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Incorporating Demographic Change in Multi-Country Macroeconomic Models: Some Preliminary Results

Ralph C. Bryant and Warwick J. McKibbin

Brookings Institution and Australian National University

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Abstract

Demographic shifts may profoundly influence the world economy, directly in the countries experiencing the demographic change and indirectly through changes in global trade, capital markets, and exchange rates. Though that point is now widely acknowledged, it is much less widely understood that existing analytical tools are inadequate for assessing the general-equilibrium and cross-border consequences of demographic change. The research reported in this paper takes preliminary steps to improve the required analytical tools. We build on theoretical work by (among others) Blanchard, P. Weil, Faruqee, Laxton, and Symansky that suggests a revised life-cycle approach to consumption and saving behavior. We use this approach to incorporate demographic structure into open-economy empirical macroeconomic models. Changes in birth and mortality rates are combined with an approximation of age-earning profiles to allow demographic shifts to influence human wealth, consumption, and asset accumulation. Our preliminary work introduces the new approach into two simplified empirical models, a two-region abridgement of the IMF's MULTIMOD model and a two-region abridgement of the McKibbin MSG3 multi-country model. This paper reports preliminary simulation results. The stylized shock on which we initially focus is an unanticipated and transitory demographic bulge, analogous to the "baby boom" experienced by some industrial nations several decades ago. With the passage of time, the shock results in population aging of the type now confronting industrial nations. One set of simulation results describes the effects when the demographic bulge occurs simultaneously in both of the two model regions. A second set considers the consequences when the shock occurs in one of the regions but not the other. Our preliminary findings strongly support the conclusion that this analytical approach is promising. They also strongly confirm the hypothesis that differences across countries in the timing and intensity of demographic shifts can have significant effects on exchange rates and cross-border trade and capital flows.

1. Introduction

Population aging is occurring at differing paces and with differing degrees of intensity in the industrialized nations of the world. Significant aging is already under way, for example, in Japan, Italy, and Germany. Major demographic changes in the United States and Canada begin in the second decade of the 21st century. With a still longer lag, the demographic trends will be manifest in developing economies as well.

These changes in the demographic structure of populations will have major economic and political consequences. Government budgets, for example, will come under severe pressure. The levels of saving and investment and the overall saving-investment balance will change significantly in many regions of the world economy. These changes will be associated with large changes in cross-border flows of financial funds and goods. Whatever the proximate impetus for the changes, major adjustments in exchange rates and balance-of-payments positions will be required. The resulting international tensions will add greatly to the complications faced by national governments as they try to cope with the pressures on their domestic fiscal budgets and appropriately revise their economic policies.

The Brookings Institution has under way a major program to study issues of population aging. As a part of this program, the two authors have organized a project on the Global Dimensions of Demographic Change. The first product of the project was a series of workshops taking stock of what the economics profession does and does not know about the macroeconomic consequences of population aging. We prepared an initial report in the fall of 1998 on that first phase of the project.[1] As described in detail in that initial report, a substantial and growing literature has called attention to population aging and other demographic changes associated with it. In that literature, it has been increasingly recognized that demographic shifts may profoundly influence the world economy, directly in the countries experiencing the demographic change and indirectly through changes in global trade, capital markets, and exchange rates.

Though the likely importance of the international dimensions is now widely acknowledged, it is much less widely understood that existing analytical tools are inadequate for assessing the general-equilibrium and cross-border consequences of demographic change. The research reported in this paper starts from the premise that an improvement in analytical tools is a necessary condition for improving understanding of the basic cross-border issues and the policy choices facing individual nations.

Section 2 of this paper contains an overview of the influences of demographic change on consumption, saving and wealth accumulation, together with a summary of the findings of the 1998 workshops on the changes needed to introduce demographic structure into existing multi-country macroeconomic models. Section 3 outlines the analytical approach we have been following in our recent research. Section 4 summarizes how we implement this analytical approach in two simplified multi-country models. Section 5 presents preliminary simulation results. Section 6 outlines our future plans.

2. Demographic Influences on Consumption, Saving, and Wealth Accumulation: An Overview

Analysis of the cross-border consequences of demographic change requires examination of many interdependencies among national economies. Such analysis therefore cannot plausibly be undertaken without use of some type of a multi-country, general-equilibrium macroeconomic framework. Explicit multi-country macroeconomic models, despite their weaknesses, are

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[1] The report is available as Brookings Discussion Paper in International Economics No. 141, "Issues in Modeling the Global Dimensions of Demographic Change," (Brookings Institution, December 1998). The paper is available on the Brookings website (www.brookings.edu).

unambiguously preferable to alternatives for conjecturing about the complex behavior of economies that rely on partial-equilibrium or implicit, unsystematic methods.

As discussed in our initial report, however, existing multi-country models are inadequate to the task in several critical ways. The most notable inadequacy is the existing models' failure to incorporate explicitly the effects of demographic changes. In at least three respects, researchers working with the multi-country models must improve their model structures. In particular, an improved analysis is required of the effects of demographic changes on: (1) consumption (including possibly patterns of consumption across different goods and services), saving, and wealth accumulation, with appropriate allowance for the openness of national economies; (2) the production/supply sides of national economies, again with appropriate allowance for openness; and (3) expenditures, transfers, and revenues in government budgets.

During the inventory-taking workshops initiated at the start of the project, the largest proportion of the dialogue focused on analytical methods for studying the consequences of demographic changes for consumption, saving, and wealth accumulation. The economics profession holds a variety of analytical views on these topics. Yet almost all researchers in this area agree that these areas are a priority topic for further research. In this paper we accordingly focus on improving the treatment of demographic effects on consumption, saving, and wealth accumulation.

As background for our own approach to these topics, we first summarize the status of the profession's understanding. For this purpose, we recapitulate some of the material in our December 1998 report.

At a very general level, since the work of Modigliani-Brumberg (1954, 1979) and Friedman (1957), economists have accepted in broad terms the idea that many households will wish to smooth their consumption across time. The degree of this intertemporal smoothing and the main factors driving it, however, continue to be much in dispute. One set of issues concerns the manner in and degree to which households or individuals voluntarily want to act as intertemporal smoothers. A second set pertains to whether constraints external to the household inhibit agents from acting as intertemporal smoothers. In an analytical model, the average degree of smoothing taking place in the model economy also depends sensitively on how the model builder chooses to treat households' expectations, particularly about future developments in labor income and wealth.

Explicit life-cycle approaches hypothesize that consumers save little in their early years, save most in their middle-to-late working years, and then may spend down their wealth accumulation after retirement. Some researchers interpret the empirical evidence as broadly supporting the life-cycle view, including the hypothesis that consumers are patient enough to begin saving for their retirement early in their working lifetimes. Other researchers, however, read the empirical evidence as providing only weak support for the life-cycle view of saving and instead supporting the hypotheses that consumers save and accumulate wealth primarily subject to the persistence of habits or to insulate consumption against uncertainty about fluctuations in income. In these latter views, the saving-for-retirement motive is much less important than habit formation or precautionary saving for uncertainty (and hence the accumulation of "buffer-stock" assets).[2]

Some researchers argue that the life-cycle hypothesis, as studied in the context of microeconomic panel data, appears unable to account for the most prominent observed changes in countries' saving behavior. For example, the life-cycle hypothesis does not seem to do a good job of explaining the pronounced decline in the saving ratio in the United States in the last several decades. Nor, apparently, can it explain the pronounced increases in saving ratios in several Asian countries (such as China, Indonesia, South Korea, Singapore, and Thailand). For these cases, one observes large time-series changes in the aggregate saving ratio -- in effect, a pronounced time trend. But the demographic changes in these countries do not seem to help in accounting for the time trends. Most

[2] Examples of researchers sympathetic to the life-cycle view of saving and consumption include Attanasio-Browning (1995) and Meredith (1995). Researchers emphasizing the importance of precautionary saving and doubting the empirical importance of the life-cycle theory include Deaton (1991, 1992), Carroll-Summers (1991), Carroll (1992, 1997), and Carroll-Samwick (1997); the papers in the international-comparison volume edited by Poterba (1994) tend to have a similar emphasis.

of the households in the U.S. economy seem to have cut their saving at the same time, and most of the households in the relevant Asian economies seem to have increased their saving at the same time.[3]

Carroll and Weil (1994) and Carroll, Overland, and Weil (2000) are partial exceptions to the view that macroeconomic analysis positing life-cycle behavior performs poorly in explaining the observed correlations between saving and growth. They argue, for example, that a model postulating a dependence of consumers' utility on comparisons of consumption to a "habit stock" determined by past consumption can do well in explaining the observed correlation between savings and growth (including in Asian countries).

Hubbard, Skinner, and Zeldes (1994) are also skeptics about the view that life-cycle models are unable to shed light on actual behavior. They use an augmented life-cycle model and show that such a model can help explain, at least for the United States, the time-series comovements of consumption and income and the high historical ratio of aggregate wealth to disposable income.

One of the complicating factors leading to differences in view about the empirical validity of the life-cycle hypothesis concerns the availability and treatment of data about government pension (social security) programs. In pay-as-you-go pension programs, a major intergenerational issue is salient. It is younger and middle-aged workers that currently pay the tax revenue into the government programs, whereas it is the elderly that currently receive the benefit payments. Moreover, the operation and fiscal balance of the pension programs is importantly influenced by regulations and provisions determining eligibility. For example, provisions setting the age for early retirement eligibility have a big effect on the actual age at which workers retire, which in turn has major effects

[3] The microeconomic analyses of saving behavior have not produced a consensus explanation for these time trends. In particular, the demographic components of the simpler versions of the life-cycle hypothesis are certainly not providing the explanation. On the relevance of life-cycle models, see among others Bosworth-Burtless-Sabelhaus (1991), Hubbard-Skinner-Zeldes (1994), Deaton-Paxson (1997), and Lusardi (1998).

on the pressures on government budgets (Gruber-Wise, 1998).[4] Unfortunately, the available microeconomic survey data typically do not collect information on payments received by households from pension systems. This omission is a major problem for studies of the age profile of saving behavior. It could be -- as argued, for example, by Meredith (1995) -- that these data omissions partially explain why the life-cycle hypothesis appears not to perform well in many microeconomic studies of saving behavior.

A further difference of view in the saving-consumption literature concerns agents that in practice may not be able to borrow and lend freely as the simplified intertemporal smoothing models presume they can. The theoretical treatment and empirical importance of such liquidity-constrained households is controversial. A number of papers provide evidence suggesting that the behavior of a sizable minority of households cannot be adequately described by intertemporal smoothing. But there is also recognition that it may not be possible to separate the features of behavior that lead to differing degrees of willingness to smooth intertemporally (e.g., the precautionary saving motive) versus the effects of external impediments such as borrowing constraints that prevent some households from intertemporal smoothing.[5]

The significance of bequests is also controversial in the specification of consumption-saving and wealth accumulation. Issues at stake include the relative importance of voluntary or strategic versus involuntary, accidental bequests. In his overview in the Clarendon Lectures, Deaton (1992, p. 217) remarks that "it now seems that bequest motives are a good deal more important than we used to

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[4] Here is another example: if the provisions in the government pension program specify a fixed age for retirement, increases in life expectancy may then increase the incentives of a worker to save through channels outside the pension system.

[5] Recent references on liquidity-constrained consumption behavior include Zeldes (1989), Deaton (1991), Attanasio-Browning (1995), and Berloffa (1997). The buffer-stock models of Carroll (1992, 1994, 1997) emphasize the precautionary-saving motive; in those models, there is an implicit constraint on intertemporal smoothing by consumers, but it does not arise because of an explicit constraint on consumers' ability to borrow against future income.

think." Kotlikoff and Summers (1981) also argued for much greater emphasis on intergenerational transfers as a driving force for aggregate capital accumulation.[6]

The treatment of agents' expectations is still another area where analytical views are heterogeneous. Few if any analysts now prefer to use backward-looking, adaptive expectations in their research. The use of forward-looking, model-consistent expectations has become common. Yet the extreme assumptions of rational, model-consistent expectations may in some ways be almost as unsatisfactory as the opposite extreme of adaptive, backward-looking expectations. Reliable empirical evidence about the pervasiveness of forward-looking behavior by individuals, households, and firms is sketchy, and behavior no doubt varies significantly across different types of agents. Although forward-looking behavior of some sort is widespread, it is still an open question what proportion of consumers (if any) makes decisions in a manner that is well captured by analyses based on model-consistent expectations.

Consumption and wealth accumulation are central to both microeconomics and macroeconomics. Yet the unresolved issues are remarkably salient -- even before one gets to the complications introduced by open national economies. Some degree of consensus may exist among researchers that the simplest versions of permanent-income or life-cycle models are not sufficient by themselves as adequate descriptions of saving and consumption behavior. For example, Deaton (1992, p. 217) summarizes that "the accumulating microeconomic evidence casts increasing doubt on the life-cycle hypothesis, or at least on the insights that come from the 'stripped-down' version, that saving is largely hump-saving for retirement, that consumption is based on lifetime resources, that aggregate wealth is accountable for by life-cycle saving, and that saving responds positively to productivity and population growth." He interprets the evidence as primarily against the low-

[6] References on the role of bequests in saving include Davies (1981), Menchik-David (1983), Abel (1985), Bernheim-Schleifer-Summers (1985), Bernheim (1991), Borsch-Supan and Stahl (1991), Altonji-Hayashi-Kotlikoff (1992), Wilhelm (1996), and Laitner-Juster (1996).

frequency intertemporal smoothing of consumption, and perceives expected future income, especially distant future income, as having "limited relevance."[7]

To many analysts, it still seems natural to suppose that large changes in the demographic structure of populations can have significant effects on consumption and saving. The effects should be larger, the more pervasive are life-cycle elements in the behavior determining saving. A household manifesting the traditional life-cycle behavior will have a hump-shaped lifetime profile for its saving rate: low saving in its early years (when, for example, children are very young), high saving in middle age in anticipation of retirement, and then low or even negative saving after retirement. Population aging due to the retirement of the baby-boom workers and lower fertility should thus, other things equal, lead to a decline in the private saving rate.[8] Developing countries that experience sharp declines in infant mortality, rising fertility, and hence a surge in the population of dependent children should likewise experience higher consumption and lower saving. A nation experiencing unusually rapid growth in the labor force as baby-boom children move into the years of working age should exhibit, other things equal, a higher private saving rate. Even if simplified life-cycle considerations are less important than precautionary saving and habit persistence and even if bequest motives importantly influence savings, some significant role for life-cycle effects may be needed in analytical models, perhaps especially those aspiring to capture cross-border spillovers.[9]

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[7] Deaton suggests that a preferable taxonomy of intertemporal choice models would lump together the (formal) permanent-income and (conventional) life-cycle models and differentiate those models sharply from approaches incorporating liquidity constraints and/or precautionary saving. For more recent surveys of the consumption-saving-wealth-accumulation literature, see Muellbauer-Lattimore (1995), Browning-Lusardi (1996), Deaton-Paxson (1997), and Attanasio (1998). Muellbauer-Lattimore (1995) discuss differences between "Euler-equation approaches" versus "solved out consumption functions," arguing that the former have been overemphasized relative to the latter in recent years.

[8] The effects on the private saving rate of increasing life expectancy could work in the opposite direction, leading households to save *more* during their working years to be able to sustain their consumption over a longer retirement period. (For this effect to be important, of course, the life-cycle motive has to be an important determinant of saving behavior.)

[9] Models with life-cycle behavior seem an especially natural analytical environment in which demographics could influence consumption and saving. But as has been pointed out to us by Delia Velculescu, models other than those with explicit life-cycle elements may also have links between demographic changes and private saving behavior. For example, Cutler, Poterba, Sheiner and Summers (1990) and Velculescu (2000) use a Ramsey growth model which allows for

Most aggregative macroeconomic models fail to allow for changes in the demographic structure of populations. The seminal contributions to growth theory in the 1950s and 1960s abstracted from such demographic factors. In effect, those models contained no children and no elderly, with the result that a faster (steady-state) rate of growth of population caused the saving rate unambiguously to rise in response to higher requirements for investment and a higher capital stock. Tobin (1967) developed a simulation model differentiating workers from retirees (but still without children). Tobin's model also predicted that faster population growth would raise the private saving rate, because the faster growth caused the population distribution to become younger (more working, saving households relative to retirees who were older and dissaving).[10] Even as theoretical growth models matured, most empirical macro models still abstracted from shifts in the demographic composition of the population. That omission persisted despite the major emphasis in consumption theory of life-cycle considerations in the microeconomic behavior of households.[11]

Another aspect of consumption which may be important for understanding the impacts of demographic shifts is changes in the composition of consumption bundles over the lifetimes of individuals. In a single good world, it is not possible to capture the impact that changes in preferences may have on the relative prices of alternative goods and services. For example, in an aging society it is likely that the demand for health services will rise, which will cause shifts in the relative prices of these services and a change in the allocation of resources to satisfy shifting

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dependency and their model predicts significant changes in saving when the proportion of young and old dependents in the population changes.

[10] Cross-country empirical studies found a positive relationship between faster growth and higher saving; see for example Modigliani (1970). Subsequent empirical work with microeconomic data questions whether the correlation observed in the cross-country comparisons can be correctly attributed to life-cycle saving behavior; see for example Paxson (1996) and Deaton-Paxson (1997).

[11] A strand in the literature on development economics was sensitive to these issues; see, for example, Coale-Hoover (1958), Mason (1987, 1988), and Taylor-Williamson (1994). The builders of empirical macroeconomic models, however, did not try to incorporate this work. Moreover, the development-economics literature did not directly address the details of how to adapt the consumption-wealth specifications in general-equilibrium macro models. The development-economics literature on population dynamics and saving is reviewed in recent papers by Higgins-Williamson (1997) and Bloom-Williamson (1997). Deaton-Paxson (1997, 1998, 1998) and Paxson (1996) are recent studies. Another important exception is the paper by Cutler, Poterba, Sheiner, and Summers (1990).

demands. Conventional single-good macroeconometric models cannot capture this type of compositional effect. Multi-sectoral multi-country models, such as the G-Cubed model of McKibbin and Wilcoxen (1999), could do so if demographics are incorporated appropriately.

Issues of aggregation are important when researchers specify the consumption-saving-wealth relationships in macroeconomic models. In particular, the aggregation issues are central for getting an adequate macro specification of the demographic influences. Macro models built up from a micro theory positing a single representative agent are not easily adapted so as to incorporate demographic changes. By definition, changes in the demographic composition of the population require analysis to acknowledge the heterogeneity of agents -- at the very least heterogeneity in age.[12] The spirit of overlapping-generations (OLG) models is to grapple directly with one or more dimensions of heterogeneity across agents. Proponents of the OLG modeling tradition tend to believe that macro-model specifications without allowance for heterogenous agents are bound to be inadequate -- see, for example, Kydland-Petersen (1997).[13]

For adequate macroeconomic analysis of the effects of demographic changes on consumption and wealth accumulation, is it essential to use explicit multi-cohort OLG models? Alternatively, for many analytical purposes, could it be sufficient to adopt one or another empirical "shortcut" to modify the existing macro models that have a more conventional life-cycle, permanent-income specification and that currently either ignore demographic composition effects or else treat them in an aggregative, representative-agent approach?

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[12] In the microeconomic empirical research on consumption, some conclusions have been found to be sensitive to whether the consuming agents are defined as "households" or as "individuals." For discussion, see Deaton-Paxson (1998, 1998), who use data for Taiwan and construct life-cycle saving profiles defined for cohorts of individuals rather than cohorts of households; their analysis shows stronger demographic effects on age profiles of saving for the individual than the household definition of cohorts.

