What is g?

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DRAFT FOR COMMENTS

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Abstract

Large secular gains in cognitive ability (the Flynn Effect) show that large, environmentally induced changes in measured cognitive ability are possible, but several studies have suggested that secular gains are not gains in general cognitive ability and are therefore not substantive. This paper extends the model of a single cognitive ability presented by Dickens and Flynn (2001) to multiple abilities. It shows that such a model can account for all the important facts about general cognitive ability without postulating any common underlying physiological cause for different mental abilities. A general intelligence factor arises in the model because people who are better at any cognitive skill are more likely to end up in environments that cause them to develop all skills. Scores on the resulting general ability factor can be highly heritable even while they are potentially subject to considerable environmental influence. Loadings of subtest scores on the general ability factor can be positively correlated with subtest heritabilities. In the model, discrimination against a social group in access to cognitively demanding environments can produce subtest score differences from other groups that are strongly correlated with both the g loadings and heritabilities of those subtests. Despite this, there is no reason to expect that meaningful secular gains should be correlated with g loadings across subtests.

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It is well established that environmental interventions can have substantial effects on measured cognitive ability (Lazar and Darlington 1982) and that family environment plays a notable role in explaining differences in cognitive ability between school age children (Plomin et al. 2001). However, it is also well established that the effects of interventions fade over time and that the fraction of variance in cognitive ability explained by shared family environment drops to insignificant levels as children become adults.[2]

The findings on secular gains in cognitive ability -- often referred to as The Flynn Effect -- are something of a contrast. These are environmentally induced gains[3] that affect people at all points in their life. There can be little doubt that large and persistent environmentally induced changes in measured cognitive ability are possible.

What is not clear is that meaningful permanent changes in cognitive ability are possible. Although the evidence is somewhat mixed,[4] most results suggest that secular gains are not gains in general intelligence or g. It has been claimed that the predictive validity of tests of cognitive ability for both academic (Jensen,1998, p. 276) and job performance (Ree et. al. 1994) is due almost entirely to the high correlation of test scores and g.[5] So a number of authors have questioned whether secular gains in measured ability reflect substantive gains in cognitive ability or simply reflect changing "bias" in the tests.[6]

Much of the skepticism about the meaningfulness of secular gains originates with those who favor a particular view of the causes of differences between people in general intelligence. This view holds that most, or all,[7] differences in general cognitive ability in

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[2] Some have suggested that this might not be true for disadvantaged populations in which the role of shared family environment is greater (Turkheimer et al. 2003).

[3] Mingroni (2004 and forthcoming) argues that the secular gains may be due to heterosis or out breading. Flynn (forthcoming, chapter 5) documents how implausible it is that heterosis could account for more than a tiny fraction of secular gains.

[4] Colom et al. (2001) Jaun-Espinosa et al. (2000) find strong correlations between g loadings and IQ gains. Jensen (1998, pp 320-321) reviews a number of studies of the relation between subtest gains and g loadings, all of which show weak positive correlations. Rushton (1999) finds that a measure of g developed on the WISC has loadings that are negatively correlated with subtest gains in several countries. But Flynn (2006) argues that IQ gains are greatest on tests of fluid g rather than crystallized g and finds a positive (though statistically insignificant) correlation between a measure of fluid g he develops and IQ gains in the same data used by Rushton. Must et al. (2002, 2003) find no correlation between g loadings and gains on two tests in Estonia, but these are achievement tests with a strong crystallized bias.

[5] See Currie and Thomas (2001) and Heckman et al. (1997) for dissenting views. See also Sternberg and Warner (1993) and Ceci (1986) and Ceci and Roazi (1994) for a discussion of the importance of g relative to practical intelligence for career success.

[6] See for example Jensen (1998), Rushton (1999), and Rushton and Jensen (2005). Wichert's et al. (2004) are the first to refer to describe secular gains as changes in bias.

[7] Most studies attribute 20 to 40 percent of the variance in adult cognitive ability to non-shared environment, but see Molenaar et al. (1993) and Jensen (1997) who suggest that what is called non-shared environmental variance may actually reflect physiological differences due to random influences on brain development.

adults reflect physiological differences that are predominantly genetically determined. It is the existence of general cognitive ability that gives rise to the pervasive correlation of performance on different measures of mental skills. Principal components analysis of scores across individual test items or subtests typically yields a first component that accounts for a large fraction of the standardized variance. Alternatively, if hierarchical factor analysis is performed, a second or third order factor emerges on which all lower order factors have positive loadings (Carroll 1993). The positive correlation between subtest loadings on the general factor and subtest heritabilities is cited as evidence of the genetic origins of general intelligence as is the high heritability of measures of general intelligence constructed as individual scores on the general intelligence factor.

The nature of general intelligence plays an important role in another debate. Jensen (1985, 1987) has shown that subtest loadings on the general intelligence factor are positively correlated with white-black differences on subtests. Dolan (2000) finds that a single factor model can adequately explain the pattern of white-black subtest differences, but Dolan et al. (2004) find that they can reject the hypothesis that the black-white gap is due to a difference on single factor in two other data sets. Rushton (1999) factor analyzes black-white differences, the extent to which subtest scores are depressed by inbreeding and secular gains and finds that the first two load on the same factor while secular gains do not. Jensen (1998), Rushton and Jensen (2005) and many others have cited the correlation of black-white subtest differences with g loadings and heritabilities as evidence that white-black differences in cognitive ability are mainly differences in general cognitive ability and largely genetic in origin. It is the existence of the correlation between black-white differences and within group heritabilities of subtests that drives the results of Jensen's (1998, p464-465) and Rowe and Cleveland (1996). Those authors conclude, using a structural equation estimation technique, that between 36 and 74 percent of the black white gap on several different tests is explained by differences between the groups in their average genetic endowment.

There are some weaknesses in the support for this view. First, there is increasing evidence that different cognitive skills are located in different parts of the brain, are activated independently, and can be damaged without direct effect on other skills (Blair 2006, Gardner 1983, Maguire et al. 2000). Second, although a number of physiological correlates of g have been found, at best they explain a small fraction of the variance in general intelligence across individuals (Jensen 1998, Chapter 6). All the obvious candidates (nerve conductive velocity, brain size, density of neuron connections, etc.) have been examined and show surprisingly little relationship to general cognitive ability. Interestingly, Colom et al. (2006) find that gray matter densities in several *specific and different* parts of the brain are related to g using the method of correlated vectors. This is a finding we will return to later. The high heritability of a wide range of measures of cognitive ability leaves no doubt that physiological differences between individuals play a large role in explaining differences in ability between people from similar backgrounds, but the failure to find any one overarching physical difference between people with widely different general measures of cognitive ability should cast doubt on the claim of a close causal link between physiology and general ability.

