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POPULATION AGING AND PUBLIC PENSION SYSTEMS: A FIRST LOOK AT THE CROSS-BORDER AND GLOBAL EFFECTS

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Ralph C. Bryant is a Senior Fellow and the Edward M. Bernstein Scholar in the Economic Studies Program of the Brookings Institution. This paper is a further interim report on a joint research project coordinated by Ralph C. Bryant at the Brookings Institution and Warwick J. McKibbin at the Australian National University. The research has been done in collaboration with Hamid Faruqee at the International Monetary Fund.

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This paper is a further interim report on a joint research project coordinated by Ralph C. Bryant at the Brookings Institution and Warwick J. McKibbin at the Australian National University. The research has been done in collaboration with Hamid Faruqee at the International Monetary Fund; Faruqee will be a co-author in a subsequent revision and elaboration of the paper. The Brookings/ANU project is supported by the Economic and Social Research Institute of the Japan Cabinet Office as part of their series of international Collaboration Projects. The views expressed in the paper are those of the authors alone and should not be attributed to any of the institutions with which they are associated. Elif Arbatli provided skillful research assistance and made numerous contributions to the development of the paper.

1. Introduction

The 2000 revisions in United Nations population projections predict that most of the developed nations will experience negative population growth rates in the 21st century, mainly due to declining fertility. Fertility rates in many countries are already below the replacement level of 2.1 births per woman. With falling fertility rates and rising life expectancy, the population of the world is expected to age more rapidly in the next 50 years than during the past half century. For example, the ratio of the elderly to the total population in all developed countries is projected to grow from 14.3 per cent in the year 2000 to 26.8 per cent by 2050. In Japan, the increase in the ratio of the elderly to the total population will be even more striking: the projected increase over the next five decades is from 17.2 per cent to 36.4 per cent. The dramatic nature of Japan's demographic shift relative to developed countries as a whole can be seen in visual terms in Figure 1, based on the revised UN projections.

Most developed countries main a public pension system in some form, . Pronounced population aging will put strong pressures on those systems. As is now widely understood, in most countries the benefit payments made to an increasing number of elderly retirees will have to be born by a relatively smaller number of nonelderly workers. A widespread expectation thus exists in political and administrative circles that population aging will pose significant challenges for policymakers in the decades to come.

A large literature devoted to demographic change has focused much of its attention on population aging and its effects on saving, investment and growth. From a policy perspective, the research has focused primarily on the increasing burdens that rising elderly dependency ratios could place on national budgets and pension systems and on the menu of possible options for reform.[1] Either for simplicity or a lack of awareness, however, most studies have failed to focus on the full effects of declining fertility rates. Declining fertility not only raises elderly dependency ratios, but also reduces youth dependency ratios (Figure 1). The effects of projected demographic changes for both the short and the long run will thus ultimately depend on whether lower youth dependency (which *lowers* total dependency) or higher elderly dependency (which *raises* total dependency) will dominate. Which of the two dominates, and the timing and magnitude of the net effects, will determine whether population aging represents a severe or moderate challenge for current and future generations.[2]

In a recent paper (Bryant, Faruqee and Velculescu (2001)), we concentrated on the analysis of youth dependency, explicitly incorporating children and child support into our theoretical framework and empirical model. We showed there that changes in youth dependency can have significant implications for the dynamic behavior of macroeconomic variables in both a closed- and open-economy context. In this paper, we build on our previous work by introducing a public pension system into our model, thereby accounting for transfers from working adults to both ends of the age distribution: child support transfers from parents to youths, and public-pension transfers to the elderly.[3]

The merit of our revised approach in this paper is that it allows us to capture the economic effects of both the early and the latter stages of the aging process. Declining youth dependency, on

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1 Well-known contributions to the literature include Aaron-Bosworth-Burtless (1989), Cutler-Poterba-Sheiner-Summers (1990), Auerbach-Cai-Kotlikoff (1991), World Bank (1994), Wise (1994), Disney (1996), Bosworth-Burtless (1998), Kohl and O'Brien (1998), and OECD (1998). Recent contributions include Velculescu (2000, 2001), and Brooks (2000a). Bryant and McKibbin (1998) identify many other pre-1998 contributions.

