BROWN CENTER on
Education Policy
at BROOKINGS

September 2013

The Algebra Imperative: Assessing Algebra in a National and International Context

Tom Loveless

INTRODUCTION

Over the past two decades, algebra has acquired elevated status within the U.S. school curriculum. Researchers have documented that readiness for both college-level mathematics and technically-oriented employment hinges on students gaining, at least by the end of high school, a basic knowledge of algebra. [1] The recognition of algebra's "gatekeeper" role within the continuum of high school math courses--that it must be taken and passed by any student who aspires to take calculus or other advanced mathematics--led Robert Moses, a 1982 MacArthur fellow, to declare algebra a civil rights issue. [2]

These developments present a challenge for policymakers: the need to measure--in a sound, trustworthy manner--national progress in learning algebra. The essay below explores how that goal can be accomplished. The essay is organized by four sections. The first section describes the current state of affairs in assessing algebra—the national and international tests that Americans rely on to measure progress. Section two presents evidence that the current battery of assessments is inadequate. Section three discusses prospects for remedying the situation. Section four concludes.

Appreciation is extended to the participants of the Brookings Meeting on Algebra/Geometry Assessments: Anne-Berit Kavli, Norwegian Directorate for Education and Training; Richard Askey, University of Wisconsin-Madison; Roger Howe, Yale University; Frederick Leung, The University of Hong Kong; Ina Mullis, Boston College; Mick Martin, Boston College; Ryan Olson, Kern Family Foundation; Morgan Polikoff, University of Southern California; Skip Fennell, McDaniel College; Dan McGrath, National Center for Education Statistics; Eugene Owen, National Center for Education Statistics. Thanks are also extended to Grover (Russ) Whitehurst, Matthew Chingos, and Morgan Polikoff for their thoughtful comments.

We also thank the Kern Family Foundation for its generous support.

I. The current state of affairs

Academic achievement in the U.S. is tracked by international, national, and state assessments. Beginning with international assessments, let's examine how algebra is assessed within the current testing regime.

Trends in International Math and Science Study (TIMSS)

The TIMSS 8th grade test in mathematics produces scores in the content domain of Algebra, but that subsection of the test is not intended to survey a broad list of topics taught in Algebra I and Algebra II. In the U.S., most of the TIMSS algebra topics are covered in a typical pre-algebra textbook. Eight algebra topics are tested on the eighth grade TIMSS. On the TIMSS survey of 8th grade math teachers, American teachers report that large majorities of students (more than 75%) are taught the TIMSS algebra topics, despite the fact that only about 47% of U.S. eighth graders are enrolled in an algebra or geometry class. Another 23% are enrolled in pre-algebra at the time they take TIMSS.[3] Teachers report that three of the TIMSS algebra topics—patterns and sequences, simple linear equations and inequalities, and equivalent representation of functions—are taught to 79 to 94% of eighth graders in the U.S. They are taught to 60 to 66% of eighth graders worldwide.[4] Quadratic equations and polynomials are algebra topics not assessed by TIMSS.[5] Pre-algebra or introductory topics are assessed, but nothing close to the complete list of topics that students encounter in a formal high school course.

Program for International Student Assessment (PISA)

The Program for International Student Assessment (PISA) tests fifteen year olds in mathematical literacy. The test is not curriculum-based and instead seeks to assess the ability of students to apply what they have learned. As a result, PISA does not report scores in the domain of algebra, but it does have "clusters" describing mathematical concepts. The cluster most closely related to algebra is called "Change and Relationships." The skills in this cluster do not easily map onto traditional topics in algebra, although most of the assessed tasks would qualify as applications or problem solving using algebra.

TIMSS Advanced

TIMSS Advanced tests students in their final year of secondary schooling (twelfth grade for most countries) who are enrolled in their nation's most advanced math and science courses. Ten countries participated in 2008 and sixteen in 1995. The math portion of the test consists of algebra, geometry, and calculus.[6] The assessment targets students who are likely to go on to study mathematics or science at the university level. The proportion of the age cohort enrolled in advanced courses varied among the 2008 TIMSS Advanced countries; for example, Slovenia, 40.5%; Sweden, 12.8%; Norway, 10.9%; Netherlands, 3.5%; Russian Federation, 1.4%. The U.S. did not participate in the 2008 TIMSS Advanced, but if the sampling population were based on calculus enrollments, about 19% of students would qualify for the test—and significantly less if enrollment in AP Calculus were the criterion for participation.

National and State Indicators of Proficiency in Algebra

The era of accountability that dawned in the U.S. in the 1990s was extended throughout the land by the No Child Left Behind Act in 2002. Tests mandated by NCLB were developed and annually administered by individual states. Defining what constitutes student proficiency was also left up to states. Most students are tested on mathematics in third through eighth grades and at least once in high school. Algebra is typically a high school or middle school subject, so it escapes the bulk of NCLB testing requirements and slips by most state accountability systems.