This paper presents an alternative view of the origin and meaning of general cognitive ability in adults. People who have superior ability in any dimension are more likely to gain access to environments that give them practice in a wide range of cognitive skills. This practice produces gains and these gains improve access so those who are good at any type of mental task are likely to do well on a wide range of mental tasks. The model below shows that this alone could account for all the facts about general cognitive ability described above. The model is similar in some respects to that of van der Maas et al. (2006), but differs in its explicit treatment of the roles of genetic endowment and environmental influences in the genesis of interpersonal differences, and in its sole concern with developed cognitive ability in adults. The specification of a model of the development of cognitive ability in children is left for the future and the integration of the model in this paper with that of van der Maas et al. might be a fruitful approach. Baltes and Nesselrode (1973) present a simulation model where correlated environmental effects produce a g factor in ability. The analysis presented here models genetic influences as well as the environmental feedback process and derives a much richer set of results.

This work builds on Dickens and Flynn 2001.[8] It extends that model of a single cognitive ability to many abilities and shows that it can account for all the important facts about general cognitive ability without postulating any common underlying physiological cause for different mental abilities. A general intelligence factor arises in the model. Scores on this factor can be highly heritable even while they are potentially subject to considerable environmental influence. Loadings of subtest scores on the general ability factor can be positively correlated with subtest heritabilities. In the model, discrimination against a socially isolated group in access to cognitively demanding environments can produce subtest score differences from other groups that are strongly correlated with both the g loadings and heritabilities of those subtests. Relatively small amounts of discrimination can give rise to large average differences in measured g. Despite this, there is no reason to expect that secular gains should be correlated with g loadings across subtests.

In the next section I provide an intuitive description of the model. The workings are illustrated with an analogy to basketball skills. The section after that presents the mathematical model and discusses several propositions about the model. The discussion of the propositions provides a more precise description of how the model explains the many facts about general ability than the intuition provided by the basketball analogy. Finally, a conclusion reviews the main results and argues that since the new model does a better job of accounting for the size and pattern of secular gains it should be preferred to the traditional model.

A Basketball Analogy

Consider the development of peoples' ability to play basketball. There are a number of physical and mental characteristics that will make a person a better basketball player. Height, and good hand-eye coordination are chief among them, but reflexes, speed, strength, endurance and the ability to anticipate where a dribbled ball will go next without looking are also important. Imagine that we knew that in very young children each of these characteristics was statistically independent of the others. If children were to grow up in an environment with no exposure to the game of basketball, and little or no exposure to other sports or athletic activities, we probably wouldn't expect much

[8] An earlier version of the model presented here was described and conjectures about its properties suggested in Dickens (2004). The model in this paper is different and the analysis of its properties more thorough.

correlation between peoples' skills when they were teenagers. Field shooting skill might be correlated with free throw ability because of their functional similarity, but we wouldn't be surprised if neither was very highly correlated with the ability to make lay-ups. The ability to make lay-ups might be correlated with the ability to rebound because height gives advantage to both, but we might not expect either skill to be highly correlated with the ability to dribble, dodge or steal a ball since those skills depend more on reflexes, coordination and speed. On the other hand, we would not be at all surprised if all those skills were highly correlated among teenagers in the US today who have all been exposed to the game of basketball to varying degrees and had varying opportunities to practice those skills.

In our 2001 paper Flynn and I made the point that a relatively small genetically induced physiological advantage (such as an extra inch or two in height) could grow into large differences in performance through feedback mechanisms. People of above average height play basketball a little better so they like the game more and play more often. As a result they get better and their improved performance makes them even more likely to choose playing basketball over other activities. This makes them better still so that some of them get chosen for teams where they get professional coaching and improve still more. The same virtuous cycle could magnify the effect of a persistent environmental advantage. A child with a next door neighbor who was a basketball fanatic would have a slight environmental advantage that could be multiplied by practice and coaching if the child became good enough to get on a team. However, unlike our genes that are always with us,[9] environmental advantages are more likely to be transitory. The neighbor may move away or make new friends. When an environmental advantage is removed there will be a tendency for the virtuous cycle to unwind. If the initial advantage is particularly short lived, the full effect from the feedback from ability to environment to ability might not have the opportunity to work itself out. Thus there will be a tendency for genetic differences to dominate the explanation of cross sectional differences even while persistent environmental changes (such as those that might exist between different generations or socially distinct groups) could still have large effects.

Even if there was no underlying correlation in the physiological basis for basketball skills, someone growing up in the US who is tall is more likely than someone who is not to play a lot of basketball. Those who play a lot will practice all the skills involved in the game and will get better at them all. Those who get particularly good at the game will get coaching in school or in after school leagues that further improves all their basketball relevant skills. Height isn't the only physical characteristic that might make one more likely to play basketball. Someone of unexceptional height with good hand-eye coordination will be better at shooting baskets and may prefer being a guard on the basketball team to running cross country. That person too will develop the full range of basketball skills (at least relative to those who play only infrequently). Even someone whose only talent was speed and endurance might tend to be more athletic than the average person and might play more basketball informally than the average person who was not athletically inclined. Such a person would likely be better at all the basketball skills than the average person who played basketball less.

[9] While our genetic endowment doesn't change, there is evidence that the effects of genetic endowment on cognitive ability change over time. None the less, there is evidence of substantial stability in genetic effects on cognitive ability (Fulker et al. 1993, McGue et al. 2002, and Plomin et al. 1994,).

Given this, we would not be surprised if data on individuals' scores on many different tests of basketball skill showed them all to be strongly positively correlated. We would not be surprised if a principal components analysis of such data yielded a single factor that explained a substantial amount of the variance in basketball playing ability and all skills loaded positively on this factor. It is quite possible that we would find other factors besides the general skill factor. Functionally related skills like free throws and field shooting, or lay-ups and rebounding, would likely be more correlated with each other than with other skills. A hierarchical factor analysis of such data might yield several first order factors such as these but people's scores on these factors would likely be correlated with each other and a general basketball ability factor would likely arise at the ultimate level. But, we would be wrong to conclude from this that there was necessarily some underlying physical difference between superior and inferior basketball players. Scores on the general ability factor would primarily reflect differing amounts of instruction and practice.

Note that someone looking for a physiological cause of general basketball ability would find that height was moderately correlated with it, but the correlation would be far from perfect. Other physiological measures would also have small correlations with ability. Any ability that contributed to the likelihood that an individual would get more instruction would have at least a weak relationship. But again, this would not be evidence for a biological basis for the correlations across abilities that instead arise from the largely social mechanisms by which skills are generated and differentiated.

Next consider which skills would be most highly correlated with general ability and therefore have the highest factor loadings. Those skills most important to superior performance in the game would be the ones that would be practiced most by those who are trying to excel. They would also be the skills most important for determining which people would be chosen for teams where they would get coaching in those skills and extra chances to practice those skills. Thus those with the most practice would have the most general skills and would be particularly likely to excel at the most important skills.

If one wanted to construct an optimal index of skills to predict success in playing basketball one would want to put the most weight on the most important skills and less weight on skills that are tangential to success. The rank order of the weights would thus be very similar, if not identical, to the loadings of the skills on the general ability factor since it is those skills that are most important for success and are most practiced by those who want to excel. Thus it would be no surprise if individuals' scores on the general ability factor were superior predictors of their overall performance as basketball players. Given that practice at any athletic activity is likely to improve skills and abilities relevant in other athletic activities we would not be surprised to find that the general basketball ability factor scores did a reasonably good job of predicting performance in other sports as well.

