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THE G-CUBED (AGRICULTURE) MODEL: A TOOL FOR ANALYZING AGRICULTURE IN A GLOBALIZING WORLD

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The G-Cubed (Agriculture) Model:
A Tool for Analyzing US Agriculture in a Globalizing World

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ABSTRACT

This paper describes the major features and structure of the G-Cubed (Agriculture) multi-country, multi-sector intertemporal general equilibrium model. It is an extension and a variation of the G-Cubed model developed by Warwick McKibbin and Peter Wilcoxen to include relatively detailed agricultural sectors and a country dis-aggregation relevant for key US agricultural markets. The paper is intended to accompany the documentation of the G-Cubed model provided by McKibbin and Wilcoxen (1995). Other background papers on the model can be found at WWW.MSGPL.COM.AU

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1. Introduction

The dependence of U.S. agriculture on the world economy has changed significantly during the past three decades. U.S. agriculture has become more closely integrated in the world economy through a variety of channels. Export markets are increasingly important for US producers. The proportion of US agricultural production exported more than doubled during the past two decades, and by 1995 accounted for nearly one quarter of U.S. agricultural production[2]. The growing importance of exports in agricultural production is a clear channel that makes US agriculture increasingly vulnerable to shocks in the world economy.

Fluctuations in export demand are a clear implication of global integration but just as important are a number of other factors that directly and indirectly affect US agriculture in an increasingly integrated world. As global financial markets have became more and more integrated and with the shift from the Bretton Woods system of fixed exchange rates to more flexible exchange rates, the world has experienced significant fluctuations in exchange rates since the early 1970s. In addition to the direct impact of exchange rate fluctuations on the demand for US agricultural output, the more integrated the world economy becomes, the more the costs of inputs for agricultural production are affected by developments in the world economy. Changes in interest rates for example are important for US agriculture and increasingly affected by developments in the world economy. US domestic demand for agricultural products is also affected by economy wide developments which are increasingly dependent on the world economy. For example Coyle, McKibbin, Wang (1998) show that the Asia currency crisis has a significant

[2]This percentage is based on the ratio of f.o.b. value of exports to gross sales at farmgate. It may overstate the share of exports in total agricultural production since there are significant value-added between farmgate and ports, especially for livestock and other high valued products.

affect on the demand for US agricultural products through a contraction in export markets but the crisis also has important effects on US interest rates through international capital flows. These changes in interest rates directly impact on US domestic demand and therefore on the demand for US agricultural products.

Recent major shocks to U.S. food and agricultural sector have frequently come from abroad as a result of the US economy becoming more open and the world economy becomes more integrated. The combination of "flexible" exchange rate system and the well integrated international markets means macroeconomic policy increasingly affects national economies through inducing changes in the value of national currencies and interest rates. In important respects, the burden of adjustment to changes in macroeconomic policy falls on the tradable sectors, of which agriculture is an important part.

Another aspect of why the changing world economy impacts importantly on the rural sector in the United States is the increasing trend of farmers in developed countries to have part-time employment outside agriculture. The growing share of income of rural households from non-farm employment reduces the relative importance of farming as primary source of income. For example, more than 60 percent of total income of US farm households come from nonagricultural employment. As a result of those changes, agricultural commodity prices and the income of farmers have become increasingly sensitive to the economic environment outside agriculture -- to domestic fiscal and monetary policies and to fluctuating exchange rates (Schuh, 1974).

Under such an economic environment, the major analytical and policy problems agricultural economists face, have to do with how U.S. agriculture adjusts to shocks in an increasingly interdependent national and world economy (Schuh, 1976). The analysis of agricultural policy therefore become more complex. Agricultural economists have to give greater attention to changes in monetary and fiscal policies both in the United States and abroad as well as developments in global financial markets if they are to understand developments in agricultural markets. Facing the above challenges for the agricultural economics profession calls for new analytical tools. Our development of the G-Cubed (Agriculture) model to explicitly deal with global agriculture markets in a multi-country, multi-sector intertemporal general equilibrium model is an attempt to develop new tools for the profession to conduct agricultural policy analysis in today's rapidly change economic environment.

The G-Cubed model was originally developed by McKibbin and Wilcoxen (1992). It combines the dynamic macroeconomic modeling approach taken in the MSG2 model of McKibbin and Sachs (1991) with the disaggregated, econometrically-estimated, intertemporal general equilibrium model of the U.S. economy by Jorgenson and Wilcoxen (1990). The Jorgenson-Wilcoxen model breaks U.S. economy down into 35 separate industries, each of which is represented by an econometrically estimated cost function.

The G-Cubed (agriculture) model has been constructed to explore the impact on US agriculture of a range of international and domestic economic shocks. The main focus of current projects is on APEC trade liberalization and the Asian economic crisis, but the model has many features that will make it useful for answering a range of issues of direct relevance to US agriculture both of a microeconomic and macroeconomic nature. It is a world model with substantial regional disaggregation and sectoral detail. In addition, countries and regions are linked both temporally and intertemporally through trade and financial markets. Like the G-Cubed model, G-Cubed (agriculture) contains a strong foundation for analysis of both short run macroeconomic policy analysis as well as long run growth consideration of alternative macroeconomic policies. Intertemporal budget constraints on households, governments and nations (the latter through accumulations of foreign debt) are imposed. To accommodate these constraints, forward looking behavior is incorporated in consumption and investment decisions. G-Cubed(agriculture) also contains substantial sectoral detail. This permits analysis of economic policies which tend to have their largest effects on small segments of the economy such as the agricultural sector. By integrating sectoral detail with a coherent macroeconomic structure the G-Cubed (agriculture) model can be used to consider the long run costs of alternative trade liberalization scenarios yet at the same time consider the macroeconomic implications of these policies over time. The response of monetary and fiscal authorities in different countries can have important effects in the short to medium run which, given the long lags in physical capital and other asset accumulation, can be a substantial period of time. Overall, the G-Cubed (agriculture) model follows the original G-Cubed model in providing a bridge between computable general equilibrium models and macroeconomic models by integrating the more desirable features of both approaches with a focus on agriculture.

G-Cubed (agriculture) is still in the process of development but it is already a large model. In its current form it contains over 9,000 equations and 175 intertemporal costate variables. Nonetheless, it can be solved using software developed for a personal computer outlined in McKibbin and Wilcoxen (1995).

The purpose of this paper is to give an overview of the first version of the G-Cubed (agriculture) model in the context of the adaptation on the G-Cubed model to focus on agriculture. It is intended as a technical document to accompany the documentation of the

Gcubed model contained in McKibbin and Wilcoxen (1995) highlighting where differences from the core model occur as well as to supplement policy-oriented application papers. It does not include a survey of related literature except where relevant. Section 2 of the paper gives a brief discussion of pro and cons of different approaches in economic modeling (drawing on McKibbin (1993)) and how the G-Cubed class of models fit into the existing modeling approaches. Section 3 provides a detailed overview of the structure of the model. A conclusion and directions for future research and model development is presented in section 4.

2. Integrating Macroeconometric and CGE models for Examining Agricultural Policy

Until recently, computable general equilibrium (CGE) models[3] and macro-econometric (ME) models[4] co-existed with very little interaction between the modelers in the two modeling streams or very little cross fertilization of the two approaches[5]. Both modeling techniques have made significant contributions to understanding various empirical aspects of economies yet attempts to combine existing models from the different approaches have been unsuccessful.[6]

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[3] These are also referred to as Applied General Equilibrium (AGE) models. Hereafter I will only use the term CGE models. See de Melo (1988), Robinson (1989) and Shoven and Whalley (1984) for an overview of CGE models. Examples of this approach include models by Dixon et al (1982), Whalley (1985), Deardorff and Stern (1985), Hertel et al (1997).

[4] See Bryant et al (1988) for a summary of the major multi-country macro-econometric models and a list of references relating to each model.

[5] Attempts have been made to reconcile the two approaches. See for example Powell (1981) and more recently Parsell, Powell and Wilcoxen (1989).

[6] A number of attempts to link macro-econometric models and CGE models do exist. See Cooper and McLaren (1983) for one such attempt using Australian models.

The distinction between the two approaches is much the same as that between microeconomic and macroeconomic theory. However, as the distinction between microeconomic and macroeconomic theory has blurred, so has the distinction between CGE models, which have begun to incorporate dynamics[7], and a new generation of macro-econometric models which are more firmly based on optimization theory[8]. In addition to the theoretical basis of the models there is also a significant range of techniques for parameterizing both schools of models. Parameters are either based on econometric estimates by the modelers or are "calibrated" by creating parameters that are based on "empirical evidence".

CGE models are derived from microeconomic optimization theory, with considerable attention to individual behavior whereas macro-econometric models are based on aggregate behavior with reliance placed heavily on correlations found in time series of aggregate data. There is by now a vast literature containing applications of computable general equilibrium models. The reader is referred to papers by Dervis et. al. (1982), de Melo (1988), Robinson (1989) and Shoven and Whalley (1984) for a detailed overview of CGE models.

The CGE modeling work descends directly from the work of Arrow and Debreu (1954). It transfers the well-known Walrasian general-equilibrium structure from an abstract

[7] The treatment of dynamics varies considerably across CGE models. Some are very simple while others are integrated more completely into behavior. Examples of dynamic CGE models include Burniaux et al (1991), Goulder and Eichengreen (1989) and Jorgenson and Wilcoxen (1990).

[8] The Multimod econometric model at the International Monetary fund outlined in Masson et al (1988) and the MSG2 multi-country model in McKibbin and Sachs (1991) are examples. It should be noted that the MSG2 model is very different to the MSG model that appears in Bryant et al (1988). That earlier model was a typical macroeconomic model whereas MSG2 is more like a dynamic CGE model.

representation of an economy into realistic models of actual economies that can be used to conduct policy evaluations by specifying production and demand functions and incorporating data reflective of the real world. They contributed quantitative results to the theoretical literature on microeconomic general equilibrium analysis and attempt to explain economic behavior by real world data. Given the focus on individual optimization, the key parameters in these type of models are parameters such as expenditure shares and the elasticities of substitution of households and firms. These parameters are sometimes "calibrated" to a data set given an assumption about functional form of utility and production functions. Or they are estimated from extensive cross sectional data on households and firms. In addition to parameter calibration, it is often the case that the data behind the model is adjusted to be consistent with the equilibrium of the model.

The applied aspects of CGE modeling descend from the fixed coefficient work of Wassily Leontief. By using input-output tables constructed for fixed coefficient models, introducing relative prices and empirical evidence on substitution in production and consumption, the CGE approach added a new dimension to this earlier modeling strategy. An early example of this type of work is Johansen (1960). During the last two decades, hundreds of such models have been built and applied to a number of policy issues, ranging from public finance and taxation, economic integration, GATT negotiations, and issues of North-South trade, to the evaluation of development strategies and energy and environment policies for almost all the major countries in the world. With the focus on micro-economic theory, CGE models are particularly well suited to analyzing questions in tax policy and international trade (see Shoven and Whalley (1984)). In addition they have played an important role in the literature on economic development (see the survey by de Melo et. al. (1982).

The advantage of this approach is the transparency of the key mechanisms in many of the models. Also considerable sectoral detail can be handled and even the results from the larger models can be understood from theoretical intuition. One problem with this approach is interpreting the time horizon over which the results are relevant. This partly relates to how long it takes for markets to clear. The early static CGE models were used for comparative static analysis of the change between equilibria given a change in some policy. They were particularly useful for analyzing the long run effects of policies. Recent work has attempted to incorporate dynamics to allow for simple adjustment between equilibria (e.g. Bourgignon et al (1989), Burniaux et.al. (1991), and Feltenstein (1986)). However these extensions have usually had macroeconomic closures which are considered unsatisfactory by macro-economists. The absence of an aggregate price level or any role for money (or nominal exchange rate fluctuations) in particular is an important omission from most CGE models.

Other attempts to introduce dynamics through explicit intertemporal optimization of agents, such as papers by Lipton and Sachs (1983), McKibbin (1986), Goulder and co-authors (1989,1990), Jorgenson and Wilcoxen (1990), McKibbin and Sachs (1991), McKibbin and Wilcoxen (1992) and Schmidt-Hebbel and Servin (1992) are more promising. The approach by McKibbin and Wilcoxen (1992) form the basis of the G-Cubed group of models.

In contrast to the approach taken in CGE models, the standard approach in macro-econometric models developed during the 1960s and 1970s was for macroeconomic theory to be used as a guide as to the appropriate variables to use in regression equations (see Bodkin et al (1991)). These variables were then tested and either included or excluded based on various tests of statistical significance. Because of the focus on aggregate relationships, it was rare that these models imposed the types of constraints across equations to satisfy conditions from microeconomic optimization theory. In some cases, the conditions of homogeneity were not even imposed. With fewer theoretical constraints or seldom any imposition of steady state conditions to impose stability, the larger econometric models tended to be explosive over long periods and were only really useful for simulations over short time horizons. This was less of a problem when the models were used for short term forecasting but was a fundamental problem for medium term policy analysis.

