The Theoretical and Empirical Structure of the G-Cubed Model

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ABSTRACT

This paper describes in detail the theoretical and empirical features of the G-Cubed multi-country, multi-sector intertemporal general equilibrium model. The G-Cubed model has been used for a number of alternative policy simulations including the implications of greenhouse gas policy, trade liberalization, tax policy and macroeconomic policy. The paper is intended as a technical guide to supplement policy-oriented papers. It does not include a survey of related literature; for that or other additional detail, including computer code for the model and analysis of a variety of policy simulations, refer to McKibbin and Wilcoxen (1994) or contact the authors.

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1 Introduction

This paper gives a detailed technical overview of the G-Cubed G-Cubed multi-country, multi-sector intertemporal general equilibrium model. The G-Cubed model was originally developed by McKibbin and Wilcoxen (1992). It combines the dynamic macroeconomic modelling approach taken in the MSG2 model of McKibbin and Sachs (1991) with the disaggregated, econometrically-estimated, intertemporal general equilibrium model of the U.S. economy by Jorgenson and Wilcoxen (1989). The Jorgenson-Wilcoxen model breaks the economy down into 35 separate industries, each of which is represented by an econometrically estimated cost function. The G-Cubed model has only 12 sectors but each sector is based on econometrically estimated cost functions.

The G-Cubed model has been constructed to contribute to the current policy debate on environmental policy and international trade with a focus on global warming policies, but it has many features that will make it useful for answering a range of issues in environmental regulation, microeconomic and macroeconomic policy questions. It is a world model with substantial regional disaggregation and sectoral detail. In addition, countries and regions are linked both temporally and intertemporally through trade and financial markets. Like MSG2, G-Cubed contains a strong foundation for analysis of both short run macroeconomic policy analysis as well as long run growth consideration of alternative macroeconomic policies. Intertemporal budget constraints on households, governments and nations (the latter through accumulations of foreign debt) are imposed. To accommodate these constraints, forward looking behavior is incorporated in consumption and investment decisions. Unlike MSG2, G-Cubed also contains substantial sectoral detail. This permits analysis of environmental policies which tend to have their largest effects on small segments of the economy. By integrating sectoral detail with the macroeconomic features of MSG2, G-Cubed can be used to consider the long run costs of alternative environmental regulations yet at the same time consider the macroeconomic implications of these policies over time. The response of monetary and fiscal authorities in different countries can have important effects in the short to medium run which, given the long lags in physical capital and other asset accumulation, can be a substantial period of time. Overall, the model is designed to provide a bridge between computable general equilibrium models and macroeconomic models by integrating the more desirable features of both approaches. Details on this integration and how G-cubed bridges the gap between CGE and traditional macro-econometric models can be found in McKibbin (1993b).

G-Cubed is still in the process of development but it is already a large model. In its current form it contains over 5,000 equations and 110 intertemporal costate variables. Nonetheless, it can be solved using software developed for a personal computer.

The purpose of this paper is to document the latest version of the G-Cubed model that has been used for a number of policy oriented papers. This paper describes in detail the theoretical and empirical features of the G-Cubed model. It is intended as a technical appendix to supplement policy-oriented papers. It does not include a survey of related literature; for that or other additional detail, including computer code for the model and analysis of a variety of policy simulations, refer to McKibbin and Wilcoxen (1994) or contact the authors. Section 2 of the paper gives a detailed overview of the structure of the model. The procedures for parameter estimation, data construction issues and parameter estimates are outlined in section 3. Section 4 summarizes a baseline projection of the model . A conclusion is presented in section 5.

2 The Structure of the Model

The key features of G-Cubed are summarized in Table 2.1. The country and sectoral breakdown of the model are summarized in Table 2.2. The model consists of eight economic regions (the United States, Japan, Australia, the rest of the OECD (ROECD), China, Oil Exporting developing countries (OPEC), Eastern Europe and states of the former Soviet Union, and all other developing countries (LDCs)) with twelve sectors in each region. There are five energy sectors (electric utilities, natural gas utilities, petroleum processing, coal extraction, and crude oil and gas extraction) and seven non-energy sectors (mining, agriculture, forestry and wood products, durable manufacturing, non-durable manufacturing, transportation and services). This disaggregation enables us to capture the sectoral differences in the impact of alternative environmental policies.

We begin by presenting the structure of a particular one of these economies: the United States. The other countries have similar structure and differ only in endowments and the values of behavioral parameters. To keep our notation as simple as possible we have not subscripted each variable by country except where needed for clarity.

Each economy or region in the model consists of several economic agents: households, the government, the financial sector and the 12 production sectors listed above. We now present an overview of the theoretical structure of the model by describing the decisions facing these agents. Throughout the discussion all quantity variables will be normalized by the economy's endowment of effective labor units. Thus, the model's long run steady state will represent an economy in a balanced growth equilibrium.

Table 2.1: Summary of Main Features

- Specification of the demand and supply sides of industrial economies;
- Integration of real and financial markets of these economies;
- Intertemporal accounting of stocks and flows of real resources and financial assets;
- Imposition of intertemporal budget constraints so that agents and countries cannot forever borrow or lend without undertaking the required resource transfers necessary to service outstanding liabilities;
- Short run behavior is a weighted average of neoclassical optimizing behavior and ad-hoc "liquidity constrained" behavior;
- The real side of the model is disaggregated to allow for production and trade of multiple goods and services within and across economies;
- Full short run and long run macroeconomic closure with macro-dynamics at an annual frequency around a long run Solow/Swan neoclassical growth model.
- The model is solved for a full rational expectations equilibrium at an annual frequency from 1993 to 2200.

Table 2.2: Overview of the G-Cubed Model

<u>Regions</u>

- United States
- Japan
- Australia
- Other OECD
- China
- LDCs
- Oil Exporting Developing Countries
- Eastern Europe and the former Soviet Union

<u>Sectors</u>

- Energy:
  - Electric Utilities
  - Gas Utilities
  - Petroleum Refining
  - Coal Mining
  - Crude Oil and Gas Extraction

- Non-Energy:
  - Mining
  - Agriculture, Fishing and Hunting
  - Forestry/ Wood Products
  - Durable Manufacturing
  - Non-Durable Manufacturing
  - Transportation
  - Services

2.1 Firms

Each of the twelve sectors is represented by a single firm in each sector which chooses it inputs and its level of investment in order to maximize its stock market value subject to a multiple-input production function and a vector of prices it takes to be exogenous. For each sector i, output (Q_i) is produced with inputs of capital (K_i), labor (L_i), energy (E_i), materials (M_i) and a sector-specific resource (R_i). Energy and Materials, in turn, are CES aggregates of inputs of intermediate goods. The nature of the sector specific resource varies across sectors. In the coal industry, for example, it is reserves of coal, while in agriculture and forestry/wood products it is land which can be transferred between these two sectors.[2] In any case, production in sector i is given by:

$$Q_i = A_{iO} \left(\sum_{j=K,L,E,M} \delta_{ij}^{1/\sigma_{iO}} X_{ij}^{(\sigma_{iO}-1)/\sigma_{iO}} \right)^{\frac{\sigma_{iO}}{(\sigma_{iO}-1)}} \tag{1}$$

where Q_i is output, X_{ij} is industry i's use of input j, and A_{iO}, δ_{ij}, and σ_{io} are parameters. Both energy and materials are aggregates of inputs of intermediate goods:

$$X_{iE} = A_{iE} \left(\sum_{j=1}^{5} \delta_{ij}^{1/\sigma_{iE}} X_{ij}^{(\sigma_{iE}-1)/\sigma_{iE}} \right)^{\frac{\sigma_{iE}}{(\sigma_{iE}-1)}} \tag{2}$$

[2] In the version of the model in this draft we have assumed an infinite supply of these resources but in future drafts we intend to explore the implications of exhaustible resources and sequestration of land for tree planting etc.

$$X_{iM} = A_{iM} \left(\sum_{j=6}^{12} \delta_{ij}^{1/\sigma_{iM}} X_{ij}^{(\sigma_{iM}-1)/\sigma_{iM}} \right)^{\frac{\sigma_{iM}}{(\sigma_{iM}-1)}} \tag{3}$$

Intermediate goods are, in turn, aggregates of imported and domestic commodities which are taken to be imperfect substitutes. Due to data limitations we assume that all agents in the economy have identical preferences over foreign and domestic varieties of each particular commodity. We represent these preferences by defining twelve composite commodities that are produced from imported and domestic goods. For commodity i the production function for composite input Y_i in terms of domestic output Q_i and imported good M_i is:

$$Y_i = A_{iY} \left(\delta_{iQ}^{1/\sigma_{iY}} Q_i^{(\sigma_{iY}-1)/\sigma_{iY}} + \delta_{iM}^{1/\sigma_{iY}} M_i^{(\sigma_{iY}-1)/\sigma_{iY}} \right)^{\frac{\sigma_{iY}}{(\sigma_{iY}-1)}} \tag{4}$$

For example, petroleum products purchased by agents in the model are a composite of imported and domestic petroleum. By constraining all agents in the model to have the same preferences over the origin of goods we require that, for example, the agricultural and service sectors have the identical preferences over domestic oil and oil imported from the middle east.[3] This accords with the input-output data we use and allows a very convenient nesting of production, investment and consumption decisions.

The full structure of production is given in Figure 2.1. This figure also shows an additional

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[3] This does not require that both sectors purchase the same amount of oil, or even that they purchase oil at all; only that they both feel the same way about the origins of oil they buy.

