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# DEMOGRAPHIC PRESSURES ON PUBLIC PENSION SYSTEMS AND GOVERNMENT BUDGETS

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This paper stems from a joint research project studying the global dimensions of demographic change coordinated by Ralph C. Bryant at the Brookings Institution and Warwick J. McKibbin at the Australian National University. The project is supported by the Economic and Social Research Institute of the Japan Cabinet Office as part of their series of international Collaboration Projects. Hamid Faruqee and Delia Velculescu at the International Monetary Fund and McKibbin and Jeremy Nguyen at the Australian National University contributed essential inputs to the ideas about the theoretical framework and their implementation in this research project. Delia Velculescu collaborated on an earlier first look at the issues of incorporating public pensions into the project's analysis. The views expressed in this paper are those of the author and collaborators alone and should not be attributed to the institutions with which they are associated. Marc de Fleurieu, Elif Arbatli, and Pablo Montagnes provided skillful, thoughtful research assistance.

## ***Abstract***

Demographic shifts profoundly influence the world economy, directly in the national economies experiencing the shifts and indirectly through changes elsewhere brought about by cross-border transactions. The research summarized in this paper is part of a project to study the global dimensions of demographic change, emphasizing macroeconomic effects working through changes in exchange rates and external-sector variables which in turn have major consequences for saving and investment flows in national economies and the world economy as a whole. Comparing alternative variants of public pension systems, this paper studies the domestic-economy and external-sector consequences of rising elderly dependency ratios brought about by earlier declines in fertility. It shows that alternative ways of operating public pension systems and managing government debt can lead to substantially different macroeconomic outcomes, especially when the openness of economies is fully integrated into the analysis. The paper also challenges the conventional wisdom that population causes unambiguously adverse macroeconomic consequences. For an open economy that is moving faster into or is further along in its demographic transition, negative consequences accompanying the demographic shift are typically cushioned because the negative effects are shared with the rest of the world. Such cushioning and sharing may not be desirable as seen from the perspective of foreigners, but it may produce sizable welfare gains for home residents.

## ***Structural Outline***

## 1. Introduction

The world at the beginning of the twenty-first century is in the middle of an unprecedented demographic transition. For much of the 20th century mortality rates for infants declined sharply and life expectancy increased for adults. Fertility rates began to fall in the second half of the 20th century, initially in the developed and more recently in developing nations. These demographic changes have already significantly influenced the age structures of populations in industrialized countries, reducing the number of youths relative to adults. The demographic transition will continue well into the 21st century. Industrialized nations will experience pronounced aging of their populations in coming decades. After varying delays, further declines in fertility and a progressive rise in elderly dependency ratios will occur in developing nations as well.[1]

Japan is the most prominent example of an industrial nation whose fertility rates have fallen dramatically and whose population is aging rapidly. Between the end of World War II and the end of the 20th century, Japanese fertility declined from over 4 to some 1.4 lifetime births per woman. The ratio of youths to the total population fell from over 45 percent to less than 21 percent between 1950 and 2000. Projections for the ratio of elderly to the total population show a near doubling over the next five decades from 17.2 percent to 36.9 percent. Figure 1, based on the revised UN projections released in 2001, visually highlights the faster and larger nature of Japan's demographic shift relative to the average shift for developed countries as a whole.

Public pension ("social security") systems are maintained in most developed and several developing nations. Many take the form of partial or full pay-as-you-go (PAYG) systems in which the current generation of workers contribute pension ("payroll") tax revenues that finance the pension benefits received by the elderly. It is now widely understood that population aging soon will put growing pressures on PAYG pension systems. At current pension-tax and pension-benefit rates, benefit payments made to an increasing number of elderly retirees will exceed system tax revenues that in any case must be paid by a relatively smaller number of nonelderly workers. In country after country, policymakers and administrators have taken, or are prospectively contemplating, actions either to make cuts in benefits, raise tax rates, or accept growing deficits in pension-system and overall government budgets.

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[1] Lee (2003) provides an overview of the demographic transition as a whole. See also United Nations, Population Division (2001) and Birdsall, Kelley, and Sinding (2001).

A growing body of theoretical and empirical research has analyzed the implications for national economies of demographic changes. Much of this work has focused on population aging and its effects on saving, investment and growth.[2] Research on the policy issues has focused on the increasing burdens on national budgets and pension systems and on the menu of possible options for reform.[3]

Much of the analysis of the relationship between population growth and economic growth initially concentrated just on the size and growth rate of the total population, paying little attention to shifts in the age structure of the population. The literature focusing on government budgets and pension systems has neglected the effects of fertility declines and falling youth-dependency ratios.

Cross-border interactions are the least studied aspects of the demographic transition. Research has seldom asked how demographic change in individual open economies influences macroeconomic developments abroad or how it influences the balance of saving and investment flows in an open national economy and in the global economy as a whole. From the outset of our project, we have been especially concerned with the spill-over effects of demographic changes in one country on foreign economies through changes in exchange rates and other external-sector variables and with the consequences for saving and investment flows in the world economy as a whole.[4]

As our own research progressed, we became increasingly dissatisfied with the treatment of demographic variables in macroeconomic models. Rather than treating youth dependency and elderly dependency as exogenous inputs to macroeconomic analysis, we sought increasingly to incorporate more of the demographics as integral, endogenous parts of the modeling framework. By paying attention to the *entire* age distribution of the population and its evolution through time, we became able

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[2] Early contributions include Coale and Hoover (1958), Leff (1969), Mason (1987, 1988), Cutler, Poterba, Sheiner, and Summers (1990), Masson and Tryon (1990), Horioka (1991), and Auerbach, Cai, and Kotlikoff (1991). For the more recent literature, see for example Masson, Bayoumi and Samiei (1995, 1998), Deaton and Paxson (1997), Bloom and Williamson (1997), Higgins and Williamson (1997), Higgins (1998), Velculescu (2003a), and Brooks (2000; 2002, 2003).

[3] Contributions include Aaron, Bosworth, and Burtless (1989), Burtless and Bosworth (1997), Bosworth and Burtless (1998), Aaron and Reischauer (2001), Burtless (2002); Wise (1994), Gruber and Wise (1999), Disney (1996); World Bank (1994), Leibfritz, Roseveare, Fore, and Wurzel (1996), Kohl and O'Brien (1998), OECD (1998); Lee and Skinner (1999), Lee and Edwards (2002), Elmendorf and Sheiner (2000), Kotlikoff, Smetters, and Walliser (2001), Brooks (2002), Velculescu (2003b, 2004). Publications about Japan include Takayama (1992, 1996, 1998, 2001), Japan Ministry of Health, Labor and Welfare (1999), Miyatake (2001), Ogura, Tachibanaki, and Wise (2001), and Clark and Mitchell (2002).

[4] Our own early work includes Bryant and McKibbin (2004), a paper that focuses on transitory "baby-boom" shocks analogous to those experienced by many industrial nations in the sixth and seventh decades of the 20th century. Faruqee and Mühleisen (2002) provide a similar analysis of macroeconomic effects in Japan. Other exceptions to the generalization that the cross-border effects of demographic changes have been ignored include Turner, Giorno, De Serres, and Vourc'h-Richardson (1998), Attanasio and Violante (2000), Brooks (2000), Borsch-Supan, Ludwig, and Winter (2003), and Fehr, Jokisch, and Kotlikoff (2003).

to analyze a broader range of demographic issues and their interactions with macroeconomic developments.

Our research significantly improves on the existing literature in two fundamental ways. First, by incorporating the later and the early stages of the aging process in a single framework – integrating the effects of rising elderly ratios and pension-system pressures with falling youth ratios and the implications for adult support of child consumption – we obtain a richer and more robust analysis of the macroeconomic consequences of demographic shifts. Second, we analyze the demographic changes in an explicitly open-economy context, capturing world general-equilibrium influences from cross-border transactions. That approach also leads to more robust conclusions about the net economic effects of demographic changes than those obtained from analytical approaches previously used in the literature.

An earlier paper concentrated on issues of youth dependency and showed that changes in youth dependency have first-order consequences for the determination of exchange rates, external imbalances, and global saving and investment flows (Bryant, Faruqee, Velculescu, and Arbatli, 2004). This paper focuses on issues of elderly dependency. It studies the cross-border and global effects of declining fertility rates with emphasis on public pensions. It is especially concerned with comparing the domestic-economy and external-sector effects of alternative variants of public pension systems on saving, exchange and interest rates, and external-sector imbalances. The current paper has conclusions analogous to those stemming from our earlier work. In particular, we show that the timing and sizes of macroeconomic effects of demographic shifts vary significantly across different types of pension-system operations.

Section 2 of the paper summarizes key features of our analytical approach, emphasizing linkages between the elderly and working adults through the operation of a public pension system. Section 3 analyzes the basic effects of sustained declines in fertility, first assuming no cross-border interactions and then subsequently assuming that the fertility shocks are asymmetric, causing significant open-economy spillovers. Section 4 studies macroeconomic effects for alternative pension systems that are continuously "balanced." Section 5 introduces "unbalanced" pension policies that bring overall government budgets and debt policies to the center of the stage. (Balanced and unbalanced pension systems are defined in section 2.) Section 6 identifies issues of intergenerational redistribution and equity. Section 7 concludes by summarizing fundamental points with a bearing on policy choices. A first appendix provides more details about the analytical approach. A second appendix provides details about baseline solutions for the model and the specification of the illustrative shocks.

## 2. Summary of Analytical Approach

Our analytical framework is a world composed of two countries with cross-border flows of goods and capital. The exchange rate linking the two currencies and economies adjusts to ensure that the global (algebraic sum of both countries) current-account balance and net-foreign-asset position is always zero. Within each economy, optimizing firms produce a single composite good, determined by an aggregate production function with capital and (productivity-augmented) labor as its arguments. The composite goods from each country are imperfect substitutes; some production in each country is exported; import demands are a function of national incomes and relative prices.

Households in each country are assumed to have identical preferences over foreign and domestic goods. The treatments of household consumption, saving, and wealth accumulation build on the overlapping generations framework of Blanchard (1985), Weil (1989), and Yaari (1965) as extended by, among others, Faruqee, Laxton, and Symansky (1997) and Faruqee (2002) to incorporate age-earnings profiles and a "bottom-up" determination of labor income. In our further extension of the framework, population growth and structure are endogenous. The population contains working adults, youth dependents (children for short), and elderly dependents who receive public pension benefits.

Our approach permits us to focus on the cross-border effects of country-specific changes in demography. A previous paper highlighted modifications in the treatment of consumption, saving, and wealth accumulation that are associated with youth dependency and the economic linkages between the child and adult populations.[5] This paper emphasizes elderly dependency and the implications of various public pension arrangements for country-specific and world levels of saving, investment, and interest rates.

The following description of the analytical framework suppresses most details and focuses on the modifications associated with elderly dependency and the economic linkages between the elderly and adult populations. Appendix 1 gives further information.[6]

### A. Population and Dependency Ratios

Previous research relying on the Blanchard-Weil-Yaari framework concentrated exclusively on the adult working population. Youth dependents and adult support for youth consumption were

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[5] Bryant, Faruqee, Velculescu and Arbatli (2004). A preliminary version of this paper was presented at the February 2003 ESRI colloquium; the 2004 version replaces the preliminary version.

[6] A separate background paper – Bryant (2004b) – presents a full exposition of our analytical framework, incorporating the various changes made since the start of our research.

ignored altogether. Elderly adults did not become "dependents" in the sense of receiving transfers from younger working adults.

To study youth dependency and elderly dependency, our extended framework modifies the earlier research in two ways. First, individuals enter the world not as adults but as dependent children, with distinct birth and mortality rates. (Adults are assumed to have a different, typically higher mortality rate than children.) After a period of life as dependent children, youths then enter the adult population and begin to supply labor. Second, after eventually reaching a threshold age, adults become elderly dependents and receive a public pension to supplement their other income flows.

The child birth rate and the mortality rates of adults and children are allowed to vary through time. To preserve the advantages of the Blanchard-Weil-Yaari framework for aggregation across individuals, however, we maintain two of its key assumptions. Elderly adults are assumed gradually to withdraw their labor from the workforce rather than discontinuously cease work at a retirement age.[7] Mortality rates are assumed to be age-invariant.[8]

The demographic dynamics of our model, though capturing key features of population structure, remain fairly simple. We denote the size of a child cohort, indexed by *s* (the time of birth) at time *t*, as *J(s,t)*. Correspondingly, *N(s,t)* is the size of an adult cohort at time *t*.

For children, the initial size of a cohort at the time of birth *s* is given by:

$$J(s,s) = b_j(s)N(s)\,, \tag{1}$$

where $b_j(s)$ is the birth (or fertility) rate at time *s*, expressed as a fraction of the contemporaneous adult population $N(s)$. Youth dependents all face the same infant/child mortality rate, $p_j(t)$. That rate can vary through time, but in any given period is the same for all youths regardless of age. The number of surviving children from the initial *J(s,s)* cohort at some later date *t* is given by:

$$J(s,t) = b_j(s)N(s)e^{-\int_s^t p_j(x)dx}\,. \tag{2}$$

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7 Hence adults continue to receive some labor income even after reaching an age at which they are defined as "elderly." After adults pass the years of their peak earnings and reach the threshold elderly age, however, their labor incomes decline continuously toward zero.

8 The assumption that mortality rates are age-invariant rather than age-specific departs seriously from reality. Blanchard (1985) himself pointed out that the evidence on mortality rates suggests low and approximately constant probabilities of death from, say, ages 20 through 40; thereafter mortality rates in real life do rise with age – sometimes modeled by "Gompertz's Law" suggesting that mortality rates after puberty rise in geometric progression as in Wetterstrand (1981), reaching rates (in the United States) in the neighborhood of 16 percent by age 80 and 67 percent by age 100. Faruqee (2003a) modifies the simplifying assumption that all adults are subject to the same age-invariant probability of death; with that modification, however, it is no longer straightforward to achieve the macroeconomic aggregation across individuals and age cohorts that is the marked advantage of the Blanchard-Weil-Yaari theoretical framework.

The total size of the child population is obtained by aggregating all child cohorts over the finite range $\Delta$ of childhood ages:[9]

$$J(t) = \int_{t-\Delta}^{t} J(s,t)ds \,. \tag{3}$$

Differentiating this expression and using the survivor formula given by equation (2) yields the following expression for the evolution of the child population:

$$\dot{J}(t) = b_j(t)N(t) - N(t,t) - p_j(t)J(t) \,, \tag{4}$$

where $N(t,t)$ denotes the outflow of youths into the adult population. More specifically, through continuity, the oldest child cohort passes into adulthood:

$$N(t,t) = J(t-\Delta\,,\ t)\ . \tag{5}$$

This cohort thus also represents the inflow of new adult workers into the economy.

Given a distinct adult mortality rate $p_n(t)$, the survivor formula for adult cohorts is given by:

$$N(s,t) = N(s,s)e^{-\int_s^t p_n(x)dx} \,. \tag{6}$$

The level and dynamics of the adult population $N$ are analogously derived:

$$N(t) = \int_{-\infty}^{t} N(s,t)ds \,; \tag{7}$$

$$\dot{N}(t) = N(t,t) - p_n(t)N(t) \tag{8}$$

Equation (8) shows that the net change in the adult population in each period depends on the inflow of new entrants to the adult population less the numbers of adults that die. The rate at which new youth entrants pass into the adult population can be viewed as an adult "birth" rate $b_n(t) \equiv N(t,t)/N(t)$. The population growth rate of adults is given by $n(t) = b_n(t) - p_n(t)$, where the adult "birth" rate $b_n(t)$ depends on the sequence of child birth rates and child mortality rates in earlier periods.

Given these population measures, the *youth dependency ratio* $\delta(t)$ is simply the ratio of the child population to the adult population:

$$\delta(t) = \frac{J(t)}{N(t)}. \qquad 0 \le \delta \le 1.. \tag{9}$$

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9 We parameterize $\Delta$ to be 18. Children are assumed to be wholly dependent for the first 18 years of their life. At the beginning of their 19th year, they enter adulthood, begin supplying labor input, and no longer receive any support payments from older adults.

Differentiating this expression yields the law of motion for the youth dependency ratio:

$$\dot{\delta}(t) = b_j(t) - [1+\delta(t)]b_n(t) + \delta(t)[p_n(t) - p_j(t)]\cdot \tag{10}$$

Intuitively, the evolution of youth dependency depends on the comparative rates of growth in the child and adult populations. This differential depends, in turn, on differences in birth and death rates for children and adults (appropriately scaled). Higher fertility rates tend to raise the youth dependency ratio, other things equal. The growth rate of the child population can be written as $\dot{J}(t) = [b_j(t) - b_n(t)]/\delta(t) - p_j(t)$, which is determined not only by the birth and death rates of children but also by their entrance into the labor force ( the adult "birth" rate).[10]

In a manner similar to the definition of the youth dependency ratio, an elderly dependency ratio is defined as the proportion of the adult population older than a certain threshold age – indexed by $i(t)$. The (fixed) difference between this index $i(t)$ and the present time, $t$, denoted by $\Lambda = t - i(t)$, reflects the number of adult years needed to reach elderly dependency status.[11] Given a fixed threshold age, the elderly dependency ratio is given by:

$$\phi(t) = \int_{-\infty}^{i(t)} \frac{N(s,t)}{N(t)} ds. \qquad 0 \le \phi \le 1. \tag{11}$$

In an economy with a constant adult "birth" rate, the elderly dependency ratio would also be constant.[12] For the case where the adult birth rate and death rate are time varying, the elderly dependency ratio evolves over time according to:

$$\dot{\phi}(t) = \frac{N(i(t),t)}{N(t)} - [p_n(t) + n(t)]\phi(t). \tag{12}$$

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10 Along a steady-state growth path, the following population relationships obtain: $\bar{b}_n = \bar{p}_n$; $\bar{b}_j = \bar{p}_n e^{\bar{p}_j \Delta}$; $\bar{\delta} = \dfrac{\bar{b}_j - \bar{p}_n}{\bar{p}_j}$.

11 For example, with a threshold age of 65 years, the adult cohort reaching 65 at time *t* became adults 47 years ago – that is, $\Lambda$= 47 (adulthood beginning in the 19th year of life).The threshold age does not imply "retirement" *per se*; there is, as noted earlier, no discontinuity of labor input at the threshold age. Rather, adults older than *i* still continue to receive some (but gradually declining) labor income even after they reach the age at which they are defined as "elderly."

12 Because death rates are modeled as age-invariant rather than age-specific, only the adult birth rate matters for the share of elderly dependents in the population. See also Faruqee (2002).

Intuitively, the change in the elderly dependency ratio is determined by the relative size of new dependents reaching the threshold age – the first term in (12) – *less* the proportion of the elderly who die during the period – $p_n(t)\phi(t)$ – and *less* a scaling term accounting for growth in the adult population base – $n(t)\phi(t)$.[13]

### B. Economy-Wide Saving and Wealth Accumulation

Adults in the model plan their consumption and wealth accumulation by solving a familiar utility-maximization problem. Adult consumption follows a modified life-cycle approach that allows for *inter vivos* transfers from parents to their children to finance child consumption. Children in the model are assumed to consume but not to earn any labor income and not to hold any financial wealth. The consumption of elderly adults in the model is financed by drawdowns of their financial wealth, by public pension benefits, and by progressively smaller amounts of labor income. The details of adult consumption are relegated to Appendix 1.

Total real domestic consumption across the entire population (working adults, elderly adults, and youth dependents), $C_{TOT}(t)$, is given by:[14]

$$C_{TOT}(t) \quad = \quad C_j(t) \ + \ C(t) \tag{13}$$

where $C_j(t)$ is aggregate child consumption and $C(t)$ is aggregate consumption for all adults (including the elderly). These two components of consumption are interconnected in a way explained in detail in Appendix 1.

Reflecting an economy-wide budget constraint for each of the countries, the aggregate stock of financial wealth (held by adults only) accumulates as:

$$F\dot{W}(t) \ = \ [r(t)FW(t) \ + \ Y(t)](1-\tau(t)) \ \ + \ PB_{ss}(t) - \tau_{SS}(t)Y(t) \ - \ C(t) \ - \ C_j(t) \quad , \tag{14}$$

where $r(t)$ is the domestic interest rate, $Y(t)$ is aggregate labor income for the entire economy, $PB_{ss}(t)$ is the aggregate total of pension benefits paid to the elderly, $\tau(t)$ represents the government's income tax

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[13] As discussed in Faruqee (2000, 2002), the model's assumption that the adult mortality rate is age-invariant has an undesirable consequence: the numbers of elderly in the total population are overestimated relative to the real-life situation in which adult mortality increases as adults become older and older.

[14] Behavioral relationships for the home country's agents and agents in the other country are identical. Accordingly this exposition omits country identifiers.

rate (applicable to both labor and capital income), and $\tau_{SS}(t)$ is the pension (payroll) tax rate. The left-hand side of equation (14) is the flow of net private saving in the economy.[15]

The fiscal authorities of each country engage in real spending on goods and services (typically, an exogenously set fraction of the national output), raise revenues by taxing the incomes of firms and households, and pay interest on the outstanding stock of government debt. Each country's government also operates a public pension system that collects revenue from a pension tax and pays out pension benefits to elderly adults. The financial sectors of the national economies, modeled in a rudimentary way, contain monies that are only high-powered money (central-bank liabilities), the demands for which depend negatively on short-term nominal interest rates and positively on national incomes. Policy reaction functions are specified for the central bank and the government fiscal authority.[16]

### C. Economic Linkages between Elderly and Working Adults through a Public Pension System

In our initial research on pensions, we implement the public pension system in a simplified pay-as-you-go (PAYG) form. On the revenue side, adult individuals throughout their working lives pay pension taxes as a given fraction – $\tau_{SS}$ , the "pension tax rate" – of their labor incomes. The pension taxes are in addition to income taxes paid on their total (labor plus capital) income. Given our treatment of age-earning profiles (see Appendix 1), individuals over the age of 65 continue doing some work and receiving a decreasing amount of labor income until the end of their lives. Hence the elderly continue to pay pension taxes after age 65 on that labor income. Since the effective labor input and labor income of the elderly declines sharply as they age beyond the threshold of 65 years, however, the pension taxes paid by elderly cohorts become negligible as they age further.[17]

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[15] Equation (14) reflects the fact that the aggregate of support transfer payments to children is equal to the total amount of child consumption. The equation also reflects the fact that aggregate financial wealth accumulates at the rate $r(t)$ – not at the rate $r(t) + p_n(t)$ – since the amount $p_n(t)FW(t)$ is not an addition to aggregate wealth, but rather a transfer from those who die to those alive implemented through the actuarially fair Yaari-style insurance companies. The variable $p_n(t)$ is the adult mortality rate. See the appendix for further details.

[16] Each central bank follows a policy rule that ensures long-run nominal stability of the national economy, either a targeting rule for (high-powered) money, a nominal-GNP-targeting rule, or a rule combining inflation targeting with real GNP targeting. Each fiscal authority uses an "intertemporal fiscal closure rule" that is a variant of debt-stock targeting (see below for further discussion).

[17] Given that the elderly in our framework never fully retire, the government's tax and pension systems are assumed not to penalize them for working in old age. The elderly start collecting the full (age-invariant) pension benefit at age 65. Elderly workers over the age of 65 continue to pay income taxes on their labor and capital income. The recipients of pension benefits, however, are assumed not to be required to pay income tax on those benefits.

After a worker reaches the threshold elderly age, he or she begins to receive a pension transfer from the government. In our initial implementation, an individual's pension benefit is not indexed to or otherwise influenced by his or her own earnings history.[18] Rather, for analytical simplicity we assume that the government sets pension benefits so that every elderly individual receives an identical payment. Part of this pension payment is a basic benefit (a "First Tier" component) that is inelastic to the behavior of aggregate labor incomes in the economy.[19] The remaining part (a "Second Tier") varies endogenously through time in proportion to contemporaneous aggregate labor income.

Seen from the perspective of an individual cohort *s*, the transfer scheme embodied in the public pension system can be summarized as:

$$ptr(s,t) = \begin{cases} -\tau_{ss}(t)y(s,t); \quad s > i(t) \\ \beta_{T1}(t) + \beta_{T2}(t)\dfrac{Y(t)}{N(t)} - \tau_{ss}(t)y(s,t); \quad s \leq i(t), \end{cases} \tag{15}$$

where $ptr(s,t)$ is the transfer amount for cohort *s* at time *t*, $\tau_{ss}(t)$ is the pension tax rate, $\beta_{T1}(t)$ is the inelastic Tier-One component of the pension, $\beta_{T2}(t)$ is the "benefit rate" used to calculate the Tier-Two pension component, $y(s,t)$ is the individual cohort's real labor income, and $Y(t)/N(t)$ is average real labor income in the economy as a whole (the aggregate variable to which the benefit rate is applied when calculating the Tier-Two component). Equation (15) states that younger generations ($s > i(t)$) pay pension taxes into the system and receive no benefits. Older agents having reached the threshold elderly age ($s \leq i(t)$) are "pensioners" and receive the two-component benefit. Since pensioners still receive modest amounts of labor income after they reach age 65, as noted above they still pay modest pension taxes on that income. An elderly individual's "net pension benefit" is the difference between the gross benefit and the pension taxes still paid on their declining labor income.

The two-tiered treatment of pension benefits is a general specification somewhat analogous to our two-part treatment of child consumption (summarized in Appendix 1). By setting the benefit rate $\beta_{T2}(t) = 0$, one can examine the implications of making the total pension benefit invariant to current conditions in the economy. Alternatively, if $\beta_{T1}(t) = 0$ whereas $\beta_{T2}(t) > 0$, an inelastic component of

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[18] In real life in (for example) the Japanese or the United States social security systems, pension payments of course are partly a function of an individual's earnings history.

[19] To keep long-run simulations with the model consistent with balanced growth paths, we have so far assumed that $\beta_{T1}(t)$ is adjusted secularly so that it grows at the same rate as labor productivity. Similarly, the nominal value of the Tier-One pension benefit is indexed to the long-run steady-state rate of inflation.

the total pension benefit is assumed absent altogether. In the general case, the total pension benefit includes non-zero components for both Tier One and Tier Two.

This pension system, though highly simplified, incorporates an essential characteristic of public pay-as-you-go systems: an individual's pension benefits cumulated from age 65 until death may be substantially greater or smaller than that individual's lifetime payments of pension taxes.

The aggregate amounts of pension taxes collected and benefits paid by the government in period *t* are:

$$PT_{SS}(t) = N(t)\tau_{SS}(t)\frac{Y(t)}{N(t)} = \tau_{SS}(t)Y(t), \tag{16}$$

$$PB_{ss}(t) = Eld(t)\left[\beta_{T1}(t) + \beta_{T2}(t)\frac{Y(t)}{N(t)}\right] = Eld(t)\beta_{T1}(t) + \phi(t)\beta_{T2}(t)Y(t), \tag{17}$$

where $PT_{ss}(t)$ and $PB_{ss}(t)$ are the real values of aggregate taxes and aggregate benefits, $Y(t)$ is aggregate real labor income in the economy, and $Eld(t)$ is the number of elderly. As above, $N(t)$ is the total adult population and $\phi(t)$ is the elderly dependency ratio. Equation (16) indicates that the total of pension taxes collected equals the pension tax on average labor income multiplied by the number of adults in the economy. Total pension benefits transferred to the elderly, equation (17), are the sum of aggregate Tier-One benefits and aggregate Tier-Two benefits, with the latter the product of the elderly dependency ratio, the Tier-Two benefit rate, and aggregate labor income in the economy.

In a continuously balanced pay-as-you-go pension system, a difference cannot exist between aggregate revenues from pension taxes and aggregate benefits paid to pensioners. This type of system could conceivably exist if the pension system were "off-budget" (kept strictly independent of other government revenues and expenditures) and the pension administrators were prohibited from accumulating or decumulating assets in a pension trust fund. To implement a continuously balanced PAYG system in our framework, one must enforce the condition $PT_{ss}(t) = PB_{ss}(t)$ in the current and all future periods. A continuously balanced status of the pension system, where equations (16) and (17) are always equal, can be written as:

$$\frac{\beta_{T1}(t)\dfrac{N(t)}{Y(t)} + \beta_{T2}(t) - \tau_{ss}(t)}{\tau_{ss}(t)} = \frac{1-\phi(t)}{\phi(t)}. \tag{18}$$

Equation (18) is a variant of the well-known condition that the net benefit-to-contribution ratio in a continuously balanced pension system must equal the support ratio – the number of working-age adults (19 through 64 years of age in our framework) relative to elderly dependents (65 years and over).