Next consider what would happen if there was a minority group that was discriminated against in being chosen for teams and was scorned by the majority group in organizing pick-up games. They would be likely to get less practice than everyone else. All their skills would suffer, but particularly those that received the most attention in practice and coaching. As already noted, these skills would be those most important for success in the game and would be the same skills that would be most highly correlated with the general ability factor. We would thus not be surprised to find a positive correlation between skills' loadings on the general ability factor and the difference in the skill scores of the minority and majority group with differences being largest on the skills with the largest factor scores.

Next, note that to the extent that physiological factors subject to substantial genetic influence help to make people more or less likely to get into environments where they will get more practice, the general ability factor could have a high heritability. It is also possible that those skills that are emphasized in practice will have a tendency to be the ones that will have the highest heritability. These are the same skills that load most heavily on the general ability factor and where a minority subject to discrimination in access to opportunities to play basketball would have the greatest deficits.

Finally, imagine what might happen if a decision was made to double the length of all basketball games, and teams were prohibited from returning a player to a game after he or she had been substituted out (making basketball more like soccer). Suppose that those playing pick-up games react by playing somewhat longer than they did before and that coaches react by scheduling extra sessions emphasizing running and endurance training so that their players can all play the full game if necessary. No doubt the average conditioning of good basketball players would improve. If people spend somewhat more time playing basketball as a result of the games being longer there would be a tendency for all their skills to improve, but the improvement in their endurance would be out of proportion to the improvement in their other skills. Any index of basketball playing skills that put much weight on endurance would show large increases, but if one were to compare the increases across all the skills to the factor loadings or heritabilities of the skills one would not expect a strong positive correlation. Still, people would be better basketball players than before – particularly under the new rules.

The relevance of the above analogy to cognitive ability is straight forward. The next section presents the formal model of cognitive ability and demonstrates that each of the results suggested by the intuition can be rigorously proved.

The Formal Model

Dickens and Flynn (2001) proposed a simple linear two-equation model to explain how both environment and genetic endowment could have large impacts on measured cognitive ability. Here that model is adapted to the case of several different measures of cognitive ability. The model in our 2001 paper was recursive, that is cognitive ability in the current period was affected by past environmental influences which had been determined in part by what was then a person's cognitive ability. While the model was recursive our analysis of the model emphasized equilibrium behavior. The equilibrium properties of a recursive model are easily found by treating it as a system of simultaneous equations where the past and present values of the endogenous variables are assumed to be equal and finding the solution. That is how this analysis will proceed.

Rather than a single cognitive ability, here several (K) cognitive abilities are modeled. These could be thought of as subtest scores from a battery of cognitive tests or as individual scores on factors at the penultimate level of a hierarchical factor analysis. Adopting the convention that upper case letters represent matrices, lower case letters vectors, and lower case Greek letters scalars, the scores for individual i measured in differences from the population mean are given by

(1) $$m_i = Ag_i + Ve_i$$

where m_i is a K length column vector of measured abilities of person *i*, g_i is an L length column vector of genetic endowments, *A* is a KxL matrix relating each measure of genetic endowment to each measured ability, e_i is a K length vector representing environmental influences on each measured ability (also measured as differences from the population average) and *V* is a KxK diagonal matrix that scales the impact of each element of e_i on the corresponding element of m_i.

Unlike the standard linear decomposition of variance of test scores, it will be assumed that *e* and *g* can be correlated. Specifically, it is assumed that e_i is given by

(2) $$e_i = Bm_i + Wz_i$$

where *B* is a KxK matrix relating person *i*'s K abilities to the K environmental influences for those abilities, z_i is an M vector of exogenous environmental influences which are uncorrelated with genetic endowment, and *W* is a KxM matrix relating those exogenous influences to person *i*'s environment for each of the K abilities. It will be assumed that g_i and z_i are standardized random variables (mean zero, variance one).

Substituting (2) into (1) and solving for the vector *m* yields

(3) $$m_i = Ag_i + VWz_i + VBm_i = (I - VB)^{-1}\left[Ag_i + VWz_i\right]$$

where *I* is a KxK identity matrix. It will be assumed that the parameters of the model are such that equation 3 has a finite solution (and thus the matrix *I-VB* can be inverted). Inspection of equation (3) shows that the expected value for all elements of the vector m_i are zero as they should be since it is measured in deviations from the population mean.

What makes this model different from the standard linear decomposition model is the presence of the Bm_i term in equation 2. Without that term (if all elements of *B* equal zero) e_i and g_i would be uncorrelated. If one or more elements of the vector *g* affected all the measured abilities then that could be the source of the correlation across abilities and the reason why factor analysis of cognitive abilities typically finds a general ability factor. Such a model would explain many of the facts about general ability and is probably the model that most advocates of g theory have in mind.

Note that in this simple model (*B=0*) one could have a general ability factor that arose because there were one or more exogenous environmental influences (z_{ij} s) that affected the environment for many different abilities (for example if the first column of matrix *W* had all positive values). This would give rise to a correlation across all measured abilities and factor analysis of the measured abilities would show a general factor. But, there would be no reason to expect this factor to be highly heritable or to be correlated with physiological characteristics unless those characteristics were caused by ability rather than being causes of it. Such a model couldn't explain very many of the salient facts about general cognitive ability.

However, with the introduction of a role for ability in shaping environment that in turn affects phenotype ability we introduce reciprocal effects between phenotype ability and environment. That induces gene x environment correlation that radically changes the nature of the model. For each ability, the solution to the model above closely resembles the equilibrium behavior of model 2 from Dickens and Flynn (2001). The only difference is the additional feedback across abilities that will enhance the multiplier effects compared to the model with only one ability. Thus this model can explain the same phenomena that the 2001 model did – the coexistence of high heritability of cognitive ability with the potential for large environmental effects. The new model allows us to consider a wider range of cognitive phenomena. It is now possible to explain all of the attributes of general ability discussed above in a model in which environment plays a major role in explaining individual differences and is entirely responsible for group differences.

To demonstrate this, a few additional assumptions will be made about the model to clarify the source of the results that will be derived. First, it will be assumed that $A=I\alpha$, and $V=I\nu$ -- that is the matrices A and V have the positive constants α and ν on the diagonal respectively and zeros everywhere else. Thus the first genetic influence and the first environmental influence affect only the first ability, the second of each affect only the second ability, and so on. It will also be assumed that W is a diagonal matrix with all positive elements on the main diagonal. Initially it will be assumed that all those elements are equal, but that assumption will be relaxed later. Combined together these assumptions imply that only its own genetic endowment and its own exogenous environmental influence directly affect each ability. Therefore any correlation between abilities arises because the environmental influences for different abilities are correlated, and this happens because people with different ability are matched to different environments as represented by the term Bm_i.