[13] Illustrations of OLG models which directly tackle the issue of different age cohorts and their saving decisions include Auerbach-Kotlikoff (1987), Rios-Rull (1994, 1996), Fougère-Mérette (1997, 1997, 1998), Storesletten (1995, 2000), and Brooks (1998).

Any researcher working in this area must take a stand on the issues summarized above. The issue of whether to pursue multi-cohort OLG models or whether to pursue an analytical shortcut approach is particularly difficult. We know from the international workshops held in 1998 that there is no consensus on which of the two routes has the greater probability of short-run success. From the perspective of the profession as a whole, both approaches will continue to be followed and further refined.

The tradeoff facing researchers about the two routes is primarily a matter of time horizon. Multi-cohort OLG approaches that explicitly keep track of different cohorts, their saving decisions, and their wealth stocks can be more rigorous theoretically. Other things being equal, a multi-cohort OLG approach thus may appear more attractive. But other things are not equal. Multi-cohort OLG models are more difficult and demanding than models that use analytical shortcuts to get demographic effects into the consumption-saving specifications in macroeconomic models. The OLG models, moreover, are likely to take much longer to advance to the stage where the models can deliver interesting empirical conclusions. The requirements of a multi-cohort specification are of course especially demanding in a model with numerous separate national economies and national currencies.

Another disadvantage of the multi-cohort OLG specification is that it might, if calibrated only to partial-equilibrium relationships derived from micro-level evidence, deliver misleading inferences about aggregative macroeconomic relationships. As Hamid Faruqee emphasized to us in a comment made after the international workshops that launched this project, it is both the virtue and the vice of a micro-level specification for individual agents that variables such as goods prices and interest rates are taken as exogenously given. Goods prices and interest rates self-evidently cannot be modeled as exogenous at an aggregative, economy-wide level. It is thus unclear whether a disaggregated OLG model, based on partial-equilibrium relationships estimated from micro-level data, will yield correct inferences and predictions for general-equilibrium, macroeconomic behavior (in the sense of being able to replicate the moments of actual macroeconomic data).

What we have labeled as "shortcuts" in the consumption specifications of macroeconomic models exist in several forms. Masson-Tryon (1990) followed one such route in their first effort to adapt the IMF Staff's MULTIMOD to study the consequences of population aging.[14] Meredith (1995) followed a similar approach in studying demographic changes and saving in Japan. A different short-cut approach to incorporating the effects of demographic changes in the equations of macroeconomic models is described in Fair-Dominguez (1991).[15] The staff in the Economics Department at the OECD experimented with a shortcut incorporation of demographics in a new "Minilink" model; see Turner and others (1998). Bryant experimented along analogous lines at Brookings, using a two-region abridgement of the IMF Staff's MULTIMOD that is a precursor of the model described later in this paper. The OECD staff and Bryant both built on a modified consumption-saving-wealth specification embodied in the Mark III revision of the IMF staff's MULTIMOD, as described in Laxton and others (1998) and Faruqee, Laxton, and Symansky (1997).

The shortcut approaches identified in the preceding paragraph specified a negative macroeconomic link between dependency ratios and saving rates. That relationship was justified in part on the basis of regression analysis with macroeconomic data. Numerous macroeconomic studies reported that negative correlation. Thus, at least at first blush, the age profile of savings appeared more consonant with the life-cycle hypothesis at the macro than at the micro level.

Table 1 replicates a survey of estimates from previous studies prepared by Guy Meredith (1995, Table 4-1).[16] It is relatively easy with macroeconomic data, as the table shows, to run

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[14] See also Masson (1992) and Masson/Bayoumi/Samiei (1995, 1998).

[15] In 1999-2000 Fair and Robin Brooks were experimenting with further extensions of that research, including applications of it to the equations in the Fair multi-country model.

[16] Meredith circulated this update of his table, which adds several papers not included in Meredith (1995), at the July 1998 workshops. A similar update table is given in the OECD study by Turner and others (1998, Table 2, p. 46). In

regressions using an aggregate saving rate as the dependent variable that yield significant coefficients on the youth dependency ratio and the elderly dependency ratio. Some of the estimated coefficients in these studies are implausibly high, suggesting for example that a 1 percentage point increase in the elderly dependency ratio could lead to a decline in the saving ratio of as much as 1 percentage point or more. But even the macroeconomic studies estimating much smaller coefficients, for example as small as -0.15 to -0.30, imply very substantial effects on saving from changes in the demographic composition of the population.

Macroeconomic estimates such as those in Table 1 are contentious, and justifiably so. Skeptics assert that when one adjusts such macroeconomic regressions properly for country intercepts and other econometric problems, one finds that the demographic effects again tend to be rather small, if not to go away altogether. The adjusted effects also seem to be quite heterogeneous across countries. Most important, as stressed above, there is substantial dissonance between these macroeconomic estimates and the microeconomic evidence based on household survey data.

3. A Modified Approach for Incorporating Demographics in Consumption and Wealth Accumulation

Hamid Faruqee and Douglas Laxton, economists at the International Monetary Fund, have recently suggested an improved "shortcut" approach for the incorporation of demographic factors into model specifications of consumption, saving, and wealth accumulation. The approach builds on their earlier work with the Blanchard (1985)-Weil (1989)-Yaari (1965) overlapping generations framework -- as in the Faruqee-Laxton-Symansky paper (1997) and the Mark III version of MULTIMOD -- but importantly modifies the manner in which demographics are treated. Our current research is exploring this approach and adapting it further to empirical multi-region models. We

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their update table, the OECD staff contrast the macroeconomic studies with the microeconomic evidence, the latter tending to find near-zero or very low effects on the saving rate from changes in dependency ratios.

believe the approach is promising and represents a marked improvement over previous shortcut efforts. The modified approach yields effects of changing demographics on saving rates that are somewhat smaller than the effects obtained in previous macroeconomic analyses. But the effects are still present, and important enough to significantly influence macroeconomic outcomes.

a) **Essentials of the Modified Blanchard Framework for Consumption and Saving**.

Blanchard's seminal theoretical paper (1985) made use of work by Yaari (1965). The model was extended by, among others, Buiter (1988) and Philippe Weil (1989). A version of the basic model has been incorporated in the IMF's MULTIMOD and in several studies by IMF economists. Variants of the model appear in numerous other studies. Zhang (1996) and Bryant and Zhang (1996a, 1996b), for example, used a version of it in their research on fiscal closure rules.

In models in which agents have finite life horizons, the analysis of retirement and elderly dependency is analytically complex. To assume in a model that individuals retire at a given age and thereafter receive zero labor income introduces a discontinuity that greatly complicates the modeling of saving, consumption, and wealth accumulation. Another discontinuity results in model environment when children enter the labor force and start to earn labor income at a specific given age. With finite-lived agents, no exact or even approximate aggregate consumption function can be derived by aggregating over the behaviors of individuals who differ in age and in consumption propensities.

Blanchard showed that a simplifying assumption can expedite aggregation and modeling and thereby allow a researcher to avoid the adoption of a more complex and analytically difficult multi-cohort OLG approach. Blanchard's key assumption was that each individual, throughout life and regardless of age, faces a constant probability of death, p. The expected life of an individual is thus $1/p$. With this assumption, researchers can choose a value for p anywhere between zero and a large number. If p is put at the limiting case of zero, individuals live forever and the model yields the infinite-horizon results familiar from still simpler models; values of *p* in the range .03 to .01 yield model "life expectancies" in the range of 33 to 100 years.[17]

The constant-probability-of-death assumption can be combined with an assumption, based on Yaari (1965), that the economy contains life insurance companies permitting agents to costlessly make annuities contracts contingent on their deaths.[18] The two assumptions together permit the derivation of an aggregate consumption function without keeping explicit track of the consumption and wealth of multiple cohorts. Aggregate consumption turns out to be a relatively simple linear function of human and non-human (financial) wealth, with the marginal propensity to consume dependent on the age-invariant probability of death and individuals' rate of time preference.[19]

Blanchard's original theoretical exposition assumed for convenience that the population is stationary and there is no growth in productivity. As Blanchard stated in a footnote and as Buiter (1988) and Weil (1989) showed in detail, however, the model can be readily adapted to cover the cases of a growing population and growth in productivity.

The original theoretical exposition included a simplified method of permitting the income and saving of individuals to decline with age. In their followup work, Faruqee, Laxton, and Symansky

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[17] The simplifying assumption is of course at variance with real life. Blanchard himself pointed out that the evidence on mortality rates suggests low and approximately constant probabilities of death from, say, ages 20 through 40; thereafter mortality rates in real life do rise with age (sometimes modeled by "Gomperty's Law" as in Wetterstrand (1981)), reaching rates (in the United States) in the neighborhood of 16 percent by age 80 and 67 percent by age 100. Model agents might be interpreted as families rather than individuals, and *p* would then be interpreted as the probability that the family ends (because, for example, members of the family die without children or current members of the family have no bequest motive). The assumption of a constant-throughout-life *p* is less inconsistent with real life if agents are interpreted as families. In subsequent papers reporting on our research, we will describe a model with youth dependency in which children have a different (age-invariant) probability of death than the (age-invariant) probability for adults. Hamid Faruqee has gone still further and shown how it is possible to modify the simplifying assumption that all adults are subject to the same age-invariant probability of death.

[18] The Blanchard-Yaari insurance-market assumption also requires a supplementary assumption that individuals are not motivated to leave bequests to survivors and cannot go into debt so as to die with negative bequests.

[19] The marginal propensity to consume is more complex if some of the assumptions in the original basic Blanchard model are relaxed (as Blanchard himself showed). In particular, if the intertemporal elasticity of substitution (IES) differs from unity and if the probability of death is time-varying (but still age invariant), the marginal propensity to consume depends on the IES and on the entire sequence of future expected values of interest rates and future mortality rates. For the case when the IES is assumed to be unity (the case of logarithmic utility) and when the age-invariant mortality rate is assumed to be constant through time, the marginal propensity to consume out of wealth reduces to the simple form of a constant, the sum of the time preference rate and the mortality rate.

(1997) introduced a more elaborate age-earning profile for individuals' incomes. They allowed for a time profile of an individual's labor income that corresponds fairly closely to empirical observations, showing a rise with age and experience when individuals are relatively young but then eventually declining with age as individuals approach retirement years and beyond. Aggregate total labor income in their model was distributed according to empirically derived age-specific weights. Faruqee, Laxton, and Symansky avoided a sharp discontinuity occurring at retirement age by approximating the lifetime age-earnings profile with a continuous curve of exponential terms that eventually declines to zero but does not have an explicit kink at retirement age.[20]

Still another refinement -- allowance for liquidity-constrained consumers -- was introduced by Faruqee, Laxton, and Symansky (1997) and subsequently adopted in MULTIMOD Mark III. As noted above, the motivation for this refinement is that capital-market imperfections prevent some agents from borrowing against their future incomes. The relevance of such a constraint is especially likely for younger individuals who have not yet established their creditworthiness and who have insufficient collateral in the form of financial wealth. These agents denied access to credit markets are assumed to have no choice but to consume out of their current resources. With the addition of an assumption that a fraction of consumers are constrained from borrowing, overall consumption depends on the behavior of intertemporally smoothing agents consuming out of lifetime ("permanent") income *and* agents consuming out of current income. The simplifying assumption that the fraction of liquidity-constrained consumers is constant over time can be roughly justified by supposing that as one cohort graduates out of the credit-constrained class another cohort of the same size is born into it.

McKibbin and Sachs (1991), McKibbin (1997), and McKibbin and Wilcoxen (1999) also allow for a constant fraction of consumption that is constrained by current incomes. The earlier

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[20] For analytical simplicity, youth dependents were ignored. Individuals were assumed to be born at age 20 and assumed to immediately start work in the labor force. See below for further comments about youth dependency.

modeling analyses by McKibbin and colleagues, however, did not incorporate Blanchard's assumption of a constant-throughout-life probability of death.

b) <u>Demographics with Endogenous Modeling of Population Growth</u>

Although the introduction of a realistic age-earnings profile by Faruqee-Laxton-Symansky (1997) permitted life-cycle behavior to be manifest in the aggregate consumption function, their approach still represented a fairly mechanical representation of life-cycle elements. The approach was a "top down" method of incorporating the age-earnings profile, in the following sense. The aggregate total of labor income was distributed across different age cohorts by the fixed age-specific weights derived from empirical observations. But the aggregate total of labor income itself was independent of changes in the demographic composition of the population.

The important innovation in Faruqee (2000a, 2000b) is to abandon the top-down approach and instead to build up the total of labor income from, so to speak, the "bottom up." The bottom-up approach permits changes in the demographic composition of the population to influence not only the allocation of total labor income across age cohorts but also the aggregate amount of total labor income itself. This modification of the Blanchard approach thereby permits more accurate study of the effects of demographic change on consumption and wealth accumulation, and hence on all macroeconomic variables.

The modified approach treats an economy-wide birth rate and death (mortality) rate as time-varying, age-invariant exogenous variables.[21] When children are ignored in the simplest version of the model economy, youth dependency does not exist. The "birth" rate in the simplest version then has to be interpreted as the arrival of new adults into the population. In effect, adults are born at, say,

21 We follow Faruqee (2000b) closely in the remainder of the exposition that follows.

age 20 and immediately enter the labor force. The model also presumes that immigration and emigration are always zero.[22]

For the model without youth dependency, the assumptions of an adult "birth" rate and a single mortality rate lead to a simple dynamic equation for the evolution of the overall population:

$$\frac{\dot{N}(t)}{N(t)} = n(t) = b(t) - p(t) \tag{1}$$

where *N* is the level of the (adult) population and *n* is the population growth rate, equal to the difference between the birth rate, *b*, and the age-invariant death rate, *p*. A dot over a variable indicates the derivative with respect to time. The number of individuals belonging to a particular cohort *s* at time *s* (the date they are born) as a proportion of the contemporaneous population is given by:

$$N(s,s) = b(s)N(s). \tag{2a}$$

The number of survivors from that cohort at a subsequent time $t \geq s$ is given by:

$$N(s,t) = N(s,s)e^{-\int_s^t p(v)dv} \tag{2b}$$

If equation (1) is integrated over time, the size of the total population at any moment in time (up to a constant of integration) is:

$$N(t) = e^{\int_{-\infty}^{t} n(v)dv} = e^{\int_{-\infty}^{t} (b(v)-p(v))dv}. \tag{3a}$$

Hence in this simplified world the population evolves according to the accumulation of past changes to its growth rate, which is just the past differences between the birth rate and the common-across-all-cohorts death rate. The population can also be defined in terms of the total of all existing individuals, summed across all cohorts (indexed by *s*):

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22 Faruqee (2000a, 2000b) observes that the assumption that old members of the adult population die at the same frequency as young adult members has the undesired effect of overstating the share of the elderly in the adult population. We roughly compensate for this problem by postulating an adult "birth" rate and a mortality rate that are somewhat above the steady-state levels that would otherwise seem appropriate.

$$N(t) = \int_{-\infty}^{t} N(s,t)ds \tag{3b}$$

To have a measurement of the age composition of the population, one can define an elderly dependency ratio as the proportion of the population above a certain threshold age level. The cohort just attaining this threshold age each period is indexed by *i(t)*. The (fixed) difference between this index *i(t)* and the present time, *t*, reflects the number of adult years needed to reach elderly dependency status. For example, members of the cohort reaching the model age of 45 years at time *t* were born and entered the labor force 45 years ago -- i.e., *t* - *i(t)* = 45.[23] Given a value for the elderly index that is constant over time, a dependency ratio can be defined as:

$$\phi(t) = \int_{-\infty}^{i(t)} \frac{N(s,t)}{N(t)} ds. \qquad 0 \le \phi \le 1. \tag{4}$$

In an economy with a constant adult birth rate, the dependency ratio would also be constant. For the case where the birth rate and death rate are time varying, the dependency ratio evolves over time according to:

$$\dot{\phi}(t) = \frac{N(i(t),t)}{N(t)} - [p(t) + n(t)]\phi(t). \tag{5}$$

Intuitively, the change in the dependency ratio is determined by the relative size of new "dependents" reaching the threshold age (the first term in (5)) *less* the proportion of the elderly who die in the period ($p(t)\phi(t)$) *less* a scaling term accounting for growth in the population base ($n(t)\phi(t)$).

c) <u>Age-Earnings Profiles from the "Bottom Up"</u>

The key new component in the approach summarized here is the introduction of age-earning profiles for labor income.

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[23] The threshold age in actual life years would be 65 if one thinks of model agents as being born at the age of 0 rather than at age 20. As discussed above, there is no discontinuity at "retirement" *per se*. Agents older than the threshold elderly age thus still continue to receive some labor income. But as agents get older, after passing the years of their peak earnings their incomes decline continuously toward zero.

The profiles entering the theoretical specification broadly match the age-earning profiles observed in actual datasets. After an individual enters the labor force, his or her labor income rises with age and experience, reaches a peak in late middle age, and then declines gradually for the rest of life. Figure 1 illustrates the typical hump shape of age-earning profiles with Japanese data for the years 1970-1997.

In real life, the labor earnings of individuals at the time of retirement fall, in some cases sharply, but often not all the way to zero. For the economy as a whole, post-retirement labor earnings of the elderly decline gradually, reaching zero for the elderly of advanced age. In the approach implemented for our model economies, it is necessary to approximate the age-earnings profile with a mathematical formulation that is continuous and permits aggregation across cohorts. Figure 2 shows two alternative curves that roughly approximate the shape of the humped age-earning profiles in actual data (with the illustrative data taken from the United States).

In the model, the shape of the age-earnings profile for individuals in the economy is assumed to be the same for all individuals and unchanged through time. The labor inputs of all cohorts are inelastically supplied, though the supply varies across cohorts depending on their age. The implicit assumption is that the underlying institutional and structural features that determine variation in relative labor earnings across age groups can be taken as fairly stable in recent historical time.[24]

The composition of the population by age can change over time. Even with the assumption of an unchanged age-earnings profile, therefore, demographic shifts can generate large changes in the earnings from labor income. Total labor income is obtained by aggregating over individuals that differ in age and experience. Hence a bottom-up aggregation over individuals permits the demographic changes to influence both the aggregate level and the age distribution of labor income.

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[24] The Japanese data in Figure 1 and US data in figure 2 suggest that this assumption is warranted (see, however, the further discussion at the end of the paper about relaxing this assumption in future work).

The hump-shaped profile of earnings by age influences both the supply side and the demand side of the model economies' behavior. On the supply side, the earnings profile is an indicator of the changes in a cohort's relative productivity and its supply of labor over its lifetime. On the demand side, the anticipated path of labor income determines the saving plans of consumers over their lifetimes. Hence through these life-cycle effects, changes in demographics significantly influence macroeconomic outcomes.

Specifically, the labor input of an individual cohort *s* at time *t* is assumed to be given by:

$$l(s,t) = \left[a_1 e^{-\alpha_1 (t-s)} + a_2 e^{-\alpha_2 (t-s)} + (1 - a_1 - a_2) e^{-\alpha_3 (t-s)}. \right] \quad (6)$$

The difference between the time index *t* and the cohort index *s* specifies the age of a particular cohort. The three exponential terms are a way of approximating the age-earnings profile. The parameters $a_1, a_2, \alpha_1, \alpha_2,$ and α_3 are specified exogenously in modeling code. They can be estimated econometrically for individual countries, for example, by using a non-linear least squares estimation procedure with actual data for age and earnings. (Figure 2 shows two examples of such estimates for U.S. data.) Loosely speaking, the first two exponential terms may be thought of as representing the decline in an individual cohort's labor supply over time as it ages and (gradually) retires. The third term can be interpreted as reflecting gains in earnings that accrue with age and experience. The restriction on the a_i terms (the third of the terms must be equal to $1 - a_1 - a_2$) embodies a normalization that the youngest cohort (for whom $s = t$) earns income equal to unity. Together the three exponential terms provide the hump-shaped profile for earnings.[25]

[25] To ensure that the sum of the exponential terms portrays a reasonable profile and that the effective amount of labor supplied is always initially increasing (when $t = s$), the following restriction on the five parameters must also hold: $a_1 > \dfrac{[\alpha_3 - a_2(\alpha_3 - \alpha_2)]}{(\alpha_3 - \alpha_1)}$.