2 This point has been made by Cutler-Poterba-Sheiner-Summers (1990), and more recently and forcefully by Burtless (2000).

3 In principle, there is a third form of "dependency burden" shouldered by workers: transfers to nonworking nonelderly adults. As participation rates in the labor force have changed over time, this third type of dependency transfer has also changed (the single largest effect is from increasing participation of women in the labor force). Eventually, we hope to find a way to incorporate this third type of dependency into our model.

the one hand, entails lower aggregate transfers to children for their support, freeing up resources for other uses, such as higher consumption per adult. Higher elderly dependency, on the other hand, reduces consumption per adult because the members of a relatively smaller labor force must provide for a larger number of retirees.

This paper contributes to the existing literature on demographic change in two ways. First, it analyzes the implications of population aging in an open-economy context, focusing on the cross border and global effects of lower fertility rates when a public pension system is operative in the economy. We examine the effects of alternative variants of public pension ("social security") systems on world saving, investment, exchange and interest rates, and the external-sector positions of individual countries. The variants include a government hands-off approach (no public pension system at all, so that individuals provide for old age entirely from their own savings) and alternative conventions by which the fiscal authorities set pension taxes, elderly benefits, and the management of imbalances between the two. Second, as mentioned above, our analytical framework explicitly incorporates children and child support as well as the elderly, and thus allows us to study the overall combined effects of changes in both youth and elderly dependency ratios. An analytical framework that accounts both for openness to the rest of the world and for declining child support due to lower youth dependency is likely to yield more robust conclusions about the net economic effects of population aging than models previously used in the literature.

Section 2 of the paper presents our theoretical framework, which is based on the Blanchard (1985)-Weil(1989)-Yaari(1965) model, modified to incorporate richer demographics, youth and elderly dependency, a child support mechanism, and a public pension system. Section 3 provides further details about the way in which the theoretical framework is incorporated into our two-region empirical model (an abridged version of the IMF staff's MULTIMOD). Section 4 presents simulation results with alternative pension arrangements but no child support for an illustrative population-aging shock, assumed to occur identically in all regions of the global economy (equivalent therefore to closed-economy analysis); this section builds intuition about the effects of various pension arrangements on domestic macroeconomic variables. Section 5 continues to exclude child support and explore the alternative pension arrangements but focuses on the cross-border effects of a demographic shock now assumed to occur asymmetrically in only one region of the world. Section 6 presents preliminary simulation results for the asymmetric, country-specific shock using the full capacities of the model to incorporate both child support and pension transfers to the elderly. Section 7 concludes and sketches future avenues for research.

2. Theoretical Framework

Building on the overlapping generations framework of Blanchard (1985), P. Weil (1989), and Yaari (1965), we extend that model to incorporate a richer demographic structure that allows for population growth and for elderly and youth dependency. We apply the extended framework to a global setting of two interdependent national economies linked together through international flows of goods and capital. Our approach permits us to focus on the cross-border effects of country-specific changes in dependency ratios, with emphasis on the implications of various public pension arrangements for the world levels of saving, investment, and interest rates.[4]

A. Population and Dependency Ratios

Previous models relying on the Blanchard-Weil-Yaari framework focus exclusively on the adult working population and assume that individuals are 'born' directly into the labor force, contribute immediately to labor and production, and never retire. Our framework allows individuals to enter the world as dependent children, with distinct birth and mortality rates. After a period of

[4] This exposition follows closely that in Bryant, Faruqee and Velculescu (2001), with the important addition of a public pension system in the model.