If equation (18) should not be satisfied, there will exist a positive or negative "financing gap" in the pension system reflecting the degree of over- or underfunding of current-period benefit payments. To analyze the interactions of demographic changes and public pension systems, it is often necessary to study cases of such over- or underfunding. In practice, such cases occur when the pension system is "on budget" instead of "off budget." Accordingly, we define a variable for the financing gap (in effect, changes in the real value of a "trust fund" for the pension system):

$$PTFGAP(t) = PT_{ss}(t) - PB_{ss}(t). \tag{19}$$

When the pension trust-fund gap is negative, the deficit in the pension system must be financed by changes elsewhere in the government's budget – either by increases in revenues from income taxes, cuts in government spending on goods and services, or increased government borrowing through additional issuance of government debt. Conversely, pension trust-fund surpluses must result in increases in government spending, reductions in other tax revenues, or retirements of government debt. A pension-system financing gap must therefore be included explicitly as part of the overall government budget identity and must be taken into account in assuring that the government budget satisfies the criterion of intertemporal consistency.

The budget identity of the fiscal authority in our framework may be written in *nominal* terms as:

$$GDEF(t) \quad \equiv \quad R(t)B(t) + P(t)\left[G(t) - T(t) - PTFGAP(t)\right] \quad . \tag{20}$$

Here *GDEF(t)* is the overall nominal deficit in the government's budget (with deficits expressed as positive numbers, surpluses as negative) and *P(t)* is the economy's overall price level. The nominal value of interest payments on the government debt is the product of the nominal interest rate *R(t)* and the outstanding stock of debt. Nominal spending on goods and services is the product of real spending *G(t)* and the price level. Revenues from income taxes are the product of the price level and real tax receipts: $T(t) = (1 - \tau(t))Y(t)$. The nominal value of the pension trust-fund gap is the product of *PTFGAP(t)* as defined in (19) and the price level.[20]

The corresponding identity for the financing of a nominal government deficit is:

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[20] For expositional simplicity, the equations prior to (20) are expressed in real rather than nominal terms. The empirical model used in the research carefully distinguishes real and nominal values and endogenously determines prices in the two national economies.

$$GDEF(t) \equiv \dot{B}(t) + \dot{M}(t) \quad ; \tag{21}$$

$\dot{B}(t)$ and $\dot{M}(t)$ are current-period changes in the nominal values of the stocks of, respectively, government debt and (high-powered) money.

In any carefully specified macroeconomic model, private agents will not hold government debt unless per capita government debt is eventually forced to grow at a rate less than the interest rate paid on the debt. To implement this constraint, our framework specifies an intertemporal fiscal closure rule that is a variant of "debt-stock targeting."[21] Our particular variant assumes that the fiscal authority sets – or has set for it – an exogenous path for a target ratio of nominal government debt to nominal GDP, *BRATIOT(t)* . Given this exogenous target ratio, an endogenous path for the nominal amount of target debt, *BT(t)*, is specified as:

$$BT(t) = BRATIOT(t)\,[NOMGDP(t)] \tag{22}$$

where *NOMGDP(t)* is nominal gross domestic product. The fiscal authority is then assumed to focus on the gap between the target path and the actual path of government debt:

$$BTGAP(t) = \log\left(\frac{B(t)}{BT(t)}\right). \tag{23}$$

As its method of ensuring intertemporal fiscal consistency, the authority is presumed to vary the tax rate on incomes, $\tau$ , up or down in response to non-zero values of *BTGAP*. In the language of control theory, the fiscal authority's response has both proportional and rate-of-change terms, but not an integral term. More precisely, the debt-stock targeting behavioral rule (alternatively referred to as the tax-rate reaction function) specifies that:

$$\dot{\tau} = \gamma_1(BTGAP(t)) + \gamma_2(B\dot{T}GAP(t)) \quad . \tag{24}$$

Higher values of $\gamma_1$ and $\gamma_2$ produce faster adjustment of the income-tax rate to a level that eventually brings *BTGAP* back to zero. (Non-zero values of *BTGAP* occur transitorily, but not permanently.) Lower values of $\gamma_1$ and $\gamma_2$ permit the government's deficit and debt to deviate for a longer time before ultimately being pulled back to levels that produce intertemporal budget consistency.[22]

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[21] The differences between alternative specifications of intertemporal fiscal closure rules are carefully described in Bryant and Zhang (1996a).

[22] Income taxes in the empirical model are paid on both labor incomes and capital incomes. The income-tax rate applies to the sum of labor and capital income; in the empirical model this income-tax total is allocated to laborers and owners of capital in proportion to the shares of labor and capital in output.

The pension system has four key policy variables: the pension tax rate $\tau_{SS}$, the inelastic Tier-One benefit $\beta_{T1}$, the benefit rate for the Tier-Two component $\beta_{T2}$, and the nominal financing gap, $PTFGAP(t)P(t)$. Assumptions about these four variables control how the model's pension systems operate in practice. A variety of alternative setups can be implemented. The most important distinction among them is between *continuously balanced* versus *unbalanced* systems.

In *continuously balanced* PAYG pension systems, the trust fund gap is treated as exogenous and is forced to stay equal to zero in every period; such systems may be characterized as independent of the rest of the government's accounts ("off-budget"). A first extreme variant in this group, the *tax-balanced system*, sets the benefit rate $\beta_{T2}$ as exogenous and constant, which means that the pension tax rate $\tau_{SS}$ must be treated as endogenous and adjusted period by period to keep the financing gap exactly on its continuously balanced path. The opposite extreme variant, the *benefit-balanced system*, reverses the roles of $\beta_{T2}$ and $\tau_{SS}$; the pension tax rate is treated as exogenous and constant, with $\beta_{T2}$ forced to adjust endogenously and continuously to keep the trust fund gap at zero. A third, intermediate variant is a combination of the other two. Rather than the extremes of keeping $\beta_{T2}$ constant while varying $\tau_{SS}$ or keeping $\tau_{SS}$ constant while varying $\beta_{T2}$, our third variant – the *intermediate-balanced system* – keeps the trust fund continuously balanced by adjusting *both* $\tau_{SS}$ and $\beta_{T2}$. Specifically, $\tau_{SS}$ is adjusted upward by enough to offset one half of any incipient trust-fund deficit in the current period and $\beta_{T2}$ is adjusted downward by enough to offset the other half of the incipient deficit (and vice versa if the trust fund in the current period runs an incipient surplus).[23]

In all variants of *unbalanced* PAYG pension systems, the systems are "on-budget." The pension tax rate and the two benefit components $\beta_{T1}$ and $\beta_{T2}$ are set exogenously (for example, at constant values) while $PTFGAP(t)P(t)$ adjusts endogenously through time, taking on non-zero values. A pension-fund actual deficit or surplus proximately leads to an overall budget deficit or surplus and therefore proximately causes an increase or decrease in the stock of the government's debt. The debt-stock targeting behavior of the fiscal authority then plays a crucial role in determining how unbalanced pension systems affect macroeconomic variables.

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[23] In real life PAYG systems, maintaining pension taxes unchanged or maintaining pension benefits unchanged would of course have quite different political and economic implications. Probably neither extreme would be politically realistic. In actual PAYG systems, moreover, it is implausible to suppose that the pension authorities vary tax rates, benefit rates, or both continuously, period by period, so as to keep the pension trust fund exactly balanced. We use the assumptions in the text as analytical benchmarks, not as realistic depictions of PAYG systems.

In one variant of an unbalanced pension system, the government is assumed not to change either its debt target path or the rule by which income taxes adjust to prevent the actual debt ratio from permanently rising or falling relative to the target ratio. In the other unbalanced variants, the government is assumed to vary its target debt ratio and/or its income-tax-rate reaction function, which then allows the outstanding stock of total government debt to reflect, much more strongly, pressures in the pension system. We label the first variant *unbalanced with zero change in the target debt ratio* (Unbalanced-0T for shorthand) and the remaining variants *unbalanced with modified target debt ratio* (Unbalanced-MT).

To gain intuition about the implications of alternative setups for the pension system, consider the thought experiment of a fertility decline leading initially to lower youth dependency and ultimately to higher elderly dependency. When the pronounced rise in the elderly dependency ratio eventually occurs, the pension system is called on to pay out more pension benefits than can be financed at the former pension tax rate. Four cases corresponding to four variants of the pension system require differing adjustments to be made.

In case 1, the extreme of a tax-balanced system, the pension authority must raise $\tau_{SS}$ sharply to keep the pension trust fund continuously balanced ( $PTFGAP(t)P(t)$ must remain at zero) without any change in $\beta_{T1}$ or $\beta_{T2}$. In case 2, the opposite extreme of a benefit-balanced system, all the adjustment pressure falls on pension benefits; $\beta_{T1}$ or $\beta_{T2}$ or both must be reduced sharply to keep $PTFGAP(t)P(t)$ at zero without any change in $\tau_{SS}$. The main burden of adjustment falls on workers in case 1, on the elderly recipients of pensions in case 2.

In variants of an unbalanced on-budget pension system, cases 3 and 4, the pension trust fund begins to run not only an incipient but also an actual growing deficit. With constant $\beta_{T1},$ $\beta_{T2}$ and $\tau_{SS}$, pension transfers to the larger numbers of elderly increasingly outstrip pension tax revenues at the unchanged tax rate. As the deficit in the pension trust fund rises, other things being equal it increases the overall budget deficit *GDEF* and hence causes an incipient increase in the outstanding stock of government debt. In case 3, suppose that the government does not change its "normal" debt-management policy (with a specified target debt ratio) despite the fact that the pension system is becoming unbalanced. In that event, the pension system is an unbalanced-0T system and the debt-targeting reaction function (24) goes to work to raise income taxes to prevent the overall budget deficit and the debt stock in the shorter run from increasing too rapidly and to prevent any permanent change in the actual ratio of debt to nominal GDP. If income taxes adjust quickly enough, the dynamic behavior of many macroeconomic variables in the economy – especially over the longer run – may be little different between case 1 and case 3.[24] In case 1, pension taxes have to rise sharply whereas in case 3 income taxes rather than pension taxes rise sharply. Since our framework does not have differential labor-supply responses to different types of taxes, the life-cycle decisions of agents about consumption and saving are not differentially influenced by whether they have to pay greater amounts of pension taxes or income taxes.[25]

But now consider a case 4 in which the pension system is allowed to be unbalanced and the fiscal authority permits an increase in its target debt ratio in response to the increased pressure on the pension system. In this unbalanced-MT system, the overall budget deficit then increases and the outstanding stock of government debt rises substantially following the shock. If the target debt ratio remains at the modified higher level, the result is a permanently higher actual debt ratio (the stock of government debt rising sharply relative to the size of the economy and remaining at the higher level). The unbalanced-MT system also causes the income tax rate and income tax revenues to be permanently higher in the long run. The dynamic behavior of most macroeconomic variables in case 4 will thus differ substantially from their behaviors in cases 1 and 3.

### D. Implementing the Framework in a Global Empirical Model

We use a stylized and simplified two-region abridgement of a larger world model for an empirical analogue of the theoretical framework. The underlying model, containing many separate countries and regions, is the IMF staff's MULTIMOD model. Our abridgement is a substantially revised and updated version of a two-region abridgement originally created in the mid-1990s. The refined two-region model is a research environment in which the global macroeconomic consequences of demographic change can begin to be systematically studied.[26]

The starting point for the empirical model is a set of equations describing the U.S. economy ("US" for short). Then a second artificial country is created, labeled for brevity as "ZZ." The ZZ

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[24] As noted above, the larger the values for the $\gamma_1$ and $\gamma_2$ coefficients in (24), the smaller will be sustained accumulations or decumulations of actual (total) debt relative to target (total) debt.

[25] In theory, the labor-supply response to a dollar of pension taxes on labor income *should* differ from the labor-supply response to a dollar of income tax on combined labor and capital income.

[26] For a description of the IMF staff's MULTIMOD Mark III, see Laxton, Isard, Faruqee, Prasad, and Turtelboom (1998). The Mark II version of MULTIMOD is presented in Masson, Meredith, and Symansky (1990). Bryant and Zhang (1996a, 1996b) describe the original abridgement. Our ultimate research agenda is to incorporate insights and specifications obtained from the stylized, abridged model back into the richer, more realistic contexts of larger world models with separate actual countries.

economy is an identical, mirror image of the United States. Thus the "world" in this stylized framework is composed of two economies, roughly like the United States, that are equal-sized, equivalently open, and identical in domestic structure. The economies are carefully linked together with the balance-sheet and income-flow identities that would have to hold in an actual world of two economies. The current-account balance and the net-foreign-asset position of the ZZ economy, for example, are exactly the negatives of the current account and the net-foreign-asset position of the US economy. The two economies are connected by a single, endogenously determined exchange rate. The exchange rate is proximately determined by a variant of the uncovered interest-parity relationship. Indirectly, the exchange rate is influenced by and in turn helps to determine all the macroeconomic variables in both economies.

The empirical model, like the theoretical framework, emphasizes the forward-looking behavior of agents and presupposes that both firms and households engage in intertemporal optimization. A partial exception stems from an allowance for a fraction of consumers whose consumption is constrained by an inability to borrow and hence are unable to smooth their consumption intertemporally. The consumption-saving sectors of the model permit an explicit assumption about the value of the consumers' elasticity of intertemporal substitution (EIS) – see equations (39) and (40) in Appendix 1.

Output of the single composite good produced in each economy is a function of capital and productivity-augmented labor. The production technology of firms is represented by constant elasticity of substitution (CES) production functions. Firms are price-taking entities that choose variable inputs and their level of investment in capital so as to maximize stock-market value. Firm investments respond to the difference between the market value and reproduction value of the capital stock (a variant of Tobin's "q" framework).[27]

The stylized model treats labor as perfectly mobile within each of the two countries but completely immobile across the countries' borders. Hence wages are equal across comparable age cohorts within each country but in general are not equal across the two countries. Over the long run labor is inelastically supplied with respect to wages and is determined by the model's demographic structural equations. Prices are sticky in the short run but flexible over a longer run. The model forces

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[27] The model's investment equations so far follow the treatment in the Mark II version of MULTIMOD (Masson, Meredith, and Symansky, 1990). Adjustment costs for investment in capital are modeled explicitly in the Mark III version of MULTIMOD (Laxton, Isard, Faruqee, Prasad, and Turtelboom 1998).

full employment of labor and capital over the long run.[28] Because the composite goods from each country are imperfect substitutes, each country exports some of its production to the other. Imports in each country are a function of national income and relative prices. Agents in a given country are assumed to have identical and time-invariant preferences over foreign and domestic goods.

The empirical model is solved with a software algorithm that imposes model-consistent ("rational") expectations. Hence agents are presumed to know the structure of the model and to correctly anticipate the entire future paths of the model's exogenous variables. Imposition of model-consistent expectations is the now-standard working assumption in most empirical work in macroeconomics and our use of this assumption is familiar ground.[29]

When using the empirical model, we first develop one or more model-consistent, steady-state baseline solutions for the evolution of the ZZ and US economies. For transparency, both economies are assumed to follow identical paths and exhibit identical behavior along these steady-state baselines. Hence the baseline exchange rate is constant over time at unity and the trade balances, current-account balances, and net-foreign-asset positions in the baseline are all constant at zero. Baseline solutions for the model typically assume that productivity growth occurs at a constant rate. Baseline steady-state rates of inflation are likewise assumed constant. The fertility (child birth) rate, the child mortality rate, and the adult mortality rate are the key exogenous demographic variables in the model. Typical baselines have these key demographic rates set at constant values. Baseline issues are discussed further in Appendix 2.

### E. <u>Illustrative Shocks to Fertility</u>

To develop analytical conclusions about the consequences of demographic shifts for macroeconomic variables and pension systems, we run shock simulations in the model, perturbing the paths of one or more of the exogenous demographic rates, and then compare the resulting shock values of endogenous variables with their baseline values. Most of the paper focuses on a shock simulation in

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[28] The current version of the model follows the treatment of prices and wages in the Mark II version of MULTIMOD. Capacity utilization can differ in the short run from long-run full use of capacity. The model includes wages and employment implicitly and hence does not explicitly track unemployment. Further refinements in the empirical model should include alternative treatments of the dynamics of prices and wages.

[29] Yet the assumption is extreme and inherently implausible for demographic shocks that begin gradually and then wane gradually over many years. Bryant has shown in previous research that it is feasible to modify the model-consistent-expectations assumption by phasing in "correct expectations" about the paths of exogenous variables with the passage of time rather than permitting expectations to be correct immediately and fully. Much interesting research is now being carried out that applies "learning" ideas to the evolution of expectations. See, for example, Evans and Honkapohja (2001) and references cited there In future research, we ultimately hope to make modifications in the assumed treatment of expectations. For the time being, we report the results with the now-familiar, full model-consistent expectations.

which the child birth rate declines sharply, remains at a low level for an extended period, and then eventually recovers enough of its earlier decline to leave the economy with a stationary population. This illustrative shock, labeled *large-cyclical*, causes for several interim decades a negative growth rate for the population as a whole. We select this fertility shock for study in part because it is roughly analogous to the recent and prospective demographic experience of Japan.[30] A second contrasting shock, labeled *smaller-gradual*, assumes that the child birth rate declines more slowly and monotonically until the population eventually reaches a zero growth rate. For these illustrative shocks to fertility, the adult-mortality and child-mortality rates remain unchanged at their baseline levels.

The exogenously specified paths for the birth rate in the two shocks are shown in the upper portion of Figure 2. The bottom part of the figure plots the corresponding endogenous growth rates for population as a whole, which of course reflect the patterns of the birth rates. Appendix 2 gives further details about the specification of the shock paths.

The assumed fertility declines have major consequences for all endogenous demographic variables. The effects on youth-dependency and elderly-dependency ratios are graphed in Figure 3. For the large-cyclical shock, the youth-dependency ratio shows a pronounced decline in the early decades. Once the birth rate stops falling further and then begins to recover, the youth-dependency ratio eventually recovers roughly half of its earlier decline. The elderly-dependency ratio during the large-cyclical shock at first rises only slowly, lagging behind the opposite movement in the youth ratio; subsequently, however, a pronounced aging of the population occurs, pushing the elderly ratio to a peak nearly double its initial level. Eventually, after the birth rate is rising again, the elderly ratio gradually reverses part of its earlier rise. For the smaller-gradual shock to fertility, the youth ratio declines monotonically and the elderly ratio rises monotonically to their eventual new steady-state levels.

Figure 4 plots the levels of adult populations and effective labor forces for the two shocks. During the earliest years of the fertility declines, for both shocks the adult populations and the effective labor forces continue to increase at the baseline positive rate of growth. And as shown in Figure 5, at first the ratios of the effective labor forces to adult populations remain stable at the baseline level. Thereafter the increases in the effective labor forces partly reflect the fact that the number of young

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30 The major decline in fertility in Japan took place in the second half of the 20$^{th}$ century. Part of our illustrative shock in this paper is motivated by Japan's past fertility experience and part by projections of its demographic future.

workers, who are less productive, decline relative to the number of older, higher-productivity workers.[31]

For the large-cyclical shock, as the workers age in the cohorts of reduced size and pass into the ranks of higher- productivity workers, the effective labor forces begin a protracted, sharp decline – at a rate much steeper than that for the adult populations as a whole. Eventually, that decline is partially reversed as the proportion of youth in the economy rises again and ultimately stabilizes at its new eventual level. The behavior of adult populations and effective labor forces is dramatically different for the smaller-gradual shock. When the economies eventually reach new steady states in which the size and composition of populations are stationary, the *levels* of populations and effective labor forces that result from the smaller-gradual shock are nearly twice as large as those that result from the large-cyclical shock (Figure 4) even though the ratios of the effective labor force to adult population have converged to the same value (Figure 5). The major differences in the demographic consequences of the two shocks cause major differences in the corresponding macroeconomic outcomes.

The next section starts with an exposition of basic macroeconomic effects resulting from fertility shocks. Section 4 then explores how the effects vary under alternative public pension systems that are balanced in their treatment of pension taxes and pension revenues. Section 5 examines unbalanced pension systems under which the macroeconomic effects raise important issues about government debt and intergenerational equity.

## 3. Basic Closed-Economy and Open-Economy Effects of Sustained Fertility Declines

The analysis in this section uses a benchmark version of the model that includes children's consumption and its financing by child-support transfers from adults. The values of the parameters governing child consumption and child-support transfers are in the middle of the plausible ranges explored in our paper focusing on youth dependency (Bryant, Faruqee, Velculescu, and Arbatli, 2004). For public pensions, the benchmark model uses an intermediate-balanced system; specific benchmark parameters are described in Appendix 2.

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[31] The initial baseline level of the *effective* labor force is some 1.7 times greater than the level of the total adult population, reflecting the calibration of the labor forces with their incorporation of the age-specific relative productivities of different aged workers. The levels of the effective labor force in the model represent, in effect, the number of labor "efficiency units," not the total number of workers. The growth rate of the *adult* populations lag behind the growth rate of the *total* (adult plus youth) populations by 18 years. For the large-cyclical shock, although the total populations begin to decline fairly soon after the onset, the adult populations and the effective labor forces thus continue to increase for a while longer even though new births of children and hence the growth rates of the total populations are falling sharply.

The analysis portrays simulation results graphically. The time paths of variables in the charts are shown as deviations from the baseline solution of the benchmark model. The units of the deviations are specified along the vertical axes of the charts. If a variable has a value of zero in a figure, at that point the variable is unchanged from its baseline path. The charts report results over sufficiently extended periods, 300 years or more, to illustrate long-run as well as short- and medium-run effects.[32]

### A. Symmetric Shocks (Closed Economy)

To gain intuition in understanding the macroeconomic consequences of demographic changes, we initially examine *symmetric* ("global") shocks in which the fertility declines occur identically in both the ZZ and US regions. When a shock is identical in both regions, the model produces identical simulation paths for both economies. External-sector balances remain at zero and the exchange rate remains unchanged at its baseline value of unity. In effect, each economy behaves as though it were completely closed, which is of course true for the world as a whole. Hence we refer to the symmetric cases (both regions experiencing the large-cyclical shock, or both experiencing the smaller-gradual shock) as "closed-economy" simulations. The closed-economy analysis facilitates interpretation of the most basic, domestic consequences of fertility declines and serves as a benchmark for analytical interpretation of open-economy effects.

Changes in effective labor forces (Figure 4) – reflecting demographic shifts, the effects of humped age-earning profiles, and the bottom-up determination of labor incomes and human wealths – critically influence the dynamic behavior of macroeconomic variables in the model. When individuals first enter the labor force, they have relatively low productivity and are relatively low savers. Then as younger workers age, gain experience, and have higher productivity, they in effect ascend the left side of the hump of the economy's age-earning profile. Individuals reach their years of peak earnings and high savings when they are in their forties and fifties. Eventually, they start to descend the right side of the humped age-earning profile, and consequently their labor incomes and saving decline. At that point, their consumption must be increasingly financed out of their privately accumulated financial wealth as supplemented by pension transfers from the government (if a pension system exists). As

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[32] Because demographic shocks often have consequences over very lengthy periods, our model simulations are carried out over long horizons, typically for more than 500 years. Numerous simulations for a variety of demographic shocks have been examined to ensure that the model eventually reaches analytically plausible outcomes (new stable steady-state growth paths) after sufficient time is allowed for all adjustments to the shocks. For the large, *permanent* demographic shocks studied in this paper, the required time for complete adjustment is often very lengthy. Adjustment times are shorter for shocks that are *transitory* (in which the exogenous demographic variables return to their initial baseline steady state values).

demographic shocks pass through the age-earning profile, the dynamic effects of the demographic movements, significant in themselves, get still further amplified.[33]

Dynamic effects from the behavior of effective labor forces become even more pronounced and significant when the model incorporates the consumption of children. During the period when the fertility declines are producing reductions in the youth population, the adults providing children with support have progressively smaller transfer payments to make to children. The demographic change thus frees up resources relative to the situation where child-support transfers are absent. The way transfers to children are distributed among adult cohorts also interacts directly with the age-earning-profile effects.

Interactions between the demographics and the age-earning profiles are of course more readily visible for the case of the large-cyclical shock. As the large-cyclical shock moves identically through the populations of the two economies, the adult populations and the effective labor forces become much smaller than they otherwise would have been (Figure 4). The ratios of the effective labor forces to the adult populations fall cyclically by large amounts and then eventually partially recover (Figure 5). In contrast, for the symmetric smaller-gradual shock, the effective labor forces and adult populations do not decline absolutely (apart from a small cyclical movement in the labor forces in the medium run) and the declines in the ratios of the two variables are gradual (again see Figures 4 and 5).

Reflecting the movement of the ratios of effective labor forces to adult populations, the economy-wide aggregate levels of human wealth, financial wealth, output, consumption, and the aggregate capital stock all decline to eventual levels that are significantly lower. But the movements of key macroeconomic variables are relatively smooth declines for the smaller-gradual shock. For the large-cyclical shock the movements have a major cyclical downswing followed by a partial reversal.

The presence in the model of youth dependency importantly determines the behavior of the real interest rate, the capital stock, and the capital-output ratio. Both in initial baselines and throughout shock simulations, the level of the real interest rate must be higher in a simulation with than without child consumption. The greater the generosity of child support, the higher must be the real interest rate. Other things being equal, the capital stock and economy-wide output and consumption are lower when children must be supported; adults have to set aside more resources to cover the needs of

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33 We first stressed the central importance of these age-earning-profile effects in a 2001 paper, published as Bryant and McKibbin (2004). See in particular the comparisons in that paper between simulations with and without the age-earning profiles present. (Excluding the effects of the age-earning profiles entails setting the three $\alpha$ coefficients in equation (25) in Appendix 1 to zero.) Interactions between demographic dynamics and efficiency labor inputs are much less significant in models that fail to incorporate age-earning profiles.

children and hence have smaller savings, leading to less capital accumulation and hence to lower output and consumption per adult. The real interest rate, reflecting the marginal product of capital, will therefore be higher, the higher are child needs.

The dynamic movements of real interest rates are shown for both shocks in Figure 6. Figure 7 is an analogous plot for the capital-output ratio. The real interest rate declines as the effective labor force declines, relatively smoothly for the smaller-gradual shock and dramatically for the large-cyclical shock. The effective labor force is lower relative to the capital stock, and hence the marginal product of capital must fall. In the case of the large-cyclical shock, when the effective labor force eventually recovers, the real interest rate also recovers; in the longest run, after the capital stock is again high relative to the labor force, the real interest rate then gradually moves down toward its final steady-state value. For the smaller-gradual shock, the interest rate partially reverses some of its earlier decline, converging from below to the final steady-state value.[34]

Human wealth, financial wealth, and adult consumption and savings are four of the most important real macroeconomic aggregates in the model. The simulated movements in two of these variables, measured in per-adult terms and as percentage deviations from baseline, are shown in Figures 8 (financial wealth) and 9 (adult consumption). Dynamic movements are partly cyclical or relatively smooth depending on which shock is examined.

For the large-cyclical shock, financial wealth per adult and human wealth per adult *rise* relative to baseline in the initial decades. The larger the assumed generosity of child support, the larger are the rises in human wealth and financial wealth relative to what would have occurred without the decline in the numbers of children. The increases in financial wealth are explained partly by the effects of the age-earning profiles on saving and partly by higher disposable incomes and savings reflecting the smaller support payments to children to be made in the shorter-run future. In the medium run, as the decline in fertility is reversed and the child population again increases, financial wealth and human wealth per adult fall steadily relative to baseline until they are well below baseline levels. Then over the long run they rise back toward baseline. Simulation paths for consumption per adult likewise show a rise in the shorter run, a sustained fall in the medium run, and an eventual rise back toward baseline.

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[34] Although the dynamic pattern of the real interest rate is broadly similar with or without youth dependency, the interest-rate movements in the medium and long runs are more pronounced when youth dependents are modeled. This greater amplitude when child support is present is due to the freeing up of resources for adults when the numbers of children are fewer. More resources for the consumption and saving of adults means that there is less need to deplete the capital stock. The capital stock relative to the effective labor force is higher in the model when child support is taken into account, which necessitates a more pronounced decline in the marginal product of capital, and hence in the real interest rate.