The National Assessment of Educational Progress (NAEP) is administered every two years in grades 4 and 8. Testing every two years was scheduled to start for grade 12 in 2013, as part of starting to report state level results, but budgetary considerations led to postponement of that plan in August 2013.[7] NAEP is commonly referred to as "the Nation's Report Card," but it is not well suited to track American students' proficiency in algebra. Algebra items constitute 30% of the eighth grade NAEP and 35% of the twelfth grade test. Scores for the algebra subscale are reported each time NAEP is administered, but at the eighth grade level, the test is given in February or March—just past the halfway mark for the year. Students taking algebra in eighth grade have not yet encountered many of the more advanced topics in Algebra I, not to mention topics addressed in Algebra II. Moreover, the algebra items on the eighth grade NAEP are not designed to assess the content of an Algebra I course comprehensively. For students who take algebra in ninth grade or later, the algebra subscale reported on the 12th grade NAEP might be indicative of what they learned in Algebra I and Algebra II, but only as an indicator of what is remembered years after taking the courses. Like TIMSS and TIMSS Advanced, NAEP is both too early (at eighth grade) and too late (at twelfth grade) in assessing algebra.

End of course exams are assessments administered immediately upon course completion. In 2012, twelve states reported giving end of course exams in Algebra I.[8] Exams vary in quality and rigor. Some are strictly multiple choice; others include constructed response items. Some assess complex algebraic concepts; others only expect mastery of basic algebra. Results from different states are not comparable.

The Common Core State Standards are intended to address the fragmented nature of state-based assessments and to standardize expectations from state to state. The hope of that intention being realized anytime soon, at least on the algebra front, is fading. The two consortia funded to create assessments—the Partnership for Assessment of Readiness for College and Careers (PARCC) and the Smarter Balanced Assessment Consortium (SBAC)—have taken different roads in assessing high school mathematics. SBAC has announced that it will not assess mathematics by course title—no end of course tests of algebra or advanced algebra—but instead will administer a culminating math test at the end of 11th grade. PARCC has promised to develop end of course tests for Algebra I and Algebra II that will be available in 2015.[9] Recent unhappiness with the cost of PARC's assessments has led to a few defections, and only 14 states and DC are currently committed to field testing PARCC in the spring of 2014. The end of course test for Algebra II will also include concepts and skills from math courses taken previously so that scores can be used as an indicator of college readiness.[10]

In sum, the current regime of international, national, and state tests do not adequately assess American students' proficiency in algebra. TIMSS 8th grade is targeted below the population of students who have completed Algebra I and Algebra II. TIMSS Advanced is targeted above them. PISA is philosophically unaligned with curriculum. NAEP assessments at 8th and 12th grades, like TIMSS, do not address all of the topics covered by Algebra I and Algebra II courses. Only 13 states administer end of course exams in Algebra I or II, and their results cannot be compared from state to state. Of the two consortia writing assessments for the Common Core State Standards, only PARCC is working on algebra tests—and only 14 states are committed to field testing the PARCC assessments once they have been developed. Today, anyone interested in knowing how well American students are learning algebra must rely on a hodgepodge of fragmented, misaligned indicators.

Let's now turn to evidence that the current regime isn't working very well.

II. Troubling signs that current algebra assessments are not enough

Today's high school graduates sport significantly higher GPAs than graduates of two decades ago, with GPAs in high school math courses rising from about 2.2 in 1990 to over 2.6 in 2005 (about 18%).[11] The ACT estimates that in 2012 only one-half of college bound seniors (46%) were ready for college level mathematics. And yet 76% of graduates had completed a core curriculum, which includes completion of algebra and geometry. The two figures suggest that about one-third of students who complete a college preparatory curriculum are nevertheless ill-prepared for college course work.[12] Students who arrive at college without a firm grasp of algebra are placed into remedial math courses. A Columbia University study in 2010 found that more than half of incoming community college students must take remedial math courses, in which students then study the arithmetic and basic algebra that should have been learned several years earlier.

The situation is no better at four-year colleges. A report from The National Center for Public Policy and Higher Education (NCPPHE) estimated that about half of all four-year college freshmen must also take remedial classes, despite today's students flowing through the K-12 system during an era of widespread support for higher academic standards and extensive school reform.

The NCPPHE put it this way:

> Over the last 15 years, many states have emphasized mastery of specific content and performance standards, as shown through grades and statewide assessments; however, this shift to standards-based performance in the schools generally has not been extended to higher levels of achievement associated with college readiness, whose indicators still focus on courses taken. The flawed assumption has been that if students take the right courses and earn the right grades, they will be ready for college.[13]

Several indicators support the suspicion that this assumption is flawed. The typical incoming college student today has taken a college-prep curriculum in high school and passed higher level courses with good grades. But he or she looks unprepared when beginning college.

The California State University System (CSU) offers a vivid example. CSU draws from the top one-third of the California's high school graduates. All students who are admitted to CSU have taken a college prep regimen of high school math courses, including Algebra I and Algebra II. Incoming students take a mandatory mathematics exam for placement if they cannot exempt out with an SAT-M score of at least 550, ACT-M of 23, or AP Calculus or AP Statistics score of 3. These exemption criteria are reasonable, well within the reach of a decent, not exceptional, math student. Those who cannot meet any of the criteria take the Entry Level Math test (ELM). The exam covers number (35%), algebra (35%), and geometry (30%).[14] In 2012, about 30% of CSU freshmen failed the ELM and were placed in remedial mathematics, despite earning a mean GPA of 3.15 in their high school college prep programs.[15] The students had passed high school exit exams, but those tests are pitched at about the 8th grade level. Nationally, high school exit exams are rarely pitched beyond the 10th grade level.