life as dependent children, youths then enter the adult population and the workforce. Eventually, they enter old age and gradually retire from the labor force.[5]

We assume that adults have a different (usually higher) mortality rate than children. We also allow for the mortality rates of adults and children to vary through time. To preserve the advantages of the Blanchard-Weil-Yaari framework for aggregation across individuals, however, we maintain that framework's assumption that mortality rates are age-invariant (for both the adult and child populations). The assumption of age-invariant mortality rates departs seriously from reality.[6]

First, we begin with a description of the population and demographic dynamics. We denote the size of a child cohort, indexed by s (s is the time of birth) at time t, as $J(s,t)$. Correspondingly, $N(s,t)$ is the size of an adult cohort at time t. The populations of children and adults are naturally interconnected (in a demographic sense). For children, the initial size of a cohort at the time of birth is given by:

$$J(s,s) = b_j(s)N(s), \tag{1}$$

where b_j is the birth (or fertility) rate, expressed as a fraction of the contemporaneous adult population N. Youth dependents all face the same infant/child mortality rate, $p_j(t)$. This rate represents the probability of a child dying in childhood; $p_j(t)$ can vary through time, but in any given period is the same for all youths, regardless of their age. The number of survivors from the initial $J(s,s)$ cohort at some later date t is given by:

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5 There is no discontinuity of labor input at the threshold "retirement" age *per se*. Adults still continue to receive some labor income even after they reach the age after which they are defined as "elderly." But as adults get older, after passing the years of their peak earnings, their incomes decline continuously toward zero.

6 Blanchard (1985) himself pointed out that the evidence on mortality rates suggests low and approximately constant probabilities of death from, say, ages 20 through 40; thereafter mortality rates in real life do rise with age (sometimes modeled by "Gomperty's Law" as in Wetterstrand (1981)), reaching rates (in the United States) in the neighborhood of 16 percent by age 80 and 67 percent by age 100. In unpublished research, Faruqee (2001) has shown how it is possible to modify the simplifying assumption that all adults are subject to the same age-invariant probability of death; with that modification, however, it is no longer straightforward to achieve the simple aggregation across individuals that is the marked advantage of the Blanchard-Weil-Yaari theoretical framework.

$$J(s,t) = b_j(s)N(s)e^{-\int_s^t p_j(v)dv}. \quad (2)$$

The total size of the child population is obtained by aggregating all child cohorts over the finite range . of childhood ages:[7]

$$J(t) = \int_{t-.}^{t} J(s,t)ds. \quad (3)$$

Differentiating this expression and using the survivor formula given by equation (2) yields the following expression for the evolution of the child population:

$$J(t) = b_j(t)N(t) - N(t,t) - p_j(t)J(t), \quad (4)$$

where $N(t,t)$ denotes the outflow of children into the adult population. More specifically, through continuity, the oldest child cohort passes into adulthood:

$$N(t,t) = J(t-.\ ,t)\ . \quad (5)$$

This cohort thus also represents the inflow of new adults or workers into the economy. Given a distinct adult mortality rate $p_n(t)$, the survivor formula for adult cohorts is given by:

$$N(s,t) = N(s,s)e^{-\int_s^t p_n(v)dv}. \quad (6)$$

The level and dynamics of the adult population N are analogously derived:

$$N(t) = \int_{-8}^{t} N(s,t)ds; \quad (7)$$

$$N(t) = N(t,t) - p_n(t)N(t). \quad (8)$$

Equation (8) shows that the net change in the adult population in each period depends on the inflow of new entrants to the adult population less the numbers of adults that die. The rate at which new youth entrants pass into the adult population can be viewed as an adult "birth" rate

[7] We parameterize . to be 18; that is, children are dependent for the first 18 years of their life and then are no longer dependent because they have entered adulthood at the beginning of their 19th year.

$b_n(t) = N(t,t)/N(t)$. The population growth rate of adults is given by $n(t) = b_n(t) - p_n(t)$, where the adult "birth" rate $b_n(t)$ depends on the fertility or child birth rate in earlier periods.