For clarity a very simple matching process will be used. It will be assumed that people split their time between environments that put no demand on their cognitive ability and a single generic cognitively demanding environment that puts demands on each ability according to the vector f. The more demanding an environment is of an ability the more it contributes to the development of that ability to be in the environment. It will be assumed that all elements of f are positive, no two are the same, and that the abilities are arrayed so that the elements of f are in descending order ($f_{i1}>f_{i2}>\ldots>f_{iK}$). Next, person i will spend a fraction $c'm_i$ more than the average person in the cognitively demanding environment (less if $c'm_i$ is negative). Thus $B=fc'$. Further, it is reasonable to assume that abilities that are most heavily used in the demanding environment would also be the ones that would be most influential in determining the amount of time spent in that environment. For simplicity it is assumed that $c=\beta f$.

The first thing to demonstrate is that with B defined this way all elements of m_i will be positively correlated with all other elements of m (from here on I will drop the i subscript for individual) and that the abilities most used in the cognitively demanding environments will have the highest correlations with the other abilities.

Proposition 1 Given the assumptions above,

(i) all elements of the true correlation matrix (C) of the elements of m will be positive,

(ii) and if $W_{ii}=W_{jj}$ then for all $i<j$ and $k \neq i$ *or* j, $C_{ik} > C_{jk}$ and $C_{ki} > C_{kj}$,

Proof
The full proof can be found in the appendix. That all elements of the correlation matrix are positive follows directly from some manipulation of the covariance matrix of the *m* s, which, as shown in appendix 1, can be written

$$(4) \quad E(mm') = (I - VB)^{-1}(AA' + VWW'V')(I - VB)^{-1}{}' = (I + \mu fc')(AA' + VWW'V')(I + \mu fc')',$$

where $\mu = \nu/(1-\nu c'f)$. Since all the terms in (4) are positive by assumption, and since the correlation is the covariance divided by the standard deviations of the two variables (which are necessarily positive) the first part of the proposition is proved. What is happening is that more intellectually able people are getting practice in all skills so they are better at all skills. This ensures that all correlations are positive.

As shown in appendix 1, the off-diagonal elements of the covariance matrix in (4) can be written as

$$(5) \quad E(mm')_{i,k} = \mu(f_i c_k[\alpha^2 + \nu^2 W_{k,k}^2] + f_k c_i[\alpha^2 + \nu^2 W_{i,i}^2]) + f_i f_k \mu^*.$$

where $\mu^* = \alpha^2\mu^2 c'c + \nu^2\mu^2 c'WW'c$. The *f*s are the cognitive demands of the demanding activity and they are proportional to the weights (*c*) that determine the amount of time a person spends in the activity. Assuming that $W_{i,i}=W_{j,j}$, if we compare the covariance of m_i and m_k with the covariance of m_j and m_k from (5), we can see that the latter will be larger than the former because f_i is larger than f_j and c_i is larger than c_j by assumption. The standard deviation of m_i will also be larger than the standard deviation of m_j, but the effect is less than proportional so the effect on the numerator of the correlation dominates and the correlation between m_i and m_k will be larger than that of m_j and m_k.

If this inequality holds strictly when $W_{i,i}=W_{j,j}$ then it will also be true for some values of $W_{i,i}$ in the neighborhood of $W_{j,j}$. This fact will be important when we consider the correlation of factor loadings with heritabilities below. The abilities most used in the demanding environment, and most important for determining time spent in the environment, will be the most highly correlated with other cognitive abilities.

Proposition 2 Given the assumptions above,

(i) The first principal component derived from the correlation matrix of abilities formed according to the system described in equations (1) and (2) above will have a positive correlation with all skills and
(ii) if $f_i > f_j$ then the loading of the *i*th skill on the first principal component will be greater than that of the *j*th skill if $W_{i,i}=W_{j,j}$.

Proof

The correlation of the first principal component with a variable is proportional to the corresponding element of the first Eigenvector of the correlation matrix. The power method (Quarteroni and Saleri, 2003, p140) for computing the first Eigenvector of a matrix uses the fact that for any non-zero vector r_0 the first Eigenvector can be approximated as $A^{\theta}r_0/\varepsilon$ with the approximation made arbitrarily close by choosing a sufficiently large value for θ. The constant ε is chosen to normalize the vector to have unit length. All elements of A must be positive as proved in proposition 1 and since the elements of r_0 can be chosen arbitrarily an r_0 can be chosen with all positive elements. In that case $A^{\theta}r_0$ will be positive for all values of θ and all values of the first Eigenvector must be positive proving part (i) of the proposition. Thus the fact that the correlation matrix has only positive elements is enough to guarantee a "g" factor that loads positively on all skills.

The first Eigenvector r of the correlation matrix C must satisfy the condition

$$(6) \quad Cr = \lambda r$$

where λ is the Eigenvalue associated with the first principal component. Since the Eigenvalues for a correlation matrix must sum to the dimension of the matrix (K), and the first Eigenvalue must be the largest, λ must be positive. Next note that equation 6 implies that

$$(7) \quad \sum_{k=1}^{K} C_{ik} r_k = \lambda r_i$$

for all i. Equation (7) implies

$$(8) \quad \sum_{k=1}^{K} r_k C_{ik} = \lambda r_i \quad \Rightarrow \quad \sum_{\substack{k=1 \\ k \neq i,j}}^{K} r_k C_{ik} = (\lambda - 1) r_i - C_{ij} r_j .$$

since $C_{ii}=1$. From proposition 1 we know that $C_{ik} > C_{jk}$ for all $i<j$ for $k \neq i,j$. Thus for $i<j$

$$\sum_{\substack{k=1 \\ k \neq i,j}}^{K} r_k C_{ik} > \sum_{\substack{k=1 \\ k \neq i,j}}^{K} r_k C_{jk} \Rightarrow (\lambda-1) r_i - C_{ij} r_j > (\lambda-1) r_j - C_{ji} r_i \Rightarrow r_i(\lambda-1+C_{ji}) > r_j(\lambda-1+C_{ij}) \Rightarrow r_i > r_j$$

where the last equivalence follows from the symmetry of the correlation matrix ($C_{ij}=C_{ji}$). Thus, the loadings of the skills on the first principal component will have the same rank ordering as the elements of f and c if all the elements on the diagonal of the W matrix are equal. So the skills most used in the cognitively demanding environment will be the ones that will be most highly correlated with the first principal component.

What we learn from the proofs of propositions 1 and 2 is that data generated by a model such as that described by equations 1 and 2 will evince a general ability factor. Further, the specific abilities that load most heavily on the general ability factor will tend to be the ones that are most used in the cognitively demanding activity, which we have assumed are the same abilities most important in determining the amount of time spent in the activity. If we think of the cognitively demanding activity as attending college (or a particularly selective college), or working in a cognitively demanding (and high paying) occupation then factor scores for the general ability factor will be very close to the ideal predictor of these outcomes. The ideal predictor would be an index with weights proportional to the *c* vector.

Now consider what would happen to the cognitive ability of a minority group that was discriminated against in access to the cognitively demanding activity. Suppose that the discrimination took the form that minorities spend a fraction δ less of their time in the cognitively demanding activity than those in the majority group with identical ability.

Proposition 3

Given the assumptions above,

(i) the minority group will have mean ability lower than that of the majority group on all abilities and

(ii) the rank order of differences, measured in majority group standard deviations, on the *i*th and *j*th abilities will be the same as that of the *i*th and *j*th loadings on the first principal component if $W_{i,i}=W_{j,j}$.