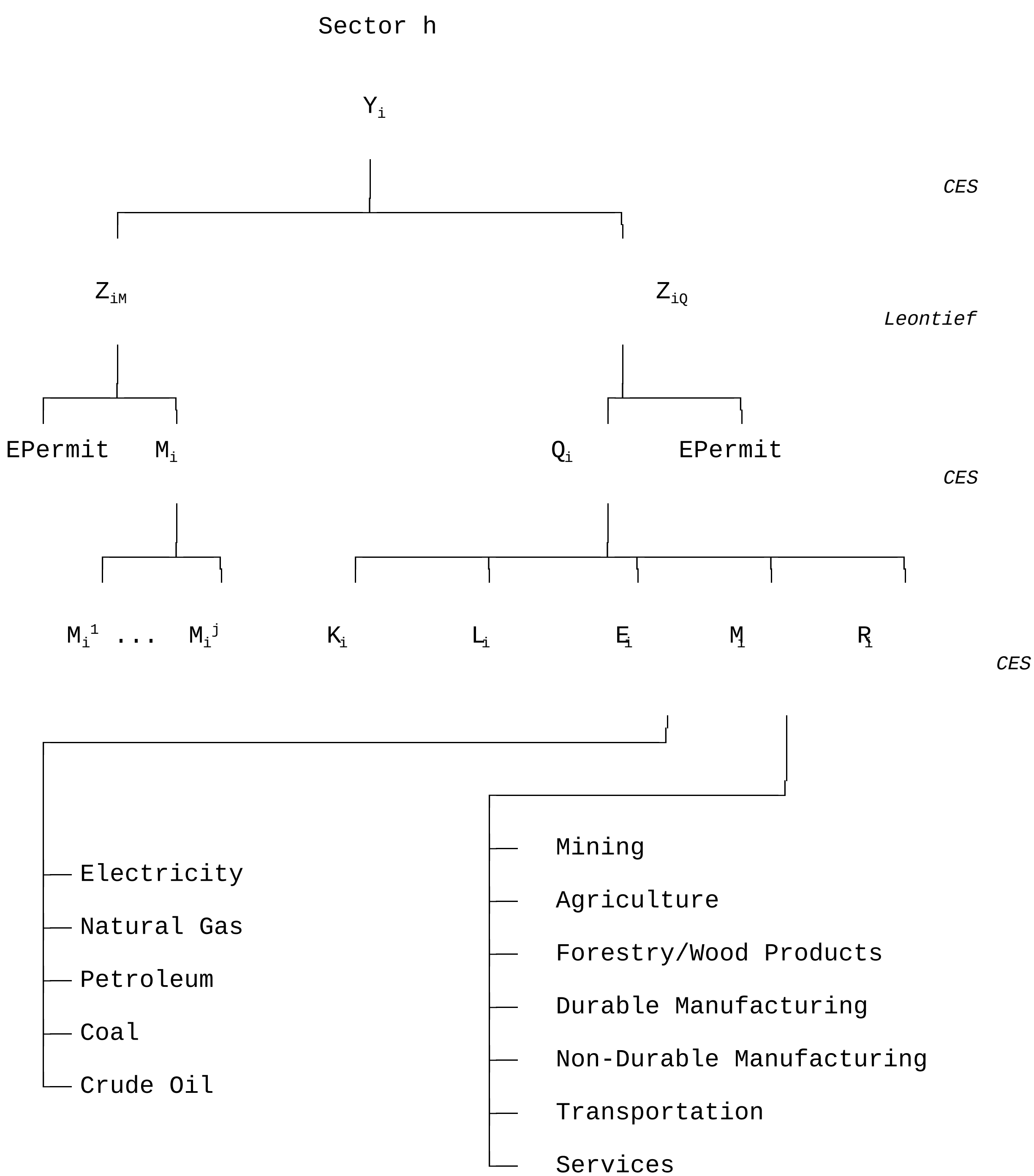
nesting in the production process in which emissions permits are combined according to a Leontief technology with inputs of imported and domestic goods. This allows for the inclusion of emission quotas associated with various inputs. One use of this would be to model a marketable permit system for carbon dioxide emissions. The permits introduce a potential cap on the use of a given input in production of the composite good. Each permit has an associated shadow value of the permit which also appears in the definition of wealth of the permit holders. In the results reported in this paper we assume an infinite supply of permits, so they are non-binding and have zero value. In future papers we will explore trading of emission rights for various types of emissions across sectors, both within and between regions. It is worth noting that setting permits on the emissions potential of both imports and domestically produced goods is equivalent to a permit system on consumption. Imposing a permit system only domestic goods would be a permit system on production. In our formulation we are able to analyze either type of policy.

In each sector the capital stock changes according to the rate of fixed capital formation (J_i) and the rate of geometric depreciation (δ_i):

$$\frac{dK_i}{dt} = J_i - \delta_i K_i \tag{5}$$

Following the cost of adjustment models of Lucas (1967), Treadway (1969) and Uzawa (1969) we assume that the investment process is subject to rising marginal costs of installation. To formalize this we adopt Uzawa's approach by assuming that in order to install J units of capital a firm must buy a larger quantity, I, that depends on its rate of investment (J/K) as follows:

Figure 2.1: Production Nesting



$$I_i = \left(1 + \frac{\phi_i}{2}\frac{J_i}{K_i}\right) J_i \tag{6}$$

where ϕ is a non-negative parameter. The difference between J and I may be interpreted many ways; we will view it as installation services provided by the capital-goods vendor. One advantage of using an adjustment cost approach is that we can vary the adjustment cost parameter for different sectors to capture the degree to which capital is sector specific.

The goal of each firm is to choose inputs of L, E, M, R, and J to maximize intertemporal net-of-tax profits. For analytical tractability, we assume that this problem is deterministic (in other words, the firm is assumed to believe its estimates of future variables with subjective certainty). Thus, the firm will maximize:[4]

$$\int_t^{\infty} (Profit - (1-\tau_4)\ P^I I\ ]\ e^{-(R(s)-n)(s-t)} ds \tag{7}$$

where all variables are implicitly subscripted by time and:

$$Profit_i = (1-\tau_2)(\ P_i^* Q_i - W_i L_i - P_i^E X_{iE} - P_i^M X_{iM} - P_i^R R_i\ ) \tag{8}$$

[4] The rate of growth of the economy's endowment of effective labor units, n, appears in the discount factor because the quantity and value variables in the model have been scaled by the number of effective labor units. These variables must be multiplied by exp(nt) to convert them back to their unscaled form.

$$R(s) \;=\; \frac{1}{s-t} \int_t^s r(v)dv \tag{9}$$

subject to equations (1) through (6). The taxes included explicitly in this specification are the corporate income tax (τ_2) and an investment tax credit (τ_4). Ad valorem and unit taxes on output are included through the difference between the purchaser's price *P* and producer price *P**. Because all real variables are normalized by the economy's endowment of effective labor units, profits are discounted adjusting for the rate of growth of population plus productivity growth (n). Solving the optimization problem facing this representative firm, we find the set of first-order conditions below:

$$\frac{dY_i}{dL_i} \;=\; \frac{W_i}{P_i} \tag{10}$$

$$\frac{dY_i}{dF_i} \;=\; \frac{P_i}{P} \tag{11}$$

$$\frac{dY_i}{dM_i} \;=\; \frac{P_i^M}{P_i} \tag{12}$$

$$\frac{dY_i}{dR_i} \;=\; \frac{P_i^R}{P_i} \tag{13}$$

$$\lambda_i \;=\; \left(1 \;+\; \phi_i \frac{J_i}{K_i} \right)(1-\tau_4)P^{I} \tag{14}$$

$$\frac{d\lambda_i}{ds} = (r+\delta_i)\lambda_i - (1-\tau_2)\frac{dQ_i}{dK_i} - \frac{\phi_i}{2}P_I(1-\tau_4)\left(\frac{J_i}{K_i}\right)^2 \tag{15}$$

where λ_i is the shadow value of an additional unit of investment in industry I (again, in a particular country).

The first four of the first-order conditions are used to solve for the demand for variable factors of production; these factors (L, E, M and R) are hired to the point where the marginal productivity of these factors equals their prices relative to the output price of the sector. We can also use these factor demand equations together with the production function to rewrite the model in terms of cost functions. In this case the price of the output at each level of the tier structure will be a function of the price of variable inputs and the quantities of available fixed factors (such as capital). The final two first-order conditions can be interpreted as follows. Integrating (15) along the optimum path of capital accumulation (J_i^*, K_i^*) gives:

$$\lambda_i(t) = \int_t^{\infty}(1-\tau_2)\left(\frac{dQ_i^*}{dK_i^*} + \Phi_i\right)e^{-(R(s)+\delta_h)(s-t)}ds \tag{16}$$

$$\Phi_i = \frac{\phi_i}{2}(1-\tau_4)P^I\left(\frac{J_i^*}{K_i^*}\right)^2 \tag{17}$$

and where dQ_i^*/dK_i^* (the marginal product of capital in production), J_i^*, and K_i^* are all evaluated along the optimal path. Φ_i is the marginal product of capital in reducing adjustment costs in investment in sector I at each point in time. Thus, λ_i is the increment to the value of the firm in sector I from a unit increase in its investment at time t. It is related to *q*, the marginal version of

Tobin's Q (Abel, 1979) as follows:

$$q_i = \frac{\lambda_i P_i}{P^I} \tag{18}$$

Thus we can rewrite the investment demand equation (14) as:

$$\frac{J_i}{K_i} = \frac{1}{\phi_i}\left(\frac{q_i}{(1-\tau_2)(1-\tau_4)} - 1 \right) \tag{19}$$

This shows that the rate of gross investment in sector h is a function of "Tobin's q" for that sector.