Given these population measures, the *youth dependency ratio* $\delta(t)$ is simply the ratio of the child population to the adult population:

$$\delta(t) = \frac{J(t)}{N(t)}. \tag{9}$$

Differentiating this expression yields the law of motion for the youth dependency ratio:

$$\delta(t) = b_j(t) - [1 + \delta(t)]b_n(t) + \delta(t)[p_n(t) - p_j(t)]. \tag{10}$$

Intuitively, the evolution of youth dependency depends on the comparative rates of growth in the child and adult populations, which in turn depend on differences in birth and death rates for children and adults (appropriately scaled). The growth rate of the child population can be written as $j(t) = [b_j(t) - b_n(t)]/\delta(t) - p_j(t)$, reflecting not only the birth and death rates of children but also their entrance into the labor force (that is, the adult 'birth' rate). From equation (10), note that higher fertility rates tend to raise the youth dependency ratio, other things equal.[8]

In a manner similar to the definition of the youth dependency ratio, we define an elderly dependency ratio as the proportion of the adult population above a certain threshold age level – indexed by *i*(*t*). The (fixed) difference between this index *i*(*t*) and the present time, *t*, which we denote by $. = t - i(t)$, reflects the number of adult years needed to reach elderly dependency status. For example, the adult cohort reaching age 65 at time *t* became adults 47 years ago -- that is, . = 47 (assuming adulthood begins in the 19th year of life). Given a fixed threshold age, the elderly dependency ratio is given by:

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[8] In steady-state, the following population relationships obtain:
$\bar{b}_n = \bar{p}$; $\bar{b}_j = \bar{p} e^{\bar{p}_j \cdot}$; $\bar{\delta} = \frac{\bar{b}_j - \bar{p}}{\bar{p}_j}$.

$$\phi(t) = \int_{-8}^{i(t)} \frac{N(s,t)}{N(t)} ds. \qquad 0 = \phi = 1. \tag{11}$$

For the case where the adult birth rate and death rate are time varying, the elderly dependency ratio evolves over time according to:

$$\phi(t) = \frac{N(i(t),t)}{N(t)} - [p_n(t) + n(t)]\phi(t). \tag{12}$$

Intuitively, the change in the elderly dependency ratio is determined by the relative size of new "dependents" reaching the threshold age -- the first term in (12) -- *less* the proportion of the elderly who die during the period -- $p_n(t)\phi(t)$ -- and *less* a scaling term accounting for growth in the adult population base -- $n(t)\phi(t)$.[9] In an economy with a constant adult 'birth' rate, the elderly dependency ratio would also be constant.[10]

B. The Adult Consumption Problem

The adult individual consumer is presumed to solve the following familiar maximization problem:

$$\max \int_t^8 e^{-(\cdot\ + p_n(v))(t-v)} u(c(s,v))dv \tag{13}$$

subject to:

$$w(s,t) = fw(s,t) + hw(s,t) \tag{14}$$

$$fw(t) = (r(t) + p_n(t))fw(s,t) + y(s,t)(1-\tau(s,t)) - v(s,t) - c(s,t) \tag{15}$$

$$hw(s,t) = \int_t^8 [y(s,v)(1-\tau(s,v)) - v(s,v)]e^{-\int_t^v (r(\propto)+p_n(\propto))d\propto} dv\,. \tag{16}$$

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[9] As discussed in Faruqee (2000a, 2000b), the model's assumption that the adult mortality rate is age-invariant has an undesirable consequence: the numbers of elderly in the total population are overestimated relative to the real-life situation in which adult mortality increases as adults become older and older.

[10] Because death rates are invariant across ages, only the adult birth rate matters for the distribution or share of elderly dependents in the population. See also Faruqee (2001).