The earnings of an individual cohort also change over time because of general growth in labor productivity, assumed to apply uniformly to all cohorts (after adjustment for their age-specific *relative* productivities). The earnings of a particular cohort are

$$y(s,t) = \left[wage(t)\right] l(s,t) \tag{7}$$

where $wage(t)$ is the economy's wage rate and the wage grows through time at a constant rate of labor-augmenting technical change, μ.

If (6) and (7) are aggregated over all individual cohorts, aggregate labor income can be written as:

$$Y(t) \;=\; \int_{-\infty}^{t} wage(t)\, l(s,t)\, N(s,t) ds \;=\; wage(t)\, L(t) \tag{8}$$

where L is aggregate labor input (adjusted for cohort-specific relative productivities). The definition of labor input for the individual cohort in (6) also permits one to write aggregate L as the sum of three components L_1, L_2, and L_3 where each component reflects an exponential term in (6). Specifically, define

$$L_k(t) = \int_{-\infty}^{t} l_k(s,t) N(s,t) ds \tag{9a}$$

where $l_k(s,t) = a_k e^{-\alpha_k (t-s)}$ and $k = 1,2,3$ so that

$$\dot{L}_k(t) \;=\; a_k b(t) N(t) - (\alpha_k + p(t)) L_k(t). \tag{9b}$$

Then one can specify a dynamic equation for aggregate labor input as

$$\begin{aligned} \dot{L}(t) \; &= \dot{L}_1(t) + \dot{L}_2 + \dot{L}_3(t) \\ &= b(t)N(t) - (\alpha_1 + p(t))L_1(t) - (\alpha_2 + p(t))L_2(t) - (\alpha_3 + p(t))L_3(t). \end{aligned} \tag{10}$$

The intuition behind (10) is that changes in aggregate labor input depend on the effective labor supply of new entrants to the labor force and on the death and relative productivity experiences of existing workers. Our model code uses a discretized version of equations (9) and (10) to describe the dynamic behavior of labor supply. The specific values of the five coefficients $a_1, a_2, \alpha_1, \alpha_2$ and α_3 obtained from estimating the age-earnings profile play an obviously important role in determining the movements of effective labor supply over time. They also play a critical role in the evolution of human wealth and consumption over time.

The specification of financial wealth, human wealth, and consumption follows lines that are by now familiar in macroeconomic models. Individuals maximize expected utility over their lifetimes subject to an intertemporal budget constraint. Financial assets for an individual or household -- $fw(s,t)$ -- are determined by its saving, equal to the difference between income and consumption:

$$f\dot{w}(s,t) = \left(r(t) + p(t)\right) fw(s,t) + y(s,t) - \tau(s,t) - c(s,t). \tag{11}$$

In this expression for the change in financial wealth, r is the real interest rate, τ is the tax rate on labor income, $y - \tau$ is disposable labor income, and c is consumption. The term $p(t)\, fw(s,t)$ enters into the intertemporal budget constraint reflecting the operation of the stylized insurance-annuities market as in Yaari (1965) and Blanchard (1985).[26]

Suppose that the utility function of the individual is of the constant relative risk aversion (CRRA) form, $u(c(s,t)) = \frac{c(s,t)^{1-\sigma} - 1}{1-\sigma}$, where σ is the coefficient of relative risk aversion, and 1/σ is the intertemporal elasticity of substitution (IES). Solution of the consumer's utility maximization problem gives the individual's consumption as a linear function of total (financial plus human) wealth:

$$c(s,t) = \frac{1}{\Psi(t)}[\, fw(s,t) + hw(s,t)]. \tag{12a}$$

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[26] In effect, agents in the model contract with the insurance company to transfer all their wealth (positive or negative) to the insurance company at their death. Given wealth in the current period, w_t , an agent receives an amount from the insurance company equal to $p_t w_t$ if still alive but pays the insurance company w_t if he dies.

Here $hw(s,t)$ is a measure of the individual's human wealth (essentially the present value of future labor income); $1/\Psi(t)$ is the marginal propensity to consume out of wealth, with $\Psi(t)$ given by:

$$\Psi(t)=\int_t^{\infty} e^{\frac{1}{\sigma}\int_t^{v}\{(1-\sigma)[(r(\mu)+p(\mu)]-(\theta+p(\mu)\}d\mu} dv\,, \tag{12b}$$

which can be written as:

$$\dot{\Psi}(t)=-1-\frac{1}{\sigma}[(1-\sigma)(r(t)+p(t))-(\theta+p(t))]\Psi(t)\,. \tag{12c}$$

The parameter θ is the individual's rate of time preference (for simplicity assumed to be constant across all agents in the economy).

The marginal propensity to consume out of wealth in the general case of the CRRA utility function thus depends on the intertemporal elasticity of substitution and on the entire sequences of future interest rates and future adult mortality rates. This dependence is readily evident in equations (12b) and (12c). In contrast, if the utility function is of the logarithmic form where the intertemporal elasticity of substitution is forced to be unity and if the adult mortality rate is assumed to be constant rather than time varying, the marginal propensity to consume out of wealth in equation (12c) reduces to a constant, the sum of the time preference rate and the mortality rate ($1/\overline{\Psi}=\theta+\overline{p}$). Individual consumption then takes the simpler form:

$$c(s,t)=\left(\theta+\overline{p}\right)\left[fw(s,t)+hw(s,t)\right]. \tag{12d}$$

For the purposes of the empirical simulations in this initial paper, we report results for the simpler case where the consumption of the individual follows (12d) ($\sigma = 1$).

The saving behavior embodied in equation (12) means that individuals eventually build up financial wealth to maintain a certain level of consumption in later years. The income from accumulation of financial wealth offsets the decline in their labor income and human wealth as their labor supply falls gradually. This behavior contrasts with more traditional life-cycle models in which the elderly are presumed to run down financial assets aggressively in later life (exhibiting negative saving rates). Those traditional models, as noted earlier, are often criticized for the presumption of negative saving by the elderly, which tends to be inconsistent with the empirical facts in many countries. In the specification used here, individuals in effect target a certain level of financial wealth as a precaution against the possibility of remaining alive without sufficient labor income.

An expression for individual human wealth is given by:

$$hw(s,t) \equiv \int_t^{\infty} [y(s,z) - \tau(s,z)] e^{-\int_z^t (r(\mu)+p(\mu))d\mu} dz \tag{13}$$

A corresponding dynamic equation for the individual's human wealth can be written:

$$\dot{hw}(s,t) = \big(r(t) + p(t)\big) hw(s,t) - \big[y(s,t) - \tau(s,t)\big]. \tag{14}$$

In a way analogous to the exponential terms used to represent the age-earnings profile in equation (6), the individual's human wealth can be expressed as the sum of three components:

$$hw(s,t) = hw_1(s,t) + hw_2(s,t) + hw_3(s,t) \tag{15a}$$

with the components defined as

$$hw_k(s,t) = \int_t^{\infty} (1-\tau(t)) wage(t) l_k(s,v) e^{-\int_v^t (r(\mu)+p(\mu))d\mu} dv \tag{15b}$$

for $k = 1,2,3.$ In equation (15b) it is assumed that labor income taxes are proportional to income and do not vary by age cohort, that is $\tau(s,t) = \tau(t) y(s,t)$.

If as before one aggregates across all individuals, total consumption for the case of logarithmic utility and a constant-through-time mortality rate -- aggregating individual consumptions from equation (12d) -- is seen to depend on a marginal propensity to consume out of total wealth (financial wealth plus human wealth), with the propensity depending on the rate of time preference and the death rate:

$$C(t) = \big(\theta + \overline{p}\big)\big[FW(t) + HW(t)\big]. \tag{16}$$

HW is aggregate human wealth and FW is aggregate financial wealth (holdings of domestic money, domestic bonds, equity claims on the domestic capital stock, and net foreign assets). For the more general case where utility is modeled with the constant relative risk aversion (CRRA) function and the mortality rate is time varying, the marginal propensity to consume out of wealth is less simple than in equation (16). For that case, aggregate consumption still depends on the sum of financial and human wealth; the marginal propensity to consume out of wealth, however, depends not only on θ but also in a nonlinear way on the intertemporal elasticity of substitution (IES, which need not be unity) and the entire future path of expected real interest rates and time-varying mortality rates *p(t)*.

The change in aggregate human wealth, a variable representing the present value of economy-wide labor income (adjusted for the varying ages and relative productivities of different cohorts), is given by:

$$\dot{WH}_t = \frac{d}{dt}\int_{-\infty}^{t} wh(s,t)N(s,t)ds \tag{17a}$$

$$= wh(t,t)b(t)N(t) \; + \; r(t)WH(t) \; - \; \left[Y(t) - T(t)\right] \tag{17b}$$

where *T* is total taxes on labor income and *Y - T* is total disposable labor income.[27]

Equations (10), (16), and (17) are the key macroeconomic relationships in the modified approach taken here. Changes in the demographic composition of the population and effective labor force significantly influence the aggregate supply of labor, given the differences across age groups summarized in the age-earnings profile. On the demand side of the economy, aggregate consumption and saving behavior -- and hence also wealth accumulation -- are strongly influenced by the life-cycle effects of the demographic changes on human wealth.

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[27] Equation (17b) shows that the incremental change in the stock of aggregate human wealth at time *t* is influenced by the additional human wealth of the newest generation born at time *t*, that is by $wh(t,t)$. The shape of the labor-earnings profile -- embodied in the five parameters $a_1, a_2, \alpha_1, \alpha_2$, and α_3 -- has a critical influence on the behavior through time of $wh(t,t)$ and hence of aggregate human wealth, *HW(t)*.

4. Our Strategy for Testing the Modified Approach in Multi-Country Empirical Models

Multi-country macroeconomic models are inherently complex. Differences among countries or regions in any given model interact in numerous ways with the model's specifications of economic behavior, thereby making it difficult to understand the consequences of changing the model's specification for any one aspect of behavior. Since alternative competing models have significant differences in the treatment of economic behavior and of particular nations or regions, comparisons across alternative multi-country models are especially complex (Bryant, Hooper and Mann, 1993).

To facilitate a clearer comparison of models and alternative treatments of economic behavior, the authors have constructed stylized and simplified versions of two existing macroeconomic models. Several years ago Bryant together with Long Zhang (1996a, 1996b) developed a stylized, two-region abridgement of the IMF staff's MULTIMOD model. McKibbin has recently developed a stylized, two-country abridgement of the McKibbin and Wilcoxen G-Cubed models (summarized in McKibbin and Vines, 2000) called the MSG3 model. These abridgements of the larger models -- labeled here for short as, respectively, the BM2R (Bryant-Multimod-2-Region) and MSG3 (McKibbin MSG3/G-Cubed 2-Region) models -- are research environments within which analytical comparisons are less complex.

Our preliminary work in this project has focused on refining these two stylized models to incorporate the modified approach for treating demographic changes and their macroeconomic consequences. Each author has made changes in his stylized model along the lines sketched above. The process has required frequent interactions to identify problems and work out their solution.

A subsequent research report will provide detailed descriptions of the BM2R and MSG3 models and our incorporation into them of endogenous demographics and the bottom-up determination of labor income and human wealth resulting from the incorporation of age-earning profiles. In this paper, we provide only a sketch of the two abridged models and identify the most important similarities and differences among them.

The starting point for both stylized models is a set of equations describing the U.S. economy (US for short). Then a second artificial country is created, labeled for brevity here as ZZ. The ZZ economy is an identical, mirror image of the United States. These two economies are carefully linked together with the balance-sheet and income-flow identities that would have to hold if the world were composed of only these two economies. The current-account balance and the net-foreign-asset position of the ZZ economy, for example, are exactly the negatives of the current account and the net-foreign-asset position of the US economy. A single exchange rate exists linking the two regions' currencies and economies.

Each region consists of several types of economic agents: households, firms, a government, and a central bank. The MSG3 model contains two production sectors, for energy and non-energy goods and services; output produced is a function of capital, productivity-augmented labor, energy, and materials. In addition there is a sector that creates capital goods which are purchased for investment purposes by both firms and households. In the BM2R model, there exists only a single composite good produced in each country, and hence only a single type of firm and production sector; output produced is a function of capital and productivity-augmented labor.[28]

The production technologies of firms in both the MSG3 and BM2R models are represented by constant elasticity of substitution (CES) production functions. Each country's goods are imperfect substitutes; each country exports some of its production to the other country. Imports in each country are a function of income and relative prices. Agents in a given country in the models are assumed to have identical, unchanging preferences over foreign and domestic goods.

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[28] The full MULTIMOD model of the IMF staff does distinguish oil from other goods. Some versions have specified non-oil commodities as a separate good. To simplify, Bryant removes these distinctions from his abridgement.

Both the MSG3 and BM2R models emphasize the forward-looking behavior of agents and presuppose that both firms and households engage in intertemporal optimization. (A partial exception in both models stems from their allowance for a fraction of consumers whose consumption is constrained by an inability to borrow and hence are unable to smooth their consumption intertemporally.) Both models require long-run evolutions of the model economies that result in steady-state, balanced-growth equilibrium paths.

The firms in the models are characterized as price-taking entities that choose variable inputs and their levels of investment in capital so as to maximize stock-market value. Firm investments in both the MSG3 and BM2R models respond to the difference between the market value and reproduction value of the capital stock (variants of Tobin's "q"). The MSG3 and BM2R models, however, differ in detail in how they model this process. For example, McKibbin adapts the cost-of-adjustment models of Lucas (1967), Treadway (1969), and Uzawa (1969) and following Hayashi (1979) allows investment also to depend on current cash flow. Bryant's approach so far in the BM2R model is less explicit about adjustment costs and follows the treatment in the Mark II version of MULTIMOD (Masson, Meredith, and Symansky, 1990; see also Meredith 1991).[29]

The specification of the household sector and effective labor supply in the BM2R and MSG3 models, as modified by the introduction of age-earning profiles and a bottom-up determination of labor income, is summarized above. When introducing the theoretical specifications in the stylized empirical models, we have attempted to do so in a comparable fashion. One significant difference stems from different underlying assumptions about the household's utility function, and hence about consumers' elasticity of intertemporal substitution (IES). The MSG3 model currently incorporates an assumption of log utility, imposing the assumption that the IES is unity. The BM2R model makes use of the CRRA utility function and would normally set the value of IES well below unity, for example

[29] Adjustment costs for investment in capital are modeled explicitly in the Mark III version of MULTIMOD (Laxton, Isard and others, 1998) and Bryant plans to amend the BM2R model accordingly in future work.

at 0.5. In the simulation results for the BM2R model reported in section 5 below the IES is assumed to be unity.[30]

Both stylized models treat labor as perfectly mobile within each of the two countries but completely immobile across the boundary separating the countries. Hence wages are equal across comparable age cohorts within each country but in general are not equal across the two countries. Over the long run, labor is inelastically supplied with respect to wages and is determined by the demographic structural equations already described. The MSG3 model allows for short-run unemployment because of sticky nominal wages, though the model converges to full employment in the long run. The BM2R model likewise forces full employment of labor and capital over the longer run.[31]

Prices are assumed to clear the goods market in the MSG3 model whereas in the BM2R model an estimated model of price stickiness is used. Thus the price stickiness in the MSG3 model is driven only by wage stickiness whereas the price dynamics in the BM2R model reflect both price and wage stickiness. This is an important difference in the models and may partially explain the larger cycle in the results for the BM2R model.

The government in each country engages in real spending on goods and services (taken as exogenous in both the BM2R and MSG3 models), can make real transfer payments to households (again taken as exogenous), raises revenues by taxing firms and households, and pays interest on its outstanding stock of debt. Both models assume that agents will not hold government debt unless per

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[30] In an earlier draft of this paper (the version of June 2001 presented in Helsinki Finland), some sensitivity analysis was undertaken on this parameter in the BM2R model, showing that alternative assumptions can make a significant quantitative difference. Further sensitivity tests on the importance of the IES will be included in subsequent papers in this project. In the early research, the MSG3 and BM2R baseline solutions have differed somewhat in their assumptions about the rate of consumers' time preference, corresponding to their sometimes different assumptions about the elasticity of intertemporal substitution.

[31] The current version of the BM2R model follows the Mark II version of MULTIMOD. Capacity utilization can differ in the short run from long-run full use of capacity. But the model does not explicitly include wages and employment, and hence does not explicitly track unemployment. The authors conjecture that some of the differences in simulation results between the two stylized models are attributable to the different specifications of the wage and price sectors in the models; we hope in future work to focus on these differences.

capita government debt is eventually forced to grow at a rate less than the interest rate paid on the debt. Both models for this exercise use a variant of a debt-stock targeting rule. The differences between alternative approaches for the specification of an intertemporal fiscal "closure rule" are carefully described in Bryant and Zhang (1996a).

The monies of the two countries appear in the models because of (implicit) transaction costs. Money demands in the models depend negatively on short-term nominal interest rates and positively on the value of aggregate output. The central bank in each country is assumed to follow a policy rule that ensures long-run nominal stability of the model's behavior. The models can enforce either a money-targeting rule, a nominal-GNP-targeting rule, or a rule combining inflation targeting with real GNP targeting. These rules are explained and analyzed in Bryant, Hooper, and Mann (1993). To facilitate comparison across the models and foster simplicity, in the first stages of this joint research both the BM2R and MSG3 models assume that the central banks follow money-targeting rules.

As a starting point for simulation experiments, we use the stylized models to develop model-consistent baseline paths for the evolution of the US and ZZ economies. In the BM2R model the steady state baseline is used whereas in the MSG3 model a baseline on the stable path to the steady state is used. Because the US and ZZ economies are identical in the baseline solutions for the two models, the exchange rate is constant over time at unity and the trade balances, current-account balances, and net-foreign-asset positions are constant at zero. Several alternative baseline solutions have been investigated. In one baseline, the birth rates and the mortality rates in both countries are held constant at 1-1/2 percent per year (.015), resulting in a population that is stationary over time. In a second baseline solution, the birth rate is held constant at 2-1/2 percent per year (.025) and the mortality rate remains constant at 1-1/2 percent per year (.015); in that baseline the two countries' populations grow at the constant rate of 1 percent a year.[32]

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[32] This second baseline underpins one of the set of curves shown in Figure 11.

The baselines typically assume that productivity growth occurs at a constant rate (2 percent per year in these initial simulations). Baseline rates of inflation are likewise assumed constant (also 2 percent per year).

We have done preliminary experiments in both models allowing for a moderate fraction (for example, one fourth) of consumers to be borrowing-constrained. Much of the preliminary experiments (and all those reported in this paper) have focused on the case where that fraction is zero. This issue is briefly discussed further below.

The two modeling approaches differ in a number of important theoretical and empirical respects. We have attempted to standardize as much as possible across the models without changing the core approaches of the models. For example in this paper the BM2R model has used a value of unity for the elasticity of substitution between labor and capital (the Cobb-Douglas case). Also for this paper we impose Cobb-Douglas production rather than those that were estimated on the MSG3 model. Despite this there are still a number of important differences in the models which show up in important ways in the differences in results. What is useful from this approach is that despite having different analytical and empirical views of the world, the two models produce many broad similarities in the results.