These macro-econometric models broke down empirically in the 1970s, in part because they relied on data periods in which events such as supply shocks were relatively unimportant. While these models were criticized for their poor tracking performance in the face of shocks that were not in their estimation sample, the Lucas Critique dealt a theoretical blow by pointing out the role of expectations and the need to worry about theoretical structure and policy regimes[9]. The modeling profession responded to this challenge by introducing rational expectations into a number of models. The multi-country models that incorporated the assumption of rational expectations such as the Liverpool, Taylor, MSG2 and Minimod (parent of Multimod) tended to be small models with relatively simple dynamics. This was partly due to the size constraints placed by the numerical techniques used to solve RE models. It was also because the long distributed lag structures in the traditional macroeconometric models made it virtually impossible to numerically solve these models including rational expectations. The main problem was that in many, if not all cases, these models were basically unstable. The instability which gradually appeared over a simulation horizon of a decade or less, manifested itself into the first year when

[9] See Lucas (1973).

attempting to introduce rational expectations. To give the saddle path stability that is required in rational expectations models required a tighter structure and more careful constraining of parameter estimates. Even today it is not a matter of size of models that prevents the use of rational expectations. The key problem is related to model stability. It is not that rational expectations was necessarily a desirable assumption to incorporate into these models. However, attempts to implement this assumption showed that the conventional style of macroeconometric model building produced models with medium term properties that were less transparent and less stable than more theoretically constrained models.

Global macro-econometric models that have been developed in the 1980s spanned the spectrum of macroeconomic and microeconomic theory with a variety of reliance on estimation versus calibration techniques. Models such as Multimod[10], Liverpool model[11], the Taylor Model[12] and perhaps GEM are closer to the microeconomic theory part of this spectrum than the derivative models of the large scale models of the 1970s such as the DRI, EPA, INTERLINK, LINK, MCM or WHARTON models[13]. The MSG2 model in contrast, lies closer to the microeconomic theory end of the spectrum because it has explicit structural parameters and it is calibrated to a data set rather than estimated over a time series of data.

In building a new model such as undertaken for this paper, it is natural to ask which approach should be followed? Total or excessive reliance on aggregate data with very little

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[10] See Masson et al (1988).

[11] See Minford et al (1986).

[12] See Taylor (1988).

[13] For an overview of these models see Bryant et al (1988).

regard to theoretical structure is the wrong modeling strategy to follow for a number of reasons. Firstly, the data at the aggregate level are so poor that theoretical restrictions are required for any useful analysis of the information contained in the data. Secondly many shocks we care about (e.g. APEC trade liberalization, currency crisis in Asia, regime changes such as shifts from fixed to floating exchange rates, moving into or out of the EMS or the shift from exchange controls to free capital mobility etc.) are not in our data samples or at least not with sufficient numbers of observations. Thirdly, for a model to be useful in a practical sense, it must be able to adequately replicate the data to a significant extent but it also must be understandable so the user gets quantitative information and a better intuition of the main issues in any analysis. This suggests a tradeoff between constraining a model to a tight theoretical structure and following data-intensive approaches to modeling. There is also an apparent tradeoff between capturing macroeconomic phenomena and sectoral detail although we will show below that recent developments in dynamic general equilibrium modeling have apparently resolved this tradeoff. Estimated models that are to be used for policy work should, in full model simulation, be able to explain recent major events rather than just give a good single equation fit on average over a thirty year sample. We also need to understand the intuition as to why the results come out as they do. Theoretical models should be subject to these same evaluations. If they do not explain statistical relationships in the data or the consequences of large observed shocks then they are not so useful for applied policy.

3. Structure of the G-Cubed (Agriculture) Model

There are now three major streams of the G-Cubed modelling research. The first is the original G-Cubed model which was constructed with funding from the Brookings Institution, US

Environmental Protection Agency and the US National Science Foundation to explore the global impacts of economic policies aimed at addressing the problem of global warming (McKibbin and Wilcoxen (1992). The second major strand is the G-Cubed (Asia Pacific) model outlined in McKibbin (1996) which was designed for exploring interdependence in the Asia Pacific region. This is a six sector and 19 region model which considerable country detail on the Western Asia Pacific region.

The latest G-Cubed research stream is the G-Cubed (agriculture model) which is the focus of this paper. The key features of G-Cubed (Agriculture) model are summarized in Table 1. The country and sectoral breakdown of the model are summarized in Table 2. In contrast to the G-Cubed model, G-Cubed (Agriculture) consists of twelve economic regions (the United States, Canada, Japan, Australia, European Union (12 member countries), Mexico, Korea, the rest of the OECD, Taiwan, ASEAN, China, and rest of the world (ROW). Each country/region consists of twelve sectors of production plus a sector that creates capital goods for firms and a sector that produces capital goods for households in each region. There is one energy sector, four primary agricultural sectors (food grains, feed grains, non-grain crops, and livestock products), mining, fishing and forestry products and 5 manufacturing sectors (processed food, durable manufacturing, textile & apparel, other non-durable manufacturing) and services.

Each region in the model consists of several economic agents: a representative household, a government, and one representative firm in each of the production sectors listed above. These agents interact in a range of markets: final goods, intermediate goods, factors of production, money markets, bond markets, foreign exchange markets and equity markets. The assumptions about how agents behave follow the G-Cubed model. Each agent is a combination of two types of behavior: intertemporally optimizing behavior and liquidity constrained behavior (or "rule of thumb" behavior). The relative weighting between the two types of agents is based on empirical evidence and the approach taken in the MSG2 model (McKibbin and Sachs, 1991). In the long run with no shocks, both types of behavior are the same but in the short run the rule of thumb behavior ignores changes in expected future income or profit streams which intertemporal optimizing behavior takes into account.

We now summarize the economic behavior of the major agents in the model as well as highlighting the role of financial markets. The reader should refer to Appendix A for the detailed algebraic expressions of the model as well as the derivations in McKibbin and Wilcoxen (1995).

Table 1: Summary of Main Features

- Specification of the demand and supply sides of modeled economies in both real and financial markets;
- Household behavior in the short run is a weighted average of neoclassical optimizing behavior and ad-hoc "liquidity constrained" behavior;
- The real side of the model is disaggregated to allow for production and trade of multiple goods and services within and across economies;
- Financial markets are integrated with real side of the economy. Each financial assets represents a claim over real resources: money over purchasing power, bonds over future tax revenues, equity over future dividend stream of a firm, and foreign assets over future exports of the debtor country;
- Imposition of intertemporal budget constraints so that agents and countries cannot forever borrow or lend without undertaking the required resource transfers necessary to service outstanding liabilities;
- Assets markets are linked globally through the international mobility of financial capital;
- Agents arbitrage between different asserts within countries and across countries - taking into account of fixity of physical capital stock in each sector in the short run;
- Labor market may not clear in the short run;
- Full short run and long run macroeconomic closure with macro-dynamics at an annual frequency around a long run Solow/Swan neoclassical growth model.
- Baseline of the model is solved for a full rational expectations equilibrium at an annual frequency from 1993 to 2070.

Table 2: Overview of the G-Cubed (Agriculture) Model

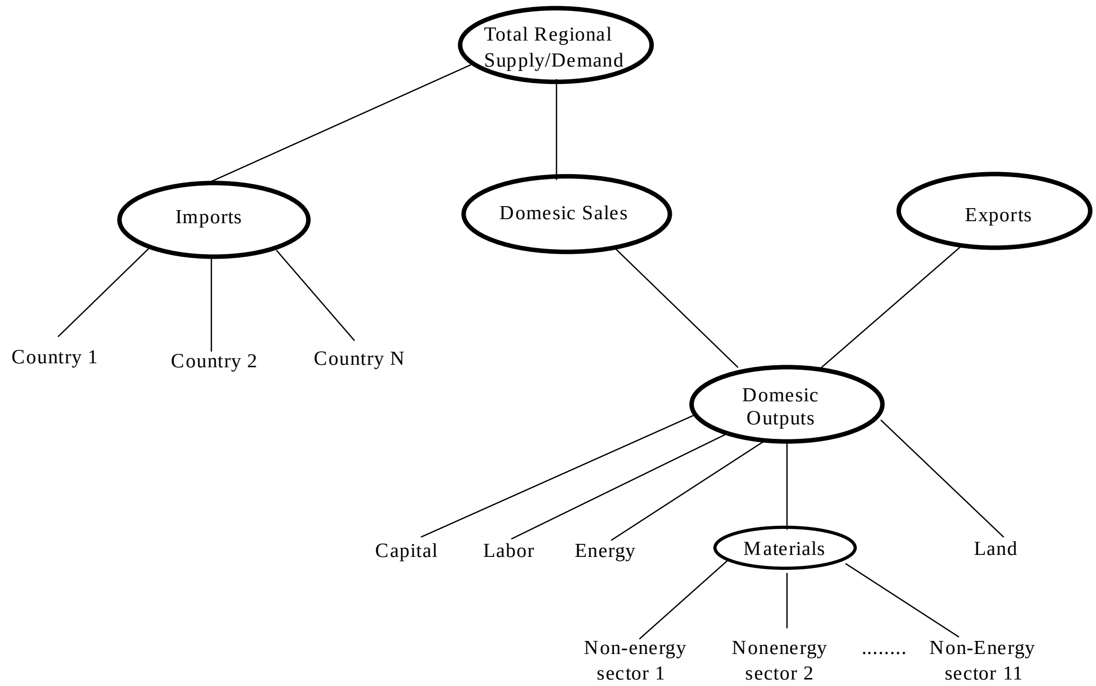
Regions		***Sectors***
United States	(U)	Energy
Canada	(C)	Mining
Japan	(J)	Forestry and Fish Products
Australia	(A)	Agriculture:
European Union	(E)	Food Grains
Mexico	(M)	Feed Grains
Rest of OECD	(O)	Non-grain Crops
Korea	(K)	Livestock Products
Taiwan	(T)	Manufacturing:
China	(H)	Processed Food
ASEAN	(N)	Durable Manufacturing
Rest of the World	(L)	Textile and Apparel
		Other Non-Durable
		Services
Agents		***Markets***
Households		Goods and Services
Firms		Factors of Production
Governments		Bond
		Equity
		Money
		Foreign Exchange

a. Firms

Each of the production sectors is represented by a single firm which chooses its variable inputs and its level of investment in order to maximize its stock market value subject to a multiple-input production function and a vector of prices it takes as exogenous. For each sector, output is produced with inputs of capital, labor, land, energy and intermediate inputs from all other sectors. Land is only used in the four agricultural sectors. Intermediate goods are, in turn, aggregates of imported and domestic commodities which are imperfect substitutes. We assume that all agents in the economy have identical preferences over foreign and domestic varieties of each particular commodity. We represent these preferences by defining twelve composite commodities that are aggregated from imported and domestic goods (the Armington assumption). For example, food grain purchased by agents in the model are a composite of imported and domestic grains. By constraining all agents in the model to have the same preferences over the origin of goods we require that, for example, the agricultural and service sectors have the identical preferences over domestic oil and oil imported from the middle east.[14] This accords with the input-output data we use and allows a very convenient nesting of production, investment and consumption decisions. The structure of production and demand in the model are showed as Figure 1.

[14] This does not require that both sectors purchase the same amount of oil, or even that they purchase oil at all; only that they both feel the same way about the origins of oil they buy.

Figure 1 Commodity/Sector Nesting in G-cubed (Agriculture) Model



In each sector the capital stock changes according to the rate of fixed capital formation less depreciation. One of the key assumptions in the model is that physical capital is costly to adjust and is sector specific at the short run. Following the cost of adjustment models of Lucas (1967), Treadway (1969) and Uzawa (1969) we assume that the investment process is subject to rising marginal costs of installation.

The goal of each firm is to choose inputs to maximize intertemporal net-of-tax profits subject to its technology and capital accumulation constraints. The solution to this intertemporal optimization problem yields the conditions that variable inputs are hired to the point where the marginal productivity of these factors equals their prices relative to the output price. Based on those conditions we write the model in terms of cost functions and derive various component demand function by Shephard's lemma. The price of the output at each level of the tier structure is also the unit cost function depending on the prices of variable inputs and the quantities of available fixed factors such as capital.

The rate of gross investment in each sector is a function of "Tobin's q" for that sector, where q is the increment to the value of the firm from an additional unit of investment. Following Hayashi (1979), we modify the investment function to improve its empirical properties by writing gross investment as a function not only of *q*, but also of the firm's current cash flow.

We assume that investment goods are supplied by a firm facing an optimization problem similar to those of the twelve industries described above. Like other industries, the investment sector demands labor and capital services as well as intermediate inputs. The investment column in the input-output table is used to parameterize the investment sector's cost function. Similar to other sectors, there is a shadow price associated with investment in the investment goods sector. Production structure of the investment good is similar to Figure 1.

b. Households

Households consume goods and services in every period and also demand labor and capital services. Household capital services consist of the service flows of consumer durables plus residential housing. Households receive income by providing labor services to firms and the government, and from holding financial assets. In addition, they may also receive transfers from their region's government.

We assume household behavior can be modeled by a representative agent in each country who maximizes an intertemporal utility function subject to the constraint that the present value of consumption be equal to the present value of after tax labor income (human wealth) plus initial financial assets. Human wealth in real terms is defined as the expected present value of a future stream of after tax on labor income. Financial wealth is the sum of real money balances, real government bonds in the hand of the public, net holdings of claims against foreign residents, and the value of capital in each sector.

The solution to this maximization problem is the familiar result that aggregate consumption is equal to a constant proportion of private wealth, where private wealth is defined as financial wealth plus human wealth.

Based on the evidence cited by Campbell and Mankiw (1987) and Hayashi (1982) we assume that only a portion of total consumption is determined by these intertemporally-optimizing consumers who calculate expected future income streams, and the remainder is determined by after tax current income. This can be interpreted as liquidity constrained behavior or a permanent income model in which household expectations regarding income are backward-looking. It implies that total consumption is a weighted average of the forward looking consumption and backward-looking consumption.