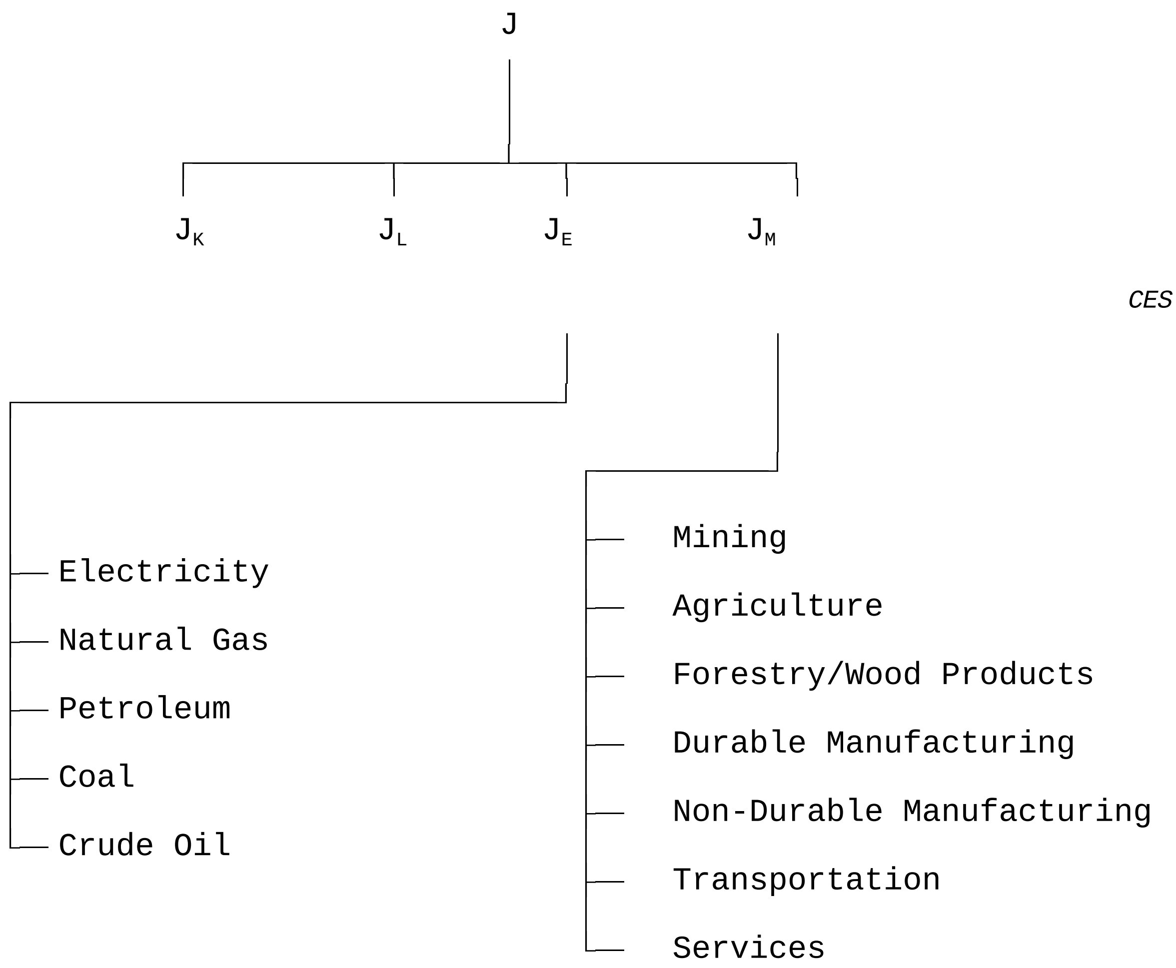
Following Hayashi (1979), we modify the investment function to improve its empirical properties by writing J_i as a function not only of q, but also of the firm's current cash flow at time t:

$$J_i = \alpha_2 \frac{1}{\phi_i}\left(\frac{q_i}{(1-\tau_2)(1-\tau_4)} - 1 \right) K_i + (1-\alpha_2) Profit_i \tag{20}$$

This improves the empirical behavior of the specification and is consistent with the existence of firms that are unable to borrow and therefore invest purely out of retained earnings. The weight on optimizing behavior, α, was taken to be 0.3 based on a range of empirical estimates reported by McKibbin and Sachs (1991).

So far we have described the demand for investment by each sector. We next assume that investment goods are supplied by a firm facing an optimization problem similar to those of the twelve industries described above (and not repeated here). Like other industries, the investment sector demands labor and capital services as well as intermediate inputs. The only difference is

Figure 2.2: Investment Nesting



that we assume there is no sector-specific resource (R) for the investment sector. The investment column in the input-output table is used to parameterize the investment sector's production function. As with the derivation above, there is a shadow "q" associated with investment in the investment goods sector. Production of the investment good is shown schematically in Figure 2.2.

2.2 Households

Households consume goods and services in every period and also demand labor and capital services. Household capital services consist of the service flows of consumer durables plus residential housing. Households receive income by providing labor services to firms and the government, and from holding financial assets. In addition, they also may receive transfers from their region's government.

Within each region we assume household behavior can be modeled by a representative agent. The behavior of the household can be thought of as a sequence of decisions. Households first decide on aggregate consumption for each period. Once this is determined expenditure is allocated across goods and services based on preferences and relative prices. We use a nested constant elasticity of substitution utility function, so income elasticities will be unity and price elasticities can differ from unity (in this version of the model, however, the unitary substitution elasticities have been imposed; this will be relaxed in future work). Aggregate household consumption (C) is nested as shown in Figure 2.3. Total private consumption is allocated between capital, labor, a basket of energy goods and a basket of non-energy goods (i.e. materials). Energy and materials are sub-aggregates of intermediate goods.

We assume household behavior can be modeled by a representative agent with an intertemporal utility function of the form:

$$U_t = \int_t^{\infty} (\ln C(s) + \ln G(s)) e^{-\theta(s-t)} ds \tag{21}$$

where C(s) is the household's aggregate consumption of goods at time s, G(s) is government consumption at s, which we take to be a measure of public goods provided, and θ is the rate of time preference.[5] The household maximizes (21) subject to the constraint that the present value of consumption be equal to human wealth (H) plus initial financial assets (F), all defined in real terms:[6]

Human wealth in real terms is defined as the expected present value of future stream of after tax labor income of households:

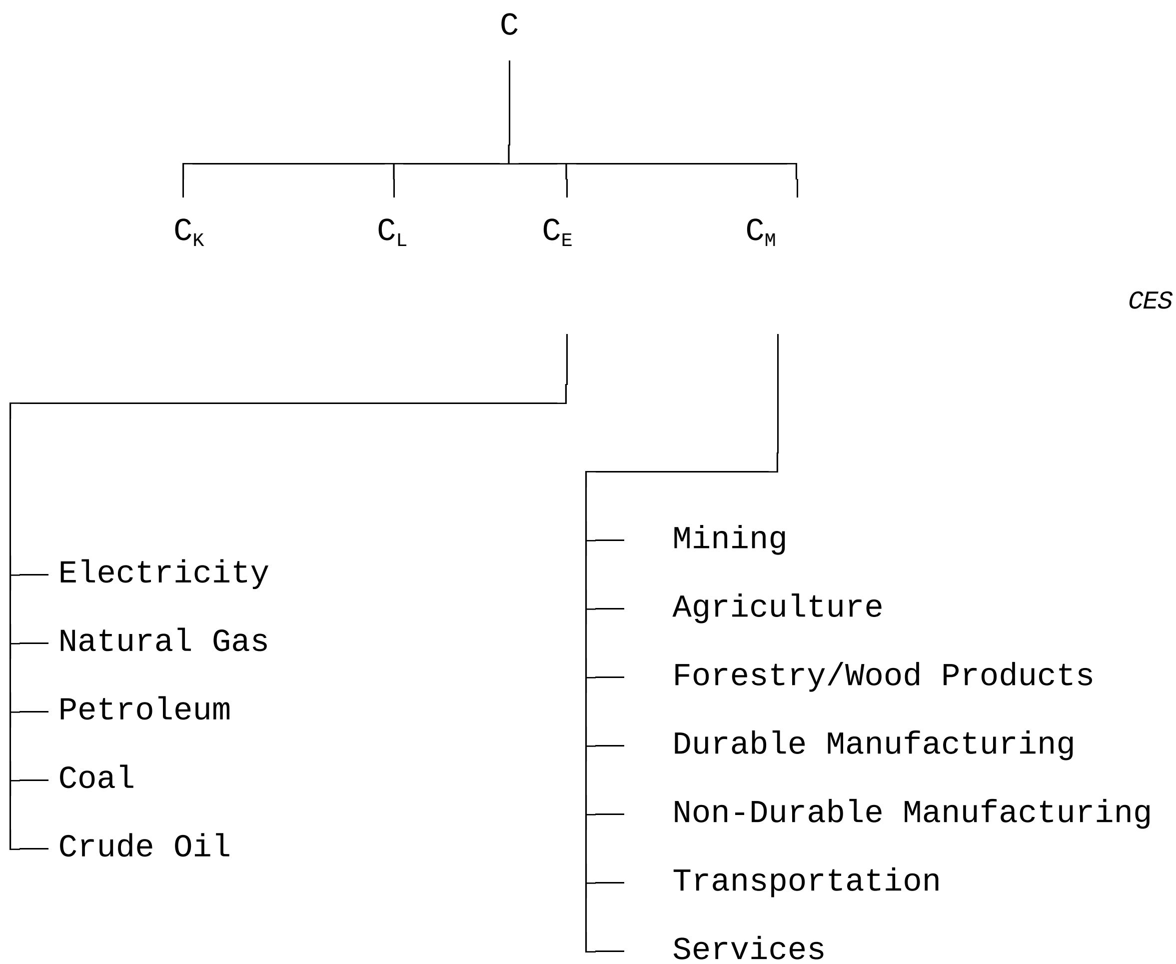
$$H_t = \int_t^{\infty} (1-\tau_1)(W(L_s^G + L_s^C + L_s^I + \sum_{h=1}^{12} L_s^h) + TR) \; e^{-(R(s)-n)(s-t)} ds \tag{22}$$

where TR is the level of government transfers, labor used directly by final consumption is L^C, labor used in producing the investment good is L^I, government employment is L^G, and employment in sector i is given by L^i.

[5] This specification imposes the restriction that household decisions on the allocations of expenditure among different goods at different points in time be separable.

[6] As before, n appears in (14) because the model's scaled variables must be converted back to their unscaled basis.

Figure 2.3: Consumption Nesting



Financial wealth is the sum of real money balance (MON/P), real government bonds in the hand of the public (B), net holding of claims against foreign residents (A) and the value of capital in each sector:

$$F = \frac{MON}{P} + B + A + q^I K^I + q^C K^C + \sum_{i=1}^{12} q^i K^i \tag{23}$$

The solution to this maximization problem is the familiar result that aggregate consumption is equal to a constant proportion of private wealth, where private wealth is defined as financial wealth plus human wealth.

$$C = \frac{\theta(F+H)}{P^c} \tag{24}$$

However, based on the evidence cited by Campbell and Mankiw (1987) and Hayashi (1982)) we assume that only a portion of consumption is determined by these intertemporally-optimizing consumers and that the remainder is determined by after tax current income (INC). This can be interpreted as liquidity constrained behavior or a permanent income model in which household expectations regarding income are backward-looking. Either way we assume that total consumption is a weighted average of the forward looking consumption and backward-looking consumption:

$$C = a_8 \frac{\theta(F_t + H_t)}{P^c} + (1 - a_8)\frac{\gamma INC}{P^c} \tag{25}$$

where a_8 is the share of optimizing consumers and γ is the marginal propensity to save for the liquidity-constrained or backward-looking households.