Proof

The assumptions stated above imply that the environment for the minority group is given by

$$(9) \quad e^m = f(c'm^m - \delta) + Wz^m .$$

Substituting (9) into (3) and using the result derived in appendix 1 that

$$(I - BV)^{-1} = I + \frac{vfc'}{1 - vc'f},$$

$$(10) \quad E(m^m) = -(I - BV)^{-1}Vf\delta = -\left(I + \frac{vfc'}{1 - vc'f}\right)vf\delta = -vf\delta/(1 - vc'f) \cdot$$

Since the majority group mean is zero the difference between the minority group and majority group means is given by (10) and is negative for all elements of *m*. Thus the first part of the proposition is proved. The ratio of any element of the expected difference to the majority group standard deviation would be

$$(11) \quad E(m^m)_j / SD_j = \frac{-\mu f_j \delta}{\sqrt{[\alpha^2 + \nu^2 W_{j,j}^2](1 + \mu 2 f_j c_j) + f_j^2 \mu *}}$$

$$= \frac{-\mu\delta}{\sqrt{[\alpha^2 + \nu^2 W_{j,j}^2](1/f_j^2 + \mu 2\beta) + \mu *}}.$$

where the derivation of the standard deviation of *j*th element of *m* can be found in appendix 1. Thus the differences between the minority and majority group means on each subtest will have the same rank order as *f* which is the same as *c* and the same as the skill loadings on the first principal component.

Can the model replicate the findings reported by Jensen (1999) and Colom et al. (2006) that the rank ordering of correlations of various subtests with certain physiological characteristics is similar to the rank ordering of the subtests' g loadings? There are no physiological characteristics in the model. Were we to model them carefully we would certainly want to allow for multiple genetic influences on each characteristic and possibly environmental influences as well. However, to demonstrate that the model being considered here can generate the sorts of relationships described in the literature a very simple model of a particular type of physiological trait will suffice. Suppose that the same genetic factor that influences an ability *i* also influences a physiological trait *y*, so that *y* measured in standard deviations from its mean is given by

$$(12) \qquad y = \varphi g_i + \varepsilon$$

where ε will be assumed to be uncorrelated with all elements of *g* and *z* and have a mean of zero.

Proposition 4
Given the assumptions above, the rank ordering of the correlation of *y* with the elements of *m*, other than the *i*th element of *m*, will be the same as the rank ordering of the correlations of *m* with the first principal component of the correlation matrix of the *m* s if $W_{j,j}=W_{k,k}$.

Proof
From equations A2 and A3 from appendix 1 we can write

$$(13) \qquad E(y\ m)_j = \begin{cases} for\ i \neq j & \alpha\varphi f_j c_i \mu \\ for\ i = j & \alpha\varphi(1 + f_j c_i \mu) \end{cases}.$$

Thus for *i* not equal to *j* the correlation of *y* and the *j*th element of *m* is equal to

$$(14) \qquad C_{y,m_j} = \frac{\alpha\varphi f_j c_i \mu}{\sqrt{(\alpha^2 + \nu^2 W_{j,j}^2)(1 + \mu 2 f_j^2 \beta) + f_j^2 \mu *}}.$$

since the standard deviation of y has been assumed to be one. So

$$(15) \qquad C_{y,m_j} > C_{y,m_k} \quad \Rightarrow \frac{\alpha\varphi c_i \mu}{\sqrt{(\alpha^2 + \nu^2 W_{j,j}^2)(1/f_j^2 + \mu 2\beta) + \mu *}} > \frac{\alpha\varphi c_i \mu}{\sqrt{(\alpha^2 + \nu^2 W_{k,k}^2)(1/f_k^2 + \mu 2\beta) + \mu *}}$$

which will be true if $f_j > f_k$ and $W_{j,j}=W_{k,k}$ and the proposition is proved. Since a superior genetic endowment for any cognitive trait will lead one to spend more time in the cognitively demanding environment one will get practice in all skills, and the most practice in the skills most important in that environment. Thus any physiological trait associated with a superior genetic endowment for any skill will have a tendency to be most highly correlated with those abilities that are most highly g loaded. Of course, as equation 13 shows, they will be most strongly correlated with the particular ability or abilities with which they are directly associated, but other than that their correlation will follow the ranking of the importance of the ability in the cognitively demanding environment.

Should secular gains in cognitive ability be g gains? That depends on the mechanism causing them. If gains were caused by people spending more time in the cognitively demanding activity then a slight recasting of proposition 3 would tell us that the rank order of gains would have to be identical to the rank order of the factor loadings of the skills. Just think of $-\delta$ as the additional time the average person is spending in the cognitively demanding environment. In that case equation 11 shows that the ranking of the gains will be the same as the ranking of the factor loadings. However, if the gains are caused at least in part by changes in the relative importance of different skills in the cognitively demanding environment then gains need not have any relationship to the factor loadings.

In time period zero before the change $f=f_0$ and in time period 1 $f_1=f+d$, the change in the expected value for m from period 0 to period 1, assuming no change in average genetic endowment, would be

$$(16) \quad E(\Delta m) = E(m_1 - m_0) = AE(g_1 - g_0) + VE(e_1 - e_0) \\ = VE(e_1 - e_0)$$

Now if τ is the average amount of time spent in the cognitively demanding environment in time period 0 and the relation $c=\beta f$ then

$$(17) \quad E(e_1 - e_0) = E(f_1((c + \beta d)' m_1 + \tau) + Wz - f(c' m_0 + \tau) - Wz) = \\ = (f + d)(c + \beta d)' E(m_1) + d\tau - fc' E(m_0) \\ = (f + d)(c + \beta d)' E(m_1 - m_0) + d\tau .$$

where the last equality follows because $m_0=0$.

Substituting (17) into (16), solving for Δm, and substituting $f+d$ for f and $c+\beta d$ for c in appendix equation A3 yields

$$(18) \quad E(\Delta m) = \left(I + \frac{\nu(f + d)(c + \beta d)'}{1 - \nu(c + \beta d)'(f + d)} \right) Vd\tau \\ = \left(\frac{\nu\tau d \left[1 - \nu(c + \beta d)'(f + d) \right] + \nu^2 (f + d)(c + \beta d)' d\tau}{1 - \nu(c + \beta d)'(f + d)} \right)$$

so

$$(19)\quad E(\Delta m)_i = \frac{\nu\tau d_i[1-\nu(c+\beta d)'(f+d)] + \nu^2\tau(f_i+d_i)(c+\beta d)'d}{1-\nu(c+\beta d)'(f+d)}$$

$$= \frac{\nu\tau d_i[1-\nu(c+\beta d)'f] + \nu\ \ \tau\nu f_i(c+\beta d)'d}{1-\nu(c+\beta d)'(f+d)}.$$

Thus the relative magnitude of the changes in any two skills depends on both the original demands and the change in demands and there is no reason for the magnitude of the changes across the skills to be strongly correlated with the g loadings. Of course if some of the secular increase is due to people spending more time in more cognitively demanding activities and the demands of those activities do not change too much then the changes could be correlated with the g loadings. But, the correlation could be weak or strong depending on which of the two mechanisms was more important for producing the secular gains and the other parameters of the model.