5. *Preliminary Simulation Experiments: A Transitory Demographic Bulge in the two Models*

In the simulations reported in this section, the focus is on a transitory demographic bulge. In the first variant, the bulge occurs simultaneously and identically in both the US economy and the ZZ economy; this variant in essence is the case of a closed world economy, since both the US and ZZ model economies are identical and are subjected to an identical demographic shift. The second variant assumes that the bulge occurs in the US economy but not in the ZZ economy; the second variant highlights the transmission of effects from an economy experiencing a demographic shock to the rest of the world.

The baby bulges -- strictly speaking, in this version of the model without youth dependency, the bulge of new 20-year-olds into the adult population -- take the following specific form. Prior to the shock, populations are stationary; the adult birth rates and mortality rates are all equal at the rate of 0.015. In year 1 the birth rate starts gradually to rise from the baseline rate. By year 8, the birth rate has risen to 0.03, a full 1-1/2 percentage points higher than in the baseline. At that point, population is growing at 1-1/2 percent per year rather than the baseline rate of 0 percent. After persisting at the higher level of .03 for 5 years, the birth rate then starts gradually to fall back. By year 22, roughly two decades after the demographic shock began, the birth rate is back to the .015 rate and remains at that rate thereafter. Throughout the shock, the death rate remains at .015. Figure 3 gives a visual plot of these series.

For the shock variant in which the demographic bulge occurs identically in both countries, populations and the effective labor forces are eventually 22-1/2 percent higher than in the baseline. The labor forces adjusted for age and relative productivities rise even higher while the demographic bulge is at its peak and waning -- see Figure 4. Figure 5 plots the elderly dependency ratios -- see equations (4) and (5) above. Initially in the baseline, slightly more than half of the population in both countries is above the threshold age defining the elderly.[33] In the simulation, as the demographic bulge occurs, the dependency ratios fall gradually until they reach a lower level of about 42 percent of the population. After an extended period of time at that lower level, the dependency ratios start to rise back toward the steady-state baseline level. After about 70 years, the dependency ratios have risen back to their initial levels.

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[33] In an economy experiencing steady-state balanced growth with a positive rather than a zero rate of growth in the population, the steady-state value of the elderly dependency ratio is well below this ratio. For example, a baseline generated by the model with a sustained 1 percent per year growth in population generates a steady-state elderly dependency ratio of approximately 0.33.

For the shock variant in which the demographic bulge occurs in the US only, the time paths in Figures 3, 4, and 5 pertain only to the US. The ZZ population and effective labor force in that variant never deviate from their baseline paths. The dependency ratio in ZZ remains throughout at its initial level.

The stylized shock studied here is analogous to the "baby boom" experienced by several industrial nations in the sixth and seventh decades of the 20^{th} century. The reversal of the baby boom with the passage of time results in population aging of the type now beginning to manifest itself in those nations.

Given both of the models' treatment of saving, investment, and financial variables as forward looking, agents immediately adjust some aspects of their behavior at the onset of the shock. The numerical algorithms used to solve the models presume that agents correctly anticipate the entire future path of the demographic shock. Thus key variables like interest rates, human wealth, the market value of capital, and forward-looking goods prices immediately jump to altered levels. For the shock variant in which only one of the regions experiences the demographic bulge, the nominal exchange rate and hence also the real exchange rate are significant "jumping" variables.

The assumption that agents, at the onset of a shock, correctly anticipate the entire future paths of exogenous variables is the now-standard working assumption about expectations in models solved with rational, model-consistent expectations. Thus our use of this assumption is familiar ground. Yet the assumption is extreme. Worse, it is inherently implausible for demographic shocks such as a baby bulge that begin gradually and then also wane gradually over many years. In previous research we have shown how it is possible to modify the model-consistent-expectations assumption by phasing in "correct expectations" with the passage of time rather than permitting expectations to be correct immediately and fully. However, for the time being we report the results here with the now familiar full model-consistent expectations.

The simulation results are shown graphically rather than in tables. All the time paths in the following figures are presented as deviations from the baseline solutions of the model (in units specified along the vertical axes of the figures). Thus if a variable has a value of zero in a figure, at that point the variable is unchanged from its baseline path.

We present the results for each model in a series of graphs. We first present a set of results for the BM2R model followed by the same results for the MSG3 model. Results are presented first for real variables, then for financial variables, and finally for international variables.

One can roughly differentiate three periods of adjustment in the results shown. Impact effects occur during the first and next few years of the shock as the population and the number of productive workers increases. (Remember that in these preliminary results we have not yet included youth dependency, so that a new member of the population is also immediately a new worker.) Given the shape of the age-earnings profile, the effective labor force increases even faster than the number of workers. Over a medium run, the (adult) birth rate stops increasing, levels out, and then starts declining; during this medium run, the demographic dynamics reverse and then pick up momentum in the opposite direction. Then over a longer run, the enlarged cohorts generated by the higher birth rate pass completely through the workforce (retirement occurs gradually and continuously) and the populations eventually re-settle onto equilibrium growth paths where the growth rates are identical with those in the baseline solutions.

Figures 6a and 7a and 6b and 7b summarize the world "closed-economy" simulation in which the demographic bulge occurs in both regions simultaneously. The figures qualified with an “a” are those for the BM2R model; those labeled “b” are for the MSG3 model. In the case of a world simulation, the exchange rate linking the two economies remains unchanged at unity throughout the entire simulation. The external balances of the two economies remain unchanged at zero values. The curves in Figures 6 and 7 show "domestic" variables. These domestic outcomes are identical in both countries.

In the results for the BM2R model (figures 6a and 7a) there are additional lines which should be ignored until the later discussion about the importance of the age earnings profiles.

Variables such as human wealth, the market value of capital, consumption, saving, and yields in asset markets are forward-looking "jump" variables. Given the assumption of model-consistent expectations, agents correctly anticipate the future events that will unfold and hence make immediate adjustments. Households, for example, initially reduce their consumption (northwest panel of Figure 6a and 6b) and increase their savings (northeast panel) knowing that the growing population will require a larger capital stock to equip a larger labor force. In the MSG3 model, the initial jump is dampened because of the adjustment costs in capital accumulation. Despite the desire to start saving for future capital requirements, the adjustment costs in the MSG3 model lead to a postponement of this decision (a similar result is noted in McKibbin and Wilcoxen (1997)). This delaying effect is much smaller in the BM2R model.

The initial movement in per capita human wealth is downward (figures 7a and 7b, NE panels). Because of the Blanchard assumption of a positive probability of death, households partly discount the future changes in their labor incomes (relative to models in which representative agents have infinite horizons). Coincident with the other initial changes, the market value of capital relative to the existing capital stock jumps up (SE panel of Figure 7a and 7b) and real interest rates adjust downwards (NW panel of Figure 7a).

As the population and labor force continue to grow, per capita human wealth and consumption fall further. At the time when growth in the population is fastest but not increasing further, the decline in per capita consumption ceases. Then as population growth starts to fall back, fewer households with low saving rates are added to the population and households perceive that, in effect, less saving will be needed than would have occurred with continued high population growth. Per capita consumption can then begin to rise back toward and then above baseline. Because of the hump-shaped age-earning profile, the *effective* size of the labor force continues to grow rapidly long after the population itself has ceased growing (Figure 4). Because of the bottom-up determination of aggregate labor income, moreover, per capita human wealth begins a long and sustained rise to a level well above baseline. After its initial jump upwards, the ratio of the market value of capital to the real capital stock falls back as the population grows; that ratio, too, then reverses course and begins a sustained cyclical rise.

The cyclical movements in per capita human wealth and consumption during the years 10 to 30 and the associated cyclical behavior of other domestic macroeconomic variables are partly due to the fact that the new members of the population resulting from the (adult) baby bulge are at first relatively low savers. Speaking loosely, these younger adults are ascending the left side of the hump of their age-earning profiles (Figures 1 and 2). As the bulge cohorts reach their years of peak earnings and high savings, another inflection point is reached. Per capita human wealth and consumption in the fourth decade of the shock begin a long decline relative to baseline. Eventually, as the baby boomers become elderly, their labor income and human wealth decline and they begin to consume out of their financial wealth. Per capita consumption in the long run returns to the baseline level of the initial steady state.

The SW panel of Figures 6a and 6b show the behavior of the real capital stock. The quite modest rise of the capital stock in the initial decades are also explained by the working in the model of the age-earning profiles and the bottom-up determination of labor income and human wealth as well as the adjustment cost approach to capital accumulation in both models. New members of society begin their lives without any financial wealth and are low savers. In the years of rapid population growth, therefore, the presence of numerous additional individuals with low saving slows down the capital accumulation that must eventually take place, leads to increases in interest rates, and explains why the capital stock declines slightly before population growth reaches its peak. Without the age-earning profiles playing a role in the model, capital accumulation would not be so weak and interest rates would not rise so much during the years 10 to 20. Once the bulge in population has occurred and the population growth rate has returned to zero (year 22 and thereafter), the baby boomers increasingly enter into their years of high saving. Financial wealth per capita and capital accumulation itself enter a period of rapid, sustained increase. By this time interest rates are falling back, even falling below the long-run baseline level to which they will ultimately return. Eventually, as the baby boomers pass into their elderly years and the elderly dependency ratio returns back to its original baseline level, capital accumulation gradually declines somewhat and interest rates rise back to their long-run level.

The role played in the model by the presence of age-earning profiles is central to understanding the simulation results. A useful way to see the importance of age-earning profiles is to compare simulations in which the profiles are and are not present. Consider, therefore, the second set of curves in Figures 6a and 7a for the BM2R model (the smooth, light curves without a marker). These curves are generated by a model run in which everything is identical except for the fact that the three "alpha" coefficients in the model -- see equations (10) and (17) above -- are set equal to zero instead of their estimated values. The effect of zeroing out these coefficients is to produce a model which does *not* have the distinctive cyclical behavior that has just been discussed. It still has unrelated cohorts being born and thus is not Ricardian. For example, without the presence of the age-earning profiles, per capita human wealth and consumption decline gradually and modestly while the demographic bulge is taking place and then gradually and slowly -- without cyclical behavior -- rise back to their baseline levels (NW and NE panels of Figure 6a). In the absence of the age-earning profiles, the capital stock would increase monotonically toward its new steady-state level and interest rates would never rise above their baseline steady-state level (NW and NE panels of Figure 6b).

Although the incorporation of age-earning profiles is probably the most distinctive and important feature of the model’s analysis of demographic shifts, many other aspects of the models also condition the results. In an earlier version of this paper, for example, we discussed the sensitivity of outcomes to the assumed value of the intertemporal elasticity of substitution. The simulation for the BM2R model shown here (using a value of the IES equal to unity) was compared with a simulation using an IES of 0.5. The differences between the two simulations corresponded qualitatively to what theory suggests should be true. When the IES is low rather than high, consumers are less willing to substitute future for present consumption. Consumers that are borrowing-constrained cannot be intertemporal smoothers. Per capita consumption under those conditions should therefore adjust somewhat less in response to shocks. Variables such as human wealth, interest rates, and the market value of capital, on the other hand, will have to exhibit greater volatility than when the IES is higher or when no consumers are borrowing-constrained.

The preceding discussion pertains to a closed world economy with no differences among its constituent parts. Our primary analytical interest, of course, is in situations where one part of the world economy experiences different shocks and different outcomes from those occurring elsewhere. Accordingly, the remainder of the charts and the discussion focus on the shock variant in which the demographic bulge occurs in the US economy only.

The three pages of Figures 8a, 9a and 10a for the BM2R model and figures 8b, 9b and 10b for the MSG3 model summarize the deviations of variables from the same baseline underlying Figures 6 and 7. The panels in the charts show two curves, one for US-economy variables (dark lines) and the other for ZZ variables (light lines). The panels showing the exchange rate (NW panel of Figures 10a and 10b) are an exception; for those panels, one of the curves is the nominal exchange rate, the other is the real exchange rate.

In the first year of the simulation following the onset of the US-only shock, the forward-looking variables in both economies exhibit immediate jumps (for the same reasons as in the worldwide closed-economy shock). In the BM2R model, in the case of interest rates and the market value of capital, the immediate adjustments are in the same direction in both economies. In the MSG3 model they move in opposite directions in both countries. For other variables such as per capita human wealth and consumption, however, the immediate adjustments are in *opposite* directions in both models. As in the closed-economy case, US households initially experience a downward jump in human wealth and consumption and raise their saving in anticipation of the fact that a growing US population will require a much larger capital stock to equip a larger labor force. Households in the rest of the world, on the other hand, modestly *increase* their consumption and *reduce* their saving, knowing that per capita human and financial wealth for ZZ residents will ultimately be boosted by the demographic shock occurring in the US.

Over the shorter and early medium run, as per capita human wealth and the market value of capital are declining in the US, a similar but much more damped cyclical movement occurs for those variables in the ZZ economy. Interest rates in the rest of the world, falling by less in the initial downward adjustment than in the US, likewise begin to increase, but again by less than in the US where the population is growing. During the medium and early longer run, ZZ variables also tend to echo the cyclical movement in counterpart US variables, but in muted degree. After the demographic bulge in the US has completely passed through the U.S. population and effective labor force (requiring the passage of some 75-90 years for the effective labor force -- see Figure 4), all US and ZZ variables are beginning an approach to their eventual new steady-state levels.

One important result immediately evident in Figures 8 is that the cyclical dynamics present in the worldwide closed-economy case are characteristic of the open-economy case as well. This result is not surprising for the economy in which the demographic shock originates. But it is less obvious that qualitatively similar cyclical movements will occur in the rest of the world. Yet, in this model environment, the US-originating shock is strongly transmitted abroad through the exchange rate, through trade volumes and prices, and through shifts in the external balances of the countries. The exchange rate and external-sector variables reflect the cyclicality originating in the US economy. This transmission of the shock through the cyclical variation in external variables, interacting with the age-earning profiles of foreign workers and their saving and consumption decisions, produces more muted but significant long swings in the ZZ economy as well. The macroeconomic outcomes in the foreign as well as shock-originating country are powerfully driven by the age-earning profiles in the model.

Consider the behavior of the nominal and real exchange rates (NW panels of Figure 10a and 10b). Both the nominal and real exchange rate initially jump upward, a depreciation of the US currency (appreciation of the ZZ currency). In the BM2R model both exchange rates then move gradually back toward or even below baseline in the rest of the first decade (an appreciation of the US currency). In the MSG3 model the nominal and real exchange rates are relatively flat for the rest of the initial decade. Over the medium run, as the rate of population growth reaches its peak and then falls back toward zero, the real value of the US currency then begins a period of strong depreciation in both models. Over the longer run, the higher output of US goods resulting from the higher population and effective labor force will lower the price of US goods relative to foreign-produced goods (given the assumption that US and ZZ goods are imperfect substitutes in consumption bundles). Thus the paths of the real exchange rate and nominal exchange rate must eventually be quite different. In the long run, the US currency must exhibit a significant *real depreciation*. Given differences in the evolution of price levels in the two countries (SW panel of Figures 9a and 9b), however, the US currency in the longer run must exhibit a *nominal appreciation* vis-a-vis the ZZ currency.

The shorter-run effects of the initial depreciation of the US currency are to reduce US imports and to stimulate US exports (increase foreign imports and reduce their exports). Thus for the first few years of the simulation, US residents do not import savings from the rest of the world but rather export some of their initially higher savings abroad. The timing and magnitude of this adjustment differs across the two models but are broadly consistent. The US trade balance and current-account balance show a modest surplus in the initial years as capital is exported from the country experiencing the demographic shock (the US). As the medium-run cyclical movements in consumption, output, and the capital stock occur, however, the US external balances move into deficit and foreign savings begin to flow net into the US to finance the forthcoming large buildup of the US capital stock. The US deficits bottom out (ZZ external surpluses stop increasing) around the time that population growth in the US has fallen back to its baseline growth rate of zero. From that point onward, the external balances (the ZZ external balances are of course exactly equal to those of the US but with opposite sign) commence a gradual and protracted move back toward baseline and their ultimate long-run steady-state ratios relative to nominal GDP.

Although the ratios to nominal GDP of the current-account balances of the two countries move back toward levels like those in the pre-shock steady state, the ultimate long-run ratios differ non-trivially from the baseline levels of exactly zero (NW panel of Figures 10a and 10b). The US current-account ratio moves to a permanent small negative number. The ZZ economy has a permanent surplus. The net-foreign-asset positions of the two economies (SW panels of Figures 10a and 10b) of course are the integral over time of the current-account balances.

As further evidence of how important a role the demographics and age-earning profiles play in the behavior of the exchange rates and external balances of the countries, consider Figure 11. The panels of that figure repeat the curves from the simulation shown in Figures 8a, 9a and 10a for the BM2R model for four variables: the real exchange rate, per capita human wealth and consumption for the US, and the US ratio of net foreign assets to nominal GDP. In addition to the curves from that simulation, shown as darker in color, each panel contains a second curve, shown in lighter color, obtained from a simulation identical in every respect except that the "alpha" coefficients in the age-earnings profile are set to zero. Again it is plain that the dominant source of the cyclicality in the model's behavior is attributable to the presence of the age-earning profiles and the bottom-up determination of labor income and human wealth.

When the age-earnings profiles are suppressed in the BM2R model, the real exchange rate after its initial jump depreciation continues gradually to depreciate slightly further. The US external balances continue in surplus until the medium run, which causes a further buildup in the US net external asset position (NE panel of Figure 11). Thereafter, as the population bulge ceases and the larger cohorts pass through the US population, the US net-foreign-asset position begins a long gradual decline. But it never turns negative, even in the very long run. In sharp contrast, when the age-earnings profile are allowed to exert their effects in the model, the initial buildup in the US net-foreign-asset position is reversed after some 6 years. The decumulation of the net-foreign-asset position is traceable to the new cohorts entering the population with their initially lower saving rates. Subsequently, when the baby boomers have ascended the left hump of their age-earning profiles and entered their high saving years, the US current-account deficit starts to decline (SW panel of Figure 8c) which in turn eventually leads to a diminution of the net-foreign-asset position relative to nominal GDP. Because of the numerous years of previous current-account deficits, however, US net foreign assets remain negative even in the long-run steady state.

To give a better sense of the differences between the simulation results for the worldwide closed-economy shock and the US-only shock, Figure 12 shows four panels of the preceding results from the BM2R model for four US variables. The broad patterns are of course similar for the US whether the shock occurs identically in both countries or occurs only in the US. But even for the US economy, differences between the two cases are noteworthy.

The strength of the cyclical effects in the model due to the age-earnings profiles depends significantly on the demographic assumptions built into the underlying steady-state baseline. Figure 13 illustrates this point by comparing, for two US variables, the simulation presented in Figures 8a, 9a and 10a with a simulation comparable in every respect except for the fact that steady-state population growth in the alternative simulation occurs at a positive rate of 1 percent per year. Only results for the BM2R model are shown. As the charts make clear, the cyclical effects produced by the age-earnings profiles are significantly different. For example, the cyclical movements in per capita human wealth have different amplitudes. The initial jump upwards in the market value of capital is greater when the baseline population is growing steadily at 1 percent. But the upward rise relative to the actual capital stock in the medium-run years is smaller and the subsequent decline back toward the long-run steady-state level is somewhat smaller as well.