The household consumption problem can be thought as a sequence of decisions. Households first decide on total consumption for each period as described above, then the total expenditure is allocated across goods and services based on preferences and relative prices. We assume that the household's preferences can be represented by a two level nested constant elasticity of substitution utility function.[15] At the top tier of the utility function total consumption is allocated across energy, a basket of non-energy goods (i.e. materials) including agricultural products, as well as labor and capital services. Household capital services consist of the service flows of consumer durables plus residential housing. At the second tier, spending on non-energy goods is further disaggregated into demands for individual commodities. The allocation of total consumption expenditure across goods and services is assumed to be separable from the intertemporal allocation.

The supply of household capital services is determined by consumers themselves who invest in household durables and housing to generate a desired flow of services according to a production function using household capital stock accumulated by previous investment. The household is assumed to maximize utility from the flow of services of durable stock by choosing an investment stream subject to quadratic costs of adjustment. As for the firm decision on optimal investment, the result is an investment function depending the shadow price of capital.

c. Government

We take each region's real government spending on goods and services as exogenous and assume that it is allocated among final goods, capital and labor services in fixed proportions, which we set to 1992 values. Total government outlays include purchases of goods and services plus interest payments on government debt, investment tax credits and transfers to households. Government revenue comes from corporate and personal income taxes, sales taxes, and by issuing

[15] This has the undesirable effect of imposing unitary income elasticities, a restriction usually rejected by data. Moreover, in the preliminary version of the model presented here, the elasticities of substitution have been constrained to be unity. In future work we plan to replace this specification with one derived from the linear expenditure system to allow income elasticities to differ from one.

government bonds.

We assume that agents will not hold government bonds unless they expect the bonds to be paid off eventually. Therefore, the government is subject to an intertemporal budget constraint that the present value of spending is restricted by the present value of future tax collections from all sources less the initial stock of existing government debt.

The implication of such a constraint is that a government running a budget deficit today must run an appropriate budget surplus at some point of time in the future. Otherwise, the government would be unable to pay interest on the debt and agents will not be willing to hold it. To ensure that the intertemporal budget constraint holds at all points in time we assume that the government levies a lump sum tax in each period equal to the value of interest payments on the outstanding debt.[16] In effect, therefore, any increase in government debt is financed by consoles, and future taxes are raised enough to accommodate the increased interest costs. Thus, any increase in the debt will be matched by an equal present value increase in future budget surpluses. Other fiscal closure rules are possible, such as requiring the ratio of government debt to GDP to be unchanged in the long run. These closures have interesting implications but are beyond the scope of this paper.

d. Financial Markets and the Balance of Payments

A key feature of the G-Cubed class of models is the integration of financial markets with the real side of global economy. There are a variety of assets available within each region including domestic money, government bonds, equity and foreign debt. Each asset represents a

[16] In the model the tax is actually levied on the difference between interest payments on the debt and what interest payments would have been if the debt had remained at its base case level. The remainder, interest payments on the base case debt, is financed by ordinary taxes.

claim over a real activity. Money is required for transactions and therefore represents a claim over purchasing power. Government bonds are a claim over the future tax collections of governments. Equity is a claim over the future dividend stream of firms. Foreign debt is a claim over the future export receipts of the debtor countries. The prices of financial assets therefore contain information about the expected future real outcomes in the economy and are used by agents in undertaking real economic activities such as investment and consumption decisions.

The twelve regions in the model are linked by flows of goods and assets. Flows of goods are determined by the import demands of households, firms and governments. These demands can be summarized in a set of bilateral trade matrices, which give the flows of each good between exporting and importing countries. There are one 12 by 12 trade matrix for each of the twelve goods.

Trade imbalances are financed by flows of assets between regions. We assume asset markets are perfectly integrated across all regions except with the rest of the world. With free mobility of capital, expected returns on loans denominated currencies of various regions must be equalized period to period according to a set of interest arbitrage relations. While we allow for exogenous risk premium in the calibration of the model there is no allowance for endogenous risk premia on the assets of alternative currencies.

Determining initial net asset positions and hence base-case international capital flows is non-trivial. We assume that capital flows are composed of portfolio investment, direct investment and other capital flows. These alternative forms of capital flows are perfectly substitutable *ex ante*, adjusting to the expected rates of return across economies and across sectors. Within an economy, the expected return to each type of assets (i.e. bonds of all maturities, equity for each sector etc) are arbitrage, taking into account the costs of adjusting physical capital stock and allowing for exogenous risk premia. Because physical capital is costly to adjust, any inflow of financial capital that is invested in physical capital (i.e. direct investment) will also be costly to shift once it is in place. The decision to invest in physical assets is based on expected rates of return. However, if there is an unanticipated shock then *ex-post* returns could vary significantly. Total net capital flows for each economy in which there are open capital markets are equal to the current account position of that country. The sum of global net flows of capital are constrained to zero.

We treat the rest of the world differently to the regions which have full internal structures. We assume that the rest of the world chooses its foreign lending in order to maintain a desired ratio of income to wealth.

e. Labor Markets

We assume that labor is perfectly mobile among sectors within each region but is immobile between regions. Thus, within each region nominal wages will be equal across sectors. The nominal wage is assumed to adjust according to labor market institutions in different countries. In the United States, wages adjust slowly according to an overlapping contracts model where nominal wages are set based on current and expected inflation and on labor demand relative to labor supply. In the long run labor supply is given by the exogenous rate of population growth, but in the short run the hours worked can fluctuate depending on the demand for labor. For a given nominal wage, the sectoral demand for labor will determine short run employment in each industry and thus economy wide unemployment will be the difference between the overall supply and the sum of sectoral demand for labor.

4. Conclusion and Directions for Future Research

The model developed in this paper is an extension of the G-Cubed multi-country model with a particular focus on agriculture. An application of the model to the Asian economic crisis can be found in Coyle, McKibbin and Wang (1998). Future research should be focused on sensitivity tests of key parameters and careful econometric estimation of the key parameters along the lines of the estimation in the original G-Cubed model. In particular the current model has been developed using Cobb-Douglas functional forms in a range of places which should be generalized as soon as possible. In particular the assumption of unitary income elasticities is a significant problem for using the model for large shocks or for projections into the medium term future.

There also needs to be further data development and resolution of discrepancies in the alternative data sources that are used in the model.

Despite the preliminary nature of the model, results presented in Coyle et al (1998) suggest the useful role the model can perform in particular in analyzing the consequences of financial adjustment in the global and US economies.

A wide variety of possible applications of the model await analysis. Each one will undoubtedly produce new questions and structural issues that also need attention in this early version of the model.

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Appendix A: Algebraic Description of the G-Cubed (Agriculture) Model

The model constructed in this study is an extension of the G-Cubed model developed by Warwick McKibbin and Peter Wilcoxen (1995) to included relatively detailed agricultural sectors. This appendix provides a detailed description of the twelve-region, twelve-sector dynamic intertemporal general equilibrium (DIGE) model used in this study.

1. Notation:

(i) Region name:

U (the United States),

J (Japan),

A (Australia),

E (12 members of European Community),

C (Canada),

M (Mexico),

O (other OECD countries),

N (Association of South East Asia Nations, includes Indonesia, Malaysia, Thailand, Philippines, and Singapore),

T (Taiwan),

H (China including Hong Kong),

K (Korea),

L (Other developing countries).

(ii) Sector name:

FGR (food grain),

GRO (feed grains),

NGC (non-grain crop),

LIV (livestock products),

FOD (processed food products),

FAF (forest and fishery products),

MNE(mining),

ENG(energy products),

TXT (textile and wearing apparel),

LMF (other light manufacture),

DUR (durable manufactures),

SEV (services).

Y (firms capital production sector),

Z (household capital production sector).

(iii) Input name:

LB (labor),

KA (capital),

EE (energy),

MT (materials),

LA (land).

(iv) Subscripts and Set Definition:

• Regions are defined in set W and indexed by r or s.

r, s ∈ W = {U, J, A, E, C, M, O, N, T, H, K, L).

R ∈ W is a subset of W excluding L.

S ∈ W is a subset of W excluding U.

• Sectors are defined in set I and indexed by i or j.

i, j ∈ I = {FGR, GRO, NGC, LIV, FOD, FAF, MNE, ENG, TXT, LMF, DUR, SEV, Y, Z).

P ∈ I is a subset of I including the twelve production sectors.

• P is divided into two subsets, PE for energy products and PN for non-energy products.

P = {PE, PN) and PE = {ENG}.

• Agricultural sectors are defined as a subset of P: PA = {FGR,GRO, NGC, LIV} ∈ P;

• Aggregate inputs are defined in set F and indexed by f. f ∈ F = {L, K, E, M, R};

(v) Conventions:

Uppercase English letter indicate variables. A variable with a bar on top is set exogenously. Greek letter or lower English letter refer to parameters, which need to be calibrated or estimated . When multiple subscripts of a variable or parameter come from the same set, the first one represents the region or sector supplying goods; the next one represents the region or sector purchasing goods.

2 Definition of Variables

Variable	Definition	No. of Variables
State variables		242
CAP_{irt}	Sector i capital stock in region r at period t	I × R (14 × 11)
$BOND_{rt}$	Government debt in region r at period t	R (11)
$WAGE_{rt}$	Average nominal wage in region r at period t	R (11)
$ASSE_{rst}$	Assets of region s hold by region r at period t	½×W ×(W-1)
Co-State (jumping) variables		175
$REXC_{rt}$	Real exchange rate in region r at period t (with U.S. dollars)	R-1(10)
TOB_{irt}	Shadow price of sector i capital in region r at period t	I × R (14 × 11)
$WELH_{rt}$	Human wealth in region r at period t	R (11)
Other Expectation variables		23
${}_{t}PRCT_{rt}$	Consumer price index in region r at period t	R (11)
${}_{t}PRID_{rt\ t}$	Aggregate domestic user price of output in region r at period t	W (12)
Inter-period endogenous variables		7239
$GDPN_{rt}$	Nominal GDP at market price of region r at period t	R (11)
$GDPR_{rt}$	Real GDP of region r at period t	R (11)
$GNPR_{rt}$	Real GNP of region r at period t	R (11)
$INCM_{rt}$	Household income in region r at period t	R (11)
$INAS_{rt}$	Nominal interest payment received in region r at period t	R (11)
$IRAS_{rt}$	Real interest payment received in region r at period t	W (12)
$CONP_{rt}$	Total private consumption in region r at period t	R (11)
$CONK_{rt}$	Consumption of capital services in region r at period t	R (11)
$CONL_{rt}$	Consumption of labor services in region r at period t	R (11)
$CONE_{rt}$	Total consumption of energy in region r at period t	R (11)
$CONO_{rt}$	Total consumption of non-energy goods in region r at period t	R (11)
$CONG_{rt}$	Total government consumption in region r at period t	R (11)
CON_{irt}	Private consumption of sector i goods in region r at period t	P × R (12 × 11)
$OUTP_{rt}$	Gross output in region r at period t	R (11)
OUY_{irt}	Total supply of sector i composite goods in region r at period t	P × R (12 × 11)
OUP_{irt}	Sector i domestic output in region r at period t	P × R (12 × 11)

Variable	Definition	No. of Variables
ENE_{irt}	Sector i demand for energy in region r at period t	P × R (12 × 11)
EN_{ijrt}	Sector i demand for energy type e in region r at period t	PE × P× R (1× 12 ×11)
OIN_{ijrt}	Sector i demand for non-energy intermediate in region r at period t	P × R (12 ×11)
OI_{ijrt}	Sector i demand for non-energy input from sector j in region r	PN×P× R (11 ×12 ×11)
EXP_{irt}	Total exports of sector i goods from region r to the world at period t	P × W (12 ×12)
IMP_{irt}	Total imports of sector i goods from the world in region r at period t	P × R (12 ×11)
TFL_{isrt}	Bilateral trade of sector i goods from region s to region r at period t	P× (W-1)×W
$EXPT_{rt}$	Total exports from region r at period t	W (12)
$IMPT_{rt}$	Total imports in region r at period t	W (12)
$EXPO_{Lst}$	Total exports from region L to region r at period t	R (11)
$IMPO_{rLt}$	Total imports in region L from region r at period t	R (11)
LAB_{irt}	Sector i demand for labor in region r at period t	P × R (12 ×11)
$LABT_{rt}$	Total employment in region r at period t	R (11)
RES_{irt}	Special resource demand in sector i at period t	PA × R (4 ×11)
$INVT_{rt}$	Total gross investment in region r at period t	R (11)
INV_{irt}	Sector i gross investment in region r at period t	I × R (14 × 11)
JNV_{irr}	Sector i gross fixed capital formation in region r at period t	I × R (14 × 11)
IIN_{irt}	Investment demand of sector i goods in region r at period t	P × R (12 × 11)
$IINL_{rt}$	Demand for labor in investment sector in region r at period t	R (11)
$IINE_{rt}$	Demand for energy in investment sector in region r at period t	R (11)
$IINO_{rt}$	Demand for materials in investment sector in region r at period t	R (11)
PRF_{irt}	Profit in sector i in region r at period t	P × R (12 × 11)
$TOBN_{irt}$	Tobin’s q in sector i of region r at period t	I × R (12 × 11)
STM_{irt}	Share market value of sector i capital stock in region r at period t	P × R (12 × 11)
PRM_{irt}	Import price of goods i in region r at period t	P × R (12 × 11)
PRX_{irt}	Export price of goods i in region r at period t	P × W (12 × 12)
PRY_{irt}	Composite goods price of sector i in region r at period t	P × R (12 × 11)
PRD_{irt}	Domestic user price of output by sector i in region r at period t	P × R (12 × 11)
PRP_{irt}	Producer price of sector i goods in region r at period t	P × R (12 × 11)
PRE_{irt}	Price of energy in sector i in region r at period t	P × R (12 × 11)