The NCPPHE report poses a good question:

> It is readily apparent why a 10th-grade equivalency is not likely to prepare students for college, but why is it that a college-prep curriculum leaves so many students without the learning skills needed for college-level study?[16]

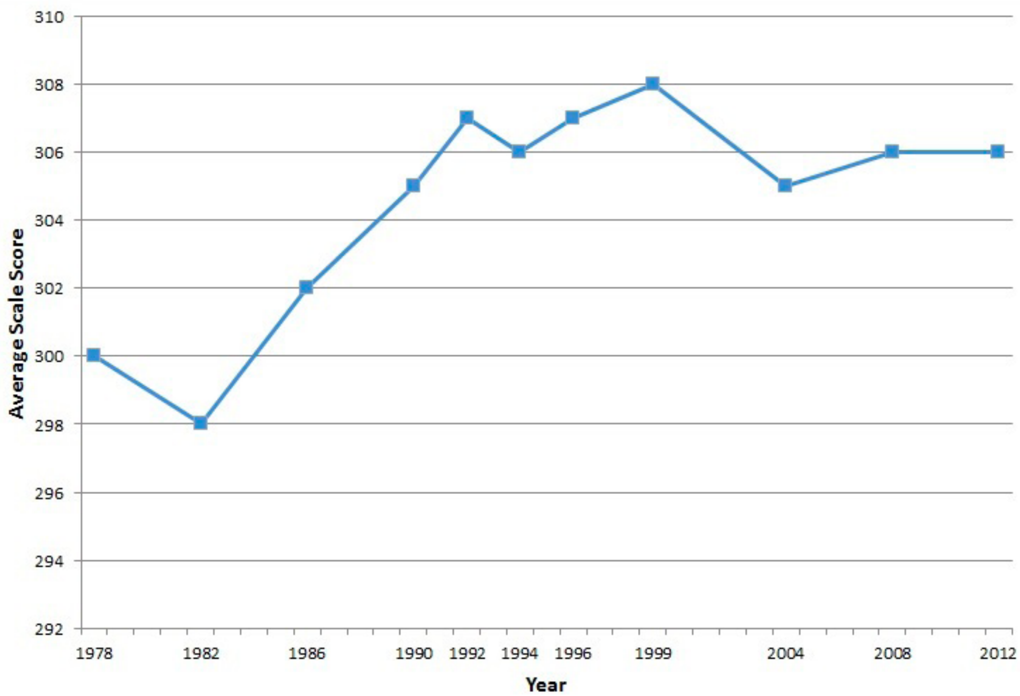
NAEP data may help answer that question.

NAEP data: Simpson's Paradox

The 2012 NAEP scores for the long term trend test (LTT) were released in June, 2013. The LTT NAEP is given to age-based samples of 9, 13, and 17 year old students. Scores in mathematics date back to 1973 (and to 1971 in reading).

Let's focus on 17 year olds because they are near graduation from high school. The 2012 math scores of 17 year olds were flat (see Figure 1), with a scale score of 306, basically unchanged from 2008. In fact, math scores of 17 year olds have been remarkably consistent for about two decades. Since 1990, scores have fluctuated within a range of 305-308. To understand just how narrow that is, consider that one standard deviation in both 1990 and 2012 was 31 points. The entire three point range is equal to less than 0.1 standard deviations, which can be statistically significant with a sample as large as NAEP's but holds no substantive significance in the real world.

FIGURE 1. NAEP MATH, 17 YEAR-OLDS (1978-2012)



Note: New assessment format began in 2004.

When the 2012 scores were released, some observers pointed out that the national scores masked underlying improvement, citing Simpson's Paradox as the source of the statistical anomaly. Simpson's Paradox occurs when aggregated data indicate trends that are the opposite of trends found in disaggregated data. And indeed the phenomenon occurs in the NAEP data for 17 year olds. When scores are disaggregated by race and ethnicity—whites, blacks, and Hispanics—a two-point gain by all three groups is evident since 1992 (see Table 1). How can that happen when the overall score fell from 307 to 306? The highest scoring group, whites, declined as a proportion of the student population from 75% to 56%. The Hispanic proportion tripled (from 7% to 22%). Despite registering a gain, Hispanic 17 year olds still scored about 20 points below whites in 2012.