Here u(.) is the utility function of the individual, assumed to be of the constant relative risk aversion (CRRA) form, $u(c(s,t)) = \frac{c(s,t)^{1-\sigma} - 1}{1-\sigma}$, where σ is the coefficient of relative risk aversion, and 1/σ is the intertemporal elasticity of substitution (IES). We assume that adults do *not* derive any utility gain from child consumption and thus do *not* determine, as part of their own consumption decision, the level at which children consume. Hence, only the consumption at time t of an individual (adult) born at time s, *c(s,t)*, enters the utility function. Since adults in this model view children as having inelastic consumption demands, reflecting basic needs for (say) food, clothing, and shelter, child consumption, which is assumed a fixed real amount, is financed through *in vivo* transfers from adults. The variable *w(s,t)* is the total wealth of that individual at time t, which in turn is the sum of *fw(s,t)*, wealth in the form of financial assets, and *hw(s,t)*, human wealth defined as the present value of expected future labor income net of transfers to children. The other variables in the preceding equations are: *r(t)*, the interest rate at time t; . , the time preference rate; *v(s,t),* the transfers to children; *y(s,t)*, the labor income at time *t* of the individual born at time *s*; and $\tau(s,t)$, the tax rate on that individual's labor income. Disposable labor income for the individual is *y(s,t)(1 -* $\tau(s,t)$).

With a positive probability of death, adult individuals discount the future at a rate higher than their pure time preference rate. As the expression in (13) shows, the effective discount rate for an individual is therefore $. + p_n(t) > 0$.[11] Furthermore, the presence of perfect annuity markets as first introduced by Yaari ensures that individual wealth grows at the rate $r(t) + p_n(t)$, where the premium paid by the perfectly competitive insurance company to an individual during his lifetime is

[11] Blanchard considers the case in which *p* is constant both across individuals and across time. We allow here for variation through time but keep the assumption, crucial for permitting aggregation across individuals, that *p* at any point in time is constant for all individuals alive at that time regardless of their age.

equal to $p_n(t)$ (equation (15)). Note that children are assumed not to possess any financial wealth nor earn any labor income.

Solving the consumer's utility maximization problem gives the individual's consumption as a linear function of total wealth:

$$c(s,t) = \frac{1}{.\ (t)} [fw(s,t) + hw(s,t)]. \tag{17}$$

where $1/.\ (t)$ is the marginal propensity to consume out of wealth and $.\ (t)$ is given by:

$$.\ (t) = \int_t^{\infty} e^{\frac{1}{\sigma}\int_t^v \{(1-\sigma)[(r(\propto)+p_n(\propto)]-(.\ +p_n(\propto)\}d\propto} dv, \tag{18}$$

which can be written as:

$$.\ (t) = -1 - \frac{1}{\sigma}[(1-\sigma)(r(t) + p_n(t)) - (.\ + p_n(t))].\ (t). \tag{19}$$

As is well known, the marginal propensity to consume out of wealth in the general case of the CRRA utility function depends on the intertemporal elasticity of substitution and on the entire sequences of future interest rates and future adult mortality rates. This dependence is readily evident in equations (21) and (22). In contrast, when the intertemporal elasticity of substitution is assumed to be unity (the case of logarithmic utility, with $\sigma = 1$) and when the adult mortality rate is assumed to be constant rather than time varying, the marginal propensity to consume out of wealth reduces to the simple form of a constant, the sum of the time preference rate and the mortality rate ($1/\bar{.} = .\ + \bar{p}_n$).[12]

We follow Faruqee (2000a, 2000b) and Faruqee-Laxton-Symansky (1997) in introducing a life-cycle component into the model, which operates via a hump shaped age-earnings profile. This

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12 For the purposes of the empirical simulations in this paper, we report results for the case where $\sigma = 1$. We believe cases where σ takes on a smaller value are more consistent with the empirical evidence (for example, where σ is 0.5 or even as small as 0.3); empirical simulations in future papers will therefore use those smaller values. We retain the assumption of $\sigma = 1$ in this paper so that results can be readily compared with the results in Bryant and McKibbin (2001), which uses the assumption of $\sigma = 1$.