Variable	Definition	No. of Variables
POI_{irt}	Price of non-energy material inputs in sector i in region r at period t (log)	P × R (12 × 11)
PRK_{irt}	Price of capital in sector i at region r in period t (log)	I × R (14× 11)
PRR_{irt}	Price of sector specific resource in sector i in region r at period t (log)	PA × R (4 × 11)
$PRIM_{rt}$	Aggregate import price of in region r at period t (log)	R (11)
$PRIX_{rt}$	Aggregate export price from region r at period t (log)	R (11)
$PREI_{rt}$	Price of energy in investment sector at region r in period t (log)	R (11)
POI_{rt}	Price of non-energy material in investment sector at region r in t (log)	R (11)
$PRII_{rt}$	Price of investment goods in region r at period t (log)	R (11)
$PRCE_{rt}$	Consumer price of energy in region r at period t (log)	R (11)
$PRCO_{rt}$	Consumer price of other materials in region r at period t (log)	R (11)
WAG_{irt}	Nominal wage rate in sector i in region r at period t (log)	P × R (12 × 11)
$EXCH_{rt}$	Nominal exchange rate with U.S. dollars in region r at period t (log)	W (12)
$NEER_{rt}$	Nominal effective exchange rate in region r at period t (log)	R (11)
$REER_{rt}$	Real effective exchange rate in region r at period t (log)	R (11)
$PGDP_{rt}$	GDP deflator in region r at period t (log)	R (11)
$WELT_{rt}$	Total wealth in region r at period t	R (11)
$ASST_{rt}$	Total net external asserts holding in region r at period t	W (12)
$MONE_{rt}$	Money balance in region r at period t (log)	R (11)
$INTR_{rt}$	Real interest rate in region r at period t	W (12)
$INTN_{rt}$	Nominal interest rate in region r at period t	R (11)
$INFL_{rt}$	Rate of inflation in region r at period t	R (11)
$TAXL_{rt}$	Lump sum tax in region r at period t	R (11)
$TAXT_{rt}$	Total government tax revenue in region r at period t	R (11)
$TAXC_{rt}$	Total cooperate tax revenue in region r at period t	R (11)
$TAXH_{rt}$	Total household income tax in region r at period t	R (11)
$TAXE_{rt}$	Total export tax in region r at period t	R (11)
$TAXM_{rt}$	Total tariff revenue in region r at period t	R (11)
TXM_{irt}	Tariff revenue from sector i goods in region r at period t	P × R (12 × 11)
$TRAN_{rt}$	Government lump sum transfer to household in region r at period t	R (11)

Variable	Definition	No. of Variables
$CURN_{rt}$	Nominal current account balance in region r at period t	R (11)
$CURR_{wt}$	Real current account balance in region r at period t	W (12)
$TBNA_{rt}$	Nominal balance of trade in region r at period t	R (11)
$TBAL_{rt}$	Real balance of trade in region w at period t	W (12)
$TBAR_{rt}$	Balance of trade in region r at period t measured in US dollars	R (11)
$SAVN_{rt}$	Nominal private saving in region r at period t	R (11)
$SAVR_{rt}$	Real private saving in region r at period t	R (11)
$DEFN_{rt}$	Nominal government budget deficit at period t	R (11)
$DEFI_{rt}$	Real government budget deficit at period t	R (11)
$EXNA_{rt}$	Total nominal exports from region r	R (11)
$IMNA_{rt}$	Total nominal imports in region r	R (11)
Exogenous variables		947
$MPOL_{rt}$	Money supply in region r at period t	R (11)
$TAXI_{rt}$	Lump sum taxes in region r at period t	R (11)
GCE_{irt}	Government spending of sector i goods at region r at period t	P × R (12 × 11)
$GCEL_{rt}$	Government consumption for labor in region r at period t	R (11)
TIM_{irt}	Tariff on sector i imports in region r at period t	P × R (12 × 11)
INS_{irt}	Changes in stock of sector i goods in region r at period t	P × R (12 × 11)
TAX_{irt}	Production tax on sector i goods in region r at period t	P × R (12 × 11)
$CURR_{l,t}$	Real current account balance in rest of the world at period t	1
$TITC_{rt}$	Investment tax credits in of period t	R (11)
TEX_{rt}	Tax (subsidies) rate on exported goods in region r at period t	R (11)
$TCOR_{rt}$	Corporate tax rate in region r at period t	R (11)
$TINC_{rt}$	Household income tax rate in region r at period t	R (11)
$TRAX_{rt}$	Government lump sum transfer to household in region r at period t	R (11)
$REST_{rt}$	Total supply of sector specific resources in region r at period t	R (11)
$SHKM_{rt}$	Money demand shock in region r at period t	R (11)
$SHKC_{rt}$	Private consumption shock in region r at period t	R (11)
$SHKI_{rt}$	Private investment shock in region r at period t	R (11)
$SHEF_{rt}$	Energy efficiency shock in region r at period t	R (11)

Variable	Definition	No. of Variables
SHY_{irt}	Supply shock in region r at period t	P × R (12 × 11)
SHL_{irt}	Supply shock in region r at period t	P × R (12 × 11)
	Total number of variables in each period t: 68×R +9×W +23×I×R+(3+R)×P×W +6×I×R + (2×PA+P×P)× R + ½×W ×(W-1) (9076)	

3 Definition of Parameters

Parameter	Definition
gr1	Rate of labor force growth
gr2	Rate of productivity growth
ρ_r (p1)	Rate of time preference plus real growth rate in region r
ν_r (p2)	Risk premium used in discounting future income streams in region r
φ_r (p8)	Ratio of money to final output in region r
η_r (p10)	Interest elasticity of the demand for money in region r
δp_r (p16)	Rate of depreciation of private capital stock in region r
θ_r (p17)	Parameter for the cost of adjustment of private sector capital stock in region r
π_r (p25)	Weight on expected inflation in wage setting in region r
α_r (p26)	Effect of labor market conditions on wage setting in region r
τ_r (p27)	Elasticity of government net transfers with respset to income in region r
ω_{sr} (p40 to p50)	Weights used in calculating effective exchange rates (column sum to 1)
Various Share parameters	
κ_r (p4)	Share of consumption determined by wealth in region r
μ_r (p5)	Propensity to consume out of current income in region r
$ß_r$ (p18)	Share of private investment determined by forward looking firms in region r
sa_{sr} (a1-an)	Shares of region r's assets hold by region r (column sum to 1)
sp_{ir} (s1)	Share of domestic production in total supply of sector i goods in region r
so_{ir} (s2)	Share of sector i in total region r's export tax (subsidy)
se_{ir} (s3)	Share of energy in domestic production of sector i good
sl_{ir} (s4)	Share of labor in domestic production of sector i good
sn_{ir} (s5)	Share of non-energy goods in domestic production of sector i good

sk_{ir} (s6)	Share of capital in domestic production of sector i good
sr_{ir} (s7)	Share of land in domestic production of sector i good
st_{ir} (s8)	Share of sector i good in total output
sik_{ir} (s9)	Initial investment rate (I/K) of sector i in region r
sie_{ijr}	Share of energy inputs in energy (E) used for investment in region r
sim_{ijr}	Share of non-energy inputs in materials (M) used for investment in region r
sce_{ijr}	Share of energy inputs in energy (E) used for consumption in region r
scm_{ijr}	Share of non-energy inputs in materials (M) used for consumption in region r
q_{ijr} (q)	Share of energy good j in the use of energy in sector i in region r
o_{ijr} (o)	Share of non-energy good j in the use of non-energy goods in sector i in region r
sx_{ir}	Share of sector i goods in the exports of country r
sm_{ir}	Share of sector i goods in the imports of country r
ss_{isr}	Share of sector i goods imported from region c in region r
Substitution elasticities	
σm_{ir} (z1)	Elasticity of substitution between domestic and import goods in sector i in region r
σs_{ir} (z2)	Elasticity of substitution between imported goods from different regions in region r
σa_{ir} (z3,z6,z9)	Elasticity of substitution in sector i between aggregate inputs in region r
σe_{ir} (z4,z7,z10)	elasticity of substitution in sector i between energy inputs in region r
σn_{ir} (z5,z8,z11)	elasticity of substitution in sector i between non-energy inputs in region r
σg_{ir} (z12)	Price elasticity of supply of sector i goods in region r
σr_{ir} (z13)	Price elasticity of land supply in sector i in region r

3. Building Block of the Model

Production, consumption, and import demand in the GCUBED-AG model are characterized by multilevel nesting. Since the model is specified in its dual form rather than in the more familiar primal format, the algebraic expression is relatively simple and transparent in its structure. The building blocks of the model are pairs of composite goods (factor) prices and the corresponding component conditional demand functions. The composite goods price is also a unit cost function that is dual to an aggregator function (Diewert, 1982). It is assumed to be positive, twice differentiable, linear homogenous, and quasiconcave. In GCUBED-AG model, it has following form (CES):

$$Q = A \times \left(\sum_{1}^{n} \delta_i^{\frac{1}{\sigma}} \times x_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{A.1}$$

where Q is the composite outlay; A is the unit parameter; x_i is the ith component in the aggregator function; δ_i is the share parameter associated with x_i , i = 1, 2, ..., n; and σ is the elasticity of substitution. When $Q = i$, the unit cost function that is dual to it has the following form:

$$C(P,1) \equiv c(P) = \frac{1}{A} \left(\sum_{1}^{n} \times \delta_i \times p_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{A.2}$$

where $C(P,Q)$ is the cost function, $c(P)$ is the unit cost function, P is the n-dimensional component price vector, and p_i is the price for component i. Taking the logarithm of both sides:

$$ln[c(P)] = -ln(A) + \frac{1}{1-\sigma} \times ln\left(\sum_{1}^{n} \delta_i \times p_i^{1-\sigma} \right) \tag{A.3}$$

By applying Shephard's lemma to the cost function $C(P,Q)$, the conditional demand function of component i in (A.1) can be derived as follows:

$$x_i(P,Q) = A^{\sigma-1} \times \delta_i \times Q \times \left(\frac{p_i}{c(P)} \right)^{-\sigma} \tag{A.4}$$

i.e. it is a function of the share parameter and the ratio of the corresponding component price to the aggregate price (unit cost) multiplied by total aggregate quantity.

There are two level nests based on cost minimization in the production and consumption tree of the model. In each nest, there are an aggregate price or unit cost function and its corresponding component demand functions similar to equations A.3 and A.4. The only differences in specification between nests are the variable names, parameter values and different types of external shocks.

4. Equation list

Production Nests (for r ∈ R, i ∈ P)

First level

$$\begin{aligned} PRP_{irt} = -S\bar{H}Y_{irt} + \left(\frac{1}{1-\sigma a_{ir}} \right) \times ln\Big\{ & sl_{ir} \times \left[exp^{(WAG_{irt} - S\bar{H}L_{irt})} \right]^{(1-\sigma a_{ir})} + se_{ir} \times \left[exp^{(PRE_{irt} - SH\bar{F}E_{rt})} \right]^{(1-\sigma a_{ir})} \\ & + sn_{ir} \times \left[exp^{POI_{irt}} \right]^{(1-\sigma a_{ir})} + sk_{ir} \times \left[exp^{PRK_{irt}} \right]^{(1-\sigma a_{ir})} + sr_{ir} \times \left[exp^{PRR_{irt}} \right]^{(1-\sigma a_{ir})} \Big\} \end{aligned} \tag{1}$$

$$LAB_{irt} = \left[exp^{(S\bar{H}L_{irt} + S\bar{H}Y_{irt})} \right]^{(\sigma a_{ir} - 1)} \times sl_{ir} \times OUP_{irt} \times \left[exp^{(WAG_{irt} - PRP_{irt})} \right]^{-\sigma a_{ir}} \tag{2}$$

$$ENE_{irt} = \left[exp^{(S\bar{H}Y_{irt} + SH\bar{E}F_{rt})} \right]^{(\sigma a_{ir} - 1)} \times se_{ir} \times OUP_{irt} \times \left[exp^{(PRE_{irt} - PRP_{irt})} \right]^{-\sigma a_{ir}} \tag{3}$$

$$OIN_{irt} = \left[exp^{\bar{SHY}_{irt}} \right]^{(\sigma a_{ir} - 1)} \times sn_{ir} \times OUP_{irt} \times \left[exp^{(POI_{irt} - PRP_{irt})} \right]^{-\sigma a_{ir}} \quad (4)$$

$$RES_{irt} = \left[exp^{\bar{SHY}_{irt}} \right]^{(\sigma a_{ir} - 1)} \times sr_{ir} \times OUP_{irt} \times \left[exp^{(PRR_{irt} - PRP_{irt})} \right]^{-\sigma a_{ir}} \qquad for\ i \in PA \quad (5)$$

$$\begin{aligned} PRF_{irt} = {} & OUP_{irt} \times exp^{(PRP_{irt} - {}_tPRDI_{rt})} - ENE_{irt} \times exp^{(PRE_{irt} - {}_tPRDI_{rt})} - LAB_{irt} \times exp^{(WAG_{irt} - {}_tPRDI_{rt})} \\ & - OIN_{irt} \times exp^{(POI_{irt} - {}_tPRDI_{rt})} - RES_{irt} \times exp^{(PRR_{irt} - {}_tPRDI_{rt})} \end{aligned} \quad (6)$$

Sectoral output OUP_i is produced by five inputs (components), four of them are variable inputs with capital as fixed input at the short run. The price for OUP_i , PRP_i, is defined by equation 1, which is the log form unit cost function similar to equation A.3, while the demands for the four variable inputs: energy, material, labor and sector specific resource are specified as component demand functions similar to equation A.4 in equations 2-5. Since all prices in the model are specified in their log form, by using the property of $x = e^{lnx}$, they are all defined as natural power when the model was implemented in computer code to take advantages of convergence properties of such functions. Capital earnings, PRF_i ,are defined in equation 6 as profit of the firm, which equals total sale revenue minus variable costs.