Once the level of overall consumption has been determined, spending is allocated among goods and services. Households demand each of the model's 12 commodities and also demand labor and capital services. Household capital services consist of the service flows of consumer durables plus residential housing. We assume that the household's preferences can be represented by a nested constant elasticity of substitution utility function.[7] At the top tier of the utility function total consumption is allocated between capital and labor services, a basket of energy goods and a basket of non-energy goods according to a CES function. At the second tier, spending on energy and materials are disaggregated into demands for individual commodities according to additional CES functions. The lower level allocation of consumption expenditure is assumed to be separable from the intertemporal allocation.

The supply of household capital services is determined by consumers themselves who invest in household capital, K^C in order to generate a desired flow of capital services, C^K according to the following production function:

$$C^{K} = \alpha K^{C} \tag{26}$$

Accumulation of household capital is subject to the accumulation equation below:

$$\frac{dK^{C}}{dt} = J^{C} - \delta^{C} K^{C} \tag{27}$$

We assume that changing the household capital stock is subject to adjustment costs so household

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[7] This has the undesirable effect of imposing unitary income elasticities, a restriction usually rejected by data. Moreover, in the preliminary version of the model presented here, the elasticities of substitution have been constrained to be unity. We are in the process of estimating the elasticities econometrically using a long time series of input-output data. In future work we plan to replace this specification with one derived from the linear expenditure system to allow income elasticities to differ from one.

spending on investment, I^C, is related to J^C by:

$$I^C = \left(1 + \frac{\Psi^C}{2}\frac{J^C}{K^C} \right) J^C \tag{28}$$

Thus the household's investment decision is to choose I^C to maximize:

$$\int_t^\infty (P^{CK} \alpha K^c - P^I I^C) e^{-(R(s)-n)(s-t)} ds \tag{29}$$

where P^{CK} is the imputed rental price of household capital.

Solving this problem yields results similar to those discussed for firms above. However, since no variable factors are used in producing capital services, the first order conditions for the problem give investment as a function of the shadow price of capital:

$$J^C = \frac{(q^C - 1)\ K^C}{\phi^C} \tag{30}$$

and an equation for the shadow price of capital itself, where we have introduced $q^C = \lambda P^{CK}/P^I$:

$$\lambda^C = \int_t^\infty (\alpha + \Phi^C) e^{-(R(s)+\delta^C)}\ ds \tag{31}$$

where:

$$\Phi^C = \frac{\phi^C}{2}\left(\frac{J^C}{K^C} \right)^2 \tag{32}$$

Thus, the treatment of household capital is very similar to that used for producing sectors.

2.3 Government

We take each region's real government spending on goods and services to be exogenous and assume that it is allocated among final goods, services and labor in fixed proportions, which we set to 1987 values. Total government outlays include purchases of goods and services plus interest payments on government debt, investment tax credits and transfers to households. Government revenue comes from sales, corporate and personal income taxes, and by issuing government debt. In addition, there can be taxes on externalities such as carbon dioxide emissions. The government budget constraint may be written in terms of the accumulation of public debt as follows:

$$\dot{B}_t = DEF_t = r_t B_t + G_t + TR_t - T_t \tag{33}$$

where B is the stock of debt, DEF is the budget deficit, G is total government spending on goods and services, TR is transfer payments to households, and T is total tax revenue net of any investment tax credit.

We assume that agents will not hold government bonds unless they expect the bonds to be paid off eventually, accordingly impose the following transversality condition:

$$\lim_{s \to \infty} B(s) e^{-(R(s)-n)s} = 0 \tag{34}$$

This restricts the per capita government debt from growing faster than the interest forever. If the government is fully leveraged, this allows (33) to be integrated and written as:

$$B_t = \int_t^{\infty} (T - G - TR) e^{-(R(s)-n)(s-t)} ds \tag{35}$$

Thus, the current level of debt will be equal to the present value of future budget surpluses.[8]

The implication of (35) is that a government running a budget deficit today must run an appropriate budget surplus as some point in the future. Otherwise, the government would be unable to pay interest on the debt and agents will not be willing to hold it. To ensure that (35) holds at all points in time we assume that the government levies a lump sum tax in each period equal to the value of interest payments on the outstanding debt.[9] In effect, therefore, any increase in government debt is financed by consols, and future taxes are raised enough to accommodate the increased interest costs. Thus, any increase in the debt will be matched by an equal present value increase in future budget surpluses. Other fiscal closure rules are possible, such as requiring the ratio of government debt to GDP to be unchanged in the long run. These closures have interesting implications but are beyond the scope of this paper.

[8] Strictly speaking, public debt must be less than or equal to the present value of future budget surpluses. For tractability we assume that the government is initially fully leveraged so that this constraint holds with equality.

[9] In the model the tax is actually levied on the difference between interest payments on the debt and what interest payments would have been if the debt had remained at its base case level. The remainder, interest payments on the base case debt, is financed by ordinary taxes.

Figure 2.4: Trade Matrix

Sector h good

Importer/ Exporter	Country 1	Country 2	....	Country n
Country 1				
Country 2				
....				
Country n				

2.4 Financial Markets and the Balance of Payments

The eight regions in the model are linked by flows of goods and assets. Flows of goods are determined by the import demands described above. These demands can be summarized in a set of bilateral trade matrices, such as the one shown in Figure 2.4, which give the flows of each good between exporting and importing countries. There is one 8 by 8 trade matrix for each of the twelve goods.

Trade imbalances are financed by flows of assets between countries. We assume asset markets are perfectly integrated across the OECD regions. With free mobility of capital, expected returns on loans denominated in the currencies of the various regions must be equalized period to period according to a set of interest arbitrage relations of the following form:

$$i_k = i_j + \frac{dE_k^j/dt}{E_k^j} \tag{36}$$

where E_k^j is the exchange rate between currencies of countries k and j. While we allow for exogenous risk premium in the calibration of the model there is no allowance for endogenous risk premia on the assets of alternative currencies.

Determining initial net asset positions and hence base-case international capital flows is non-trivial. We assume that capital flows are composed of portfolio investment, direct investment and other capital flows. These alternative forms of capital flows are perfectly substitutable ex ante, adjusting to the expected rates of return across economies and across sectors. Within an economy, the expected return to each type of asset (i.e. bonds of all maturities, equity for each sector etc) are arbitraged, taking into account the costs of adjusting physical capital stock and allowing for exogenous risk premia. Because physical capital is costly to adjust, any inflow of financial capital that is invested in physical capital (i.e. direct investment) will also be costly to shift once it is in place. The decision to invest in physical assets is based on expected rates of return. However, if there is an unanticipated shock then ex-post returns could vary significantly. Total net capital flows for each economy in which there are open capital markets are equal to the current account position of that country. The global net flows of private capital are constrained to zero.

We treat the OPEC region differently to the regions which have have full intneral stuctures. We assume that OPEC chooses its foreign lending in order to maintain a desired ratio of income to wealth.

2.5 Labor Market Equilibrium

We assume that labor is perfectly mobile among sectors within each region but is immobile between regions. Thus, within each region wages will be equal across sectors. The wage is assumed to adjust slowly according to an overlapping contracts model where nominal wages are set based on current and expected inflation and on labor demand relative to labor supply. In the long run labor supply is given by the exogenous rate of population growth, but in the short run the hours worked can fluctuate depending on the demand for labor. For a given nominal wage, the sectoral demand for labor will determine short run employment in each industry and thus economy wide unemployment will be the residual between the overall supply, and sectoral demand for labor.

2.6 Money Demand

Finally, we assume that money enters the model via a constraint on transactions.[10] We use a money demand function in which the demand for real money balances is a function of aggregate output and short-term nominal interest rates:

$$\frac{MON}{P} = Y \cdot i^{\epsilon} \tag{37}$$

where Y is aggregate output, I is the interest rate and ϵ is the interest elasticity of money demand. Following McKibbin and Sachs (1991) we take ϵ to be -0.6. The supply of money is determined by the balance sheet of the central bank and is exogenous.

[10] Unlike other components of the model we simply assume this rather than deriving it from optimizing behavior. Money demand can be derived from optimization under various assumptions: money gives direct utility; it is a factor of production; or it must be used to conduct transactions. The distinctions between these models are unimportant for our purposes.

2.7 Parameterization

G-Cubed's parameters fall into three classes: substitution elasticities, share parameters, and other parameters.[11] Most of the substitution elasticities are estimated from U.S. data and are discussed in more detail in section 3. We impose the restriction that substitution elasticities are equal across regions and use the U.S. estimates everywhere. The share parameters (δ's in the derivations above) are derived from regional input-output data (a stylized input-output table is shown in Figure 2.5) and generally differ from one region to another. The share parameters for the United States are taken from a 1987 U.S. input-output table prepared by the Bureau of Labor Statistics. Those for Japan, Australia, China and the Former Soviet Union have been taken from input-output tables for each region. The share parameters for the other regions are calculated by adjusting U.S. share parameters to account for actual final demand components from non-U.S. data. In effect, we are assuming that all regions share a similar but not identical production technology. In addition, the regions also differ in their endowments of primary factors and patterns of final demands.

[11] A complete description of the sources of data used to parameterize G-Cubed is contained in McKibbin and Wilcoxen (1994) Appendices C and D.

Figure 2.5: Stylized Interindustry Accounting Matrix for country k.

	1	2	3	12	C	I	G	X	M	tot
1	N_{k1}^{1k}			N_{k4}^{1k}	C_k^1	I_k^1	G_k^1	X_k^1	IM_k^1	Y_{k1}
2										
3										
12	N_{k1}^{12k}			N_{k12}^{12k}	C_k^{12}	I_k^{12}	G_k^{12}	X_k^{12}	IM_k^{12}	Y_k^{12}
R	R_{k1}				---	---	---	---	---	
K	K_{k1}			K_{k12}	K^C	K^I	---	---	---	
L	L_{k1}			L_{k12}	L^C	L^I	L^G	---	---	
Y										

Trade shares are based on the United Nations SITC (Standard Industry Trade Classification) data for 1987 with sectors aggregated from 4 digit levels to map as closely as possible to the industries defined in the model.[12] Trade price elasticities are currently imposed to be unity; these will be estimated in future work.

2.8 Solution Algorithm

G-Cubed is solved using software developed by McKibbin (1986) for solving large models with rational expectations on a personal computer.[13] The model has approximately 2100 equations in its current form with 47 costate variables. To describe the solution procedure we begin by observing that from a mathematical standpoint, G-Cubed is a system of simultaneous equations which can be written in the form:

$$Z_t = F(Z_t, S_t, C_t, X_t) \tag{38}$$

$$S_{t+1} - S_t = G(Z_t, S_t, C_t, X_t) \tag{39}$$

[12] A full mapping of SITC and SIC codes is contained in a technical appendix available from the authors by request.