A last point stands out in the simulations when the demographic shock occurs only in the US. Welfare in the rest of the world -- at least if narrowly interpreted by a measure such as real per capita total consumption (private plus government) -- is *improved* in the long run by the demographic bulge in the US. (There is a transitory period in the early medium run, a bit longer than a decade, in which real per capita consumption in the rest of the world falls below baseline.) Welfare consequences in the US, again measured by the crude rubric of real per capita consumption, are complex. As the demographic bulge occurs, real per capita consumption has to fall well below baseline. At the height of the bulge and while it is waning, on the other hand, real per capita total consumption in the US rises well above baseline. Ultimately, in the BM2R model US per capita real consumption declines to a point significantly -- some 2-1/2 percent -- below baseline. In contrast, in the MSG3 model it remains forever above baseline.

At first glance, this result may seem puzzling. Why should the average US consumer be ultimately worse off than in the steady-state baseline? The intuitive way to understand this result is that US consumers at the time the shock occurs are better off in an open economy than they would otherwise be in a closed economy, given their positive rate of time preference and given that in this Blanchard-type world they further discount the future because of an above-zero probability of death. US consumers alive at the time the demographic shock begins wish to keep up their consumption, even if their discounted gains come at the expense of future US generations. The future US generations alive many decades later have to pay investment income to foreigners because of the borrowing from foreigners that facilitated the buildup of the US capital stock required by the US population bulge. In the long run, foreigners earn net income on their permanent creditor net-foreign-asset position. Future generations of foreigners are thus better off on average in terms of their consumption per capita than they otherwise would have been. Seen in the light of these considerations, the outcomes for per capita consumptions in the two countries are not puzzling. US consumers in the shorter run have higher welfare, in the sense that the US can keep consumption per capita above the levels that would otherwise have to prevail in a closed economy. But the higher welfare in the shorter run occurs at the expense of their children and grandchildren.

6. Tentative Conclusions

The preliminary findings in our joint research -- the results reported here are illustrative of those findings -- are promising. As in the initial efforts of Faruqee (2000a), a Blanchard-type approach with the proposed modifications for a "bottom-up" modeling of age-earnings profiles has proven feasible and produces plausible conclusions.

Our research shows that demographic effects can be incorporated into a framework that endogenously determines all the key macroeconomic variables involved in consumption, saving, and wealth accumulation. Most importantly, the framework permits study of external-sector macroeconomic variables and spillover influences on foreign countries. As illustrated in the preceding discussion, exchange rates and cross-border trade and capital flows can be powerfully influenced by underlying demographic changes.

More work is of course needed to refine and test our approach further. Children and youth dependency need to be introduced in the models. Our presumption is that the incorporation of youth dependency will have major influences on the dynamic effects of demographic change. We also plan to incorporate an explicit simplified pension-social-security system in the models. Tax revenues will be collected from working cohorts of the population and transfers will be made to elderly members. Any imbalances between revenues and transfers will result in government budget imbalances and changes in government debt stocks, which in turn will have significant macroeconomic effects. A further refinement we would like to make is a relaxation of the assumption that an economy’s age-earnings profile is unchanged through time.

Despite the need for such improvements, we now feel confident that significant progress can be obtained along the lines developed in this paper. We thus plan to continue with development of our two-region stylized models, working out identified problems and improving the models' abilities to capture key features of demographic influences on macroeconomic behavior.

Our simplified approach ignores immigration and emigration. Immigration is a quantitatively significant phenomenon for the United States and several European nations. Immigration has not been, at least not yet, a significant factor in the evolution of Japanese demographics. As of now, we do not see an easy way of remedying the drawback that our model ignores the flows of people across national borders.

In addition to the refinements already identified, we believe a great deal can be learned about the impacts of demographic change within our stylized models by undertaking sensitivity analyses. These analyses could focus on, for example, the modeling of investment decisions, the role of wage institutions in the short run, the implications of alternative assumptions about the elasticity of intertemporal substitution, the importance of alternative specifications of fiscal closure rules, and the importance of the degree of stickiness and liquidity constraints in the economy. Figures 7 and 11 above illustrate the type of insights that can be generated with such sensitivity analyses.

Eventually, our research must move to a second main stage. The two-region stylized models need to be expanded to include specific countries and to incorporate some of the main differences in macroeconomic behavior that are different across countries. The countries to receive first emphasis should be Japan and, probably, the European Union nations. Further work will also be needed on the existing equations for the United States. Ultimately, the multi-country models need to incorporate the macroeconomic behavior of several developing-country regions, again putting emphasis on the interactions between demographics and key macroeconomic variables.

The McKibbin-Wilcoxen MSG3 and G-cubed models will easily lend themselves to extension and modification along these lines. The insights obtained from the MSG3 stylized model will point the way to the required modifications. Analogously, the insights obtained from the BM2R stylized model can be readily adapted for incorporation in the IMF staff's MULTIMOD.

Though the second stage of the research should capture some of the most important differences among major countries or regions, the focus should continue to be on careful analysis of the macroeconomic interactions among the countries and regions. In particular, special attention should be given to exchange rates, current-account imbalances, and net-foreign-asset positions.

The promising results of the project so far suggest a large agenda of possibilities for future work. Our approach may yield important insights about the global dimensions of demographic change well before it will be possible to implement successfully an explicit multi-cohort OLG approach in multi-country empirical models.

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Table 1
Summary of Previous Studies' Estimates of Demographic Effects on Saving Rates

		Effect on Saving Rate of a 1 Percentage Point Rise in Demographic Ratio	
Study	Data Source	Youth Dependency Ratio	Elderly Dependency Ratio
Aggregate cross-section studies:			
Modigliani (1970)	....	-0.20 (3.7)	-0.88 (3.1)
Modigliani and Sterling (1983)	....	-0.13 (1.4)	-0.51 (4.3)
Feldstein (1980)	....	-0.77 (3.9)	-1.21 (2.7)
Horioka (1986)	21 OECD countries 1976-82 average	-0.92 (4.2)	-1.61 (4.0)
Graham (1987)	24 OECD countries, 1975 or 1970-80 average	-0.87 (2.9)	0.12 (0.3)
Koskela and Viren (1989)	23 countries, 1979-83 average	-0.73 (1.7)	-0.76 (0.8)
Horioka (1991)	14 OECD countries, 1980-88 average	-0.44 (1.7)	-1.09 (2.4)
OECD (1990)	14 OECD countries, 1980-88 average	....	-0.93 (2.4)
Time series studies:			
Shibuya (1987)	1966-83 (Japan)	....	-0.34 (3.8)
Horioka (1991)	1956-87 (Japan)	-0.30 (5.1)	-1.13 (3.7)
Masson and Tryon (1990)	1969-87 (pooled) G7 plus small industrial countries	-1.10	
Weil (1994)	1960-85 (pooled)	-0.3	-0.5
Masson-Bayoumi-Samiei (1995)	1972-93 (pooled) 21 industrial countries	-0.15 to -0.30	
Meredith (1995)	simulation model	-0.30	

Source: see Meredith (1995). Figures in parentheses are estimated t-statistics. The youth dependency ratio here is defined as the ratio of the population aged 0-19 to population aged 20-64. The elderly dependency ratio is defined as the ratio of the population over 64 to population aged 20-64 (except in the OECD (1990) study where it is the ratio of population over 64 to total population). See Meredith's original table for additional footnote information.

Figure 1. Age Earnings Profiles, Japanese Data 1970-1997

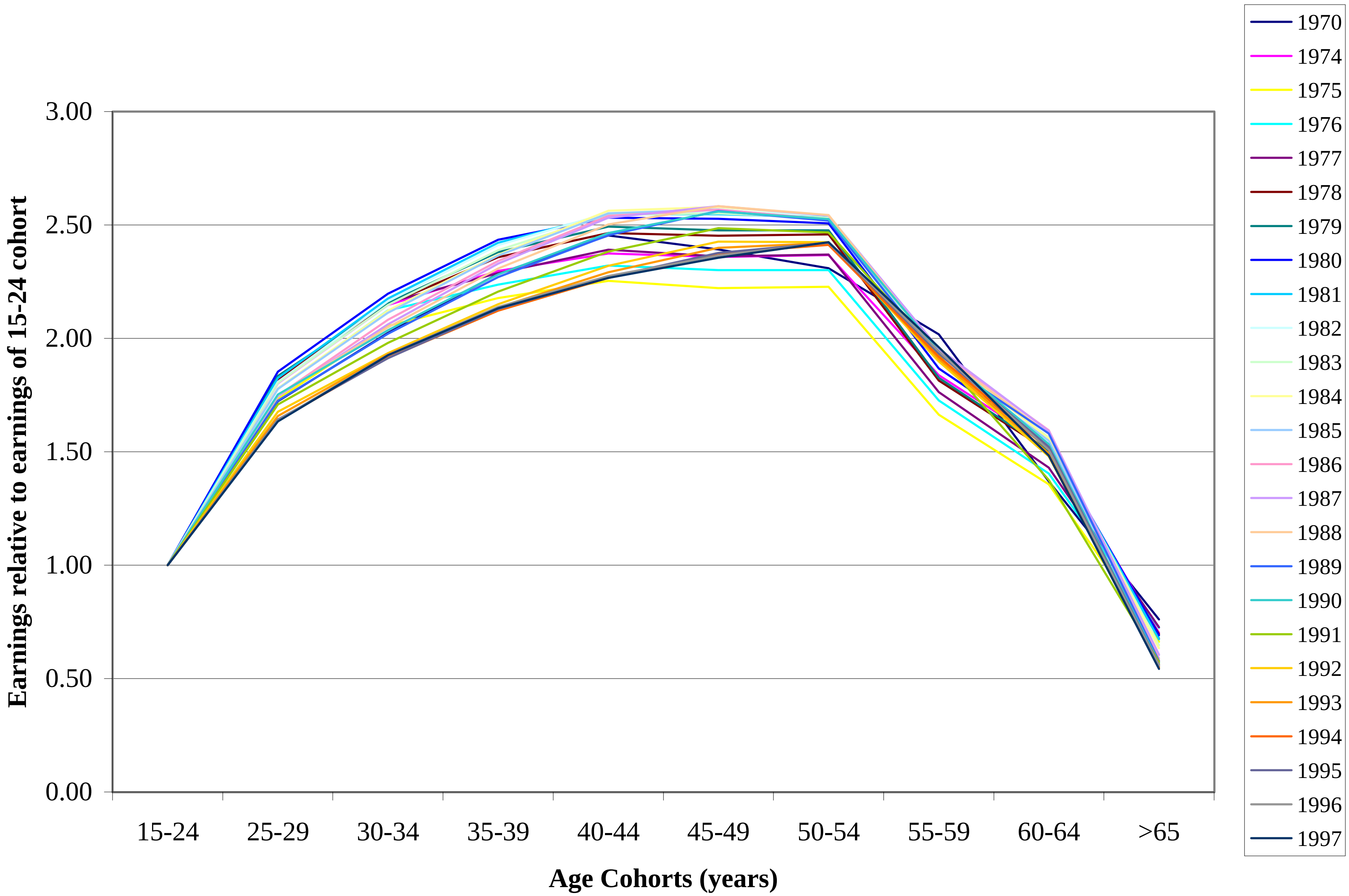


Figure 2. Alternative Approximations of an Age-Earnings Profile, U.S. Data 1980-1995

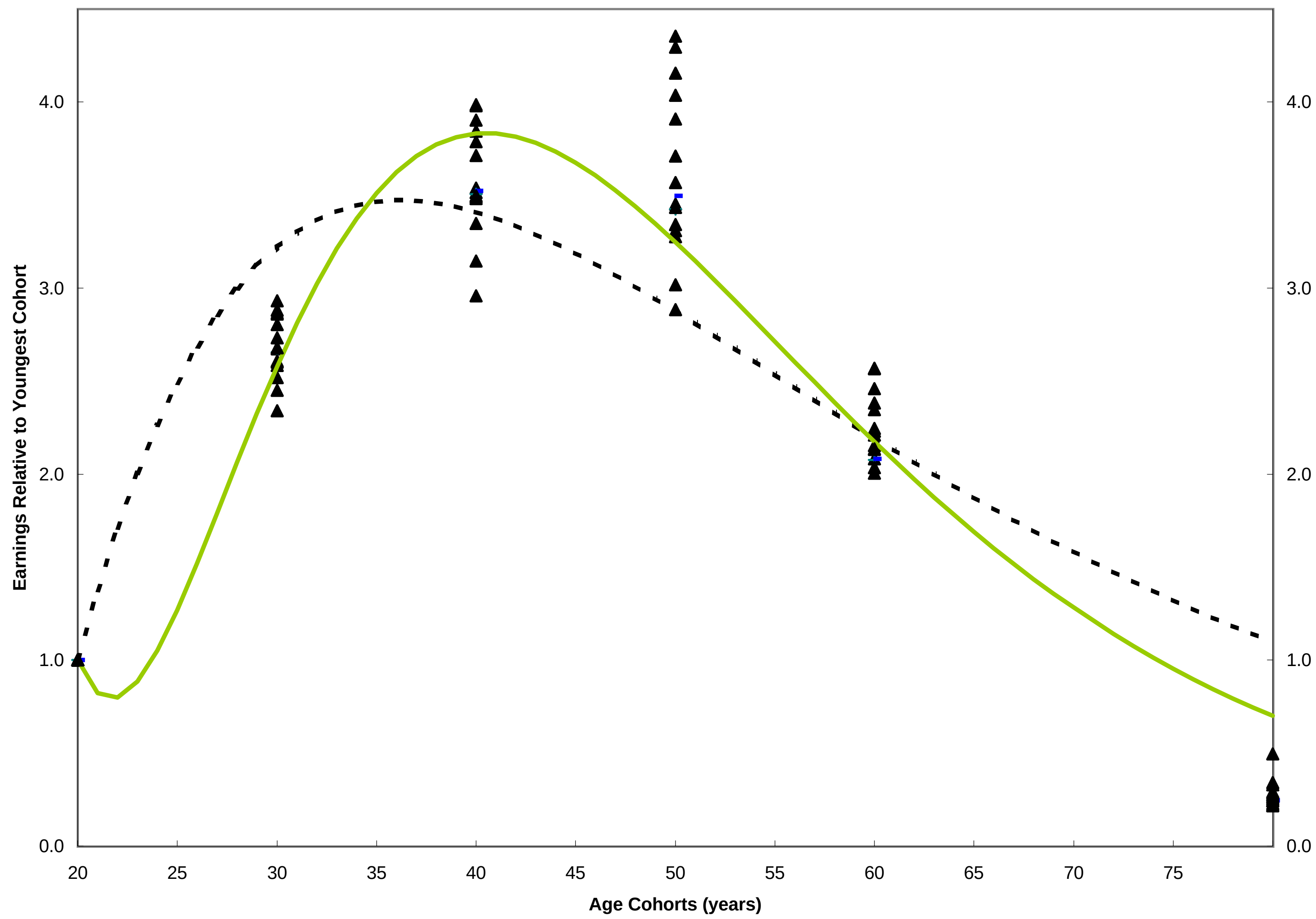


Figure 3
Transitory Baby Bulge: Birth Rates, Mortality Rates, Population Growth Rates