Second level

$$PRE_{irt} = \left(\frac{1}{1 - \sigma e_{ir}} \right) \times ln \left\{ \sum_{j \in PE} q_{ijr} \times \left[exp^{PRY_{jrt}} \right]^{(1 - \sigma e_{ir})} \right\} \quad (7)$$

$$POI_{irt} = \left(\frac{1}{1 - \sigma n_{ir}} \right) \times ln \left\{ \sum_{j \in PN} o_{ijr} \times \left[exp^{PRY_{jrt}} \right]^{(1 - \sigma n_{ir})} \right\} \quad (8)$$

$$EN_{ijrt} = q_{ijr} \times ENE_{irt} \times \left[exp^{(PRY_{jrt} - PRE_{irt})} \right]^{-\sigma e_{ir}} \quad for\ j \in PE \tag{9}$$

$$OI_{ijrt} = o_{ijr} \times OIN_{irt} \times \left[exp^{(PRY_{jrt} - POI_{irt})} \right]^{-\sigma n_{ir}} \quad for\ j \in PN \tag{10}$$

The energy input, ENE_i, is aggregated over each type of energy. Its price, PRE_i, is specified as the unit cost function (equation 7) and the demands for each type of energy, EN_{ij}, are specified as corresponding component demand functions (equation 9).

The material intermediate demand, OIN_i, is split into each non-energy commodity, OI_{ij}, in a similar way as aggregate energy input. Its price, POI_i, is the unit cost (equation 8) and demand for each sector goods are specified as component demand functions (equation 10).

Investment Goods Production Nests (for r ∈ R, i = Y)

Fist level

$$\begin{aligned} PRII_{rt} = & \left(\frac{1}{1 - \sigma a_{ir}} \right) \times Ln \Big\{ sl_{ir} \times \left[exp^{(WAGE_{rt} - \bar{SHL}_{irt})} \right]^{(1 - \sigma a_{ir})} + se_{ir} \times \left[exp^{(PREI_{rt} - \bar{SHEF}_{rt})} \right]^{(1 - \sigma a_{ir})} \\ & + sn_{ir} \times \left[exp^{(PROI_{rt})} \right]^{(1 - \sigma a_{ir})} + sk_{ir} \times \left[exp^{PRK_{irt}} \right]^{(1 - \sigma a_{ir})} \Big\} \quad for \quad i = Y \end{aligned} \tag{11}$$

$$IINL_{rt} = \left[exp^{(\bar{SHL}_{irt} + \bar{SHY}_{irt})} \right]^{(\sigma a_{ir} - 1)} \times sl_{ir} \times INVT_{rt} \times \left[exp^{(WAGE_{rt} - PRII_{rt})} \right]^{-\sigma a_{ir}} \quad for\ i = Y \tag{12}$$

$$IINE_{rt} = \left[exp^{(\bar{SHEF}_{irt} + \bar{SHY}_{irt})} \right]^{(\sigma a_{ir} - 1)} \times se_{ir} \times INVT_{rt} \times \left[exp^{(PREI_{rt} - PRII_{rt})} \right]^{-\sigma a_{ir}} \quad for\ i = Y \tag{13}$$

Second level

$$PREI_{rt} = \left(\frac{1}{1-\sigma e_{ir}}\right) \times ln\left\{ \sum_{j \in PE} se_{jir} \times \left[exp^{PRY_{jrt}} \right]^{(1-\sigma e_{ir})} \right\} \qquad for\ i=Y \tag{14}$$

$$PROI_{rt} = \left(\frac{1}{1-\sigma n_{ir}}\right) \times ln\left\{ \sum_{j \in PN} sn_{jir} \times \left[exp^{PRY_{jrt}} \right]^{(1-\sigma n_{ir})} \right\} \qquad for\ i=Y \tag{15}$$

$$IIN_{irt} = \begin{cases} sie_{ijr} \times IINE_{irt} \times \left[exp^{(PRY_{jrt} - PREI_{rt})} \right]^{-\sigma e_{ir}} & for\ i,j \in PE \\ sim_{ijr} \times IINO_{irt} \times \left[exp^{(PRY_{jrt} - PROI_{rt})} \right]^{-\sigma n_{ir}} & for\ i,j \in PN \end{cases} \tag{16}$$

The investment (capital) goods are produced by four inputs and have a similar structure as other sectoral outputs. At the first level, its price, $PRII_i$, is defined as the unit cost function (equation 10) while the three variable input demands (energy, materials and labor) are specified as component demand functions (equations 11 - 13). At the second level, the aggregate energy and material inputs are further allocated into their components based on cost minimization. Equations 14 and 15 define the prices while equation 16 gives the corresponding component demand functions.

Consumption Nests (for r ∈ R, i = Z)

First level

$$PRCT_{rt} = \left(\frac{1}{1-\sigma a_{ir}}\right) \times ln\left\{ sl_{ir} \times \left[exp^{(WAGE_{rt} - \bar{SHL}_{irt})} + se_{ir} \times \left[exp^{(PRCE_{rt} - \bar{SHEF}_{irt})} \right]^{(1-\sigma a_{ir})} \right]^{(1-\sigma a_{ir})} \right.$$
$$\left. + sn_{ir} \times \left[exp^{PRCO_{rt}} \right]^{(1-\sigma a_{ir})} + sk_{ir} \times \left[exp^{PRK_{irt}} \right]^{(1-\sigma a_{ir})} \right\} \qquad for\ i=Z \tag{17}$$

$$CONL_{rt} = \left[exp^{(\bar{SHL}_{irt} + \bar{SHY}_{irt})} \right]^{(\sigma a_{ir} - 1)} \times sl_{ir} \times CONP_{rt} \times \left[exp^{(WAGE - PRCT_{rt})} \right]^{-\sigma a_{ir}} \quad for\ i = Z \tag{18}$$

$$CONE_{rt} = \left[exp^{(\bar{SHEF}_{rt} + \bar{SHY}_{irt})} \right]^{(\sigma a_{ir} - 1)} \times se_{ir} \times CONP_{rt} \times \left[exp^{(PRCE_{rt} - PRCT_{rt})} \right]^{-\sigma a_{ir}} \quad for\ i = Z \tag{19}$$

$$CONO_{rt} = \left[exp^{\bar{SHY}_{irt}} \right]^{(\sigma a_{ir} - 1)} \times sn_{ir} \times CONP_{rt} \times \left[exp^{(PRCO_{rt} - PRCT_{rt})} \right]^{-\sigma a_{ir}} \quad for\ i = Z \tag{20}$$

$$CONK_{rt} = 0.05 \times CAP_{irt} \quad for\ i = Z \tag{21}$$

Second level

$$PRCE_{rt} = \left(\frac{1}{1 - \sigma e_{ir}} \right) \times ln \left\{ \sum_{j \in PE} se_{jir} \times \left[exp^{PRY_{jrt}} \right]^{(1 - \sigma e_{ir})} \right\} \quad for\ i = Z \tag{22}$$

$$PRCO_{rt} = \left(\frac{1}{1 - \sigma n_{ir}} \right) \times ln \left\{ \sum_{j \in PN} sn_{jir} \times \left[exp^{PRY_{jrt}} \right]^{(1 - \sigma n_{ir})} \right\} \quad for\ i = Z \tag{23}$$

$$CON_{irt} = \begin{cases} sce_{ijr} \times CONO_{irt} \times \left[exp^{(PRY_{jrt} - PRCE_{rt})} \right]^{-\sigma e_{ir}} & for\ i,j \in PE \\ cim_{ijr} \times CONO_{irt} \times \left[exp^{(PRY_{jrt} - PRCO_{rt})} \right]^{-\sigma n_{ir}} & for\ i,j \in PN \end{cases} \tag{24}$$

The consumer's aggregate consumption bundle is aggregated over four type of commodities: energy, material, labor and capital services. The aggregate consumer price defined in equation 17 is the unit cost function of such bundle, while consumer demand for each type goods are specified as component demand functions (equation 18-20). The consumption of capital services is flow of services from consumer durables and residential housing, which is fixed at 0.05 in the model (equation 21). The unit cost of aggregate energy and material consumption is defined in equations 22 and 23, while consumption on each type of energy and non-energy commodities are specified as component demand functions (equation 24).

Import Nests (two level Arminton function for r ∈ R, i ∈ P)

Fist level

$$PRY_{irt} = ln(1 + \bar{TAX}_{irt}) + \left(\frac{1}{1 - \sigma m_{ir}}\right) \times ln\left\{ sp_{ir} \times \left[exp^{PRD_{irt}} \right]^{(1 - \sigma m_{ir})} + (1 - sp_{ir}) \times \left[exp^{PRM_{irt}} \right]^{(1 - \sigma m_{ir})} \right\} \quad (25)$$

$$OUP_{irt} = sp_{ir} \times OUY_{irt} \times \left[exp^{(PRD_{irt} - PRY_{irt})} \right]^{-\sigma m_{ir}} \quad (26)$$

$$IMP_{irt} = (1 - sp_{ir}) \times OUY_{irt} \times \left[exp^{(PRM_{irt} - PRY_{irt})} \right]^{-\sigma m_{ir}} \quad (27)$$

Second level

$$PRM_{irt} = \left(\frac{1}{1 - \sigma s_{ir}}\right) \times ln\left\{ \sum_{s \in W} ss_{isr} \times \left[exp^{(EXCH_{st} - EXCH_{rt} + PRX_{ist}) + ln(1 + \bar{TIM}_{irt})} \right]^{(1 - \sigma s_{ir})} \right\} \quad (28)$$

$$TFL_{isrt} = ss_{isr} \times IMP_{irt} \times \left\{ (1 + \bar{TIM}_{irt}) \times exp^{(EXCH_{st} - EXCH_{rt} + PRX_{ist} - PRM_{irt})} \right\}^{-\sigma s_{isr}} \quad (29)$$

Total supply of each of the 12 composite commodities, OUY_i , is an aggregation of domestically produced goods OUP_i , and goods imported from rest of the world, IMP_i, according to cost minimization. Its price, PRY_i, is specified as the unit cost function (equation 25), while demands for OUP_i and IMP_i are specified as component demand functions similar to equations A.3 and A.4, respectively (equations 26 and 27). At the second level, the total regional imports by sectors are aggregate over different sources. Therefore, the price of imports in each region are unit cost function (equation 28), which aggregate other region's export prices plus import tariff, while the imports by sources (bilateral trade flows) are specified as component demand functions (equation 29).