[13] For a more detailed description of the algorithm, see Appendix C of McKibbin and Sachs (1991). The software developed for solving this model has been written in the GAUSS programming language.

$$C_{t+1} - C_t = H(Z_t,S_t,C_t,X_t) \tag{40}$$

where Z is a vector of endogenous variables, S is a vector of state variables, C is a vector of costate variables, X is a vector of exogenous variables, and F, G and H are vector functions. The first step in constructing the baseline is to use numerical differentiation to linearize (38) though (40) around the model's database (which is for 1987). We then transform the model into its minimal state space representation by using (38) to find a set of equations f() that allow us to eliminate Z from (39) and (40):

$$S_{t+1} - S_t = G(f(S_t,C_t,X_t),S_t,C_t,X_t) \tag{41}$$

$$C_{t+1} - C_t = H(f(S_t,C_t,X_t),S_t,C_t,X_t) \tag{42}$$

The linearized model is then in the form:

$$dS_{t+1} = (I+G_Zf_S+G_S)dS_t + (G_Zf_C+G_C)dC_t + (G_Zf_X+G_X)dX_t \tag{43}$$

$$dC_{t+1} = (I+H_Zf_C+H_C)dC_t + (H_Zf_S+H_S)dS_t + (H_Zf_X+H_X)dX_t \tag{44}$$

The eigenvalues of this system of equations are then calculated to ensure that the condition for saddle-point stability is satisfied (that is, that the number of eigenvalues outside the unit circle are equal to the number of costate variables). Following that we compute the model's stable manifold as follows. For convenience, define Γ:

$$\Gamma = (I + H_Z f_C + H_C)^{-1} \tag{45}$$

Using Γ we can rewrite (44) to give dC_t in terms of the other variables:

$$dC_t = \Gamma dC_{t+1} - \Gamma(H_Z f_S + H_S) dS_t - \Gamma(H_Z f_X + H_X) dX_t \tag{46}$$

Substituting (46) into (43) gives:

$$\begin{aligned} dS_{t+1} = {} & (I + G_Z f_S + G_S - \Gamma(H_Z f_S + H_S)) dS_t + (G_Z f_C + G_C)\Gamma dC_{t+1} \\ & + (G_Z f_X + G_X - \Gamma(H_Z f_X + H_X)) dX_t \end{aligned} \tag{47}$$

Applying (46) recursively and using (47) allows us to find an expression for the stable manifold for the costate variables in terms of changes in current state variables and all current and future changes in the exogenous variables. The expression will have the following form:

$$dC_t = \Phi \; dS_t + \sum_{i=t}^{T} \Theta_i dX_i + \Omega \; dC_T \tag{48}$$

where Φ, Θ_i, and Ω are matrices of constants. We evaluate Φ, Θ, and Ω numerically; in general, their closed-form expressions will be quite complicated. Once this is found the model can be solved quickly and easily for different experiments because the new values of the costate variables can be calculated simply by evaluating (48). These values can then be inserted into (38) to calculate the other endogenous variables.

3 Parameter Estimation

This section describes the data sources and methods used to estimate the behavioral parameters in G-Cubed. We begin by using each agent's optimization problem to derive a set of equations which can be estimated from available data and which identify all behavioral parameters needed by the model. We then describe how we used data from a variety of sources to construct a consistent time series data set appropriate for estimating these equations. Following that we present and discuss our estimation results.

3.1 Deriving Estimable Equations

The first step in estimating the behavioral parameters appearing in G-Cubed is to derive estimable equations from the optimization problem faced by each agent. On the production side, each of G-Cubed's twelve industries (shown in Table 3.1) is represented by a tier-structured constant elasticity of substitution (CES) production function. At the top tier, output is a function of inputs of capital, labor, energy, materials and sector-specific resources. At the second tier, energy is a CES function of inputs from sectors 1 through 5 while materials is a CES function of inputs from sectors 6 through 12. We will refer to the top CES function as the "output node", and the energy and materials functions as the "energy node" and the "materials node".

The production function representing each node has the following form:

$$Q = A \left(\sum_{i=K,L,E,M,R} \delta_i^{1/\sigma} X_i^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)} \tag{49}$$

where Q is output associated with the node, X_i is the quantity of input I, and A, δ and σ are parameters which vary across industries and across nodes within each industry. Parameter A reflects the level of technology, parameter σ is the elasticity of substitution and the δ_i reflect the weights of

Table 3.1: Industry Definitions

1	Electric utilities
2	Gas utilities
3	Petroleum refining
4	Coal mining
5	Crude oil and gas extraction
6	Mining
7	Agriculture, fishing and hunting
8	Forestry and wood products
9	Durable manufacturing
10	Nondurable manufacturing
11	Transportation
12	Services

different inputs in production. Without loss of generality, we constrain the δ's to sum to one.

For the energy and materials nodes, all inputs are variable and it is convenient to express the production function in its dual form:

$$C = \frac{Q}{A} \left(\sum \delta_i \, p_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \qquad (50)$$

where C is the cost of the node's output, Q is the quantity of output, p_i is the price of input I, and the remaining terms are parameters having the same interpretation as they did in the production function. Since this expression has constant returns to scale it can be rewritten as a unit cost function:

$$c = \frac{1}{A} \left(\sum \delta_i \, p_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \qquad (51)$$

where c is the node's cost per unit of output. Assuming that the energy and materials nodes earn zero profits, c will be equal to the price of the node's output. Using Shepard's Lemma to derive demand equations from the cost function gives:

$$X_i = \delta_i \, p_i^{-\sigma} \, \frac{Q}{A} \left(\sum \delta_i \, p_i^{1-\sigma} \right)^{\frac{\sigma}{1-\sigma}} = \delta_i \, Q \, A^{\sigma-1} \left(\frac{p_i}{p_o} \right)^{-\sigma} \tag{52}$$

where c has been replaced by p_o. Multiplying both sides by p_i and dividing by p_oQ gives the share of costs devoted to input I:

$$s_i = \delta_i \, A^{\sigma-1} \left(\frac{p_i}{p_o} \right)^{1-\sigma} \tag{53}$$

Taking the logarithm of both sides:

$$\ln s_i = \ln\delta_i + (\sigma-1)\ln A + (1-\sigma)(\ln p_i - \ln p_o) \tag{54}$$

This form is convenient for estimation: it is nearly linear and only requires data on prices and cost shares.

The output node must be treated differently because it includes capital, which is not variable in the short run.[14] We assume that the firm chooses variable inputs to maximize its restricted profit function, π:

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[14] It also includes sector-specific resources which are not variable in the short run. However, data limitations discussed below prevented us from estimating demands for sector specific resources in this version of G-Cubed.

$$\pi = p_o Q - \sum p_j X_j \tag{55}$$

where the summation is taken over all variable inputs but excludes capital. It is convenient to rewrite the production function as:

$$Q = A\left(\delta_k^{1/\sigma} X_k^{(\sigma-1)/\sigma} + \sum \delta_j^{1/\sigma} X_j^{(\sigma-1)/\sigma} \right)^{\frac{\sigma}{\sigma-1}} \tag{56}$$

where X_k is the quantity of capital owned by the firm, δ_k is the distributional parameter associated with capital, and j ranges over inputs other than capital. Inserting this into the restricted profit function gives:

$$\pi = p_o\, A\left(\delta_k^{1/\sigma} X_k^{(\sigma-1)/\sigma} + \sum \delta_j^{1/\sigma} X_j^{(\sigma-1)/\sigma} \right)^{\frac{\sigma}{\sigma-1}} - \sum p_j X_j \tag{57}$$

Optimal inputs of variable factors will satisfy a set of first order conditions including the following one for input j:

$$\delta_j^{1/\sigma} X_j^{-1/\sigma} p_o A\left(\delta_k^{1/\sigma} X_k^{(\sigma-1)/\sigma} + \sum \delta_i^{1/\sigma} X_i^{(\sigma-1)/\sigma} \right)^{1/(\sigma-1)} = p_j \tag{58}$$

Taking ratios of the first order conditions for inputs I and j and rearranging gives:

$$X_i = X_j \frac{\delta_i}{\delta_j} \left(\frac{p_j}{p_i} \right)^{\sigma} \tag{59}$$

Substituting this into the first order condition for input j and rewriting gives the firm's factor demand equation for input j:

$$X_j = \delta_j \; p_j^{-\sigma} \delta_k^{1/(\sigma-1)} X_k \left((p_o A)^{1-\sigma} - \sum \delta_i \; p_i^{1-\sigma} \right)^{\sigma/(1-\sigma)} \tag{60}$$

There will be three of these equations: one each for labor, energy and materials. If we assume the firms are price takers, this expression can be used to estimate the output node values of A, σ and the δ parameters from data on prices, inputs and industry capital stocks.

3.2 Constructing the Data Set

In order to estimate the equations derived above we needed a time series data set on prices, industry outputs, and inputs to industries by commodity. Since G-Cubed's industries span production in each region, this amounts to requiring a time series of input-output tables. This section describes how we constructed such a series.

The principal source of raw data was the set of benchmark input-output tables produced by the Bureau of Economics Analysis at the U.S. Department of Commerce. These tables are produced about every five years and are based on detailed surveys such as the Census of Manufactures. However, the industry classifications and other standards used in building the tables have varied a great deal over the years so it was necessary to transform the tables to a consistent basis before they could be used. In this section we discuss the structure of input-output tables and describe the methods we used to build a consistent data set.

An input-output table summarizes all market transactions in an economy in a particular year. Most input-output tables are structured according to the conventions described in the System of National Accounts (SNA) proposed by the United Nations. Under the SNA, all purchases of goods, services, and primary inputs are summarized by a "use" table. There is one row in the table for each good or service, and one column for each buyer. The buyers include both industries and final demand sectors. The SNA collects all information on production of goods and services into a "make" table. There is one row in the make table for each industry and one column for each good or service. The make table allows for joint production: one industry could produce several products.

If there were a one-to-one correspondence between industries and products, the make table would be diagonal and the use table could be interpreted as showing transactions between industries. At the twelve sector level of aggregation used in G-Cubed, the make table is very close to diagonal. We take advantage of this at estimation stage by treating each industry as though it produced a single product.