TABLE 1. NAEP MATH SCORES, 17 YEAR-OLDS (BY RACE, 1978-2012)

Year	White	Black	Hispanic
2012	314	288	294
2008	314	287	293
2004[1]	311	284	292
2004[2]	313	285	289
1999	315	283	293
1996	313	286	292
1994	312	286	291
1992	312	286	292
1990	309	289	284
1986	308	279	283
1982	304	272	277
1978	306	268	276

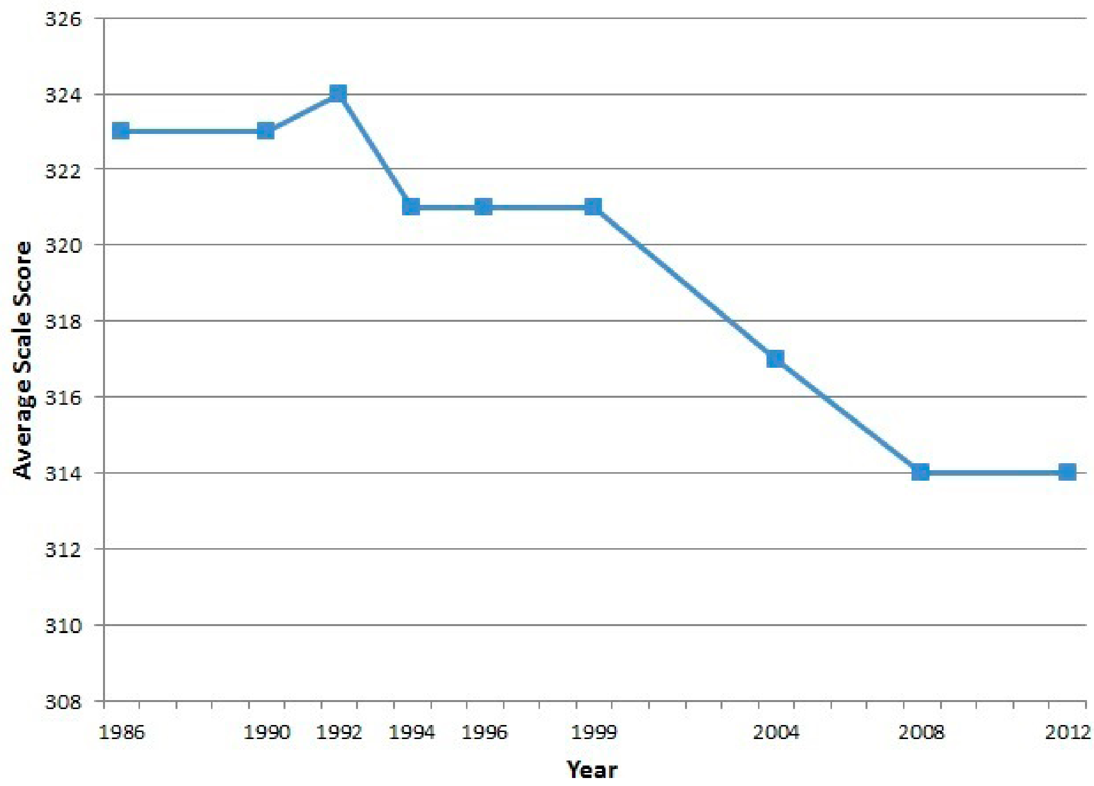
1. New assessment format
2. Original assessment format

On any test, a lower scoring group that grows as a proportion of the tested population (even if that group is improving) can hold down the overall average. In the case of NAEP from 1992 to 2012, it's not much—the difference between a two point gain for subgroups and a one point loss nationally is a scant three points—but when the sign of an indicator switches from positive to negative, it gets people's attention. Simpson's Paradox is also evident over the entire NAEP history, with substantial gains by whites (8 points), blacks (20 points), and Hispanics (18 points) hidden under the smaller national gain (only 6 points).

NAEP Performance of Students Who Have Completed Algebra II

NAEP data can be sliced along several dimensions. Let's examine the math scores of 17 year olds who reported that they have taken a second-year course in algebra (in other words, Algebra II or Advanced Algebra). Historically, these have been high achieving, college bound students. As illustrated in Figure 2, scores for this group of students fell by 10 points, 324 to 314, from 1992 to 2012. The steady decline of scores has occurred while the percentage of students taking Algebra II has increased (see Table 2). In 1986, Algebra II had been taken by less than half of all seventeen year olds (44%) but now completion of the course is commonplace (76%). In 1986, completing Algebra II told the outside world something important: here is a student who is solid in mathematics and prepared for college level courses in the subject. That message has been undermined.

FIGURE 2. NAEP MATH, 17 YEAR-OLDS WHO HAVE COMPLETED SECOND YEAR ALGEBRA (1986-2012)



Note: New assessment format began in 2004.

NAEP data cannot tell us why the scores of Algebra II completers have declined. The fact that the fall off in scores coincided with the boom in Algebra II enrollments suggests it is due to more low performing students taking the course in 2012 than in previous years. The course may be drawing from a less prepared pool of students. But that hypothesis is only suggested. Perhaps today's Algebra II textbooks are of lower quality or Algebra II teachers are not as effective—there are several plausible theories. The point here is that the completion of Algebra II has lost some of its luster as a credentialing mechanism, as a signal to prospective colleges and employees of a student's accomplishments in learning mathematics. In 1986, students who had "Algebra II" on their high schools transcripts scored about 21 points above the typical 17 year old. Today, it's 8 points.