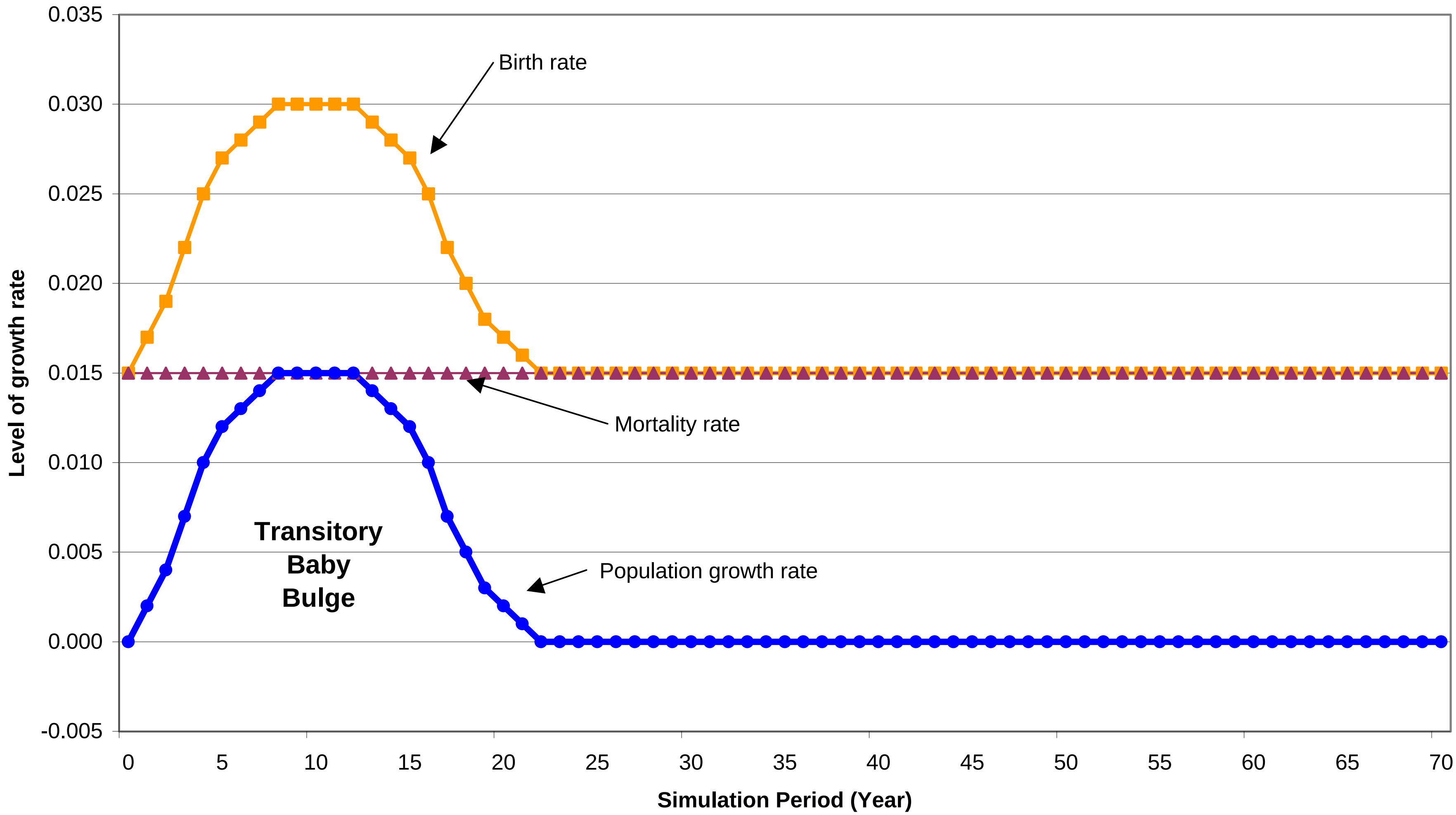


Figure 4
Population and Labor Force

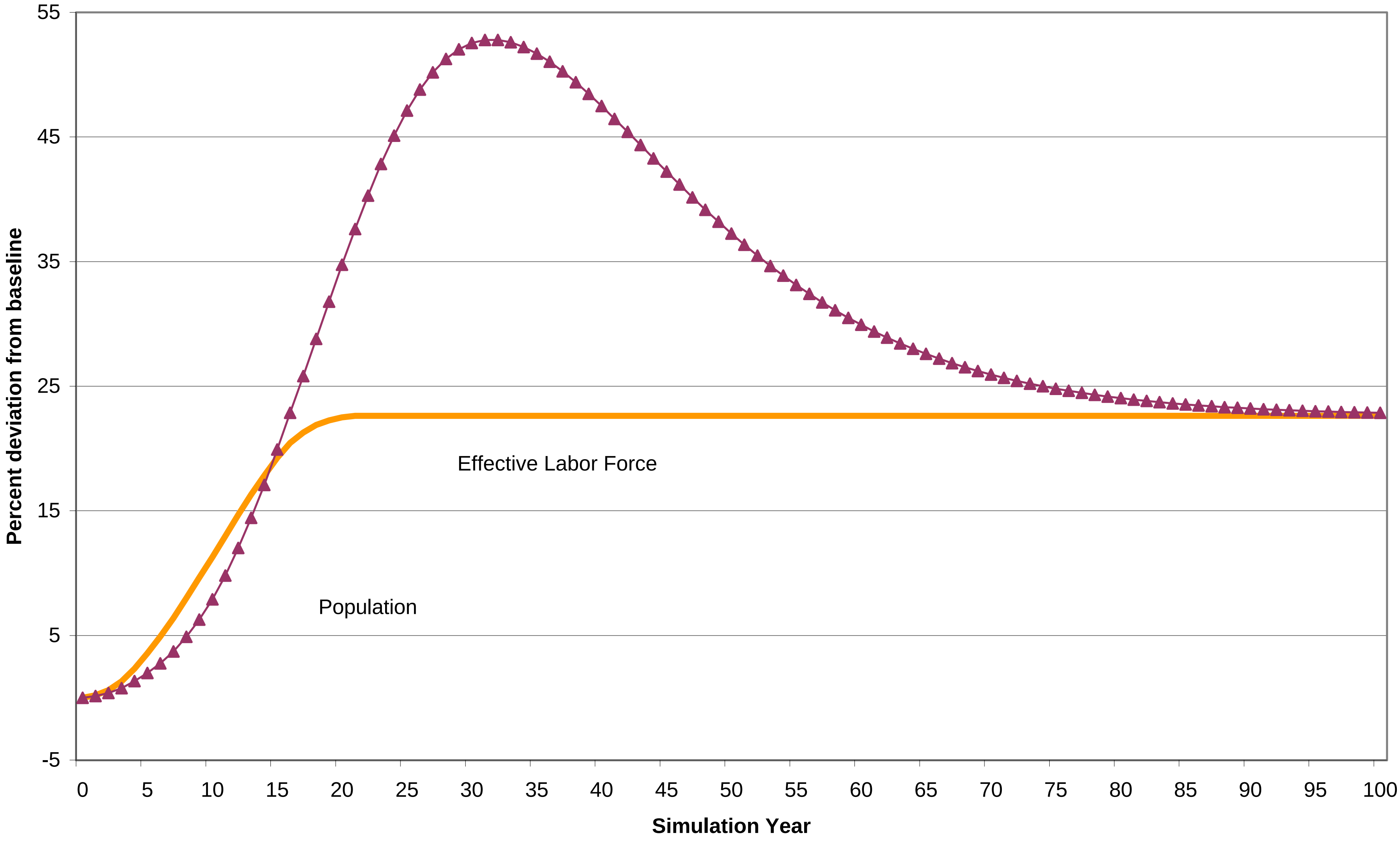


Figure 5
Elderly Dependency Ratios

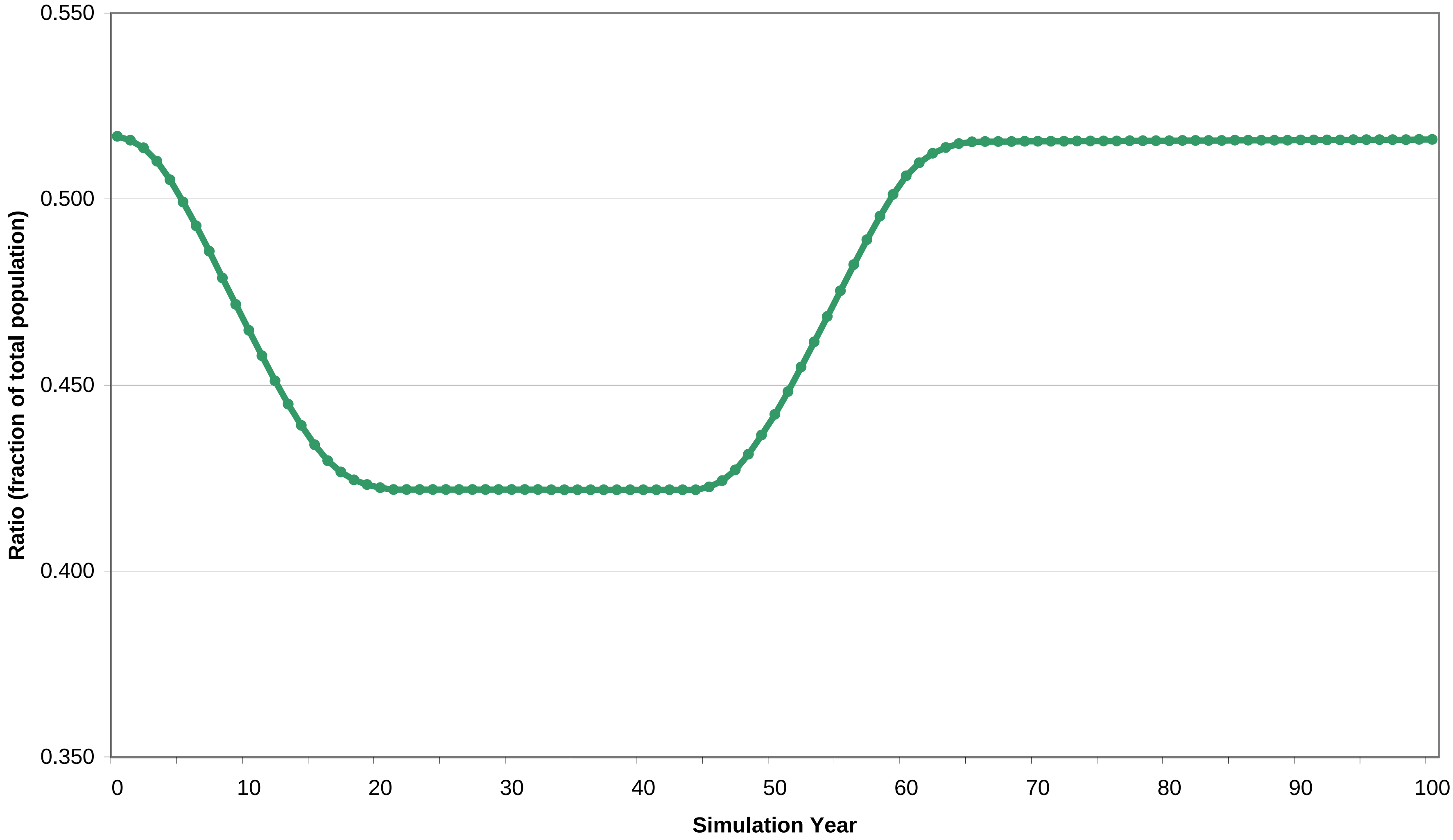


Figure 6a: Real Implications of a Baby Bulge in Both Regions Simultaneously - BM2R Model

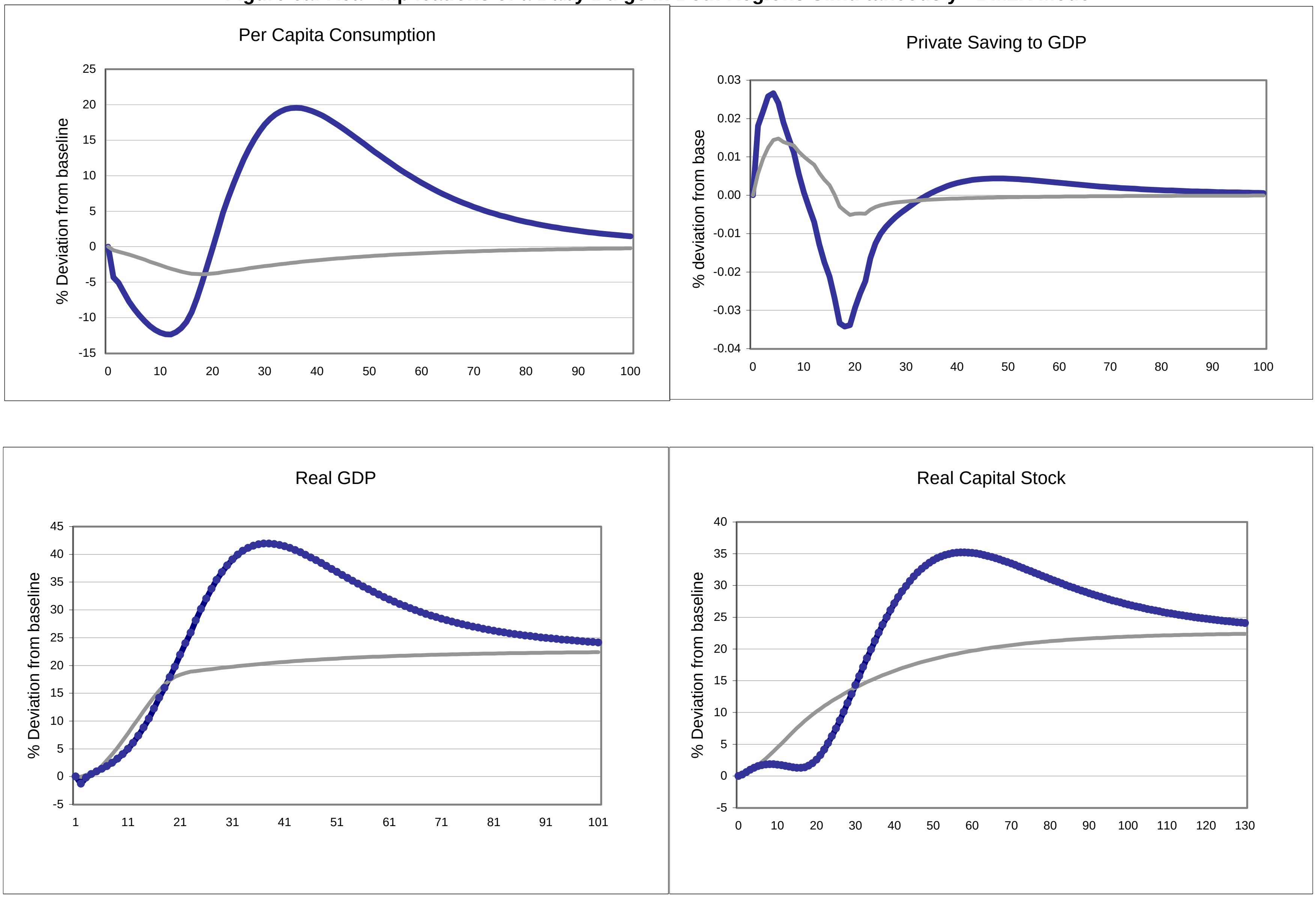


Figure 6b: Real Implications of a Baby Bulge in Both Regions Simultaneously - MSG3 Model

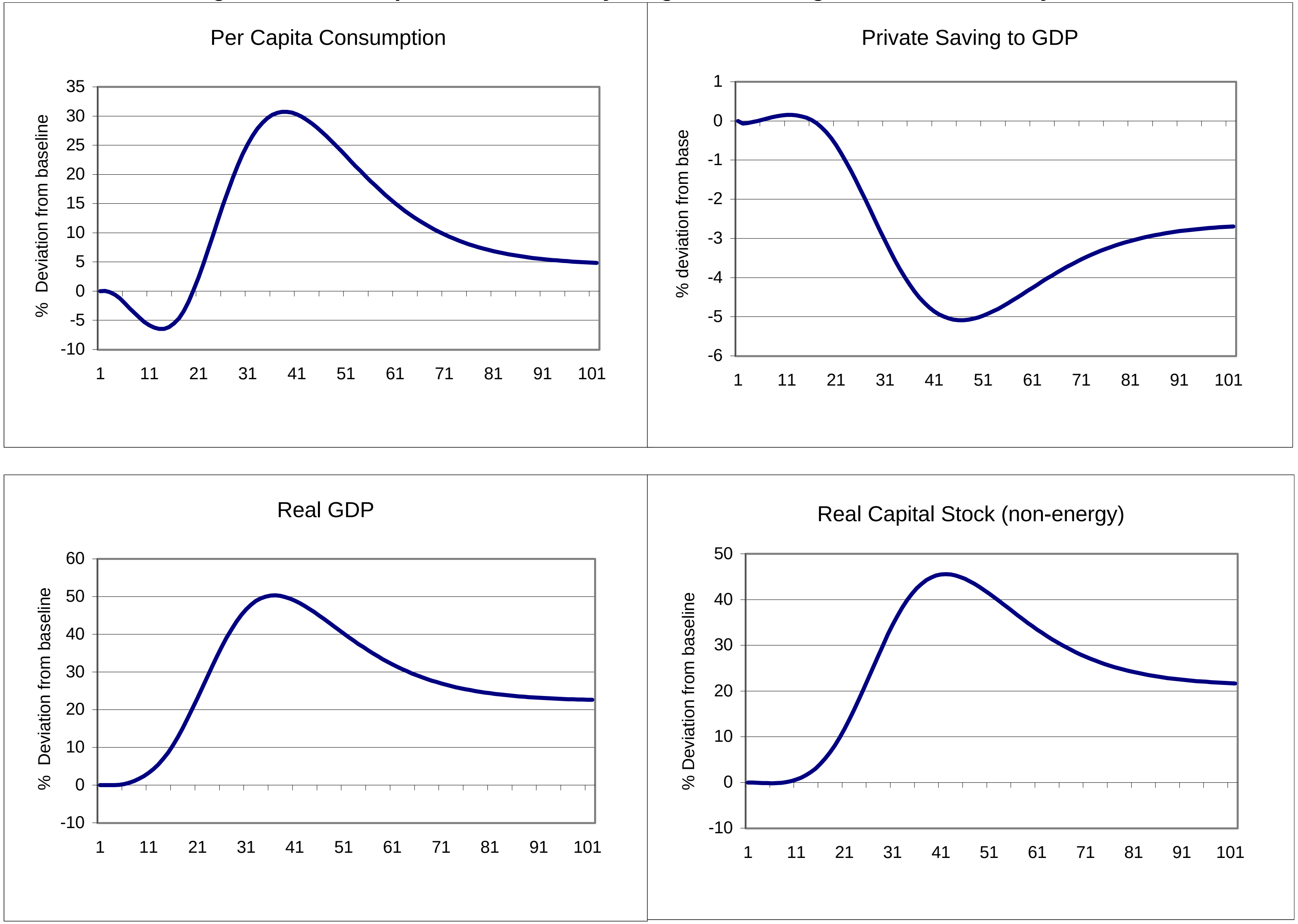


Figure 7a: Financial Implications of a Baby Bulge in Both Regions Simultaneously - BM2R Model

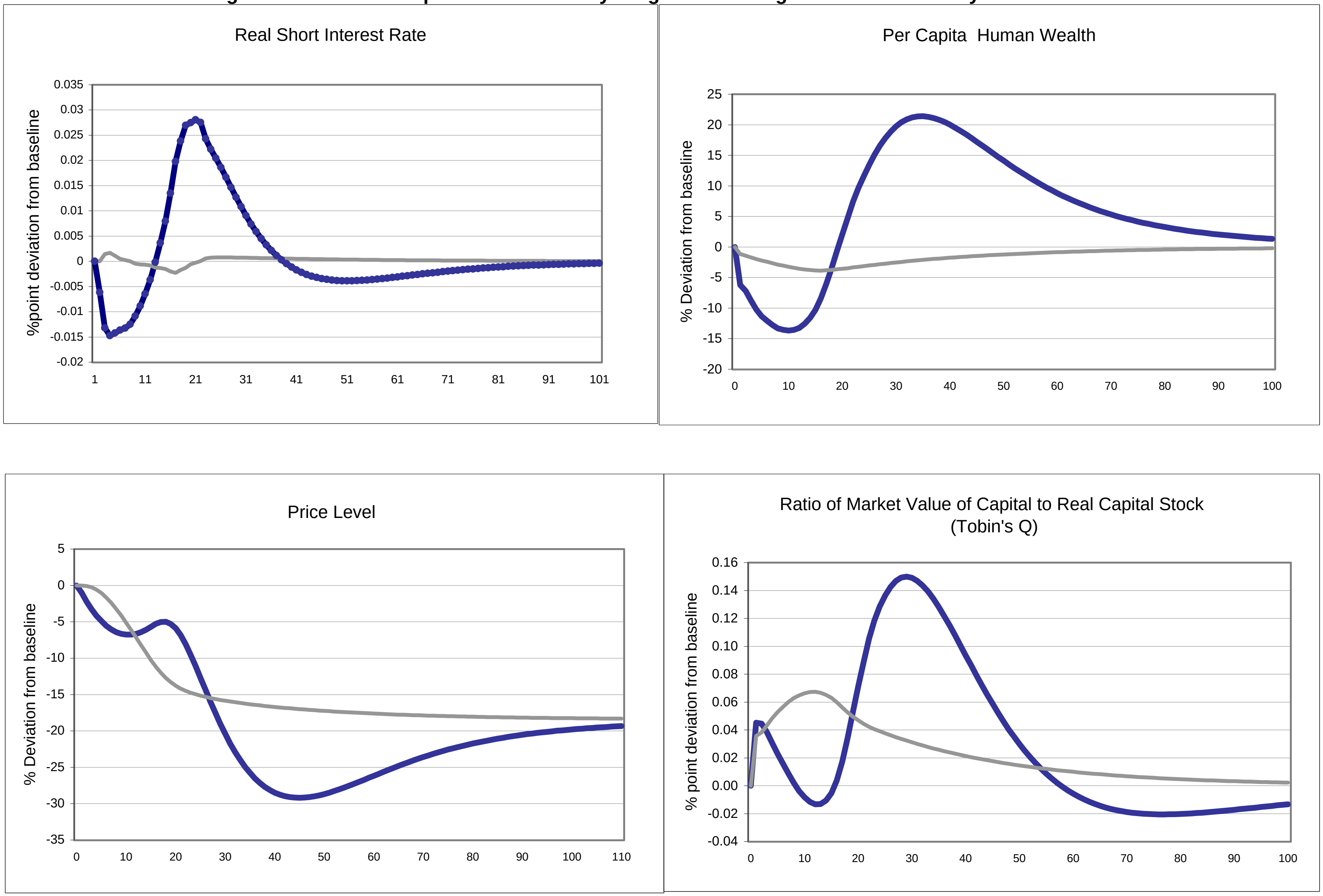


Figure 7b: Financial Implications of a Baby Bulge in Both Regions Simultaneously - MSG3 Model