The structure of G-Cubed's use table is shown in Table 3.2. Five final demand sectors were included: consumption, investment, government spending, exports and imports. The value added portion of the table included two inputs: capital and labor.[15] Summing across the each of the first twelve rows of the table gives the value of commodity output; summing down each of the first twelve columns gives the value of industry output.

In order to estimate G-Cubed's behavioral parameters we needed a consistent time series of input-output tables of the form shown in Table 3.2. To construct the series we began with the benchmark input-output transactions tables produced by the Bureau of Economic Analysis (BEA)

[15] In principle there should also be a primary factor row for the sector specific resource. However, it was not possible to compute inputs of sector specific resources from Bureau of Economic Analysis data. We plan to address this problem in future work.

Table 3.2: The Structure of the G-Cubed Use Table

	1	...	12	C	I	G	X	M
1								
...		A				B		
12								
K								
		C				D		
L								

A) Interindustry transactions.
B) Industry sales to final demand sectors.
C) Purchases of primary factors by industries.
D) Purchases of primary factors by final demand sectors.

for years 1958, 1963, 1967, 1972, 1977 and 1982.[16] The conventions used by the BEA have changed over time, so the raw tables are not completely comparable. In addition, the level of industry detail available on magnetic tape varies from about 80 sectors for early tables to 500 or more sectors for the later ones. The first step in constructing the data set was to convert the older benchmark tables to the 1982 conventions and to a uniform number of sectors. This process is quite involved and not terribly interesting, so we will not describe it here; see McKibbin and Wilcoxen (1994) for a complete discussion. Once the tables were consistent, they were aggregated to 12 sectors. Table 3.4 shows the relationship between G-Cubed sectors and the Standard Industrial Classification and the industry definitions used by the Bureau of Economic Analysis.

[16] A benchmark table also exists for 1947 but it has inadequate final demand detail for our purposes.

Table 3.4: G-Cubed Sectors in terms of BEA and SIC Categories

	G-Cubed Industry Name	BEA Codes	SIC Codes
1	Electric utilities	68.01, 78.02, 79.02	491, B
2	Gas utilities	68.02	492, B
3	Petroleum refining	31	29
4	Coal mining	7	12
5	Crude oil and gas extraction	8	13
6	Mining	5, 6, 9, 10	10, 14
7	Agriculture, fishing and hunting	1, 2, 4	01, 02, 07, 09
8	Forestry and wood products	3, 20, 21	04
9	Durable manufacturing	13, 22, 23, 35-64	24, 25, 32-39
10	Nondurable manufacturing	14-19, 24-30, 32-34	20-23, 26-28, 30, 31
11	Transportation	65	40-42, 44-47, A
12	Services	66, 67, 69, 70-77, 68.03, 78.01, 78.03, 78.04, 79.01, 79.03	48, 494-497, 50-65, 67, 70, 72, 73, 75, 76, 78-84, 86-89, C

A) Includes local government transit, for which no SIC code exists.
B) Includes part of SIC 493 (Combined Services).
C) Includes government enterprises other than local transit and electric utilities.

3.2.1 Consumer Durables

A well-known flaw in the National Income and Product Accounts (and the benchmark input-output tables as well) is that purchases of consumer durables are treated as consumption rather than investment. Since we include an explicit model of consumer investment in G-Cubed, this approach is not satisfactory. The next adjustment we made to the data set was to transfer consumer durables from consumption to investment.

At first it might appear that this adjustment is trivial to make: the entry for durables in the consumption column could simply be shifted to investment. However, this is incorrect because of the way transportation and wholesale and retail trade margins are treated in input-output accounting. In particular, the trade margins associated with durables are included in the trade row of the consumption column, not in the value of durables. These margins are substantial: in 1982, the value of consumer durables at purchaser's prices (after margins) was 61% higher than the value at producer prices (before margins). In other words, shifting only the producer-price value of durables would understate consumer investment in durables by about 40%.

Some, but not all, of the benchmark tables include information on the value of margins associated with consumer durables. In particular, this data was available for 1982. Under the assumption that the ratios of trade and transportation margins to the producer price value of durables has remained relatively constant, we used the 1982 ratios to infer the margins for other years. We then shifted the entire durable entry in the consumption column to investment, and also shifted the inferred trade and transportation margins from consumption to the corresponding rows of the investment column. The final step was to increase the capital entry in the consumption column to account for the imputed services flows from both consumer durables and housing. This will be discussed in more detail below.

3.2.2 Value Added

From the standpoint of estimating cost and production functions, the least satisfactory part of the benchmark input-output tables are the value added rows. In the early years, labor and capital are not disaggregated. In all years, the methods used by BEA to construct implicit price deflators for labor and capital are subject to various methodological problems. One example is that the income of proprietors is not split between capital and imputed labor income correctly. Primary factors often account for half or more of industry costs so it is particularly important that this part of the data set be constructed as carefully as possible.

We chose to address these problems as follows. For each benchmark year we take the total value of primary factor inputs from the corresponding input-output table. We then split those values up into labor and capital inputs using a data set constructed by Dale Jorgenson and his colleagues.[17] We also use the Jorgenson data set to calculate price indices for labor and capital by industry.

The capital service flows appearing in the Jorgenson data set include several extra imputations not found in the National Income and Product Accounts. First, additional capital income was imputed to the Other Transportation Equipment sector in certain years in which the NIPA data show a net loss. This was necessary to ensure that the sector's price of capital was always nonnegative. Second, extra capital income was imputed to Government Enterprises to account for the capital used by nonprofit organizations like the Postal Service. Some of this imputation was then transferred to the electric utilities sector when the government enterprises sector was decomposed into utilities and other activities. Third, capital input to the final demand category for consumption was adjusted to include the imputed values of owner occupied housing and consumer durables.

3.2.3 Price Data

The remaining data needed are prices for each good in each benchmark year. We obtain these from the output and employment data set constructed by the Office of Employment Projections

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[17] This data set is the work of several people over many years. In addition to Dale Jorgenson, some of the contributors were L. Christensen, Barbara Fraumeni, Mun Sing Ho and Dae Keun Park. The original source of the data is the Fourteen Components of Income tape produced by the Bureau of Economic Analysis. See Ho (1989) for more information.

at the Bureau of Labor Statistics. The BLS data set divides production into 224 industries; the correspondence between G-Cubed sectors and those in the BLS data set are shown in Table 3.5.

Table 3.5: Relationship Between G-Cubed and BLS Sectors

	G-Cubed Sector	BLS Sector
1	Electric utilities	155,214,219
2	Gas utilities	156
3	Petroleum refining	138,139
4	Coal mining	7
5	Crude oil and gas extraction	8,9
6	Mining	6,10
7	Agriculture, fishing and hunting	1-3,5
8	Forestry and wood products	4,30-36
9	Durable manufacturing	37-103
10	Nondurable manufacturing	104-137,140-144
11	Transportation	145-152,218
12	Services	153,154,157-166, 168-211, 213,216,220

Prices for G-Cubed classifications were formed from the indicated groups of BLS sectors by Divisia aggregation.

3.3 Estimating the Parameters

The first step in estimating the parameters was to construct Divisia price and quantity indices for the energy and materials nodes in each industry's cost function (since the aggregate prices of energy and materials to each sector were unobservable). Next, the energy and materials nodes were estimated under the assumption that the inputs to those nodes were variable in the short run. Finally, the output node was estimated under the restriction that capital was fixed in the short run. To determine the significance of holding capital fixed we also estimated the output node under the assumption that all inputs, including capital, were variable in the short run.[18]

3.3.1 Energy and Materials Nodes

Because we had relatively few observations on interindustry trade it was necessary to estimate the energy and materials nodes in a sequence of steps. The estimating equation derived in the first section of this chapter was:

$$\ln s_i = \ln\delta_i + (\sigma - 1)\ln A + (1 - \sigma)(\ln p_i - \ln p) \tag{61}$$

We began by estimating a version of this in which some of the parameters were not identified:

$$\ln s_i = \alpha_i + (1 - \sigma)(\ln p_i - \ln p) \tag{62}$$

In this expression s_i is the share of input I in the total value of the node being estimated, α_i is an estimated parameter which is implicitly a function of the underlying behavioral parameters A and σ appearing in (13), σ is the elasticity of substitution, p_i is the price of input I and p is the price of the aggregate. The advantage of this approach is that (14) is linear in its parameters.

For each energy and materials node we estimated an appropriate system of equations of the form shown above. We dropped any equations for which the value of s_i was very small and constrained the elasticity to be equal across the remainder.[19] Each system of equations was estimated

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[18] Although it is clearly incorrect to assume that capital is variable in the short run, this assumption seems to be common in the empirical literature on cost and production functions.

[19] Dropping these equations was necessary precisely because the elasticity was constrained to be equal across equations. Otherwise, very small inputs could have unreasonably large effects

Table 3.6: Non-Zero Energy and Materials Inputs
(Columns correspond to inputs; rows to industries)

	1	2	3	4	5	6	7	8	9	10	11	12
1		X	X	X					X	X	X	X
2		X			X				X	X	X	X
3	X	X	X		X				X	X	X	X
4	X		X	X				X	X	X	X	X
5	X	X	X		X				X	X	X	X
6	X	X	X	X		X			X	X	X	X
7	X	X	X				X		X	X	X	X
8	X	X	X	X			X	X	X	X	X	X
9	X		X	X		X			X	X	X	X
10	X	X	X	X	X		X		X	X	X	X
11	X		X						X	X	X	X
12	X	X	X						X	X	X	X

by the method of maximum likelihood. Table 3.6 indicates which inputs were non zero in each industry.

After obtaining preliminary estimates of σ from (14), we fixed the σ's and estimated (13) for each node using maximum likelihood. This produced preliminary estimates of A and δ_i. Finally, we used the preliminary estimates of σ, A and δ_i as a starting guess for one further regression in which we estimated (13) without constraints. This gave us the final set of parameters for the energy and materials nodes. The results are shown in Table 3.7.

on the estimated elasticity.

3.3.2 The Output Node

We were able to estimate the output node directly without having to break the problem into a series of steps. For each industry we estimated a system of simultaneous factor demands for labor, energy and materials. Each demand equation had the form derived in equation (52), repeated below:

$$X_j = \delta_j \, p_j^{-\sigma} \delta_k^{1/(\sigma-1)} X_k \left((p_o A)^{1-\sigma} - \sum \delta_i \, p_i^{1-\sigma} \right)^{\sigma/(1-\sigma)} \tag{63}$$

where X_k is the industry's stock of capital. There was no demand equation for capital itself since it was assumed to be fixed in the short run. The results appear in Table 3.7.