TABLE 2. PERCENTAGE OF 17 YEAR-OLDS WHO COMPLETED SECOND YEAR ALGEBRA (1986-2012)

Year	All Students	White	Black	Hispanic
2012	76	79	69	69
2008	71	73	65	64
2004	69	69	66	62
1999	64	67	57	43
1996	63	66	52	49
1994	58	61	52	43
1992	54	57	45	37
1990	51	53	46	36
1986	44	47	32	30

Is there any chance that Simpson's Paradox is at work again? Are there underlying gains among the advanced algebra students that are masked by aggregation? It doesn't appear to be so, at least not along racial/ethnic lines. Table 3 disaggregates scores by the same three groups shown above—blacks, whites, and Hispanics. All three groups have experienced declines since 1986, the earliest year such data are available, —whites by seven points (327 to 320), blacks by four points (299 to 295), and Hispanics by three points (305 to 302). The magnitude of the aggregated decline is greater, but all of the subgroups display declines as well.

TABLE 3. NAEP MATH, 17 YEAR-OLDS WHO HAVE COMPLETED SECOND YEAR ALGEBRA (BY RACE, 1986-2012)

Year	White	Black	Hispanic
2012	320	295	302
2008	321	294	301
2004	322	295	299
1999	325	296	314
1996	325	301	310
1994	326	299	306
1992	327	303	313
1990	327	306	310
1986	327	299	305

Note: The new assessment format began in 2004.

In terms of credentialing, note that blacks and Hispanics pay the highest price for Algebra II completion not meaning what it once meant. In 2012, black students who completed

Algebra II scored 295 on NAEP, well below (11 points) the math score of the average 17 year old, regardless of coursework. That is 0.3 standard deviations below average, equivalent to about the 38th percentile. Nearly 62% of the 17 year old population has higher math scores. The legitimacy of any credential signaling knowledge of a particular subject is difficult to maintain when 62% of the population knows more about the subject than the holders of that credential.

How can this happen? How can students in college prep courses score so poorly? A recent study of high school transcripts sheds light on the problem. Researchers at NCES examined the high school transcripts of 17,800 students who graduated in 2005. Here is the stated purpose of the study:

> The 2005 National Assessment of Educational Progress (NAEP) High School Transcript Study (HSTS) found that high school graduates in 2005 earned more mathematics credits, took higher level mathematics courses, and obtained higher grades in mathematics courses than in 1990. The report also noted that these improvements in students' academic records were not reflected in twelfth-grade NAEP mathematics and science scores. Why are improvements in student coursetaking not reflected in academic performance, such as higher NAEP scores?[18]

The researchers analyzed the textbooks students used in Algebra I, Geometry, and integrated math courses, coding the topics covered by each book. *Education Week* quoted NCES Commissioner Sean P. "Jack" Buckley as stating, "We found that there is very little truth in labeling for high school Algebra I and Geometry courses."[19] Not only do courses called "Algebra I" vary considerably in mathematical content, but labels such as "honors" and "regular" provide no guidance as to the rigor of courses. Courses with the same name can be miles apart in terms of the demands placed on students. Less than one in five students (18%) who took an Algebra I course called "honors" actually encountered a curriculum that the researchers coded as "rigorous."[20]

The NCES study was confined to high school algebra courses, omitting Algebra I courses that are taken in middle school. Research suggests that they, too, may not be exactly as advertised by course titles. Enrollment in eighth grade advanced math (i.e., algebra and above) has boomed. In 1990, only 16% of eighth graders took advanced math. Such courses were populated by the very top math students.

By 2011, enrollment had jumped to 47% (see Table 4). Today's advanced classes in eighth grade include large numbers of low achieving math students. Indeed, in 2005 approximately 29% of students scoring at the 10th percentile and below on NAEP were enrolled in an Algebra I or more advanced course for eighth grade math. These students function at about the third grade level or below, unable to demonstrate even minimal proficiency on NAEP items involving fractions, decimals, and percentages. It is difficult to imagine a "real" algebra course that could cover the topics conventionally taught in Algebra I while simultaneously addressing these students' needs.[21]

TABLE 4. ENROLLMENT IN 8TH GRADE ADVANCED MATH

Year	All Students
2011	47%
2009	44%
2007	43%
2005	42%
2003	33%
2000	27%
1996	25%
1992	20%
1990	16%

Note: Spaces indicate when response categories changed.

Source: NAEP Data Explorer

To summarize, the current system of validating mastery of algebra rests on assumptions about course completion. Students and parents assume that completing college prep math courses with good grades signifies that students are adequately prepared for college level mathematics. Teachers assume that students who have completed Algebra I must be ready for Algebra II. Colleges and employees assume that students who have completed Algebra II with good grades know the subject. Increasingly, those assumptions are proving to be unfounded. A lot of people are being fooled. Better information is needed—and needed as early as possible in a student's academic career—about how well students have learned algebra. New assessments are needed to replace the current regime.

III. Prospects for the Future: International, National and State Level Assessments

At the International Level

A new international algebra assessment is not currently feasible primarily because of insufficient interest outside the United States. The role of algebra as a gatekeeper course for attending college is peculiar to the U. S. Similarly, the literature documenting long term economic returns for individuals learning algebra is based on data collected in the U.S. Most countries do not partition the secondary mathematics curriculum by topics, as in the U.S., but instead teach integrated courses that interweave algebra, geometry, number, statistics, and other mathematics. Thus, the idea of "algebra" as a distinct body of knowledge needing special assessment has limited international appeal. Moreover, many countries have their own high stakes assessments for matriculation from lower to higher secondary education or for assigning students to academic streams within upper secondary schooling. Teachers, students, parents, and policymakers are focused on the results from those assessments and trust them as valid indicators of mastery.[22]

At the U.S. National and State Levels

For the next several years, national and state assessments will be dominated by the unfolding of the Common Core State Standards. As described above, the two consortia developing tests aligned with the Common Core are taking different approaches to assessing high school mathematics. PARCC is developing end of course exams for Algebra I, Algebra II, and Geometry. SBAC is developing a comprehensive test, encompassing content learned either in a traditional sequence (i.e., Algebra I-Geometry-Algebra II) or a sequence of integrated math courses (Integrated I, II, and III). The SBAC exam will be given at the end of eleventh grade.

