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Ralph C. Bryant, Hamid Faruqee, Delia Velculescu, and Elif Arbatli

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Ralph C. Bryant is a Senior Fellow and the Edward M. Bernstein Scholar in the Economic Studies Program of the Brookings Institution. Hamid Faruqee is a Senior Economist in the International Monetary Fund's European Department. Delia Velculescu is a Senior Economist at the International Monetary fund, and Alif Arbatli is a Graduate Assistant in the Department of Economics at the Johns Hopkins University.

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Ralph C. Bryant, Hamid Faruqee, Delia Velculescu, and Elif Arbatli

Brookings Institution, International Monetary Fund, and Johns Hopkins University

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Structural Outline

1. Introduction

Mortality rates for infants and children fell sharply for much of the 20th century, while life expectancy increased for adults. Somewhat later fertility rates began to decline, first in developed nations and more recently in developing countries. These major demographic shifts have already had significant effects on the age structure of the populations of most of the industrialized world, reducing the number of youths relative to adults. The transition will continue into the 21st century. The populations of most developed nations will have much higher elderly dependency ratios in the coming decades. After varying delays, further declines in fertility and a progressive aging of populations will occur in developing nations as well.

Japan is the most prominent example of an industrial nation whose fertility rates have fallen and whose population is aging. Between the end of World War II and the end of the 20th century, Japanese fertility declined from over 4 to some 1.4 lifetime births per woman. The share of youths in the total population fell from over 45 percent to less than 21 percent between 1950 and 2000. Projections for the share of elderly in the total population show a near doubling over the next five decades from 17.2 percent to 36.9 percent. The dramatic nature of Japan's demographic shift relative to developed countries as a whole can be seen in visual terms in Figure 1, based on the revised UN projections released in 2001.

Changes in youth dependency are even more important for developing nations. For several decades after 1950, youth dependency in many developing nations actually *increased* substantially. Projections for the 21st century tend, however, to show marked falls in fertility, declining youth dependency, and eventual population aging – though lagging behind by several decades the experiences of developed nations.[1]

Substantial research in both theoretical and empirical macroeconomics has been devoted to analyzing the implications for national economies of these profound demographic changes. The focus of much of this work has been on population aging and its effects on saving, investment and growth.[2] Unfortunately, much of the analysis of the relationship between population and economic growth initially concentrated just on the size and growth rate of the total population, paying little attention to

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[1] Lee (2003) provides an overview of the demographic transition for the world as a whole. See also United Nations, Population Division (2001) and Birdsall, Kelley, and Sinding (2001).

[2] See, for example, Cutler, Poterba, Sheiner, and Summers (1990); Bosworth, Burtless and Sabelhaus (1991); Auerbach, Cai, and Kotlikoff (1991); Deaton and Paxson (1997); Velculescu (2003a, 2003b, 2004); and Brooks (2000). Bosworth-Bryant-Burtless (2004) provide a survey.

shifts in the age structure of the population. From a policy perspective, research has largely focused on the increasing burdens that rising elderly dependency ratios will place on national budgets and pension systems and on the menu of possible options for reform.[3]

The least studied aspects of the demographic shifts are the cross-border interactions: how demographic change in individual open economies influences macroeconomic developments abroad and the global balance of saving and investment flows. In our own work we have been especially concerned with the spill-over effects of demographic changes in one country on foreign economies through changes in exchange rates and other external-sector variables and with the consequences for saving and investment flows in the world economy as a whole.[4]

As our research progressed, we became increasingly dissatisfied with the treatment of demographic variables in macroeconomic models. Rather than treating youth dependency, elderly dependency, and the sizes of the population and labor force as exogenous inputs to macroeconomic analysis, we sought increasingly to incorporate more of the demographics as integral, endogenous parts of the modeling framework.[5] By paying attention to the *entire* age distribution of the population and its endogenous evolution through time, we became able to analyze a broad range of demographic issues and their interaction with macroeconomic developments.

To integrate the effects of youth dependency, we explicitly incorporated children and child support into our overlapping generations framework. That enabled us to focus attention on the proximate effects of falling fertility as they caused declines in youth dependency ratios. Our approach assumes that individuals are classified as youths for the first 18 years of life and do not participate in economic activity except as consumers of resources that are transferred to them by the adult population. Child consumption does not enter the adult utility function; we abstract from such intergenerational links through utility to simplify the analysis. Essentially, child consumption in our model is financed as if a tax were imposed on adults for child support. We explicitly model how the burden of adult support is divided between younger and older adults, and allow for degrees of

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3 Well-known contributions include Aaron, Bosworth, and Burtless (1989); World Bank (1994); Wise (1994); Disney (1996); Bosworth and Burtless (1998); Kohl and O'Brien (1998); and OECD (1998).

4 Our own early work includes Bryant and McKibbin (2004), a paper that focuses on transitory "baby-boom" shocks analogous to those experienced by many industrial nations in the sixth and seventh decades of the 20th century. Faruqee and Mühleisen (2003) provide a similar analysis of macroeconomic effects in Japan. Bryant (2004a) analyzes elderly dependency and policy issues facing public pension systems. Other exceptions to the generalization that the cross-border effects of demographic changes have been ignored include Turner, Giorno, de Serres, Vourc'h, and Richardson (1998), Attanasio and Violante (2000), Brooks (2003), Borsch-Supan, Ludwig, and Winter (2003), and Fehr, Jokisch, and Kotlikoff (2003).

5 Fertility (birth) rates and mortality rates are the only demographic variables we model as exogenous.

variability of child support relative to adult consumption. The simplified approach we adopt to incorporate children into our theoretical framework seems a reasonable first attempt to account for the economic impacts of youth dependency.

Allowing for youth dependency turns out to have profound implications for key macroeconomic variables. Why? The key point is that the consumption-saving behavior of individual adults who provide in-vivo transfers to children is dramatically different, in theory and in practice, from the behavior of otherwise identical individuals without financial responsibilities for child support. If a demographic shock occurs lowering fertility rates and the numbers of children, the financial burden on adults is reduced and resources are freed for additional adult consumption or saving. That reallocation of resources radically changes the transitional dynamics and the ultimate steady state of the economy compared to what it would otherwise be in an analysis that ignores children. Ignoring youth dependency suppresses this major source of macroeconomic effects.

Evidence exists in the literature suggesting that youth dependency matters for the determination of national saving, investment, and foreign capital accumulation. Mason (1987, 1988) finds that changes in the growth rate of the population and in the youth dependency ratio can have opposite effects on aggregate saving. Bloom and Williamson (1997), Higgins and Williamson (1997), and Williamson (1998) investigate the implications of youth dependency for growth in East Asia. They argue that youth dependency has a significant role in savings, investment and foreign capital dependence – an idea discussed earlier by Coale and Hoover (1958).[6] Numerous empirical studies, some going back to the 1980s, have identified a negative macroeconomic link between dependency ratios and saving rates.[7] Bloom, Canning, and Sevilla (2001) and Bosworth, Bryant, and Burtless (2004) survey some of the issues.

Our research differs from previous work in two respects. We model the age structure of populations, demographic shocks, and the consequences for macroeconomic variables in a genuinely multi-nation and general-equilibrium framework. And our framework allows us to nest the presence or

6 See also Higgins (1998). The conclusion that high growth rates in East Asia before 1990 can be explained primarily by dependency-ratio effects has been questioned. But the view that changes in youth dependency can have significant macroeconomic effects is well supported.

7 Table 2 in Bryant and McKibbin (1998), reproducing an updated version of a table by Meredith (1995), identifies many of the empirical studies of the effects of dependency ratios on saving rates and summarizes their main findings. The studies include those of Feldstein (1980), Modigliani and Sterling (1983), Masson and Tryon (1990), Horioka (1991), Masson-Bayoumi-Samiei (1995, 1998), and Meredith (1995). Recent empirical evidence suggesting that both youth and elderly dependency ratios have a significant negative effect on savings is summarized by Loayza, Schmidt-Hebbel and Serven (2000), using a large macroeconomic database released by the World Bank covering 150 countries over the period between 1965-1994. (The economically appropriate cut-off age for defining youths varies significantly between developed and developing countries, and within developing countries themselves.)

absence of youth dependency in a single integrated model, which in turns permits us analytically to isolate the effects of several dimensions of youth dependency. Our analysis sheds additional light on the conclusions reached in the earlier contributions to the literature on youth dependency. Whereas the earlier literature relies on reduced-form and linear time-series approaches, moreover, our approach is structural and points the way to a more reliable method for studying the macroeconomic effects of youth dependency.[8]

We emphasize fertility declines as the illustrative demographic shifts in this paper. We isolate the economic effects of changes in youth dependency and study how such changes cause significant shifts in the composition of the adult work force, in elderly dependency, and in key macroeconomic aggregates such as production and consumption. Our research highlights the cross border effects in open economies and strongly reinforces the presumption that changes in youth dependency have first-order consequences for the determination of exchange rates, external imbalances, and global saving and investment flows.

Section 2 summarizes key features of our analytical approach, with emphasis on youth dependency and child support. Section 3 describes benchmark simulation results for a fertility decline with and without youth dependency. The results of a global, symmetric ("closed-economy") shock are contrasted with asymmetric shocks that differ in magnitude between two countries inter-linked through trade and capital flows. Section 4 carries out sensitivity analysis with alternative specifications for how adults support children's consumption. Section 5 contains concluding remarks. A first appendix gives more details about the theoretical framework. A second appendix provides details about baseline solutions for the model and the specification of the illustrative shocks. A third appendix discusses the sensitivity of our results to alternative assumptions about the intertemporal elasticity of substitution in consumption, a key behavioral parameter governing adult consumption.

2. Analytical Approach

Our analytical framework is a world composed of two countries with cross-border flows of goods and capital. The exchange rate linking the two currencies and economies adjusts to ensure that the global (algebraic sum of both countries) current-account balance and net-foreign-asset position is always zero. Within each economy, optimizing firms produce a single composite good, determined by

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[8] As discussed in Bryant and McKibbin (1998), the macroeconomic estimates of relationships between dependency ratios and saving rates have used a reduced-form approach. Skeptics argue that the existing macroeconomic research has failed to allow properly for econometric problems and cannot be reconciled with the microeconomic evidence based on household survey data.

an aggregate production function with capital and (productivity-augmented) labor as its arguments. The composite goods from each country are imperfect substitutes; some production in each country is exported; import demands are a function of national incomes and relative prices.

Households in each country are assumed to have identical preferences over foreign and domestic goods. The treatments of household consumption, saving, and wealth accumulation build on the overlapping generations framework of Blanchard (1985), Weil (1989), and Yaari (1965) as extended by, among others, Faruqee, Laxton, and Symansky (1997) and Faruqee (2002) to incorporate age-earnings profiles and a "bottom-up" determination of labor income. In our further extension of the framework, population growth and structure are endogenous. Households are comprised of both adults and youth dependents (children for short).

In the following description of the analytical framework, we suppress most details and summarize only the modifications in the treatment of consumption, saving, and wealth accumulation that are associated with youth dependency and the economic linkages between the child and adult populations. Appendix 1 provides a fuller account.[9]

A. Economy-Wide Saving and Wealth Accumulation

The significance of youth dependency in the model is immediately apparent from the definition of total real domestic consumption $C_{TOT}(t)$ (across both the youth dependent and adult populations):

$$C_{TOT}(t) \quad = \quad C_j(t) \, + \, C(t) \quad . \tag{1}$$

where $C_j(t)$ is aggregate child consumption and $C(t)$ is aggregate adult consumption.[10] These two components of consumption are interconnected. Children are assumed to consume but not to earn any income and not to hold any financial wealth. Child consumption is thus exclusively financed by *inter vivos* transfers from parents to children. Adult consumption, meanwhile, is formulated in a familiar life-cycle approach but modified to account for the transfers to children. The details of adult consumption are relegated to Appendix 1.

Reflecting an economy-wide budget constraint for each of the countries, the aggregate stock of financial wealth (held only by adults) accumulates as:

$$\dot{FW}(t) \; = \; [r(t)FW(t) \; + \; Y(t)](1-\tau(t)) \;\; + \;\; PB_{ss}(t) - \tau_{SS}(t)Y(t) \; - \; C(t) \; - \; C_j(t), \tag{2}$$

9 A separate background paper – Bryant (2004b) – presents a full exposition of our analytical framework, incorporating the various changes made since the start of our research.

10 Behavioral relationships for the home country's agents and agents in the other country are identical. Accordingly this exposition omits country identifiers.

where r(t) is the domestic interest rate, $Y(t)$ is aggregate labor income, $\tau(t)$ represents the government's income tax rate (applicable to both labor and capital income), $PB_{ss}(t)$ is the economy-wide real value of pension benefits, and $\tau_{SS}(t)$ is the pension (payroll) tax rate. The left-hand side of equation (2) is the flow of net private saving in the economy.[11]

The fiscal authorities of each country engage in real spending on goods and services, raise revenues by taxing the incomes of firms and households, and pay interest on the outstanding stock of government debt. Each country's government also operates a pay-as-you-go public pension ("social security") system that collects revenue from a pension tax and pays out pension benefits to elderly adults.[12] The financial sectors of the national economies, modeled in a rudimentary way, contain monies that are only high-powered money (central-bank liabilities), the demands for which depend negatively on short-term nominal interest rates and positively on national incomes. Policy reaction functions are specified for the central bank and the government fiscal authority.[13]

B. Economic Linkages between Youth Dependents and Adults

The introduction of youth dependent consumption into the model requires addressing two central issues: how the consumption of youth dependents is determined, and how and by whom that consumption is financed.

First, aggregate child consumption $C_j(t)$ is defined simply as the sum over individual child consumption levels, $c_j(s,t)$, scaled by the number of children $J(s,t)$ from each age cohort (indexed by s):

$$C_j(t) = \int_{t-\Delta}^{t} c_j(s,t) J(s,t) ds \tag{3}$$

where Δ, set to 18 in our implementation, denotes the age-range of the youth dependent population.[14]

[11] Equation (2) reflects the fact that the aggregate of support transfer payments to children is equal to the total amount of child consumption. The equation also reflects the fact that aggregate financial wealth accumulates at the rate $r(t)$, not at the rate $r(t) + p_n(t)$, since the amount $p_n(t)FW(t)$ is not an addition to aggregate wealth, but rather a transfer from those who die to those alive implemented through the actuarially fair Yaari-style insurance companies. The variable $p_n(t)$ is the adult mortality rate. See the appendix for further details.

[12] The modeling of transfers from working adults to elderly dependents through the public pension system is described in Appendix 1.

[13] Each central bank follows a policy rule that ensures long-run nominal stability of the national economy, either a targeting rule for (high-powered) money, a nominal-GNP-targeting rule, or a rule combining inflation targeting with real GNP targeting. Each fiscal authority uses an "intertemporal fiscal closure rule" that is a variant of debt-stock targeting: the income tax rate is varied up or down to ensure that, over a medium run, the actual path of the government's debt converges to an exogenously-specified target path for the debt.

[14] Details regarding the population dynamics for the adult and youth dependent population are relegated to the appendix. The value of $c_j(s,t)$ is the same for every child cohort rather than varying across child cohorts.

Second, because children do not earn any income, they are dependent on transfers from their parents to finance this consumption. In particular, aggregate youth-dependent consumption must equal the aggregate of parent-child transfers *V(t)*, where *V(t)* is the sum of individual parent-child transfers $v(s,t)$ from each adult cohort $N(s,t)$ over all the relevant cohorts (indexed by s):

$$C_j(t) \quad = \quad \int_{-\infty}^{t} v(s,t)N(s,t)ds \quad = \quad V(t) \quad . \tag{4}$$

The key behavioral implications of youth dependency in the model rest on the treatment of the right-hand side variables in equations (3) and (4).

Individual child consumption $c_j(s,t)$ can be divided into two components, one that is inelastic and fixed, the other that endogenously varies through time in response to conditions in the economy. The inelastic component is unaffected by changes in the economic situation of parents – i.e., it reflects, so to speak, the "basic needs" of children for (say) food, clothing, and shelter. The second component is "discretionary" and modeled as a constant fraction of adult consumption per capita. The total resources consumed by each youth dependent, $c_j(s,t)$, is thus given by:

$$c_j(s,t) = c_1 + c_2\, C(t)/N(t) \tag{5}$$

where c_1 is the fixed, basic-needs component and c_2 is the parameter determining the responsiveness of the time-varying, endogenous component of child consumption to adult consumption per adult.

The formulation in (5) is appealing because it nests several possible ways to model child consumption. By setting $c_2 = 0$, for example, one can study the implications of child consumption that is inelastic to conditions in the economy.[15] Conversely, by setting $c_1 = 0$ with $c_2 > 0$, one can analyze the case where the basic-needs component of child consumption is absent altogether. For a given value of the basic-needs component c_1, the parameter c_2 determines the generosity and sensitivity of discretionary child consumption relative to the consumption of an average adult. By varying the value of c_2, one can thereby investigate the effects of making the discretionary component of child consumption more or less generous than average adult consumption, and hence more or less volatile (in absolute terms). Using this general formulation yields the following expression for aggregate child consumption:

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[15] The model framework allows labor productivity in the economy to grow at a constant, steady-state rate. For model simulations in which productivity growth is positive, we assume that the basic-needs component of child consumption, c_1, is not fixed absolutely but rather increases at the constant rate of productivity growth.

$$C_j(t) = c_1 J(t) + c_2 (\frac{C(t)}{N(t)}) J(t) = c_1 J(t) + c_2 \delta(t) C(t) \tag{6}$$

where J(t) is the size of the child population and $\delta(t)$ is the youth-dependency ratio, defined as the ratio of children to adults (i.e., $\delta(t) \equiv J(t)/N(t)$).

Our treatment of the second central issue is to assume that child consumption is entirely financed through *inter vivos* transfers from adults. Youth dependents do not earn labor income and have no financial wealth.[16] In our framework, adult transfers to children are thus, in economic effect, tantamount to "child tax" payments earmarked for child support. Alternatively stated, adult parents are assumed *not* to derive any direct utility gain from child consumption and thus *not* to determine, as an integral part of their own consumption decision, the level at which children consume. Our simplified formulation, embodying a rule-of-thumb standard for setting child consumption and treating adults as the only decisionmakers in the economy, allows us to avoid fundamentally altering the consumer problem facing adult agents.[17]

We further assume that transfers are cohort-specific across adults. Specifically, transfers are assumed to be *hump-shaped*, rising with the age of the parent initially before declining. The hump-shaped profile allocates transfers most heavily to middle-aged adults with larger families of dependent children and less to younger adults with growing families and older adults with grown children. For a specific approximation of a hump-shaped profile, we adopt the following:

$$v(s,t) = v(t)[z_1 e^{-\omega_1 (t-s)} + (1-z_1) e^{-\omega_2 (t-s)}]; \quad z_1 > 1, \quad \omega_2 \geq \omega_1 \geq 0 \quad . \tag{7}$$

The weighting function in (7), in square brackets, is normalized relative to the youngest adult cohort (where $s = t$). In other words, $v(t)$ represents the transfers that the *newest* cohort of parents provide to their (newly born) children. Parents in older cohorts provide relatively more or less support than $v(t)$ depending on their age, according to the weighting function.

The specification in (7) nests several more simplified cases used in our initial explorations of youth dependency. If both curvature parameters are set to zero ($\omega_1 = \omega_2 = 0$), support payments for children are completely age-invariant (every adult age cohort makes exactly equivalent transfers). For

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16 Thus when a youth becomes an adult and enters the workforce at the beginning of the 19th year of life, he or she starts out with zero financial wealth.

17 See Appendix 1 for a description of the adult consumer's problem. Our treatment of child consumption is a major step forward from the practice of ignoring it entirely, but self-evidently is unrealistic and needs to be modified in future extensions of the research. For a more realistic formulation of child consumption in which child consumption directly enters the parents' utility function (but which cannot be easily adopted in our model), see for example Becker and Barro (1988).

the special case in which $\omega_1 = \omega_2$ and both parameters are positive, (7) produces an exponentially declining rather than hump-shaped distribution of transfers across cohorts; that is, the newest cohort of adults provides the largest amount, with the amount declining exponentially for older and older cohorts. The most plausible configuration of the parameters matching the pattern of age-specific fertility is the hump-shaped case where $\omega_2 \geq \omega_1 \geq 0$.[18]

With the cohort distribution of adult transfers in (7), the economy-wide total for adult transfers can be written in terms of two components, where $b_n(t)$ is the rate of new young entrants into the labor force (so to speak, an adult "birth" rate) and $p_n(t)$ is the adult mortality rate:

$$V(t) = V_1(t) + V_2(t); \tag{8}$$

$$\dot{V}_1(t) = \frac{\dot{v}(t)}{v(t)} V_1(t) + z_1 b_n(t) N(t) v(t) - (\omega_1 + p_n(t)) V_1(t) \tag{9}$$

$$\dot{V}_2(t) = \frac{\dot{v}(t)}{v(t)} V_2(t) + (1 - z_1) b_n(t) N(t) v(t) - (\omega_2 + p_n(t)) V_2(t) \tag{10}$$

Using the adding-up condition in (4) between aggregate child consumption and total adult transfers that $C_j(t) = c_1 J(t) + c_2 \delta(t) C(t) \;=\; V(t)$, one can write the law of motion for *v*, individual transfers by the newest adult cohort, as:

$$\frac{\dot{v}(t)}{v(t)} = \frac{c_1 \dot{J}(t) + c_2 \delta(t) C(t) \left[\frac{\dot{\delta}(t)}{\delta(t)} + \frac{\dot{C}(t)}{C(t)} \right]}{V(t)} - \frac{b_n(t) N(t) v(t)}{V(t)} + \omega_1 \frac{V_1(t)}{V(t)} + \omega_2 \frac{V_2(t)}{V(t)} + p_n(t) \quad . \tag{11}$$

C. <u>Implementing the Framework in a Global Empirical Model</u>

For our empirical analogue of the theoretical framework, we use a stylized and simplified two-region abridgement of a larger world model. The underlying model, containing many separate countries and regions, is the IMF staff's MULTIMOD model. Our abridgement is a substantially revised and updated version of a two-region abridgement originally created in the mid-1990s. The

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[18] The case of exponentially declining weights takes into account the likelihood that the children of middle-aged adult cohorts may have already themselves reached adulthood and are no longer dependent on parent-child transfers. But it fails to recognize that family sizes tend to grow in the early years of adulthood after youths have come of age and begin to have children.

refined two-region model is a research environment in which the global macroeconomic consequences of demographic change can begin to be systematically studied.[19]

The starting point for the empirical model is a set of equations describing the U.S. economy ("US" for short). Then a second artificial country is created, labeled for brevity as "ZZ." The ZZ economy is an identical, mirror image of the United States. Thus the "world" in this stylized framework is composed of two economies, roughly like the United States, that are equal-sized, equivalently open, and identical in domestic structure. The economies are carefully linked together with the balance-sheet and income-flow identities that would have to hold in an actual world of two economies. The current-account balance and the net-foreign-asset position of the ZZ economy, for example, are exactly the negatives of the current account and the net-foreign-asset position of the US economy. The two economies are connected by a single, endogenously determined exchange rate. The exchange rate is proximately determined by a variant of the uncovered interest-parity relationship. Indirectly, the exchange rate is influenced by and in turn helps to determine all the macroeconomic variables in both economies.

The empirical model, like the theoretical framework, emphasizes the forward-looking behavior of agents and presupposes that both firms and households engage in intertemporal optimization. A partial exception stems from an allowance for a fraction of consumers whose consumption is constrained by an inability to borrow and hence are unable to smooth their consumption intertemporally. The consumption-saving sectors of the model permit an explicit assumption about the value of the consumers' elasticity of intertemporal substitution (EIS) – see equations (38) and (39) in Appendix 1.

Output of the single composite good produced in each economy is a function of capital and productivity-augmented labor. The production technology of firms is represented by constant elasticity of substitution (CES) production functions. Firms are price-taking entities that choose variable inputs and their level of investment in capital so as to maximize stock-market value. Firm investments respond to the difference between the market value and reproduction value of the capital stock (a variant of Tobin's "q" framework).[20]

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[19] For a description of the IMF staff's MULTIMOD Mark III, see Laxton, Isard, Faruqee, Prasad, and Turtelboom (1998). The Mark II version of MULTIMOD is presented in Masson, Meredith, and Symansky (1990). Bryant and Zhang (1996a, 1996b) describe the original abridgement. Our ultimate research agenda is to incorporate insights and specifications obtained from the stylized, abridged model back into the richer, more realistic contexts of larger world models with separate actual countries.