profile suggests that effective individual labor input varies according to age, initially increasing as experience and seniority are accumulated, and eventually declining after the peak years of productivity have been reached. Figure 2 illustrates the typical hump shape of age-earning profiles with Japanese data for the years 1970-1997. As we use the concept here, the age-earning profile of labor input reflects both changes in the participation rates of cohorts by age and changes over time (initially, increases and then subsequently, decreases) in the relative productivities of cohorts. Given our incorporation of an age-earnings profile in the model, the demographic composition of the population influences not only the allocation of total labor income across age cohorts but also the aggregate amount of total labor income itself.[13]

Specifically, the labor input of an individual cohort *s* at time *t* is assumed to be given by:

$$l(s,t) = \left[a_1 e^{-\alpha_1 (t-s)} + a_2 e^{-\alpha_2 (t-s)} + (1 - a_1 - a_2) e^{-\alpha_3 (t-s)}. \right] \tag{20}$$

The three exponential terms are a way of approximating the age-earnings profile. The parameters $a_1, a_2, \alpha_1, \alpha_2,$ and α_3 are specified exogenously in the modeling code. They can be estimated econometrically for individual countries, for example, by using a non-linear least squares estimation procedure with actual data for age and earnings. Loosely speaking, the first two exponential terms may be thought of as representing the decline in an individual cohort's labor supply over time as it ages and (gradually) retires. The third term can be interpreted as reflecting gains in earnings that accrue with age and experience. The restriction on the a_i terms (the third of the terms must be equal to $1 - a_1 - a_2$) embodies a normalization that the youngest cohort (for whom $s = t$) earns

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[13] As Faruqee (2000a,b) and Bryant and McKibbin (2001) discuss in detail, the age-earnings profile has significant implications for both the steady state and the dynamics of the model when a demographic shock occurs.

income equal to unity. Together the three exponential terms provide the hump-shaped profile for earnings.[14]

The earnings of an individual cohort also change over time because of general growth in labor productivity, assumed to apply uniformly to all cohorts (after adjustment for their age-specific *relative* productivities). The earnings of a particular cohort are:

$$y(s,t) = \left[wage(t)\right] l(s,t), \tag{21}$$

where $wage(t)$ is the economy's wage rate and the wage grows through time at the rate of labor-augmenting technical change, $\propto$.

The chief merit of the Blanchard-Weil-Yaari approach is its simple and elegant way of going from the individual to the aggregate economy-wide level without having to keep track of the behavior of each age-cohort. The assumption that the probability of death is age independent, combined with the fair insurance-market assumption, allows for the straightforward derivation of aggregate levels of model variables.[15]

If (20) and (21) are aggregated over all individual cohorts, aggregate labor income can be written as:

$$Y(t) \;=\; \int_{-8}^{t} wage(t)\, l(s,t)\, N(s,t) ds \;=\; wage(t)\, L(t), \tag{22}$$

where L is aggregate labor input adjusted for cohort-specific relative productivities. The definition of labor input for the individual cohort in (20) also permits one to write aggregate L as the sum of three components L_1, L_2, and L_3 where each component reflects an exponential term in (20).

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14 To ensure that the sum of the exponential terms portrays a reasonable profile and that the effective amount of labor supplied is always initially increasing (when $t = s$), the following restriction on the five parameters must also hold: $a_1 > \dfrac{[\alpha_3 - a_2(\alpha_3 - \alpha_2)]}{(\alpha_3 - \alpha_1)}$.

15 The Blanchard-Yaari insurance-market assumption also requires a supplementary assumption that individuals are not motivated to leave bequests to survivors and cannot go into debt so as to die with negative bequests.