The heritability of an ability is the fraction of variance due to genetic variance. In this model with $W_{ii}=\omega$ for all i that will be

$$(20)\quad h_i^2 = \frac{\left[(I-VB)^{-1}(AA')(I-VB)^{-1}{}'\right]_{i,i}}{\left[(I-VB)^{-1}(AA'+VWW'V')(I-VB)^{-1}{}'\right]_{i,i}} = \frac{\alpha^2}{\alpha^2+\nu^2\omega^2}\ \forall\ i.$$

Thus with $W=I\omega$ the heritability of all subtests will be constant. Appendix 2 shows that $W_{i,i}<W_{j,j} \Rightarrow h^2_i>h^2_j$. Since all the results about the relative rankings of different quantities above held with strict inequality when $W=\omega I$ they will continue to hold for some values where the diagonal elements of W depart from equality. Thus it is possible for the model above to have subtest heritabilities with the same ranking as factor loadings and majority-minority differences.

But why would the rankings of the Ws be the inverse of the rankings of the fs? Mechanically, since the variance of the systematic component of environment Bm is larger for the abilities with the largest values of f if we normalize e by scaling the elements of f and W proportionally to standardize the es the Ws will take on a rank order that is the inverse of the order of f. But, this doesn't really answer the question as there is no strong reason to prefer standardized valued for environmental effects in the equation for ability than environment measured in any other metric. However, there are substantive reasons why we might expect the variance of the exogenous effects to have the reverse order of magnitude of the endogenous effects. If variables that are less characteristic of the cognitive activity are more likely to be used in non-cognitive activities then they will be more subject to shocks from outside the system described by equations 1 and 2. In that case the standard deviation of those shocks (the $W_{i,i}$ s) will be larger. For example, memory is an ability that is less correlated with g than many other cognitive skills, but is ubiquitous in human activities -- probably more so than verbal skills or quantitative skills. On the other hand, more esoteric and more highly g loaded skills, such as the ability to imagine and manipulate mental models of three dimensional objects, are probably less commonly called upon in activities that don't have a large cognitive component. If this is true to any degree then there will be a tendency for g loadings and subtest heritabilities to be positively correlated.

Can all of the results demonstrated above be obtained in a model such as that just described? The following numeric example shows the answer is yes. To capture the intuition as to how the variance of exogenous environmental effects could be larger for the cognitive abilities that are least representative of the cognitive task equation 2 will be slightly modified to

$$(2') \quad e = F\left[C'm + Wz_i\right].$$

The covariance matrix of the abilities thus becomes

$$(4') \quad E(mm') = (I - VFC')^{-1}(AA' + VFWW'F'V')(I - VFC')^{-1}{}'.$$

The model is implemented with twenty abilities – five cognitive abilities and fifteen non-cognitive abilities. There are seventeen activities that people divide their time between: one cognitive activity that puts demands only on the five cognitive abilities, fifteen non-cognitive activities each of which puts demands on its own unique non-cognitive activity and one of the cognitive activities, and a final non-cognitive activity that buts no demands on any of the activities being modeled (and is thus the residual category for time allocation). Five of the non-cognitive activities put demands on the cognitive ability with the lowest demands in the cognitive activity. Four put demands on the cognitive ability with second lowest demands in the cognitive activity. Three put demands on the cognitive ability with the third lowest demands in the activity, and so on. The only other deviation from the assumptions made in the previous model is to allow the $W_{i,i}$ for the cognitive activity to be one third the size of that for the non-cognitive activities. The same effect could have been approximated by tripling the number of non-cognitive activities.

The model is analyzed using the computer program presented in Appendix 3. That program first computes the results presented in table 1 below using equation (4') and extensions of the formula developed in equations (9) through (20).

Table 1
Results for Model with Normalization of *m* and *e**

Cognitive Abilities	Effect of Demanding Environment (f)	Loading on First Principal Component	h^2	Majority-Minority Gap in Majority SDs	Change in Effect of Demanding Environment (d)	Secular Gains
1	1.0	.82	.77	1.07	.20	.56
2	0.9	.79	.72	1.01	.24	.63
3	0.8	.76	.67	0.94	.10	.53
4	0.7	.73	.63	0.87	.24	.67
5	0.6	.69	.59	0.79	.20	.66

* Results computed with parameter values of α=.7, β=1, ν=.15, δ=3, τ=1, $W_{\text{cognitive}}$ =1, $W_{\text{non-cognitive}}$ =3. See appendix 3 for the program used to do computations.

With these parameter values the first principal component explains 58% of the standardized variance in the five abilities. The results show that the rank ordering of the effect of the demanding environment, the first factor loadings, the heritabilities and the majority-minority score gap are all the same. At the same time, the secular gains on the different tests are determined far more by the change in the effect of the demanding environment than the base effect of such environments.

Conclusion

Advocates of the view that cognitive ability can be shaped by environment have pointed to secular gains as strong evidence that large environmentally induced changes are possible. Skeptics have argued that since secular gains are not g gains they are not likely to be substantive. But that begs the question; what is g?

Here a model of cognitive ability has been presented in which the correlations of different abilities that give rise to a g factor result not from underlying biological causes, but from the mechanism by which abilities are reinforced with practice. People who are good at any cognitive skill are more likely to spend more time in activities that put demands on all their cognitive abilities. These demands give them additional practice which improves their skills. The skills that are used most in more demanding activities, and are therefore important in determining the amount of time spent in such activities, are the ones that are most highly correlated with other skills, and load most heavily on the g factor. When discrimination reduces one group's access to more cognitively demanding activities this will reduce their opportunities for practice and they will have lower ability. The deficits will be largest on the skills most used in the activities to which they have reduced access. In the models presented above, this produced losses that were greatest on the most g loaded abilities. In some versions of the model these were also the abilities that were most highly heritable.

Is there any reason to prefer the model presented in this paper to the biological g model described in the opening of the last section? The biological g model has a very difficult time explaining both the magnitude and pattern of secular gains. In the biological g model environment plays a relatively small roll in the determination of cognitive abilities because heritabilities are relatively high. Secular gains of a standard deviation or more would require exogenous improvements in the environment of considerably more than a full standard deviation. This seems implausible. But if somehow the genome was improving (due say to decreased inbreeding) then secular gains should have been g gains. Again, this doesn't seem to be the case.

It was this paradox – large IQ gains vs. high estimates of heritability – that lead Flynn and I to develop the model presented in our 2001 paper. The reciprocal relationship between phenotype ability and environment multiplies the effects of both exogenous environmental and genetic endowment. Environmental shocks that differentiate people in similar populations tend to be short lived and therefore don't gain the full benefit of the multiplier unlike genetic effects or environmental changes between generations or between relatively segregated social groups.