[20] The model's investment equations so far follow the treatment in the Mark II version of MULTIMOD (Masson, Meredith, and Symansky, 1990). Adjustment costs for investment in capital are modeled explicitly in the Mark III version of MULTIMOD (Laxton, Isard, Faruqee, Prasad, and Turtelboom, 1998).

The stylized model treats labor as perfectly mobile within each of the two countries but completely immobile across the countries. Hence wages are equal across comparable age cohorts within each country but in general are not equal across the two countries. Over the long run labor is inelastically supplied with respect to wages and is determined by the model's demographic structural equations. Prices are sticky in the short run but flexible over a longer run. The model forces full employment of labor and capital over the long run.[21] Because the composite goods from each country are imperfect substitutes, each country exports some of its production to the other. Imports in each country are a function of national income and relative prices. Agents in a given country are assumed to have identical and time-invariant preferences over foreign and domestic goods.

The empirical model is solved with a software algorithm that imposes model-consistent ("rational") expectations. Hence agents are presumed to know the structure of the model and to correctly anticipate the entire future paths of the model's exogenous variables. Imposition of model-consistent expectations is the now-standard working assumption in most empirical work in macroeconomics and our use of this assumption is familiar ground.[22]

When using the empirical model, we first develop one or more model-consistent, steady-state baseline solutions for the evolution of the ZZ and US economies. For transparency, both economies are assumed to follow identical paths and exhibit identical behavior along these steady-state baselines. Hence the baseline exchange rate is constant over time at unity and the trade balances, current-account balances, and net-foreign-asset positions in the baseline are all constant at zero. Baseline solutions for the model typically assume that productivity growth occurs at a constant rate. Baseline steady-state rates of inflation are likewise assumed constant. The fertility (child birth) rate, the child mortality rate, and the adult mortality rate are the key exogenous demographic variables in the model. Typical baselines have these key demographic rates set at constant values. Baseline issues are discussed further in Appendix 2.

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[21] The current version of the model follows the treatment of prices and wages in the Mark II version of MULTIMOD. Capacity utilization can differ in the short run from long-run full use of capacity. But the model includes wages and employment implicitly and hence does not explicitly track unemployment. Further refinements in the empirical model should include alternative treatments of the dynamics of prices and wages.

[22] Yet the assumption is extreme and inherently implausible for demographic shocks that begin gradually and then wane gradually over many years. It is feasible to modify the model-consistent-expectations assumption by phasing in "correct expectations" about the paths of exogenous variables with the passage of time rather than permitting expectations to be correct immediately and fully. Much interesting research is now being carried out that applies "learning" ideas to the evolution of expectations. See, for example, Evans and Honkapohja (2001) and references cited there In future research, we ultimately hope to make modifications in the assumed treatment of expectations. For the time being, we report the results with the now-familiar, full model-consistent expectations.

D. <u>Illustrative Shocks to Fertility</u>

To develop analytical conclusions about the consequences of demographic shifts for macroeconomic variables, we run shock simulations in the model, perturbing the paths of one or more of the exogenous demographic rates, and then compare the resulting shock values of endogenous variables with their baseline values.[23] Most of the paper focuses on a shock simulation in which the child birth rate declines sharply, remains at a low level for an extended period, and then eventually recovers enough of its earlier decline to leave the economy with a stationary population. This illustrative shock, labeled *large-cyclical*, causes for several interim decades a negative growth rate for the population as a whole. We select this fertility shock for study in part because it is roughly analogous to the recent and prospective demographic experience of Japan.[24] A second contrasting shock, labeled *smaller-gradual*, assumes that the child birth rate declines more slowly and monotonically until the population eventually reaches a zero growth rate. For these illustrative shocks to fertility, the adult-mortality and child-mortality rates remain unchanged at their baseline levels.

The exogenously specified paths for the birth rate in the two shocks are shown in the upper portion of Figure 2. The bottom part of the figure plots the corresponding endogenous growth rates for population as a whole, which of course reflect the patterns of the birth rates. Appendix 2 gives further details about the specification of the shock paths.

The assumed fertility declines have major consequences for all endogenous demographic variables. The effects on youth-dependency and elderly-dependency ratios are graphed in Figure 3. For the large-cyclical shock, the youth-dependency ratio shows a pronounced decline in the early decades. Once the birth rate stops falling further and then begins to recover, the youth-dependency ratio eventually recovers roughly half of its earlier decline. The elderly-dependency ratio during the large-cyclical shock at first rises only slowly, lagging behind the opposite movement in the youth ratio; subsequently, however, a pronounced aging of the population occurs, pushing the elderly ratio to a peak nearly double its initial level. Eventually, after the birth rate is rising again, the elderly ratio gradually reverses part of its earlier rise. For the smaller-gradual shock to fertility, the youth ratio declines monotonically and the elderly ratio rises monotonically to their eventual new steady-state levels.

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[23] Fertility (birth) and mortality rates are the exogenous demographic variables; the resulting population growth rates and structures are determined endogenously.

[24] The major decline in fertility in Japan took place in the second half of the 20th century. Part of our illustrative shock in this paper is motivated by Japan's past fertility experience and part by projections of its demographic future.

Figure 4 plots the levels of adult populations and effective labor forces for the two shocks. During the earliest years of the fertility declines, for both shocks the adult populations and the effective labor forces continue to increase at the baseline positive rate of growth. And as shown in Figure 5, at first the ratios of the effective labor forces to adult populations remain stable at the baseline level. Thereafter the increases in the effective labor forces partly reflect the fact that the number of young workers, who are less productive, decline relative to the number of older, higher-productivity workers.[25]

For the large-cyclical shock, as the workers age in the cohorts of reduced size and pass into the ranks of higher- productivity workers, the effective labor forces begin a protracted, sharp decline – at a rate much steeper than that for the adult populations as a whole. Eventually, that decline is partially reversed as the proportion of youth in the economy rises again and ultimately stabilizes at its new eventual level. The behavior of adult populations and effective labor forces is dramatically different for the smaller-gradual shock. When the economies eventually reach new steady states in which the size and composition of populations are stationary, the *levels* of populations and effective labor forces that result from the smaller-gradual shock are nearly twice as large as those that result from the large-cyclical shock (Figure 4) even though the ratios of the effective labor force to adult population have converged to the same value (Figure 5). The major differences in the demographic consequences of the two shocks cause major differences in the corresponding macroeconomic outcomes.

3. <u>Simulating a Decline in Fertility With and Without Youth Dependency</u>

This section of the paper focuses on basic macroeconomic effects that result from fertility shocks, highlighting the differences between effects with and without youth dependency. Section 4 then conducts various forms of sensitivity tests for alternative specifications of child support.

The analysis in this section uses two variants of our analytical model. One of the variants uses benchmark assumptions for children's consumption and its financing by child-support transfers from adults. The other variant zeroes out children's consumption and adult support altogether. Specific

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[25] The initial baseline level of the *effective* labor force is some 1.7 times greater than the level of the total adult population, reflecting the calibration of the labor forces with their incorporation of the age-specific relative productivities of different aged workers. The levels of the effective labor force in the model represent, in effect, the number of labor "efficiency units," not the total number of workers. The growth rate of the *adult* populations lag behind the growth rate of the *total* (adult plus youth) populations by 18 years. For the large-cyclical shock, although the total populations begin to decline fairly soon after the onset, the adult populations and the effective labor forces thus continue to increase for a while longer even though new births of children and hence the growth rates of the total populations are falling sharply.

values of benchmark parameters and the construction of baseline solutions for the two variants are discussed in Appendix 2.[26]

The analysis portrays simulation results graphically. The time paths of variables in the charts are shown as deviations from the baseline solutions of the variant models. The units of the deviations are specified along the vertical axes of the charts. If a variable has a value of zero in a figure, at that point the variable is unchanged from its baseline path. The charts report results over sufficiently extended periods, 300 years or more, to illustrate long-run as well as short- and medium-run effects.[27]

A. Symmetric Shocks (Closed Economy)

Our main purpose is to study the macroeconomic consequences of demographic changes when shocks are asymmetric across countries. To gain intuition, however, it is first helpful to examine *symmetric* shocks in which the fertility declines occur identically in both the ZZ and US economies. When a shock is identical in both regions, the nature of the shock is, so to speak, “global” and the model then produces identical simulation paths for both economies. External-sector balances remain at zero and the exchange rate remains unchanged at its baseline value of unity. In effect, each economy behaves as though it were completely closed, which is of course literally true for the world as a whole. Hence we refer to the symmetric cases (both regions experiencing the large-cyclical shock, or both experiencing the smaller-gradual shock) as "closed-economy" simulations. The closed-economy analysis facilitates interpretation of the most basic, domestic consequences of fertility declines and serves as a benchmark for analytical interpretation of open-economy effects.

Consider again Figure 4. Changes in effective labor forces – reflecting demographic shifts, the effects of humped age-earning profiles, and the bottom-up determination of labor incomes and human wealths – critically influence the dynamic behavior of macroeconomic variables in the model. When individuals first enter the labor force, they have relatively low productivity and are relatively low savers. Then as younger workers age, gain experience, and have higher productivity, they in effect ascend the left side of the hump of the economy's age-earning profile. Individuals reach their years of

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[26] Both variants of the model include the same benchmark assumptions for balanced public pension systems. The background paper (Bryant, 2004b) contrasts four variants of the model: a benchmark version that includes pensions and youth dependency, and versions that omit pensions, omit youth dependency, or omit both together.

[27] Because demographic shocks often have consequences over very lengthy periods, our model simulations are carried out over long horizons, typically for more than 500 years. Numerous simulations for a variety of demographic shocks have been examined to ensure that the model eventually reaches analytically plausible outcomes (new stable steady-state growth paths) after sufficient time is allowed for all adjustments to the shocks. For the large, *permanent* demographic shocks studied in this paper, the required time for complete adjustment is often very lengthy. Adjustment times are shorter for shocks that are *transitory* (in which the exogenous demographic variables return to their initial baseline steady state values).

peak earnings and high savings when they are in their forties and fifties. Eventually, they start to descend the right side of the humped age-earning profile; consequently, their labor incomes and saving decline. At that point, their consumption must be increasingly financed out of their privately accumulated financial wealth as supplemented by pension transfers from the government (if a pension system exists). As demographic shocks pass through the age-earning profile, the dynamic effects of the demographic movements, significant in themselves, get still further amplified.[28]

Dynamic effects from the behavior of effective labor forces become even more pronounced and significant when the model incorporates the consumption of children. During the period when the fertility declines are producing reductions in the youth population, the adults providing children with support have progressively smaller transfer payments to make to children. The demographic change thus frees up resources relative to the situation where child-support transfers are absent. The way transfers to children are distributed among adult cohorts also interacts directly with the age-earning-profile effects.

Interactions between the demographics and the age-earning profiles are of course more apparent for the case of the large-cyclical shock. As the large-cyclical shock moves identically through the populations of the two economies, the adult populations and the effective labor forces become much smaller than they otherwise would have been (Figure 4). The ratios of the effective labor forces to the adult populations fall cyclically by large amounts and then eventually partially recover (Figure 5). In contrast, for the symmetric smaller-gradual shock, the effective labor forces and adult populations do not decline absolutely (apart from a small cyclical movement in the labor forces in the medium run) and the declines in the ratios of the two variables are gradual and smooth (again see Figures 4 and 5).

Reflecting the movement of the ratios of effective labor forces to adult populations, the economy-wide aggregate levels of human wealth, financial wealth, output, consumption, and the aggregate capital stock all decline to eventual levels that are significantly lower. The movements of key macroeconomic variables are relatively smooth declines for the smaller-gradual shock. In contrast, for the large-cyclical shock the movements have a major cyclical downswing followed by a partial reversal.

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[28] We first stressed the central importance of these age-earning-profile effects in a 2001 paper, published as Bryant and McKibbin (2004). See in particular the comparisons in that paper between simulations with and without the age-earning profiles present. (Excluding the effects of the age-earning profiles entails setting the three α coefficients in equation (22) in Appendix 1 to zero.) Interactions between demographic dynamics and efficiency labor inputs are much less significant in models that fail to incorporate age-earning profiles.

The dynamic movements of key variables (deviations of simulation levels from baseline levels) are illustrated for the closed-economy cases in Figures 6 through 9. For a particular variable, each figure reports the results for four symmetric-shock simulations: effects for the large-cyclical shock with and without youth dependency, and effects for the smaller-gradual shock with and without youth dependency. The simulations that include youth dependency have solid curves; dashed curves indicate that youth dependency is excluded.

The presence or absence in the model of youth dependency importantly conditions the dynamic behavior of the real interest rate, the capital stock, and the capital-output ratio. Both in initial baselines and throughout shock simulations, the *level* of the real interest rate must be higher in a simulation with than without child consumption. The greater the generosity of child support, the higher must be the real interest rate. Other things being equal, the capital stock and economy-wide output and consumption are lower when children must be supported. Adults have to set aside more resources to cover the needs of children and hence have smaller savings, leading to less capital accumulation and hence to lower output and consumption per adult. The real interest rate, reflecting the marginal product of capital, will therefore be higher, the higher are child needs.[29]

Real interest rates decline as the effective labor force declines (Figure 6). The capital stock declines much less relative to baseline than either the effective labor force or output; thus the capital-output ratio increases substantially (Figure 7). These changes occur relatively smoothly for the smaller-gradual shock but cyclically and by much larger amounts when the shock itself is large and cyclical. Because the effective labor force is lower relative to the capital stock, the marginal product of capital must fall. For the large cyclical shock, the effective labor force eventually recovers somewhat; accordingly the real interest rate also recovers and the cyclical upswing in the capital-output ratio is partially reversed; in the longest run, after the capital stock is again high relative to the labor force, the real interest rate gradually moves down and the capital-output ratio gradually moves up toward their final steady-state values.

Although the dynamic pattern of the real interest rate is broadly similar with and without youth dependency, the interest-rate movements in the medium run are more pronounced when youth dependents are modeled. This greater amplitude when child support is present is due to the freeing up of resources for adults when the numbers of children are fewer. More resources for the consumption

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[29] To illustrate the size of the differences, the *level* of the baseline short-term real interest rate in the benchmark simulation that includes youth dependency is 6.54 percent, some 119 basis points greater than the baseline short-term real interest rate of 5.35 percent in the simulation for which youth dependency is excluded. The *level* of the baseline capital-output ratio with youth dependency included is 2.08, lower than the baseline capital-output ratio of 2.28 prevailing in the simulation without youth dependency.

and saving of adults means that there is less need to deplete the capital stock. The capital stock relative to the effective labor force is higher in the model when child support is taken into account, which necessitates a more pronounced decline in the marginal product of capital, and hence in the real interest rate.

Human wealth, financial wealth, and adult consumption and savings are four of the important real macroeconomic variables in the model. The simulated movements in two of these variables, measured in per-adult terms, are shown in Figures 8 (financial wealth) and 9 (adult consumption). Both with and without youth dependency, dynamic movements are partly cyclical or relatively smooth depending on which of the two shocks is examined.

For the large-cyclical shock, financial wealth per adult and human wealth per adult *rise* relative to baseline in the initial decades when the youth dependency ratio is falling sharply. The larger the assumed generosity of child support, the larger are the rises in human wealth and financial wealth relative to what would have occurred without the decline in the numbers of children. The increases in financial wealth are explained partly by the effects of the age-earning profiles on saving and partly by higher disposable incomes and savings reflecting the smaller support payments to children to be made in the shorter-run future. In the medium run, as the decline in fertility is reversed and the child population again increases, financial wealth and human wealth per adult fall steadily relative to baseline until they are well below baseline levels. Then over the long run they rise back toward baseline. Simulation paths for consumption per adult likewise show a rise in the shorter run, a sustained fall in the medium run, and an eventual rise back toward baseline. The cyclical swings in consumption per adult are more pronounced when adults make transfers to support child consumption. The broad pattern of cyclical movement for each of the variables human wealth, financial wealth, and consumption is qualitatively similar for the large-cyclical shock to the pattern of movements in the ratio of the effective labor force to the adult population (Figure 5).

For the smaller-gradual shock, macroeconomic variables such as financial wealth per adult and adult consumption per adult manifest much smoother and smaller adjustments in the shorter and medium runs. In the long run, however, they eventually reach the same steady-state values as those for the large-cyclical shock. This outcome occurs because the demographics for the two shocks themselves eventually converge on identical steady states.

B. Asymmetric Shocks (Open Economy)

The preceding analysis makes clear that demographic events powerfully influence macroeconomic outcomes within domestic economies. The behavioral dynamics identified in the closed-economy simulations are no less important when economies are open and demographic changes are asymmetric. But the consequences of *asymmetric* changes – country-specific variants of demographic shifts in which one part of the world economy experiences different shocks and different outcomes from those occurring elsewhere – are significantly modified by the openness and interactions of national economies. Our primary goal is to study asymmetric demographic changes which cause significant cross-border spillovers. Thus with the preceding closed-economy summary as background, we now focus on open-economy analysis by imposing the large-cyclical shock in fertility on the ZZ economy but assuming that the US economy experiences the smaller-gradual shock.

For this asymmetric-shock case, the path for the ratio of the effective labor force to adult population exhibiting the large-cyclical behavior in Figure 5 should now be understood as the path experienced in the ZZ economy. In contrast, the smaller-gradual path for that ratio in Figure 5 is the path experienced by the US economy. Because of the asymmetry in shocks, the ZZ and US economies will now have very different evolutions of key variables such as interest rates, the capital stock, human wealth, savings and financial wealth, and consumption.

The charts that accompany the following, Figures 10 through 20, compare two model simulations. One includes youth dependency with benchmark assumptions about adult support for child consumption. The other excludes youth consumption altogether. The asymmetric fertility shock, large-cyclical in the ZZ economy and small-gradual in the US economy, is identical for both simulations. For each simulation, curves are shown for the ZZ and the US economies. The ZZ paths for the variable in a chart are plotted with thicker, more prominent curves than those for the corresponding US paths. As before, the solid curves indicate the simulation with youth dependency included and dashed curves the exclusion of youth dependency.[30]

During the first two decades of the shock, before the adult population and the labor force have yet experienced the consequences of the fertility decline, real interest rates in the ZZ and US economies change relatively little from their baseline values (Figure 10). Thereafter, the cumulating sharp fall in the ZZ effective labor force leads to a progressively larger fall in the ZZ real interest rate. The extent of the fall in the ZZ real interest rate and its subsequent reversal, however, is significantly damped because of the ZZ economy's openness to the rest of the world. The real interest rate in the US shows an analogous, but much smaller, decline than that in the ZZ economy; the downward movement in the US real interest rate is significantly larger for its smaller-gradual shock than in the closed-economy case; the US rate also has significantly more cyclical behavior, reflecting the interactions

[30] Details about the shock simulations and underlying baselines are provided in Appendix 2.

with the ZZ interest rate. As with the symmetric shock (Figure 6), and for the same reasons, interest-rate movements in the medium and long runs are more pronounced when youth dependents are taken into account rather than assumed absent.[31]

Figure 11 shows movements in the capital-output ratios. In the medium and longer runs, the US economy now experiences a *lower* rise in its capital-output ratio than in the closed-economy, smaller shock (compare with Figure 7). With the larger-cyclical shock in the ZZ economy, the ZZ capital-output ratio must rise substantially *higher* in the medium run than in the closed-economy case; furthermore, the ZZ capital-output ratio remains at a permanently higher level forever. These different capital-stock and output evolutions in the two economies are associated with major differences in saving and external-sector behaviors.

Movements in human wealth and financial wealth in both economies continue to be driven by the interactions of the fundamental demographic forces and age-earning profiles. But with a much larger fertility decline affecting the ZZ than the US economy, the relative positions of the two economies are very different. As seen in Figure 12, for example, ZZ financial wealth per adult in the shorter run rises relative to baseline even more strongly than in the closed-economy case and then diminishes only very gradually in the medium and longer runs. In marked contrast, US financial wealth per adult does not stay on a roughly level path in the short and medium runs but rather falls sharply in the medium and long runs. The differences between the two economies are larger when the model allows for the effects of youth dependency than when it does not. Significantly, ZZ financial wealth per adult (and ZZ savings per adult) is consistently much higher than it would be for the large shock in the closed-economy case. The opposite is true for the US economy; financial wealth per adult (and savings per adult) fall well *below* relative to the outcomes for the smaller shock in the closed-economy case. The differences in saving behavior and hence in financial wealth between the ZZ and US economies are thus attributable not merely to their different-sized demographic shocks but also to major influences working through the exchange rate and external-sector transactions.

The model enforces a variant of the uncovered interest parity condition as part of the behavior determining the nominal exchange rate. Hence a sizable interest differential between the two economies – once it opens after the initial years of the asymmetric shock (Figure 10) – puts strong pressure on the real and nominal exchange rates. The ZZ currency begins a sustained appreciation,

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31 Nominal short-term interest rates in the two economies follow qualitatively similar paths as the real interest rates, with similar conditioning by the openness of the economies.

first in nominal terms, then with a lag in real terms (Figure 13).[32] By the ninth or tenth decade of the shock, both the nominal and real values of the ZZ currency have appreciated by a large amount. The real exchange value of the ZZ currency appreciates substantially further over the next several decades, reaching a peak appreciation of some 100 percent before reversing and falling back. In the new long-run steady state, the real exchange rate settles at a level very much higher than in the baseline solution. Allowing for youth dependency in the model increases the degree to which the ZZ currency appreciates in the medium and long runs. (For the symmetric cases in which both economies experience identical shocks, the nominal and real exchange rates of course never deviate from the baseline value of unity.)

To understand why the asymmetric shock results in a real exchange rate permanently higher in the ultimate than in the initial steady state, recall that the asymmetric fertility declines are transitory in terms of demographic *rates of growth* but have permanent effects on the *relative levels* of demographic and macroeconomic variables. In particular, as the larger shock causes the ZZ fertility rate to fall faster and progressively below baseline, the ZZ population and effective labor force begin to fall further and further below baseline. The ratio of the ZZ to the US effective labor force and the ratio of the two countries' populations fall correspondingly. Once the decline in the ZZ fertility rate is partially reversed, eventually the ZZ and US population growth rates again become equal. The ratio of the ZZ to the US effective labor force, however, remains permanently smaller (Figure 4). In the medium run and in the new long-run steady state in which the aggregate sizes of the capital stock and labor force in the ZZ economy are well below baseline whereas the corresponding macroeconomic aggregates in the US economy are above baseline, therefore, the quantity of ZZ-produced goods is markedly smaller than the quantity of US-produced goods. Figure 14 emphasizes this result by plotting the evolution over time of the ratio of populations and GDPs in the two economies. Given unchanged preferences in each economy for the two types of goods, relative prices in the world economy must change. A permanent real appreciation of the ZZ currency (an improvement in the ZZ economy's real terms of trade) is an integral part of the required change in relative prices.[33]

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[32] An upward movement in the exchange rate in the model represents an appreciation of the ZZ currency (a depreciation of the US currency).

[33] One can legitimately question the model's working assumption that preferences for domestically-produced and foreign-produced goods remain unchanged over time (alternatively stated, that the substitutability in demand of domestic and foreign goods is independent of large changes in the relative sizes of economies). Yet most of the existing analysis of international trade operates with the assumptions built into our analytical framework. Products are assumed to have, in effect, *spatially-determined* characteristics that make them imperfect substitutes (an electronic device produced in Japan not being a perfect substitute for a functionally similar electronic device produced in the United States, a wine from France not being a perfect substitute for an Australian wine made with the same grape , etc.). If one questions the conventional treatment of goods preferences as inappropriate, such doubts could also lead to doubts about the strength of the exchange-

Large changes in exchange rates generate powerful expenditure-switching incentives between the two economies. Thus by the fourth decade of the shock, the ZZ economy begins to import substantially more of the now relatively cheaper goods produced in the US. ZZ exports to the US relative to baseline are inhibited by the appreciation of the ZZ currency. The ratios of real imports to real GDP in the two economies are shown in Figure 15. After the underlying demographic variables have begun to converge to their ultimate steady-state values, the ZZ import ratio has risen some 19 percentage points above its baseline value of 12.9 percent; it ultimately falls back to some 12 percentage points above baseline. The US import ratio reaches a level some 7 percentage points below its baseline level of 12.9 percent, ultimately coming to rest at some 6 percentage points below baseline. As would be expected from the differential effects on the real and nominal exchange rates, taking youth dependency into account *increases* the degree to which the ZZ economy raises imports and the US economy cuts back on imports.