Much of the literature on cost and production functions fails to account for the short run fixity of capital. Rather than using (63), the common approach is to use factor demands of the form:

$$X_i = \delta_i \, p_i^{-\sigma} \frac{Q}{A} \left(\sum \delta_i \, p_i^{1-\sigma} \right)^{\frac{\sigma}{1-\sigma}} \tag{64}$$

As shown in the first section of this chapter, this expression is correct only if all inputs are variable in the short run. Equation (64) differs from (63) in one very important respect: (64) has constant returns to scale. This implies that Q is exogenous in (64), both in terms of economic interpretation and econometric specification. (The firm's supply curve will be perfectly horizontal so the firm will be indifferent about its scale of output.) In (63), however, Q is implicitly endogenous while the price of output (p) is exogenous. In other words, the firms described by (63) have upward-sloping marginal cost curves.

Using (64) instead of (63) may bias the estimated elasticity of substitution downward in a relatively inflexible econometric specification such as the CES. This comes about because fixity of the capital stock would be misinterpreted as a lack of substitutability among inputs. To gauge the importance of using the correct specification (63) rather than (64) we estimated the output node using both specifications. These results are shown in Table 3.8.

3.3.3 Estimation Results

Table 3.7 presents parameter estimates for the energy, materials and output nodes of each industry's production function. The values in parentheses are standard errors.[20] Each column corresponds to a different industry while each row corresponds to a parameter. The first seven rows are parameters in the energy node: SE is the elasticity of substitution, AE is the technology parameter A, and D1 through D5 are the δ parameters for each fuel. Parameters for the materials node appear in subsequent rows and have similar interpretations. The last six rows contain parameter estimates for the output tier under the fixed-capital stock assumption. D15 through D18 are the δ parameters for capital, labor, energy and materials, respectively.

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[20] t-statistics are not reported because for most of the parameters zero is not an interesting or relevant null hypothesis. Virtually all of the parameters are significantly different from zero, as would be expected.

Table 3.7: Estimation Results, Fixed Capital Stock

	1	2	3
SE	0.2000	0.9325 (0.3473)	0.2000
AE	1.0978 (0.0225)	1.2968 (1.6823)	0.9997 (0.0016)
D1	0.0000	0.0000	0.0060 (0.0009)
D2	0.3300 (0.0179)	0.6426 (0.0309)	0.0175 (0.0012)
D3	0.2356 (0.0475)	0.0000	0.1045 (0.0033)
D4	0.4344 (0.0318)	0.0000	0.0000
D5	0.0000	0.3574 (0.0309)	0.8720 (0.0050)
SM	1.0000	0.2000	0.2000
AM	1.0000	1.0748 (0.0205)	1.0535 (0.0050)
D6	0.0000	0.0000	0.0000
D7	0.0000	0.0000	0.0000
D8	0.0000	0.0000	0.0000
D9	0.1539 (0.0209)	0.0999 (0.0163)	0.0863 (0.0109)
D10	0.0500 (0.0034)	0.0424 (0.0033)	0.2168 (0.0142)
D11	0.3627 (0.0125)	0.0807 (0.0265)	0.3125 (0.0143)
D12	0.4334 (0.0116)	0.7769 (0.0302)	0.3844 (0.0190)
SO	0.7634 (0.0765)	0.8096 (0.0393)	0.5426 (0.0392)
AO	1.1188 (0.0363)	1.2626 (0.0072)	0.9791 (0.0020)
D15	0.3851 (0.0277)	0.2466 (0.0053)	0.0736 (0.0025)
D16	0.2150 (0.0137)	0.1332 (0.0095)	0.0555 (0.0047)
D17	0.2585 (0.0364)	0.5799 (0.0118)	0.7592 (0.0102)
D18	0.1413 (0.0051)	0.0403 (0.0030)	0.1118 (0.0034)

Table 3.7, continued: Estimation Results, Fixed Capital Stock

	4	5	6
SE	0.1594 (0.1208)	0.1372 (0.0339)	1.1474 (0.1355)
AE	1.0290 (0.0042)	0.9920 (0.0159)	0.9401 (0.0685)
D1	0.1028 (0.0062)	0.1137 (0.0149)	0.5129 (0.0185)
D2	0.0000	0.0448 (0.0069)	0.1727 (0.0139)
D3	0.1007 (0.0190)	0.1077 (0.0116)	0.2891 (0.0195)
D4	0.7965 (0.0238)	0.0000	0.0253 (0.0043)
D5	0.0000	0.7337 (0.0286)	0.0000
SM	0.5294 (0.0187)	0.2000	2.7654 (0.0278)
AM	1.0258 (0.0037)	1.0442 (0.0079)	0.9815 (0.0035)
D6	0.0000	0.0000	0.1510 (0.0121)
D7	0.0000	0.0000	0.0000
D8	0.0240 (0.0033)	0.0000	0.0000
D9	0.4034 (0.0152)	0.1461 (0.0212)	0.2946 (0.0143)
D10	0.1157 (0.0055)	0.0417 (0.0037)	0.1318 (0.0065)
D11	0.0437 (0.0056)	0.0353 (0.0075)	0.0570 (0.0105)
D12	0.4133 (0.0146)	0.7769 (0.0243)	0.3656 (0.0226)
SO	1.7030 (0.0380)	0.4934 (0.0310)	1.0014 (0.3146)
AO	1.3681 (0.0638)	1.7834 (0.0785)	0.0001 (0.0009)
D15	0.3669 (0.0242)	0.5849 (0.0095)	0.2302 (0.8571)
D16	0.3058 (0.0142)	0.1670 (0.0068)	0.3214 (0.3698)
D17	0.1088 (0.0093)	0.0497 (0.0069)	0.0698 (0.0896)
D18	0.2185 (0.0035)	0.1984 (0.0049)	0.3786 (0.3979)

Table 3.7, continued: Estimation Results, Fixed Capital Stock

	7	8	9
SE	0.6277 (0.0510)	0.9385 (0.1380)	0.8045 (0.0582)
AE	1.0208 (0.0157)	1.2990 (0.7145)	3.8779 (1.5069)
D1	0.1488 (0.0204)	0.3489 (0.0228)	0.5019 (0.0251)
D2	0.0258 (0.0066)	0.0993 (0.0097)	0.0000
D3	0.8254 (0.0267)	0.5377 (0.0233)	0.3013 (0.0070)
D4	0.0000	0.0141 (0.0045)	0.1968 (0.0236)
D5	0.0000	0.0000	0.0000
SM	1.7323 (0.1052)	0.1757 (0.0000)	0.2000
AM	0.9924 (0.0072)	1.0046 (0.0025)	1.0287 (0.0033)
D6	0.0000	0.0000	0.0265 (0.0032)
D7	0.5350 (0.0178)	0.0583 (0.0043)	0.0000
D8	0.0000	0.5934 (0.0117)	0.0000
D9	0.0225 (0.0015)	0.0792 (0.0112)	0.6592 (0.0115)
D10	0.1997 (0.0125)	0.0594 (0.0033)	0.0913 (0.0036)
D11	0.0278 (0.0016)	0.0615 (0.0058)	0.0436 (0.0015)
D12	0.2151 (0.0054)	0.1483 (0.0069)	0.1794 (0.0114)
SO	1.2830 (0.0469)	0.9349 (0.0802)	0.4104 (0.0193)
AO	0.8650 (0.0051)	0.9741 (0.0107)	1.0124 (0.0029)
D15	0.1382 (0.0101)	0.1140 (0.0130)	0.0682 (0.0011)
D16	0.2471 (0.0113)	0.2747 (0.0087)	0.3402 (0.0027)
D17	0.0194 (0.0020)	0.0251 (0.0033)	0.0312 (0.0016)
D18	0.5953 (0.0022)	0.5862 (0.0087)	0.5604 (0.0018)

Table 3.7, continued: Estimation Results, Fixed Capital Stock

	10	11	12
SE	1.0000	0.2000	0.3211 (0.0449)
AE	1.0000	1.0379 (0.0054)	1.0086 (0.0052)
D1	0.3492 (0.0105)	0.0581 (0.0060)	0.4313 (0.0062)
D2	0.2374 (0.0124)	0.0000	0.1619 (0.0055)
D3	0.2962 (0.0145)	0.9419 (0.0060)	0.4068 (0.0047)
D4	0.0304 (0.0025)	0.0000	0.0000
D5	0.0868 (0.0078)	0.0000	0.0000
SM	0.0573 (0.0000)	0.2000	3.0056 (0.0728)
AM	1.0412 (0.0034)	1.1182 (0.0117)	0.9867 (0.0008)
D6	0.0000	0.0000	0.0000
D7	0.1841 (0.0095)	0.0000	0.0000
D8	0.0000	0.0000	0.0000
D9	0.0591 (0.0020)	0.1300 (0.0032)	0.0938 (0.0046)
D10	0.5263 (0.0053)	0.0550 (0.0041)	0.1349 (0.0106)
D11	0.0487 (0.0018)	0.3673 (0.0274)	0.0347 (0.0012)
D12	0.1817 (0.0047)	0.4477 (0.0219)	0.7366 (0.0128)
SO	1.0044 (0.0117)	0.5368 (0.0700)	0.2556 (0.0272)
AO	0.9496 (0.0057)	0.9236 (0.0138)	1.0000 (0.0164)
D15	0.1034 (0.0038)	0.1263 (0.0082)	0.1942 (0.0033)
D16	0.2613 (0.0027)	0.4876 (0.0055)	0.4764 (0.0129)
D17	0.0167 (0.0016)	0.0776 (0.0089)	0.0312 (0.0008)
D18	0.6186 (0.0015)	0.3086 (0.0054)	0.2982 (0.0109)

Table 3.8: Estimation Results, Variable Capital Stock

	1	2	3
SO	0.8662 (0.0100)	0.7812 (0.0010)	1.0381 (0.0089)
AO	1.2518 (0.0546)	1.2353 (0.0339)	1.0884 (0.0308)
D15	0.4777 (0.0112)	0.2355 (0.0066)	0.1177 (0.0049)
D16	0.2604 (0.0046)	0.1368 (0.0087)	0.1382 (0.0046)
D17	0.1399 (0.0081)	0.5867 (0.0178)	0.5533 (0.0116)
D18	0.1220 (0.0040)	0.0411 (0.0041)	0.1908 (0.0040)