We define:

$$L_k(t) = \int_{-8}^{t} l_k(s,t)N(s,t)ds, \tag{23}$$

where $l_k(s,t) = a_k e^{-\alpha_k (t-s)}$ and $k = 1,2,3$ so that

$$L_k(t) \ = \ a_k b_n(t)N(t) - (\alpha_k + p_n(t))L_k(t). \tag{24}$$

Then one can specify a dynamic equation for aggregate labor input as

$$\begin{aligned} L(t) \ &= L_1(t) \ + L_2 + \ L_3(t) \\ &= b_n(t)N(t) - (\alpha_1 + p_n(t))L_1(t) - (\alpha_2 + p_n(t))L_2(t) - (\alpha_3 + p_n(t))L_3(t). \end{aligned} \tag{25}$$

The intuition behind (25) is that changes in aggregate labor input depend on the effective labor supply of new entrants to the labor force and on the death and relative productivity experiences of existing workers. Our model code uses a discretized version of equations (24) and (25) to describe the dynamic behavior of labor supply. The specific values of the five coefficients $a_1, a_2, \alpha_1, \alpha_2$ and α_3 obtained from estimating the age-earnings profile play an obviously important role in determining the movements of effective labor supply over time. They also play a critical role in the evolution of human wealth and consumption over time.

When one aggregates over individual consumption functions, total adult consumption *C* is given by:

$$C(t) = \int_{-8}^{t} c(s,t)N(s,t)ds = \frac{1}{.\ (t)}[FW(t) + HW(t)], \tag{26}$$

where *C(t)* is aggregate adult consumption, . *(t)* is given by equations (21) and (19), and *FW(t)* and *HW(t)* are aggregate financial and human wealth.

Since we assume that adults transfer to each child a constant real amount of resources, which we denote by c_j[16], total consumption for the entire economy (across both populations, youths plus adults) *X(t)* is given by:

$$X(t) = c_j J(t) + C(t). \tag{27}$$

Given that each child consumes a constant amount c_j, the following condition -- requiring that the consumption of children be exactly matched by the aggregate of transfers from adults to children -- must always be satisfied:

$$c_j J(t) = \int_{-8}^{t} v(s,t) N(s,t) ds. \tag{28}$$

In equation (28), $v(s,t)$ represents lump-sum transfers that adults from a given cohort s provide to their children. We allow support transfer payments for children to be cohort specific and, at least after a certain adult age, to decline over time. The specific formulation which we adopt here, which follows Bryant, Faruqee and Velculescu (2001), is:

$$v(s,t) = v(t)e^{-.\ (t-s)}; \quad .\ = 0. \tag{29}$$

This specification allows older adults to contribute less than younger adults to the support of the youth population, reflecting the fact that the children of older adults may have already themselves reached adulthood and are no longer dependent on parent-child transfers.[17]

When $v(s,t)$ is given by equation (29), the adding-up constraint in equation (28) implies that child transfers evolve through time according to:

$$\frac{v(t)}{v(t)} = \frac{\delta(t)}{\delta(t)} + \left[1 - \frac{v(t)}{c\delta(t)}\right] b_n(t) + . \tag{30}$$

Financial wealth evolves according to:

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[16] When long-run productivity growth is nonzero, we allow for real consumption for children to evolve over time in balance with the economy.

[17] The appendix to Bryant-Faruqee-Velculescu (2001) indicates how the specification in equation (29) can be improved by introducing a hump-shaped profile for the age distribution of child support.

$$FW(t) = r(t)FW(t) + (Y(t) - .\ (t)) - c_j J(t) - C(t), \tag{31}$$

where *Y(t)* is aggregate labor income, and *T(t)* represents aggregate taxes on labor income. Equation (31) reflects the fact that aggregate financial wealth accumulates at the rate *r(t)* (and not at the rate $r(t) + p_n(t)$), since the amount $p_n(t)FW(t)$ is not an addition to aggregate wealth, but rather a transfer from those who die to those alive through the fair insurance companies. Equation (31) makes use of the fact -- see equation (28)-- that the aggregate of support transfer payments to children is equal to the total amount of child consumption.