For the initial decades of the shock, the ZZ real trade balance relative to real GDP changes little. As the shock progresses thereafter, however, the expenditure-switching effects cause the ZZ economy to run a larger and larger *deficit* on real trade account (Figure 16). This net import of real resources from abroad provides a cushion of support to the ZZ economy that permits it to sustain a significantly higher level of consumption than would otherwise be possible. The US economy experiences the opposite effect: it must export real resources abroad and correspondingly curtail its consumption relative to what would otherwise be possible.

The medium-run trade deficit of the ZZ economy is *not* associated with a deficit on current account. The ZZ economy not only imports more from abroad. The ZZ economy also saves more relative to baseline so that ZZ financial wealth *rises* relative to baseline (Figure 12).[34] And a fraction of that higher financial wealth is invested abroad at the higher interest rates available abroad. Hence the ZZ economy over the medium run starts to earn a higher flow of investment income from abroad. The net investment income payments received are more than enough to offset the increased deficit on trade account, with the result that the ZZ economy in the medium run begins to experience a

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rate and terms-of-trade effects reported here. Note, however, that even if one were to believe that shrinkages in the relative quantities of similar goods produced at home versus abroad were to induce somewhat higher elasticities of substitution between home and foreign goods *merely because home goods were more scarce in the world*, it would require very great changes in preferences to completely offset the exchange-rate and terms-of-trade shifts that would otherwise be caused by the relative shrinkages in home-produced goods.

[34] Numerous channels cause private saving to rise in the ZZ economy. Among them is the fact that the population aging caused by the large fertility decline requires the government authority operating the intermediate-balanced pension system to raise pension taxes and reduce pension benefits (relative to baseline), which in turn is an incentive for increases in private saving.

significant current-account *surplus* (Figure 17). The surplus reaches a peak during the eighth decade of the shock. Interestingly, the current-account surplus thereafter falls and even returns close to balance for several decades as the two economies move toward their new long-run steady states. Eventually, in the very long run the ratio of the ZZ current balance to nominal GDP settles at a moderate surplus ratio.[35]

The net foreign asset positions of the two economies, shown relative to nominal GDPs in Figure 18, are the integral over time of the current-account imbalances. The ZZ economy – despite the large shock it experiences, causing the economy's output and aggregate consumption to fall far below the levels that would have been observed without the shock – accumulates a large *positive* net foreign asset position, on which it earns a sizable return.

The openness of the ZZ and US economies thus decisively influences the macroeconomic consequences of the demographic shocks. Domestic variables in both economies are strongly influenced by cross-border transactions. Because of the openness of the economy, ZZ domestic variables are partly cushioned from the full impacts of the large ZZ fertility decline. As a counterpart, US variables are adversely buffeted by the larger demographic shock emanating from the ZZ economy. An important component of these cushioning and buffeting effects is associated with the changes in exchange rates. The permanent appreciation in the real value of the ZZ currency enables the ZZ economy to enjoy a large permanent improvement in its real terms of trade with the rest of the world. The opposite effect, a deterioration in real terms of trade, contributes to the adverse effects on the US economy.

Effects on the real exchange rate, trade balances, current-account balances, and net-foreign-asset positions of the two economies are larger when analysis takes into account the economic effects of children than when it does not. For example, the positive effects on ZZ saving and financial wealth resulting from the fertility decline are larger because resources are freed up as support payments to children become smaller. A fraction of the incrementally freed resources from lower child consumption are saved rather than consumed. The ZZ currency appreciates by a larger amount. The associated net capital flows permit the ZZ current-account surplus to be larger by the medium run than it would otherwise be without youth dependency taken into account. The resources freed up by declining numbers of children, partly invested abroad, increase the cushioning effects on the ZZ

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[35] Note that the vertical scales for Figures 16 and 17, chosen to make it easier to identify the dynamic movements of the curves, are dissimilar; the range of the vertical distance in Figure 16 is 30 percentage points of GDP (absolute deviations of the ratio of from -20.0 to 10.0) whereas it is only 3 percentage points in Figure 17.

economy from its openness to the rest of the world. Similarly, the effects of youth dependency increase the degree to which the US economy is influenced by spillovers from the ZZ economy.[36]

It is essential when interpreting the implications of cross-border spillovers to differentiate carefully between aggregate levels of variables and their per-adult (or per-capita) values. The large demographic shock occurring in the ZZ economy will inevitably cause major negative effects on ZZ aggregate output and consumption. As shown in Figure 19, the ZZ path for aggregate real consumption by the medium run must accordingly fall much further below baseline than real aggregate consumption in the US. Yet as is also clear from Figure 19, the ZZ path for aggregate real consumption is significantly *above* the path that would be experienced in the hypothetical case where the ZZ economy is completely closed and therefore unable to cushion its large-cyclical shock through transactions with the rest of the world. The openness of the economy works to mitigate the size of the negative effects on the aggregates.[37]

Now consider the per-adult or per-capita values of such macroeconomic variables. Economy-wide aggregates cannot be straightforwardly used to make normative or welfare judgments about the consequences of demographic shocks such as a decline in fertility. Per capita or per adult measures are likely to be, at least for some purposes, a more useful focus for normative comparisons of pre-shock and post-shock outcomes. Figure 20 plots the percentage deviations from baseline for adult consumption per adult in both economies. Those curves provide a suggestive indication of the relative welfare effects in the two countries. Notwithstanding the fact that the demographic shock in the ZZ economy is much larger than in the US economy, ZZ per adult consumption is actually *higher* than per adult consumption in the US. The difference is sizable in the initial decades of the shock. It is even more marked in the new long-run steady state. Moreover, as Figure 20 reveals, the cushioning effects are so substantial when measured in per-adult (or per-capita) terms that individual adults in the ZZ economy are significantly better off relative to baseline than individual adults in the US economy. Indeed, US adult consumption per adult is markedly *lower* than in the baseline despite the fact that the US population, aggregate US real GDP, and aggregate US consumption are all at higher levels than in the no-shock baseline. When analysis ignores youth dependency and child consumption, adverse spillover effects on US per-adult consumption are even greater than when youth dependency is taken into account.

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[36] When the analysis takes into account public pensions, many of the macroeconomic effects and cross-border spillovers are somewhat smaller than for model variants that exclude public pensions. See Bryant (2004b, 2004a).

[37] Analogous generalizations are true for aggregate real GDPs.

4. Alternative Specifications of Child Support: Sensitivity Analysis

The preceding discussion demonstrates that analytical conclusions about the macroeconomic effects of demographic change, not least the cross-border spillovers, can be significantly influenced by whether youth dependency is modeled or not. The particular features of the modeling of youth dependency condition the quantitative size of its effects. In this section, we report the results of sensitivity analysis along three dimensions: varying the total level of generosity of child support, varying the composition of child support between basic-needs and endogenous components, and varying the age distribution among adult cohorts of the burden of child support.

We discuss results only for the asymmetric, open-economy shock where the ZZ economy experiences the large-cyclical decline in fertility and the US economy experiences the smaller-gradual decline. Except where noted explicitly, the parameters used in the analysis are the same as those underlying the simulations discussed in section 3.

A. Generosity of Child Support

The analysis of section 3 implicitly showed that the generosity of child support influences the size of effects. We now show this conclusion explicitly by reporting a bracketing range of assumptions about the magnitude of $c_j(s,t)$ in equation (5) – and hence $C_j(t)$ in equation (6). In section 3, for the simulation including child consumption, the baseline level of $c_j(s,t)$ was set at 50 percent of the average amount of individual adult consumption in the baseline; of that amount, two fifths was a fixed, basic-needs component while three fifths was time-varying and dependent in simulations on variation in adult consumption. Expressed in terms of equations (5) and (6), c_1 was fixed as 20 percent of initial baseline per-adult consumption and varied over time only with growth of productivity; the coefficient c_2 was set at 0.3. Total consumption for each child was thus set to one half of initial baseline per-adult consumption. The lower bound for child consumption, self-evidently, is zero (both c_1 and c_2 set to zero) and the simulation with those assumptions was the other set of curves in Figures 6 through 20.

An intermediate assumption about the generosity of child support, between the two cases already considered, fixes baseline per-child consumption at 25 percent of baseline per-adult consumption. A high-end assumption sets the total of baseline per-child consumption at 75 percent of baseline per-adult consumption.[38] To contrast these additional cases with the earlier simulations, it is convenient to continue with the benchmark assumption that, for the given total of child consumption, two fifths takes the form of basic needs while three fifths endogenously varies with adult consumption.

As the level of child support is raised from zero (no economic effects of children in the model) to larger and larger fractions of adult consumption, the effects of the fertility-decline shock on domestic macroeconomic variables become larger. The intuition behind this generalization is straightforward: the higher is the ratio of child to adult consumption, the larger is the amount of real resources freed up by a decline in the number of children (those resources no longer being devoted to child consumption). Hence real interest rates must fall further, capital-output ratios must move more, and the effects on per-adult human wealth, financial wealth, and consumption are all bigger. The dependence of the size of real interest rate movements and of the variations in the private saving ratio on the generosity of child support, for example, are shown in Figures 21 and 22.

An analogous generalization applies to external-sector variables. The higher is child consumption relative to adult consumption, the more the ZZ currency appreciates in real and nominal terms. Hence the more the ZZ economy can afford to run a larger real trade deficit (because ZZ national saving is higher, because some of that higher saving is invested abroad, which in turn means that net investment income from abroad becomes larger). With higher rather than low or zero child consumption, the ZZ current-balance surplus is bigger and hence the ZZ net foreign asset position (Figure 23) becomes substantially more positive. Figure 23, plotting the ZZ ratio of the current-account balance to nominal GDP, illustrates the size of these differential external-sector effects.

As expected, the level at which adults are assumed to support child consumption powerfully influences their own consumption paths. This conclusion is highlighted in Figure 24. When a fertility decline occurs, a high level of adult support for children's consumption frees up large amounts of resources for the adults, some part of which raises adult consumption immediately but another part of which is saved, building up adults' income and consumption in the future. For ZZ adults, the quantitative differences among simulations are considerable for alternative levels of child support. The differences become large by the medium run, determining whether per-adult consumption will fall substantially below baseline (child support assumed completely absent) or can hover above baseline (child support assumed at some one third or more of adult consumption). The differences remain very large over the longest run.

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[38] Cutler, Poterba, Sheiner and Summers (1990) used an estimate of child support relative to adult consumption generated by Lazear and Michael (1980), then added expenditures on public education to the Lazear-Michael estimate, and came up with a calculation that child consumption is approximately 72% of adult consumption.

B. Invariant Basic-Needs versus Endogenous Time-Varying Child Support

Our earliest efforts to incorporate youth dependency into macroeconomic models made the simplifying assumption that all child consumption was "basic needs," fixed at some proportion of initial baseline adult consumption. In subsequent work, we developed the specification in equations (5) and (6) that, for any given total level of child support, divides the total between a time-invariant basic-needs component and a discretionary component that is time varying and endogenously dependent on adult consumption. As explained earlier, this richer specification nests several alternative ways to model child consumption.

We illustrate the alternatives in what follows by contrasting the benchmark-case simulation used in the earlier discussion with two additional simulations, one in which the entire amount of child consumption is basic needs and the other in which the entire amount varies endogenously with adult consumption. These additional simulations are extremes, bracketing the range of possibilities. For the purposes of this comparison, the total generosity of child support for all three simulations is set at 50 percent of baseline adult consumption.[39]

To illustrate the sensitivity of results to alternative compositions of child support between the basic-needs and the endogenous-to-adult-consumption components, we report results for the simulations for just three variables: the ZZ ratio of private savings to GDP (Figure 25), the ZZ ratio of net foreign assets to nominal GDP (Figure 26), and ZZ real adult consumption per adult (Figure 27). The differential consequences for these variables are similar to those for most other variables in the model.

As the time-varying endogenous component of child consumption increases relative to the basic-needs component, the sizes of effects on most variables relative to baseline are diminished, with the differences tending to be small in the shorter run but becoming more pronounced in the medium and then longer runs. The largest within-country differences are for variables such as the private savings ratio (Figure 25) and per-adult financial wealth. The differences in effects are especially noticeable in the exchange rate (real and nominal) and in the external-sector variables (for example, Figure 26), again primarily for the medium and long runs.

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[39] In the benchmark-case simulation, as described earlier, c_1 in equation (5) is set as 0.2 times initial baseline adult consumption and c_2 is set at 0.3. In the simulation where child consumption is all basic needs, c_1 is set at 0.5 times initial baseline adult consumption and $c_2 = 0$. For the simulation in which child consumption has no basic-needs component, $c_1 = 0$ and c_2 is set at 0.5.

If one interprets per-adult consumption (Figure 27) as a rough indicator of welfare effects, the largest favorable effects for the ZZ economy are associated with the assumption that all child consumption is basic needs and the smallest favorable effects for the opposite extreme assumption. The benchmark case between the extremes, for which the assumptions seem a priori more plausible, yields intermediate results. The intuition for these results is again straightforward. A reduction in the relative importance of the basic-needs component of child consumption should moderate the effects on macroeconomic variables because a fertility decline "releases" the largest amount of resources to adults for other uses in the case where all child consumption is assumed to take the form of basic needs.

Because our initial results for fertility declines and youth dependency (reported in an earlier version of this paper circulated in 2001) employed the specification that child consumption was entirely the basic-needs component, those early results tended slightly to overestimate the consequences of youth dependency. As Figures 25 through 27 reveal, the extent of overestimation was relatively minor, especially for shorter-run effects. Nonetheless, we feel more comfortable with estimates that assume a significant fraction of child consumption varies endogenously with adult consumption. Pending the availability of better underlying empirical evidence about the composition of child consumption and parents' child-support decisions, our future research will continue to postulate that the basic-needs component of child consumption is only half or less of the total.

C. <u>Alternative Distributions Among Adults of Child Support</u>

We have conducted a variety of sensitivity tests about the age distribution of child support among adult cohorts, specified in equation (7). These tests suggest that a hump-shaped profile for the age distribution, in addition to being more plausible in its own right, also yields effects on macroeconomic variables that appear more reasonable. The effects on key variables from hump-shaped profiles tend to fall in between those from the extreme of a flat contribution from every adult and the extreme of weights that decline exponentially with age with the largest contribution made by the youngest adult cohort. Typically, the effects on macroeconomic variables follow a similar pattern qualitatively, but with the smallest effects associated with the flat-distribution assumption, the largest effects with the exponentially-declining weights assumption, and the effects with the hump-shaped profile falling in between. In future work we plan to restrict attention to one variant or another of the hump-shaped profile (cases in which $\omega_2 \geq \omega_1 \geq 0$).[40]

[40] Our initial efforts during 2001-2002 to study the effects of youth dependency did not empirically implement the richer specification shown in equation (7) for the age distribution of child support among adult cohorts. The initial analysis used either the extreme assumption that the age distribution is completely flat (every adult cohort makes the same contribution regardless of age) or some variant of the artificial assumption that the youngest adult cohort makes the largest

In one set of sensitivity tests, we experimented with choices for the ω_1 and ω_2 parameters that leave unchanged the benchmark identification of which adult cohorts bear the largest relative size of the burden of child support (in our estimates, adults in the 26-32 year age range) but raise or lower their *relative* share of the total burden. In effect, these sensitivity tests raise or lower the peak of the hump-shaped distribution without shifting the peak to younger or older adults. So long as there is already a modest-sized hump in the distribution (which is the case for our empirical estimates), it seems to make relatively little difference to the simulation results if the peak is somewhat raised or somewhat lowered.[41]

Differences among assumptions matter more, however, if we choose parameters that shift the peak of the hump profile to the left or to the right rather than up or down (for any given total generosity of the child-support burden). Shifting the peak of the hump to the left pushes more of the relative burden on to adults younger than ages 28-30. Shifting the peak to the right increases the relative contributions of older adults. Speaking loosely, as the peak of the hump shifts rightwards, grandparents and older middle-aged parents provide relatively more of the child support.

Figures 28 through 30 give the flavor of the differential consequences of shifting the age distribution to the right and the left. Each of these figures reports the curves for a variable for both countries. The curves toward the top of each figure pertain to the ZZ variable; US curves are at the bottom. Outcomes for the US as well as the ZZ variable are shown as a reminder of how differently the asymmetric shocks affect the two economies.

When younger adults are assumed to take on more of the burden of child support relative to the benchmark case, the effects on private saving and financial wealth are increased, the exchange rate and external-sector variables are more strongly affected, the ZZ economy builds up a larger net foreign asset position over time, and accordingly ZZ per-adult consumption rises further above baseline. If the burden of child support falls more on middle-aged parents and grandparents, the effects are dampened relative to those in the benchmark case. Differences due to varying assumptions about the age distribution of child support tend to be smaller in the short run, but are more sizable over the medium

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contribution with contributions by older adult cohorts declining exponentially with their age. Equation (7), as explained earlier, nests those earlier simplified cases. With a suitable choice of the ω_1 and ω_2 parameters, equation (7) lends itself to the more plausible hump-shaped pattern for the age distribution among adults of child support. To save space, we do not report here the simulations for the flat-distribution or the exponentially-declining weights assumptions.

[41] Because the results of these tests show small differences, we omit an account of these tests here.

and long runs as the differential effects on saving and external asset positions cumulate to larger amounts.[42]

5. <u>Concluding Remarks</u>

The analytical approach developed here and in our other papers yields insights into the cross-border dimensions of the macroeconomic consequences of demographic change that are not yet attainable in other approaches. Our primary focus is on exchange rates, external-sector imbalances, international capital flows, and hence spillover effects to other countries.

For an open economy asymmetrically experiencing fertility declines and population aging, negative consequences accompanying the demographic shifts are typically cushioned because the negative effects are shared with the rest of the world. That cushioning and sharing may not be desirable as seen from the perspective of foreigners, but it can produce sizable welfare gains for home residents.

The open-economy dimensions are first-order in importance. Failing to take into account the powerful macroeconomic effects working through exchange rates and cross-border transactions leads to a seriously inaccurate assessment of the net impacts of demographic change.

Careful analysis of the macroeconomic consequences of demographic change needs to focus on both ends of the age distribution: *youth dependency* (transfers from adults to children) and *elderly dependency* (transfers from working adults to older adults through public pensions). Introduction of youth dependency into macroeconomic models generates significantly different inferences about the economic behavior of countries linked to the global economy. Models focusing only on the behavior of adult populations, ignoring children and child support, miss important aspects of how economies respond to demographic changes. Similarly, transfers to elderly dependents through public pension systems, and alternative ways in which the pension systems are managed, have large effects on the evolution of open economies. The effects of public pensions are particularly important if an imbalance in pension tax revenues and pension benefit payments causes large changes in the stock of outstanding government debt.

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[42] The benchmark assumptions used for the hump-shaped profile for the age distribution, based on empirical estimates, are $\omega_1 = 0.084; \omega_2 = 0.102$. The parameters in the simulation where younger adults assume more child support are $\omega_1 = 0.1318; \omega_2 = 0.160$. For the simulation shifting more of the burden on to older adults, the parameter values are $\omega_1 = 0.070; \omega_2 = 0.085$.

This paper concentrates on youth dependency effects and their open-economy aspects. The consumption-saving behavior of individual adults who provide in-vivo transfers to children differs significantly from the behavior of otherwise identical individuals without financial responsibilities for child support. As lower fertility rates reduce the financial burden on existing adults, resources are freed for additional consumption and saving. That reallocation of resources changes the transitional dynamics and the ultimate steady state of the economy compared to what it would otherwise be in an analysis that suppresses the macroeconomic effects of children. Youth dependency's implications for consumption and saving in turn generate significant differential effects on exchange rates and external-sector variables. A large fertility decline induces relatively higher saving, part of which goes into increased assets held abroad. Exchange rates, interest rates, the trade balance and the current account in a well specified macroeconomic model are all powerfully influenced by changes in youth dependency. In turn, large changes in the real terms of trade of a country tend to cause sizable differential feedbacks on its output and consumption. Thus inferences about the evolution of key macroeconomic variables in an open economy – and judgments about policy implications and economic welfare – can be critically influenced by whether or not children are explicitly introduced into the analysis.

We further show in this paper that particular choices made about the modeling of child support and its age distribution across adult cohorts can have significant quantitative influences on analytical and policy conclusions. Higher levels of child support and skewness of that support toward invariant basic needs (away from time-varying dependence on adults' own consumption) both tend to magnify the size of macroeconomic effects. Assuming that younger adult cohorts carry a higher (lower) burden of child support also tends to heighten (diminish) the size of macroeconomic effects. Improvements in our admittedly rudimentary treatment of adult support for child consumption and further empirical research should help analysts in forming better judgments about the details of how youth dependency is incorporated in macroeconomic models.

Many challenges remain for research into the interactions between demographic change and macroeconomic outcomes. The effects need to be carefully studied and refined in more disaggregated, empirically realistic multi-country models. Nonetheless, we like to believe that our research is a significant step forward toward a better understanding of the complexities of the demographic transition the world is now facing. Our general approach should be especially useful to other researchers who are analyzing demographic change in economies experiencing large changes in fertility rates and dependency ratios. Researchers aspiring to build macroeconomic models to study demographic shifts in developing nations, for example, have no choice but to confront the analytical challenges of introducing youth dependency into their models.

Appendix 1:
Further Background on Analytical Approach

This appendix provides further details about our analytical framework, indicating how we treat population dynamics (for both the adult and youth dependent populations), age-earnings profiles for the labor force, economic linkages between the elderly and working adults through public pension systems, and the consumption decision made by adults.

A. Population Dynamics and Youth Dependency

Our augmented framework introduces both youth dependency and elderly dependency. Unlike the standard Blanchard-Weil-Yaari framework, individuals enter the economy not as adults but as dependent children. After a period of life as dependents, youths enter the adult population and begin to supply labor in the workforce. After eventually reaching a threshold age, adults become elderly dependents and receive a public pension to supplement their other income flows. Adults have a different, typically higher mortality rate than children. We also allow for the child birth rate and the mortality rates of adults and children to vary through time. To preserve the advantages of the Blanchard-Weil-Yaari framework for aggregation across individuals, however, we maintain that framework's assumptions that mortality rates are age-invariant and that elderly adults gradually instead of discontinuously withdraw their labor from the workforce.[43]

We denote the size of a child cohort, indexed by s (s is the time of birth) at time t, as $J(s,t)$. Correspondingly, $N(s,t)$ is the size of an adult cohort at time t. The populations of children and adults are interrelated as both evolve through time.

[43] The assumption that mortality rates are age-invariant rather than age-specific departs seriously from reality. Blanchard (1985) himself pointed out that the evidence on mortality rates suggests low and approximately constant probabilities of death from, say, ages 20 through 40; thereafter mortality rates in real life do rise with age – sometimes modeled by "Gompertz's Law" suggesting that mortality rates after puberty rise in geometric progression as in Wetterstrand (1981), reaching mortality rates (in the United States) in the neighborhood of 16 percent by age 80 and 67 percent by age 100. Faruqee (2003a) modifies the simplifying assumption that all adults are subject to the same age-invariant probability of death; with that modification, however, it is no longer straightforward to achieve the macroeconomic aggregation across individuals and age cohorts that is the marked advantage of the Blanchard-Weil-Yaari theoretical framework.

For children, the initial size of a cohort at the time of birth *s* is:

$$J(s,s) = b_j(s)N(s)\,, \tag{12}$$

where $b_j(s)$ is the birth (or fertility) rate at time *s*, expressed as a fraction of the contemporaneous adult population $N(s)$. Youth dependents face the infant/child mortality rate, $p_j(t)$, which can vary through time but is the same for all youths regardless of age. The number of survivors from the initial *J(s,s)* cohort at some later date *t* is given by:

$$J(s,t) = b_j(s)N(s)e^{-\int_s^t p_j(x)dx} \tag{13}$$

Aggregation of all child cohorts over the finite range Δ of childhood ages and then differentiating, using the survivor formula given by equation (13) yields the following expression for the evolution of the total child population, *J(t)*:[44]

$$\dot{J}(t) = b_j(t)N(t) - N(t,t) - p_j(t)J(t)\,, \tag{14}$$

where $N(t,t)$ denotes the outflow of youths into the adult population. More specifically, through continuity, the oldest child cohort passes into adulthood:

$$N(t,t) = J(t-\Delta\,,\ t) \tag{15}$$

This cohort thus also represents the inflow of new adult workers into the economy.