Capital and Investment (for r ∈ R)

$$CAP_{ir\ t+1} = JNV_{irt} + (1 - \delta p_r - gr1 - gr2) \times CAP_{irt} \qquad for \quad i \in I$$
$$CAP_{ir0} \quad given \tag{30}$$

$$TOBN_{irt} = TOB_{irt} \times \frac{exp^{(tPRID_{rt} - PRII_{rt})}}{(1 - TI\bar{T}C_{irt})} \qquad for \quad i \in I \tag{31}$$

$$INV_{irt} = \beta_r \left(1 + \frac{TOBN_{irt} - 1}{2} \right) \times (TOBN_{irt} - 1) \times \left(\frac{CAP_{irt}}{\theta} \right) + (1 - \beta_r) \times \Big[TI\bar{T}C_{rt} \times INV_{irt}$$
$$+ (1 - TC\bar{O}R_{rt}) \times PRO_{ir} \times exp^{(PRID_{rt} - PRII_{rt})} \Big] + SH\bar{K}I_{rt} \qquad for \quad i \in P \wedge i = Y \tag{32}$$

$$INV_{irt} = JNV_{irt} \times \left(1 + \frac{\theta_r}{2} \times \frac{JNV_{irt}}{CAP_{irt}} \right) \qquad for \quad i = Z \tag{33}$$

$$JNV_{irt} = \left(\frac{1}{1 + \theta_{ir} \times sik_{ir}} \right) \times (INV_{irt} + \frac{\theta_{ir}}{2} \times sik_{ir}^{\ 2} \times CAP_{irt}) \qquad for \ i \in P \tag{34}$$

$$JNV_{irt} = CAP_{irt} \times \left(\frac{TOBN_{irt} - 1}{\theta_r} \right) \qquad for \quad i \in y,z \tag{35}$$

$$TOB_{ir\ t+1} = (1.0 + INTR_{rt} + \delta p_r) \times TOB_{irt} - (1 - \overline{TCOR}_{rt}) \times exp^{(PRP_{irt} - {}_tPRID_{rt})} \quad for\ i \in P$$
$$\times \left[sk_{ir} \times \left(\frac{OUP_{irt}}{CAP_{irt}} \right) \times \left(exp^{\overline{SHY}_{irt}} \right)^{(\sigma a_{ir} - 1)} \right]^{(\frac{1}{\sigma a_{ir}})} - (1 - \overline{TITC}_{irt}) \times exp^{(PRII_{rt} - {}_tPRID_{rt})} \times \frac{\theta_r}{2} \times \left(\frac{JNV_{irt}}{CAP_{irt}} \right)^2 \tag{36}$$

$$TOB_{ir\ t+1} = (1.0 + INTR_{rt} + \delta p_r) \times TOB_{irt} - (1 - \overline{TCOR}_{rt}) \times exp^{(PRII_{rt} - {}_tPRID_{rt})} \quad for\ i = Y$$
$$\times \left[sk_{ir} \times \left(\frac{INVT_{rt}}{CAP_{irt}} \right) \times \left(exp^{\overline{SHY}_{irt}} \right)^{(\sigma a_{ir} - 1)} \right]^{(\frac{1}{\sigma a_{ir}})} - (1 - \overline{TITC}_{irt}) \times exp^{(PRII_{rt} - {}_tPRID_{rt})} \times \frac{\theta_r}{2} \times \left(\frac{JNV_{irt}}{CAP_{irt}} \right)^2 \tag{37}$$

$$TOB_{ir\ t+1} = (1 + INTR_{rt} + \delta p_r) \times TOB_{irt} - 0.05 \times exp^{(PRK_{irt} - {}_tPRID_{rt})}$$
$$- (1 - \overline{TITC}_{irt}) \times exp^{(PRII_{rt} - {}_tPRID_{rt})} \times \frac{\theta_r}{2} \times \left(\frac{JNV_{irt}}{CAP_{irt}} \right)^2 \quad for\ i = Z \tag{38}$$

$$PRK_{irt} = PRP_{rt} + \left(\frac{\sigma a_{ir} - 1}{\sigma a_{ir}} \right) \times \overline{SHY}_{irt} + \frac{1}{\sigma a_{ir}} \times ln\left(sk_{ir} \times \frac{OUP_{irt}}{CAP_{irt}} \right) \quad for\ i \in P \tag{39}$$

$$PRK_{irt} = PRII_{rt} + \left(\frac{\sigma a_{ir} - 1}{\sigma a_{ir}} \right) \times \overline{SHY}_{irt} + \frac{1}{\sigma a_{ir}} \times ln\left(sk_{ir} \times \frac{INVT_{rt}}{CAP_{irt}} \right) \quad for\ i = Y \tag{40}$$

$$PRK_{irt} = PRCT_{rt} + \left(\frac{\sigma a_{ir} - 1}{\sigma a_{ir}} \right) \times \overline{SHY}_{irt} t + \frac{1}{\sigma a_{ir}} \times ln\left(sk_{ir} \times \frac{CONP_{rt}}{CONK_{rt}} \right) \quad for\ i = Z \tag{41}$$

Equation 30 describes the motion of capital stocks in the economy. It states that capital stock in each of the 12 production sectors as well as in both the investment and consumption sector at period t equals last period's capital stock plus fixed capital formation , *JNV,* minus depreciation.

Because all real variables in the model are normalized by effective labor units of the economy, the changes of capital are adjusted by the growth of labor force plus productivity growth. Equation 31 defines Tobin's q (marginal). It is the ratio of shadow price of capital (unit value of new investment) and the replacement cost of capital (the price of capital goods, *PRII*) adjusted by investment credits. By Hayashi's identity (Hayashi, 1982), the marginal Tobins'q is also equal to the average q , the ratio of the value of the firm to its capital stock.

Equation 32 determines investment level in each of the 12 production sectors and investment goods production. It states that firms' investment at time t are determined by Tobin's q, investment credits, the firm's tax adjusted current cash income flow, and subject to a rising marginal cost of installation, i.e. the cost of investment increase as the ratio of investment and current capital stock increase. When firms earn more cash returns and have more retained earnings, their investment will increase. Equation 33 determines investment levels in household consumption for durable goods and residential housing. It is assumed as functions of Tobin's q only because there are no variable factors are used in producing capital service from durable goods. Equations 34 and 35 determine the amount of gross fixed capital formation in each sector at period t. It implies that production as well as household durable consumption in the model can not adjust instantaneously to price changes because desired capital stock allocation across sectors are only attained gradually over time. The speed of adjust depend on the ratio of the size of investment and current capital stock as well as the installation cost of new capital.

Equations 36, 37 and 38 define the shadow prices of new capital (marginal value for the firm or household from an additional unit of investment) at time t. This equation is actually solved in integral form with the current value of TOBQ equal to the integral of the increment to future marginal products of capital of an additional unit of investment, plus the marginal returns of capital in reducing adjustment cost in investment. The economic meaning of this equation can also be understood by rearranging as follows:

$$INTR_{rt} \times TOB_{ir\,t} = (1 - \overline{TCOR}_{rt}) \times exp^{(PRP_{irt} - PRID_{rt})} \times \left[sk_{ir} \times \left(\frac{OUP_{irt}}{CAP_{irt}} \right) \times \left(exp^{\overline{SHY}_{irt}} \right)^{(\sigma a_{ir} - 1)} \right]^{\left(\frac{1}{\sigma a_{ir}}\right)}$$
$$+ (1 - \overline{TITC}_{irt}) \times exp^{(PRII_{rt} - PRID_{rt})} \times \frac{\theta_r}{2} \times \left(\frac{JNV_{irt}}{CAP_{irt}} \right)^2 + (TOB_{ir\,t+1} - TOB_{irt}) - \delta p_r \times TOB_{irt} \qquad \text{(A.5)}$$

Equation (A.5) reveals that the required return to capital (term in left hand) for firm making investment decision needs equal to the sum of after tax marginal revenue of the added capital (first term in right hand) and the marginal returns of capital in reducing adjustment cost in investment (second term in right hand), plus capital gain (third term in right hand) net of depreciation loss (fourth term in right hand).

Equations 39-41 define current prices for capital stock of each sector in the model. It is the unit cost of sector output or aggregate investment (consumption) multiply by the capital share of output per capital unit.

Other Price Equations ($r \in R$)

$$PRD_{irt} = Ln(exp^{PRP_{irt}}) \qquad for \quad i \in P \qquad (42)$$

$$PRX_{irt} = ln\left\{ \frac{(1 + \overline{TEX}_{irt}) \times exp^{PRY_{irt}}}{(1 + \overline{TAX}_{irt})} \right\} \qquad for \quad i \in P \qquad (43)$$

$$PRX_{ist} = \sigma w_i \times EXP_{ist} \times \sum_{r \in R} ss_{isr} \times (PRX_{irt} + EXCH_{rt} - EXCH_{st}) \qquad for \quad i \in P, s = L \tag{44}$$

$$PRID_{rt} = \sum_{i \in P} st_{irt} \times \begin{cases} PRD_{irt} & for \ r \in R \\ PRX_{irt} & for \ r = L \end{cases} \tag{45}$$

$$PRIX_{rt} = \sum_{i \in P} sx_{irt} \times PRX_{irt} \tag{46}$$

$$PRIM_{rt} = \sum_{i \in P} sm_{irt} \times PRM_{irt} \tag{47}$$

Equation 42 defines user price of output. It equals sector output price in current version of the model. Equation 43 defines export price in all regions except rest of the world, in which equals the composite goods price adjusted by sector production taxes plus export taxes (subsidies). Equation 44 specifies prices of exports from rest of the world. Since there is no production structure is specified for this region, its export prices are defined as trade weighted average of other region's export prices, multiply by prices elasticities and the region's total sectoral exports to the world, which represents a downward plopping demand curve for rest of the world's exports in other regions. Equations 45 - 47 define the aggregate price index for each region's domestic products, exports and imports, respectively.

Other Trade Flow Equations

$$TFL_{isrt} = ss_{isr} \times sm_{ir} \times (EXPT_{rt} - TBAR_{rt}) \times exp^{({}_t PRID_{Ut} - EXCH_{st} - PRX_{ist})} \qquad for \quad r = L \tag{48}$$

$$EXP_{irt} = \sum_{s \in W} TFL_{irst} \qquad for \ i \in P, r \in W \quad r \neq s \tag{49}$$

$$IMPT_{rt} = \begin{cases} \sum_{i \in P} exp^{(PRM_{irt} - {}_t PRID_{rt})} \times IMP_{irt} & for \ r \in R \\ \sum_{s \in R} IMPO_{rst} \times exp^{(REXC_{st})} & for \quad r = L \end{cases} \tag{50}$$

$$EXPT_{rt} = \begin{cases} \sum_{i \in P} exp^{(PRX_{irt} - {}_t PRID_{rt})} \times EXP_{irt} & for \ r \in R \\ \sum_{s \in R} EXPO_{rst} \times exp^{(REXC_{rt})} & for \quad r = L \end{cases} \tag{51}$$

$$IMPO_{rst} = \sum_{i \in P} IMP_{irst} \times exp^{(PRX_{irt} - {}_t PRID_{rt})} \qquad for \quad s = L \tag{52}$$

$$EXPO_{rst} = \sum_{i \in P} IMP_{irst} \times exp^{(PRX_{ist} - {}_t PRID_{st})} \qquad for \quad s = L \tag{53}$$

Equation 48 specifies bilateral trade flows between rest of the world and other regions in the model, while equation 49 gives each region's total sectoral exports, which is the sum of exports to all destinations. Equations 50 and 51 are aggregated imports and exports in each region respectively. For structured regions, they are aggregated by sectors, while for the rest of the world, they are aggregate by regions. The imports by source and exports by destination for rest of the world are specified in equation 52 and 53.

Household Wealth, Income and Consumption (for r ∈ R)

$$\begin{aligned} INCM_{rt} = {} & \overline{TRAN}_{rt} - \overline{TAXL}_{rt} + (1 - \overline{TINC}_{rt}) \times \Big\{ exp^{(WAGE_{rt} - {}_tPRID_{rt})} \times (IINL_{rt} \times + CONL_{rt} + \overline{GCEL}_{rt}) \\ & + \sum_{i=Y,Z} exp^{(PRK_{irt} - {}_tPRID_{rt})} \times CAP_{irt} + \sum_{i \in PA} exp^{(PRR_{irt} - {}_tPRID_{rt})} \times RES_{irt} \Big\} \\ & + \sum_{i \in P} (1 - \overline{TINC}_{rt}) \times exp^{(WAG_{irt} - {}_tPRID_{rt})} \times LAB_{irt} + \sum_{i \in P} (1 - \overline{TCOR}_{rt}) \times PRO_{irt} \\ & + (1 - \overline{TINC}_{rt}) \times \left(INTR_{rt} \times BOND_{rt} + \frac{IRAS_{rt}}{exp^{REXC_{rt}}} \right) \end{aligned} \tag{54}$$

$$\begin{aligned} WELH_{r\,t+1} = {} & (1 + v_r + INTR_{rt} - gr1) \times WELH_{rt} + TAXH_{rt} - TRAN_{rt} - exp^{(WAGE_{rt} - PRID_{rt})} \\ & \times (IINL_{rt} + CONL_{rt} + \overline{GCEL}_{rt}) - \sum_{i \in P} exp^{(WAG_{irt} - PRID_{rt})} \times LAB_{irt} \end{aligned} \tag{55}$$

$$WELT_{rt} = BOND_{rt} + \frac{ASST_{rt}}{exp^{REXC_{rt}}} + \varphi_r \times exp^{(MONE_{rt} - {}_tPRID_{rt})} + \sum_{i \in I} STM_{irt} + WELH_{rt} \tag{56}$$

$$\begin{aligned} CONP_{rt} = {} & \kappa_r \times (\rho_r - gr1) \times WELT_{rt} \times exp^{({}_tPRID_{rt} - {}_tPRCT_{rt})} + (1 - \kappa_r) \times \mu_r \\ & \times INCM_{rt} \times exp^{({}_tPRID_{rt} - {}_tPRCT_{rt})} + \overline{SHKC}_{rt} \end{aligned} \tag{57}$$

Equation 52 defines households' current income, which is the sum of their labor earnings, payments from assets they holding, plus transfer from their region's government minus taxes. The motion of human wealth is specified in equation 55. It states that human wealth in next period equals human wealth in current period minus present labor income, plus household income tax minus government transfer. This equation is actually solved in integral form assuming rational expectations. In this form, the value of human wealth in real terms in period t is the expected present value of the future stream of household's after tax labor income. Equation 56 gives total wealth in region r, which is the sum of real money balance, government bonds, net holding of assets from abroad, value of capital in all sectors, plus human wealth. Note that human wealth is adjusted by taxes so that the value of government debt in wealth is partially offset by the discounted value of future tax liabilities implied by government debt. The level of aggregate consumption is determined by equation 57, which is a weighted average of the forward looking consumption based on rational expectation and backward-looking consumption determined by after tax current income.