The cyclical ups and downs in consumption per adult are more pronounced when adults make transfers to support child consumption. The broad pattern of cyclical movement for each of the variables human wealth, financial wealth, and consumption is qualitatively similar for the large-cyclical shock to the pattern of movements in the ratio of the effective labor force to the adult population (Figure 5).

For the smaller-gradual shock, macroeconomic variables such as financial wealth per adult and adult consumption per adult manifest much smoother and smaller adjustments in the shorter and medium runs. In the long run, however, they eventually reach the same steady-state values as those for the large-cyclical shock. This outcome occurs because the demographics for the two shocks themselves eventually converge on identical steady states.

### B. <u>Asymmetric Shocks (Open Economy)</u>

Our primary interest is in analysis of *asymmetric* (country-specific) variants of demographic shocks in which one part of the world economy experiences different shocks and different outcomes from those occurring elsewhere and in which open-economy interactions are critical determinants of the effects within each economy. With the preceding closed-economy summary as background, we now focus on open-economy analysis by imposing the large-cyclical shock in fertility on the ZZ economy but assuming that the US economy experiences the smaller-gradual shock.

The preceding summary makes clear that demographic events powerfully influence macroeconomic outcomes in domestic economies. The behavioral dynamics identified in the closed-economy simulations are no less important when demographic shifts are asymmetric. But the consequences of asymmetric demographic shocks are significantly modified by the openness of national economies.

The charts that accompany the following text discussion, Figures 10 through 20, contain three curves, each pertaining to a particular variable. As an easy reminder of the closed-economy analysis, the dashed curve shows the path for the variable resulting when the large-cyclical shock hits both economies identically. The other two curves in Figures 10 through 20, one for ZZ and the other for US, plot the paths for the variable in each economy when the shock is asymmetric (the outcomes for the two economies now of course being quite different). The ZZ path for a variable in the charts is plotted with a thicker, more prominent curve than that for the US path.

Refer again to Figure 5. In the asymmetric-shock case to be described in what follows, the path for the ratio of the effective labor force to adult population exhibiting the large-cyclical behavior in Figure 5 should now be understood as the path experienced in the ZZ economy. In contrast, the smaller-gradual path for that ratio in Figure 5 is the path experienced by the US economy. Because of the asymmetry in shocks, the ZZ and US economies will now have very different evolutions of key variables such as interest rates, the capital stock, human wealth, savings and financial wealth, and consumption.

Deviations of the real interest rates from baseline for the two economies are plotted in Figure 10. During the first two decades of the shock, before the adult populations and labor forces have yet experienced the consequences of the fertility decline, the real interest rates change relatively little. Thereafter, the cumulating sharp fall in the ZZ effective labor force leads to a progressively larger fall in the ZZ real interest rate. Notice, however, that the extent of the fall in the ZZ real interest rate and its subsequent reversal is significantly damped because of the ZZ economy's openness to the rest of the world. The real interest rate in the US shows an analogous, but much smaller, decline than that in the ZZ economy; the downward movement in the US real interest rate, however, is significantly *larger* than in the closed-economy case for its smaller-gradual shock (compare with Figure 6); the US rate also has significantly more cyclical behavior, reflecting the interactions with the ZZ interest rate.[35]

Figure 11 portrays movements in the capital-output ratios. In the medium and longer runs, the US economy now experiences a *lower* rise in its capital-output ratio than in the closed-economy, smaller shock (compare with Figure 7). The ratio in the ZZ economy must rise substantially *higher* in the medium run than in the closed-economy case with the larger-cyclical shock; furthermore, the capital-output ratio in the ZZ economy remains at a permanently higher level forever. These different capital-stock and output evolutions in the two economies are associated with major differences in saving and external-sector behaviors.

Movements in human wealth and financial wealth in both economies continue to be driven by the interactions of the fundamental demographic forces and age-earning profiles. But with a much larger fertility decline affecting the ZZ than the US economy, the relative positions of the two economies are very different. As seen in Figure 12, for example, ZZ financial wealth per adult in the shorter run rises relative to baseline even more strongly than in the closed-economy case and then diminishes only very gradually in the medium and longer runs. In marked contrast, US financial wealth per adult does not stay on a roughly level path in the short and medium runs but rather falls sharply in the medium and long runs. Significantly, ZZ financial wealth per adult (and ZZ savings per adult) is consistently much higher than it would be for the large shock in the closed-economy case. The opposite is true for the US; financial wealth per adult (and savings per adult) fall well *below*

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[35] Nominal short-term interest rates in the two economies follow qualitatively similar paths as the real interest rates, with similar conditioning by the openness of the economies.

relative to the outcomes for the smaller shock in the closed-economy case. The differences in saving behavior and hence in financial wealth between the ZZ and US economies are thus attributable not merely to their different-sized demographic shocks but also to major effects working through the exchange rate and external-sector transactions.

The model enforces a variant of the uncovered interest parity condition as part of the behavior determining the nominal exchange rate. Hence a sizable interest differential between the two economies – once it opens after the initial years of the asymmetric shock (Figure 10) – puts strong pressure on the real and nominal exchange rates. The ZZ currency begins a sustained appreciation, first in nominal terms, then with a lag in real terms (Figure 13).[36] By the ninth or tenth decade of the shock, both the nominal and real values of the ZZ currency have appreciated by a large amount. The real exchange value of the ZZ currency appreciates substantially further over the next several decades, reaching a peak appreciation of some 100 percent before reversing and falling back. In the new long-run steady state, both the nominal and the real exchange rates settle at levels very much higher than in the baseline solution. (For the symmetric cases in which both economies experience identical shocks, the nominal and real exchange rates never deviate from the baseline value of unity.)

To understand why the asymmetric shock results in a real exchange rate permanently higher in the ultimate than in the initial steady state, recall that the asymmetric fertility declines are transitory in terms of demographic *rates of growth* but have permanent effects on the *relative levels* of demographic and macroeconomic variables. In particular, as the larger shock causes the ZZ fertility rate to fall faster and progressively below baseline, the ZZ population and effective labor force begin to fall further and further below baseline. The ratio of the ZZ to the US effective labor force and the ratio of the two countries' populations fall correspondingly. Once the decline in the ZZ fertility rate is partially reversed, eventually the ZZ and US population growth rates again become equal. The ratio of the ZZ to the US effective labor force, however, remains permanently smaller (the two levels are shown in Figure 4). In the medium run and in the new long-run steady state in which the aggregate sizes of the capital stock and labor force in the ZZ economy are well below baseline whereas the corresponding macroeconomic aggregates in the US economy are above baseline, therefore, the quantity of ZZ-produced goods is markedly smaller than the quantity of US-produced goods. Figure 14 emphasizes this result by plotting the evolution over time of the ratio of GDPs in the two economies. Given unchanged preferences in each economy for the two types of goods, relative prices in the world

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[36] An upward movement in the exchange rate in the model represents an appreciation of the ZZ currency (a depreciation of the US currency).

economy must change. A permanent real appreciation of the ZZ currency (an improvement in the ZZ economy's real terms of trade) is an integral part of the required change in relative prices.[37]

Large changes in exchange rates generate powerful expenditure-switching incentives between the two economies. Thus by the fourth decade of the shock, the ZZ economy begins to import substantially more of the now relatively cheaper goods produced in the US. ZZ exports to the US relative to baseline are inhibited by the appreciation of the ZZ currency. The ratios of real imports to real GDP in the two economies are shown in Figure 15. After the underlying demographic variables have begun to converge to their ultimate steady-state values, the ZZ import ratio has risen some 18 percentage points above its baseline value of 12.9 percent; it ultimately falls back to some 11 percentage points above baseline. The US import ratio reaches a level some 7 percentage points below its baseline level of 13 percent, ultimately coming to rest at some 6 percentage points below baseline.

For the initial decades of the shock, the ZZ real trade balance relative to real GDP changes little. As the shock progresses thereafter, however, the expenditure-switching effects cause the ZZ economy to run a larger and larger *deficit* on real trade account (Figure 16). This net import of real resources from abroad provides a cushion of support to the ZZ economy that permits it to sustain a significantly higher level of consumption than would otherwise be possible. The US economy experiences the opposite effect: it must export real resources abroad and correspondingly curtail its consumption relative to what would otherwise be possible.

The medium-run trade deficit of the ZZ economy is *not* associated with a deficit on current account. The ZZ economy not only imports more from abroad. The ZZ economy also saves more relative to baseline so that ZZ financial wealth *rises* relative to baseline (Figure 12).[38] And a fraction

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[37] One can legitimately question the model's working assumption that preferences for domestically-produced and foreign-produced goods remain unchanged over time (alternatively stated, that the substitutability in demand of domestic and foreign goods is independent of large changes in the relative sizes of economies). Yet most of the existing analysis of international trade operates with the assumptions built into our analytical framework. Products are assumed to have, in effect, *spatially-determined* characteristics that make them imperfect substitutes (an electronic device produced in Japan not being a perfect substitute for a functionally similar electronic device produced in the United States, a wine from France not being a perfect substitute for an Australian wine made with the same grape , etc.). If one questions the conventional treatment of goods preferences as inappropriate, such doubts could also lead to doubts about the strength of the exchange-rate and terms-of-trade effects reported here. Note, however, that even if one were to believe that shrinkages in the relative quantities of similar goods produced at home versus abroad were to induce somewhat higher elasticities of substitution between home and foreign goods *merely because home goods were more scarce in the world*, it would require very great changes in preferences to completely offset the exchange-rate and terms-of-trade shifts that would otherwise be caused by the relative shrinkages in home-produced goods.

[38] Numerous channels cause private saving to rise in the ZZ economy. Among them is the fact that the population aging caused by the large fertility decline requires the government authority operating the intermediate-balanced pension system to raise pension taxes and reduce pension benefits (relative to baseline), which in turn is an incentive for increases in private saving.

of that higher financial wealth is invested abroad at the higher interest rates available abroad. Hence the ZZ economy over the medium run starts to earn a higher flow of investment income from abroad. The net investment income payments received are more than enough to offset the increased deficit on trade account, with the result that the ZZ economy in the medium run begins to experience a significant current-account *surplus* (Figure 17). The surplus reaches a peak during the eighth decade of the shock. Interestingly, the current-account surplus thereafter falls and even returns close to balance for several decades as the two economies move toward their new long-run steady states. Eventually, in the very long run the ratio of the ZZ current balance to nominal GDP settles at a moderate surplus ratio.[39]

The net foreign asset positions of the two economies, shown relative to nominal GDPs in Figure 18, are the integral over time of the current-account imbalances. The ZZ economy – despite the large shock it experiences, causing the economy's output and aggregate consumption to fall far below the levels that would have been observed without the shock – accumulates a large *positive* net foreign asset position, on which it earns a sizable return.

The openness of the ZZ and US economies thus decisively influences the macroeconomic consequences of the demographic shocks. Domestic variables in both economies are strongly influenced by cross-border transactions. Because of the openness of the economy, ZZ domestic variables are partly cushioned from the full impacts of the large ZZ fertility decline. As a counterpart, US variables are adversely buffeted by the larger demographic shock emanating from the ZZ economy. An important component of these cushioning and buffeting effects is associated with the changes in exchange rates. The permanent appreciation in the real value of the ZZ currency enables the ZZ economy to enjoy a large permanent improvement in its real terms of trade with the rest of the world. The opposite effect, a deterioration in real terms of trade, contributes to the adverse effects on the US economy.[40]

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[39] Note that the vertical scales for Figures 16 and 17, chosen to make it easier to identify the dynamic movements of the curves, are greatly dissimilar; the range of the vertical distance in Figure 16 is 30 percentage points of GDP (absolute deviations of the ratio of from -0.2 to 0.1) whereas it is only 3 percentage points in Figure 17.

[40] Effects on the real exchange rate, trade balances, current-account balances, and net-foreign-asset positions of the two economies are larger when analysis takes into account the economic effects of children than when it does not. For example, the positive effects on ZZ saving and financial wealth resulting from the fertility decline are larger because resources are freed up as support payments to children become smaller. A fraction of the incrementally freed resources from lower child consumption are saved rather than consumed. The ZZ currency appreciates by a larger amount. The associated net capital flows permit the ZZ current-account surplus to be larger by the medium run than it would otherwise be without youth dependency taken into account. The resources freed up by declining numbers of children, partly invested abroad, increase the cushioning effects on the ZZ economy from its openness to the rest of the world. Similarly, the effects of youth dependency increase the degree to which the US economy is influenced by spillovers from the ZZ economy. When

It is essential when interpreting the implications of cross-border spillovers to differentiate carefully between aggregate levels of variables and their per-adult (or per-capita) values. The large demographic shock occurring in the ZZ economy will inevitably cause major negative effects on ZZ aggregate output and consumption. As shown in Figure 19, the ZZ path for aggregate real consumption by the medium run must accordingly fall much further below baseline than real aggregate consumption in the US. Yet as is also clear from Figure 19, the ZZ path for aggregate real consumption is significantly *above* the path that would be experienced in the hypothetical case where the ZZ economy is completely closed and therefore unable to cushion its large-cyclical shock through transactions with the rest of the world. The openness of the economy works to mitigate the size of the negative effects on the aggregates.[41]

Now consider the per-adult or per-capita values of such macroeconomic variables. Economy-wide aggregates cannot be straightforwardly used to make normative or welfare judgments about the consequences of demographic shocks such as a decline in fertility. Per capita or per adult measures are likely to be, at least for some purposes, a more useful focus for normative comparisons of pre-shock and post-shock outcomes. Figure 20 plots the percentage deviations from baseline for adult consumption per adult in both economies. Those curves provide a suggestive indication of the relative welfare effects in the two countries. Notwithstanding the fact that the demographic shock in the ZZ economy is much larger than in the US economy, ZZ per adult consumption is actually *higher* than per adult consumption in the US. The difference is sizable in the initial decades of the shock. It is even more marked in the new long-run steady state. Moreover, as Figure 20 reveals, the cushioning effects are so substantial when measured in per-adult (or per-capita) terms that individual adults in the ZZ economy are significantly better off not only relative to individual adults in the US economy but better off absolutely relative to the no-shock baseline. Indeed, US per-adult consumption is markedly *lower* than in the baseline despite the fact that the US population, aggregate US real GDP, and aggregate US consumption are all at higher levels than in the no-shock baseline.

The qualitative conclusions stressed here have great relevance for policy debates about population aging, and more generally about the consequences of demographic transitions. We return to these conclusions at the end of the paper.

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the analysis takes into account public pensions, many of the macroeconomic effects and cross-border spillovers are somewhat smaller than for model variants that exclude public pensions.

41 Analogous generalizations are true for aggregate real GDPs.

## 4. "Balanced" Public Pension Systems

The analysis now focuses on alternative public pension systems. The primary emphasis is on how the openness of economies shapes the outcomes and policy choices under alternative pension systems when demographic shocks are asymmetric across countries. This section examines pension systems that are continuously balanced. Section 5 compares unbalanced and balanced systems.

The presence or absence of a public pension system in an economy, and the nature of its operations, can significantly affect private saving behavior. The pressures of population aging on public pension systems and hence on overall government budget imbalances can critically influence government saving or dissaving. National saving, the net sum of private saving and government saving, and the operation of a public pension system are thus two subjects inevitably entangled together.

Unbalanced public pension systems influence private and national savings especially strongly (section 5 below). But even balanced PAYG public pension systems may have significant savings effects. To summarize, a public pension scheme can somewhat discourage individuals from saving for their old age outside the pension system. Intuitively, if the present discounted value of an individual's current and future pension taxes ("social-security contributions") will be roughly equal to the present discounted value of expected future pension benefits, the operation of a public pension system will not much change the individual's lifetime human wealth. But the private saving that forward-looking individuals would otherwise do on their own without a pension system tends to be partially displaced by their pension taxes under a pension scheme. If an individual entering the labor force at the beginning of worklife were to expect future pension benefits to exceed current and future pension taxes (both being appropriately discounted to present value), then the individual's saving outside the pension system (for short, his private saving) might fall by a still larger amount. Conversely, if discounted pension taxes were expected to exceed discounted future pension benefits, his private saving would rise above what it would be with the present values of pension taxes and pension benefits expected to be equal.

A further consideration affecting saving outside the pension system is that the operation of a public pension system providing assured annuities in old age may weaken the precautionary motive to save when longevity is uncertain. The preceding generalizations are consistent with a public pension system providing a net *positive* correction for a tendency for some, possibly many, individuals to save too little for retirement and ill health in old age (notwithstanding the public pension system's partial discouragement of private saving outside the pension system).

Effects of alternative pension systems on private saving depend in part on the degree to which individuals and households are forward-looking in their consumption, saving, and wealth accumulation. A consensus interpretation of the empirical evidence on this matter does not exist. Hence judgments about it remain controversial. Our analytical model, as noted above, errs on the side of specifying sophisticated forward-looking behavior. One may doubt that most individuals and households are as forward-looking and perspicacious as assumed by model-consistent expectations and nonetheless see merit in examining saving and pension behavior utilizing that benchmark assumption.

The text and charts in this section compare the intermediate-balanced, the tax-balanced, and the benefit-balanced pension systems. For reasons given earlier, the tax-balanced and benefit-balanced systems represent the two extremes for balanced systems. Section 3's benchmark simulations assumed the presence of the intermediate-balanced system; thus consequences of the fertility shocks using that intermediate assumption are already familiar. The exposition here concentrates on the asymmetric shock case and open-economy interactions as they influence the ZZ economy.

Because flows of private saving are the most important determinant of the accumulation of financial wealth, consider first Figure 21. Like Figures 8 and 12 in section 3, Figure 21 plots deviations of simulation ratios from the baseline ratio for financial wealth per adult in the ZZ economy. The solid curves in Figure 21 indicate the consequences for ZZ financial wealth per adult of the asymmetric fertility shock. The dashed curves show how the large-cyclical fertility shock would affect the ZZ ratio if the ZZ economy were closed to the rest of the world.

The openness of an economy makes a substantial difference for private saving behavior and financial wealth. For the same large-cyclical shock hitting the ZZ economy, private saving and financial wealth per adult are much higher when cross-border transactions are present than they would be in a closed economy ( a fact already evident in Figure 12). It is also evident from Figure 21 that the medium- and long-run behavior of private saving and financial wealth differ substantially among the three balanced pension systems. Open-economy saving and financial wealth in the medium and long runs are substantially *higher* under the benefit-balanced than under the intermediate-balanced system. Saving and financial wealth are markedly *lower* under the tax-balanced than under the intermediate-balanced system.

The closed-economy (dashed) curves for effects on saving and financial wealth reveal an analogous ranking among the three balanced pension systems: private savings are highest with the benefit-balanced extreme and lowest with the tax-balanced extreme. The differences among the three pension systems, however, are much less marked for the closed-economy case; effects across the three balanced systems move essentially in parallel and fairly closely together. Notice, too, that cyclical movements – reflecting the underlying cycle in the ratio of the effective labor force to the adult population – are prominent in the closed-economy case but much less evident in the asymmetric, open-economy case.[42]

After the fertility decline begins to generate rapid population aging, the tax-balanced pension system must cope with the increasing elderly dependency by sharply raising the pension tax rate, thereby collecting more revenue through pension taxes. The pension tax rate and pension tax revenue must also rise under the intermediate-balanced system but by only about half as much. (Some of these pension-tax rate increases are partially reversed in the long run after the population and dependency ratios reach their ultimate new steady-state paths.) In the benefit-balanced system, of course, the pension tax rate does not change at all. The degree to which the pension tax rate must rise in the tax-balanced and intermediate-balanced systems is roughly invariant as to whether the economy is open or closed. When the economy has cross-border interactions with the rest of the world, the ZZ income tax rate by the medium run falls by some 2½ to 3 percentage points (falling the most for the benefit-balanced pension system and the least for the tax-balanced system). When the economy is closed, on the other hand, the income tax rates have a very small upward and downward swing but thereafter remain slightly *above* baseline (regardless of which balanced pension system is in operation).[43]

The cushioning of the ZZ economy attributable to cross-border transactions, which among many other things influences the amount of taxes collected by the ZZ government, is implicitly reflected in Figure 22. The variable plotted in the figure is deviations from baseline for the ratio of *total* tax revenue raised (the sum of pension taxes and income taxes) to nominal GDP.[44] For the tax-balanced system, the closed-economy case (dashed curve) requires a very large cyclical rise and subsequent partial reversal in the tax-income ratio. With open-economy cushioning, however, it need not rise nearly as much (the peak value of the ratio is only some 4¾ percent higher instead of 6½ percent). For the opposite extreme of the benefit-balanced system, the tax-income ratio is largely flat in the closed-economy case; with open-economy cushioning, however, it *falls* significantly relative to

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42 Curves for US financial wealth per adult (not shown in Figure 21) follow a lower and much more cyclical path in the asymmetric, open-economy case than would otherwise pertain if the US economy were closed and had to absorb internally all the adjustments to the smaller-gradual fertility shock. (The qualitative difference for the US between the open and closed cases can be seen by comparing the curve for the smaller-gradual shock in Figure 8 with the US curve for the asymmetric case in Figure 12.)

43 Figures 26 and 27 below, which pertain to unbalanced as well as balanced pension system, plot the paths of the ZZ pension tax rate and the ZZ income tax rate separately.

44 The path of this tax-income ratio for each balanced pension system is conditioned both by changes in pension taxes and income taxes in the numerator and by changes in nominal GDP in the denominator.

baseline and remains 1½ percentage points lower permanently. The outcome under the intermediate-balanced system is of course in between, with an upswing followed by a downswing of less than half the size of the swings under the tax-balanced system; again the ratio for the open-economy case is well below what it would be if the large-cyclical shock had to be absorbed entirely within a closed ZZ economy.

An intuitive way to understand the differing curves for the tax-income ratio in Figure 22 is to recall that the large fertility shock reduces the denominator of the ratio (nominal GDP) by a smaller amount through time when the economy is open than the reduction that would occur if the ZZ economy had to deal with the large shock without any cushioning effects from the rest of the world. For any one of the three balanced pension systems, therefore, the income tax rate can be *lower* in the open- than in the closed-economy case given that the less-reduced GDP raises sufficient revenues to keep the overall government budget in balance even though the income tax rate is lower.

A transparent way of comparing pension benefits themselves across the three balanced systems is to examine the pension benefit paid to each elderly individual person in relation to per-adult average labor income. For this ratio, plotted in Figure 23, for any one of the three systems there is virtually no difference between the open-economy and closed-economy outcomes. Cross-border effects do little to alter the magnitudes of benefits and the benefit ratio.

But the three pension systems do of course produce dramatic differences in benefits. Because the pension tax rate must rise sharply in the tax-balanced system, the burden of adjustment falls on workers paying the pension tax. In the medium run under the tax-balanced system, recipients of pensions even receive payments representing a modestly *higher* fraction of average per-worker labor income. Conversely, the benefit rate for the Tier-Two component and total benefits have to fall sharply under the benefit-balanced system. The benefit-balanced system thus places the adjustment burden entirely on pension recipients, the oldest of adults. As always, the intermediate-balanced system falls roughly in the middle between the two extremes.

Choosing among balanced pension systems thus obviously entails difficult issues of intergenerational equity. Those issues arise even more insistently with unbalanced pension systems and are discussed in Section 6.

The nature of effects stemming from the ZZ economy's external transactions and their cushioning influences on the economy are qualitatively similar across the three balanced pension systems but differ quantitatively in significant ways. The movements of the nominal and real exchange rates, for example, show roughly the same pattern as seen already in Figure 13 (the benchmark case of the intermediate-balanced pension system). But the appreciation of the ZZ currency in the medium and longer runs, both in real and nominal terms, is somewhat higher under the benefit-balanced than under the intermediate-balanced system and modestly lower under the tax-balanced system.

The differences in exchange-rate movements across the balanced pension systems in turn somewhat modify the behavior of external transactions. Relative to the benchmark case of the intermediate-balanced system (Figures 15 through 18), the ability of the ZZ economy to weather the large fertility shock by importing real resources net from the rest of the world through running a trade deficit is marginally greater when the pension system is benefit-balanced but marginally less when it is tax-balanced. The differences across the three systems as manifested in the evolutions of the ZZ economy's net foreign asset position are plotted in Figure 24. Because savings are highest under the benefit-balanced system (Figure 21), more savings can be exported abroad; hence the economy receives marginally greater net investment income from abroad (which more than offsets the marginally greater trade deficit). The ZZ economy can run a somewhat greater current-account surplus, and hence it can build up through time a somewhat stronger net foreign asset position under the benefit-balanced system. For the tax-balanced system relative to the intermediate-balanced system, the differences run the other way (marginally smaller and less-beneficial cushioning effects through the external sector). In the very long run differences in the external-sector effects across the three balanced systems gradually diminish.

As with many other macroeconomic variables, the basic effects of the fertility shock on adult consumption per adult (or consumption of all residents per capita) are qualitatively similar across the balanced pension systems – see Figure 25. Quantitative differences are fairly small in the shorter run. For the first two decades the tax-balanced system is slightly better for average per-adult consumption across the whole economy; in particular the tax-balanced system permits markedly higher consumption for the elderly. Thereafter, however, the ranking of the three systems changes. The benefit-balanced system starts to do best among the three for average per-adult consumption across the whole economy; by the medium and long runs, this dominance is marked. The extreme of the tax-balanced system delivers a markedly lower average adult consumption per adult (and consumption per capita) over the medium and long runs.[45]

[45] Within the adult population, of course, the elderly who receive pensions may still for a long time do better under the tax-balanced system relative to younger adults than under balanced pension procedures that adjust benefits downwards. Section 6 returns to this issue.

## 5. "Unbalanced" Pension Systems and Government Debt

Original planning blueprints usually specify that public pension systems should remain balanced, if not continuously then at least over time. Yet real-life governments exhibit inertia in implementing such designs, especially if the public pension system is "on budget" instead of "off budget." Politicians prefer to let problems build up. They defer necessary policy responses until the evidence is unambiguous that policy reform needs to be taken. For example, as population aging occurs and a PAYG pension system starts to have benefit payments that exceed revenues collected, a government may be slow to adjust its pension policies. Political consensus cannot readily be generated for pension tax increases, pension benefit cuts, or some combination of the two. The course of least resistance is to postpone difficult decisions. In the meantime, actual deficits in the pension system accumulate and still larger prospective deficits loom across the horizon.

One cannot capture the behavior of public pension systems in many developed countries today without acknowledging that, at least at some times and in some circumstances, the systems may become unbalanced. Before drawing conclusions about alternative pension systems, one accordingly needs to examine unbalanced on-budget systems in which the pension tax rate and pension benefit components are set independently while imbalances in the pension system cause the overall government budget to move into deficit or surplus.

To illustrate consequences of our fertility shocks when on-budget pension systems become unbalanced, this section shows results for two alternatives: an unbalanced system with zero change in the target debt ratio (*Unbalanced-0T*) and a specific example of an unbalanced system with a modified target debt ratio (*Unbalanced-MT*). These correspond to the “case 3” and “case 4” situations identified in section 2C.

Section 4 discussed how population aging stemming from a fertility shock causes incipient deficits in a balanced PAYG system. In the unbalanced-0T and unbalanced-MT systems, the incipient deficits become actual deficits and the overall budget deficit correspondingly moves into actual deficit, causing an incremental rise in government debt and in the ratio of that debt to nominal GDP (hereafter the “debt ratio”).

The unbalanced-0T system requires the government not to change its existing debt-management policy and therefore not to permit any change in its *target* debt ratio. Thus the fiscal authority begins to raise the income tax rate to reverse an emerging overall budget deficit and the associated incremental rise in government debt. By assumption, the fiscal authority prevents any permanent change in the actual debt ratio. The unbalanced-0T system may be characterized as "unbalanced in the pension system but only transitorily unbalanced for the government budget as a whole."

Conversely, an unbalanced-MT system permits sustained imbalances in the overall government budget and even permanent changes in the debt ratio. For the shock studied here, therefore, the fiscal authority is assumed to permit the overall budget to run continuing and increasing deficits during the transition period when population aging puts growing pressure on the pension system. Hence the government's debt accumulates to much higher levels in relation to the economy. As the deficits in the pension system and the overall budget develop, the fiscal authority "lets things roll," riding with the situation and not reacting to the rising debt ratio. But then eventually, as the deficits and debt mount to very high levels, the fiscal authority is assumed to get "cold feet." It acknowledges that the government budget cannot be moved into intertemporal balance and the debt ratio cannot be stabilized unless the fiscal authority somehow raises more revenues (increases pension or income taxes) or cuts some form of expenditures (pension benefits or other government spending).