Given a distinct adult mortality rate $p_n(t)$, the survivor formula and dynamics for adult cohorts are given by:

$$N(s,t) = N(s,s)e^{-\int_s^t p_n(x)dx} \tag{16}$$

$$\dot{N}(t) = N(t,t) - p_n(t)N(t) \tag{17}$$

Equation (17) shows that the net change in the adult population in each period depends on the inflow of new entrants to the adult population less the numbers of adults that die. The rate at which new youth entrants pass into the adult population can be viewed as an adult "birth" rate $b_n(t) \equiv N(t,t)/N(t)$. The population growth rate of adults is given by $n(t) = b_n(t) - p_n(t)$, where the adult "birth" rate $b_n(t)$ depends on the sequence of child birth rates in earlier periods.

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[44] We parameterize Δ to be 18. Children are assumed to be wholly dependent for the first 18 years of their life. At the beginning of their 19th year, they enter adulthood, begin supplying labor input, and no longer receive any support payments from older adults.

Given these population measures, the *youth dependency ratio* $\delta(t)$ is simply the ratio of the child population to the adult population:

$$\delta(t) = \frac{J(t)}{N(t)}, \qquad 0 \le \delta \le 1. \tag{18}$$

Differentiating this expression yields the law of motion for the youth dependency ratio:

$$\dot{\delta}(t) \;=\; b_j(t) \;-\; [1+\delta(t)]b_n(t) \;+\; \delta(t)[p_n(t) - p_j(t)] \tag{19}$$

Intuitively, the evolution of youth dependency depends on the comparative rates of growth in the child and adult populations.[45] This differential depends, in turn, on differences in birth and death rates for children and adults (appropriately scaled). Higher fertility rates (i.e., arrival rates of children) tend to raise the youth dependency ratio, while higher child mortality rates tend to lower this ratio, other things equal.[46]

We define an elderly dependency ratio as the proportion of the adult population older than a certain threshold age – indexed by $i(t)$. For example, i might be, say, 47 years of adult life (measured from the moment of entering adulthood at the beginning of the 19th year of life).[47] Given an index value for i that is unchanged through time, the elderly dependency ratio $\phi(t)$ is defined as:

$$\phi(t) = \int_{-\infty}^{i(t)} \frac{N(s,t)}{N(t)} ds. \qquad 0 \le \phi \le 1. \tag{20}$$

In an economy with a constant adult "birth" rate, the elderly dependency ratio would also be constant. For the case where the adult birth rate and death rate are time varying, the elderly dependency ratio evolves according to:

$$\dot{\phi}(t) = \frac{N(i(t),t)}{N(t)} \;-\; [p_n(t) + n(t)]\phi(t). \tag{21}$$

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45 The growth rate of the child population can be written as: $\dot{J}(t) = [b_j(t) - b_n(t)]/\delta(t) - p_j(t)$ which is determined not only by the birth and death rates of children but also by their entrance into the labor force – i.e, the adult "birth" rate, b_n.

46 In steady-state, the following population relationships obtain: $\bar{b}_n = \bar{p}_n$; $\bar{b}_j = \bar{p}_n e^{\bar{p}_j \Delta}$; $\bar{\delta} = \dfrac{\bar{b}_j - \bar{p}_n}{\bar{p}_j}$.

47 If measured from the year of actual birth, the threshold age would be 65 years. The threshold age does not imply "retirement" *per se*; as noted above, there is no discontinuity of labor input at the threshold age. Rather, adults older than i still continue to receive some (but gradually declining) labor income even after they reach the age at which they are defined as "elderly."

Intuitively, the change in the elderly dependency ratio is determined by the relative size of new dependents reaching the threshold age – the first term in (21) – *less* the proportion of the elderly who die during the period – $p_n(t)\phi(t)$ – and *less* a scaling term accounting for growth in the adult population base – $n(t)\phi(t)$.[48]

B. Age-Earnings Profiles and Aggregate Labor Income

The adult consumption problem follows the familiar formulation widely used since Blanchard (1985) but with two major modifications: (1) the inclusion of age-earnings profiles to introduce a life-cycle path to labor income, and (2) the explicit inclusion of parent-child transfers in the adult budget constraint. We elaborate on these two issues and other aspects of the consumer's problem in what follows.

In real-life economies, labor earnings display a hump-shaped pattern across age groups, rising initially as younger workers accumulate experience and seniority, peaking at later middle age, and declining after the peak years of productivity have been passed and workers supply less labor and eventually retire. We account for this life-cycle age-earnings profile by varying effective individual labor input according to age. Moreover, our analytical framework incorporates such a hump-shaped profile in a way that permits a "bottom-up" rather than "top-down" determination of aggregate labor income.

The age-earnings profile is mathematically approximated by specifying the labor input of an individual cohort *s* at time *t* with three exponential terms:

$$l(s,t) = \left[a_1 e^{-\alpha_1 (t-s)} + a_2 e^{-\alpha_2 (t-s)} + (1 - a_1 - a_2) e^{-\alpha_3 (t-s)}. \right] \tag{22}$$

Loosely speaking, the first two terms in (22) may be thought of as representing the decline in an individual cohort's labor supply over time as it ages and (gradually) retires. The third term can be interpreted as reflecting gains in earnings that accrue with age and experience. The restriction on the a_i coefficients on the exponential terms (the third of the coefficients must be equal to $1 - a_1 - a_2$) embodies a normalization that the youngest cohort (for whom $s = t$) has effective labor input equal to unity.

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48 Because death rates are modeled as age-invariant rather than age-specific, only the adult birth rate matters for the share of elderly dependents in the population. As discussed in Faruqee (2000, 2002), the model's assumption that the adult mortality rate is age-invariant has an undesirable consequence: the numbers of elderly in the total population are overestimated relative to the real-life situation in which adult mortality increases as adults become older and older.

Assuming that differences in earnings are attributable to these age-specific differences in labor productivity and labor supply, the relative labor earnings of a particular cohort can be written as:

$$y(s,t) = \left[wage(t)\right] l(s,t), \tag{23}$$

where $wage(t)$ is the economy's average wage rate which grows over time at the rate of labor-augmenting technical change. Consequently, individual labor earnings can change over time because of general growth in labor productivity, assumed to apply uniformly to all cohorts (i.e., after adjustment for their age-specific *relative* productivities). The time-path for individual relative earnings, meanwhile, is assumed to match the cross-sectional age-earnings pattern observed in the data. Given (22) and (23), the parameters $a_1, a_2, \alpha_1, \alpha_2$, and α_3 are calibrated to match non-linear least squares estimates on cross-sectional data for age and earnings for Japan and the United States obtained by Faruqee (2000). The relative productivity parameters are further specified to be exogenous (i.e., fixed over time) in the modeling code.

If (22) and (23) are aggregated over all individual cohorts, aggregate labor income can be written as:

$$Y(t) \;=\; \int_{-\infty}^{t} wage(t)\, l(s,t)\, N(s,t) ds \;=\; wage(t)\, L(t), \tag{24}$$

where L is aggregate labor input adjusted for cohort-specific relative productivities. The definition of labor input for the individual cohort in (22) also permits one to write aggregate L as the sum of three components L_1, L_2, and L_3 where each component reflects an exponential term in (22). Details are in the background paper. Intuitively, changes in aggregate labor input depend on the effective labor supply of new entrants to the labor force, the relative productivity experiences and deaths of existing workers, and the general pace of labor-augmenting technical progress. The specific values of the five coefficients $a_1, a_2, \alpha_1, \alpha_2$ and α_3 in (22) obtained from estimating the age-earnings profile play a critical role in determining the movements of effective labor supply and the evolution of human wealth and consumption over time.

C. <u>Economic Linkages between Elderly and Working Adults through a Public Pension Scheme</u>

The simplified governments in our analytical framework purchase real goods and services (typically, an exogenously set fraction of the national output), raise revenue by taxing the incomes of firms and households, and pay interest on the outstanding stock of their debts. The government also operates a public pension ("social security") system that collects pension-tax revenue from workers and makes pension transfer payments to the elderly.

We implement the public pension system in a simplified pay-as-you-go (PAYG) form, identical for both countries. On the revenue side, adult individuals throughout their working lives pay pension taxes as a given fraction – τ_{SS}, the "pension tax rate" – of their labor incomes. The pension taxes are in addition to income taxes paid on their total (labor plus capital) income. Given our treatment of age-earning profiles, individuals over the age of 65 continue doing some work and receiving a decreasing amount of labor income until the end of their lives. Hence the elderly continue to pay pension taxes after age 65 on that labor income.[49] Since the effective labor input and labor income of the elderly declines sharply as they age beyond the threshold of 65 years, however, the pension taxes paid by elderly cohorts become negligible as they age further.

After a worker reaches the threshold elderly age, he or she begins to receive a pension transfer from the government. In our initial implementation, an individual's pension benefit is not indexed to or otherwise influenced by his or her own earnings history.[50] Rather, for analytical simplicity we assume that the government sets pension benefits so that every elderly individual receives an identical payment. Part of this pension payment is a basic benefit (a “First Tier” component) that is inelastic to the behavior of aggregate labor incomes in the economy.[51] The remaining part (a “Second Tier”) varies endogenously through time in proportion to contemporaneous aggregate labor income.

Seen from the perspective of an individual cohort *s*, the transfer scheme embodied in the public pension system can be summarized as:

$$ptr(s,t) = \begin{cases} -\tau_{ss}(t)y(s,t); \quad s > i(t) \\ \beta_{T1}(t) + \beta_{T2}(t)\dfrac{Y(t)}{N(t)} - \tau_{ss}(t)y(s,t); \quad s \le i(t), \end{cases} \tag{25}$$

where $ptr(s,t)$ is the transfer amount for cohort *s* at time *t*, $\tau_{ss}(t)$ is the pension tax rate, $\beta_{T1}(t)$ is the inelastic Tier-One component of the pension, $\beta_{T2}(t)$ is the “benefit rate” used to calculate the Tier-Two

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49 Given that the elderly in our framework never fully retire, the government’s tax and pension systems are assumed not to penalize them for working in old age. The elderly start collecting the full (age-invariant) pension benefit at age 65. Elderly workers over the age of 65 continue to pay income taxes on their labor and capital income. The recipients of pension benefits, however, are assumed not to be required to pay income tax on those benefits.

50 In real life in (for example) the Japanese or the United States social security systems, pension payments of course are partly a function of an individual’s earnings history.

51 To keep long-run simulations with the model consistent with balanced growth paths, we have so far assumed that $\beta_{T1}(t)$ is adjusted secularly so that it grows at the same rate as labor productivity. Similarly, the nominal value of the Tier-One pension benefit is indexed to the long-run steady-state rate of inflation.

pension component, $y(s,t)$ is the individual cohort's real labor income, and $Y(t)/N(t)$ is average real labor income in the economy as a whole (the aggregate variable to which the benefit rate is applied when calculating the Tier-Two component). Equation (25) states that younger generations ($s > i(t)$) pay pension taxes into the system and receive no benefits. Older agents having reached the threshold elderly age ($s \leq i(t)$) are "pensioners" and receive the two-component benefit. Since pensioners still receive modest amounts of labor income after they reach age 65, as noted above they still pay modest pension taxes on that income. An elderly individual's "net pension benefit" is the difference between the gross benefit and the pension taxes still paid on their declining labor income.

The two-tiered treatment of pension benefits is a general specification somewhat analogous to our two-part treatment of child consumption in equation (5). By setting the benefit rate $\beta_{T2}(t) = 0$, one can examine the implications of making the total pension benefit invariant to current conditions in the economy. Alternatively, if $\beta_{T1}(t) = 0$ whereas $\beta_{T2}(t) > 0$, an inelastic component of the total pension benefit is assumed absent altogether. In the general case, the total pension benefit includes non-zero components for both Tier One and Tier Two.

The aggregate amounts of pension taxes collected and benefits paid by the government in period t are:

$$PT_{SS}(t) = N(t)\tau_{SS}(t)\frac{Y(t)}{N(t)} = \tau_{SS}(t)Y(t), \tag{26}$$

$$PB_{ss}(t) = Eld(t)\left[\beta_{T1}(t) + \beta_{T2}(t)\frac{Y(t)}{N(t)}\right] = Eld(t)\beta_{T1}(t) + \phi(t)\beta_{T2}(t)Y(t), \tag{27}$$

where $PT_{ss}(t)$ and $PB_{ss}(t)$ are the real values of aggregate taxes and aggregate benefits, $Y(t)$ is aggregate real labor income in the economy, and $Eld(t)$ is the number of elderly. As above, $N(t)$ is the total adult population and $\phi(t)$ is the elderly dependency ratio. Equation (26) indicates that the total of pension taxes collected equals the pension tax on average labor income multiplied by the number of adults in the economy. Total pension benefits transferred to the elderly, equation (27), are the sum of aggregate Tier-One benefits and aggregate Tier-Two benefits, with the latter the product of the elderly dependency ratio, the Tier-Two benefit rate, and aggregate labor income in the economy.

In a continuously balanced pay-as-you-go pension system, a difference cannot exist between aggregate revenues from pension taxes and aggregate benefits paid to pensioners. To implement a continuously balanced PAYG system in our framework, one must therefore enforce the condition $PT_{ss}(t) = PB_{ss}(t)$ in the current and all future periods. If pension contributions and benefits are not identical, on the other hand, there will exist a positive or negative "financing gap" in the pension system reflecting the degree of over- or underfunding of current-period benefit payments. To analyze the interactions of demographic changes and public pension systems, it is often necessary to study cases of such over- or underfunding. Accordingly, we define a variable for the financing gap (changes in the real value of the "trust fund" for the pension system):

$$PTFGAP(t) = PT_{ss}(t) - PB_{ss}(t). \tag{28}$$

When the pension trust-fund gap is negative, the deficit in the pension system must be financed by changes elsewhere in the government's budget – either by increases in revenues from income taxes, cuts in government spending on goods and services, or increased government borrowing through additional issuance of government debt. Conversely, pension trust-fund surpluses must result in increases in government spending, reductions in other tax revenues, or retirements of government debt. A pension-system financing gap must therefore be included explicitly as part of the overall government budget identity and must be taken into account in assuring that the government budget satisfies the criterion of intertemporal consistency.

The budget identity of the fiscal authority in our framework may be written in *nominal* terms as:

$$GDEF(t) \equiv R(t)B(t) + P(t)\left[G(t) - T(t) - PTFGAP(t)\right] \quad . \tag{29}$$

Here *GDEF(t)* is the overall nominal deficit in the government's budget (with deficits expressed as positive numbers, surpluses as negative) and *P(t)* is the economy's overall price level. The nominal value of interest payments on the government debt is the product of the nominal interest rate *R(t)* and the outstanding nominal stock of debt, *B(t)*. Nominal spending on goods and services is the product of real spending *G(t)* and the price level. Revenues from income taxes are the product of the price level and real tax receipts: $T(t) = (1 - \tau(t))Y(t)$. The nominal value of the pension trust-fund gap is the product of *PTFGAP(t)* as defined in (29) and the price level.[52]

The corresponding identity for the financing of a nominal government deficit is:

$$GDEF(t) \equiv \dot{B}(t) + \dot{M}(t) \quad ; \tag{30}$$

[52] For expositional simplicity, the equations prior to (29) are expressed in real rather than nominal terms. The empirical model used in the research carefully distinguishes real and nominal values and endogenously determines prices in the two national economies.

$\dot{B}(t)$ and $\dot{M}(t)$ are current-period changes in the nominal values of the stocks of, respectively, government debt and (high-powered) money.

In any carefully specified macroeconomic model, private agents will not hold government debt unless per capita government debt is eventually forced to grow at a rate less than the interest rate paid on the debt. To implement this constraint, our framework specifies an intertemporal fiscal closure rule that is a variant of "debt-stock targeting."[53] Our particular variant assumes that the fiscal authority sets – or has set for it – an exogenous path for a target ratio of nominal government debt to nominal GDP, *BRATIOT(t)* . Given this exogenous target ratio, an endogenous path for the nominal amount of target debt, *BT(t)*, is specified as:

$$BT(t) = BRATIOT(t)\,[NOMGDP(t)] \tag{31}$$

where *NOMGDP(t)* is nominal gross domestic product. The fiscal authority is then assumed to focus on the gap between the target path and the actual path of government debt:

$$BTGAP(t) = \log\left(\frac{B(t)}{BT(t)}\right). \tag{32}$$

As its method of ensuring intertemporal fiscal consistency, the authority is presumed to vary the tax rate on incomes, τ , up or down in response to non-zero values of *BTGAP*. In the language of control theory, the fiscal authority's response has both proportional and rate-of-change terms, but not an integral term. More precisely, the debt-stock targeting behavioral rule (alternatively referred to as the tax-rate reaction function) specifies that:

$$\dot{\tau} = \gamma_1(BTGAP(t)) + \gamma_2(\dot{BTGAP}(t)) \quad . \tag{33}$$

Higher values of γ_1 and γ_2 produce faster adjustment of the income-tax rate to a level that eventually brings *BTGAP* back to zero. (Non-zero values of *BTGAP* occur transitorily, but not permanently.) Lower values of γ_1 and γ_2 permit the government's deficit and debt to deviate for a longer time before ultimately being pulled back to levels that produce intertemporal budget consistency.[54]

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[53] The differences between alternative specifications of intertemporal fiscal closure rules are carefully described in Bryant and Zhang (1996a).

[54] Income taxes in the empirical model are paid on both labor incomes and capital incomes. The income-tax rate applies to the sum of labor and capital income; in the empirical model this income-tax total is allocated to laborers and owners of capital in proportion to the shares of labor and capital in output.

The pension system has four key policy variables: the pension tax rate τ_{SS}, the inelastic Tier-One benefit β_{T1}, the benefit rate for the Tier-Two component β_{T2}, and the nominal financing gap, $PTFGAP(t)P(t)$. Assumptions about these four variables control how the model's pension systems operate in practice. In *continuously balanced* PAYG pension systems, the trust fund gap is treated as exogenous and is forced to stay equal to zero in every period. A first extreme variant in this group, the *tax-balanced system*, sets the benefit rate β_{T2} as exogenous and constant, which means that the pension tax rate τ_{SS} must be treated as endogenous and adjusted period by period to keep the financing gap exactly on its continuously balanced path. The opposite extreme variant, the *benefit-balanced system*, reverses the roles of β_{T2} and τ_{SS}; the pension tax rate is treated as exogenous and constant, with β_{T2} forced to adjust endogenously and continuously to keep the trust fund gap at zero. A third, intermediate variant is a combination of the other two; this *intermediate-balanced system* keeps the trust fund continuously balanced by adjusting *both* τ_{SS} and β_{T2}. Specifically, τ_{SS} is adjusted upward by enough to offset one half of any incipient trust-fund deficit in the current period and β_{T2} is adjusted downward by enough to offset the other half of the incipient deficit (and vice versa if the trust fund in the current period runs an incipient surplus).[55]

In all variants of *unbalanced* PAYG pension systems, the pension tax rate and the two benefit components β_{T1} and β_{T2} are set exogenously (for example, at constant values) while $PTFGAP(t)P(t)$ adjusts endogenously through time, taking on non-zero values. A pension-fund actual deficit or surplus proximately leads to an overall budget deficit or surplus and therefore proximately causes an increase or decrease in the stock of the government's debt. The debt-stock targeting behavior of the fiscal authority then plays a crucial role in determining how unbalanced pension systems affect macroeconomic variables.[56]

D. Adult Consumption

The individual adult consumer in our framework can be described as solving the following maximization problem:

55 In real life PAYG systems, maintaining pension taxes unchanged or maintaining pension benefits unchanged would of course have quite different political and economic implications. Probably neither extreme would be politically realistic. In actual PAYG systems, moreover, it is implausible to suppose that the pension authorities vary tax rates, benefit rates, or both continuously, period by period, so as to keep the pension trust fund exactly balanced. Our assumptions are analytical benchmarks, not realistic depictions of PAYG systems.

56 For further discussion of alternative pension systems and their effects, see Bryant (2004a).

$$\max \int_t^{\infty} e^{-(\theta + p_n(x))(t-x)} u(c(s,x))dx \tag{34}$$

subject to:

$$w(s,t) = fw(s,t) + hw(s,t) \tag{35}$$

$$\begin{aligned} \dot{fw}(t) = &\left[(r(t) + p_n(t))\, fw(s,t) + y(s,t)\right](1 - \tau(s,t)) \\ &+ pben(s,t) - \tau_{SS}(s,t)y(s,t) \ - \ c(s,t) \ - \ v(s,t) \end{aligned} \tag{36}$$

$$hw(s,t) \equiv \int_t^{\infty} [y(s,x)(1 - \tau(s,x) - \tau_{SS}(s,x)) - \ v(s,x)] e^{-\int_t^{v} (r(\mu) + p_n(\mu)) d\mu} dx \quad . \tag{37}$$

Here u(.) is the utility function of the individual adult, assumed to be of the constant relative risk aversion (CRRA) form, $u(c(s,t)) = \frac{c(s,t)^{1-\sigma} - 1}{1 - \sigma}$, where σ is the coefficient of relative risk aversion, and 1/σ is the intertemporal elasticity of substitution. Adults are assumed not to derive explicit utility gain from child consumption; hence only the consumption at time t of an individual adult born at time s, *c(s,t)*, directly enters the utility function in (34). The total wealth of the adult individual at time t, *w(s,t)* is the sum of *fw(s,t)*, wealth in the form of financial assets, and *hw(s,t)*, human wealth defined as the present value of expected future labor income net of taxes on labor income and *in vivo* transfers to children. Transfers made by adults to support the consumption of youth dependents, *v(s,t),* are discussed in section 2B of the text above. The other variables in the preceding equations are: *r(t)*, the real interest rate at time t; *θ*, the time preference rate; *y(s,t)*, the pre-tax labor income at time *t* of the individual born at time *s*; $\tau(s,t)$, the income-tax rate on that individual's combined labor and capital income; *pben(s,t)*, the pension benefit (if any) received at time *t* by that individual[57]; and $\tau_{SS}(s,t)$, the pension tax rate on labor income. Disposable income for the individual after taxes is $\left[(r(t) + p_n(t))\, fw(s,t) \ + \ y(s,t)\right](1 - \tau(s,t)) + pben(s,t) - \tau_{SS}(s,t)y(s,t)$. The individual's saving in the period – the net change in financial wealth given by (36) – is disposable income *less* taxes *less* own consumption *less* transfer payments for child support.[58]

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57 It is assumed that pension benefits received by the elderly are not subject to income tax.

58 With a positive probability of death, adult individuals discount the future at a rate higher than their pure time preference rate. The effective discount rate for an individual is therefore $\theta + p_n(t) > 0$. The presence of perfect annuity markets as first introduced by Yaari ensures that individual wealth grows at the rate $r(t) + p_n(t)$, where the premium paid by the perfectly competitive insurance company to an individual during his lifetime is equal to $p_n(t)$ (equation (36)).

Solving the utility maximization problem gives adult consumption as a linear function of total wealth:

$$c(s,t) = \frac{1}{\Psi(t)}[\,fw(s,t) + hw(s,t)]. \tag{38}$$

where $1/\Psi(t)$ is the marginal propensity to consume out of wealth. The dynamics of $\Psi(t)$ are given by:

$$\dot{\Psi}(t) = -1 - \frac{1}{\sigma}[(1-\sigma)(r(t) + p_n(t)) - (\theta + p_n(t))]\Psi(t) \tag{39}$$

The marginal propensity to consume out of wealth in the general case of the CRRA utility function depends, as is well known, on the intertemporal elasticity of substitution (EIS) and on the entire sequences of future interest rates and future adult mortality rates. This dependence is readily evident in equation (39). In contrast, when the EIS is assumed to be unity (the case of logarithmic utility, with $\sigma = 1$) and when the adult mortality rate is assumed to be constant rather than time varying, the marginal propensity to consume out of wealth reduces to the simple form of a constant, the sum of the time preference rate and the mortality rate ($1/\overline{\Psi} = \theta + \overline{p}_n$).[59]

When one aggregates over the consumption functions of individual decision-making units (equation (38)), total adult consumption *C* is given by:

$$C(t) = \int_{-\infty}^{t} c(s,t)N(s,t)ds = \frac{1}{\Psi(t)}[FW(t) + HW(t)], \tag{40}$$

where *C(t)* is aggregate adult consumption and *FW*(*t*) and *HW*(*t*) are aggregate financial and human wealth. The evolution of *FW*(*t*) is described in equation (2) in the text.