Government Budget (for r ∈ R)

$$TXM_{irt} = TIM_{irt} \times \sum_{s \in W} exp^{(EXCH_{st} - EXCH_{rt} + PRX_{ist} - {}_tPRID_{rt})} \times IMP_{isrt} \qquad for\ r \in W \quad s \neq r \tag{58}$$

$$TAXM_{rt} = \sum_{i \in P} TXM_{irt} \tag{59}$$

$$TAEX_{rt} = \sum_{i \in P} TEX_{irt} \times exp^{(PRX_{irt} - {}_tPRID_{rt})} \times EXP_{irt} \tag{60}$$

$$\begin{aligned} TAXC_{rt} = & -TI\overline{T}C_{rt} \times exp^{(PRII_{rt} - {}_tPRID_{rt})} \times \sum_{i \in P} INV_{irt} + TC\overline{O}R_{rt} \times \sum_{i \in P} \{ PRO_{irt} \\ & + T\overline{A}X_{irt} \times exp^{(PRP_{irt} - {}_tPRID_{rt})} \times OUP_{irt} + exp^{(PRM_{irt} - {}_tPRID_{rt})} \times IMP_{irt} \} \end{aligned} \tag{61}$$

$$TAXH_{rt} = TAXL_{rt} + \overline{TINC}_{rt} \times exp^{(WAGE_{rt} - PRID_{rt})} \times (IINL_{rt} + CONL_{rt} + \overline{GCEL}_{rt}) + \overline{TINC}_{rt} \times \sum_{i \in P} exp^{(WAG_{irt} - PRID_{rt})} \times LAB_{irt} \tag{62}$$

$$TAXL_{rt} = INTR_{rt} \times BOND_{rt} + \overline{TAXI}_{rt} \tag{63}$$

$$TRAN_{rt} = \overline{TRAX}_{rt} - \tau_{r} \times OUTP_{rt} \tag{64}$$

$$TAXT_{rt} = TAXC_{rt} + TAXH_{rt} + TAXE_{rt} + TAXM_{rt} \tag{65}$$

$$CONG_{rt} = \overline{GCEL}_{rt} \times exp^{(WAGE_{rt} - PRID_{rt})} + \sum_{i \in P} exp^{(PRY_{irt} - PRID_{rt})} \times \overline{GCE}_{irt} \tag{66}$$

$$DEFR_{rt} = CONG_{rt} + TRAN_{rt} + INTR_{rt} \times BOUD_{rt} - TAXT_{rt} \tag{67}$$

$$DEFN_{rt} = CONG_{rt} + TRAN_{rt} + INTN_{rt} \times BOUD_{rt} - TAXT_{rt} \tag{68}$$

Equations 58-63 determine government revenue from tariffs, export taxes, corporate taxes, household income taxes, and lump sum taxes, respectively, while equations 64 and 65 defines government transfer to households and total government tax revenue respectively. Total government spending and its allocation among composite goods and labor services are fixed in real terms (equation 66) at the base year level, while real and nominal government budget deficit are given in equations 67 and 68.

Financial Markets (for r ∈ R)

$$BOND_{r\,t+1} = DEFR_{rt} + BOND_{rt} \times (1 - gr1 - gr2) \qquad BOUD_{r0} \quad given \tag{69}$$

$$MONE_{rt} = ln(OUTP_{rt}) + {}_tPRID_{rt} + \eta_r \times INTN_{rt} + \overline{SHKM}_{rt} \tag{70}$$

$$STM_{irt} = \frac{TOB_{irt}}{(1 - \overline{TITC}_{irt})} \times CAP_{irt} \qquad for \;\; i \in P \wedge i = Y \tag{71}$$

$$INTN_{rt} = \frac{1}{-\eta_r} \times \{{}_tPRID_{rt} + ln(OUTP_{rt}) + \overline{SHKM}_{rt} - \overline{MPOL}_{rt}\} \tag{72}$$

$$INTR_{rt} = \begin{cases} INTN_{rt} - ({}_tPRID_{r\,t+1} - {}_tPRID_{rt}) & r \in R \\ INTR_{st} & r = L, s = U \end{cases} \tag{73}$$

$$REXC_{r\,t+1} = REXC_{rt} - (INTR_{rt} - INTR_{st}) \qquad for \;\; s = U \;\; r \in S \tag{74}$$

$$REXC_{rt} = EXCH_{rt} + ({}_tPRID_{rt} - {}_tPRID_{st}) \qquad for \;\; s = U \;\; r = L \tag{75}$$

$$EXCH_{rt} = \begin{cases} REXC_{rt} - ({}_tPRID_{rt} - {}_tPRID_{st}) & for \;\; s = U \;\; r \in S; r \neq L \\ 0 & for \;\; r = L \end{cases} \tag{76}$$

$$NEER_{rt} = \begin{cases} -\sum_{s \in S} \omega_{sr} \times EXCH_{st} & for \quad r = U \\ \omega_{u,r} \times EXCH_{rt} - \sum_{s \in R} \omega_{sr} \times (EXCH_{st} - EXCH_{rt}) & for \quad s \in R; s \neq r \end{cases} \tag{77}$$

$$REER_{rt} = \begin{cases} -\sum_{s \in S} \omega_{sr} \times REXC_{st} & for \quad r = U \\ \omega_{u,r} \times REXC_{rt} - \sum_{s \in R} \omega_{sr} \times (REXC_{st} - REXC_{rt}) & for \quad s \in R; s \neq r \end{cases} \tag{78}$$

$$\begin{aligned} ASSE_{Us\ t+1} &= (1 - gr1 - gr2) \times ASSE_{Ust} - CURR_{st} \\ &+ (1 - gr1 - gr2) \times \sum_{s \in W} exp^{REXC_{st}} \times (ASSE_{sr\ t+1} - ASSE_{srt}) \qquad r,s \in S \\ & ASSE_{rs0} \quad given \quad for \quad r \in R, s \in W \end{aligned} \tag{79}$$

$$\begin{aligned} ASSE_{rs\ t+1} &= (1 - gr1 - gr2) \times exp^{REXC_{rt}} \times ASSE_{rst} - \left(\frac{sa_{rs} \times CURR_{st}}{exp^{REXC_{rt}}} \right) \\ & ASSE_{rs0} \quad given \quad for \quad r,s \in S \end{aligned} \tag{80}$$

$$ASSE_{rst} = exp^{REXC_{st}} \times (-ASSE_{srt}) \qquad for \ r \in R, s \in W \tag{81}$$

$$ASST_{rt} = \begin{cases} \sum_{s \in W} exp^{REXC_{rt}} \times ASSE_{rst} & r \in R \wedge r \neq s \\ \sum_{s \in R} -exp^{REXC_{st}} \times ASSE_{srt} & for \quad r = L \end{cases} \tag{82}$$

Equation 69 describes the motion of bonds, which are issued by the government to finance its budget deficits. It equals previous debt outstanding plus current period budget deficit. Equation 70 define real money demand, which is a function of aggregate output and short-term nominal interest rates. Firm equity market value is defined in equation 71, which equals the value of the firm at period t, i.e. the firms current capital stock multiply by the shadow price of capital adjusted by investment credits. It is the firm's future earing potential. The nominal interest rate is defined by equation 72 which is the inverse of the money demand function given in equation (70). Total money supply is determined by the balance sheet of the central bank and set exogenously. The real interest rate is the nominal interest adjusted by expected aggregate price level changes (equation 73).

Equations 74 and 75 specify the real exchange rate for all structured regions and rest of the world respectively. Equation 74 is also known as uncovered interest arbitrage, that the interest differential is equal to the expected exchange rate change. It is actually solved in integral form so that the current exchange rate is the interest differential plus the expected future exchange rate. This can be recursively solved forward to also write it as the current exchange rate equal to the sum of expected future interest differentials plus the equilibrium real exchange rate. When foreign interest rates are higher than the domestic interest rate, the real value of the home currency is expected to depreciate relative to its long-run value and vice-versa. Equation 76 defines nominal exchange rate, which is the real exchange rate adjusted by the difference in aggregate price level between regions with the United States. Equations 77 and 78 define nominal and real effective exchange rate. Please note that all exchange rates in the model are defined in their natural log forms, therefore a negative sign before them means their inverse.

Equations 79 and 80 describe the motion of U.S. and other region's holding of assets in region s respectively. In the U.S., it equals U.S. previous assets holdings in region s plus the difference between region s's current account balance and changes of region s holding of other region's assets. In other regions, it equals the region's previous assets holdings in region s minus a constant portion of the region's real current account balance. Equation 81 establishes the symetric property between two region's foreign assets holding, while equation 82 gives each region's total net foreign assets holding.

Balance of Payment (for r ∈ R)

$$IMNA_{rt} = \sum_{i \in p} exp^{(PRM_{irt} - PRIM_{rt})} \times IMP_{irt} \tag{83}$$

$$EXNA_{rt} = \sum_{i \in p} exp^{(PRX_{irt} - PRIX_{rt})} \times EXP_{irt} \tag{84}$$

$$TBNA_{rt} = EXNA_{rt} - IMNA_{rt} \tag{85}$$

$$TBAL_{rt} = EXPT_{rt} - IMPT_{rt} \qquad for \quad r \in W \tag{86}$$

$$TBAR_{rt} = \begin{cases} exp^{REXC_{rt}} \times (EXPT_{rt} - IMPT_{rt}) & for \quad r \in S; r \neq L \\ CURR_{rt} - IRAS_{rt} & for \quad r = L \end{cases} \tag{87}$$

$$INAS_{rt} = exp^{REXC_{rt}} \times INTN_{rt} \times \sum_{s \in W} ASSE_{rst} \qquad r \in R \wedge r \neq s \tag{88}$$

$$IRAS_{rt} = \begin{cases} exp^{REXC_{rt}} \times INTR_{rt} \times \sum_{s \in W} ASSE_{rst} & r \in R \wedge r \neq s \\ \sum_{s \in R} exp^{REXC_{st}} \times INTR_{st} \times (-ASSE_{srt}) & for \quad r = L \end{cases} \tag{89}$$

$$CURN_{rt} = TBAL_{rt} + \frac{INAS_{rt}}{exp^{REXC_{rt}}} \qquad for \quad r \in R \tag{90}$$

$$CURR_{rt} = \begin{cases} -\sum_{s \in S} CURR_{st} & for \ r = U \\ TBAR_{rt} + IRAS_{rt} & for \ r \in S \end{cases} \tag{91}$$

Equations 83 and 84 calculate nominal imports and exports, while equations 85 and 86 give the balance of trade in both nominal and real terms measured in domestic currencies respectively. Each region's balance of trade in real terms measured by U.S. dollars is given by equation 87. Equations 88 and 89 calculate real and nominal payments from holding other regions' assets for each region in the model, while their nominal and real current account balances are given in equations 90 and 91 respectively.

There are two terms in each region's current account: balance of trade and payment from holding of foreign assets. It implies that trade imbalances are financed by flows of assets between regions and net capital flows for each region at period t are equal to their current account position. The global net flows of capital are constrained to zero at each period by equation 91.

Market Equilibrium (for r ∈ R)

Flows *(intra-period equilibrium)*:

$$OUY_{irt} = CON_{irt} + IIN_{irt} + \overline{INS}_{irt} + \overline{GCE}_{irt} + EXP_{irt} + \begin{cases} \sum_{j \in P} EN_{jit} & for\ i \in PE \\ \sum_{j \in P} OI_{jit} & for\ i \in PN \end{cases} \tag{92}$$

$$LABT_{rt} = CONL_{rt} + IINL_{rt} + \overline{GCEL}_{rt} + \sum_{i \in P} LAB_{irt} \tag{93}$$

$$\begin{aligned} WAGE_{r\ t+1} &= Wage_{rt} + \pi_r \times ({}_tPRCT_{r\ t+1} - {}_tPRCT_{rt}) + (1 - \pi_r) \\ &\times ({}_tPRCT_{rt} - {}_tPRCT_{r\ t-1}) + \alpha_r \times ln(LABT_{rt}) \qquad WAGE_{r0}\ \ given \end{aligned} \tag{94}$$

$$WAG_{irt} = WAGE_{rt} \qquad for\ i \in P \tag{95}$$

$$\overline{REST}_{rt} = \sum_{i \in PA} RES_{irt} \tag{96}$$

$$PRR_{irt} = PRP_{irt} + \sigma r_{ir} \times ln(\overline{REST}_{rt}) \qquad for\ \ i \in PA \tag{97}$$

$$INVT_{rt} = \sum_{i \in I} INV_{irt} \tag{98}$$

$$SAVN_{rt} = INVT_{rt} + DEFN_{rt} + CURN_{rt} \tag{99}$$

$$SAVR_{rt} = INVT_{rt} + DEFR_{rt} + CURR_{rt} \tag{100}$$

Equation 92 establishes the intra-period equilibrium condition in commodity markets. The sum of private and government consumption, investment demand and changes in inventory, intermediate input and exports must equals total composite goods supplied in each and every sector.

Equations 93-95 define equilibrium in labor market. Total employment is given in equation 93, which is the sum of labor demand by private household, the government, investment and each of the 12 sectors of production. In equation 94, the economy-wide wage is adjusted slowly according to current and expected inflation and labor demand relative to labor supply, the latter is determined exogenously by the growth rate of labor force. It implies the labor market may not clear in the short run. For a given nominal wage, demand for labor will determine short run employment, and the residual between total demand and supply will be the economy wide unemployment level. The parameter π_r in equation 94 is weight of backward-looking versus forward-looking price expectations and it varies from region to region.

Since the model assume labor is perfectly mobile among sectors within each region but is immobile between regions, therefore, nominal wages across sectors within each region will be equalized as specified in equation 95 but real wages may differ.