Our specific example of this unbalanced-MT system assumes that the fiscal authority gradually raises its target debt ratio – set exogenously in accord with its debt-targeting fiscal closure rule; see equations (22), (23), and (24) – so that the target ratio rises from its baseline value of 35 percent to a level of 80 percent of nominal GDP, more than double the initial ratio. Once the actual debt ratio exceeds twice the level it otherwise would have been, the fiscal authority reevaluates the situation, refuses to allow its target debt ratio to rise sharply further, and thus begins to raise the income tax rate enough to make the actual ratio converge to the target ratio of 80 percent.[46]

The assumed behaviors of the fiscal authorities in the unbalanced-0T and unbalanced-MT examples are not realistic characterizations of actual circumstances in many countries. But these examples are useful benchmarks in examining the range of options facing fiscal and pension authorities and serve as illustrations of how differences in the behaviors of fiscal and pension authorities can produce significantly different macroeconomic outcomes.

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[46] Both the ZZ and US regions operate public pension systems. Because the asymmetric demographic shock is larger in the ZZ economy than in the US economy, more severe pressures occur on the pension system in ZZ than in the US. For the unbalanced-MT simulation illustrated here, moreover, only the ZZ fiscal authority – but not the US fiscal authority – is assumed to raise its target debt ratio. We calculate the path for the incremental ZZ government debt implied by the rising path of *BRATIOT* in the unbalanced-MT simulation to be similar to the accumulation of the deficits in the ZZ pension system that occurs in the unbalanced-0T simulation of the same shock. A problematic feature of our unbalanced-MT example (and indeed of the simulations for the balanced and the unbalanced-0T systems as well) is that agents in the model correctly anticipate at the outset of the shock, well before the population aging actually occurs, exactly how the fiscal and pension authorities will behave in the future at the time that population aging puts pressure on the pension system.

Figures 26, 27, and 28 summarize key aspects of the differing behaviors of the ZZ pension and fiscal authorities under the two unbalanced pension systems. For ease of comparison, those figures also include the authorities' behavior under the three balanced systems. The figures plot the levels of the pension tax rate, the income tax rate, and the tax-income ratio (pension and income taxes together as a percent of nominal GDP). The demographic shock determining the simulation results continues to be the asymmetric decline in fertility (large-cyclical in ZZ and smaller-gradual in US).

The pension tax rate (Figure 26), as discussed already, must be raised sharply under the tax-balanced and less sharply under the intermediate-balanced systems during the long period in which the elderly dependency ratio and population aging are increasing. In the extreme of the benefit-balanced system and in the unbalanced systems, however, the pension tax rate remains at its pre-shock value of 12.8 percent.

The income tax rate moves under the unbalanced pension systems in a dramatically different way than under the balanced systems (Figure 27). Reflecting the background operation of the debt-targeting reaction function, the income tax rate in the unbalanced-0T and unbalanced-MT simulations must substantially rise. Under any of the balanced systems, it falls and then partially reverses in a gradual cyclical swing, ultimately ending up well below the baseline level of 20.1 percent. Although the income tax rate rises under both unbalanced systems, the time patterns are very different. Because under the unbalanced-0T assumptions the fiscal authority does not adjust its target debt ratio or the response coefficients in its debt-targeting reaction function, the income tax rate begins to increase as soon as the rising elderly dependency ratio puts pressure on the pension system. If one compares the unbalanced-0T path for the income tax rate in Figure 27 with the tax-balanced path of the pension tax rate in Figure 26, it can be seen that the qualitative movements are essentially similar. Speaking loosely, the income tax rate in the unbalanced-0T simulation is doing the work in preventing the emergence of deficits in the overall government budget that the pension tax rate is doing in the tax-balanced simulation to prevent the emergence of pension-system deficits.

Under the unbalanced-MT regime, in contrast, the incremental rise in the income tax rate in the first several decades of the population aging is much less than that required with the unbalanced-0T system. As the government's overall budget deficit widens and the debt ratio rises, the fiscal authority in effect sits on its hands and refuses to let the income tax rate rise. When the fiscal authority is finally forced to halt the burgeoning deficits and the rise in the debt ratio, however, the income tax rate must then rise very rapidly. Furthermore it must climb to a much higher level and then partially falls back to a significantly more elevated level than in the unbalanced-0T simulation.

The outcomes for the total of government tax revenue as a percent of nominal GDP vary widely across the five pension systems (Figure 28), reflecting complex combinations of interactions of macroeconomic developments with the differing pension tax rates and income tax rates. The movements of the total tax to income ratios for the tax-balanced and the unbalanced-0T systems are, as expected, almost indistinguishable. The general burden of taxes relative to the economy is lowest under the benefit-balanced system; indeed, the ultimate steady-state tax-income ratio is some 1½ percentage points less than the 28½ percent ratio in the initial baseline. Given the large, permanent increase in the debt ratio under the unbalanced-MT system, by far the largest rise in the tax-income ratio over the medium and long runs occurs under that system (an eventual permanent increase from the initial 28½ percent to over 32 percent). Note, however, that the tax burden under the unbalanced-MT regime is significantly *less* for the first several decades than under the tax-balanced and unbalanced-0T systems and about the same as or slightly less than under the intermediate-balanced system.

The major differences between the two unbalanced systems manifest themselves strongly in private saving and its accumulation into financial wealth. The fertility decline in the presence of cross-border spillovers boosts private saving relative to the size of the economy (Figure 29) and leads to more rapid accumulation of financial wealth per adult (Figure 30) for the reasons familiar from the earlier analysis. But the dynamic patterns and ultimate levels of saving and financial wealth are quite different when the fiscal authorities permit large budget deficits and increases in the debt ratio. Private saving in the unbalanced-MT case rises extremely rapidly during the period when the pension-system and overall budget deficits are becoming progressively larger. Private saving rises so strongly in part to offset the government's dissaving (although the offset is only partial; the analytical framework does *not* entail "Ricardian equivalence"). As soon as the fiscal authority is forced to hit the brakes and prevents any further increase in the target debt ratio, however, income taxes start to rise very sharply and private saving falls very quickly (again partially offsetting the government's move toward less dissaving). As seen in Figure 28, the eventual level of income taxes in relation to the economy is higher. Hence under the unbalanced-MT policies the saving ratio by the medium run and ultimately even financial wealth per adult are lowest of all among the five pension systems.

The consequences of the differing saving behaviors have the expected consequences for the economy's capital stock and its ability to produce output. The ZZ economy experiences the largest cyclical increases in the capital stock and the capital-output ratio (and the highest long-run value of the capital-output ratio) if the pension system is operated as benefit-balanced. Increases in the capital stock and capital-output ratio are lower under the unbalanced-0T system than any of the balanced systems, and lowest of all for the policies that permit a big upward modification in the target debt ratio (Figure 31).

The ZZ government's choice of how to operate the public pension system marginally influences the behavior of the nominal and real exchange rates as the economy interacts with the rest of the world in adjusting to its demographic shock (Figure 32). It is shown in a background paper for this project (Bryant, 2004b) that introduction of public pension systems into the analytical model (relative to a model variant without pensions) unambiguously *reduces* the degree to which the ZZ currency appreciates in response to asymmetric fertility shocks. When the pension system is unbalanced and the ZZ fiscal authority permits a substantial rise in its target debt ratio, the diminution in the size of the appreciation of the ZZ currency that would otherwise occur is especially noticeable.

As would be expected from the differential effects on the real and nominal exchange rates, the pension system associated with the smallest real appreciation of the ZZ currency – the unbalanced-MT system – is also associated with the least large increase in the ZZ import ratio, the smallest fall in the US import ratio, and the least large deficit in the external-trade account (Figure 33). If one were to judge pension systems by their influence on the size of the net import of real resources by the ZZ economy and hence their ability to cushion ZZ consumption, in other words, the benefit-balanced system performs best and the unbalanced-MT system is the least satisfactory. A similar ranking applies to the sizes of net investment income flows from abroad. ZZ savings are least robust under the unbalanced-MT regime and hence result in the smallest net inflow. Thus under the unbalanced-MT policies the current-account balance as a percent of nominal GDP (Figure 34) correspondingly shows the smallest medium-run increase of all five pension systems; the balanced systems all produce larger medium-run increases than the unbalanced systems; the benefit-balanced system with its higher savings is associated with the biggest surplus ratio. The ZZ net-foreign-asset position (Figure 35), the stock counterpart of the period-by-period current-account flows, is a complex net outcome from all the external-sector transactions. The balanced pension systems produce stronger net-foreign-asset positions than the unbalanced. In the very long run the unbalanced-MT evolution progressively deteriorates and eventually turns negative so that the ZZ economy has a sizable net-foreign-*liability* position.

The final comparative graphs, Figures 36 and 37, refer again to adult consumption per adult. The cushioning effects of the openness of the economy, stressed already in section 3, are a matter of first-order importance. Even for the unbalanced-MT pension system with its least favorable effects on national saving, the capital stock, and the net-foreign-asset position for the ZZ economy, the cushioning effects are strong enough to raise medium-run and long-run adult consumption per adult *above* the initial baseline for all but a few decades. For any of the balanced pension systems, the net favorable effects on consumption measured per adult or per capita are striking. To be sure, the *aggregate* sizes of ZZ output and consumption relative to the rest of the world are diminished (as shown in Figure 14 and emphasized in section 3). But *per adult* and *per capita* consumption are significantly augmented, not diminished.

## 6. Intergenerational Issues

Adjustments to demographic shocks have major distributional consequences, both across countries at any given time and intertemporally across generations. Our analytical framework cannot focus precisely on distributional issues because it does not keep track of the consumption and wealth accumulation of individual age cohorts. Even in our model, however, some inferences about distributional consequences are possible.

A major distributional effect across countries has already been identified. The counterpart of the favorable cushioning effects of openness on the ZZ economy in the presence of the asymmetric demographic shock is an *unfavorable* sharing of the demographic developments as appraised from the perspective of the US economy. Despite the fact that US population and output loom relatively larger in the world economy in absolute terms after the asymmetric demographic shock (shown in Figure 19 for the benchmark case), the effects on US adult consumption per adult are unambiguously adverse (Figure 20). This adverse US outcome in per capita terms obtains for the short, the medium, and the long runs. It is present for any of the balanced or unbalanced pension systems. The least adverse per capita outcome for the US occurs when the ZZ fiscal authority permits its budget deficit and debt issuance to accumulate (unbalanced-MT) rather than keeping its overall budget in approximate balance.

Distributional effects across generations within any one economy are much more subtle but equally significant. Averaged across all adults in the ZZ economy (younger adults working full time and elderly combined), the effects on adult consumption per adult relative to the no-shock baseline are an *improvement* in the shorter run, a major protracted cyclical *worsening* in the medium run, and a large permanent *improvement* in the long run (Figure 36). But the presence or absence of a pension system, and the choice of variant for operating the pension system, makes a noticeable difference in how various cohorts of adults will appraise the outcomes.

Contrast, for example, the evolutions of the ZZ economy under the intermediate-balanced and unbalanced-MT pension systems. As the fertility decline and subsequent population aging first influence the economy, the cohorts of older workers then alive and consuming may be contented if the fiscal and pension authorities are prepared to let the budget deficit roll on, financing the needed increases in pension benefits by issuing more government debt rather than by raising taxes (whether pension taxes or income taxes). Because so-called Ricardian equivalence does not hold in the model, individuals working today do not raise their saving (lower their consumption) in full anticipation of higher taxes that ultimately will have to be paid in the future because of today's deficits and accumulating debt. Increased private saving, in other words, does not fully offset government dissaving. Older adults currently working, and even more so current recipients of pension benefits no longer working, recognize that they may be dead by the time tax rates will be raised. Hence today's workers may well prefer to consume part of the windfall resulting from the fiscal authority's behavior today, albeit at the expense of future generations. Cohorts and generations are not altruistically linked in our analytical framework, and the welfare of future generations does not influence the current generation’s utility. Per capita consumption for older adults is therefore higher in the shorter run when an unbalanced-MT rather than a balanced pension system is in operation. A tax-balanced system or (to a lesser degree) an intermediate-balanced system, moreover, will permit higher consumption of older adults than is possible under a benefit-balanced system.

When economic welfare is judged from the perspective of the youngest workers and future generations of adults, the appraisal goes in the other direction. Young workers and even more so future generations would prefer today's government to keep the government's overall budget in balance by raising more taxes (pension or income) or reducing expenditures (pension benefits or other government spending) rather than issue increased debt to finance deficits today. Future generations are the cohorts that will ultimately have to bear most of the debt burden of higher shorter-run debt issuance under the unbalanced-MT policy. To oversimplify but to capture the essence of the intergenerational tension: the losers from government budget deficits today are future workers and future taxpayers whereas the gainers are today's workers, today's taxpayers, and to some extent the future owners of the economy's capital stock and foreign assets. The less favorable effects for future generations from unbalanced-MT policies relative to balanced pension-system policies (in particular relative to the intermediate-balanced or benefit-balanced systems) are clearly visible in Figure 36.

As a suggestive indication that today's pension recipients and older workers are likely to have opposite preferences about the choice of pension system than younger workers or cohorts yet unborn, Figure 37 has the same information as in Figure 36 except that it portrays the initial years of the simulation more clearly by including fewer years on the horizontal axis and adjusting the scale on the vertical axis. For the first decades of the shock, the unbalanced-MT policy of permitting budget deficits and debt to accumulate allows the highest level of per-adult consumption. For these first few initial years, in fact, the ranking of the five pension systems is completely reversed from the ranking that pertains in the medium and long runs. Remember, too, that the variable plotted in Figures 36 and 37 is per-adult consumption averaged across all adults, young and old. If it were possible to compare today's consumption ranking among the five pension systems for only elderly adults with a ranking pertinent to cohorts yet unborn, the contrast would be much more striking.

Issues of intergenerational equity are highly complex.[47] A substantial and growing literature continues to address them. Consensus is elusive even among economists.[48] Other disciplines introduce still more dissent.[49] Notwithstanding all the difficulties, resolving the issues of intergenerational equity is at the heart of comparing and choosing among alternative pension systems.

## 7. Concluding Observations

The analytical approach developed in this research project yields important insights into the cross-border dimensions of the macroeconomic consequences of demographic change. The analysis focuses on exchange rates, external-sector imbalances and interactions, and hence the spillover effects to other countries. The cross-border macroeconomic effects, though disregarded in almost all studies so far, are much too important to justify suppression. Ignoring the powerful influences working through exchange rates and cross-border transactions leads to a seriously inaccurate assessment of the net impacts of demographic change.

Careful analysis of the macroeconomic effects of demographic change must focus on both ends of the age distribution: *youth dependency* (transfers from adults to children) and *elderly dependency* (transfers from working adults to older adults, especially through public pensions). The previous paper

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[47] Here is a brief identification of just one of the complexities. The most important determinant of long-run growth in an economy is probably its secular rate of technical progress (productivity). The analytical framework in this paper assumes that the secular rate of technical progress is given exogenously. If the rate of technical progress in the future will be quite high (say, in excess of 3 percent per year), future generations will be relatively well off – can enjoy quite high incomes and consumption – almost regardless of how much or how little saving is done in the shorter run. Conversely, if the rate of technical progress will be very low, the incomes and consumption of future generations will probably be disappointing even if the flow of saving in the shorter run should be high. Whether it seems appropriate to ask today's workers and taxpayers to make sacrifices in order to keep national savings high today thus should depend in part on one's expectations for secular growth in productivity. (The economist Herb Stein was known to remark, however, that most people save for their children independently of whether or not they expect their children to be richer than they are.)

[48] An important set of papers is in the volume edited by Portney and Weyant (1999); see especially Cline (1999), Lind (1999), Nordhaus (1999), and Schelling (1999). Other references include Deaton and Paxson (1994), Lind (1995), Ball and Mankiw (1995), Corak (1998), and Shiller (1999).

[49] For example see Parfit (1982), Cowen and Parfit (1992), Laslett and Fishkin (1992), and Shaviro (1997).

dealing primarily with youth dependency showed that incorporating youth dependency into macroeconomic models – including the particular way in which children's consumption and adult transfers for child support are analytically modeled – leads to significantly different inferences about the effects of demographic change. This paper focusing on elderly dependency and public pensions leads to analogous conclusions: alternative ways of operating public pension systems and managing government debt can lead to substantially different macroeconomic outcomes, especially when the openness of economies is fully integrated into the analysis.

Our previous research has shown that transfers to children from parents and transfers to the elderly through public pensions often have partially offsetting effects on key macroeconomic variables. The offsetting pulls of the effects of youth dependency and elderly dependency in the presence of a marked decline in fertility reveal themselves in, for example, external-sector transactions. With youth dependency taken into account, reductions in the number of children in the economy frees up resources for adult use that otherwise would have gone into child support. Thus adult consumption, saving, and financial wealth can increase in the shorter run, which together with the boost that these changes give to the exchange rate in turn increases the net import of real resources and the net inflow of investment income from abroad. Taking elderly dependency and public pensions into account pushes the external-sector transactions in the other direction. The exchange rate appreciates less with public pensions in the model, and the ability of the open economy to experience net imports of real resources and a net inflow of investment income from abroad are somewhat dampened. Lower child dependency ratios resulting from declines in fertility mitigate some of the negative consequences of the subsequent population aging.[50]

Three fundamental points stemming from this project's analysis of the global consequences of demographic change merit special emphasis from a policy perspective. These conclusions deal with policy choices among alternative pension systems, policymakers' perspectives on issues of intergenerational equity, and policy assessments of the welfare consequences of population aging. To conclude this paper, we identify each of the three in turn.

Policy choices about public pension systems should be shaped by analysis that looks well into the future as well as at short-run pressures. Longer-run economic welfare, appropriately discounted back to present value today, depends importantly on choosing pension policies that encourage – or at least do not unduly penalize – national saving. For a single open economy that is moving faster into or

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[50] These effects contrasting model simulations with and without youth dependency and with and without public pensions are discussed in detail in Bryant (2004b).

is further along in its demographic transition, policymakers should also give substantial weight to encouraging the economy's ability to cushion the effects of its demographic transition by sharing them with the rest of the world.

These criteria of encouraging national saving and supporting the cushioning available through cross-border transactions have important implications for choices about the operation of PAYG public pension systems. Other things equal, policymakers should prefer a pension system that stays balanced over pension policies that permit imbalances to cumulate between pension revenues and pension tax revenues. Given the two criteria, moreover, policymakers should be especially wary of allowing population aging to produce large overall budget deficits and mounting increments to government debt. These conclusions emerge robustly from the analysis in sections 4 and 5 of this paper.

The preceding observations, however, fail to address the difficult issues of intergenerational equity. Criteria for choosing the operation of a public pension system that maximizes national saving or strongly supports the ability of the economy to exploit the cushioning available from cross-border transactions do not take into account the distributional effects that favor or disfavor the individuals alive today relative to future generations. Individuals alive today are voters. Generations as yet unborn do not vote in today's elections. The analytical and political issues of intergenerational equity thus just have to be central to policy choices about public pension systems. Facing these intergenerational aspects squarely is politically difficult, but obfuscation about them is probably worse than confronting them.

Finally, consider again the assessment of welfare consequences stemming from population aging. Much conventional wisdom asserts that population aging causes unambiguously adverse economic consequences. At best, that conventional wisdom is simplistic. In important respects, it is fundamentally wrong, especially for an open economy that is significantly ahead of other economies in its demographic transition. An economy that has experienced a fertility decline and is currently experiencing population aging – the situation of Japan early in the twenty-first century – is able to diffuse part of its larger shock into other nations. In effect, it “shares” the shock with the rest of the world. In per capita terms, the outcome can be significantly better for that country's residents. Discussions of population aging pay far too little attention to this fundamental point.

Analyses of national welfare of course cannot rely exclusively on per capita measures of economic variables. For some political or security purposes, for example, it may be necessary to stress aggregate economy-wide data for a nation *relative to nations in the rest of the world*. If a country experiences a fertility decline sooner or faster than the rest of the world, its population, GDP, and consumption will become smaller proportions of the world total. In our model, the ZZ economy as a whole shrinks in size relative to the US economy after its larger demographic shock. With relatively fewer ZZ real resources available for investment or consumption, the ZZ government and the nation as a whole might well be supposed to have lessened influence in the world. In real life, Japan with its earlier and faster fertility decline, leading to faster population aging and a relative shrinking of its population and GDP, may conceivably experience diminished effects on its power and security positions in the world.

Thus one should not ignore the consequences of population aging for a nation's aggregate macroeconomic variables. But neither should one forget the powerful effects on per-capita measures of economic welfare, which caution against simplified adverse judgments. From the perspectives of individuals in an open economy, conclusions about the welfare consequences of population aging may point in the opposite direction from those based solely on macroeconomic aggregates. Again, for an open economy asymmetrically experiencing fertility declines and population aging, negative consequences accompanying the demographic shifts are typically cushioned because the negative effects are shared with the rest of the world. That cushioning and sharing may not be desirable as seen from the perspective of foreigners, but it can produce sizable welfare gains for home residents.[51]

The analysis summarized in our research papers so far is too abstract and too qualitative to permit confident application of the generalizations to today's conditions in Japan or in other fast aging nations. But the qualitative analytical conclusions emerging from the model are robust. It seems plausible to believe that the qualitative influences stressed here have played, and will continue to play, a significant role in the macroeconomic evolution of the Japanese economy and Japan's real exchange rate.

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[51] Burtless (2002) gives additional reasons, from a domestic (closed-economy) perspective, why population aging has favorable as well as problematic consequences.

# Appendix 1: Further Background on Analytical Approach

This appendix provides further details about our analytical framework, indicating how we treat age-earnings profiles for the labor force, economic linkages between youth dependents and adults, and the consumption decision made by adults.

## A. Age-Earnings Profiles and Aggregate Labor Income

The adult consumption problem follows the familiar formulation widely used since Blanchard (1985) but with two major modifications: (1) the inclusion of age-earnings profiles to introduce a life-cycle path to labor income, and (2) the explicit inclusion of parent-child transfers in the adult budget constraint. We elaborate on these two issues and other aspects of the consumer's problem in what follows.

In real-life economies, labor earnings display a hump-shaped pattern across age groups, rising initially as younger workers accumulate experience and seniority, peaking at later middle age, and declining after the peak years of productivity have been passed and workers supply less labor and eventually retire. We account for this life-cycle age-earnings profile by varying effective individual labor input according to age. Moreover, our analytical framework incorporates such a hump-shaped profile in a way that permits a "bottom-up" rather than "top-down" determination of aggregate labor income.

The age-earnings profile is mathematically approximated by specifying the labor input of an individual cohort *s* at time *t* with three exponential terms:

$$l(s,t) = \left[ a_1 e^{-\alpha_1 (t-s)} + a_2 e^{-\alpha_2 (t-s)} + (1 - a_1 - a_2) e^{-\alpha_3 (t-s)} . \right] \tag{25}$$

Loosely speaking, the first two terms in (25) may be thought of as representing the decline in an individual cohort's labor supply over time as it ages and (gradually) retires. The third term can be interpreted as reflecting gains in earnings that accrue with age and experience. The restriction on the $a_i$ coefficients on the exponential terms (the third of the coefficients must be equal to $1 - a_1 - a_2$) embodies a normalization that the youngest cohort (for whom $s = t$) has effective labor input equal to unity.

Assuming that differences in earnings are attributable to these age-specific differences in labor productivity and labor supply, the relative labor earnings of a particular cohort can be written as:

$$y(s,t) = \left[wage(t)\right] l(s,t), \tag{26}$$

where $wage(t)$ is the economy's average wage rate which grows over time at the rate of labor-augmenting technical change. Consequently, individual labor earnings can change over time because of general growth in labor productivity, assumed to apply uniformly to all cohorts (i.e., after adjustment for their age-specific *relative* productivities). The time-path for individual relative earnings, meanwhile, is assumed to match the cross-sectional age-earnings pattern observed in the data. Given (25) and (26), the parameters $a_1, a_2, \alpha_1, \alpha_2$, and $\alpha_3$ are calibrated to match non-linear least squares estimates on cross-sectional data for age and earnings for Japan and the United States obtained by Faruqee (2000). The relative productivity parameters are further specified to be exogenous (i.e., fixed over time) in the modeling code.

If (25) and (26) are aggregated over all individual cohorts, aggregate labor income can be written as:

$$Y(t) \;=\; \int_{-\infty}^{t} wage(t)\, l(s,t)\, N(s,t) ds \;=\; wage(t)\, L(t), \tag{27}$$

where $L$ is aggregate labor input adjusted for cohort-specific relative productivities. The definition of labor input for the individual cohort in (25) also permits one to write aggregate $L$ as the sum of three components $L_1$, $L_2$, and $L_3$ where each component reflects an exponential term in (25). Details are in the background paper (Bryant, 2004b). Intuitively, changes in aggregate labor input depend on the effective labor supply of new entrants to the labor force, the relative productivity experiences and deaths of existing workers, and the general pace of labor-augmenting technical progress. The specific values of the five coefficients $a_1, a_2, \alpha_1, \alpha_2$ and $\alpha_3$ in (25) obtained from estimating the age-earnings profile play a critical role in determining the movements of effective labor supply and the evolution of human wealth and consumption over time.

### B. Economic Linkages between Youth Dependents and Adults

The introduction of youth dependency into a macroeconomic model requires treatment of two central issues: (i) how the consumption of youth dependents is determined, and (ii) how and by whom that consumption is financed.

Our recent approach divides consumption by children into two components, one that is inelastic and fixed, the other that endogenously varies through time in response to conditions in the economy. The inelastic component is unaffected by changes in the economic situation of adult parents; it reflects, so to speak, the "basic needs" of children for (say) food, clothing, and shelter. The second component is "discretionary" and modeled as a constant fraction of economy-wide parental income; hence this portion of consumption per youth varies with consumption per adult over time. The total resources consumed by each youth dependent, $c_j(s,t)$, is thus given by:

$$c_j(s,t) = c_1 + c_2\, C(t)/N(t) \tag{28}$$

where $c_1$ is the fixed, basic-needs component and $c_2$ is the coefficient of proportionality determining the responsiveness of the time-varying, endogenous component of child consumption to adult consumption per adult.[52]

The formulation in (28) is appealing because it nests several possible ways to model child consumption. By setting $c_2 = 0$, for example, one can study the implications of making all child consumption inelastic to conditions in the economy.[53] Conversely, by setting $c_1 = 0$ with $c_2 > 0$, one can analyze the case where the basic-needs component of child consumption is absent altogether. For any given value of the basic-needs component $c_1$, the parameter $c_2$ determines the generosity and sensitivity of discretionary child consumption relative to the consumption of an average adult. By varying the value of $c_2$, one can thereby investigate the effects of making the discretionary component of child consumption more or less generous than average adult consumption, and hence more or less volatile (in absolute terms).

Our treatment of the second central issue is to assume that child consumption is entirely financed through *in vivo* transfers from adults. Youth dependents do not earn labor income and have no financial wealth.[54] In our simplified framework, adult transfers to children are thus, in economic effect, like tax payments earmarked for child support – see the way that *v(s,t)* enters in equation (37) below but is not present in equation (35) below. Alternatively stated, adult parents are assumed *not* to derive any direct utility gain from child consumption and thus *not* to determine, as an integral part of their own consumption decision, the level at which children consume. Our simplified formulation,

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52 The value of $c_j(s,t)$ is the same for every child cohort rather than varying across child cohorts.

53 The model framework allows labor productivity in the economy to grow at a constant, steady-state rate. For model simulations in which productivity growth is positive, we assume that the basic-needs component of child consumption, $c_1$, is not fixed absolutely but rather increases at the constant rate of productivity growth.

54 Thus when a youth becomes an adult and enters the workforce at the beginning of the 19th year of life, he or she starts out with zero financial wealth.

embodying a rule-of-thumb standard for setting child consumption and treating adults as the only decisionmakers in the economy, allows us to avoid fundamentally altering the way the model treats the consumer problem facing adult agents.[55]

Given the preceding assumptions, for consistency it must be true in each period that total consumption by children, $C_j(t)$, is exactly matched by the aggregate of support transfers from adults. Given the general formulation of individual child consumption given by (28), this condition is:

$$C_j(t) = \int_{t-\Delta}^{t} c_j(s,t)J(s,t)ds = c_1 J(t) + c_2\delta(t)C(t) \;=\; \int_{-\infty}^{t} v(s,t)N(s,t)ds = V(t) \tag{29}$$

where the youth-dependency ratio, $\delta(t)$, is as shown in equation (9) in the text above, and $v(s,t)$ is again the lump-sum transfers that adults from a particular cohort provide to their children.