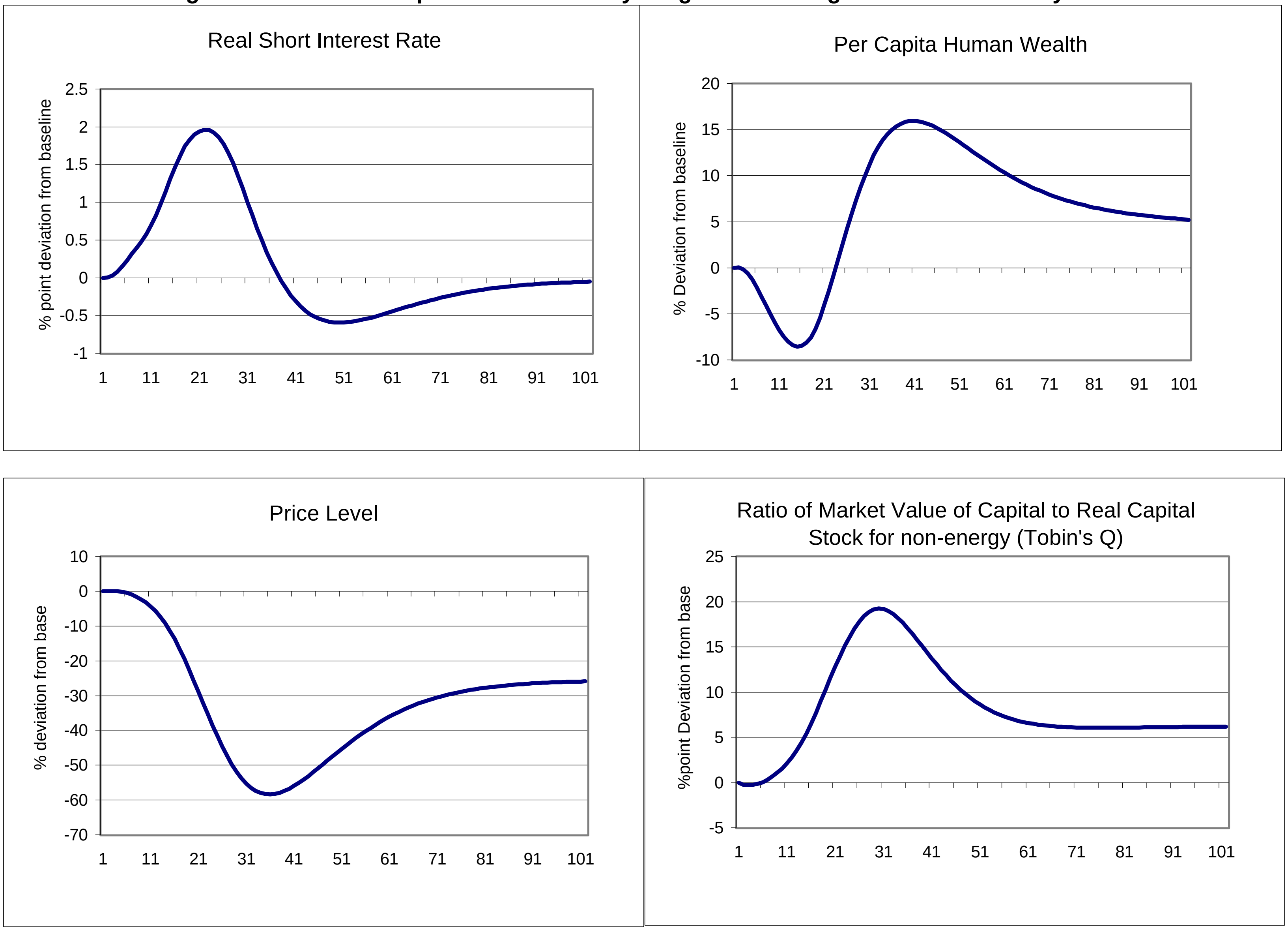


Figure 8a: Real Implications of a Baby Bulge in the US Region Only-BM2R Model

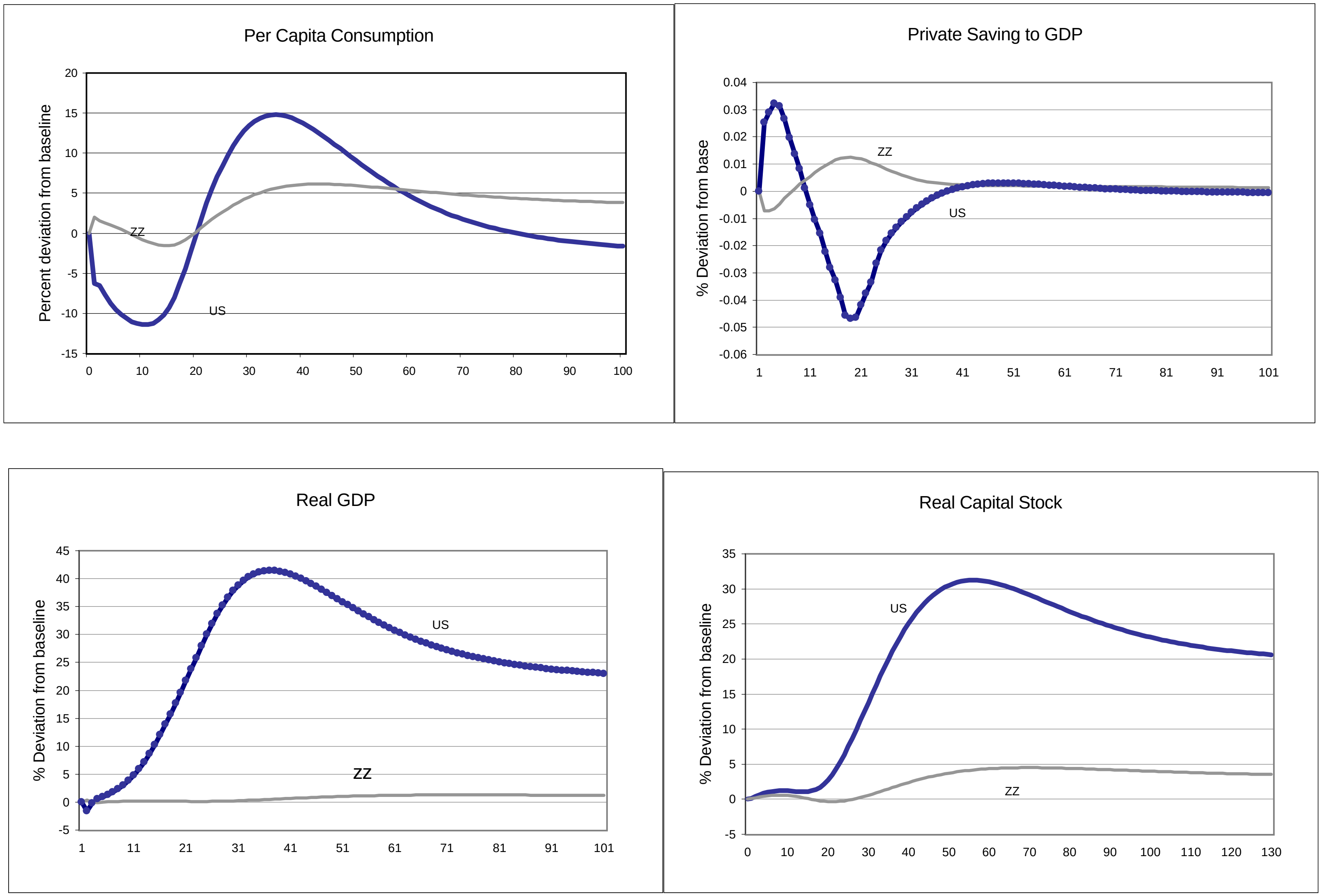
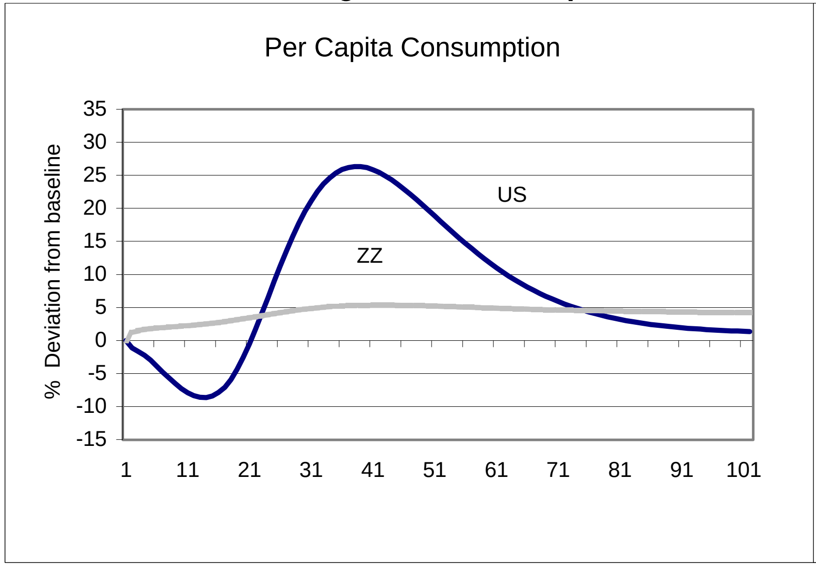
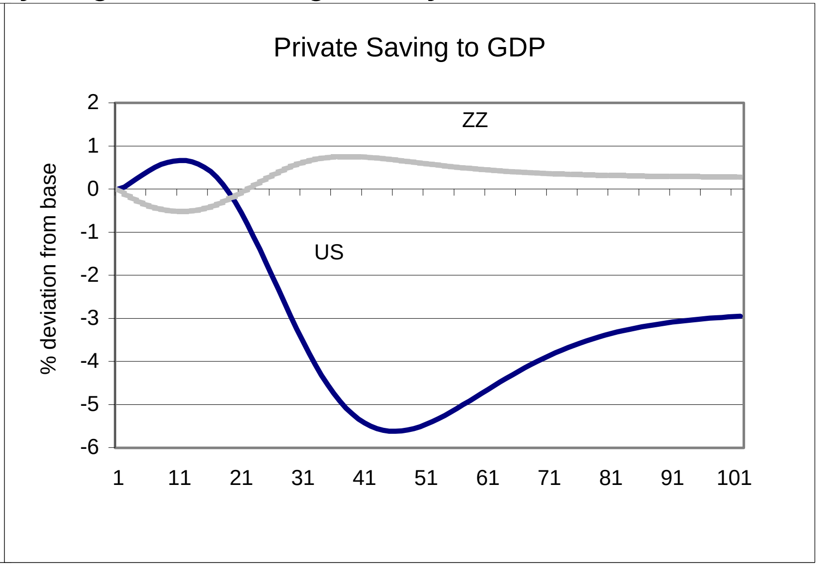
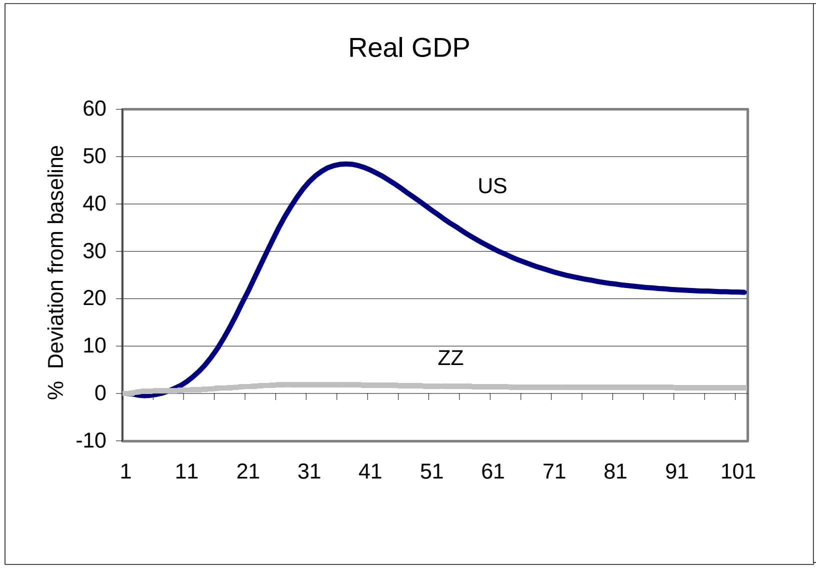


Figure 8b: Real Implications of a Baby Bulge in the US Region Only - MSG3 Model







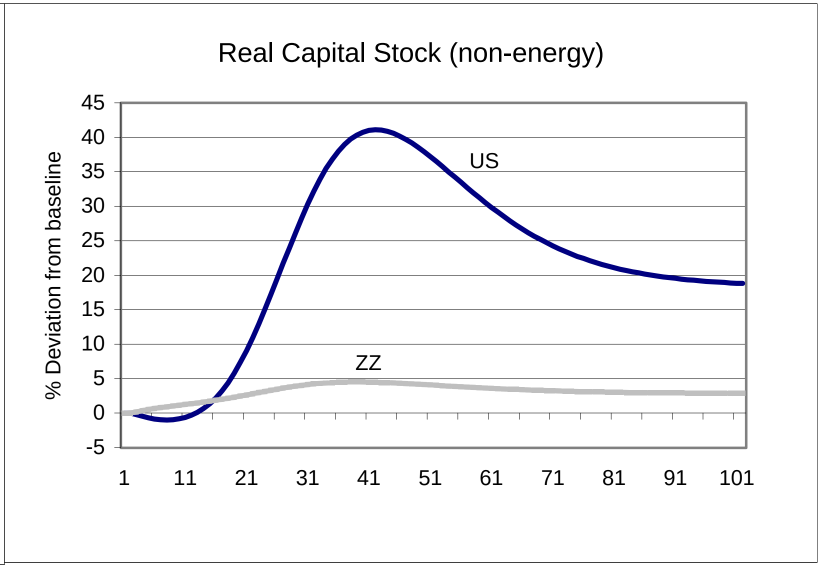


Figure 9a: Financial Implications of a Baby Bulge in the US Region Only-BM2R Model

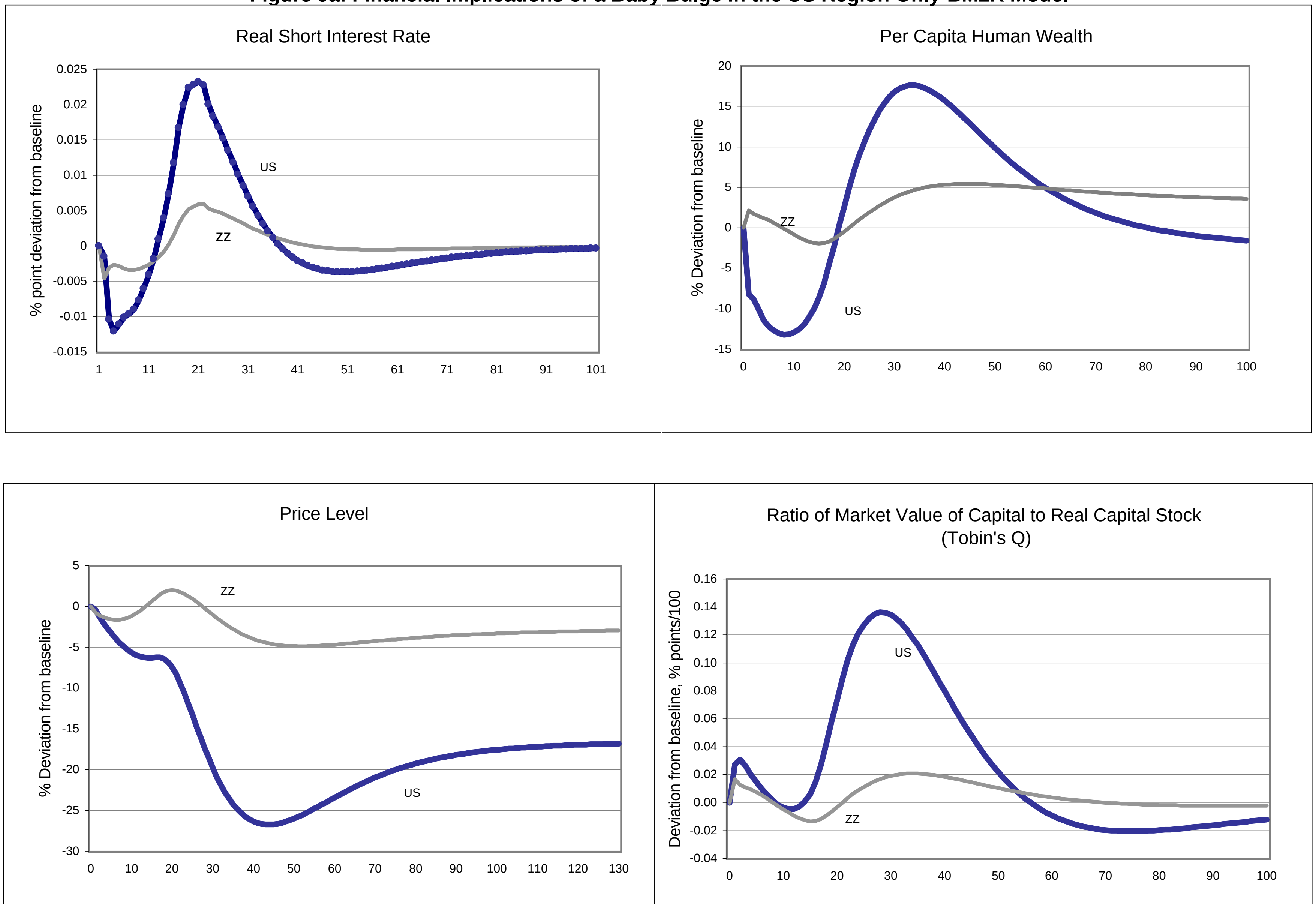


Figure 9b: Financial Implications of a Baby Bulge in the US Region Only - MSG3 Model

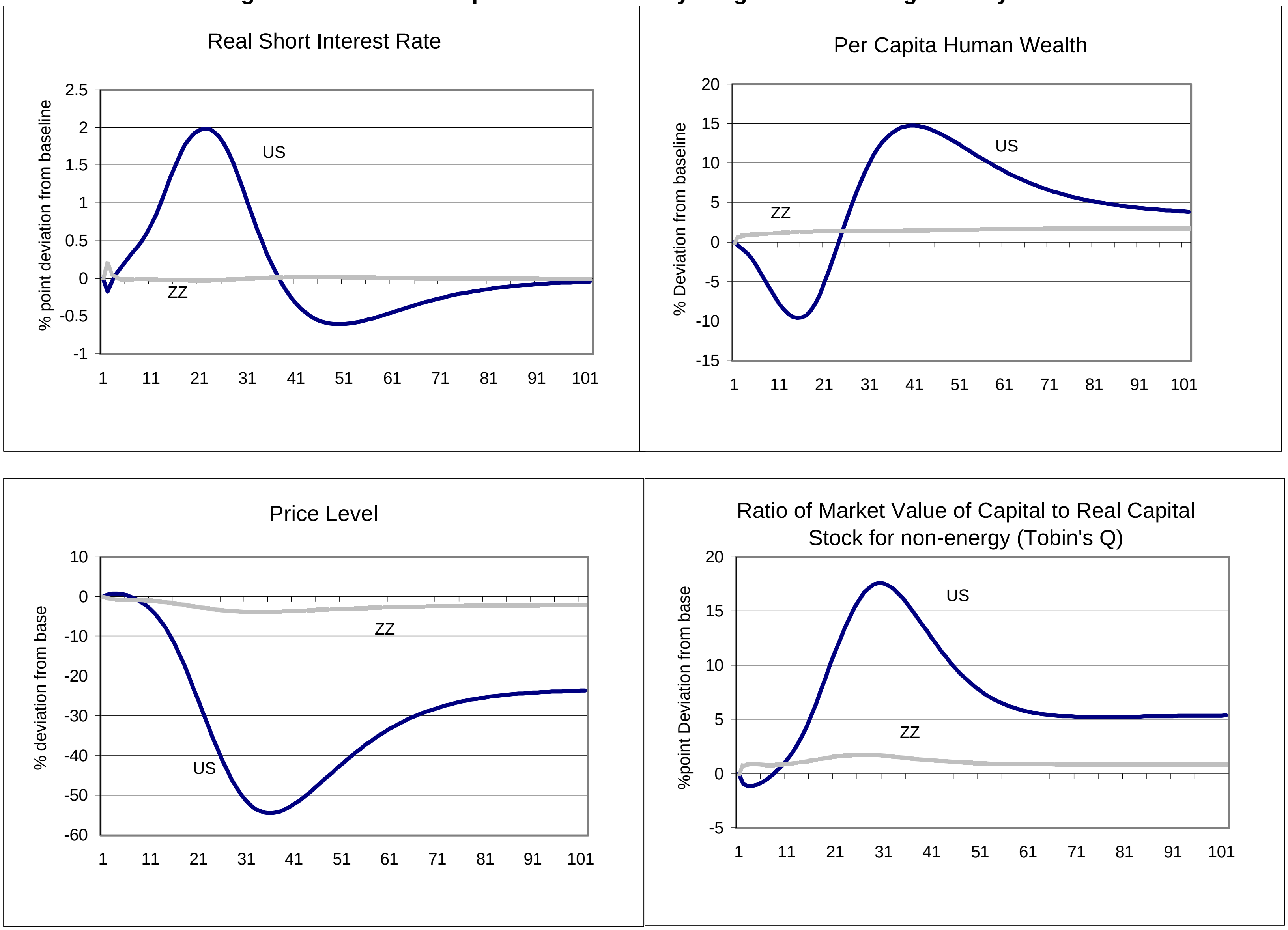


Figure 10a: International Implications of a Baby Bulge in the US Region Only-BM2R Model