Table 3.8, continued: Estimation Results, Variable Capital Stock

	4	5	6
SO	0.9903 (0.0006)	0.9537 (0.0074)	1.0014 (0.0006)
AO	1.3910 (0.1009)	2.1141 (0.2181)	0.0001 (0.0000)
D15	0.1939 (0.0066)	0.4779 (0.0046)	0.2305 (0.0148)
D16	0.3947 (0.0079)	0.1891 (0.0078)	0.3213 (0.0089)
D17	0.1612 (0.0039)	0.0426 (0.0025)	0.0698 (0.0131)
D18	0.2501 (0.0037)	0.2904 (0.0126)	0.3784 (0.0054)

Table 3.8, continued: Estimation Results, Variable Capital Stock

	7	8	9
SO	1.1503 (0.0140)	0.9465 (0.0196)	1.0432 (0.0108)
AO	0.8864 (0.0096)	0.9643 (0.0160)	0.9856 (0.0188)
D15	0.1738 (0.0037)	0.1243 (0.0081)	0.1133 (0.0028)
D16	0.2106 (0.0057)	0.2898 (0.0074)	0.3240 (0.0031)
D17	0.0254 (0.0021)	0.0179 (0.0012)	0.0184 (0.0011)
D18	0.5902 (0.0022)	0.5679 (0.0045)	0.5444 (0.0032)

Table 3.8, continued: Estimation Results, Variable Capital Stock

	10	11	12
SO	0.9832 (0.0001)	0.8602 (0.0136)	0.9428 (0.0043)
AO	0.9362 (0.0199)	0.9353 (0.0366)	0.9954 (0.0253)
D15	0.1019 (0.0018)	0.1622 (0.0048)	0.2217 (0.0080)
D16	0.2620 (0.0025)	0.4737 (0.0063)	0.4789 (0.0047)
D17	0.0154 (0.0008)	0.0553 (0.0035)	0.0219 (0.0014)
D18	0.6208 (0.0024)	0.3088 (0.0091)	0.2776 (0.0092)

Some of the parameters appearing in the table without standard errors were imposed. Most of these are δ parameters for inputs which were negligible. A few elasticities were imposed because their estimated values came out negative or because it was impossible to get the estimation procedure to converge otherwise.

3.4 Some Conclusions Regarding Parameter Estimation

The principal finding in Table 3.7 is that many of the nodes have elasticities fairly far from unity. For the output elasticities, in particular, statistical tests would strongly reject the hypothesis that the output node is Cobb-Douglas. From this we conclude that it is essential to estimate these elasticities in order to obtain meaningful simulation results.

Comparing Tables 3.7 and 3.8 shows that the treatment of capital has a very significant effect on the estimated elasticities of substitution. The estimates in Table 3.8 are not all biased downward but virtually all are biased toward unity, some substantially so. In sector 3. for example, the variable-capital elasticity estimate is 1.0381 while in the fixed-capital case it is 0.5426. Sector 3 is petroleum refining and the lower estimate obtained in the fixed-capital case seems more reasonable than an elasticity of unity. The results for other sectors are similarly intuitive. We conclude that it is essential to treat capital fixity correctly in order to obtain useful estimates of elasticities of substitution.

The main limitation of this approach is that there are very few benchmark input-output tables, so our data set contains few observations. In future work we plan to extend the data set back to include the 1947 input-output table and forward to include benchmark tables built by the Bureau of Labor Statistics for 1987 and 1990. This would improve the parameter estimates by increasing the number of data points substantially.

4 Generating a Baseline from 1990 to 2020

In order to construct baseline projections for G-Cubed we begin by making assumptions about the future course of several key exogenous variables:

- population growth by region;
- productivity growth by sector by region;
- energy efficiency improvements by sector by region;
- tax rates by region;
- fiscal spending patterns on each sector output by region;
- monetary policy by region
- short term nominal interest rate or growth path for money balances.
- real price of oil;
- other exogenous shifts in spending patterns..

We take the underlying long-run rate of world population growth plus productivity growth to be 2.5 percent per annum. We also assume that the long-run real interest rate is 4.5 percent in the baseline. Our remaining assumptions are listed in Table 4.1. Some of these are not much more than rough guesses; the model would benefit from further refinement in this area.

These assumptions pose a small problem: the solution of the model in 1990--the base period for the simulation--will not necessarily give values of variables that are equal to the values contained in the model database for 1990. In particular, it is unlikely that the costate variables based on current and expected future paths of exogenous variables in the *model*, will equal the *actual* values of those variables in the database for 1990. The real expectations held by agents in 1990 almost certainly differ from what we assume about future variables in the model. This discrepancy is inconvenient when interpreting the model's results.

Table 4.1: Regional Assumptions Used in Generating the Baseline

	USA	Japan	Aust	ROECD	China	LDCs	EEB
Population growth	0.5%	0.0%	0.8%	0.7%	1.5%	1.0%	0.5%
non-energy productivity growth	2.0%	2.5%	2.2%	2.3%	4%	2.5%	2.0%
energy sector productivity growth	1.5%	2.0%	1.7%	1.8%	4%	2.5%	1.5%
energy efficiency growth	1%	1%	1%	1%	1%	1%	1%
tax rates	1990 levels	1990 levels	1990 levels	1990 levels	1990 levels	1990 levels	1990 levels
fiscal spending	1990 shares	1990 shares	1990 shares	1990 shares	1990 shares	1990 shares	1990 shares
monetary policy (fixed money growth rate)	2.9 %	1.25 %	1.64 %	3.98 %	12.84 %	6.48 %	23.81 %
real oil price	1990 levels	1990 levels	1990 levels	1990 levels	1990 levels	1990 levels	1990 levels

To solve this problem we add a set of constants, one for each costate variable, to the model's costate equations. For example, the constants for Tobin's q for each sector in each country are added to the arbitrage equation for that q. These constants can be interpreted as risk premia. Similarly, constants for each real exchange rate are added to the real interest arbitrage equation for each country, and a constant for human wealth is added to the equation for human wealth.

One additional problem is to solve for both real and nominal interest rates consistently since the real interest rate is the nominal interest rate from the money market equilibrium less the ex ante expected inflation rate. To produce the expected inflation rate implicit in the database for 1990 we add a constant to the equation for nominal wages in each country. This can be interpreted as a shift in the full employment level of unemployment in these equations. This technique of getting the database value of expected inflation for 1990 is equivalent to using the full model to solve for the natural rate of unemployment in each country.

We calculate the values of the constants by using Newton's Method to find a vector of constants that will make the model's costate variables in 1990 exactly equal their 1990 base case values. After adding the constants we can show that the model will reproduce the base year values for 1990 conditional on all of the following: the dynamic variables inherited from 1989; the assumed future paths of all exogenous variables in the model; and the constants added to the costate equations.

We can then solve the model for each period after 1990 given any shocks to variables, information sets or initial conditions. The new steady state of the model will be a function of the intertemporal constants since these constants act like risk premia in a variety of places. The path of the model as well as its equilibrium is therefore affected by these adjustments. Preliminary tests underlying the results in this report suggest that the dynamic properties of the model in the short term are not significantly affects by these adjustments, so that calculations of percentage deviations from baseline are not much different from our technique of calculating percentage deviations from an artificial baseline.

After adjusting the costate equations, we then solve the model forward to 2045. Although a vast number of variables are generated in the baseline, here we concentrate on the path of carbon emissions. Table 4.2 shows the shares of each region in global carbon emissions originating from fossil fuels. Figure 4.1 shows the annual path of carbon emissions from use of fossil fuels in each region from 1990 to 2020. Figure 4.1 and Table 4.2 highlight the more rapid growth of carbon emissions from the developing country regions of the model relative to the OECD regions. Global emissions of carbon are projected to rise from 5,388 million metric tons of carbon in 1990 to 11,752 million tons in 2020. United States emissions over this same period rise from 1339 million tons in 1990 to 1,854 million tons of carbon in 2020.

Table 4.2: Share of Each Region in Global Carbon Emissions

	1990	2000	2010	2020
USA	24.9	21.1	16.7	12.7
Japan	5.9	5.1	4.2	3.5
Australia	1.4	1.4	1.3	1.1
Other OECD	19.0	17.4	15.5	13.6
China	11.3	14.5	15	16.2
LDCs	18.8	19.1	21.9	23.9
Eastern Europe and former Soviet Union (EEB)	18.7	21.4	25.5	29.0

5 Conclusion

G-Cubed is a detailed, comprehensive world economic model suitable for analyzing the effects of environmental policies on international trade. In this model, the world economy has been disaggregated into eight regions with twelve industries in each. International trade is represented by a detailed set of bilateral trade matrices. Intertemporal optimization is used to model saving, investment, and asset market arbitrage. Where appropriate, the existence of liquidity constrained agents is taken into account. Behavioral parameters have been estimated from time-series data wherever possible. All temporal and intertemporal budget constraints are satisfied at all times.

Some of the model's results turn on a few key assumptions. For example, our use of an Infinitely-lived representative agent drives savings behavior and causes the supply of savings to be very elastic in the long run. Other formulations of the household optimization problem could change this.

A second caveat is that any simulations involving absolute targets for carbon emissions or other pollution would depend very heavily on the assumptions we make about future rates of labor force growth, productivity growth, and technical change. This problem is not unique to G-Cubed--all analyses of very long run policies must confront it. However, more work is clearly needed to identify the best possible set of such assumptions.

A related area where G-Cubed could be improved is in its treatment of the income elasticities of consumer demand. Our current formulation implies that budget shares will be independent of income, a fact which is clearly inconsistent with empirical studies. In future work we intend to address this problem by reformulating the consumer model.

A fourth caveat is that our parameter estimates for several countries, particularly the LDC's, are derived from time-series estimates from U.S. data. This is an unfortunate necessity brought on by the lack of time series input-output data for many developing countries. Additional data would substantially improve G-Cubed's representation of non-OECD production. A related point is that G-Cubed does not include any special treatment of the informal sector in developing countries.

Despite these caveats, G-Cubed provides a rigorous, empirically-based tool for studying economic policy in an international context. To date our applications have been on carbon dioxide abatement but the model can be used to analyze a much wider range of issues including: other environmental policies, energy policy, tax policy, monetary and fiscal policy, or trade policy. It has already been used to study the Uruguay Round of the GATT, other regional trading arrangements, and other issues related to productivity growth and trade. These exercises only scratch the surface of the model's potential applications.

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