Equation 96 gives the total demand for sector specific resources (land in current model), while its price depend on prices of its output and total land supply, which is fixed exogenously (equation 97). Equation (97) is an inverted demand function for land.

Equation 98 calculate total investment demand in each region, it must equal the sum of household saving, government saving (budget surplus) and foreign savings (current account deficit) in both nominal and real terms (equations 99 and 100).

Stock (steady state equilibrium)

$$\frac{JNV_{ir}^{ss}}{CAP_{ir}^{ss}} = \delta p_r + gr1 + gr2 \qquad for\ i \in I \tag{101}$$

$$TOB_{ir}^{ss} = \frac{(1-\overline{TITC}_{ir}) \times \frac{\theta}{2} \times (\delta p_r + gr1 + gr2)^2 + (1-\overline{TCOR}_r) \times \left(sk_{ir} \times \frac{OUP_{ir}^{ss}}{CAP_{ir}^{ss}} \right)^{\frac{1}{\delta a_{ir}}}}{INTR_r^{ss} + \delta p_r} \qquad for\ i \in I \tag{102}$$

$$BOND_r^{ss} = \frac{DEFR_r^{ss}}{gr1 + gr2} \qquad \wedge \qquad \lim_{T \to \infty} \frac{BOND_{rT}}{\prod_{t=0}^{T} (1 + INTR_{rt} - gr1 - gr2)} = 0 \tag{103}$$

$$ASSE_{rs}^{ss} = \begin{cases} -CURR_s^{ss} & for\ r = U, s \in S \\ -\frac{sa_{rs} \times exp^{-REXC_r^{ss}} \times CURR_s^{ss}}{1 - exp^{REXC_r^{ss}} \times (1 - gr1 - gr2)} & for\ r \in S, s \in W \end{cases} \quad \wedge$$

$$INTR_r^{ss} = INTR_s^{ss} \ for\ r = U, s \in S \tag{104}$$

$$WELH_r^{ss} = \frac{(1-\overline{TINC}_r) \times exp^{WAGE_r^{ss}} - PRID_r^{ss} \times LABT_r^{ss} + TRAN_r^{ss}}{INTR_r^{ss} - gr1}$$

$$\alpha_r \times ln\left(LABT_r^{ss}\right) = 0 \quad \to \quad LABT_r^{ss} = 1 \tag{105}$$

There are a number of major dynamics linking flow and stock variables in the model. Investment leads to accumulation of capital stock, fiscal deficits leads to the accumulation of government debt, trade deficits lead to the accumulation of foreign debt. As long as the transversality condition are satisfied, these types of motion equations will be finite and well-defined.

Equation 101 define the steady state equilibrium for capital stock. It states that at stead state equilibrium, the rate of gross capital formation must equal the rate of depreciation plus labor force growth. The steady state condition for the shadow price of capital is specified in equation 102. Equation 103 specify the steady state and transversality condition for government bonds. Transversality condition imposes a long-term budget constraints on public sector borrowing. Because households will not hold government bonds unless they expect the bonds to be paid off eventually, a government running a budget deficit today must run an appropriate budget surplus at some point in the future. Otherwise, the government would be unable to pay interest on the debt and households will not be willing to hold it. Thus, the current level of debt will be equal to the present value of future budget surpluses.[1] therefore, any increase in the government debt will be matched by an equal present value increase in future budget surpluses.

A similar condition is specified for the change in the net foreign assets position of each region in equation 104. It implies a region must service its net international debt by running trade surplus in the steady state. However, no end point condition is imposed to force each region's net debt position to zero. As long as regions need to pay interest on their foreign borrowing, there will be an automatic adjustment mechanism to prevent countries from infinite international borrowing. As national wealth declines because of growing foreign debt, households will cut back on their consumption, thereby resulting a fall in imports and a rise in exports (run trade surplus). Because capital are assumed perfectly mobile among regions, expected return on loans by all region must be equalized and converge to an international interest rate (equals U.S. interest in the model). Short term nominal interest rate at each region are flexible in short run to clear the money market. The

[1] Strictly speaking, public debt must be less than or equal to the present value of future budget surpluses. For tractability we assume that the government is initially fully leveraged so that this constraint holds with equality.

steady-state condition for human wealth is given in equation 105. It requires full employment in the long run (as mentioned earlier, all variable in the model are scaled by labor endowment, therefore total employment equals one mean full employment).

Aggregate Price Level and National Products (for r ∈ R)

$$OUTP_{rt} = \sum_{i \in P} exp^{(PRD_{irt} - PRID_{rt})} \times OUP_{irt} \tag{106}$$

$$\begin{aligned} GDPN_{rt} = {} & exp^{PRCT_{rt}} \times CONP_{rt} + exp^{PRID_{rt}} \times CONG_{rt} + exp^{PRII_{rt}} \times INVT_{rt} \\ & + exp^{PRID_{rt}} \times TBNA_{rt} + exp^{PRII_{rt}} \times \sum_{i \in P} \overline{INS}_{irt} \end{aligned} \tag{107}$$

$$GDPR_{rt} = CONP_{rt} + CONG_{rt} + INVT_{rt} + TBNA_{rt} + \sum_{i \in P} \overline{INS}_{irt} \tag{108}$$

$$GNPR_{rt} = GDPR_{rt} + \frac{INAS_{rt}}{exp^{REXC_{rt}}} \tag{109}$$

$$PGDP_{rt} = ln\left(\frac{GDPN_{rt}}{GTPR_{rt}}\right) \tag{110}$$

$$INFL_{rt} = PRCT_{rt} - PRCT_{r\ t-1} \qquad PRCT_{r0} \quad given \tag{111}$$

Equation 106 defines gross output for each region, while nominal and real gross domestic product (GDP) are calculated in equations 107 and 108 respectively. Real GDP plus payment received from holding of foreign assets equals gross national product (GNP, equation 109). Finally, equations 110 and 111 define the GDP deflator and a consumer price inflation index respectively.

Appendix B

Derivation of CES Unit Cost Functions and Component Demand Functions[2]

This section provide a detailed step by step derivations of CES unit cost function and corresponding component demand functions used in the model. We summary all derivations in the following lemma and its three corollaries.

Lemma 1: A CES aggregator function has the form

$$Q = A \times (\sum_{\Theta}^{\varphi} \delta_{\varkappa}^{\frac{1}{\sigma}} \times x_{\varkappa}^{\frac{\sigma-\Theta}{\sigma}})^{\frac{\sigma}{\sigma-\Theta}} \qquad \text{(B.1)}$$

where Q is the composite outlay; A is the unit parameter; x_i is the ith component in the aggregator function; δ_i is the share parameter associated with x_i , i = 1, 2, ..., n; and σ is the elasticity of substitution. When Q = 1, the unit cost function that is dual to it has the following form:

$$C(P,1) \equiv c(P) = \frac{1}{A} (\sum_{\Theta}^{\varphi} \times \delta_{\varkappa} \times p_{\varkappa}^{\Theta-\sigma})^{\frac{\Theta}{\Theta-\sigma}} \qquad \text{(B.2)}$$

where C(P,Q) is the cost function, c(P) is the unit cost function; P is the n dimension component price vector, and p_i is the price for component i.

Proof: The cost minimization problem can be defined as:

$$Min \quad C(P,1) = \sum_{\Theta}^{\varphi} p_{\varkappa} x_{\varkappa} \qquad \text{(B.3)}$$

$$s.t \quad A \times (\sum_{\Theta}^{\varphi} \delta_{\varkappa}^{\frac{1}{\sigma}} \times x_{\varkappa}^{\frac{\sigma-\Theta}{\sigma}})^{\frac{\sigma}{\sigma-\Theta}} = 1 \qquad \text{(B.4)}$$

The first order conditions are:

[2]A thorough treatment of the duality between cost and the aggregator function is presented in (Diewert, 1982).

$$\frac{p_{\varkappa}}{p_{\varpi}} = \frac{\delta_{\varkappa}^{\frac{1}{\sigma}}}{\delta_{\varpi}^{\frac{1}{\sigma}}} \times (\frac{x_{\varkappa}}{x_{\varpi}})^{-\frac{\Theta}{\sigma}} \qquad \textit{for } i, j = 1, 2, \ldots n \textit{ and} \tag{B.5}$$

$$(\sum_{1}^{n} \delta_i^{\frac{1}{\sigma}} \times x_i^{\frac{\sigma-1}{\sigma}}) = (\frac{1}{A})^{\frac{\sigma-1}{\sigma}} \tag{B.6}$$

multiplying both sides of (B.5) by x_i/x_j for all i, we have

$$\frac{p_i x_i}{p_j x_j} = \frac{\delta_i^{\frac{1}{\sigma}}}{\delta_j^{\frac{1}{\sigma}}} \times (\frac{x_i}{x_j})^{\frac{\sigma-1}{\sigma}} \qquad \textit{fori}, j = 1, 2, \ldots, \imath \tag{B.7}$$

Summing (B.7) i from 1 to n, and plus one for both side for i = j, we have

$$\frac{\sum_{i}^{n} p_i x_i}{p_j x_j} = \frac{(\sum_{1}^{n} \delta_i^{\frac{1}{\sigma}} x_i^{\frac{\sigma-1}{\sigma}})}{\delta_j^{\frac{1}{\sigma}} x_j^{\frac{\sigma-1}{\sigma}}} \tag{B.8}$$

Substitute (B.3) and (B.6) into (B.8), resulting in

$$\frac{c(p)}{p_j x_j} = \frac{(\frac{1}{A})^{\frac{\sigma-1}{\sigma}}}{\delta_j^{\frac{1}{\sigma}} x_j^{\frac{\sigma-1}{\sigma}}} \tag{B.9}$$

Solving x_j from (B.9), we have

$$x_j = (\frac{1}{A})^{1-\sigma} \times (\frac{\delta_j^{\frac{1}{\sigma}} \times c(p)}{p_j})^{\sigma} \tag{B.10}$$

Multiplying both sides of (B.10) by p_j, and summing j from 1 to n, we have

$$c(P) = \sum_{1}^{n} p_j x_j = (\frac{1}{A})^{1-\sigma} \times c(P)^{\sigma} \times \sum_{1}^{n} (\delta_j \times p_j^{1-\sigma}) \tag{B.11}$$

Solving c(P) from (B.11), we obtain exactly (B.2).

Q.E.D.

Corollary 1 The cost function

$$C(P,Q) \equiv Q \times c(P) = \frac{Q}{A} \left(\sum_{1}^{n} \times \delta_i \times p_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{B.12}$$

Proof: It follows directly from the linear homogeneity property of the CES aggregator function.

Corollary 2 The conditional demand function of component i in (B.1) can be defined as

$$x_i(P,Q) = A^{\sigma-1} \times \delta_i \times Q \times \left(\frac{p_i}{c(P)} \right)^{-\sigma} \tag{B.13}$$

i.e. it is a function of the share parameter and the ratio of aggregate price (unit cost) and component price, multiplying by total aggregate outlay.

Proof: Based on Shephard's lemma, taking partial derivatives of (B.12) with respect to p_j, we have

$$\begin{aligned} \frac{\partial C(P,Q)}{\partial p_j} &= \frac{Q}{A} \left(\sum_{1}^{n} \delta_j \times p_j^{1-\sigma} \right)^{\frac{\sigma}{1-\sigma}} \times \delta_j \times p_j^{-\sigma} \\ &= \frac{Q}{A} \times \delta_j \times p_j^{-\sigma} \times c(P)^{\sigma} \times \left(\frac{1}{A}\right)^{-\sigma} \\ &= A^{\sigma-1} \times \delta_i \times Q \times \left(\frac{p_i}{c(P)} \right)^{-\sigma} \quad using\ (B.2) \end{aligned} \tag{B.14}$$

It is (B.13).

Q.E.D.

Corollary 3 If the aggregator function is linear homogenous, then the unit cost function (composite price index) c(P) can be replaced by an average price identity:

$$P_{average} \equiv \frac{\sum_{1}^{n} p_i x_i}{Q} \equiv c(P) \tag{B.15}$$

It follows directly from the property of the cost function that is the dual of the aggregator function:

$$C(P,Q) \equiv \sum_{1}^{n} p_i x_i \equiv Q \times c(P) \tag{B.16}$$

Lemma 2 The elasticity of substitution, σ in the CES aggregator function is the negative elasticity of conditional component demand with respect to its own price.

Proof: Taking natural logs for both sides of equation (B.13), we have

$$\ln x_i(P,Q) = (\sigma - 1) \times \ln A + \ln\delta_i - \sigma \times \ln[\, p_i - c(P)] + Ln\ Q$$

Taking partial derivative of *Ln* x_i(P,Q) with respect to *Ln* p_i, results in the elasticity in its log form:

$$\sigma = -\frac{\partial Ln\ x_i(P,Q)}{\partial Ln\ p_i} \tag{B.17}$$

Q. E. D.

Denote the unit cost function *c(P)* by p_o and multiplying both sides of equation B.13 by p_i then dividing by p_oQ, resulting cost share devoted to input i as follows:

$$s_i = \delta_i\, A^{\sigma-1} \left(\frac{p_i}{p_o}\right)^{1-\sigma} \tag{B.18}$$

Taking the logarithm of both sides:

$$\ln s_i = \ln\delta_i + (\sigma-1)\ln A + (1-\sigma)(\ln p_i - \ln p_o) \tag{B.19}$$

This form is convenient for econometric estimation: it is nearly linear and only requires data on prices and cost shares.

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