Aggregate human wealth is a stock variable representing the present value of economy-wide labor income (adjusted for the varying ages and relative productivities of different cohorts). Its change through time is given by:

$$\dot{HW}_t = \frac{d}{dt}\int_{-\infty}^{t} hw(s,t)N(s,t)ds$$

[59] The net response of consumption to changes in the real interest rate depends on the relative strength of substitution and income effects. With a low rather than high value of EIS, consumers act less strongly to shift their consumption intertemporally; the substitution effect is thus smaller relative to the income effect. For any given shock to the economic system, the real interest rate thus must adjust by a larger amount the lower is the value of EIS. Empirical evidence, in our view, is more consistent with σ having a value smaller than unity (for example, 0.5 or even as small as 0.3) than with the assumption that $\sigma = 1$ (logarithmic utility).

$$= \ hw(t,t)b_n(t)N(t) \ + \ r(t)HW(t) \ - \ Y(t)(1-\tau(t)) \ - PTFGAP(t) - \ C_j(t) \ . \qquad (41)$$

Equation (41) shows that the incremental change in aggregate human wealth at time *t* is influenced by the additional human wealth of the newest adult cohort coming of age at time *t*, $hw(t,t)$. The shape of the labor-earnings profile – embodied in the five parameters $a_1, a_2, \alpha_1, \alpha_2$, and α_3 in equation (22) – has a critical influence through time on the behavior of $hw(t,t)$ and hence of aggregate human wealth, *HW(t)*.

Income taxes and child-support transfers influence aggregate human wealth. Any imbalance in the pension trust fund between pension tax revenues and benefit payments, the variable *PTFGAP(t)* in (41), also affects the dynamics of aggregate human wealth. When the pension system is continuously balanced so that current-period pension taxes exactly cover current-period pension benefits paid to the elderly ($PTFGAP(t) = 0$), the *direct* effect of the pension system on aggregate human wealth nets out to zero. However, even for the continuously balanced cases where *PTFGAP* never differs from zero, the inclusion of a public pension system in the framework has important *indirect* effects on human wealth (and hence on consumption and other key macroeconomic variables) through the evolution of *hw(t,t)*, the human wealth of the newest cohort just entering adulthood and the workforce (see below).

Given the age-earnings profile, adult support of child consumption, and the public pension system, the dynamics of human wealth for the new cohort of individuals entering adulthood and working life are:

$$hw(t,t) = \left[\sum_{k=1}^{3} hw_i(t,t) \right] - hw_{v1}(t,t) - hw_{v2}(t,t) \ + \ hw_{ss}(t,t) \qquad (42)$$

Equation (42) defines the human wealth of the newest individual cohort entering the adult population at time *t* as the sum of six components. The first three components derive from the concave time profile of labor income as described in equations (22) through (24) and are given by:

$$\dot{hw}_k(t,t) = [r(t) + p_n(t) + \alpha_k] hw_k(t,t) - a_k wage(t)[1-\tau(t) - \tau_{ss}(t)]; \quad k \in \{1,2,3\} \ . \qquad (43)$$

The $hw_k(t,t)$ equations allow for both the income and pension taxes that are paid on labor income. The fourth and fifth components of (42) reflect the impact of parent-child transfers on human wealth, with $v(t)$ as determined in equation (7) in the text above:

$$\dot{hw}_{v1}(t,t) = (r(t) + p_n(t) + \omega_1) hw_{v1} - \ z_1 v(t) ; \qquad (44)$$

$$\dot{hw}_{v2}(t,t) = (r(t) + p_n(t) + \omega_2) hw_{v2} - \ (1-z_1) v(t) \ . \qquad (45)$$

The final component, $hw_{ss}(t,t)$, is included to reflect the real value of the stream of pension benefits that the new adult cohort expects to receive eventually after reaching the threshold elderly age (47 years in the future, $t+\Lambda$), discounted back to the present:

$$\dot{hw}_{ss}(t,t) = [r(t)+p_n(t)]hw_{ss}(t,t) \quad - \quad [\beta_{T1}(t+\Lambda)+\beta_{T2}(t+\Lambda)\frac{Y(t+\Lambda)}{N(t+\Lambda)}]e^{-\int_t^{t+\Lambda}(r(s)+p_n(s))ds} \tag{46}$$

The second term in square brackets in (46) is the per-elderly gross pension benefit expected in the future at the date today's new adults expect to reach the threshold elderly age.

Note that even for a "pure" PAYG case in which aggregate pension taxes continuously equal aggregate pension benefits, the discounted stream of an *individual* new worker's future pension taxes will not necessarily be equal to the discounted stream of pension benefits that that worker expects to receive from age 65 until death. Consequently, even a continuously balanced PAYG pension system affects individual consumption and saving behavior through its effect on individual human wealth. The effects for particular individuals on human wealth, consumption, and saving are influenced even more strongly when the trust-fund gap in the pension system is not continuously zero.

Appendix 2:
Baseline Solutions and Specification of Illustrative Shocks

The empirical analysis in the paper starts by developing one or more model-consistent, steady-state baseline solutions for the evolution of the ZZ and US economies. To provide analytical transparency, each economy is assumed to follow identical paths and exhibit identical behavior along these steady-state baselines.

The fertility (child birth) rate, the child mortality rate, and the adult mortality rate are the key exogenous demographic variables. The initial baselines in this paper assume that in both economies the child birth rate has a constant value of 0.02504 (the new child cohort each year being slightly more than 2-1/2 percent of the adult population that year). The child mortality and adult mortality rates are constant at, respectively, .0075 and .015; the adult mortality rate implies that individuals at age 19 entering the workforce expect to live roughly another 67 years. These baseline assumptions about fertility and child mortality have the consequence that youths enter the baseline adult labor force at the constant rate .02 (2 percent per year), which in turn results in the total population growing slowly in the baseline at the constant rate .005 (1/2 percent per year). Labor productivity (labor-augmenting technical change) and the price level both grow in the baselines at 1/2 percent per year.

The intertemporal elasticity of substitution in consumption (EIS) is set at 0.5. Our preferred working value for the EIS is this value of 0.5, but we have studied values as low as 0.3 and as high as unity (logarithmic utility). For illustrative simulations comparing alternative values of the EIS, see Appendix 3. Our experimental results for alternative values of the EIS make it clear that analytical and policy judgments depend sensitively on how researchers calibrate their models for this key parameter.

One third of consumers are assumed to be borrowing constrained and therefore consume only out of current-period income rather than smoothing intertemporally. The elasticity of substitution in the CES production function is set at unity (the Cobb-Douglas case).

For expositional simplicity, we assume that central banks use a money-targeting reaction function in setting the short-term nominal interest rate. We use values for the coefficients in the governments' tax-rate reaction functions that cause government debts in shock simulations to return fairly promptly to their target, baseline paths.

The analysis in section 3 of the paper seeks to isolate the effects of demographic shifts with and without youth dependency. Thus two slightly different baseline solutions for the model are prepared. For shorthand here, we label these as "C-IB"– a baseline where children and the intermediate-balanced version of the pension system are both included – and "0-IB"– an otherwise similar baseline except that children are not included.

For the C-IB baseline, we use a set of benchmark assumptions about child support. The total generosity of baseline individual child support is equivalent to one half the amount of the average for baseline individual adult consumption. Of this amount, two fifths is assumed to be a fixed, basic-needs component; the remaining three fifths is assumed to be time-varying and dependent on variation in baseline adult consumption. In terms of equation (5) in the text, c_1 is determined as 20 percent of baseline $C(t)/N(t)$ and varies over time only with growth of productivity; the coefficient c_2 is set at 0.3. The coefficients determining the distribution of child support across different ages of parent cohorts $-\omega_1$ and ω_2 in equation (7) – are set at estimated values of, respectively, 0.084 and 0.102, resulting in an age distribution that is hump-shaped and concentrated most heavily on adults in their late twenties.

Baseline solutions of the model depend on the baseline values set for the pension tax rate τ_{SS} and the pension benefit parameters β_{1T} and β_{2T}. For an illustrative pension benefit in baselines, we use a value roughly the order of magnitude of the average size of benefits relative to average wages in both the Japanese and U.S. public pension systems. If the pension trust fund in the baseline is assumed to be kept exogenously fixed at zero, if the Tier-One component of pensions is set at 10% of average per-worker labor income, and if the Tier-Two benefit rate β_{2T} is set at 0.22, the baseline pension tax rate consistent with those assumptions typically falls into the range 0.12 to 0.14. This pension tax rate also is broadly in line with actual experiences in the Japanese and U.S. public pension systems.

When shocks are applied to the model, the "intermediate-balanced" variant of a balanced pension system maintains a continuous equality between pension tax revenues and pension benefit payments by adjusting β_{T2} and τ_{SS} in opposite directions (Appendix 1). For either the C-IB or 0-IB case, therefore, τ_{SS} is adjusted upward by enough to offset one half of any incipient trust-fund deficit in the current period and β_{T2} is adjusted downward by enough to offset the other half of the incipient deficit (and vice versa if the trust fund in the current period runs an incipient surplus). In the shock-free C-IB and 0-IB baselines themselves, τ_{SS} and β_{2T} remain constant at values of, respectively, 0.1276 and 0.32.

Throughout the analysis in sections 3 and 4, the adult mortality rate and the child mortality rate remain unchanged at the baseline values of, respectively, .0075 and .015. The analysis specifies two shocks to the birth rate. One is large-cyclical, the other smaller-gradual (see again Figure 2). In the large-cyclical fertility shock, the child birth rate falls persistently from its baseline level of roughly 2.5 percent to about 1.0 percent over a period of some 5-1/2 decades; it remains at that low rate over the next three decades; then it gradually rises back to a new level of about 1.7 percent; it remains stationary at that level thereafter. Given the mortality rates, the eventual birth rate results in a steady-state population that is stationary. For the smaller-gradual fertility shock, the birth rate starts from the same initial baseline rate but then declines slowly and monotonically. The birth rate eventually reaches the same steady-state rate of 1.7 percent as that reached finally in the large-cyclical shock. Thus the smaller-gradual shock also results ultimately in a stationary population. The two shocks reach the ultimate steady-state rate at the same point in time; the lengths of the time period over which the shocks occur is therefore identical. Unlike in the large-cyclical shock, however, the birth rate in the smaller-gradual shock does not follow a U-shaped demographic pattern of sharp fall and then partial recovery.

As shown clearly in Figures 2 and 4, the large-cyclical shock measured in terms of its effects on the total population is much larger in magnitude than the smaller-gradual shock. Population growth becomes negative by the third decade of the simulation and falls all the way to a negative rate of 1/2 percent per year during the period when the birth rates are at their lowest point.[60] As the birth rate then recovers, the population growth rate gradually becomes less negative, eventually reaching a new steady-state rate of zero as the birth rates level out at their new steady-state rates.[61]

Figure 3, discussed briefly in the text, indicates the large consequences of the fertility shocks for the youth-dependency and elderly-dependency ratios. By coincidence, the two ratios are nearly the same in the initial baseline (both children and elderly being roughly 40 percent the size of the adult population). From that 40 percent ratio, in the large-cyclical shock elderly individuals ultimately reach a peak plateau of some 63 percent of the adult population. After that peak is reached, the elderly ratio falls back substantially, eventually settling at about 50 percent of the adult population (significantly above its initial baseline level, but well below the intermittent peak reached when the demographic shock has its greatest effect on the relative numbers of elderly). The youth-dependency ratio in the large-cyclical shock has a reverse cyclical swing to that in the elderly ratio, about as marked but occurring earlier in time. When the new steady-state situation is eventually reached, youths have fallen to only 29 percent of the adult population. Under the smaller-gradual shock, the two dependency ratios reach the same long-run steady-state paths but do so gradually and monotonically without any cyclical swing.

As discussed in the text, the situation is *symmetric, closed-economy* if the ZZ and the US economies both experience the large-cyclical shock, or if they both experience the smaller-gradual shock. (When a shock is identical in both regions, the model produces identical simulation paths for both economies. Each economy behaves as though it were completely closed; external-sector balances remain at zero and the exchange rate remains unchanged at its baseline value of unity.) The situation is *asymmetric with cross-border spillovers* when the ZZ economy experiences the large-cyclical shock and the US economy experiences the smaller-gradual shock.

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60 This path for population growth is broadly analogous to that projected by the Japan National Institute of Population and Social Security Research, Ministry of Health and Welfare, medium variant projections (1997, 2002) and further discussed in Takahashi, Kaneko, Ishikawa, Ikenoue, and Mita (1999).

61 The shock is constructed so that the economy experiencing it in the long run ultimately settles into a new steady state in which not only the total population but also its distribution by age are stationary.

Appendix 3:
Alternative Assumptions about Adult Consumer Behavior

Our analytical framework permits an explicit assumption about the value of the consumers' elasticity of intertemporal substitution (EIS) – see equations (38) and (39) in Appendix 1. This parameter plays an important role in determining several aspects of the model's behavior in simulation experiments. We briefly highlight this sensitivity in this appendix.

The situation in which the EIS is unity is the often-used assumption of logarithmic utility. If a researcher postulates values of the EIS progressively less than unity, the model's simulations of the effects on many macroeconomic variables often grow progressively larger.

The benchmark case reported in section 3 of the paper embodied an assumed value of the EIS of 0.5. To highlight the effects of varying the EIS, we show here several charts each containing a curve for that benchmark simulation plus two additional curves for simulations identical with the benchmark case except for the assumed value of the EIS. One of the additional simulations uses a high value for the EIS of unity. The other drops the EIS all the way to the value of 1/3.[62]

Figure A3-1 again plots the deviations from baseline of the ZZ real short-term interest rate in response to our standardized asymmetric fertility decline. The curve for the benchmark simulation (EIS value of 1/2) is the same curve shown in Figures 10 and 21. Assuming a lower (higher) value of the EIS means that the strength of the intertemporal substitution effect in response to interest-rate changes diminishes (increases) relative to the income effect. With a lower (higher) EIS, moreover, consumers respond absolutely less (more) to interest-rate changes. For a given size shock in the model, therefore, interest rates are forced to change by larger amounts for lower values of the EIS. In the example here, cutting the value of the EIS to 1/3 nearly doubles the size of the medium-run cyclical dip in the real interest rate. Raising the EIS to unity cuts the size of the cyclical dip roughly in half.

Effects on private saving and financial wealth are also highly sensitive to the value of the EIS. Note in Figure A3-2, for example, that ZZ private savings rises very strongly in the shorter and medium runs when the EIS is as low as 1/3, and still quite strongly for the benchmark value of 1/2. But when the EIS is as high as unity, the private saving ratio *falls* in the short and early medium run, a very marked difference in behavior.

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[62] We believe cases where the EIS takes on a value markedly less than unity are more consistent with the available empirical evidence (studies in which researchers have tried to estimate the value). For example, plausible estimate of the EIS appear to be as low as 0.5 or even 0.3.

In the external sector of the ZZ economy, when the EIS is set lower than in the benchmark case the ZZ currency must then appreciate more, the ZZ current-account balance must move into a larger surplus (shown in Figure A3-3), and hence the ZZ net foreign asset ratio must rise to still more positive values. Opposite effects result when the EIS is raised above the benchmark value. Interestingly, the sign of the nominal trade balances and current-account balances in the short run can be reversed for a value of the EIS as high as unity (see again Figure A3-3). Thus even the direction – and most definitely the magnitude – of net capital flows associated with asymmetric fertility declines can be sensitive to the model's assumption about the value of the EIS.

Most consequentially of all, for values of the EIS lower than the benchmark assumption, ZZ per-adult consumption rises more initially, has a smaller cyclical downswing during the medium-run playing out of the shock, and is ultimately significantly higher in the long-run steady state (Figure A3-4). In contrast, an EIS value as high as unity exacerbates the medium-run cyclical dip in per-adult consumption and results in an outcome for the long-run steady state that is much less favorable.

Note that the effects of progressively lowering the EIS tend to be non-linear. For example, changes in the effects resulting from a drop in the assumed elasticity from 1/2 to 1/3 are, for some variables, virtually as large as the changes in effects from dropping it all the way from unity to 1/2.

These differential results for alternative values of the EIS make it clear that analytical and policy judgments can sensitively depend on how researchers calibrate their models for this key parameter. In our future research we plan to study this issue further.[63]

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[63] From our research so far, model results are relatively insensitive to alternative assumptions about the fraction of consumers that are borrowing constrained. Faruqee and Laxton (2000) explain why an assumption about the fraction of consumption that is borrowing-constrained is less consequential than the assumption about the value of the EIS.

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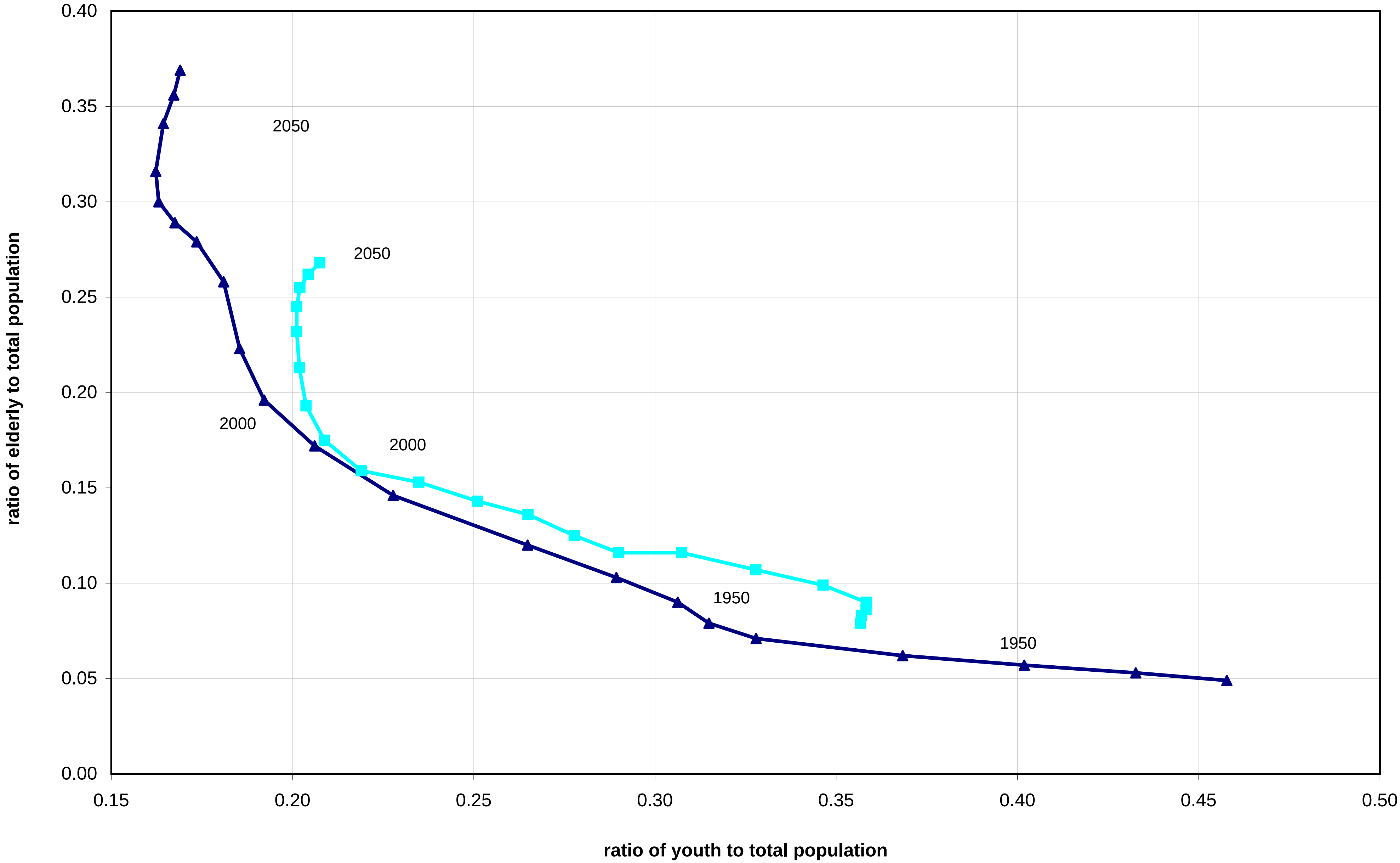
Figure 1. Evolution of Youth Dependency and Elderly Dependency: Japan versus All Developed Countries (1950-2050)



Source: United Nations (2001)
Youth: Younger than 20 year-old. Elderly: Older than 65 years old.

Figure 2. Two Illustrative Shocks for Fertility Decline and Implications for the Endogenous Growth Rate of Total Population

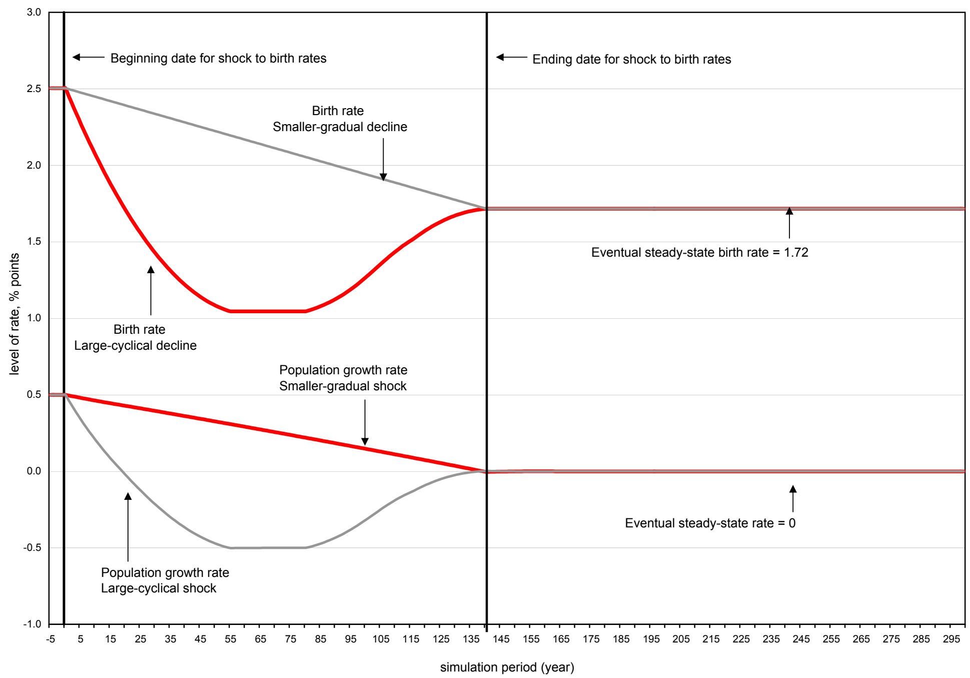


Figure 3. Youth and Elderly Dependency Ratios for the Two Illustrative Shocks

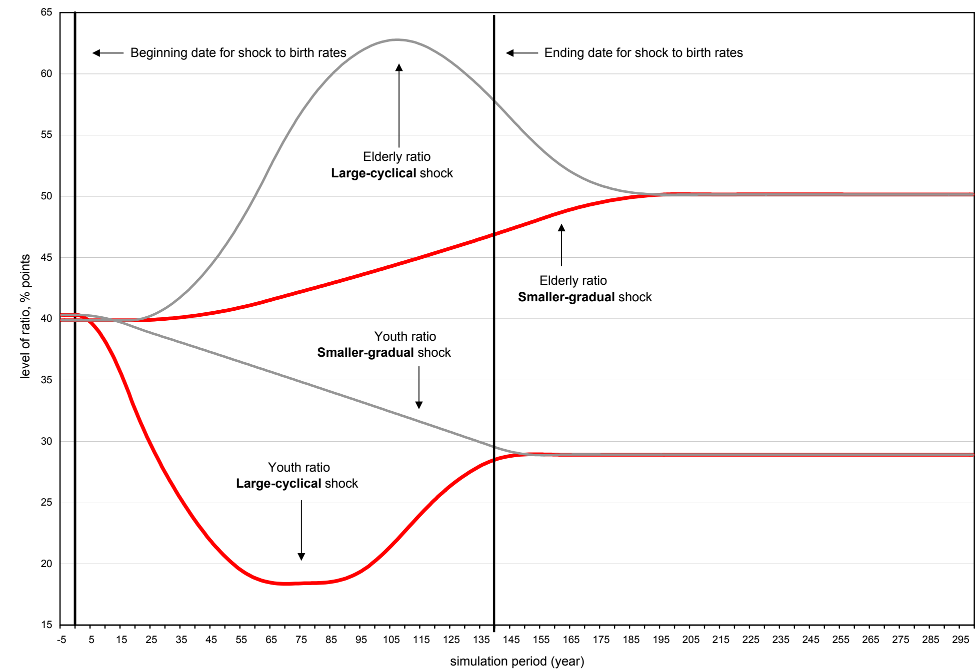


Figure 4. Adult Populations and Effective Labor Forces for the Two Illustrative Shocks