We further assume that support transfers for children are cohort specific and that the age distribution across adult cohorts is hump-shaped. This assumption about the age distribution of child support follows the typical pattern of family size; household numbers usually rise at early stages of adulthood and then subsequently fall as children eventually mature and become independent. The hump-shaped profile, in other words, allocates transfers most heavily to middle-aged adults with larger families of dependent children. For a specific approximation of a hump-shaped profile, we adopt the following:

$$v(s,t) = v(t)[z_1 e^{-\omega_1(t-s)} + (1-z_1)e^{-\omega_2(t-s)}]; \quad z_1 > 1, \quad \omega_2 \geq \omega_1 \geq 0 \quad . \tag{30}$$

The weighting function in (30), in square brackets, is normalized relative to the youngest adult cohort (where $s = t$). In other words, $v(t)$ represents the transfers that the *newest* cohort of adult parents provide to their (newly born) children. Parents in older cohorts provide relatively more or less support than $v(t)$ depending on their age, according to the weighting function.

The specification in (30) nests the still more simplified cases used in our earlier papers exploring youth dependency. In particular, when both curvature parameters are set to zero ($\omega_1 = \omega_2 = 0$), support payments for children are completely age-invariant (every adult age cohort makes exactly equivalent transfers). For the special case in which $\omega_1 = \omega_2$ and both parameters are positive, (30) produces an exponentially declining rather than hump-shaped distribution of transfers

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[55] Our treatment of child consumption is a major step forward from the practice of ignoring it entirely, but self-evidently is unrealistic and needs to be modified in future extensions of the research. For a more realistic formulation of child consumption in which child consumption directly enters the parents' utility function (but which cannot be easily adopted in our model), see for example Becker and Barro (1988).

across cohorts; that is, the newest cohort of adults provides the largest amount, with the amount declining uniformly for older and older cohorts. The most plausible configuration of the parameters are the hump-shaped profiles for which $\omega_2 \geq \omega_1 \geq 0$.[56]

With the cohort distribution of adult transfers in (30), the economy-wide total for adult transfers can be written in terms of two components:[57]

$$V(t) = V_1(t) + V_2(t); \tag{31}$$

$$\dot{V}_1(t) = \frac{\dot{v}(t)}{v(t)} V_1(t) + z_1 b_n(t) N(t) v(t) - (\omega_1 + p_n(t)) V_1(t); \tag{32}$$

$$\dot{V}_2(t) = \frac{\dot{v}(t)}{v(t)} V_2(t) + (1 - z_1) b_n(t) N(t) v(t) - (\omega_2 + p_n(t)) V_2(t). \tag{33}$$

Using the adding-up condition in (29) between aggregate child consumption and total adult transfers that $C_j(t) = c_1 J(t) + c_2 \delta(t) C(t) \; = \; V(t)$, one can write the law of motion for *v*, individual transfers by the newest adult cohort, as:

$$\frac{\dot{v}(t)}{v(t)} = \frac{c_1 \dot{J}(t) + c_2 \delta(t) C(t) \left[ \frac{\dot{\delta}(t)}{\delta(t)} + \frac{\dot{C}(t)}{C(t)} \right]}{V(t)} - \left[ \frac{b_n(t) N(t) v(t)}{V(t)} + \omega_1 \frac{V_1(t)}{V(t)} + \omega_2 \frac{V_2(t)}{V(t)} + p_n(t) \right]. \tag{34}$$

### C. <u>Adult Consumption</u>

The individual adult consumer is presumed to solve the following familiar maximization problem:

$$\max \int_t^{\infty} e^{-(\theta + p_n(x))(t-x)} u(c(s,x)) dx \tag{35}$$

subject to:

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[56] The case of exponentially declining weights takes into account the likelihood that the children of middle-aged adult cohorts may have already themselves reached adulthood and are no longer dependent on parent-child transfers. But it fails to recognize that family sizes tend to grow in the early years of adulthood after youths have come of age and begin to have children.

[57] The specific equations for the two components of equation (31) are $V_1(t) \equiv \int_{-\infty}^{t} z_1 e^{-\omega_1 (t-s)} N(s,t) ds$ and $V_2(t) \equiv \int_{-\infty}^{t} (1 - z_1) e^{-\omega_2 (t-s)} N(s,t) ds$.

$$w(s,t) = fw(s,t) + hw(s,t) \tag{36}$$

$$\begin{aligned}\dot{fw}(t) = &\left[(r(t) + p_n(t))\, fw(s,t) + y(s,t)\right](1-\tau(s,t)) \\ &+ pben(s,t) - \tau_{SS}(s,t)y(s,t) \ - \ c(s,t) \ - \ v(s,t)\end{aligned} \tag{37}$$

$$hw(s,t) \equiv \int_t^{\infty}[y(s,x)(1-\tau(s,x)-\tau_{SS}(s,x)) - \ v(s,x)]e^{-\int_t^{v}(r(\mu)+p_n(\mu))d\mu}dx \, . \tag{38}$$

Here *u(.)* is the utility function of the individual, assumed to be of the constant relative risk aversion (CRRA) form, $u(c(s,t)) = \frac{c(s,t)^{1-\sigma} - 1}{1-\sigma}$, where σ is the coefficient of relative risk aversion, and 1/σ is the intertemporal elasticity of substitution. Adults are assumed not to derive explicit utility gain from child consumption (discussed above). Hence only the consumption at time *t* of an individual adult born at time *s*, *c(s,t)*, directly enters the utility function in (35). The variable *w(s,t)* is the total wealth of the adult individual at time t, which in turn is the sum of *fw(s,t)*, wealth in the form of financial assets, and *hw(s,t)*, human wealth defined as the present value of expected future labor income net of taxes on labor income and *in vivo* transfers to children. Transfers made by adults to support the consumption of youth dependents, *v(s,t)* in (37) and (38), are discussed above. The other variables in the preceding equations are: *r(t)*, the real interest rate at time t; $\theta$, the time preference rate; *y(s,t)*, the pre-tax labor income at time *t* of the individual born at time *s*; $\tau(s,t)$, the income-tax rate on that individual's combined labor and capital income; *pben(s,t)*, the pension benefit (if any) received at time *t* by that individual;[58] and $\tau_{SS}(s,t)$, the pension tax rate on labor income. The public pension system is discussed below. Disposable income for the individual after taxes is $\left[(r(t) + p_n(t))\, fw(s,t) \ + \ y(s,t)\right](1-\tau(s,t)) + pben(s,t) - \tau_{SS}(s,t)y(s,t)$. The individual's saving in the period – the net change in financial wealth given by equation (37) – is after-tax disposable income *less* own consumption *less* transfer payments for child support.

Solving the utility maximization problem gives adult consumption as a linear function of total wealth:

$$c(s,t) = \frac{1}{\Psi(t)}[\, fw(s,t) + hw(s,t)]. \tag{39}$$

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[58] It is assumed that pension benefits received by the elderly are not subject to income tax.

where $1/\Psi(t)$ is the marginal propensity to consume out of wealth. The dynamics of $\Psi(t)$ are given by:

$$\dot{\Psi}(t) = -1 - \frac{1}{\sigma}[(1-\sigma)(r(t)+p_n(t)) - (\theta + p_n(t))]\Psi(t) . \tag{40}$$

The marginal propensity to consume out of wealth in the general case of the CRRA utility function depends, as is well known, on the intertemporal elasticity of substitution (EIS) and on the entire sequences of future interest rates and future adult mortality rates. This dependence is readily evident in equation (40). In contrast, when the EIS is assumed to be unity (the case of logarithmic utility, with $\sigma = 1$) and when the adult mortality rate is assumed to be constant rather than time varying, the marginal propensity to consume out of wealth reduces to the simple form of a constant, the sum of the time preference rate and the mortality rate ($1/\overline{\Psi} = \theta + \overline{p}_n$ ).[59]

When one aggregates over the consumption functions of individual decision-making units (equation (39)), total adult consumption *C* is given by:

$$C(t) = \int_{-\infty}^{t} c(s,t)N(s,t)ds = \frac{1}{\Psi(t)}[FW(t) + HW(t)], \tag{41}$$

where *C(t)* is aggregate adult consumption and *FW*(*t*) and *HW*(*t*) are aggregate financial and human wealth. The evolution of *FW*(*t*) is described in equation (14) in the main text.

Aggregate human wealth is a stock variable representing the present value of economy-wide labor income (adjusted for the varying ages and relative productivities of different cohorts). Its change through time is given by:

$$\dot{HW}_t = \frac{d}{dt}\int_{-\infty}^{t} hw(s,t)N(s,t)ds$$

$$= hw(t,t)b_n(t)N(t) + r(t)HW(t) - Y(t)(1-\tau(t)) - PTFGAP(t) - C_j(t) . \tag{42}$$

Equation (42) shows that the incremental change in aggregate human wealth at time *t* is influenced by the additional human wealth of the newest adult cohort coming of age at time *t*, $hw(t,t)$. The shape of

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[59] The net response of consumption to changes in the real interest rate depends on the relative strength of substitution and income effects. With a low rather than high value of EIS, consumers act less strongly to shift their consumption intertemporally; the substitution effect is thus smaller relative to the income effect. For any given shock to the economic system, the real interest rate thus must adjust by a larger amount the lower is the value of EIS. Empirical evidence, in our view, is more consistent with $\sigma$ having a value smaller than unity (for example, 0.5 or even as small as 0.3) than with the assumption that $\sigma = 1$ (logarithmic utility).

the labor-earnings profile – embodied in the five parameters $a_1, a_2, \alpha_1, \alpha_2$, and $\alpha_3$ in equation (25) – has a critical influence through time on the behavior of $hw(t,t)$ and hence of aggregate human wealth, *HW(t)*.

Income taxes and child-support transfers influence aggregate human wealth. Any imbalance in the pension trust fund between pension tax revenues and benefit payments, the variable *PTFGAP(t)* in (42), also affects the dynamics of aggregate human wealth. When the pension system is continuously balanced so that current-period pension taxes exactly cover current-period pension benefits paid to the elderly ($PTFGAP(t) = 0$), the *direct* effect of the pension system on aggregate human wealth nets out to zero. However, even for the continuously balanced cases where *PTFGAP* never differs from zero, the inclusion of a public pension system in the framework has important *indirect* effects on human wealth (and hence on consumption and other key macroeconomic variables) through the evolution of *hw(t,t)*, the human wealth of the newest cohort just entering adulthood and the workforce (see below).

Given the age-earnings profile, adult support of child consumption, and the public pension system, the dynamics of human wealth for the new cohort of individuals entering adulthood and working life are:

$$hw(t,t) = \left[ \sum_{k=1}^{3} hw_i(t,t) \right] - hw_{v1}(t,t) - hw_{v2}(t,t) \ + \ hw_{ss}(t,t) \ . \qquad (43)$$

Equation (43) defines the human wealth of the newest individual cohort entering the adult population at time *t* as the sum of six components. The first three components derive from the concave time profile of labor income as described in equations (25) through (27) and are given by:

$$\dot{hw}_k(t,t) = [r(t) + p_n(t) + \alpha_k] hw_k(t,t) - a_k wage(t)[1 - \tau(t) - \tau_{ss}(t)]; \ \ \mathrm{k} \in \{1,2,3\} \ . \qquad (44)$$

The $hw_k(t,t)$ equations allow for both the income and pension taxes that are paid on labor income. The fourth and fifth components of (43) reflect the impact of parent-child transfers on human wealth, with *v*(*t*) as determined in equation (30) above:

$$\dot{hw}_{v1}(t,t) = (r(t) + p_n(t) + \omega_1) hw_{v1} - \ z_1 v(t) ; \qquad (45)$$

$$\dot{hw}_{v2}(t,t) = (r(t) + p_n(t) + \omega_2) hw_{v2} - \ (1 - z_1) v(t) . \qquad (46)$$

The final component, $hw_{ss}(t,t)$, is included to reflect the real value of the stream of pension benefits that the new adult cohort expects to receive eventually after reaching the threshold elderly age (47 years in the future, $t + \Lambda$), discounted back to the present:

$$\dot{hw}_{ss}(t,t) = [r(t) + p_n(t)]hw_{ss}(t,t) \quad - \quad [\beta_{T1}(t+\Lambda) + \beta_{T2}(t+\Lambda)\frac{Y(t+\Lambda)}{N(t+\Lambda)}]e^{-\int_t^{t+\Lambda}(r(s)+p_n(s))ds} \tag{47}$$

The second term in square brackets in (47) is the per-elderly gross pension benefit expected in the future at the date today's new adults expect to reach the threshold elderly age. Note that even for balanced PAYG pension systems in which aggregate pension taxes continuously equal aggregate pension benefits, the discounted stream of an *individual* new worker's future pension taxes will not necessarily be equal to the discounted stream of pension benefits that that worker expects to receive from age 65 until death. Consequently, even a continuously balanced pension system affects individual consumption and saving behavior through its effect on individual human wealth. The effects for particular individuals on human wealth, consumption, and saving are especially strong in unbalanced pension systems when the trust-fund gap is not continuously zero.

# Appendix 2:
# Baseline Solution and Specification of Illustrative Shocks

The empirical analysis in the paper starts by developing one or more model-consistent, steady-state baseline solutions for the evolution of the ZZ and US economies. To provide analytical transparency, each economy is assumed to follow identical paths and exhibit identical behavior along these steady-state baselines.

The fertility (child birth) rate, the child mortality rate, and the adult mortality rate are the key exogenous demographic variables. The initial baselines in this paper assume that in both economies the child birth rate has a constant value of 0.02504 (the new child cohort each year being slightly more than 2½ percent of the adult population that year). The child mortality and adult mortality rates are constant at, respectively, .0075 and .015; the adult mortality rate implies that individuals at age 19 entering the workforce expect to live roughly another 67 years. These baseline assumptions about fertility and child mortality have the consequence that youths enter the baseline adult labor force at the constant rate .02 (2 percent per year), which in turn results in the total population growing slowly in the baseline at the constant rate .005 (½ percent per year). Labor productivity (labor-augmenting technical change) and the price level both grow in the baselines at ½ percent per year.

The intertemporal elasticity of substitution in consumption (EIS) is set at 0.5. Our preferred working value for the EIS is this value of 0.5, but we have studied values as low as 0.3 and as high as unity (logarithmic utility).[60] One third of consumers are assumed to be borrowing constrained and therefore consume only out of current-period income rather than smoothing intertemporally. The elasticity of substitution in the CES production function is set at unity (the Cobb-Douglas case).

For expositional simplicity, we assume that central banks use a money-targeting reaction function in setting the short-term nominal interest rate. We use values for the coefficients in the governments' tax-rate reaction functions – the $\gamma_1$ and $\gamma_2$ coefficients in (24) are set respectively at 0.04 and 0.30 – that cause government debts in shock simulations to return fairly promptly to their target, baseline paths.

The baselines in this paper use a set of benchmark assumptions about child consumption financed by child-support transfers from adults that emerged from the analysis in Bryant, Faruqee, Velculescu, and Arbatli (2004). The total generosity of baseline individual child support is equivalent

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60 For illustrative simulations comparing alternative values of the EIS, see Appendix 3 in Bryant, Faruqee, Velculescu, and Arbatli (2004). Our experimental results for alternative values of the EIS make it clear that analytical and policy judgments depend sensitively on how researchers calibrate their models for this key parameter.

to one half the amount of the average for baseline individual adult consumption. Of this amount, two fifths is assumed to be a fixed, basic-needs component; the remaining three fifths is assumed to be time-varying and dependent on variation in baseline adult consumption. In terms of equation (28), $c_1$ is determined as 20 percent of baseline $C(t)/N(t)$ and varies over time only with growth of productivity; the coefficient $c_2$ is set at 0.3. The coefficients determining the distribution of child support across different ages of parent cohorts – $\omega_1$ and $\omega_2$ in equation (30) – are set at estimated values of, respectively, 0.084 and 0.102, resulting in an age distribution that is hump-shaped and concentrated most heavily on adults in their late twenties.

Baseline solutions of the model also depend on the values set for the pension tax rate $\tau_{SS}$ and the pension benefit parameters $\beta_{1T}$ and $\beta_{2T}$. For an illustrative pension benefit in baselines, we use a value roughly the order of magnitude of the average size of benefits relative to average wages in both the Japanese and U.S. public pension systems. If the pension trust fund in the baseline is assumed to be kept exogenously fixed at zero, if the Tier-One component of pensions is set at 10 percent of average per-worker labor income, and if the Tier-Two benefit rate $\beta_{2T}$ is set at 0.22, the baseline pension tax rate consistent with those assumptions typically falls into the range 0.12 to 0.14. This pension tax rate also is broadly in line with actual experiences in the Japanese and U.S. public pension systems. The alternative pension systems are described in section 2C of the text.

Throughout the analysis in sections 3, 4, and 5, the adult mortality rate and the child mortality rate remain unchanged at the baseline values of, respectively, .0075 and .015. The analysis specifies two shocks to the birth rate. One is large-cyclical, the other smaller-gradual (see again Figure 2). In the large-cyclical fertility shock, the child birth rate falls persistently from its baseline level of roughly 2.5 percent to about 1.0 percent over a period of some 5½ decades; it remains at that low rate over the next three decades; then it gradually rises back to a new level of about 1.7 percent; it remains stationary at that level thereafter. Given the mortality rates, the eventual birth rate results in a steady-state population that is stationary. For the smaller-gradual fertility shock, the birth rate starts from the same initial baseline rate but then declines slowly and monotonically. The birth rate eventually reaches the same steady-state rate of 1.7 percent as that reached finally in the large-cyclical shock. Thus the smaller-gradual shock also results ultimately in a stationary population. The two shocks reach the ultimate steady-state rate at the same point in time; the lengths of the time period over which the shocks occur is therefore identical. Unlike in the large-cyclical shock, however, the birth rate in the smaller-gradual shock does not follow a U-shaped demographic pattern of sharp fall and then partial recovery.

As shown clearly in Figures 2 and 4, the large-cyclical shock measured in terms of its effects on the total population is much larger in magnitude than the smaller-gradual shock. Population growth becomes negative by the third decade of the simulation and falls all the way to a negative rate of ½ percent per year during the period when the birth rates are at their lowest point.[61] As the birth rate then recovers, the population growth rate gradually becomes less negative, eventually reaching a new steady-state rate of zero as the birth rates level out at their new steady-state rates.[62]

Figure 3, discussed briefly in the text, indicates the large consequences of the fertility shocks for the youth-dependency and elderly-dependency ratios. By coincidence, the two ratios are nearly the same in the initial baseline (both children and elderly being roughly 40 percent of the adult population). From that 40 percent ratio, in the large-cyclical shock elderly individuals ultimately reach a peak plateau of some 63 percent of the adult population. After that peak is reached, the elderly ratio falls back substantially, eventually settling at about 50 percent of the adult population (significantly above its initial baseline level, but well below the intermittent peak reached when the demographic shock has its greatest effect on the relative numbers of elderly). The youth-dependency ratio in the large-cyclical shock has a reverse cyclical swing to that in the elderly ratio, about as marked but occurring earlier in time. When the new steady-state situation is eventually reached, youths have fallen to only 29 percent of the adult population. Under the smaller-gradual shock, the two dependency ratios reach the same long-run steady-state paths but do so gradually and monotonically without any cyclical swing.

As discussed in the text, the situation is *symmetric, closed-economy* if the ZZ and the US economies both experience the large-cyclical shock, or if they both experience the smaller-gradual shock. (When a shock is identical in both regions, the model produces identical simulation paths for both economies. Each economy behaves as though it were completely closed; external-sector balances remain at zero and the exchange rate remains unchanged at its baseline value of unity.) The situation is *asymmetric with cross-border spillovers* when the ZZ economy experiences the large-cyclical shock and the US economy experiences the smaller-gradual shock.

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[61] This path for population growth is broadly analogous to that projected by the Japan National Institute of Population and Social Security Research (1997, 2002) and further discussed in Takahashi, Kaneko, Ishikawa, Ikenoue, and Mita (1999).

[62] The shock is constructed so that the economy experiencing it in the long run ultimately settles into a new steady state in which not only the total population but also its distribution by age are stationary.

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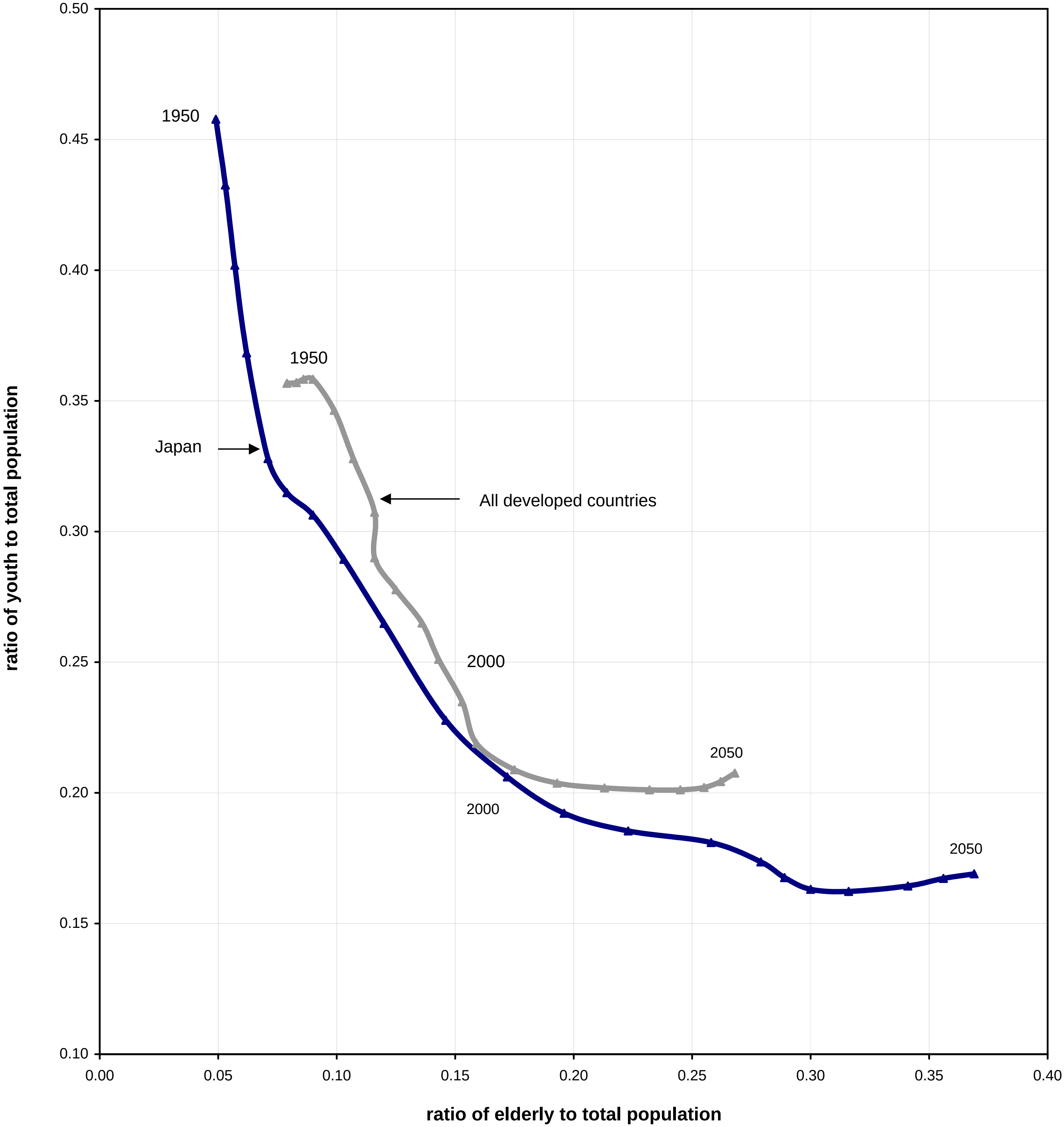
**Figure 1. Evolution of Elderly Dependency and Youth Dependency: Japan versus All Developed Countries (1950-2050)**




source: United Nations (2001)
youth: younger than 20 years old; elderly: older than 65 years old.