Flynn (forthcoming) has argued that the pattern of cognitive gains on different tests at different times can be explained by the changing technology and culture. In the early 20th century, the scientific world view, with its emphasis on using logic to deal with non-concrete problems on classifying the world spread from a few intellectuals to the general population. Also, the rise of mass production required a literate and numerate workforce while increasing numbers of technical workers required more sophisticated skills. All skills rose together. In the second half of the 20^{th} century the switch away from an industrial economy to an information economy gave rise to increasing demands for flexibility in adapting to new working environments. This produced a big rise in the demand for on-the-spot problem solving skills. At the same time calculators and computers were reducing the need for people to be good at routine calculation. Thus the ability to solve novel problems increased enormously while more mundane skills, such as vocabulary and arithmetic, stalled. This story fits neatly with the models presented here. They can do a better job than the physiological g models of explaining both the magnitude and the pattern of cognitive gains.

The implication is that secular gains are substantive and adult cognitive ability malleable. In fact, if the gains weren't substantive it would be hard for them to be so big. Large gains are possible because of the feedback between environment and phenotype ability. Gains in ability cause people to get themselves into better environments that produce higher ability. If the gains weren't substantive then there would be no reason for them to cause further improvements in environment and there would be no continuing feedback.

In the models presented here all the correlation of abilities is explained by the feedback mechanism. This was done to clearly illustrate the alternative mechanism. However, it is possible that there are underlying biological capabilities that differ between people and affect more than one mental skill. If so, then the g pattern we observe in the correlation of abilities would reflect both the underlying physiological sources of correlation as well as the reinforcement mechanism. The relative contribution of the two would depend both on the importance of the physiological process as well as the importance of the environmental feedback effects. Elsewhere Flynn and I (Dickens and Flynn 2001, p362) have argued that the best evidence suggests that at least 40% of the effects of genetic endowment on IQ come through environmental feedback. That would suggest an important roll for the mechanisms described in this paper for explaining the g phenomena.

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Appendix 1

Proof of Proposition 1

Equivalent to equation (3) from the text, the solution to the system of simultaneous equations in given in (1) and (2) can be obtained by repeated substitution. Substituting (2) into (1) and dropping the subscript for individual yields

$$(A1)\quad m = Ag + VWz + VBm\,.$$

We can then substitute this equation into itself repeatedly to arrive at

$$\begin{aligned}(A2)\quad m &= Ag + VWz + VB[Ag + VWz + VBm] \\ &= Ag + VWz + VB[Ag + VWz] + (VB)^2[Ag + VWz +] + \ldots \\ &= (I + \sum_{i=1}^{\infty}(VB)^i)[Ag + VWz].\end{aligned}$$

Substituting *fc'* for *B* and recalling that $V=\nu I$ we can write

$$(A3)\quad I + \sum_{i=1}^{\infty}(VB)^i = I + \sum_{i=1}^{\infty}\nu^i(fc')^i = I + \nu fc'\sum_{i=1}^{\infty}(\nu c' f)^{i-1} = I + \frac{\nu fc'}{1-\nu c' f} = I + \mu fc'.$$

where $\mu=\nu/(1-\nu c'f)$. This follows since the sequence $(fc')^i = fc'\ldots fc' = fc'(c'f)^{i-1}$ reduces to the matrix *fc'* times the scalar constant *c'f* to some power and since $\sum_{i=0}^{\infty} x^i = \frac{1}{1-x}$ for $x<1$. The constant $\nu c'f$ must be less than 1 by the assumption that the system of equations has a finite solution. If the condition did not hold the *m* s would all be infinitely positive or negative. Substituting (A3) into (A2) and comparing the result to (3) in the text we see that $(I - BV)^{-1} = I + \frac{\nu fc'}{1-\nu c' f} = I + \mu fc'$ when $B=fc'$.

Combining (A2) and (A3) the covariance matrix for the *m* s can be written

$$\begin{aligned}(A4)\quad E(mm') &= (I + \mu fc')(AE(gg')A' + VWE(zg')A' \\ &\quad + AE(gz')W'V' + VWE(zz')W'V')(I + \mu fc')' \\ &= (I + \mu fc')(AA' + VWW'V')(I + \mu fc')' \\ &= (\alpha^2 I + \nu^2 WW') + \alpha^2\mu[fc' + cf'] + \\ &\quad \nu^2\mu[fc'WW' + WW'cf'] + \mu^*\, ff'\end{aligned}$$

where μ^* is the scalar constant $\alpha^2\mu^2c'c+\nu^2\mu^2c'WW'c$. The second equality follows from the first because $E(gg')=E(zz')=I$, by the assumptions that *g* and *z* are standardized and independent, and $E(gz')=E(zg')=0$ by the assumption that the g s and z s are uncorrelated. Thus the i,j th element ($i\neq j$) of the covariance matrix of the m s can be written

$$(A5)\quad E(mm)_{i,j}=\mu(f_i c_j[\alpha^2+\nu^2W_{j,j}^2]+f_j c_i[\alpha^2+\nu^2W_{i,i}^2])+f_i f_j\mu^*.$$

Since the first term in the last equality in (A4) contains only diagonal elements it disappears for these off diagonal elements ($i\neq j$). The other terms in (A4) are matrices with all positive elements. Since the correlation of any two m s is their covariance divided by their standard deviations all correlations must be positive and the first part of the proposition is proved.

Next note that the variance of the ith element of m is

$$(A6)\quad E(mm')_{i,i}=[\alpha^2+\nu^2W_{i,i}^2](1+\mu 2f_i c_i)+f_i^2\mu^*.$$

Now define $C_{i,k}=E(mm')_{i,k}/(E(mm')_{i,i}E(mm')_{j,j})^{.5}$ as the correlation of m_{ij} and m_{ik} then for all k not equal to i or j

$$(A7)\,C_{ik}>C_{jk}\Rightarrow\frac{\mu f_i c_k[\alpha^2+\nu^2W_{k,k}^2]+\mu f_k c_i[\alpha^2+\nu^2W_{i,i}^2]+f_i f_k\mu^*}{\sqrt{([\alpha^2+\nu^2W_{i,i}^2](1+\mu 2f_i c_i)+f_i^2\mu^*)([\alpha^2+\nu^2W_{k,k}^2](1+\mu 2f_k c_k)+f_k^2\mu^*)}}$$

$$>\frac{\mu f_j c_k[\alpha^2+\nu^2W_{k,k}^2]+\mu f_k c_j[\alpha^2+\nu^2W_{j,j}^2]+f_j f_k\mu^*}{\sqrt{([\alpha^2+\nu^2W_{j,j}^2](1+\mu 2f_j c_j)+f_j^2\mu^*)([\alpha^2+\nu^2W_{k,k}^2](1+\mu 2f_k c_k)+f_k^2\mu^*)}}$$

$$\Downarrow$$

$$\frac{\mu\beta[2\alpha^2+\nu^2(W_{k,k}^2+W_{i,i}^2)]+\mu^*}{\sqrt{([\alpha^2+\nu^2W_{i,i}^2](1/f_i^2+\mu 2\beta)+\mu^*)}}>\frac{\mu\beta[2\alpha^2+\nu^2(W_{k,k}^2+W_{j,j}^2)]+\mu^*}{\sqrt{([\alpha^2+\nu^2W_{j,j}^2](1/f_j^2+\mu 2\beta)+\mu^*)}}.$$

The third inequality follows from the second after both sides are multiplied by the standard deviation of m_k divided by f_k, the numerator and denominator of the left-hand-side are divided by f_i , the numerator and denominator of the right-hand-side are divided by f_j, and terms are collected. Inspecting this third inequality it can be seen that if $W_{i,i}=W_{j,j}$ the two sides are identical except for the f_i and f_j terms in the denominators. Since $f_i>f_j$ if $j>i$ by assumption, and since all terms in (A7) are positive, $C_{i,k}>C_{j,k}$ for all k not equal to i or j if $j>i$ and $W_{i,i}=W_{j,j}$. Since this is a strict inequality it will still hold for some values $W_{i,i}$ in the neighborhood of $W_{j,j}$. Thus the second part of proposition 1 is proved.

