0195**	0.0362***
	(0.00819)	(0.00809)
Bachelor's degree attainment rate for		
population 25 and older, 2000	-0.00699***	-0.00364
	(0.00260)	(0.00257)
Population, 2000	1.47e-10	1.65e-10
	(1.07e-10)	(1.06e-10)
GDP/Worker, 2000	6.86e-05***	8.62e-05***
	(1.89e-05)	(1.87e-05)
Constant	-0.00490***	-0.00719***
	(0.00141)	(0.00140)
Observations	356	356
Adjusted R-squared	0.432	0.554

Standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1. Regressions include state fixed effects. IPO data include those issued from 2000 to 2006. Source of IPO data: Kenney, Martin and Donald Patton. 2010. Firm Database of Initial Public Offerings (IPOs) from June 1996 through 2006 (Version B).

The regression results shown in Appendix Table 4 report the relationship between doctoral programs in science and innovation indicators. Having a top-ranked (90th percentile) science program is associated with significantly more patents overall and per resident from 2007 to 2011. Doctoral programs outside of science have a weak negative effect, controlling for science programs, and medium or low-ranked science programs have a small positive effect that is insignificant when controlling for the other factors in the regression model.

The first two columns regress the rate of patenting and then the level of patenting on the independent variables, including controls for population, share of jobs in tech sector, science degree attainment rate, and the average number of inventors per patents—a measure of collaboration. The second two columns repeat those regressions but add state fixed effects, which filters out the average number or rate of patenting in each state. In theory, this controls for state laws and history.

The last two columns repeat the first two but use the number of eminent scientists working at pubic or academic institutions in the metropolitan area in 1906 as an instrument for the contemporary number of top-ranked science programs. The list was put together in 1906 by a Columbia University professor and the results were published in the prestigious journal *Science*.^x This method is used to address the concern that a high rate of patenting may cause the local universities to gain in prominence, thus biasing the earlier results through reverse causality. The use of an instrumental variable is meant to identify the causal effect of top-research universities by using variation in the distribution of eminent scientists a century

earlier. For this to mitigate the bias, the distribution of eminent academic scientists must not have any correlation with patenting other than through its effect on research universities. Since these scientists were employed at research universities and institutions (Harvard, U.S. Geological Society, Princeton, Johns Hopkins, UC Berkeley, Smithsonian Institute, etc), this seems like a reasonable assumption, but their presence may have also sparked industrial developments that led both to patenting and higher quality research universities. So, this evidence must be interpreted as only suggestive of a causal link between universities and patenting.

The results are uniform in finding that high-ranking research universities are strongly correlated with the level and rate of patenting. Likewise, tech-sector employment is a very strong predictor as well. Two other variables—science education attainment and the average number of collaborators per patent—strongly predict the rate of patenting but not the level. Since both measures are of intensity, the contrast is not surprising. Collaboration and a high rate of scientifically trained workers are both more common in more populous metro areas.

	Patents		Patents		Patents	
	per		per		per	
	million		million		million	
	residents,		residents,		residents,	
	2007-	Patents,	2007-	Patents,	2007-	Patents,
	2011	2007-2011	2011	2007-2011	2011	2007-2011
	1	2	3	4	5	6
Number of doctoral programs in MSA	-9.599**	-13.93**	-8.515*	-14.91	-9.599**	-13.93*
	(4.246)	(5.506)	(4.844)	(9.772)	(4.820)	(7.342)
Number of science doctoral programs in MSA	8.251	6.253	5.926	6.474	8.251	6.253
	(6.641)	(8.613)	(8.027)	(14.40)	(7.415)	(11.74)
Number of top-ranked doctoral programs in MSA	24.70***	110.5***	27.30***	109.7***	24.70***	110.5***
	(6.144)	(7.968)	(5.110)	(32.68)	(6.510)	(28.12)
Population, 2010	5.93e-06	0.000390***	8.25e-06	0.000395***	5.93e-06	0.000390***
	(1.86e-05)	(2.41e-05)	(2.20e-05)	(4.65e-05)	(2.15e-05)	(4.11e-05)
Share of jobs in tech sector, 2010	9,004***	10,661***	8,522***	11,727**	9,004***	10,661**
	(1,020)	(1,322)	(3,112)	(5,517)	(2,569)	(4,878)
Bachelor of Science Degree Attainment Rate	3,518***	584.7	4,118***	1,432	3,518***	584.7
	(680.4)	(882.5)	(841.1)	(1,388)	(821.9)	(820.5)
Average number of inventors per patent, average 2007-2011	87.21***	22.14	47.65	2.052	87.21***	22.14
	(19.83)	(25.71)	(34.49)	(33.31)	(26.02)	(21.41)
Constant	-539.2***	-420.7***	-451.3***	-458.0***	-539.2***	-420.7***
	(66.24)	(85.92)	(98.70)	(166.9)	(90.92)	(116.0)
Controls for state of MSA			yes	yes		
Instrument for top science programs					yes	yes
Observations	356	356	356	356	356	356
Adjusted R-squared	0.563	0.798	0.681	0.787	0.563	0.798

Standard errors in parentheses, clustered at state level. *** p<0.01, ** p<0.05, * p<0.1. Source: Brookings analysis of data from Strumsky Patent Database, Census Bureau, Moody's Analytics, and NRC. Instrument is the number of "eminent" scientists working in the metropolitan area in 1900see text for historic source. Finally, Appendix Table 5 revisits the analysis from Appendix Table 1 (the relationship between patents and productivity growth), but adds SBIR funding at the metropolitan area and a control variable for the share of patents that are funded by the government. The SBIR data was available from 1983 to 2010, so the panel is compressed to 1990, 2000, and 2010. SBIR funding has a robust and strong positive relationship with productivity growth in this specification. The share of patents funded by the federal government has a positive but insignificant relationship with growth. This suggests that SBIR projects boost metro area growth beyond their effects on patents. Including this variable does not meaningfully change the effect of patents on growth.