**Figure 2. Two Illustrative Shocks for Fertility Decline and Implications for the Endogenous Growth Rate of Total Population**

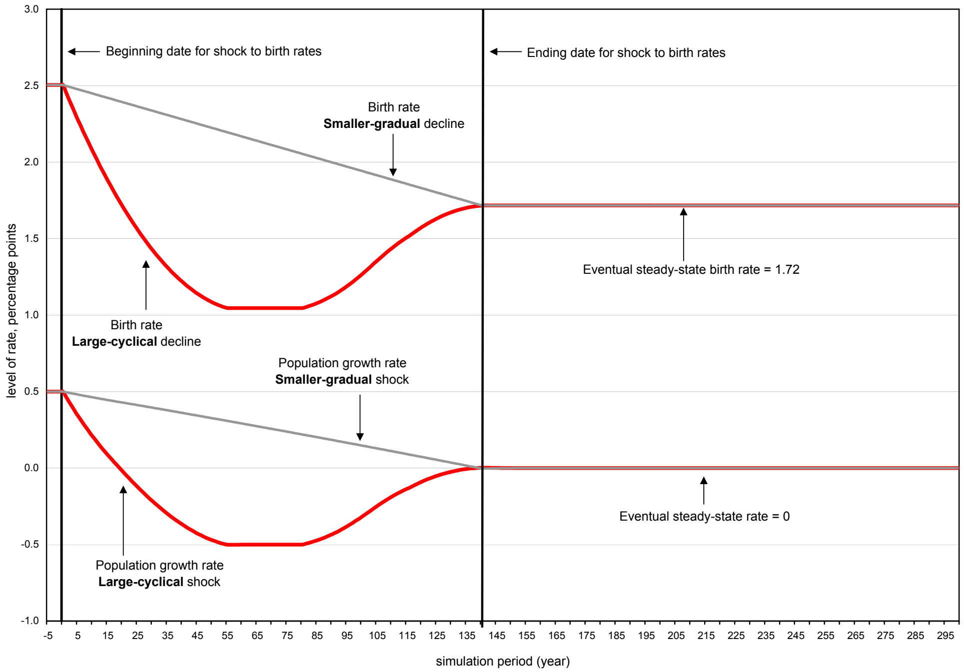



**Figure 3. Youth and Elderly Dependency Ratios for the Two Illustrative Shocks**

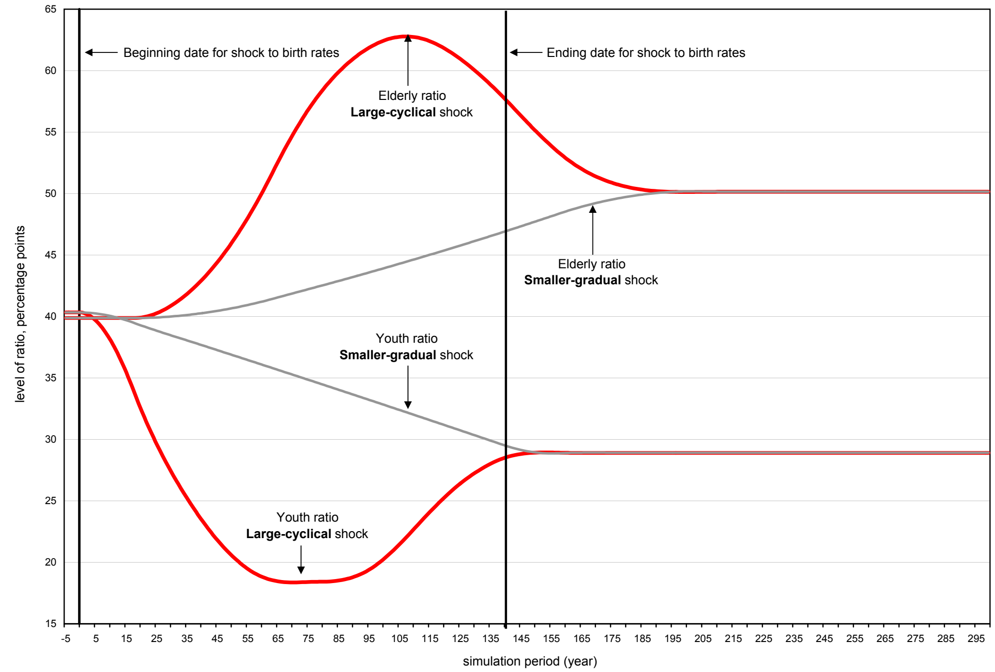



**Figure 4. Adult Populations and Effective Labor Forces for the Two Illustrative Shocks**

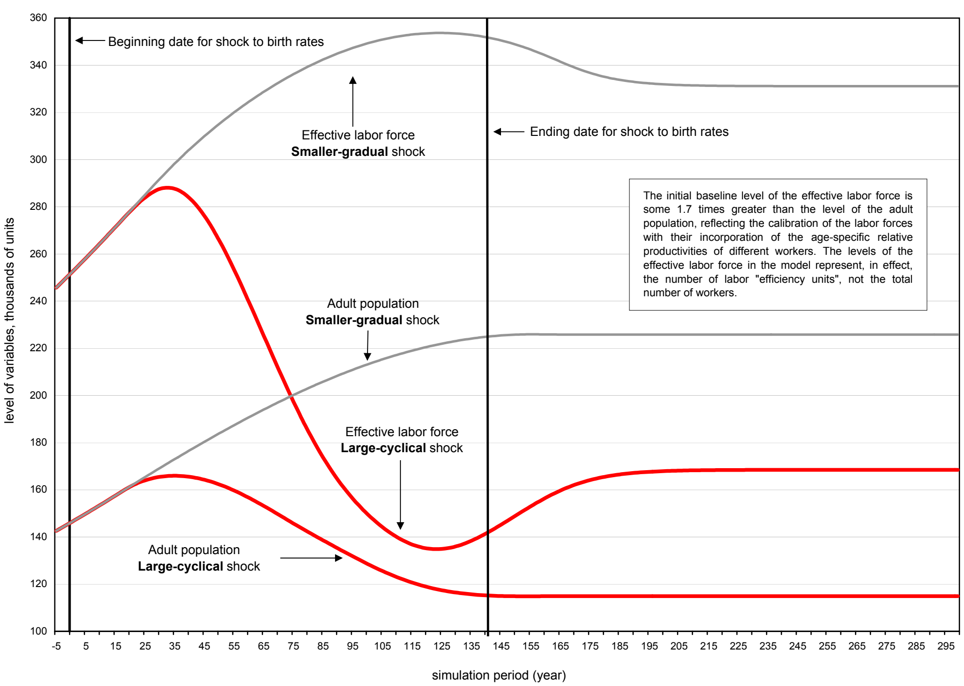



**Figure 5. Ratio of Effective Labor Forces to Adult Population for the Two Illustrative Shocks**

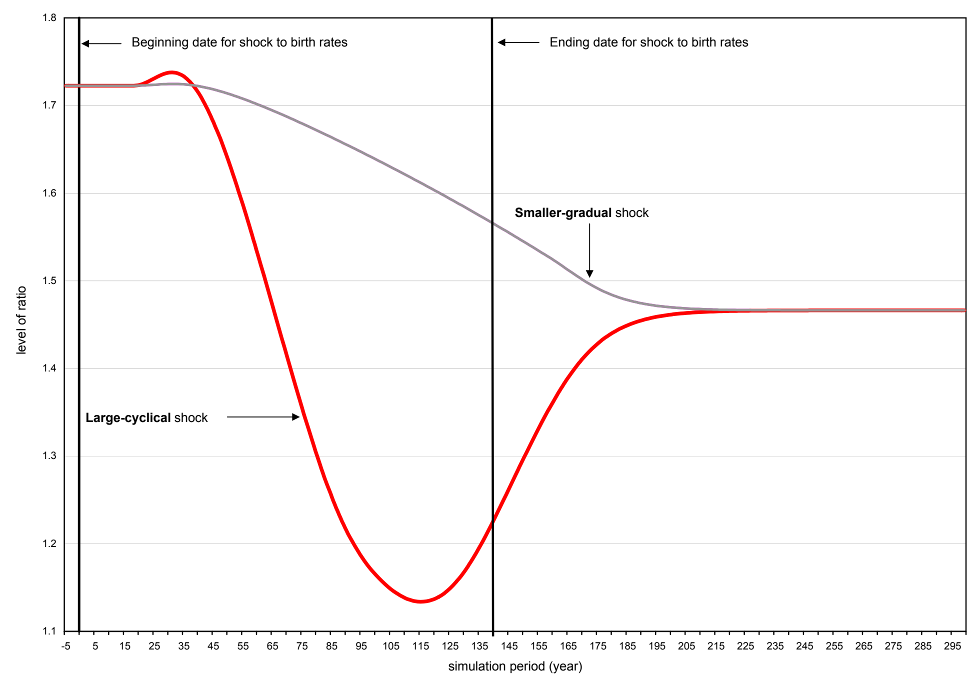


**Figure 6. Real Short-term Interest Rates**
**Symmetric (Closed-Economy) Shocks**

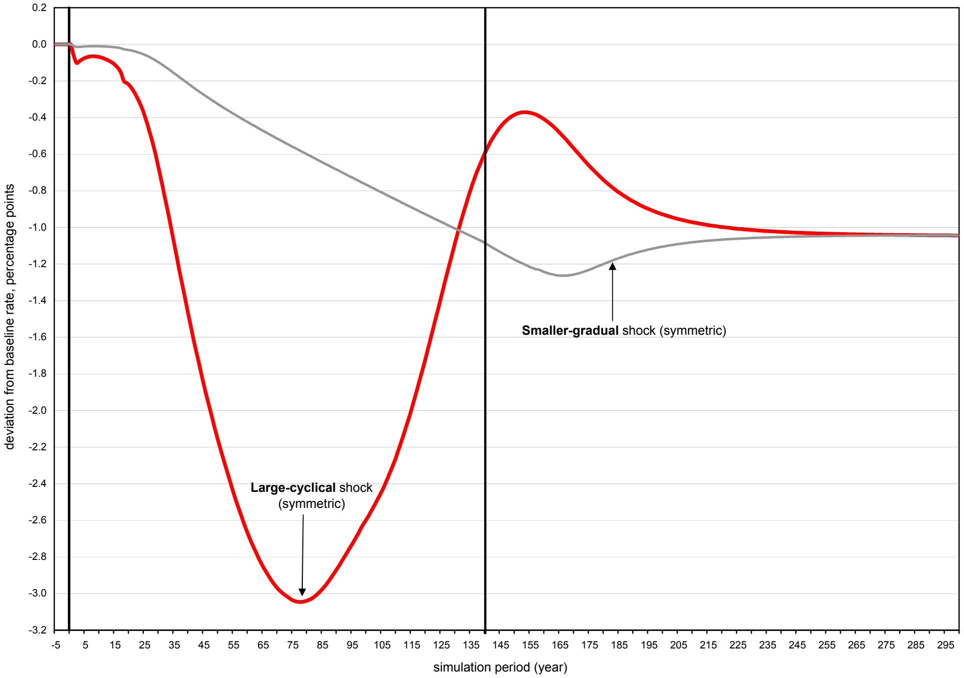



**Figure 7. Capital-Output Ratios**
**Symmetric (Closed-Economy) Shocks**

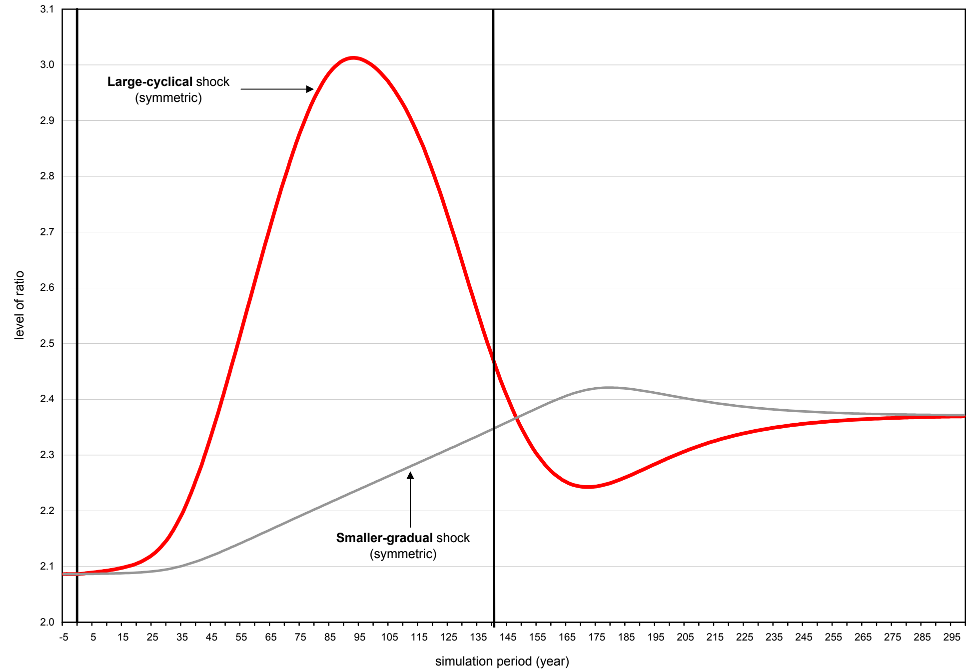



**Figure 8. Financial Wealths per Adult**
**Symmetric (Closed-Economy) Shocks**

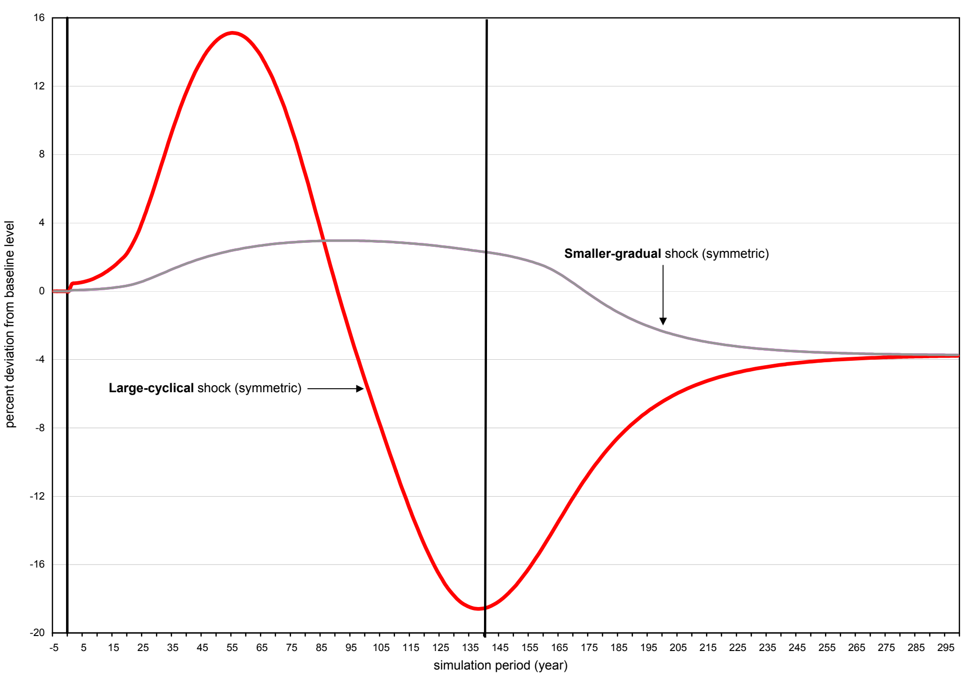



**Figure 9. Real Adult Consumptions per Adult**
**Symmetric (Closed-Economy) Shocks**

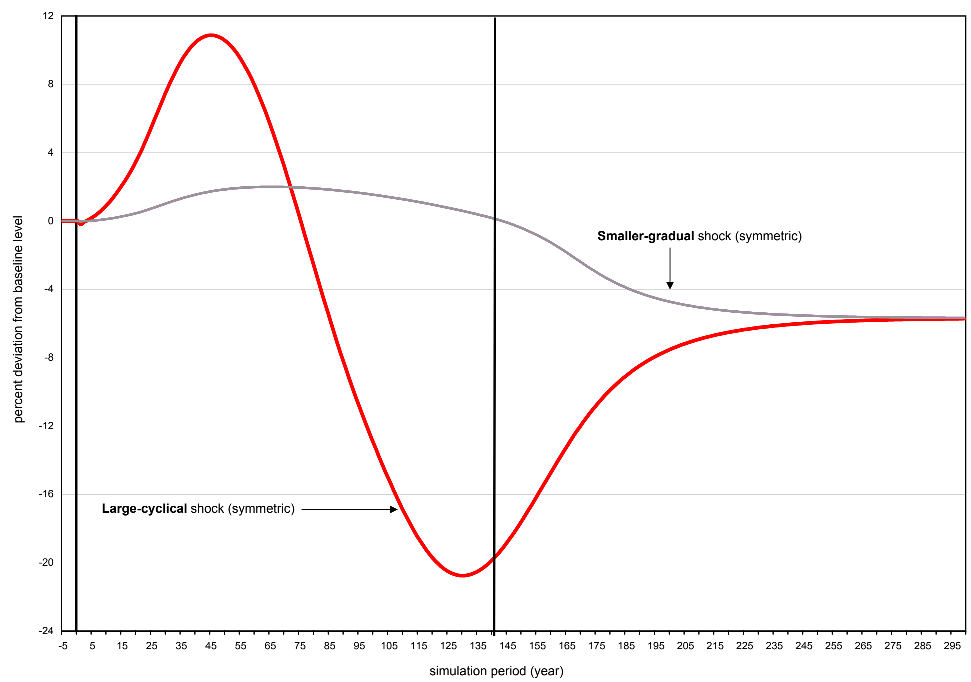


**Figure 10. Real Short-term Interest Rates**

**Asymmetric Shock with Cross-Border Spillovers Compared with Symmetric Closed-Economy Shock**

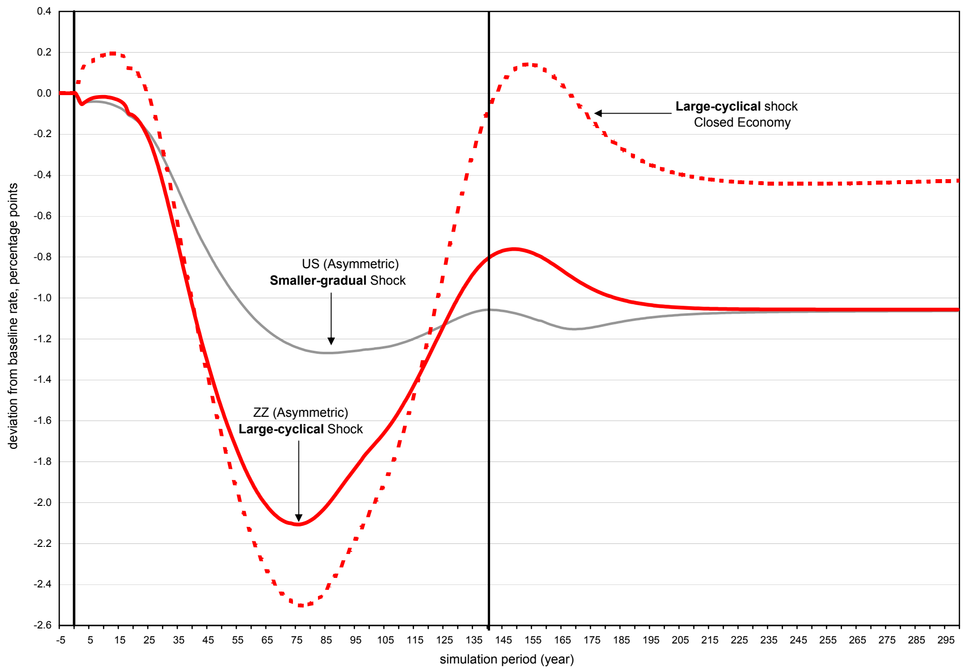



**Figure 11. Capital-Output Ratios**

**Asymmetric Shock with Cross-Border Spillovers Compared with Symmetric (Closed-Economy) Shock**

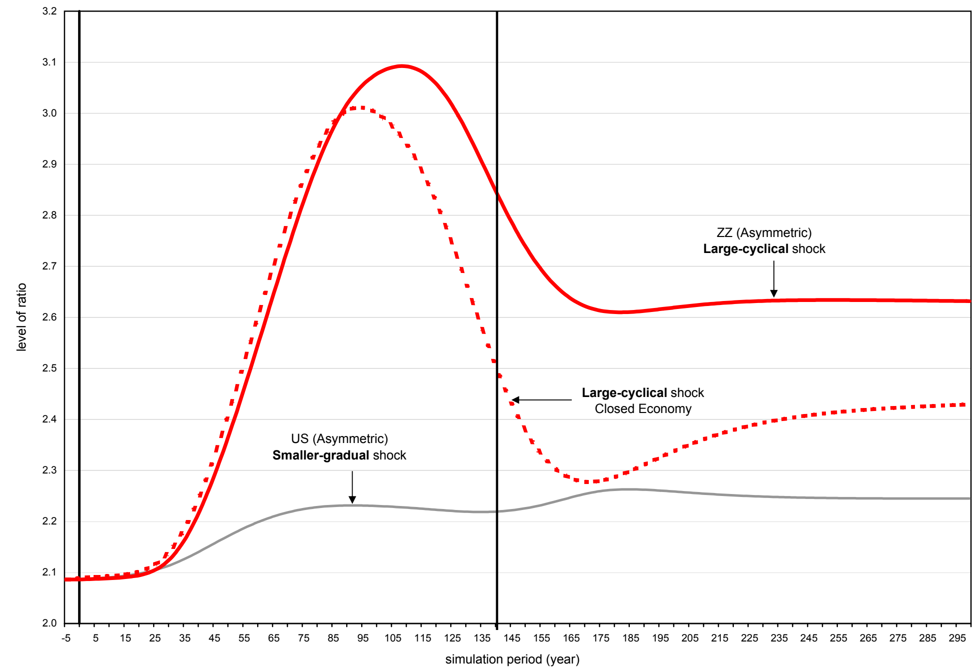



**Figure 12. Financial Wealths per Adult**

**Asymmetric Shock with Cross-Border Spillovers Compared with Symmetric (Closed-Economy) Shock**

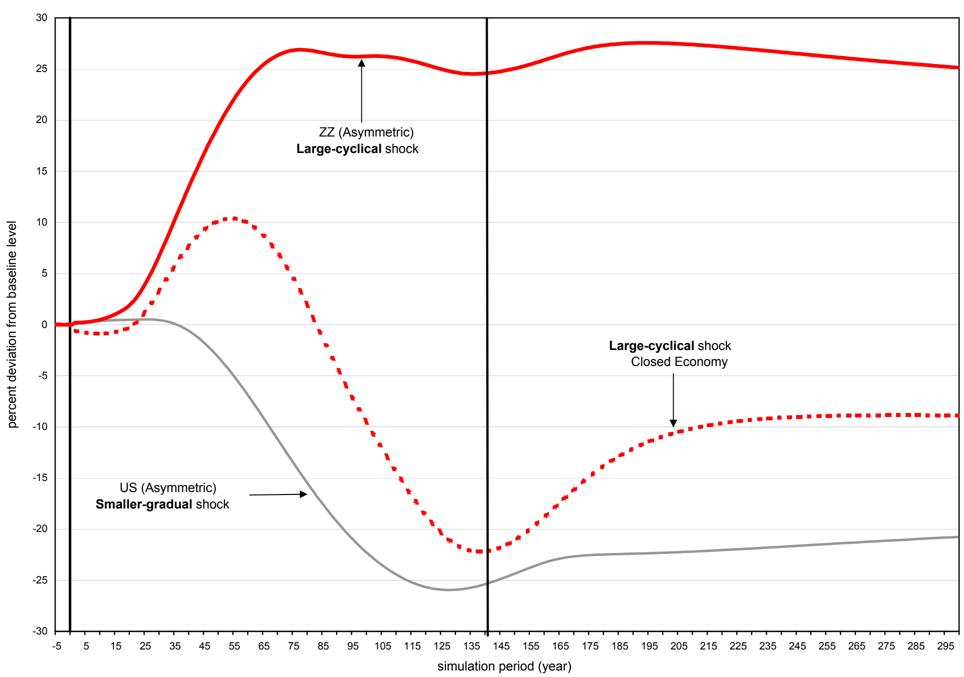


**Figure 13. Nominal and Real Exchange Rates**
**Asymmetric Shock with Cross-Border Spillovers Compared with Symmetric (Closed-Economy) Shock**

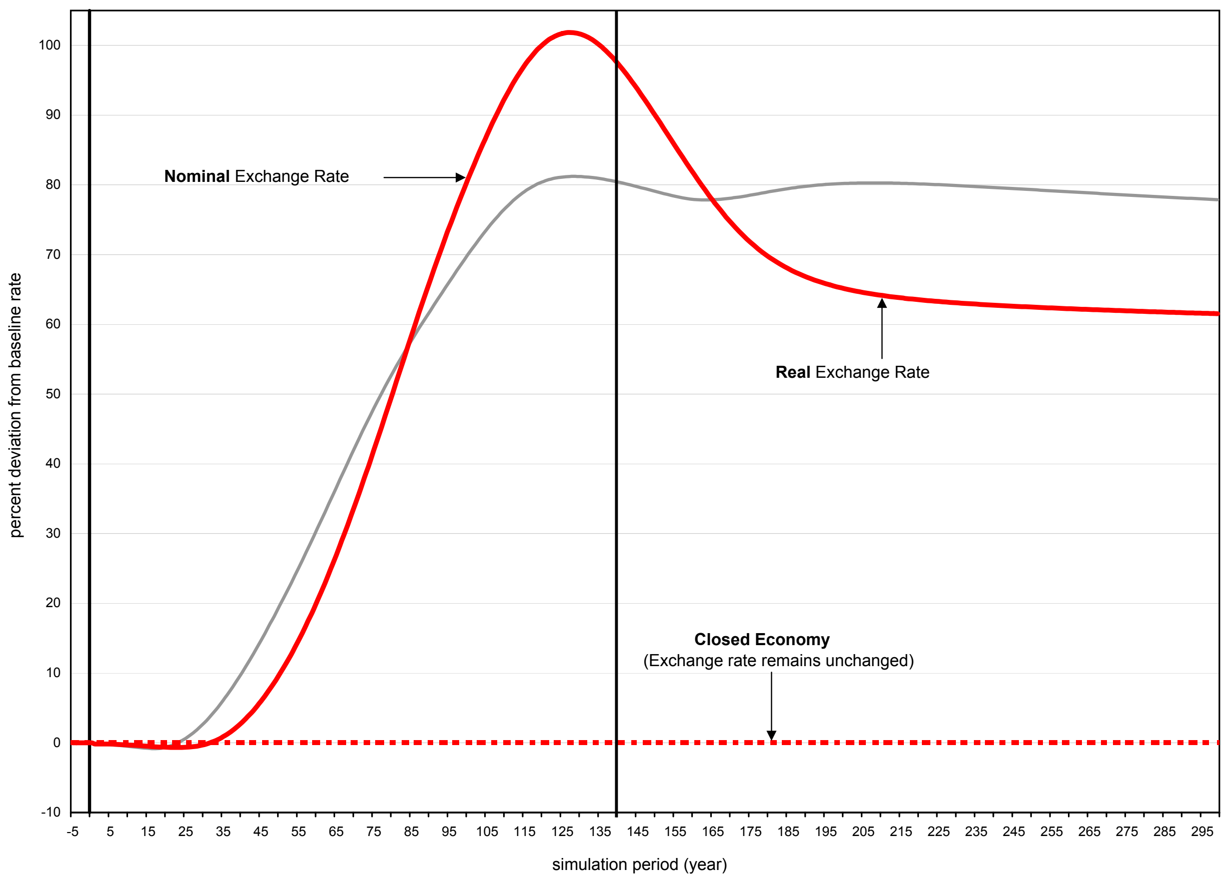



**Figure 14. ZZ Economy Relative to US Economy**
**Ratios of Population and Real GDP after Asymmetric Shock with Cross-Border Spillovers**

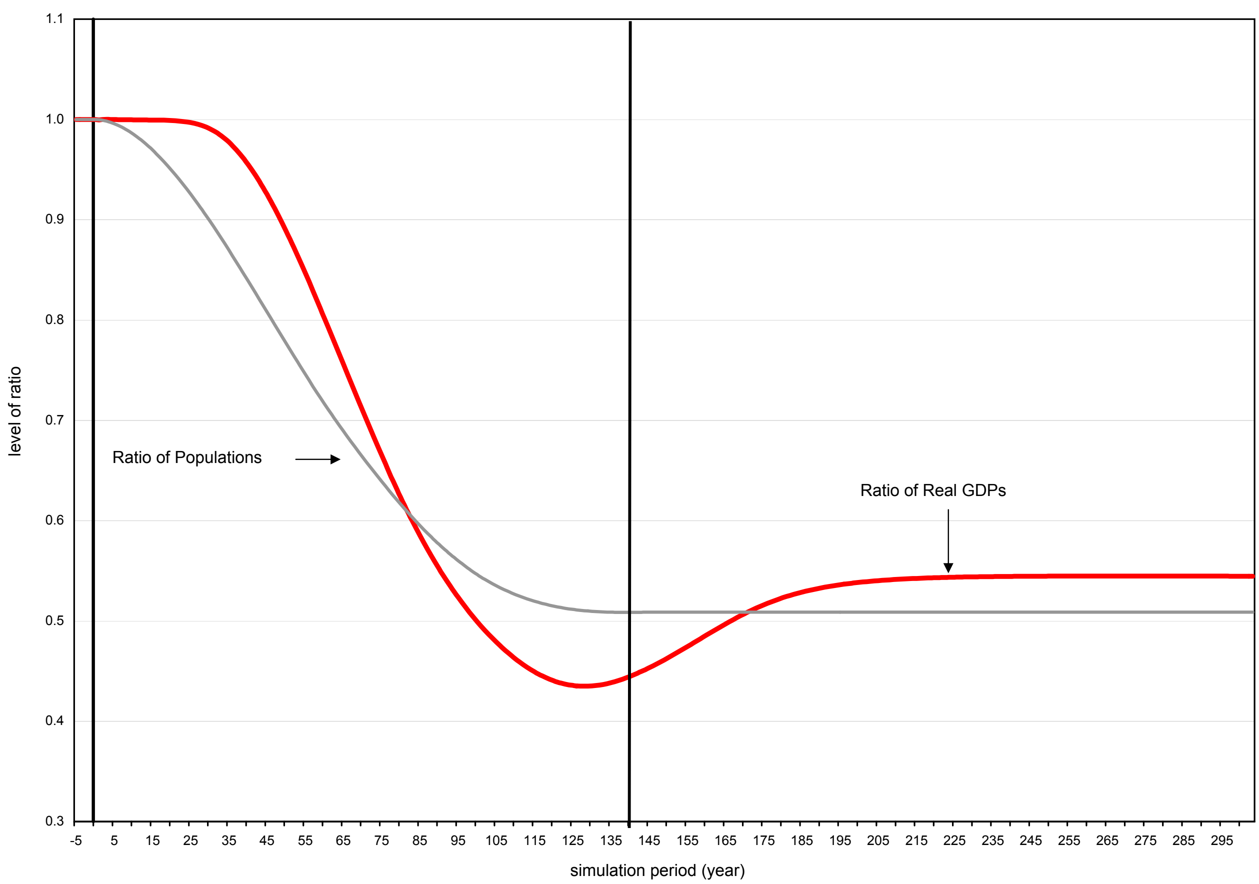


**Figure 15. Ratios of Real Imports to Real GDP**
**Asymmetric Shock with Cross-Border Spillovers Compared with Symmetric (Closed-Economy) Shock**