Appendix 2

Analysis of Heritability When the Diagonal Elements of $W_{i,i}$ are Not All Equal

Using equation 6 from appendix 1 and equation 20 from the text

$$(A8)\quad h_i^2 = \alpha^2\phi_i \Big/ \left(\alpha^2\phi_i + \nu^2 W_{i,i}^2\left[1+2\beta\mu f_i^2\right] + \nu^2\mu^2 f_i^2 c'WW'c\right)$$

where $\phi_i = (1+f_i^2\,[2\beta\mu+\mu^2 c'c])$. Thus

$$\begin{aligned}(A9)\quad h_i^2 > h_j^2 \Rightarrow\ & \phi_i\left(\nu^2 W_{j,j}^2\left[1+2\beta\mu f_j^2\right] + \nu^2\mu^2 f_j^2 c'WW'c\right)\\ & > \phi_j\left(\nu^2 W_{i,i}^2\left[1+2\beta\mu f_i^2\right] + \nu^2\mu^2 f_i^2 c'WW'c\right),\end{aligned}$$

where the second version of the inequality is obtained by substituting (A8) into the first inequality, multiplying both sides of the inequality by the denominators of each side and then subtracting the term $\alpha^2\phi_i\phi_j$ from both sides.

From equation 20 in the text we know that $h^2_i = h^2_j$ when $W_{i,i} = W_{j,j}$ so if the derivative of the left hand side of the inequality in (A9) with respect to $W_{j,j}$ is greater than that of the right hand side we will know that $W_{j,j} > W_{i,i} \Rightarrow h^2_i > h^2_j$. Taking derivatives of both sides of (A9) with respect to $W^2_{j,j}$ yields

$$(A10)\quad \phi_i\left(\nu^2\left[1+2\beta\mu f_j^2\right] + \nu^2\mu^2 f_j^2 c_j^2\right) > \phi_j \nu^2\mu^2 f_i^2 c_j^2$$

$$\Downarrow$$

$$\begin{aligned}&\nu^2\left[1+2\beta\mu f_j^2\right] + \nu^2\mu^2 f_j^2 c_j^2 + \nu^2 f_i^2\left[2\beta\mu + \mu^2 c'c\right]\left[1+2\beta\mu f_j^2\right] + \nu^2\mu^2 f_i^2 c_j^2 f_j^2\left[2\beta\mu + \mu^2 c'c\right]\\ &> \nu^2\mu^2 f_i^2 c_j^2 + \nu^2\mu^2 f_i^2 c_j^2 f_j^2\left[2\beta\mu + \mu^2 c'c\right]\end{aligned}$$

$$\Downarrow$$

$$\nu^2\left[1+2\beta\mu f_j^2\right] + \nu^2\mu^2 f_j^2 c_j^2 + \nu^2 f_i^2\left[2\beta\mu + \mu^2 c'c\right]\left[1+2\beta\mu f_j^2\right] > \nu^2\mu^2 f_i^2 c_j^2.$$

The last inequality holds since the term $\nu^2 f_i^2 \mu^2 c'c$ on the left hand side of the inequality is greater than $\nu^2\mu^2 f_i^2 c_j^2$ on the right. Thus $W_{j,j} > W_{i,i} \Rightarrow h^2_i > h^2_j$.

Appendix B
Program to Solve Model with Different $W_{i,i}$ s

This program is written in GAUSS. A GAUSS interpreter is built into the console version of OX which is available free to academic users at http://www.doornik.com/download.html .

```
alpha=.7;      @ Direct Effect of Genetic Endowment on Ability @
nu=.15;        @ Direct Effect of Environment on Genetic Ability @
beta=1;       @ Ratio of Coefficients of ability to Activity Demands @
omega=3;       @ SD of Non-cognitive Environmental Innovations @
omega1=1;      @ SD of Cognitive Environmental Innovations @
delta=3;       @ Black disadvantage in access to cognitive activity @
d={.2,.24,.1,.24,.2};@ Change in demands of cognitive activity @
d=d*1.2;
tau=1;         @ Average time spent in each activity @
psi=1;         @ Correlation of genetic endowment with physical trait @

@ Pad d with zeros for non-cognitive activities @
d=(d|zeros(15,1))~zeros(20,15);
@ Coefficients for Genetic Endowment in Ability Equation @
A=Eye(20)*alpha;
@ Coefficients for Environment in Ability Equation @
v=eye(20)*nu;

/* Set up Matrix of Activity Demands  */
@ Cognitive activity puts demands of .6 .7 .8 .9 and 1 on 5 cognitive abiliities @
@ and no demands on non-cognitive abilities. Non cognitive activites put demands @
@ of 1 on their unique ability. Zeros are place holders for demands of non- @
@ cognitive activities on cognitive abilities   @
F=(seqa(.6,.1,5)|zeros(15,1))~(zeros(5,15)|1*eye(15));

@ Add demands of non-cognitive activities on cognitive abilities. @
@ Five activities put demands of .5 on first ability, four on second @
@ three on third, two on fourth and one on fifth. Those abilities  @
@ that are least "characteristic" of the cognitive activity are the  @
@ ones most likely to be used in a non-cognitive activity.   @
k=2;
for i (1,5,1);
        for j (1,i,1);
                F[j,k]=.5;
                k=k+1;
        endfor;
endfor;

@ create C, W and B matricies @
C=beta*F;
W=omega*eye(16);
W[1,1]=omega1;
B=F*C';

"multiplier";
mult=inv(eye(20)-V*B);
mult[1:5,1:5];
"Covariance";
covmat=mult*(A*A'+V*F*W*W'F'V')*mult';
covmat;
"Correlation Matrix";
cormat=(covmat./sqrt(diag(covmat)))./sqrt(diag(covmat)');
cormat;
"Fraction of Variance Explained by First PC";
{va,ve}=eigrs2(cormat[1:5,1:5]);
va[5]/5;
"First Factor Loading";
ve[.,5]'sqrt(va[5]);
"Heritability";
num=mult*A*A'mult';
```

```
(diag(num)./diag(covmat))';
"Black disadvantage in SDs ";
delta/sqrt(C[1:5,1]'covmat[1:5,1:5]*C[1:5,1]);
"White-Black Difference";
disad=delta|zeros(15,1);
(mult*V*F*disad)';
"Change in cognitive abilities";
C1=F+d;
(inv(eye(20)-V*(F+d)*C1')*V*d*ones(16,1)*tau)';
```