2010	Ln MSA Productivity				
	1	2	3	4	
Ln value of SBIR Funding Received, lagged 10 years	0.00922**	0.00918**	0.00860**	0.00913**	
	(0.00391)	(0.00387)	(0.00389)	(0.00389)	
		-			
Log number of patents, lagged 10 years	0.0220*	0.0764***	0.0101		
	(0.0118)	(0.0276)	(0.0123)		
Log number of patent claims, lagged 10 years		0.0949***		0.0344***	
		(0.0241)		(0.0103)	
Average claims per patent, lagged 10 years			0.0143***		
			(0.00437)		
Log of GDP/Worker, lagged 10 years	0.191***	0.190***	0.189***	0.191***	
	(0.0282)	(0.0279)	(0.0280)	(0.0280)	
Log of Predicted Productivity based on industries	0.102***	0.104***	0.104***	0.103***	
	(0.00866)	(0.00859)	(0.00861)	(0.00861)	
Log of population, lagged 10 years	0.0735***	0.0743***	0.0779***	0.0644***	
	(0.0246)	(0.0243)	(0.0244)	(0.0242)	
Bachelor's degree attainment rate, lagged 10 years	0.0136	0.0174	0.0132	0.0118	
	(0.0113)	(0.0112)	(0.0112)	(0.0110)	
Log of Tech-sector Employment, lagged 10 years	0.0276*	0.0274*	0.0247*	0.0238	
	(0.0150)	(0.0149)	(0.0150)	(0.0149)	
Share of patents funded by federal government, lagged 10					
years	0.105	0.121	0.111	0.105	
	(0.102)	(0.101)	(0.101)	(0.101)	
Year 2000	0.00352	0.0155	0.0136	0.00968	
	(0.0101)	(0.0105)	(0.0105)	(0.0103)	
	-		-	-	
Year 1990	0.0652***	-0.0412**	0.0479***	0.0545***	
	(0.0170)	(0.0179)	(0.0177)	(0.0173)	
Constant	3.485***	3.481***	3.485***	3.477***	
	(0.119)	(0.118)	(0.118)	(0.118)	
Observations	1,082	1,082	1,082	1,082	
Adjusted R-squared Standard errors in parentheses *** p<0.01 ** p<0.05 * p<0	0.931	0.932	0.932	0.932	

Appendix Table 5. Panel Regression of Productivity on SBIR Funding and Patents for Metropolitan Areas, 1980-

2010

Standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1. Source: Brookings analysis of Strumsky Patent Database, Moody's Analytics, U.S. Census Bureau. All columns include metropolitan area fixed effects and decennial year effects. All variables shown are standardized to have mean zero, except lagged productivity and the decennial year binary variables.

ⁱⁱⁱ Small Business Innovation Research, available at http://www.sbir.gov/sbirsearch/technology (December 2012).

^{ix} As a robustness check, the average number of patents per worker in 1990, 2000, 2010 was calculated and correlated with average unemployment. Again, there was a very strong negative correlation, suggesting that patenting lower unemployment, but it because insignificant one the average educational attainment rate was included. Yet, the growth rate in patenting does seem to reduce unemployment, even accounting for education, but a consistently high level of patenting only reduces unemployment by attracting highly educated workers.

^x James McKeen Cattell, "A Statistical Study of American Men of Science. III. The Distribution of American Men of Science," *Science* 24 (623) (1906): 732-742.

ⁱ U.S. Patent and Trademark Office (USPTO), available at

http://www.uspto.gov/web/offices/ac/ido/oeip/taf/h_counts.htm (December 2012).

ⁱⁱ Louis Johnston and Samuel H. Williamson, "What Was the U.S. GDP Then?" MeasuringWorth, 2011, available at (<u>http://www.measuringworth.com/usgdp</u> (December 2012).

^{iv} For example, assume average U.S. productivity in the oil and gas sector is \$100,000 per worker. Assume productivity in the health care sector is \$40,000 per worker. If 50 percent of a metro area's workers are in the oil and gas sector and 50 percent are in health care, then average metro industrial productivity is \$50,000 plus \$20,000, or \$70,000.

^v USPTO and U.S. Department of Commerce, "Intellectual Property and the U.S. Economy: Industries in Focus." ^{vi} Minnesota Population Center. *National Historical Geographic Information System: Version 2.0.* Minneapolis, MN: University of Minnesota 2011, available at http://www.nhgis.org.

^{vii} This analysis was suggested, at a high level, from comments by Rob Atkinson and Josh Lerner who both wondered how patent class changes affected growth in particular regions.

^{viii} Jonathan Rothwell, Jose Lobo, Deborah Strumsky, "The Role of Invention in U.S. Metropolitan Productivity" SSRN Working Paper 2175310 (2012). Available at SSRN: <u>http://ssrn.com/abstract=2175310</u>