To get an idea of what the future holds for algebra testing, Table 5 displays information on states' current end of course testing, also identifying those states that belong to PARCC and are committed to field testing.

TABLE 5. STATES WITH END OF COURSE EXAMS FOR ALGEBRA COURSES (2011-2012)

State	Algebra I	Algebra II
Arkansas*	X	
Delaware		X
Florida	X	
Indiana	X	
Louisiana*	X	
Maryland*	X	
Mississippi*	X	
Missouri	X	X
North Carolina	X	
Oklahoma	X	X
Tennessee*	X	X
Virginia	X	X
Washington	X	

* Member of PARCC and committed to 2014-2015 field test.

Source: Center on Education Policy, *Exit Exam Survey of State Departments of Education, 2011*, pp. 4-9. Data for 2011-2012 updated from CEP website.

If the committed PARCC states are added to the non-PARCC states currently giving their own end of course algebra tests, a total of 22 states may be administering end of course exams in Algebra I by 2015—and 19 states in Algebra II. That is the best case scenario, one that assumes both independent and SBAC states keep their existing algebra exams in place and, in the case of SBAC states, give them in addition to the consortium's eleventh grade comprehensive math test. That scenario is probably a stretch. It also, of course, does not address the problem of being unable to compare results of states with different tests.

Creation of an open algebra exam

With government-sponsored assessment programs unlikely to develop a national algebra assessment anytime soon, what is the alternative? An open algebra test is one intriguing idea. Stanford economist Eric A. Hanushek proposes open tests as a way of addressing widespread reports of cheating on tests and the gaming of accountability systems.[23] The idea is to create a bank of test items large enough to cover all of the objectives of a course.

Hanushek illustrates the idea with fourth grade math:

> Imagine 1,500 questions for fourth grade math that cover the entire scope of appropriate material from basic to advanced topics. Next, make all of the test items – not just sample items – publicly available and encourage teachers to teach to the test, because the items cover the full range of the desired curriculum. Making the items public will also ensure the quality of the test items. One could invite feedback ratings or open sourcing to provide a path to improving the questions over time. Then, move to computerized adaptive testing, where answers to an initial set of questions move the student to easier or more difficult items based on responses. This testing permits accurate assessments at varying levels while lessening test burden from excessive questions that provide little information on individual student performance. Such assessments would not be limited to minimally proficient levels that are the focus of today's tests, and thus they could provide useful information to districts that find current testing too easy. Students would be given a random selection of questions, and the answers would go directly into the computer – bypassing the erasure checks, the comparison of responses with other students, and the like.[24]

Algebra I and Algebra II are prime candidates for such an approach. Topics in both introductory and advanced algebra are well-defined by current end of course tests and textbooks. Algebra frameworks based on the Common Core have been written by PARCC. The Final Report of the National Mathematics Advisory Panel, a group that included mathematicians, math educators, psychologists, and policy analysts—offers a list of the major topics in algebra (See Appendix A). These sources are not in complete agreement on the most important algebra topics that students should learn, but they are similar enough to make a consensus description of an algebra item pool a feasible task.

A discussion of the technical development of an open algebra test is beyond the scope of this paper. The steps necessary to get an open test algebra project off the ground—convening a planning committee, acquiring funding, settling on a list of topics to be assessed, development of test items, field testing the assessment, refining the assessment, implementation, disseminating results—are probably the same as for any testing program. The word "probably" is important here because the creation of this type of assessment has never been attempted before. The fact that the test would be open adds a new element. Technical and political challenges may arise that traditional state, national, or international tests have not faced.

Two principles should guide the development of such a test so as to ensure its integrity.

1. Fidelity of Content. The assessment must measure a body of knowledge broadly and authoritatively recognized as "algebra." It will be important to gain the participation of professional groups concerned with teaching and learning algebra—the Mathematical Association of America, American Mathematical Society, and National Council of Teachers of Mathematics—to certify the content of the assessment

2. Trust in Results—the results of the test must be viewed as legitimate by students, teachers, parents, employers, and higher education. Part of that legitimacy will come from validation by other measurements, but the test results must also prove useful. For example, a valid indicator that students have not learned particular skills or concepts in an Algebra I course can serve as an early warning system to students who could then seek help to shore up deficiencies before taking Algebra II, to Algebra II teachers who are selecting topics for review activities during the first few weeks of the school year, and to schools that are deciding how to design a new after-school tutoring program. Post-secondary institutions would place their trust in an instrument telling them what their own placement exams show--that students know or do not know the mathematics they are supposed to know coming out of high school.