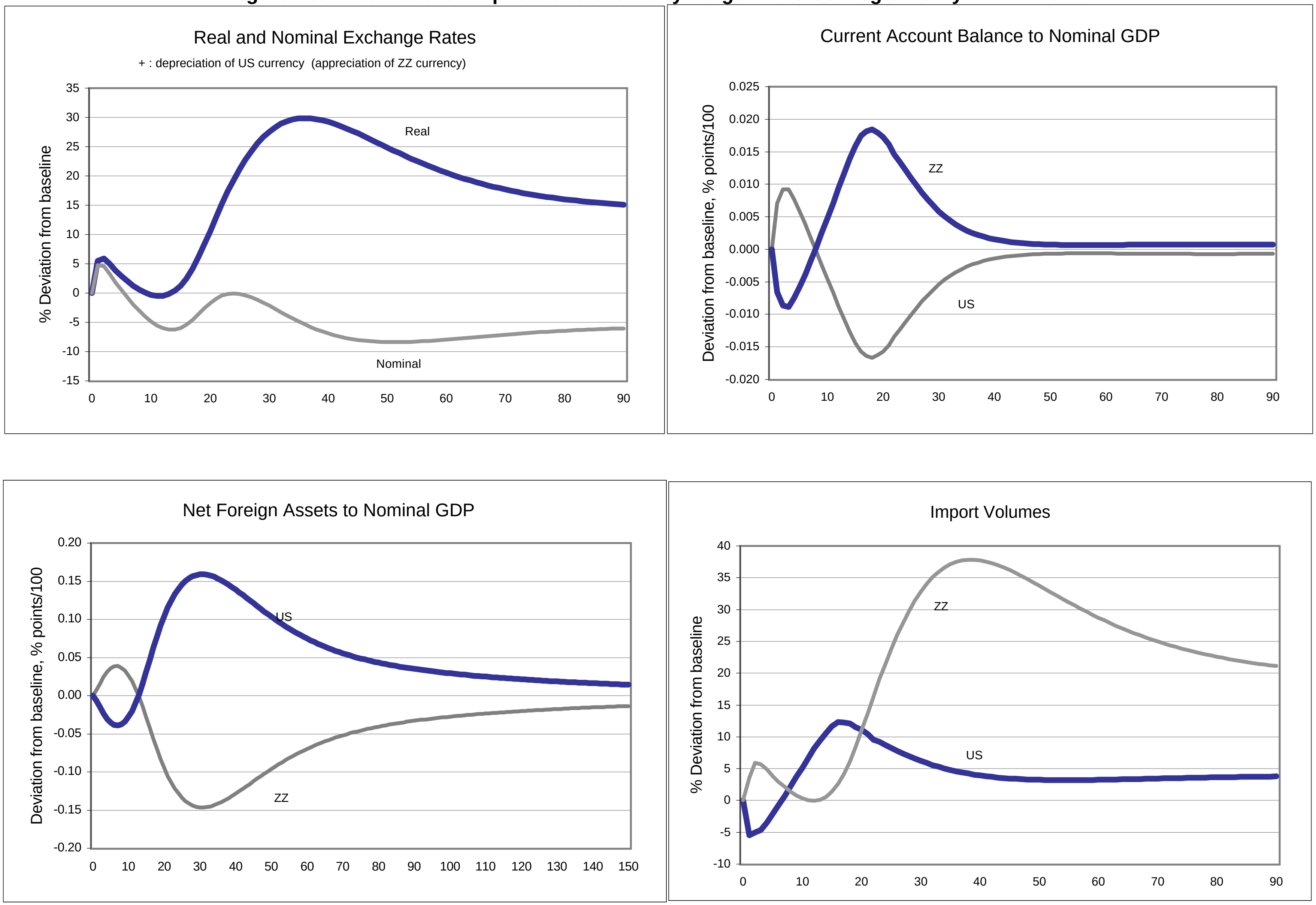


Figure 10b: International Implications of a Baby Bulge in the US Region Only - MSG3 Model

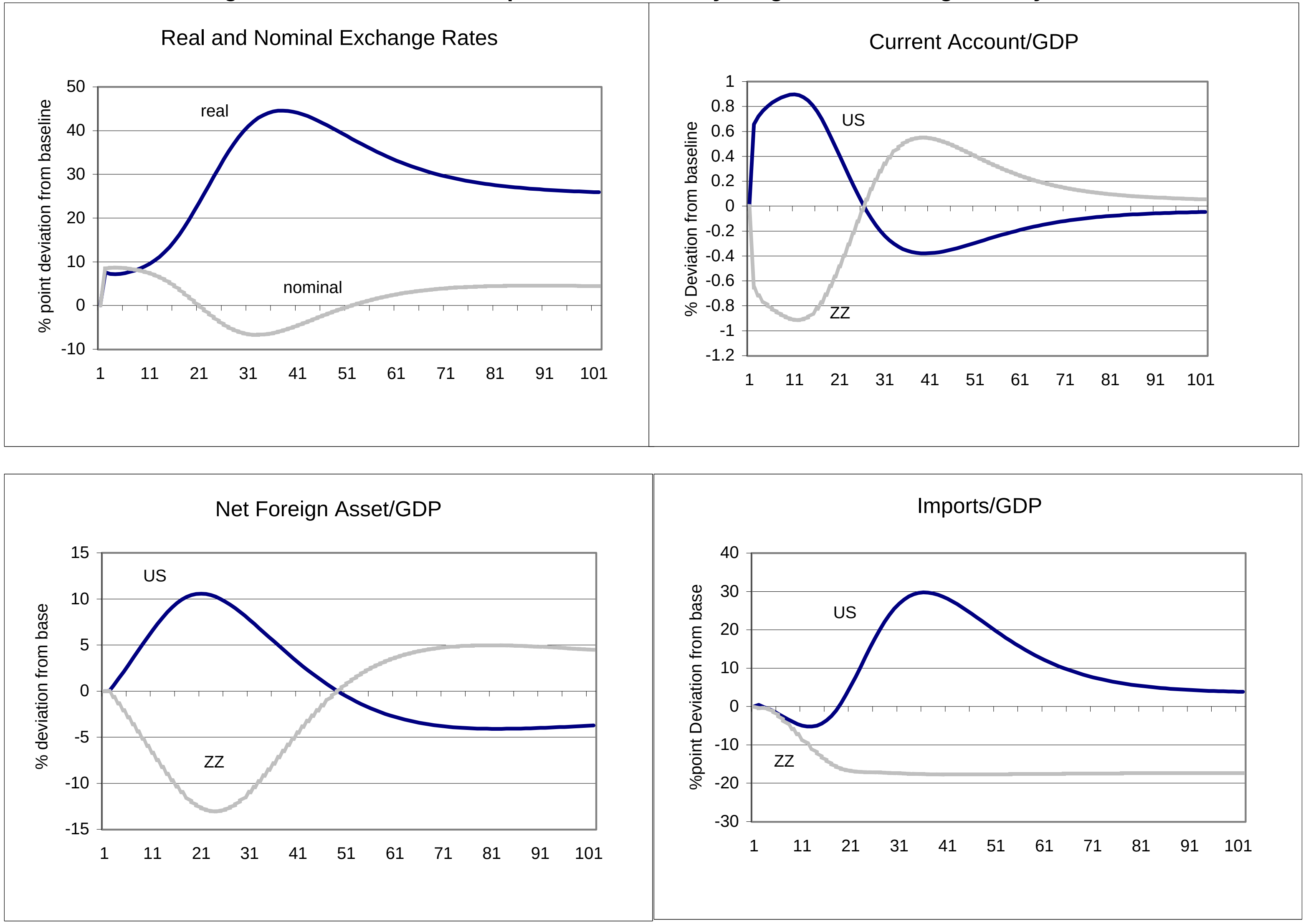


Figure 11: Baby Bulge in US Only, Effects With and Without Age-Earning Profiles - BM2R Model

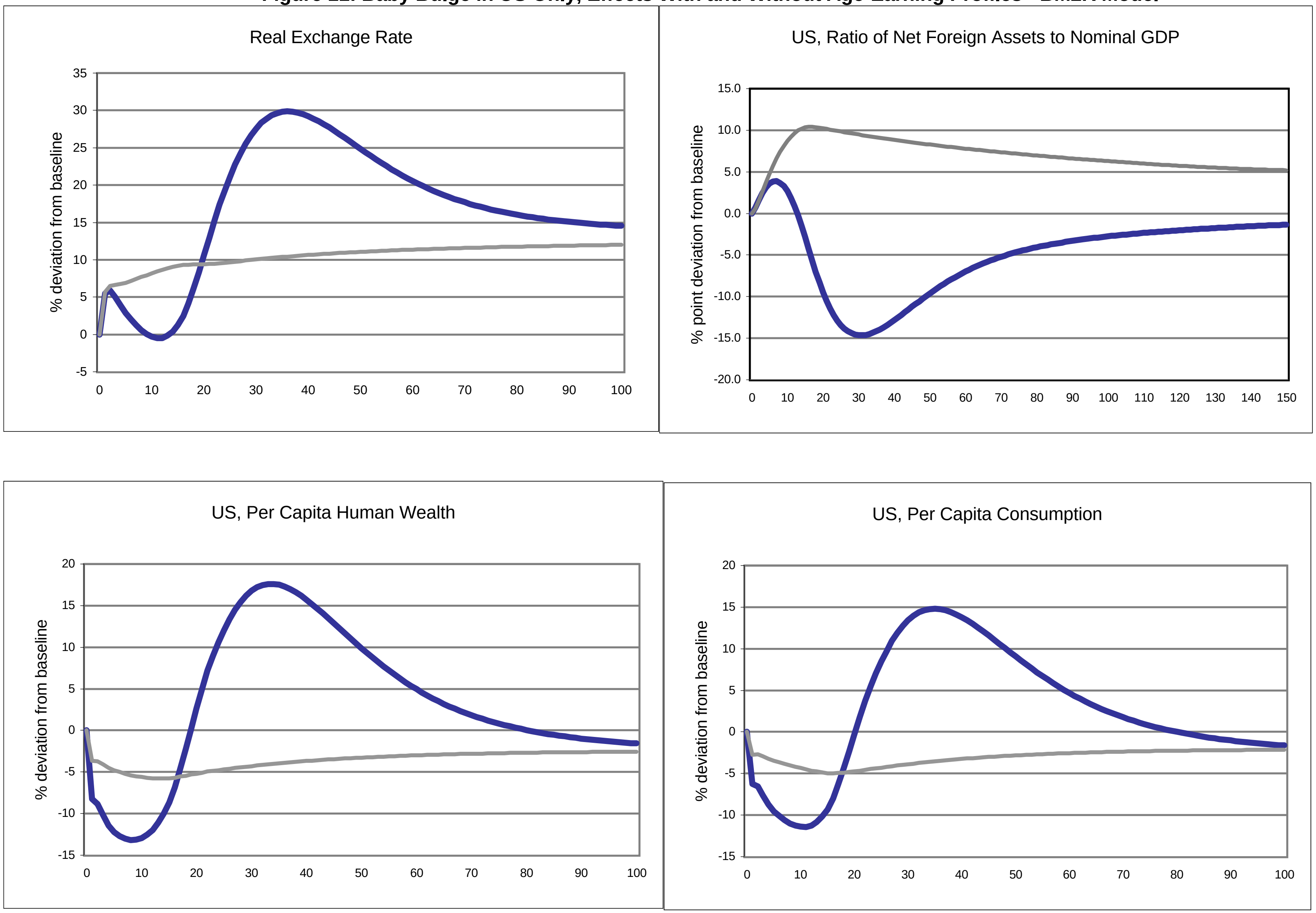


Figure 12: US Variables, Comparison of Shock in Both Regions versus in US Only - BM2R Model

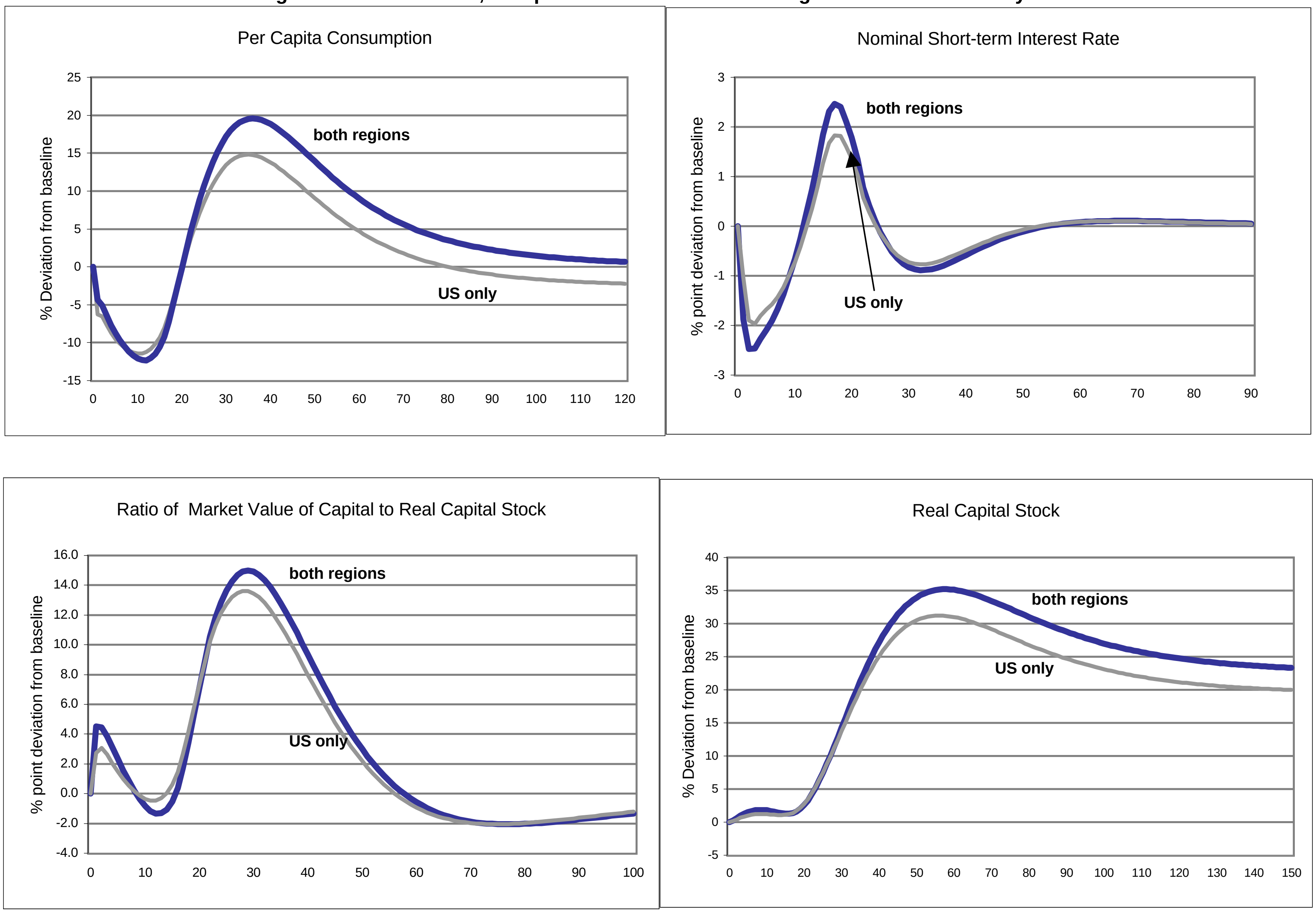
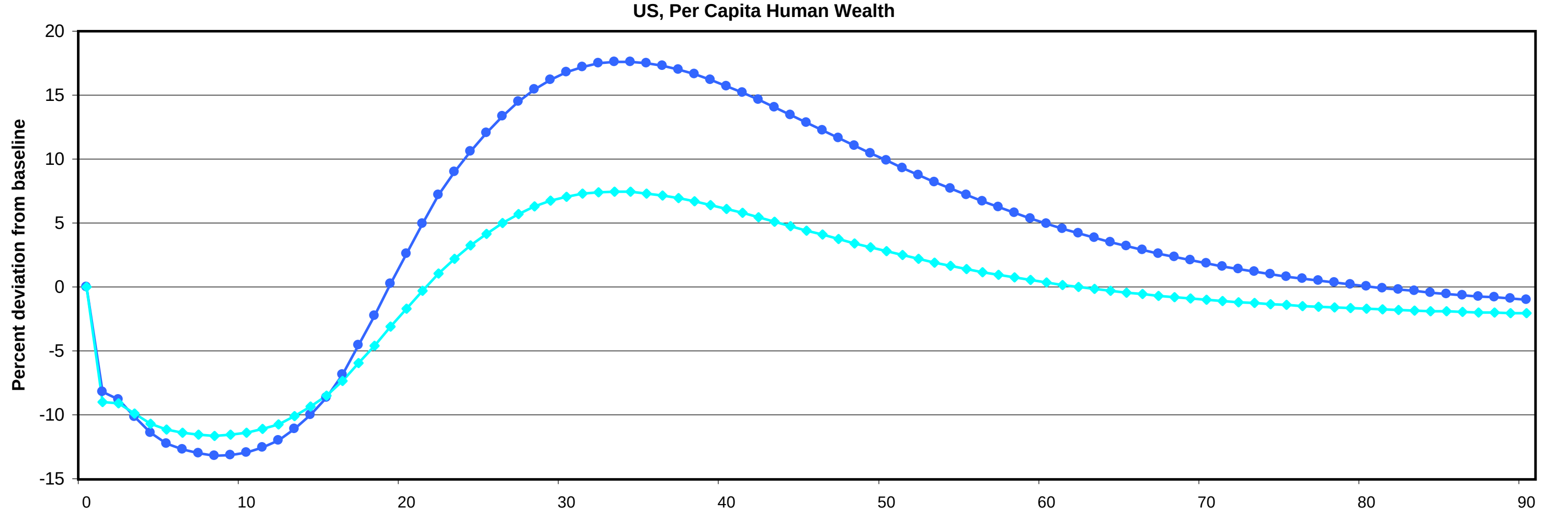


Figure 13: Long-Run Stationary Population versus Long-Run Population Growth at 1% per Year, BM2R Model

Baby Bulge in US Only

US, Per Capita Human Wealth



US, Ratio of Market Value of Capital to Real Capital Stock