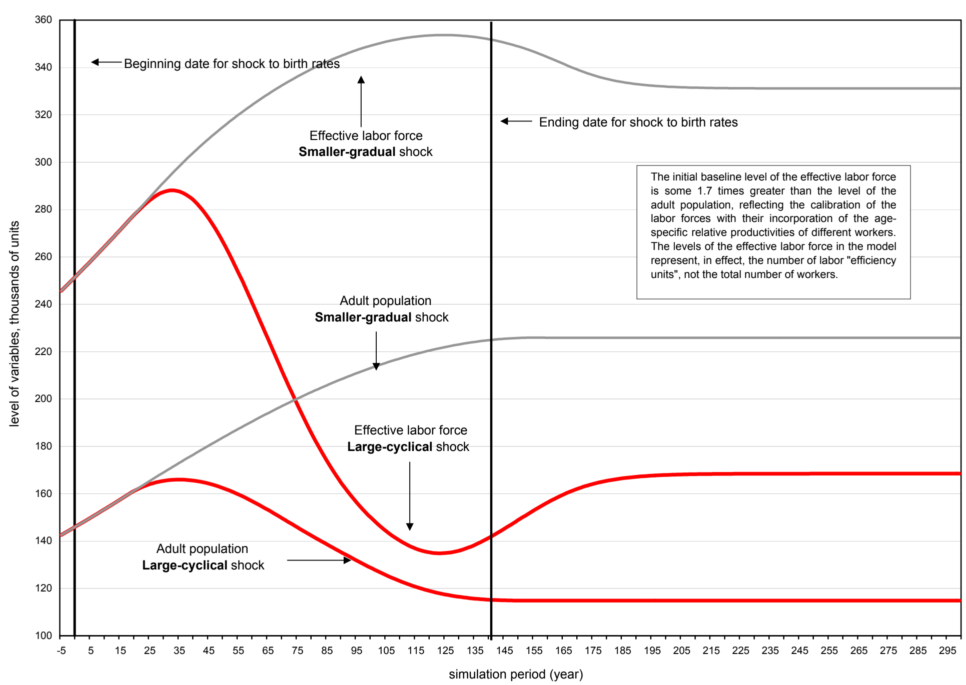


Figure 5. Ratio of Effective Labor Forces to Adult Population for the Two Illustrative Shocks

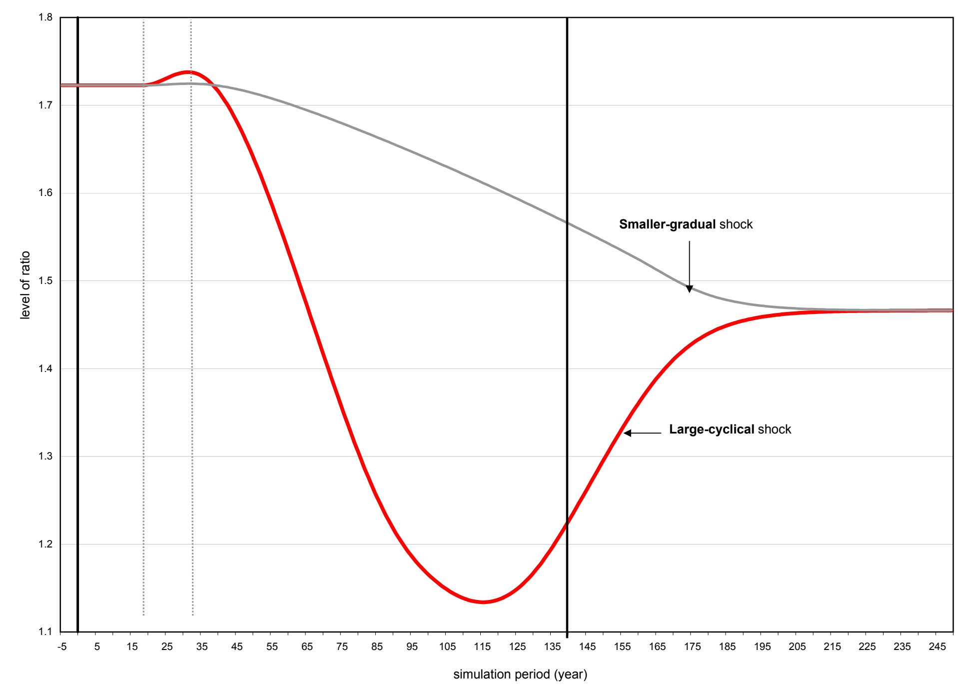


Figure 6. Real Short-term Interest Rates in Both Economies
Symmetric (Closed Economy) Shocks

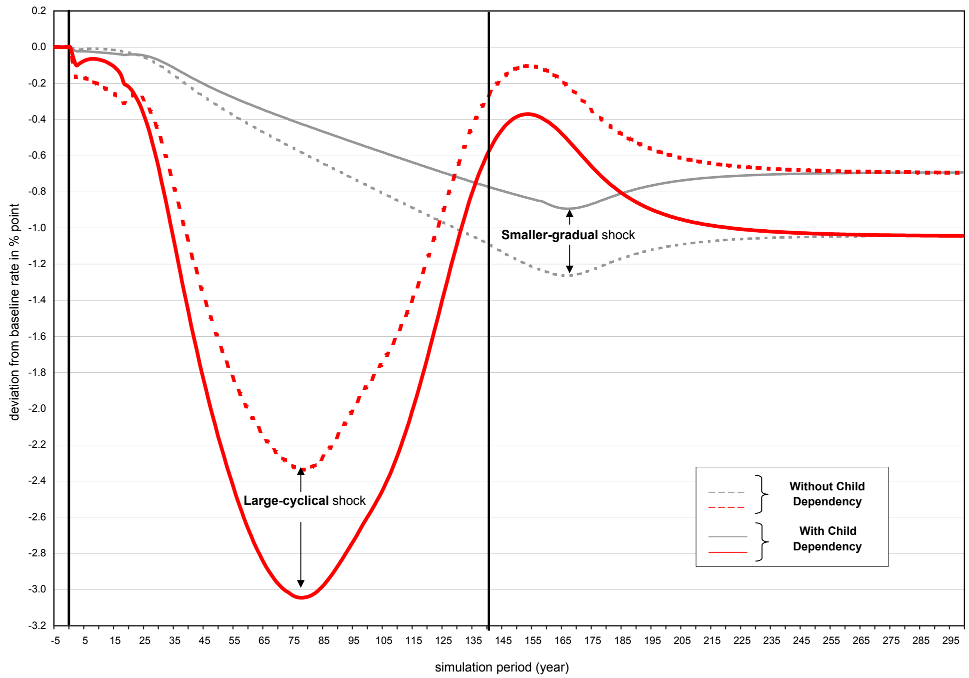


Figure 7. Capital-Output Ratios in Both Economies
Symmetric (Closed Economy) Shocks

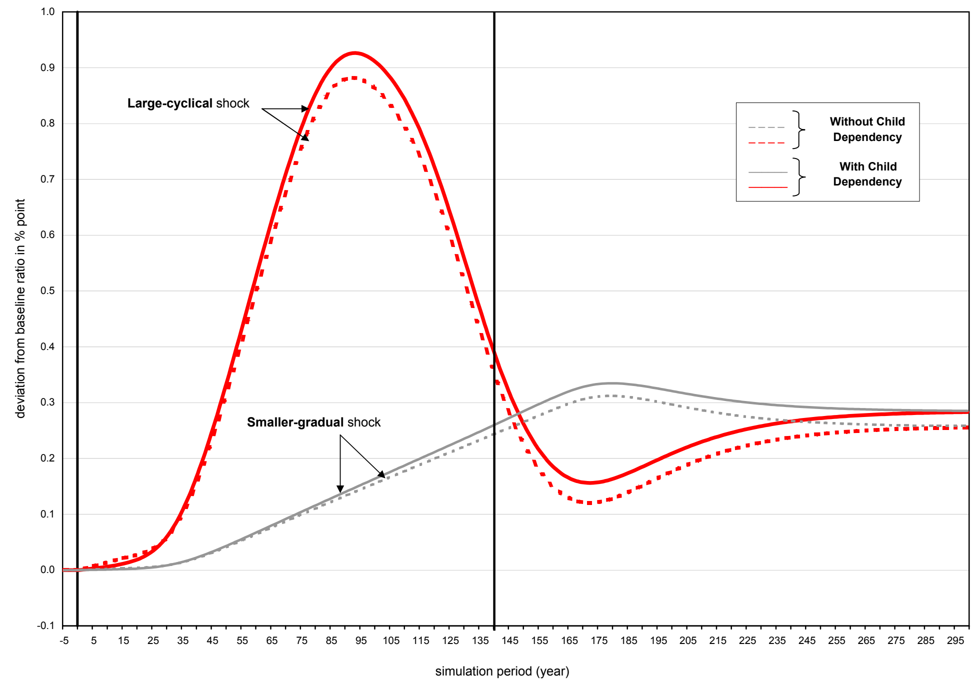


Figure 8. Financial Wealths per Adult in Both Economies
Symmetric (Closed Economy) Shocks

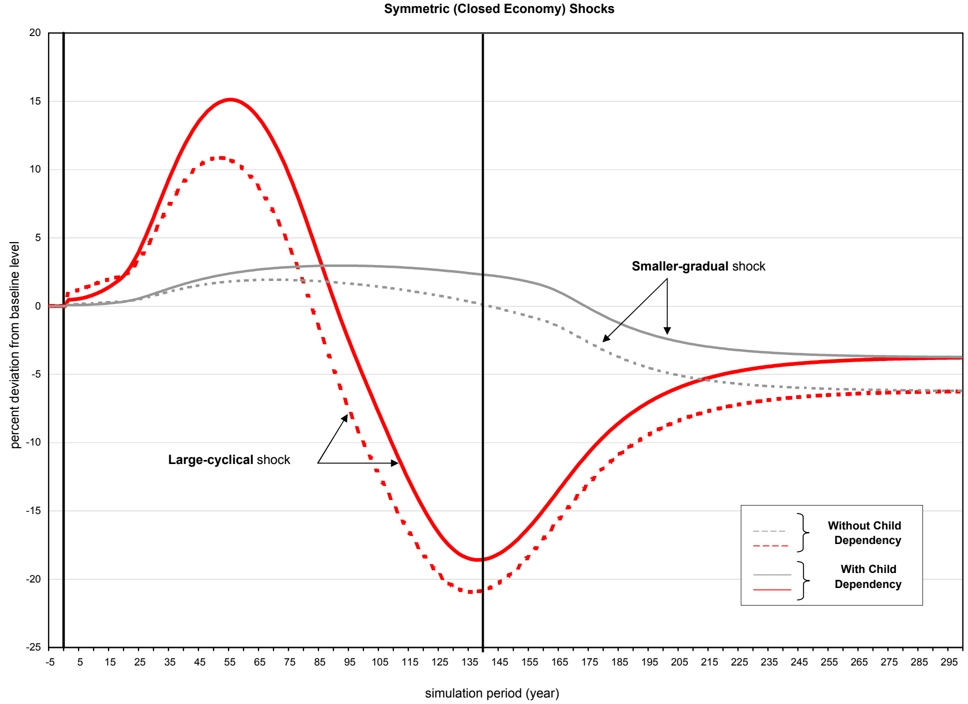


Figure 9. Real Adult Consumption per Adult in Both Economies
Symmetric (Closed Economy) Shocks

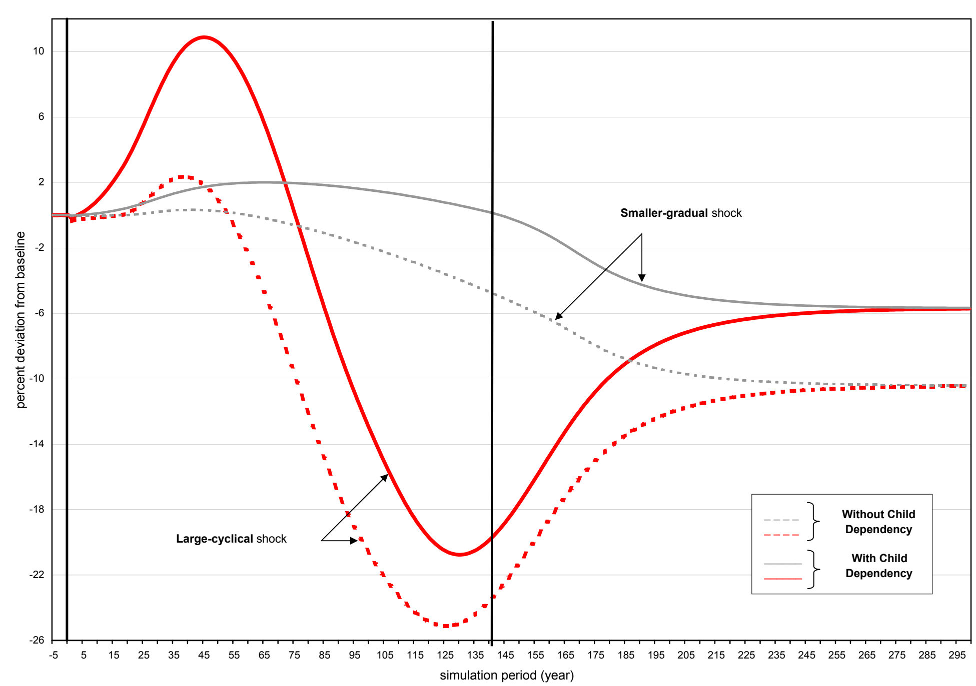


Figure 10. Real Short-term Interest Rates, With and Without Youth Dependency
Asymmetric Shock with Cross-Border Spillovers

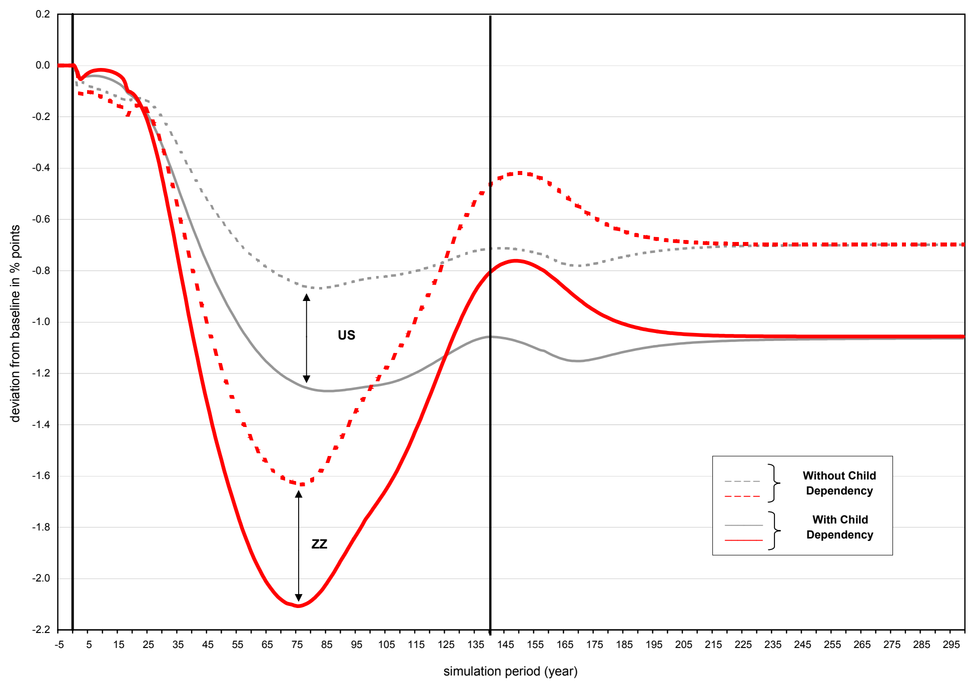


Figure 11. Capital Output Ratio, With and Without Youth Dependency
Asymmetric Shock with Cross-Border Spillovers

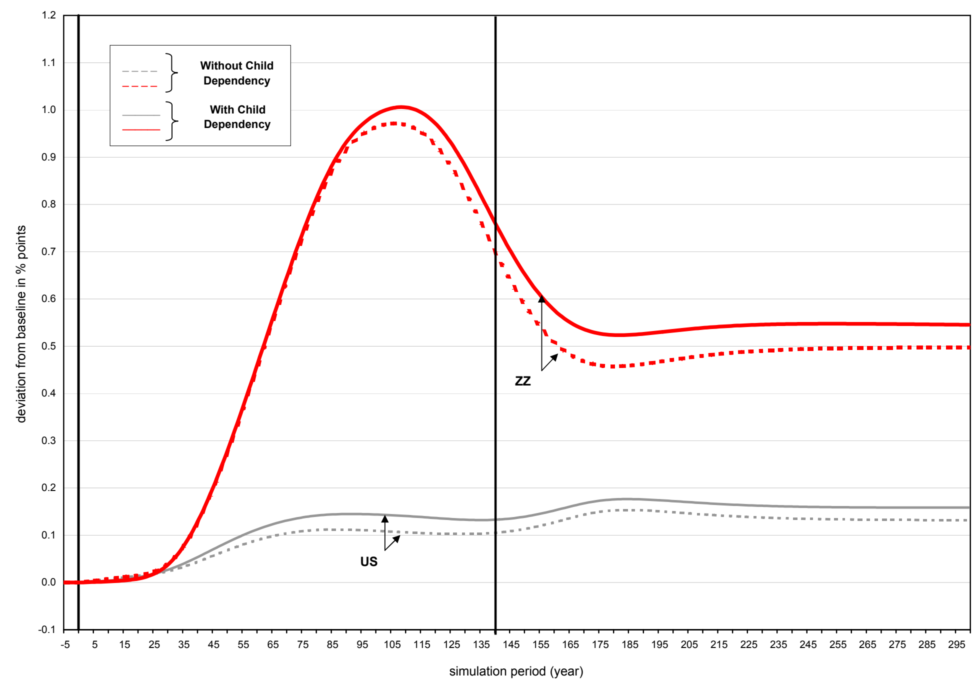


Figure 12. Financial Wealths per Adult, With and Without Youth Dependency.
Asymmetric Shock with Cross-Border Spillovers

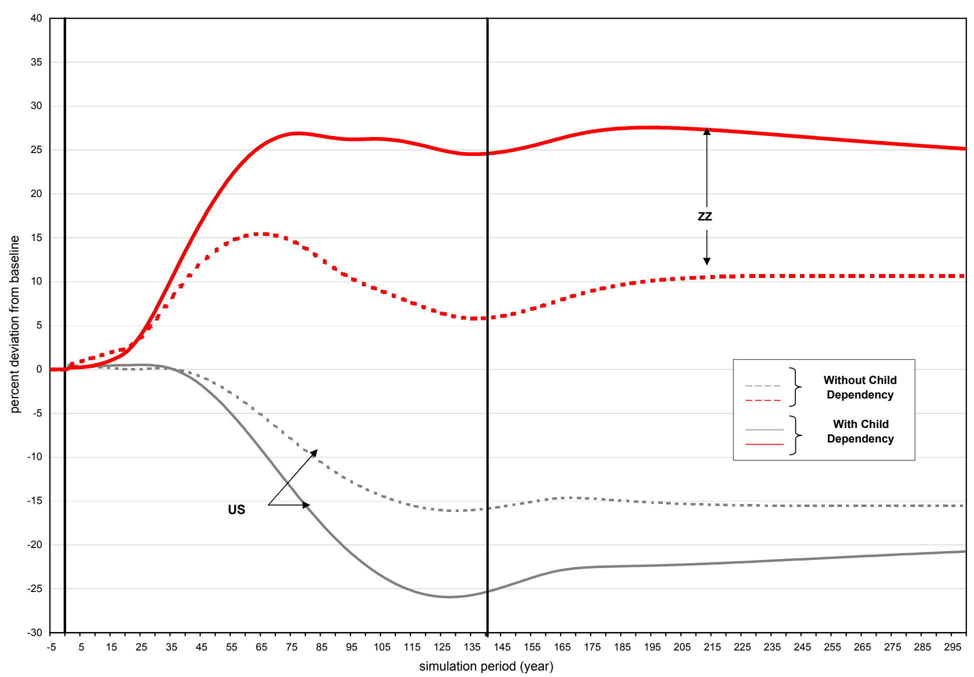


Figure 13. Real and Nominal Exchange Rates, With and Without Youth Dependency.
Asymmetric Shock with Cross-Border Spillovers

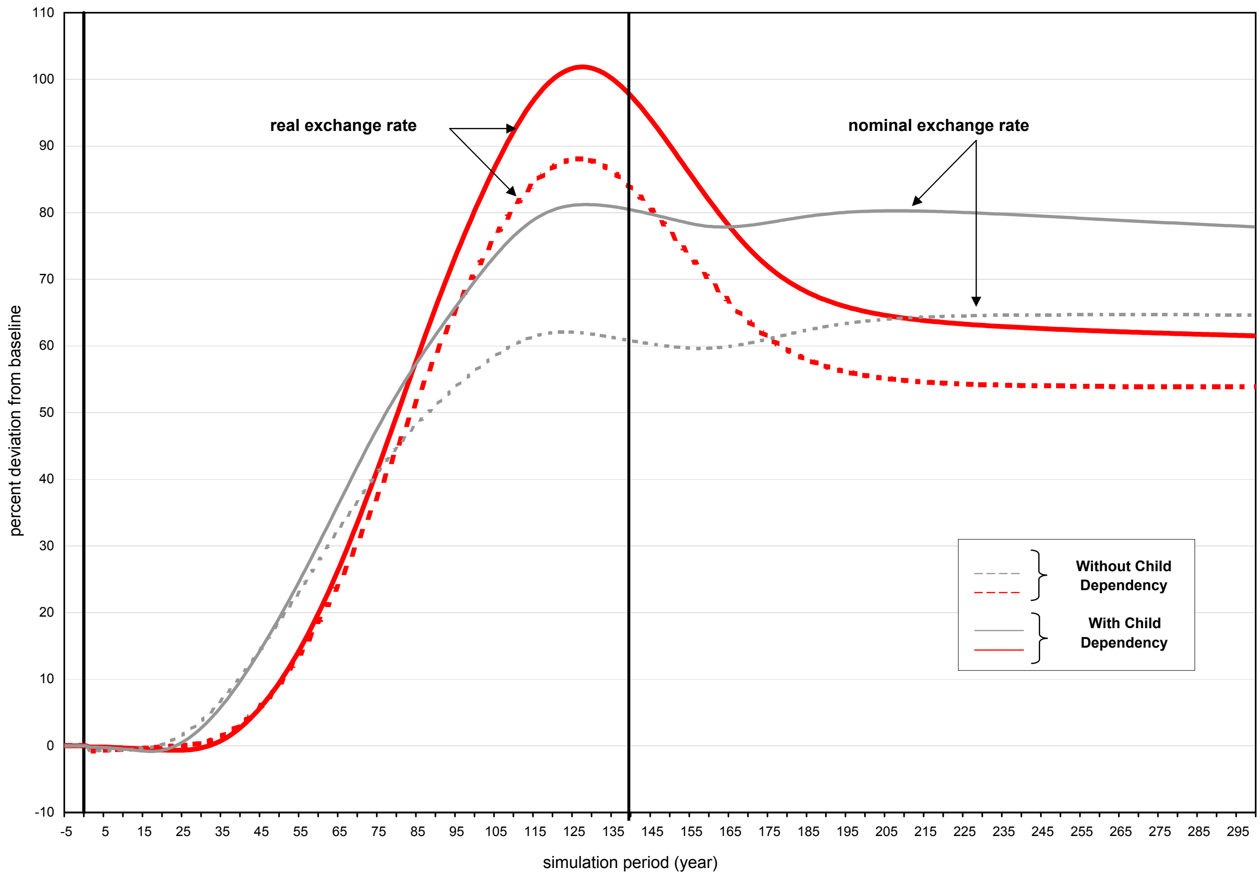


Figure 14. Populations and GDPs: Relative Size of ZZ to US
With and Without Youth Dependency
Asymmetric Shock with Cross-Border Spillovers