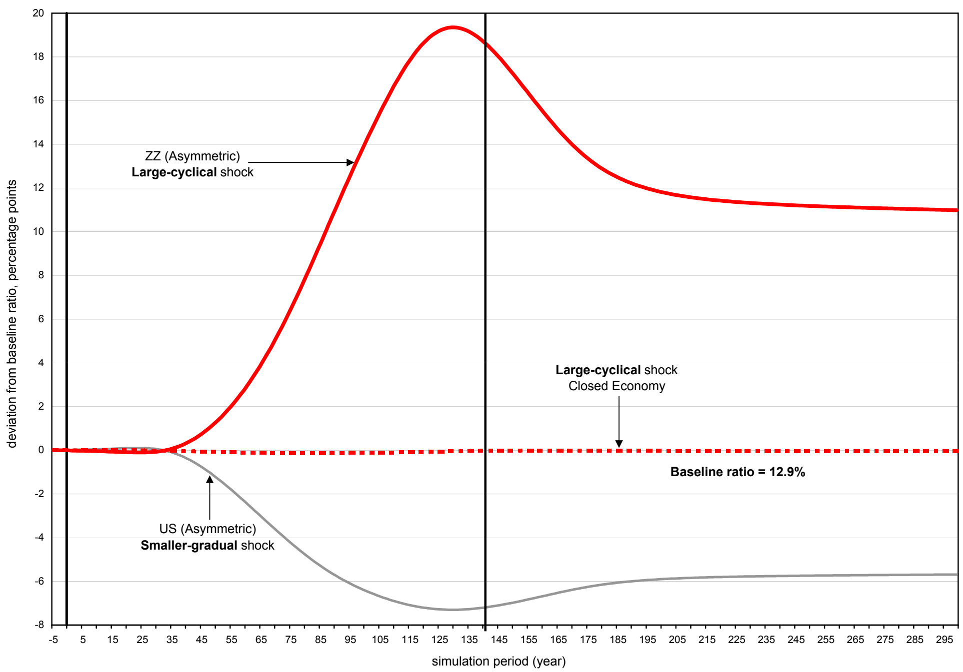



**Figure 16. Ratios of Real Trade Balance to Real GDP**
**Asymmetric Shock with Cross-Border Spillovers Compared with Symmetric (Closed-Economy) Shock**

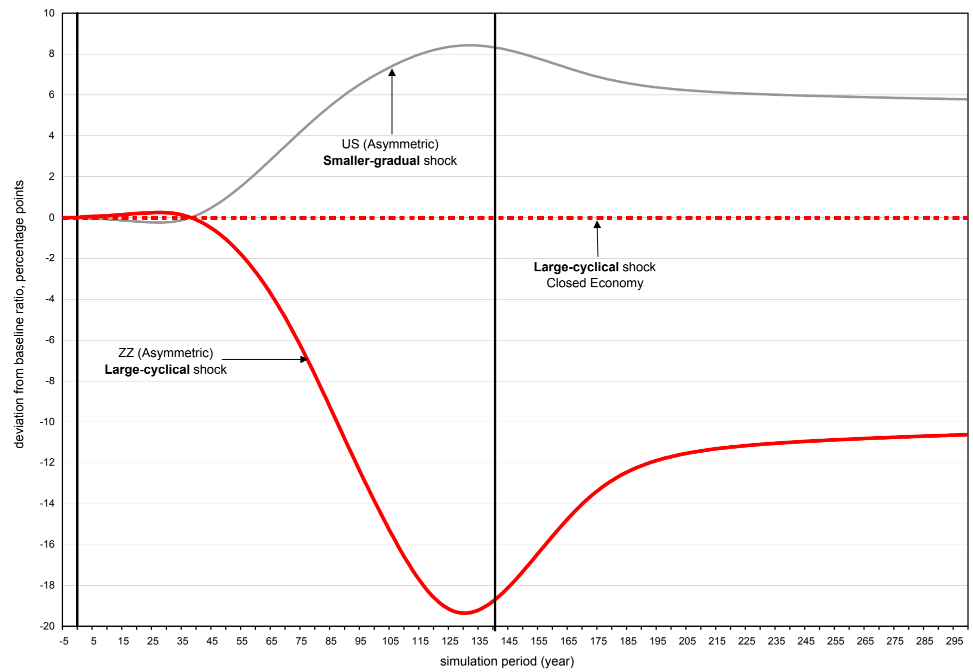



**Figure 17. Ratios of Current Account Balance to Nominal GDP**
**Asymmetric Shock with Cross-Border Spillovers Compared with Symmetric (Closed-Economy) Shock**

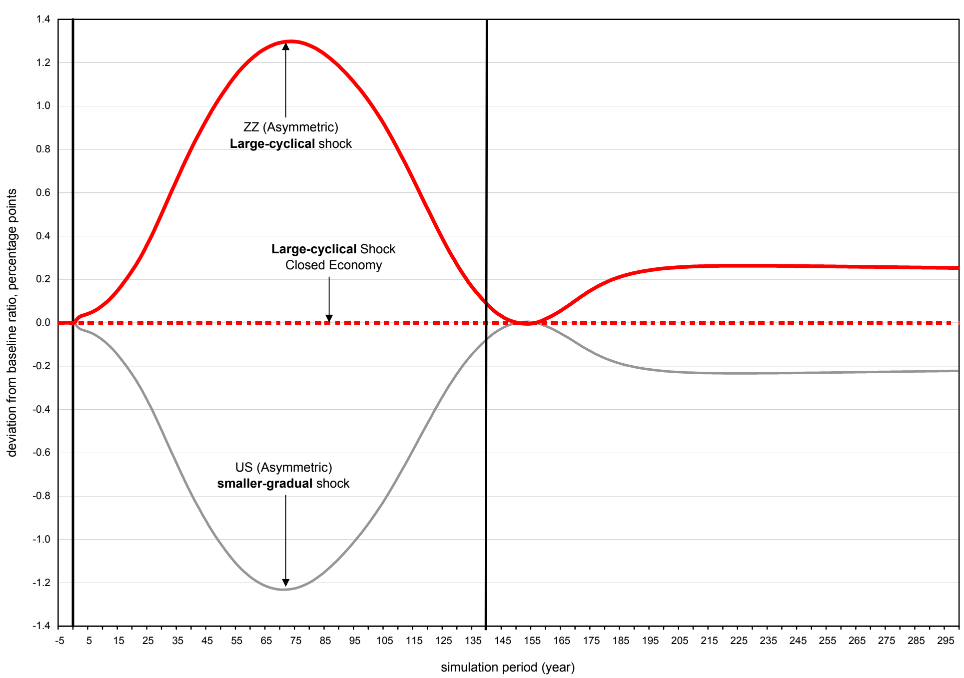



**Figure 18. Ratios of Net Foreign Assets to Nominal GDP**
**Asymmetric Shock with Cross-Border Spillovers Compared with Symmetric (Closed-Economy) Shock**

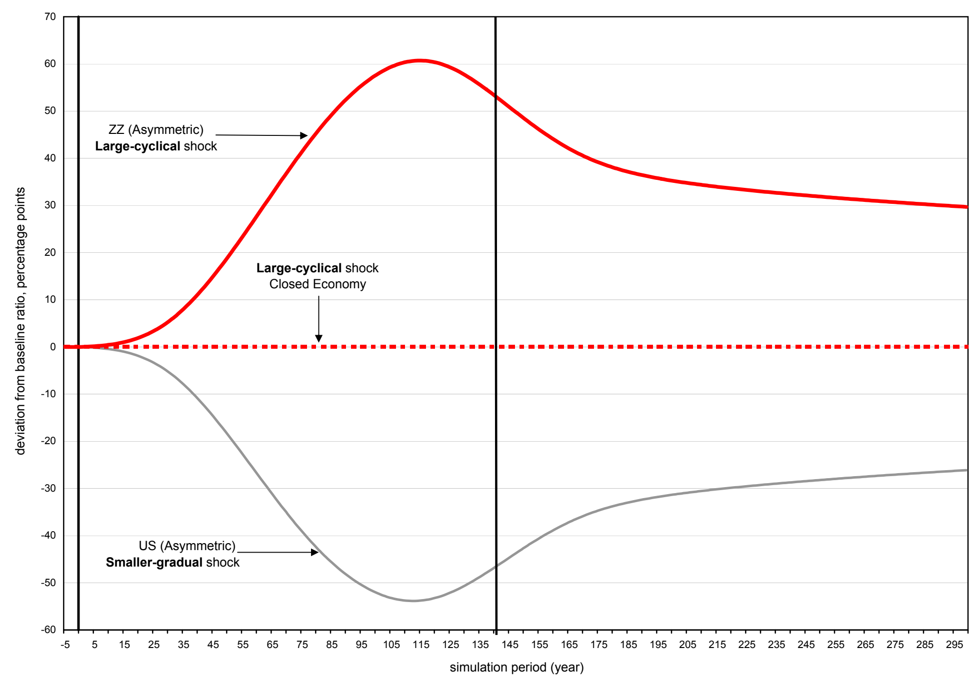


**Figure 19. Aggregate Real Adult Consumptions (Economy-Wide)**
**Asymmetric Shock with Cross-Border Spillovers Compared with Symmetric (Closed-Economy) Shock**

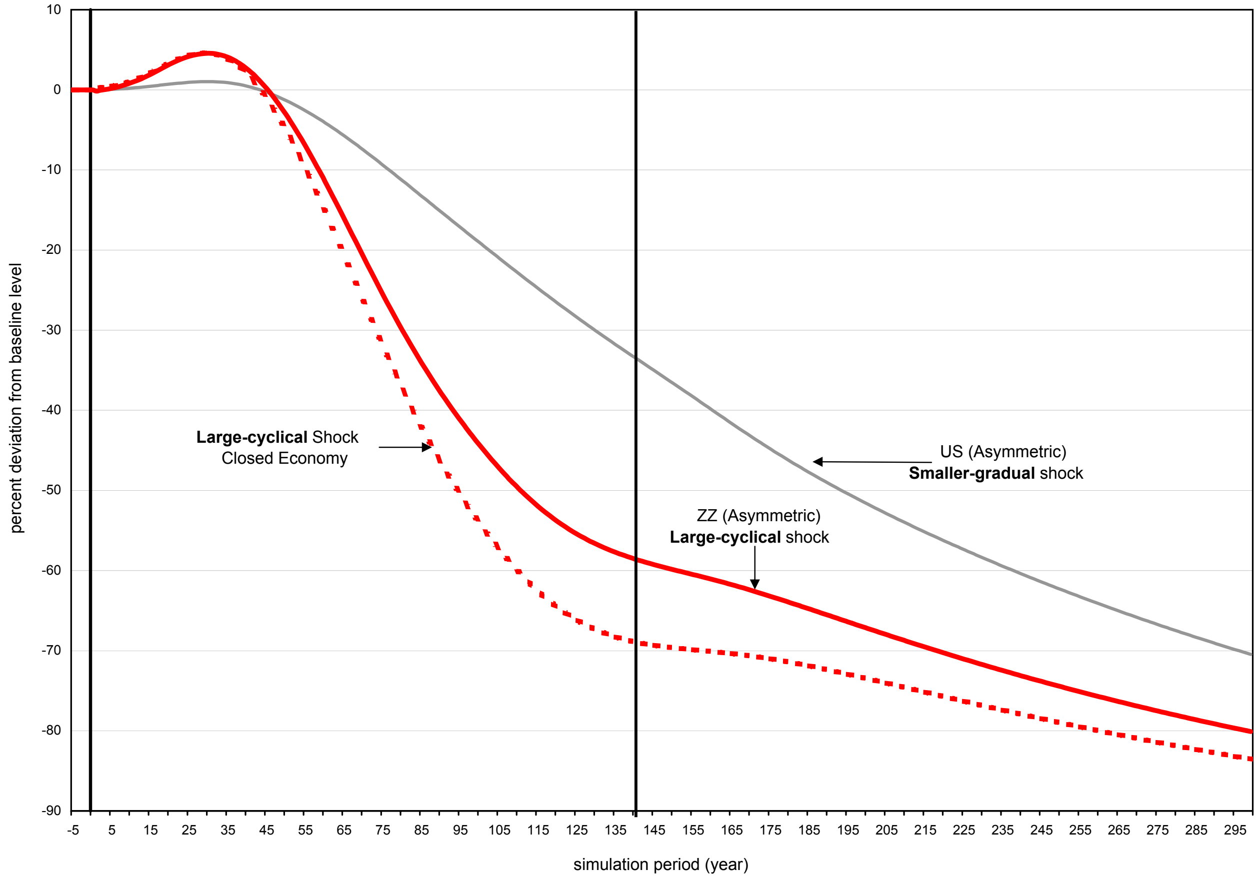



**Figure 20. Real Adult Consumptions per Adult**
**Asymmetric Shock with Cross-Border Spillovers Compared with Symmetric (Closed-Economy) Shock**

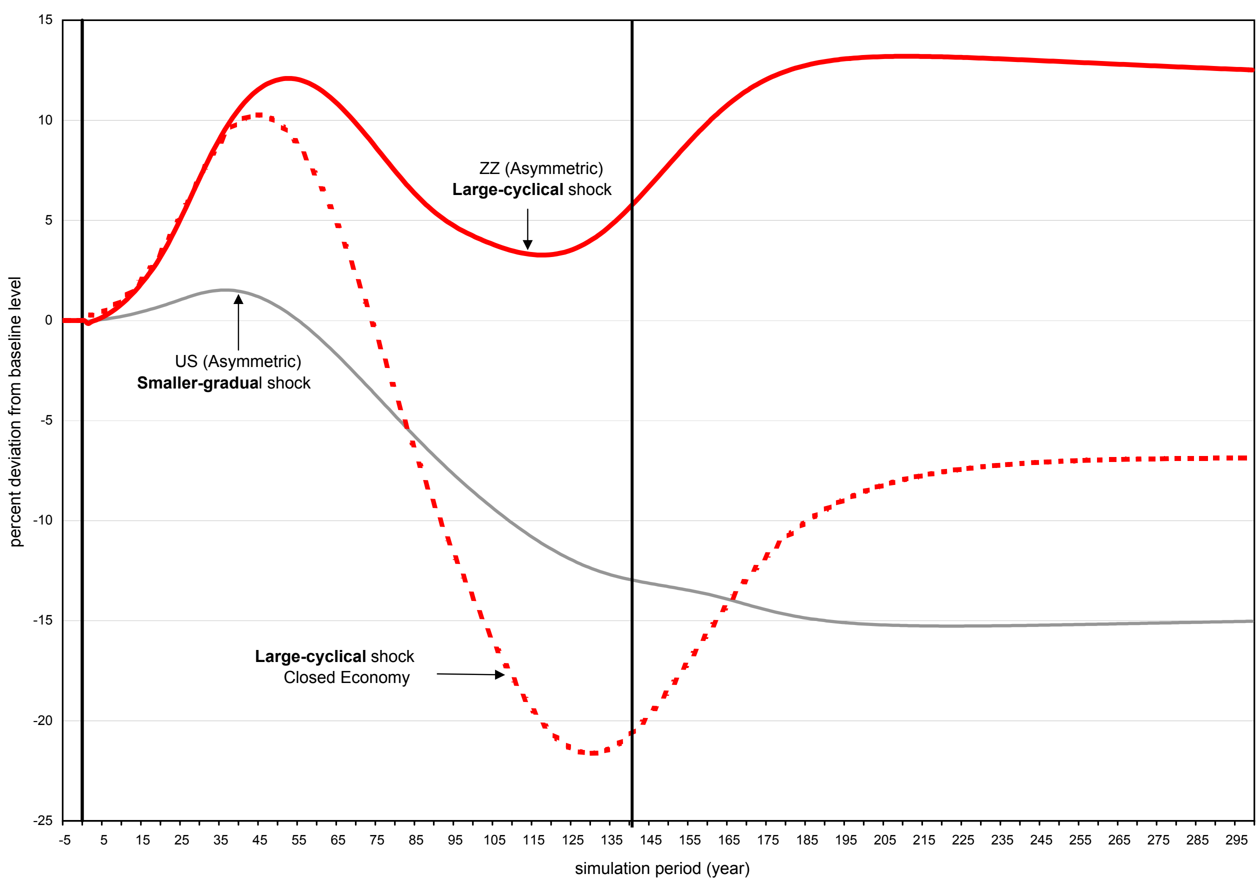


**Figure 21. ZZ Financial Wealths per Adult under Balanced Pension Systems**
**Asymmetric Shock with Cross-Border Spillovers Compared with Symmetric (Closed-Economy) Shock**

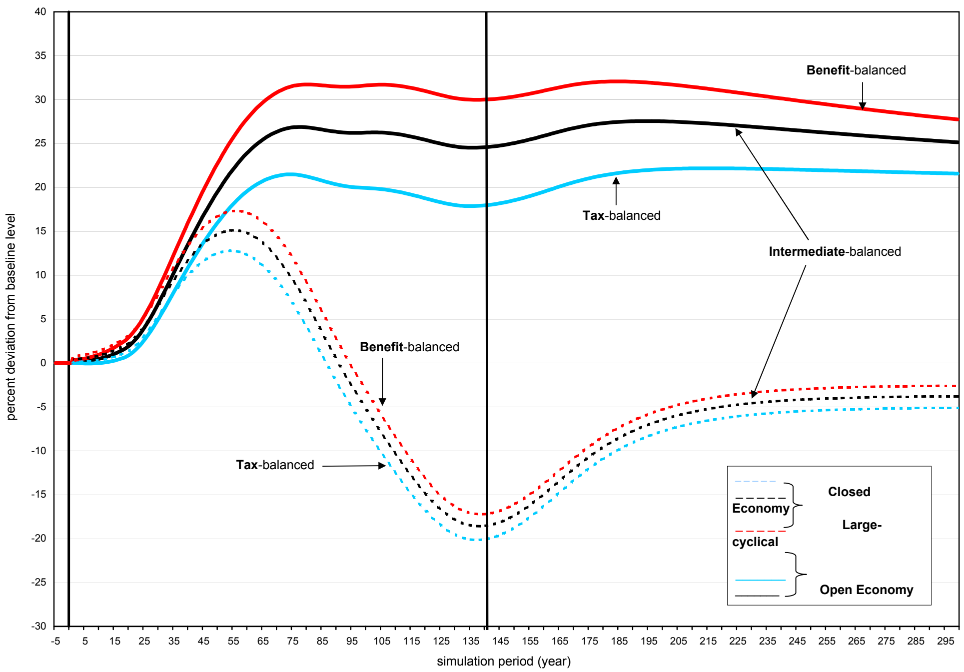



**Figure 22. ZZ Balanced Systems, Ratios of Total (Pension plus Income)Taxes to Nominal GDP**
**Asymmetric Shock with Cross-Border Spillovers Compared with Symmetric (Closed-Economy) Shock**

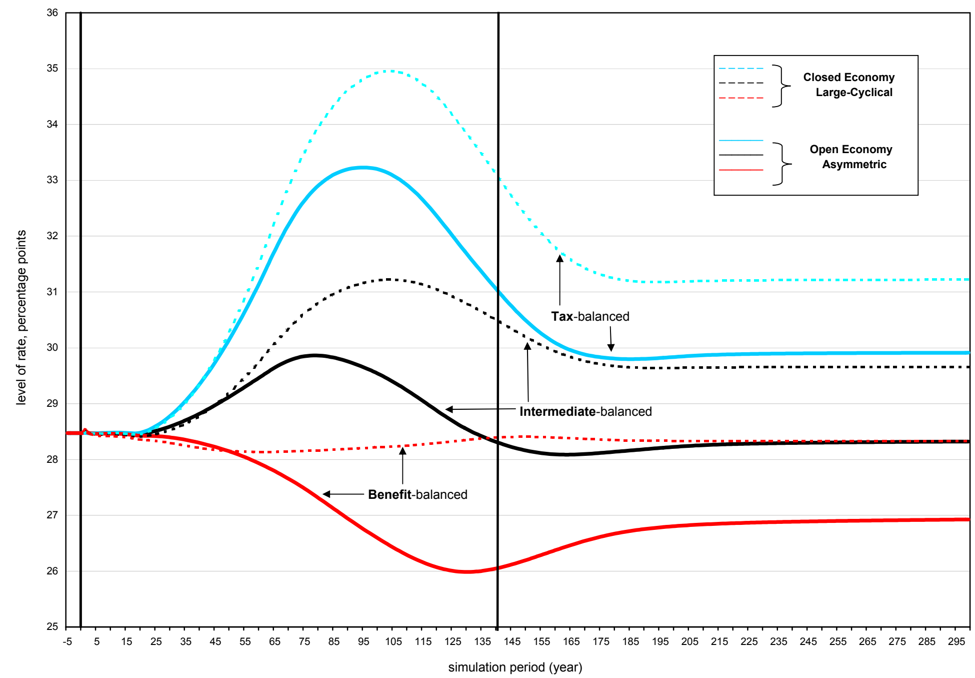



**Figure 23. ZZ Ratios of Pension Benefits per Elderly to Labor Income per Adult Worker**
**Asymmetric Shock with Cross-Border Spillovers Compared with Symmetric (Closed-Economy) Shock**

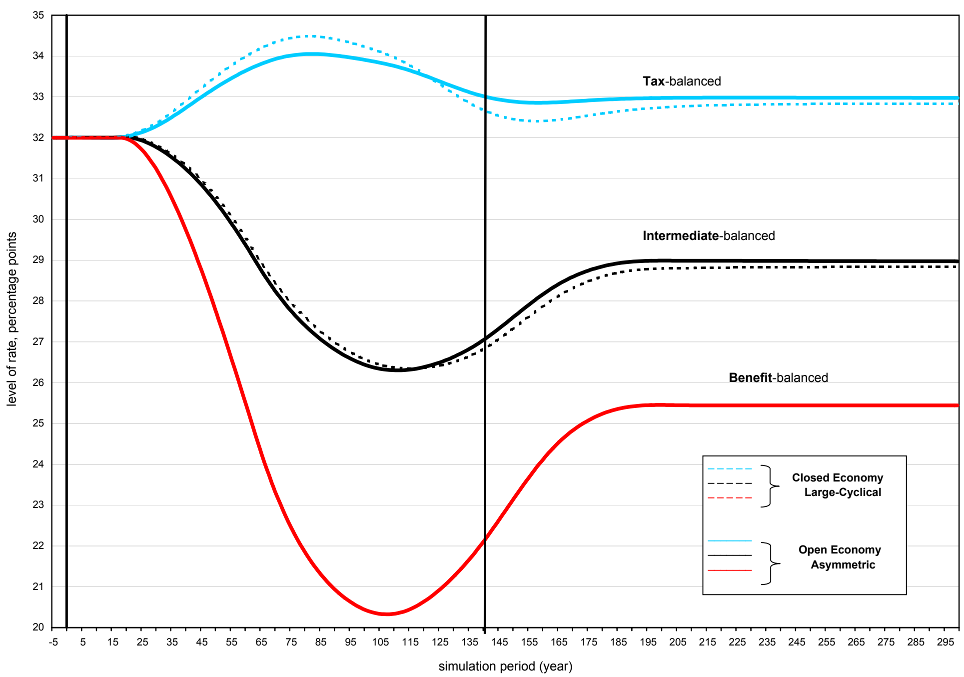


**Figure 24. ZZ: Ratios of Net Foreign Assets to Nominal GDP, Balanced Pension Systems**
**Asymmetric Shock with Cross-Border Spillovers**

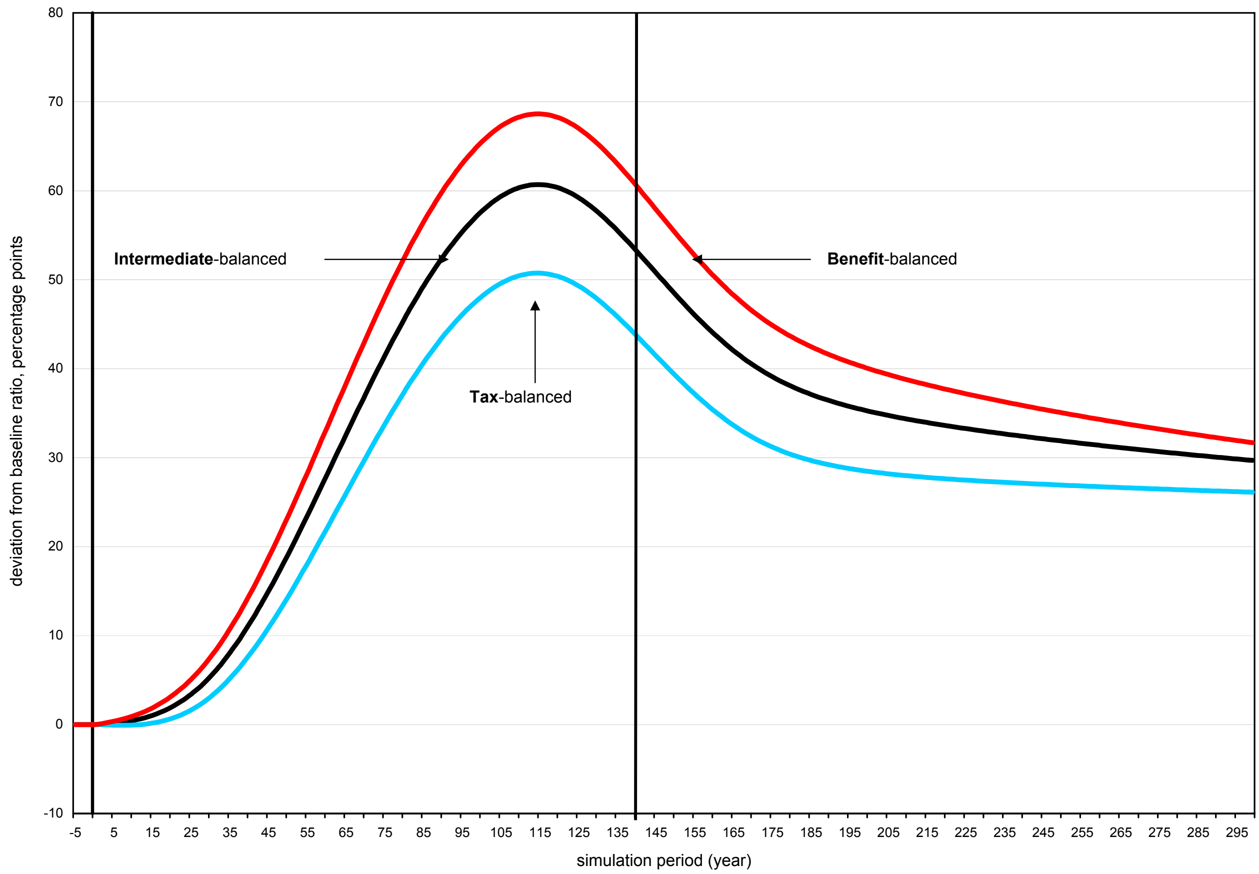



**Figure 25. ZZ: Real Adult Consumptions per Adult, Balanced Pension Systems**
**Asymmetric Shock with Cross-Border Spillovers**