IV. Conclusion

In the past two decades, algebra has grown in importance in the U.S. math curriculum. Research has documented algebra's gatekeeper status, as a body of mathematics that students must learn to gain access to advanced courses in high school, to non-remedial math classes in four-year colleges and universities, and to technically-oriented careers. Students and educators got the message. Algebra I, a math course that twenty years ago was only taken by our most mathematically gifted eighth graders, is now completed by approximately half of all students before entering high school. Algebra II, a course once reserved for college bound students with a mathematical bent, is now taken by nearly three-quarters of all high school graduates.

Make no mistake about it, these enrollment trends are praiseworthy accomplishments. Expecting students to take more rigorous courses--raising the bar on what we expect students to learn in math—is at the center of improving K-12 education in the U.S. But such accomplishments are diminished if teachers aren't teaching and students aren't learning the content advertised by the titles of advanced courses. And evidence from NAEP and NCES transcript studies suggest that there is a growing gap between the titles of courses and what students actually learn in them. The danger is that grade inflation, the often discussed phenomenon of students receiving higher and higher grades for mediocre academic achievement, has been joined by course inflation. Completing advanced math courses does not mean what it once meant because course titles no longer signify the mathematics that students have studied and learned.

This paper has discussed international, national, and state assessments as tools for giving a more accurate picture of achievement in algebra. Prospects are not good that government-sponsored international or national tests will focus on algebra anytime soon, nor are they good that impending assessments based on the Common Core will remedy the situation. Open tests of algebra, an intriguing but inchoate idea at this point in the world of assessments, may offer the best hope for restoring the legitimacy of algebra courses and tracking the future progress of American students in learning algebra.

End Notes

1. Richard J. Murnane and Frank Levy, *Teaching the New Basic Skills* (The Free Press, 1996).
2. The Algebra Project, http://www.algebra.org/. For algebra's gatekeeper role, note that the most commonly cited source of evidence, Clifford Adelman's (1999) *Answers in the Toolbox*, actually identifies completing a course beyond algebra II as the most consequential predictor for college completion. The study also fails to adequately control for selection effects. Adelman, C. 1999. *Answers in the Tool Box: Academic Intensity, Attendance Patterns, and Bachelor's Degree Attainment*. Washington, DC: U.S. Department of Education.
3. Course data from 2011 NAEP, NAEP Data Explorer, available at: http://nces.ed.govnationsreportcard/naepdata/report.aspx
4. Martin, Michael O., Ina V.S. Mullis, and Pierre Foy. *TIMSS 2007 International Mathematics Report*. Chestnut Hill: TIMSS & PIRLS International Study Center, Lynch School of Education, Boston College, 2008, Exhibit 5.10.
5. TIMSS assesses fourteen topics in Geometry, with the six most advanced of these taught to about half of all eighth grade students internationally; Martin, Michael O., Ina V.S. Mullis, and Pierre Foy. *TIMSS 2007 International Mathematics Report*. Chestnut Hill: TIMSS & PIRLS International Study Center, Lynch School of Education, Boston College, 2008, Exhibit 5.11.
6. Garden, Robert A., et al. *TIMSS Advanced 2008 Assessment Frameworks*. Chestnut Hill: TIMSS & PIRLS International Study Center, Lynch School of Education, Boston College, September 2006, 13-15.
7. http://www.nagb.org/naep/assessment-schedule.html
8. Center on Education Policy, *High School Assessments Survey of State Departments of Education*. Washington, DC: Center on Education Policy, 2011, 4-9.
9. The PARCC Model Content Frameworks for Mathematics include a high school section that provides assessment guidance for Algebra I, Geometry, and Algebra II and Mathematics I, Math II, Math III.
10. Gewertz, Catherine. "Testing Group Picks 'College Readiness' Exam: Students will be assessed at end of Algebra 2 or Math 3." *Education Week*, January 9, 2013. http://www.edweek.org/ew/articles/2013/01/09/15parcc.h32.html?qs=PARCC+algebra
11. Schneider, Mark. *Math in American High Schools: The Delusion of Rigor*. Washington, DC : American Enterprise Institute, 2009.
12. ACT, Inc. *The Condition of College and Career Readiness: 2012*. ACT, Inc., 2012, http://media.act.org/documents/CCCR12-NationalReadinessRpt.pdf
13. *Beyond the Rhetoric: Improving College Readiness Through Coherent State Policy*. Washington, DC: The National Center for Public Policy and Higher Education, June 2010.
14. Educational Testing Service. *California State University Placement Tests: Topics on the ELM Test*. 2013. http://www.ets.org/csu/about/elm/elm_topics.
15. The California State University. *Fall 2012 Final Regularly Admitted First-time Freshmen Remediation Systemwide*. 2012. http://www.asd.calstate.edu/remediation/12/Rem_Sys_fall2012.htm.
16. *Beyond the Rhetoric: Improving College Readiness Through Coherent State Policy*. Washington, DC: The National Center for Public Policy and Higher Education, June 2010.
17. A similar phenomenon is evident with the NAEP scores of eighth graders enrolled in algebra I. See Loveless, Tom. *The Misplaced Math Student: Lost in Eighth-Grade Algebra*. Washington, DC: The Brookings Institution, 2008.
18. Brown, Janis, Kathryn Schiller, Stephen Roey, Robert Perkins, William Schmidt, and Richard Houang. *Algebra I and Geometry: Results from the 2005 High School Transcript Mathematics Curriculum Study*. Washington, DC: National Center for Education Statistics, Institute of Education Sciences, U.S. Department of Education, March 2013, 1.
19. Sparks, Sarah D. "Algebra, Geometry Classes Vary in Rigor, Say Study." *Education Week*, March 27, 2013.