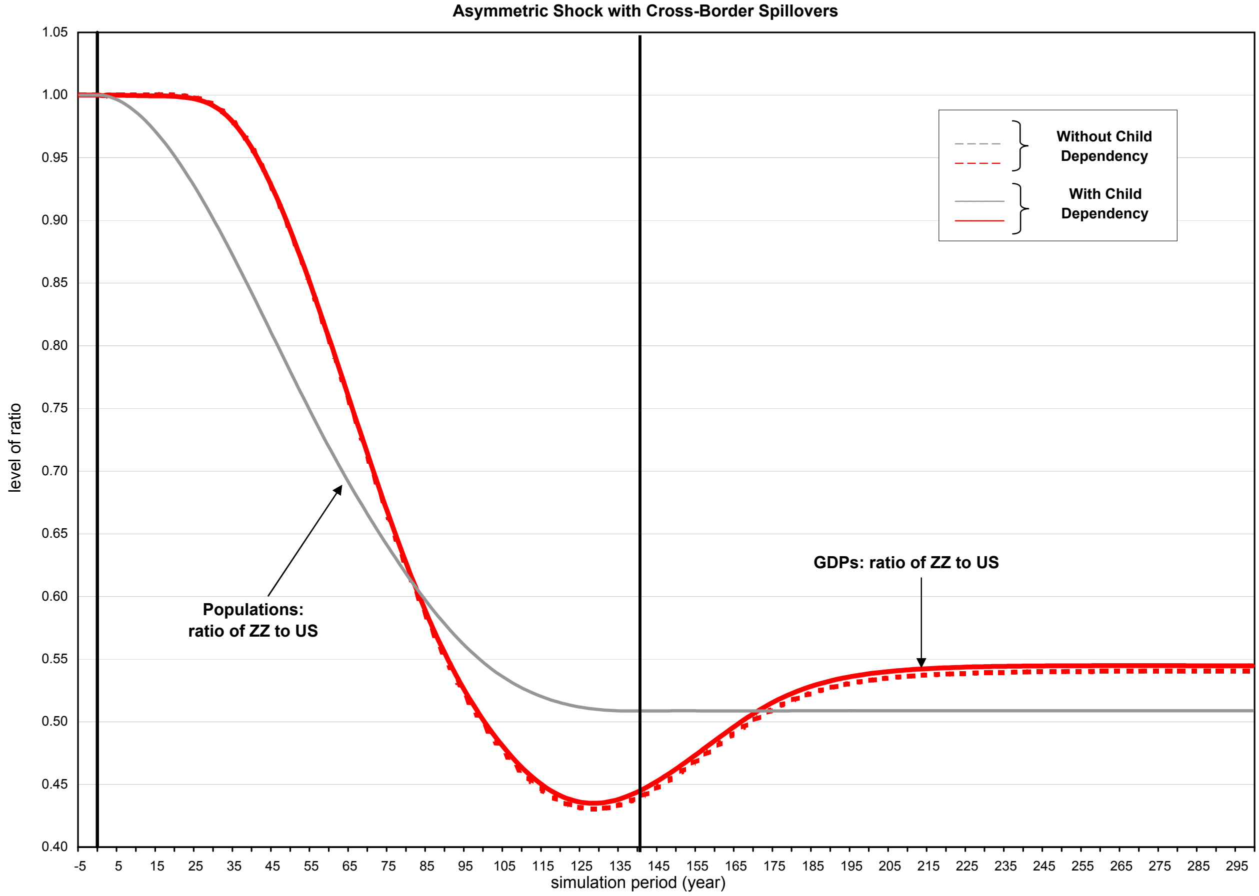


Figure 15. Ratios of Real Imports to Real GDP, With and Without Youth Dependency
Asymmetric Shock with Cross-Border Spillovers

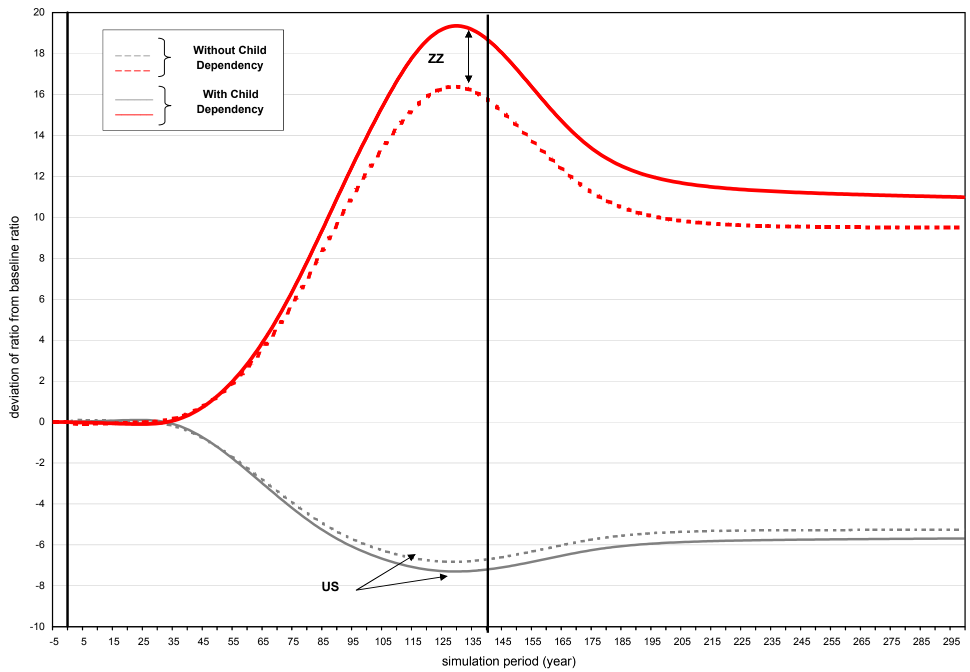


Figure 16. Ratios of Real Trade Balance to Real GDP, With and Without Youth Dependency
Asymmetric Shock with Cross-Border Spillovers

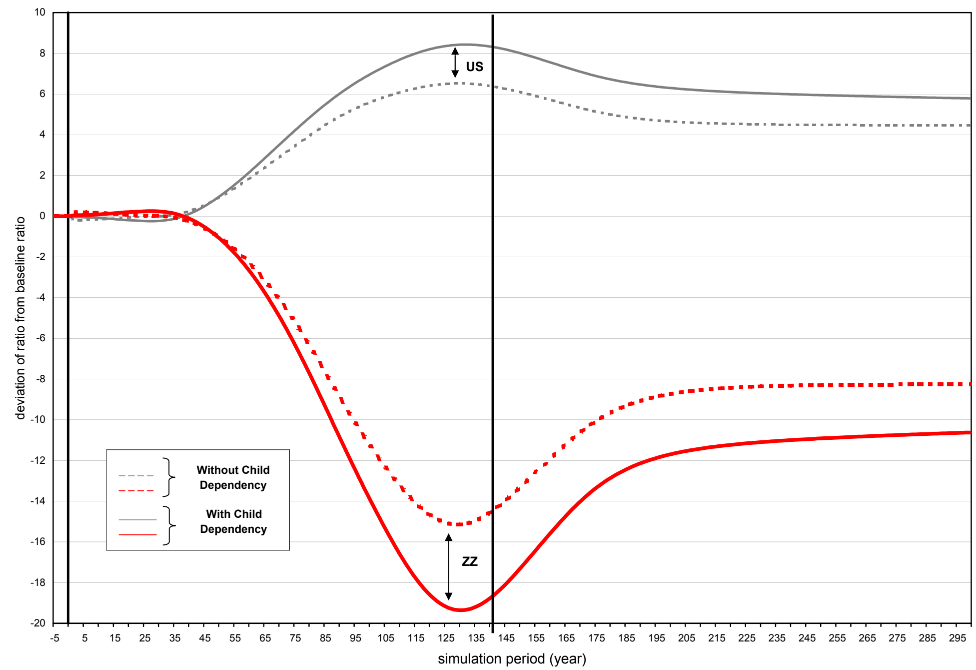


Figure 17. Ratios of Current Account Balance to Nominal GDP, With and Without Youth Dependency
Asymmetric Shock with Cross-Border Spillovers

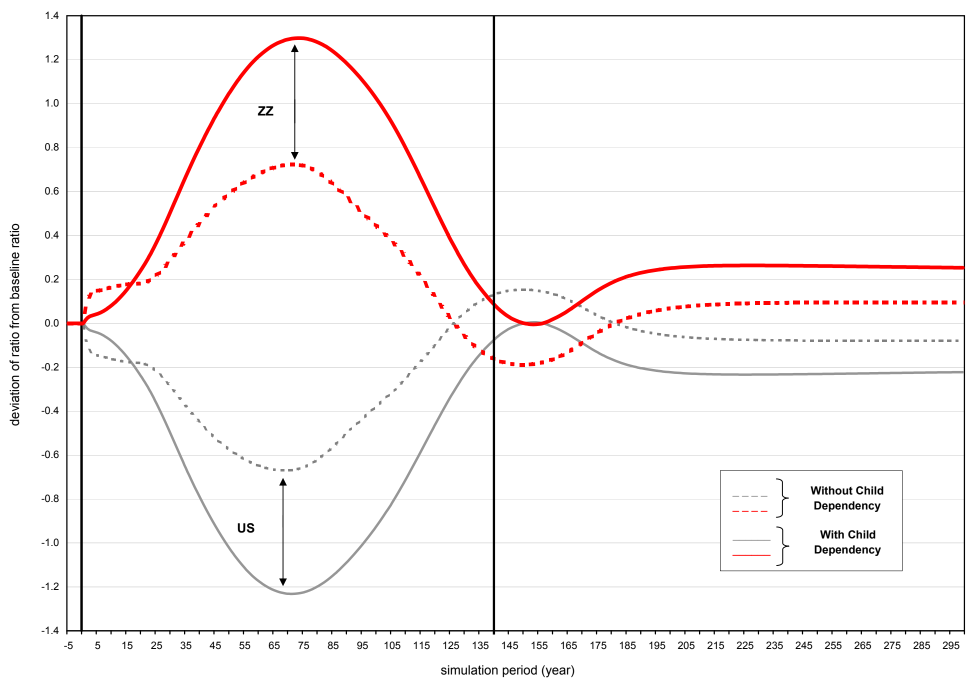


Figure 18: Ratios of Net Foreign Assets to Nominal GDP, With and Without Youth Dependency
Asymmetric Shock with Cross-Border Spillovers

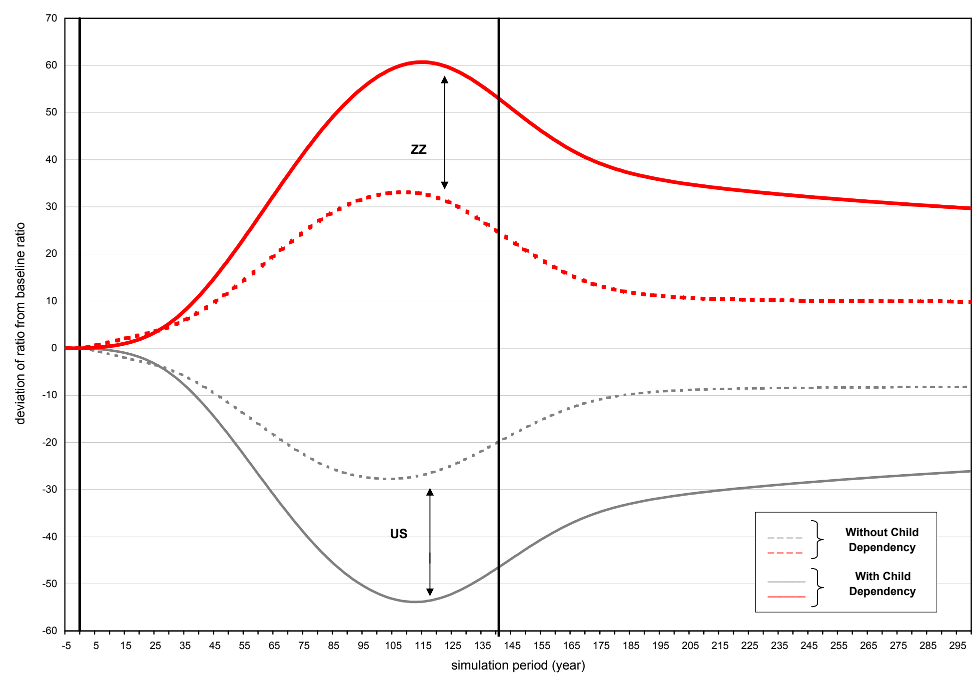


Figure 19. Aggregate Real Adult Consumptions (Economy-Wide)
Asymmetric Shock with Cross-Border Spillovers Compared with Symmetric (Closed-Economy) Shock

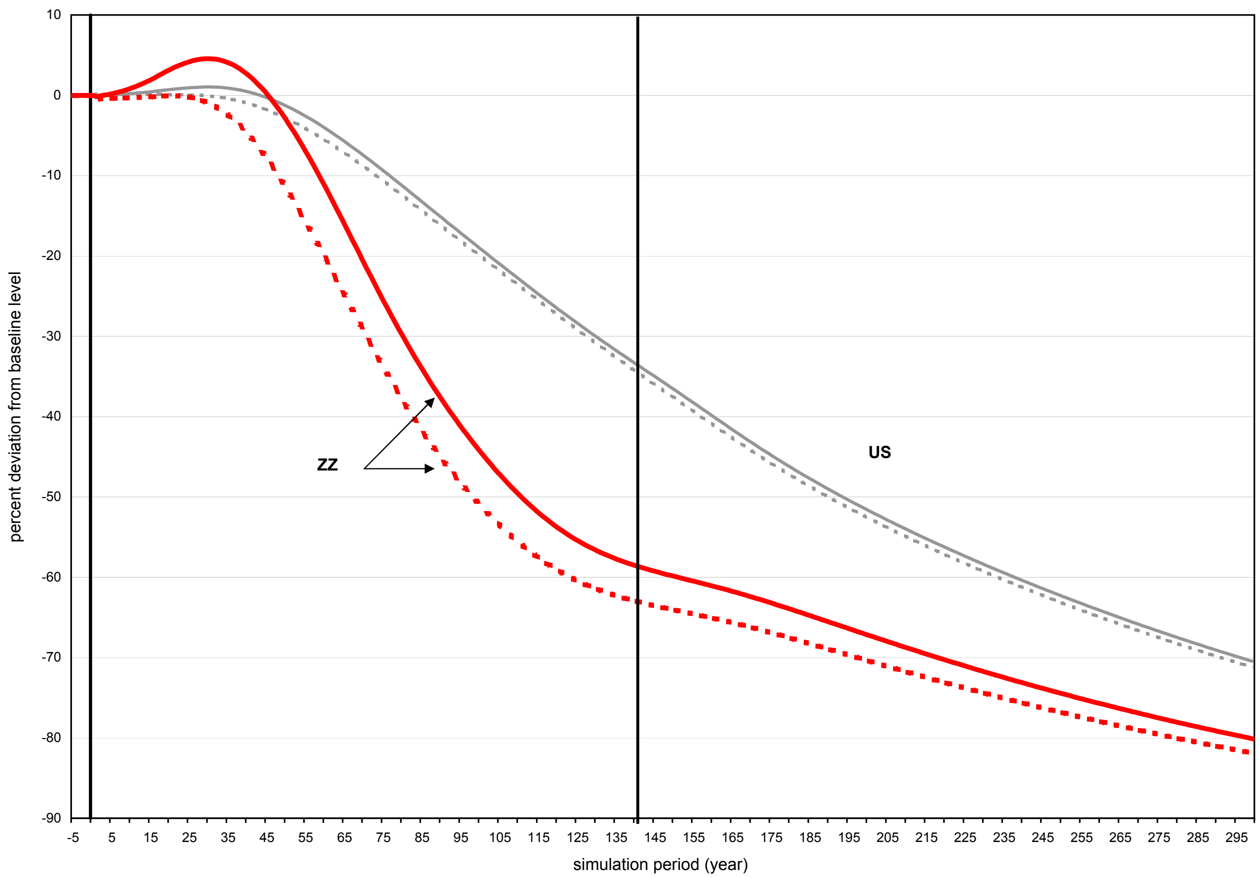


Figure 20. Real Adult Consumption per Adult, With and Without Youth Dependency
Asymmetric Shock with Cross-Border Spillovers

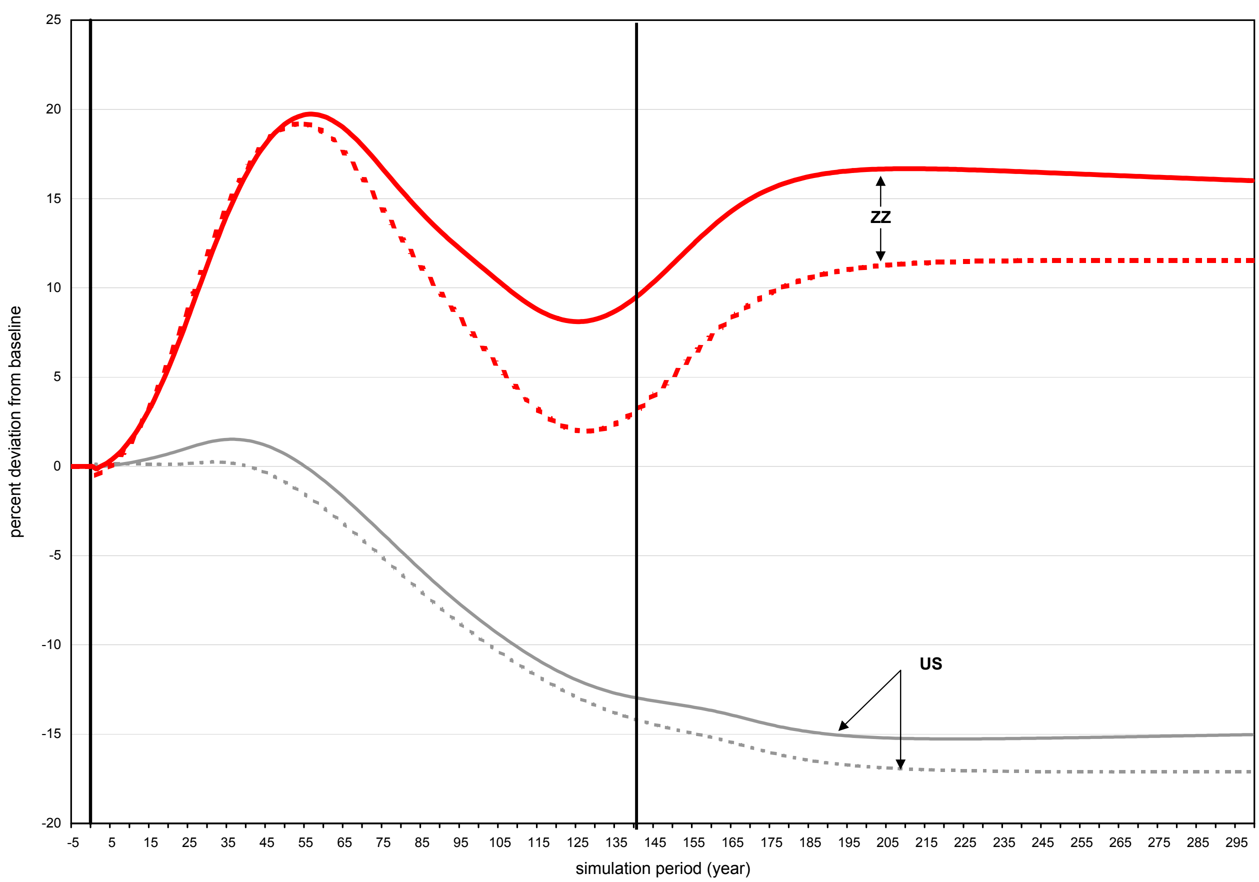


Figure 21. ZZ Real Short-term Interest Rate
with Alternative Generosities of Child Support
Asymmetric Shock with Cross-Border Spillovers

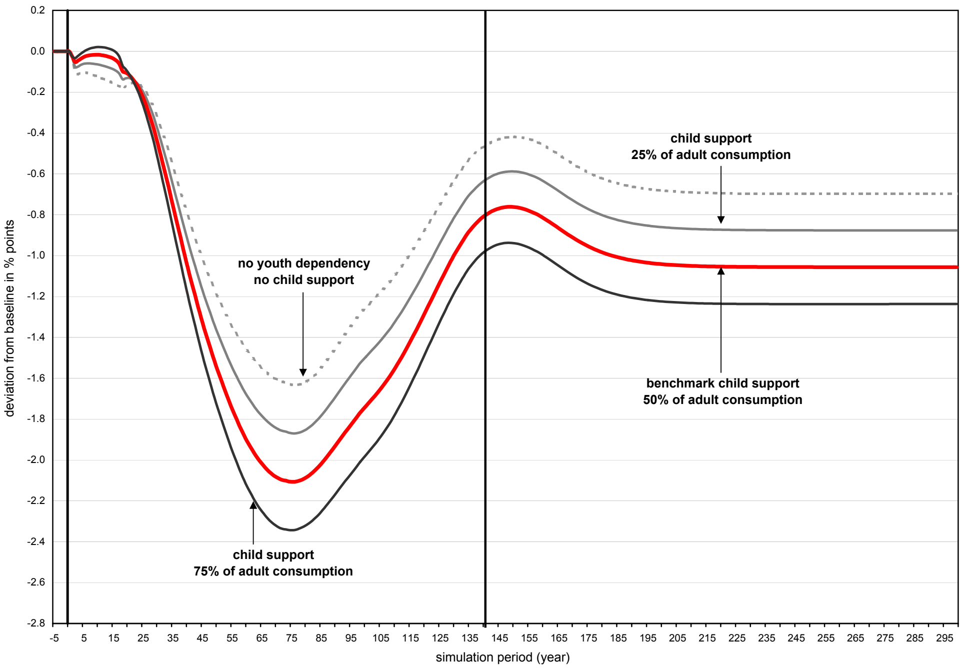


Figure 22. ZZ Private Saving Ratio
with Alternative Generosities of Child Support
Asymmetric Shock with Cross-Border Spillovers

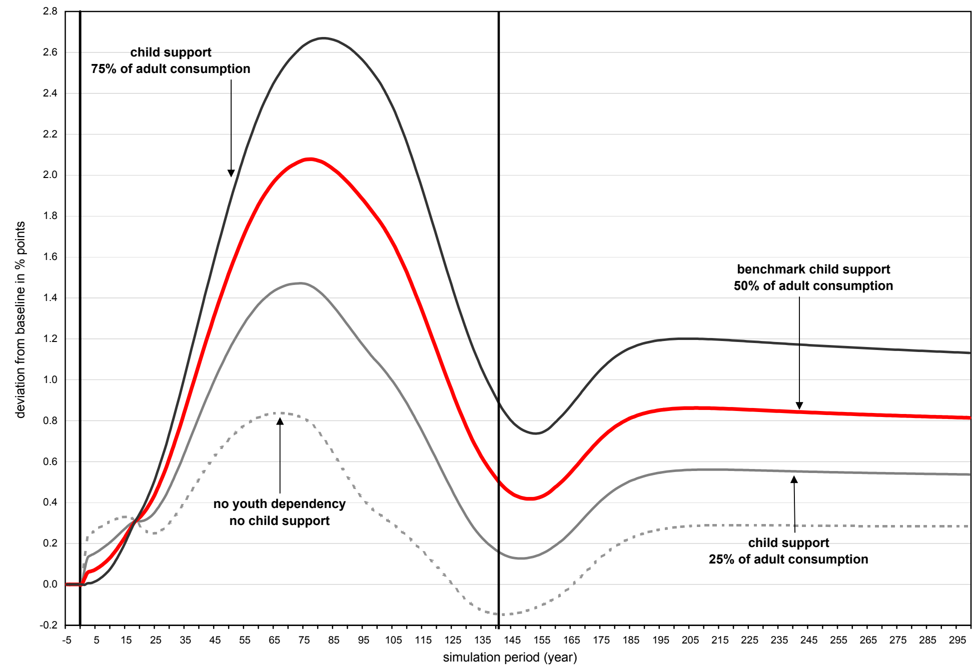


Figure 23. ZZ Ratio of Net Foreign Asset to Nominal GDP
with Alternative Generosities of Child Support
Asymmetric Shock with Cross-Border Spillovers