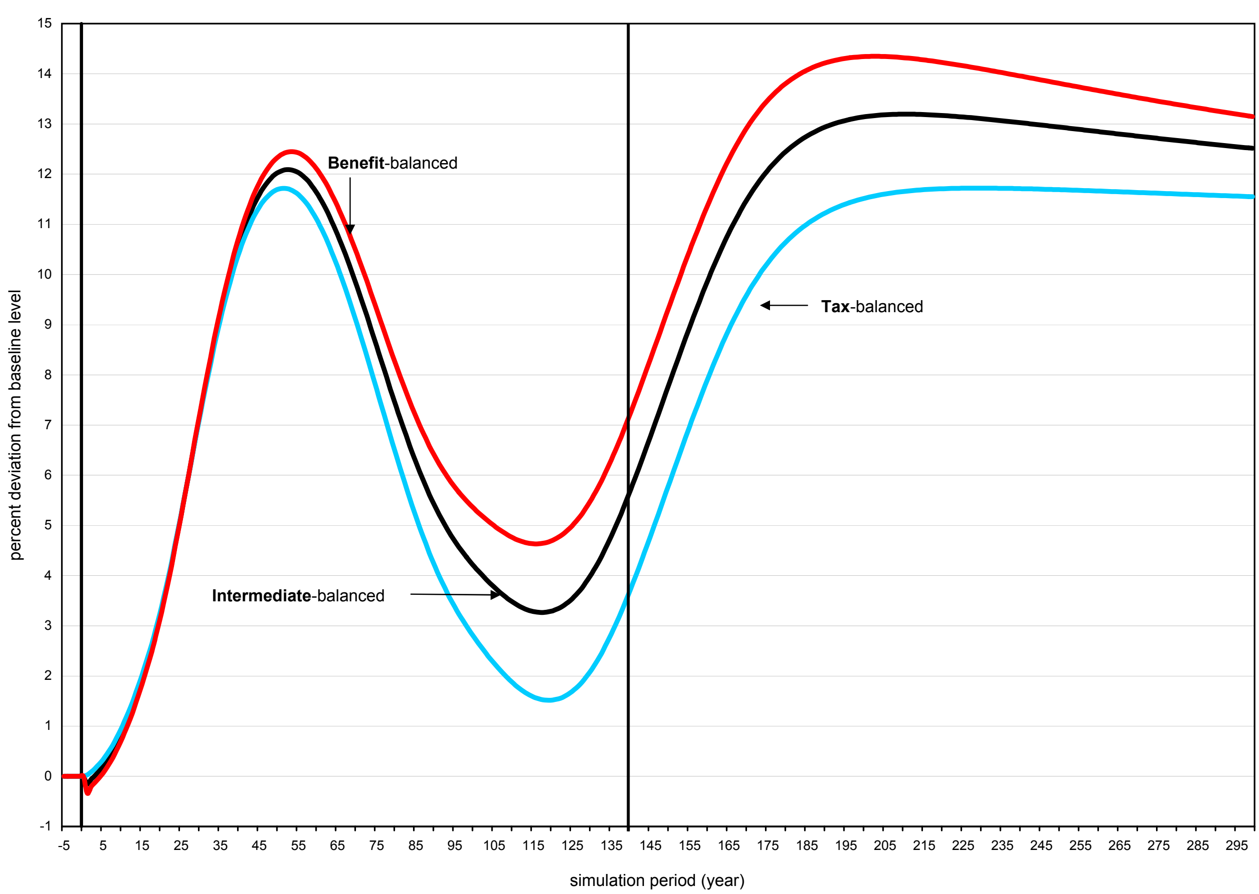


**Figure 26. ZZ: Pension Tax Rate under Balanced and Unbalanced Pension Systems**
**Asymmetric Shock with Cross-Border Spillovers**

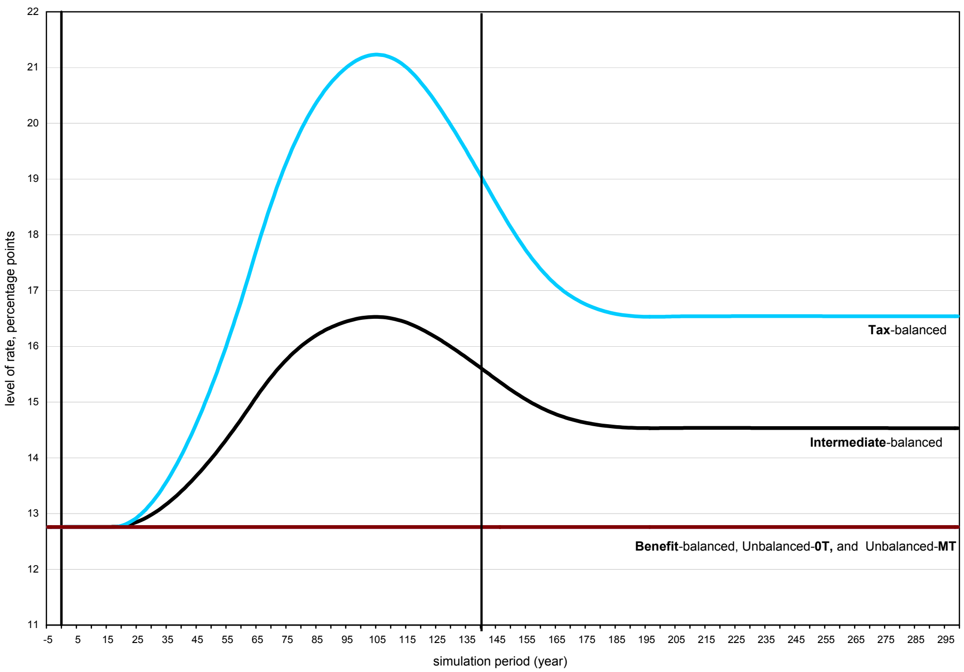



**Figure 27. ZZ: Income Tax Rate under Balanced and Unbalanced Pension Systems**
**Asymmetric Shock with Cross-Border Spillovers**

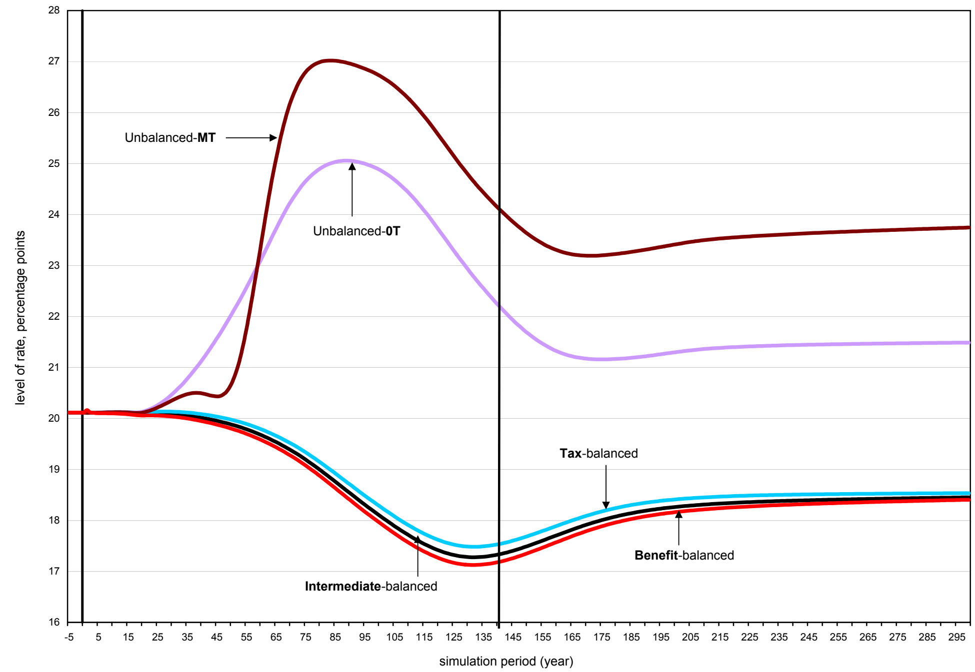



**Figure 28. ZZ: Ratios of Total Taxes to Nominal GDP under Balanced and Unbalanced Systems**
**Asymmetric Shock with Cross-Border Spillovers**

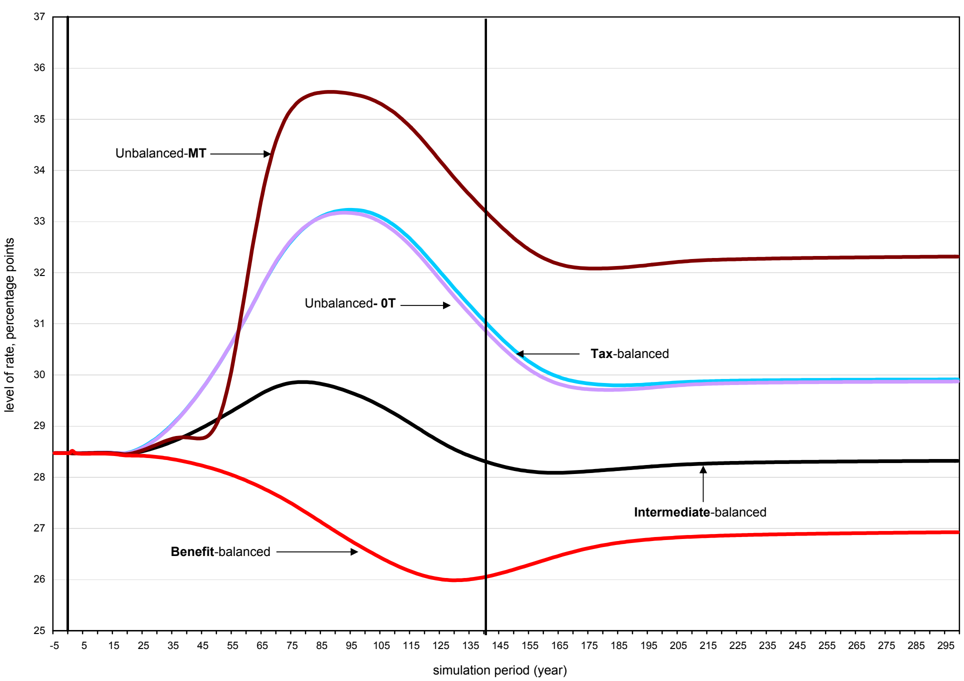


**Figure 29. ZZ: Private Saving Ratio under Balanced and Unbalanced Pension Systems**
**Asymmetric Shock with Cross-Border Spillovers**

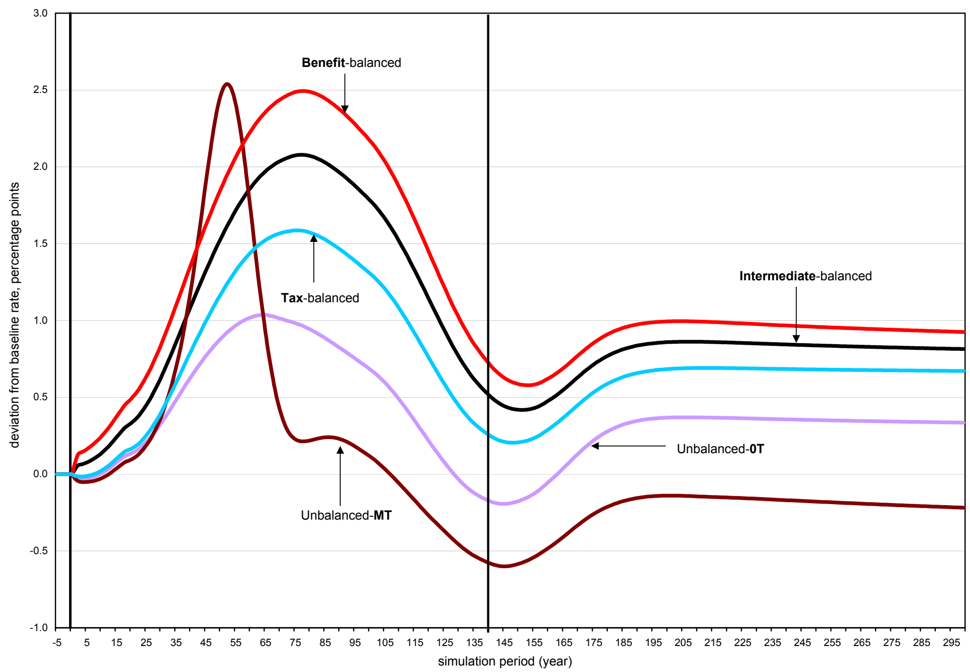



**Figure 30. ZZ: Financial Wealth per Adult under Balanced and Unbalanced Pension Systems**
**Asymmetric Shock with Cross-Border Spillovers**

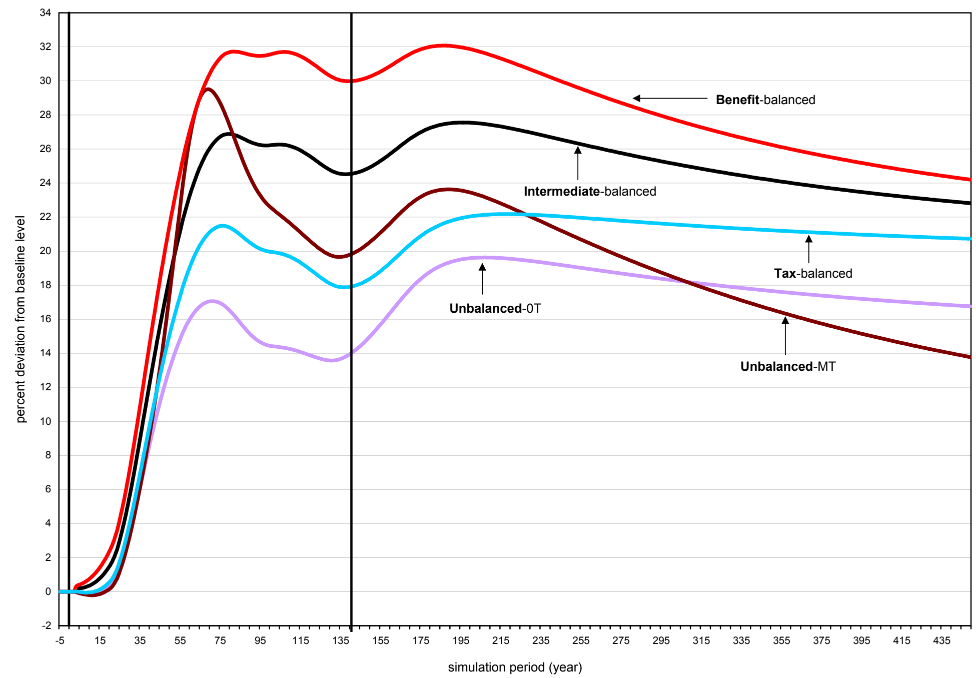



**Figure 31. ZZ: Capital-Output Ratios under Balanced and Unbalanced Pension Systems**
**Asymmetric Shock with Cross-Border Spillovers**

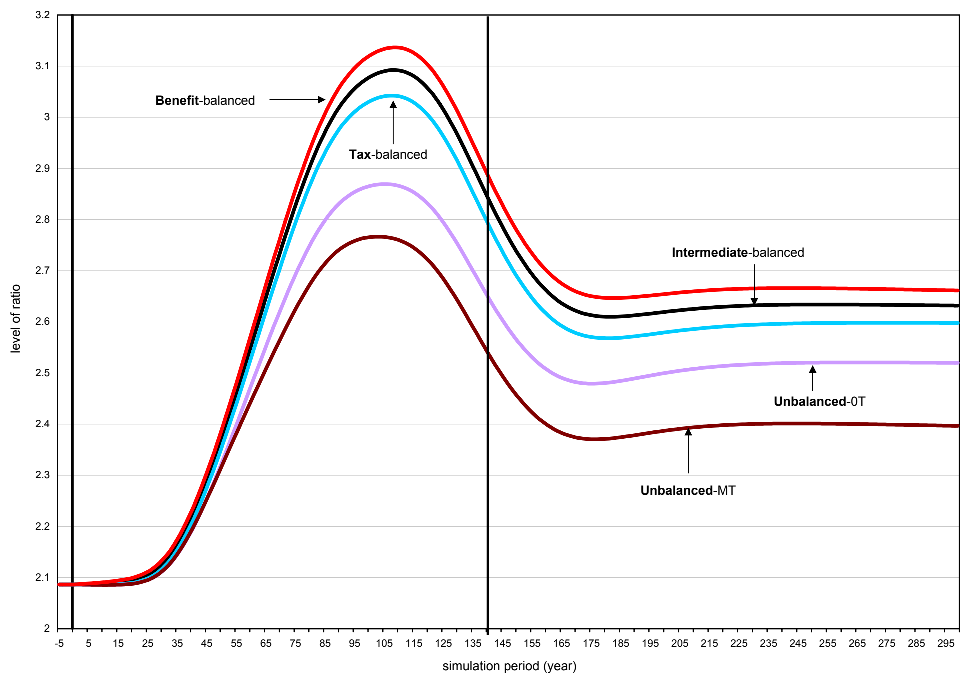


**Figure 32. Nominal and Real Exchange Rates, Balanced and Unbalanced Pension Systems**
**Asymmetric Shock with Cross-Border Spillovers**

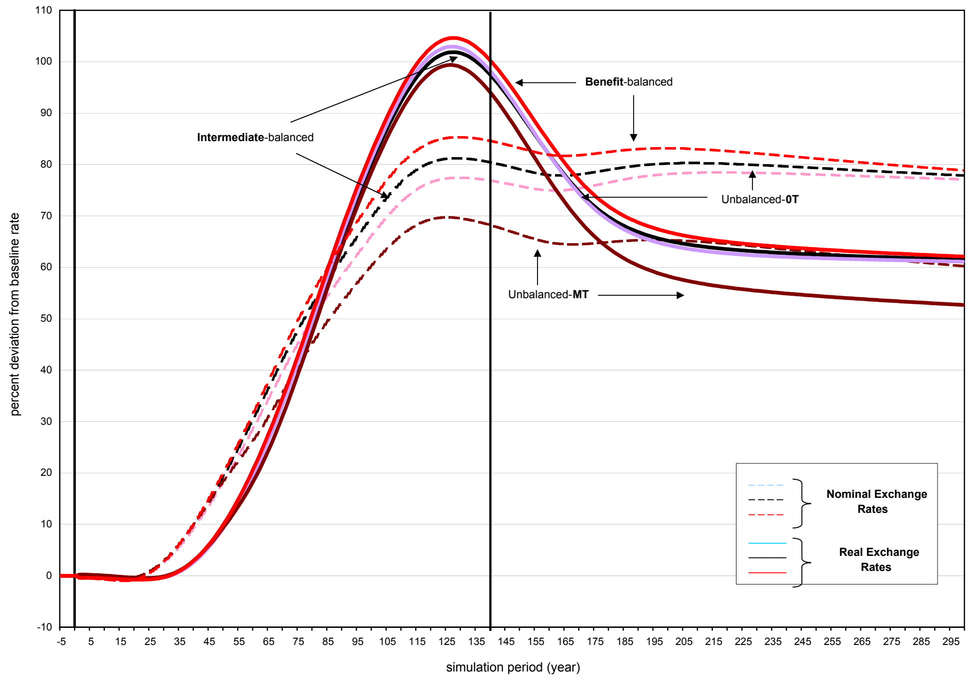



**Figure 33. ZZ: Ratios of Nominal Trade Balance to Nominal GDP, Balanced and Unbalanced**
**Asymmetric Shock with Cross-Border Spillovers**

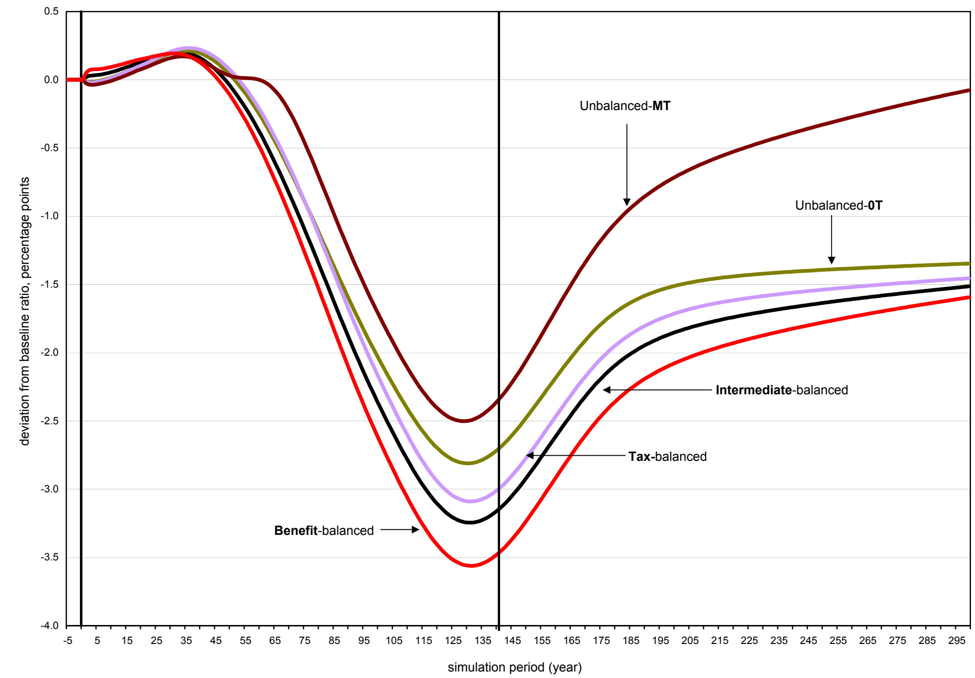



**Figure 34. ZZ: Ratios of Current Account to Nominal GDP, Balanced and Unbalanced Systems**
**Asymmetric Shock with Cross-Border Spillovers**

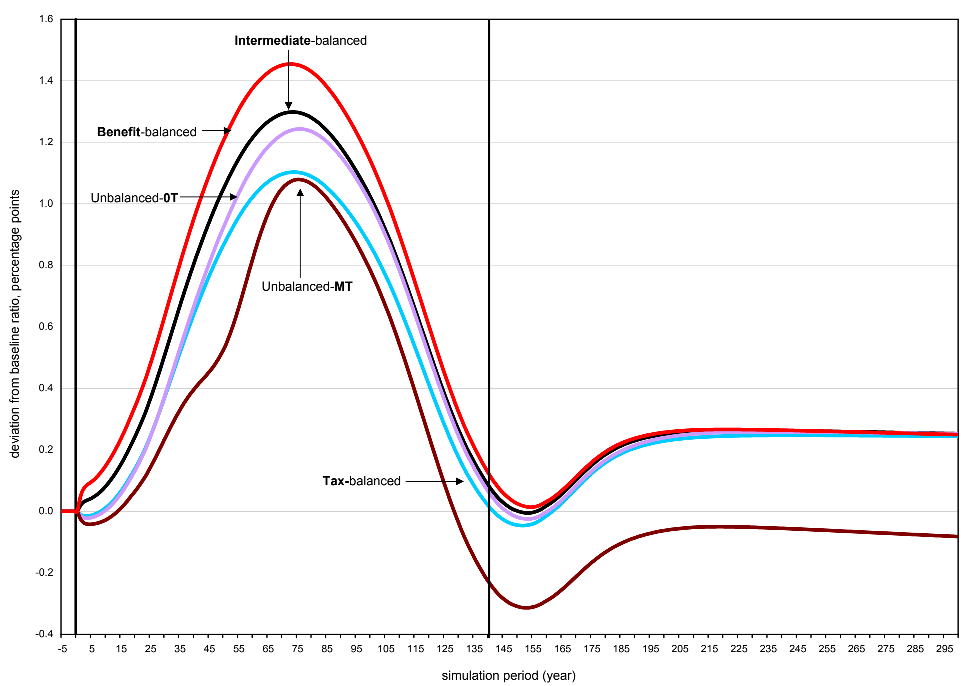



**Figure 35. ZZ: Ratios of Net Foreign Assets to Nominal GDP, Balanced and Unbalanced Systems**
**Asymmetric Shock with Cross-Border Spillovers**

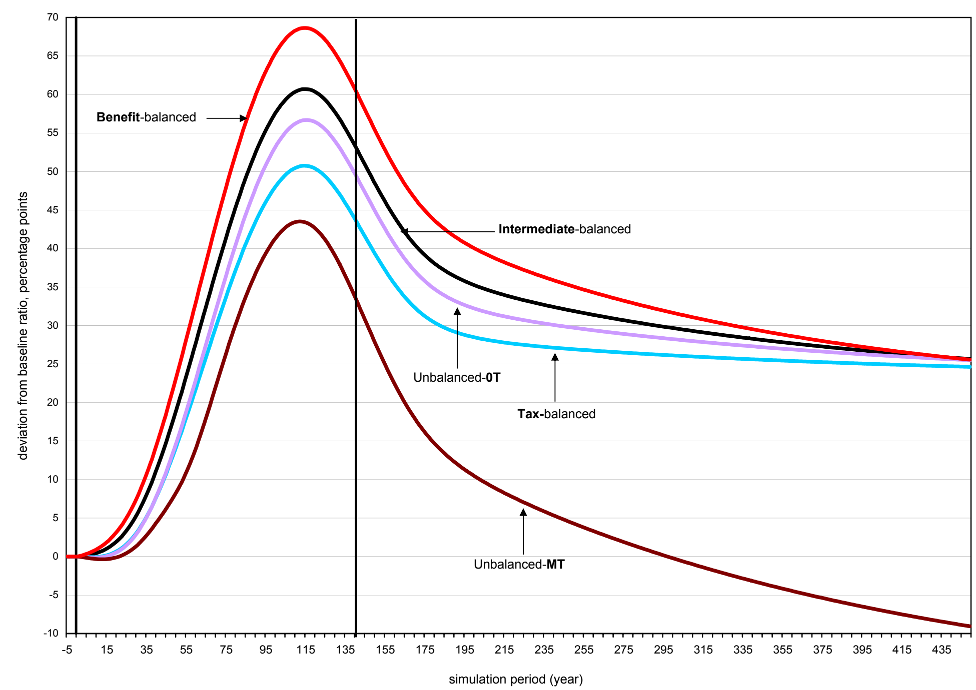


**Figure 36. ZZ: Adult Consumption per Adult, Balanced and Unbalanced Pension Systems**
**Asymmetric Shock with Cross-Border Spillovers**

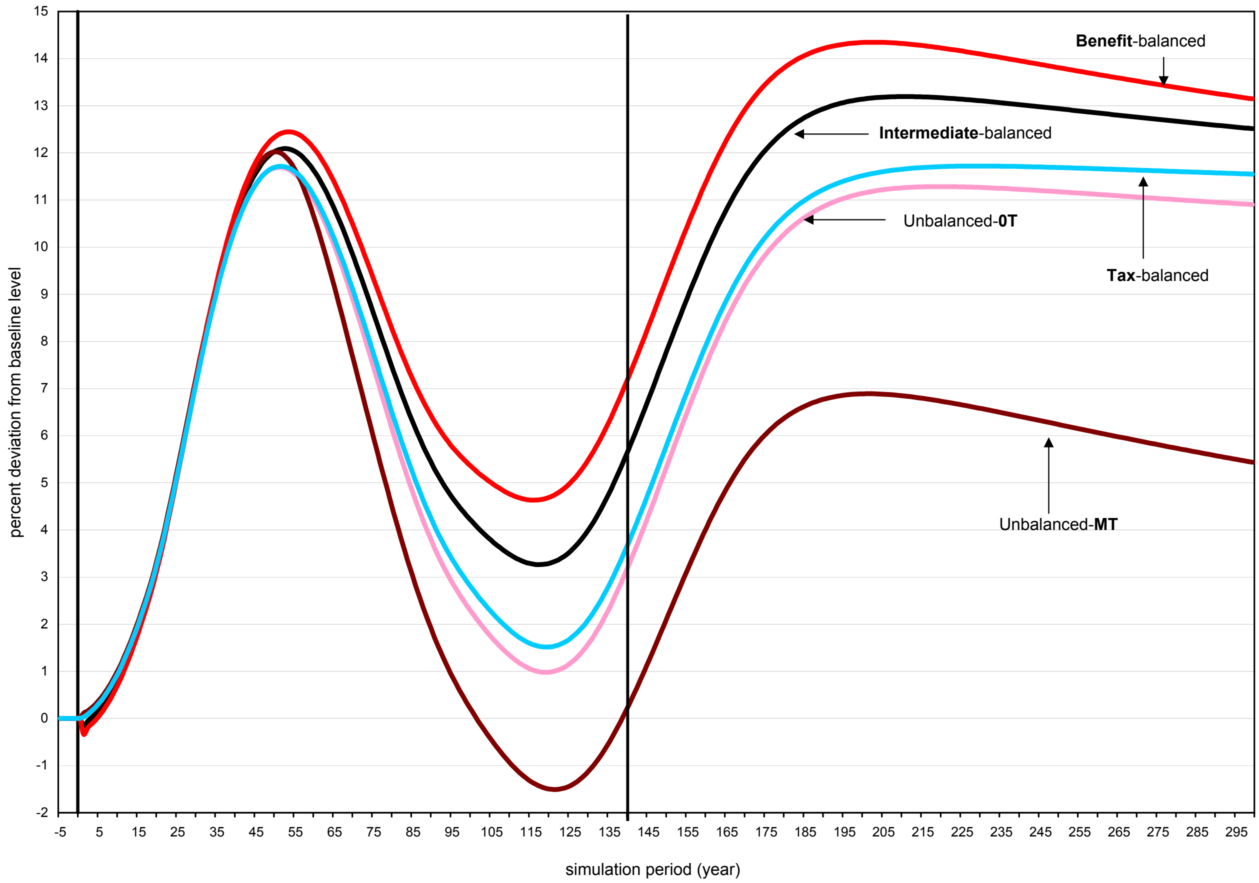



**Figure 37. ZZ: Adult Consumption per Adult Balanced and Unbalanced Pension Systems**
**Initial Decades - Asymmetric Shock with Cross-Border Spillovers**