20. See Figure B., page 5. Brown, Janis, Kathryn Schiller, Stephen Roey, Robert Perkins, William Schmidt, and Richard Houang. *Algebra I and Geometry: Results from the 2005 High School Transcript Mathematics Curriculum Study*. Washington, DC: National Center for Education Statistics, Institute of Education Sciences, U.S. Department of Education, March 2013, 1.
21. Loveless, Tom. *The 2008 Brown Center Report on American Education: How Well Are American Students Learning?* Washington, DC: The Brookings Institution , 2008.
22. Conclusions about the feasibility of an international test of algebra are largely drawn from a focus group discussion held at the Brookings Institution, Washington, DC, March 1, 2013.
23. Hanushek, Eric A. "Why Not Have Open Tests?" Stanford University. September 14, 2011. http://hanushek.stanford.edu/opinions/why-not-have-open-tests
24. Ibid.

Appendix A:

Task Group Reports of the National Mathematics Advisory Panel

Results and Conclusions
The Major Topics in School Algebra

The Major Topics in School Algebra that were developed by the Task Group on Conceptual Knowledge and Skills are shown in this section. The teaching of Algebra, like the teaching of all of school mathematics, must ensure that students are proficient in computational procedures, can reason logically and clearly, and can formulate and solve problems. For this reason, the topics listed below should not be regarded as a sequence of disjointed items, simply to be committed to memory. On the contrary, teachers and textbook writers should emphasize the connections as well as the logical progression among these topics. The topics comprise both core and foundational elements of school algebra—those elements needed for study of school algebra itself and those elements needed for study of more advanced mathematics courses. The total amount of time spent on covering them in single-subject courses is normally about 2 years, although algebra content may be and is often structured in other ways in the secondary grades. What is usually called Algebra I would, in most cases, cover the topics in Symbols and Expressions, and Linear Equations, and at least the first two topics in Quadratic Equations. The typical Algebra II course would cover the other topics, although the last topic in Functions (Fitting Simple Mathematical Models to Data), the last two topics in Algebra of Polynomials (Binomial Coefficients and the Binomial Theorem), and Combinatorics and Finite Probability are sometimes left out and then included in a precalculus course. It should be stressed that this list of topics reflects professional judgment as well as a review of other sources.

Symbols and Expressions

- Polynomial expressions
- Rational expressions
- Arithmetic and finite geometric series

Linear Equations

- Real numbers as points on the number line
- Linear equations and their graphs
- Solving problems with linear equations
- Linear inequalities and their graphs
- Graphing and solving systems of simultaneous linear equations

Quadratic Equations

- Factors and factoring of quadratic polynomials with integer coefficients
- Completing the square in quadratic expressions
- Quadratic formula and factoring of general quadratic polynomials
- Using the quadratic formula to solve equations

Functions

- Linear functions
- Quadratic functions—word problems involving quadratic functions
- Graphs of quadratic functions and completing the square
- Polynomial functions (including graphs of basic functions)
- Simple nonlinear functions (e.g., square and cube root functions; absolute value; rational functions; step functions)
- Rational exponents, radical expressions, and exponential functions
- Logarithmic functions
- Trigonometric functions
- Fitting simple mathematical models to data

Algebra of Polynomials

- Roots and factorization of polynomials
- Complex numbers and operations
- Fundamental theorem of algebra
- Binomial coefficients (and Pascal's Triangle)
- Mathematical induction and the binomial theorem

Combinatorics and Finite Probability

- Combinations and permutations, as applications of the binomial theorem and Pascal's Triangle

Source: Fennell, Francis "Skip", et al. "Chapter 3: Report of the Task Group on Conceptual Knowledge and Skills." In Foundations for Success: Report of the *National Mathematics Advisory Panel*, by National Mathematics Advisory Panel. U.S. Department of Education , 2008.

Governance Studies
The Brookings Institution
1775 Massachusetts Ave., NW
Washington, DC 20036
Tel: 202.797.6090
Fax: 202.797.6144
www.brookings.edu/
governance.aspx

Editor
Sarah Whitfield

Production & Layout
Sarah Whitfield
Beth Stone

About the Brown Center on Education Policy

Established in 1992, the Brown Center on Education Policy conducts research and provides policy recommendations on topics in American education. The Brown Center is part of The Brookings Institution, a private nonprofit organization devoted to independent research and innovative policy solutions. For more than 90 years, Brookings has analyzed current and emerging issues and produced new ideas that matter - for the nation and the world.

Brown Center on Education Policy
The Brookings Institution
1775 Massachusetts Ave. NW
Washington DC, 20036
202.797.6090
202.797.6144 (f)
http://www.brookings.edu/brown.aspx

Email your comments to browncenter@brookings.edu