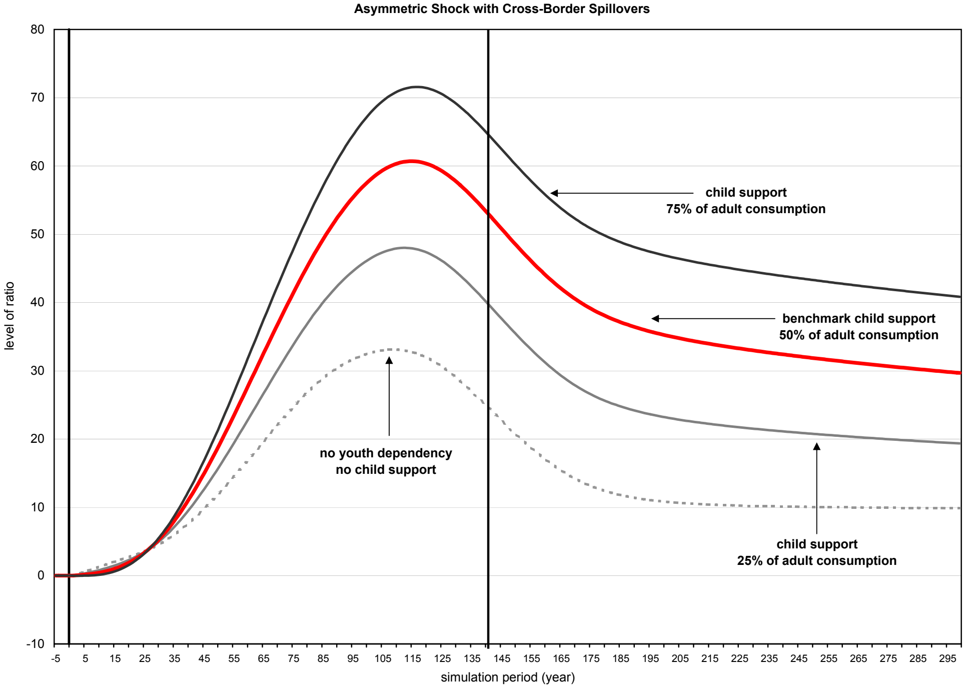


Figure 24. ZZ Real Adult Consumption per Adult
with Alternative Generosities of Child Support
Asymmetric Shock with Cross-Border Spillovers

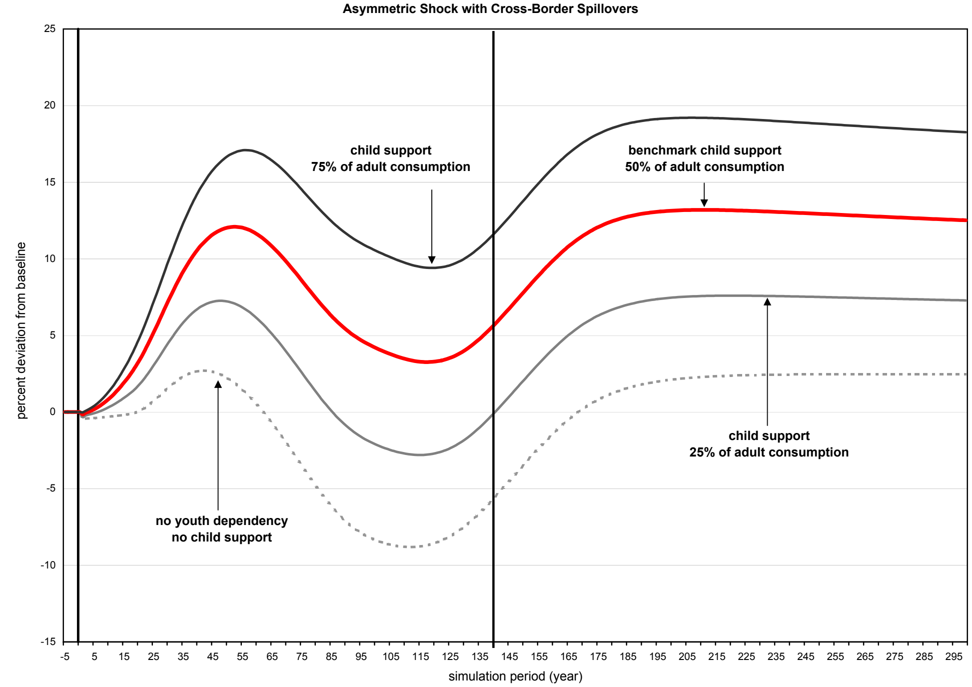


Figure 25. ZZ Private Saving Ratio
Varying the Basic-Needs Proportion of Child Support
Asymmetric Shock with Cross-Border Spillovers

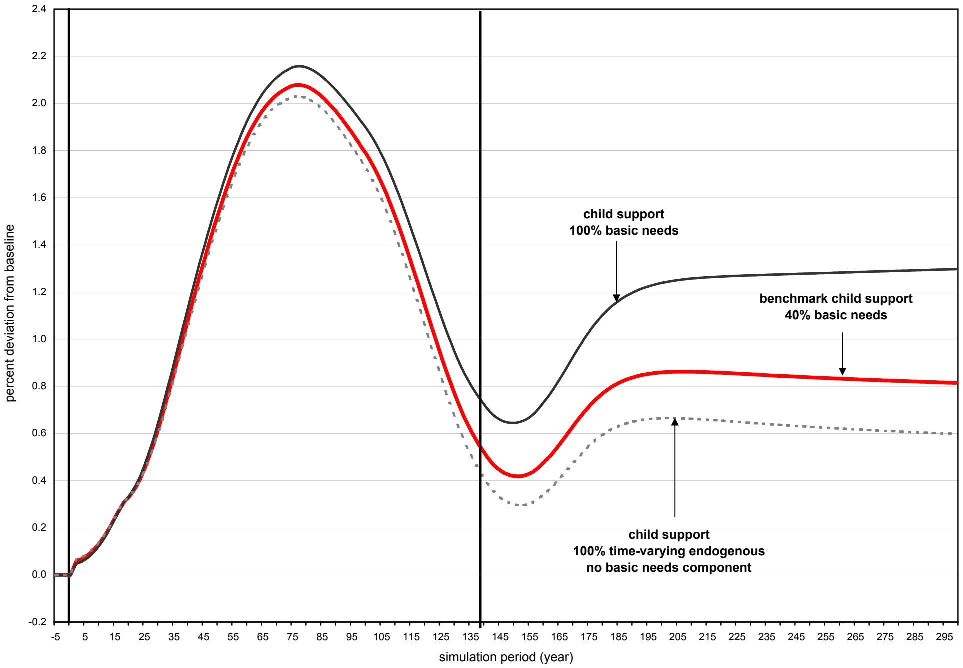


Figure 26. ZZ Ratio of Net Foreign Assets to Nominal GDP
Varying the Basic-Needs Proportion of Child Support
Asymmetric Shock with Cross-Border Spillovers

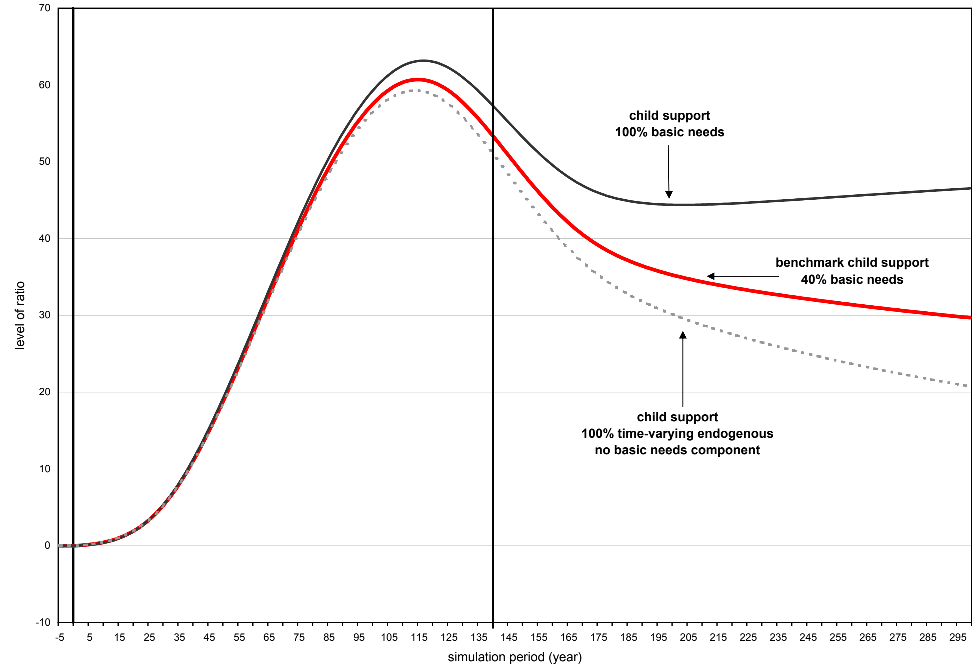


Figure 27. ZZ Real Adult Consumption per Adult
Varying the Basic-Needs Proportion of Child Support
Asymmetric Shock with Cross-Border Spillovers

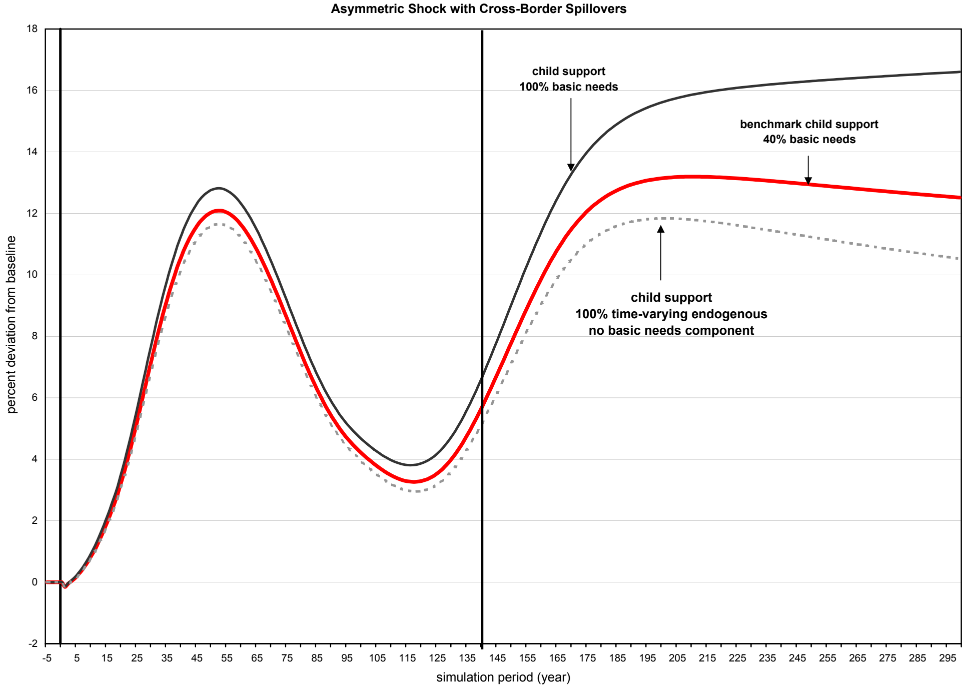


Figure 28. ZZ and US Private Saving Ratios
Varying the Age Distribution of Child Support
Asymmetric Shock with Cross-Border Spillovers

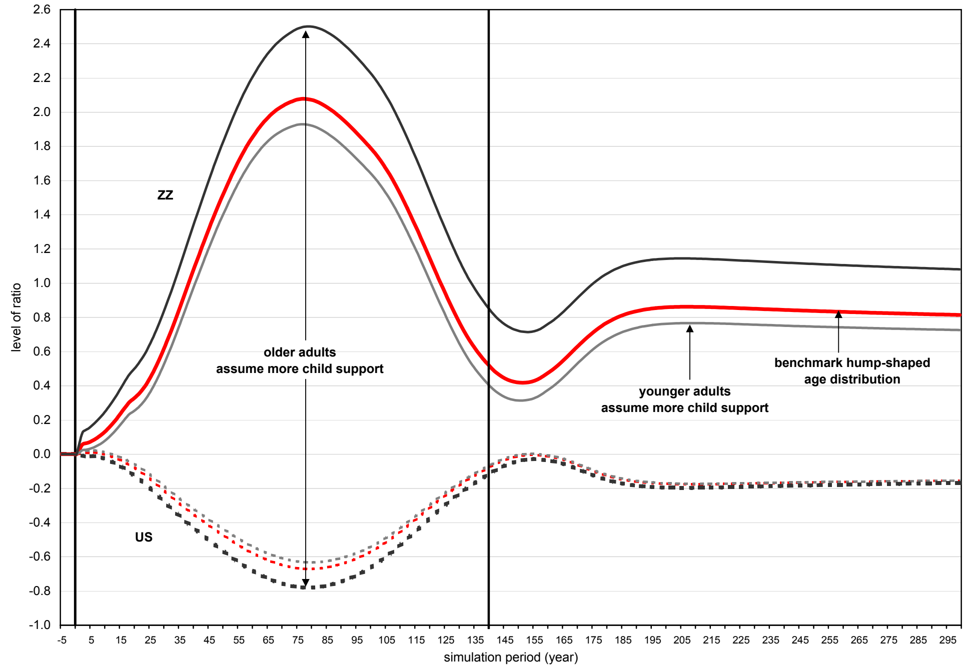


Figure 29. ZZ and US Ratios of Net Foreign Assets to Nominal GDP
Varying the Age Distribution of Child Support
Asymmetric Shock with Cross-Border Spillovers

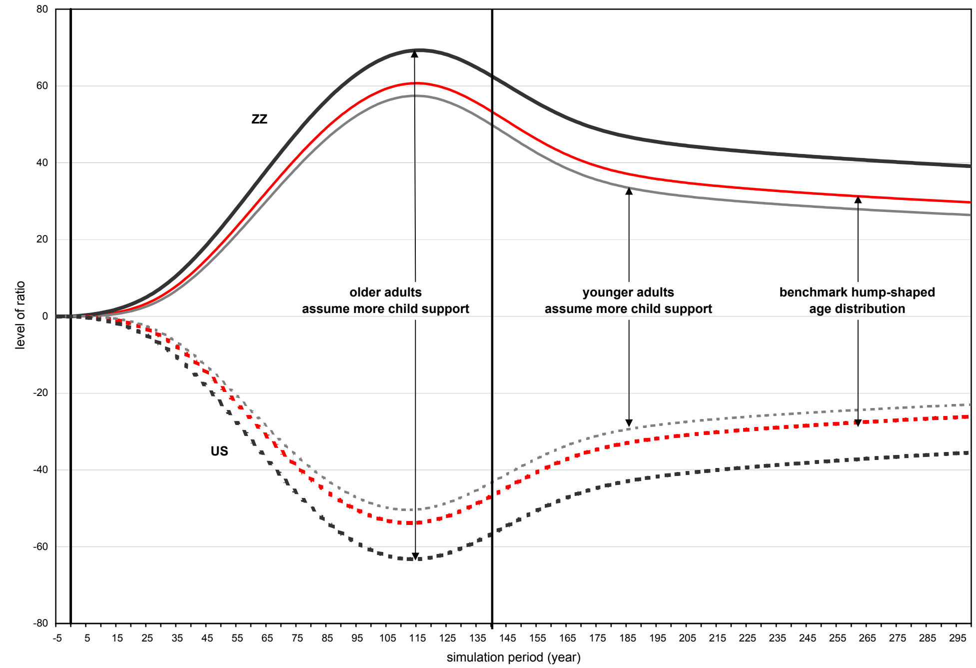


Figure 30. ZZ and US Real Adult Consumption per Adult
Varying the Age Distribution of Child Support
Asymmetric Shock with Cross-Border Spillovers

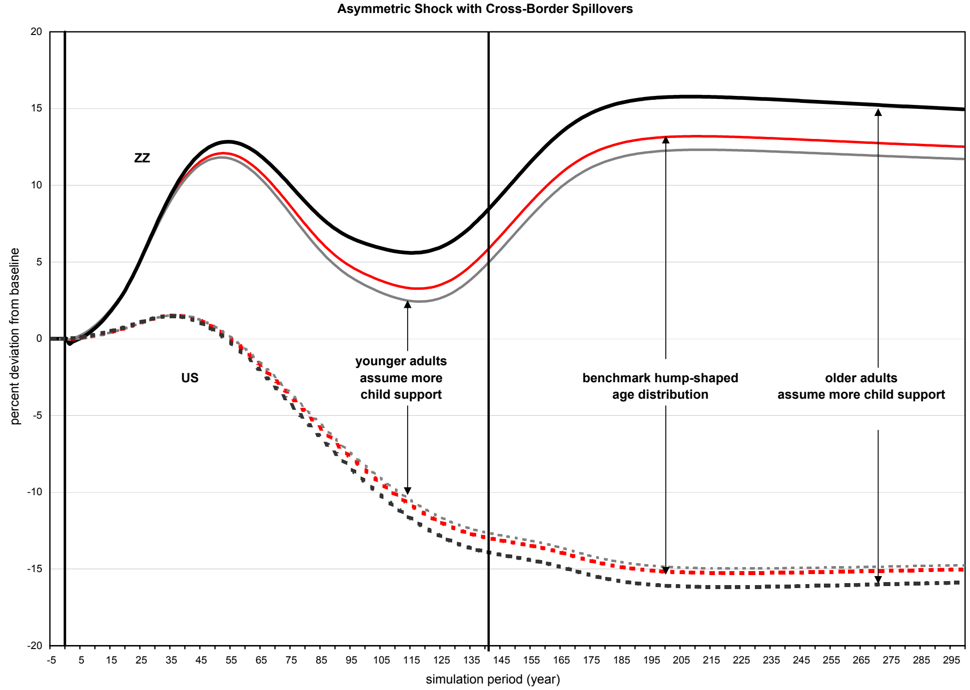


Figure A3-1: ZZ Real Short-term Interest Rate
Varying the Intertemporal Elasticity of Substitution in Consumption
Asymmetric Shock with Cross-Border Spillover

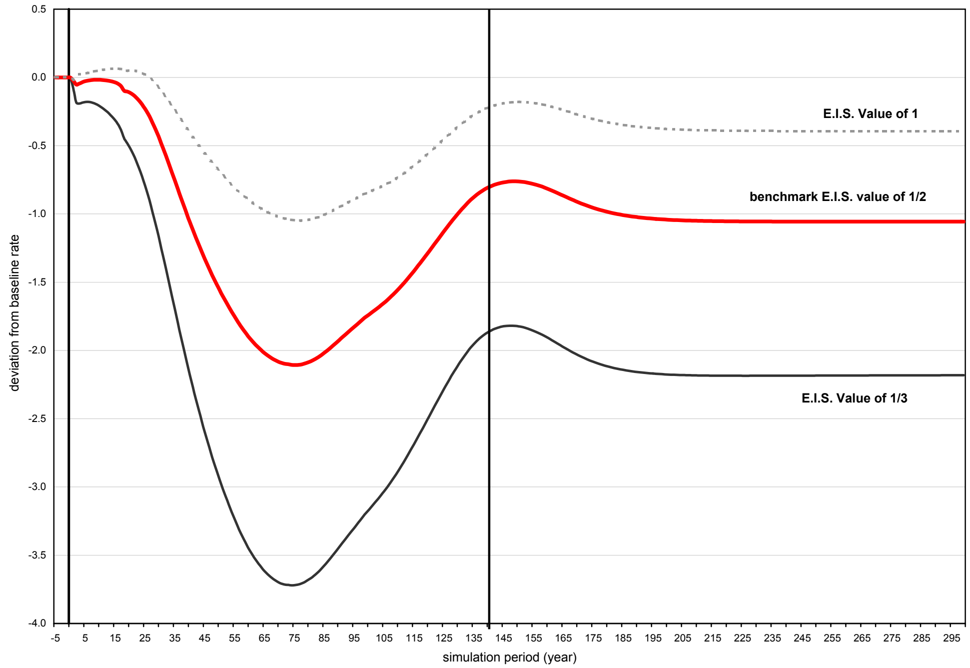


Figure A3-2: ZZ Private Saving Ratio
Varying the Intertemporal Elasticity of Substitution in Consumption
Asymmetric Shock with Cross-Border Spillover

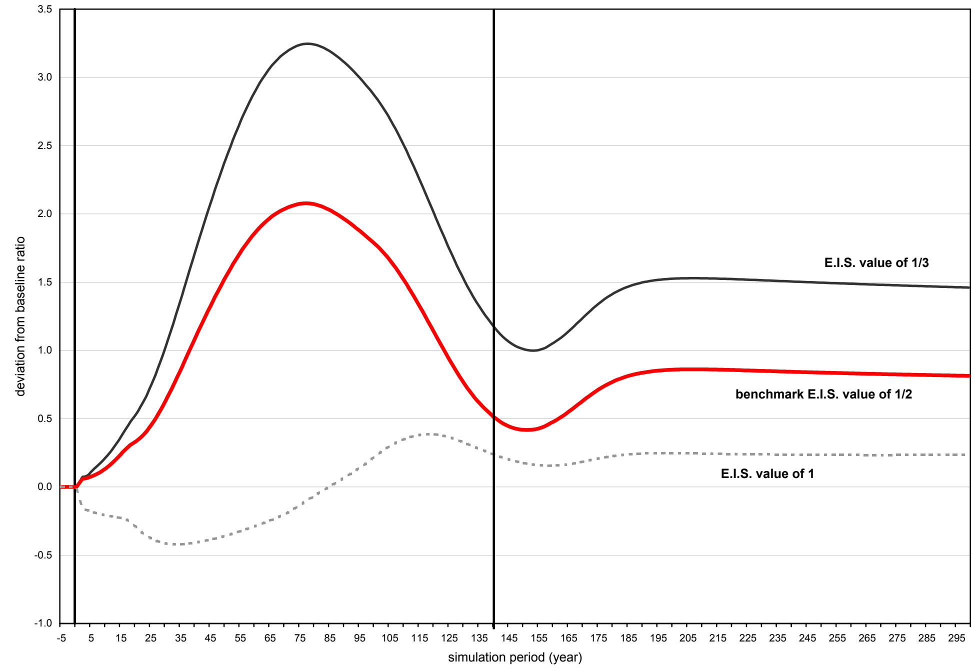


Figure A3-3: ZZ Ratio of Current Account Balance to Nominal GDP
Varying the Intertemporal Elasticity of Substitution in Consumption
Asymmetric Shock with Cross-Border Spillovers

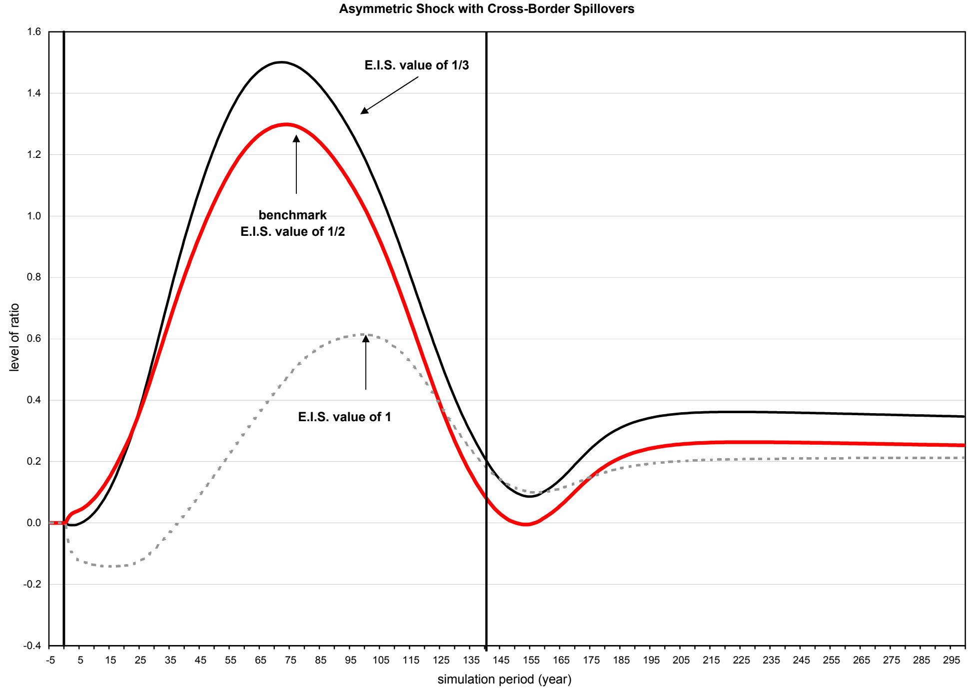


Figure A3-4: ZZ Real Adult Consumption per Adult
Varying the Intertemporal Elasticity of Substitution in Consumption
Asymmetric Shock with Cross-Border Spillovers