The change in aggregate human wealth, a variable representing the present value of economy-wide labor income (adjusted for the varying ages and relative productivities of different cohorts), is given by:

$$HW_t = \frac{d}{dt}\int_{-8}^{t} hw(s,t)N(s,t)ds \;= hw(t,t)\,b_n(t)N(t) \;+\; r(t)HW(t) \;-\; [Y(t) - .\ (t) - c_j J(t)]. \tag{32}$$

Equation (32) shows that the incremental change in the stock of aggregate human wealth at time *t* is influenced by the additional human wealth of the newest generation born at time *t*, that is by $hw(t,t)$. The shape of the labor-earnings profile -- embodied in the five parameters $a_1, a_2, \alpha_1, \alpha_2$, and α_3 -- has a critical influence through time on the behavior of $hw(t,t)$ and hence of aggregate human wealth, *HW(t)*, as shown below.

With the age-earnings profile given by equation (20), and the formulation of child transfers discussed above, the dynamics of human wealth for the new cohort of individuals entering adulthood and working life are:

$$hw(t,t) = \left[\sum_{k=1}^{3} hw_k(t,t)\right] - hw_v(t,t); \tag{33}$$

$$hw_k(t,t) = (r(t) + p_n(t) + \alpha_k)hw_k(t,t) - a_k w(t)(1-\tau(t)); \quad \text{k . } \{1,2,3\} \tag{34}$$

$$hw_v(t,t) = (r(t) + p_n(t) + .\ )hw_v(t,t) - v(t) \tag{35}$$

Equation (33) defines the human wealth of the newest individual cohort entering the adult population at time *t* as the sum of four components. The first three derive from the concave time profile of labor income as described earlier; the fourth component reflects the impact of parent-child transfers on human wealth, with v(t) being described by equation (30).

C. Public Pension System

In previous versions of our theoretical approach, the government fiscal authority in a country engaged in real spending on goods and services, raised revenues by taxing the incomes of firms and households, and paid interest on the outstanding stock of the government's debt. The innovation in this paper is that now each government, in addition to its other activities, operates a pay-as-you-go (PAYG) public pension system that collects pension-tax revenue from workers and makes pension transfer payments to the elderly.

We implement the public pension ("social security") system in a simplified form to keep the effects as transparent as possible. On the revenue side, we assume that adult individuals throughout their working lives pay pension taxes as a given fraction (the "pension tax rate") of their labor incomes. The pension taxes are in addition to income taxes paid on their total (labor plus capital) income. Recall that, given our treatment of age-earning profiles, individuals in the model over the age of 65 continue doing some work and receiving some labor income until the end of their lives. Hence we assume that the elderly continue to pay pension taxes after age 65 to the degree that they are still doing some work and receiving some labor income.[18] Since the effective labor input and hence labor income of the elderly declines sharply as they age beyond the threshold of 65 years, the

[18] Elderly workers over the age of 65 also continue to pay income taxes on their labor and capital income.

pension taxes paid by the elderly become increasingly negligible as the individuals grow older and older.

As a worker reaches the threshold age of 65, he or she becomes eligible for a public pension ("social security benefits") and begins to receive a pension transfer from the government.[19] We do not allow an individual's pension benefit to be indexed to or otherwise influenced by his or her own earnings history, as is true in real life in (for example) the Japanese or the United States social security systems. Rather, for analytical simplicity we assume that the government sets pension benefits simply in proportion to average labor income. In each year past the threshold age of 65, therefore, every elderly person receives an identical benefit, calculated as a given fraction (the "elderly benefit rate") of that year's average labor income in the economy as a whole. Our simplified pension system incorporates an essential characteristic of public pay-as-you-go systems: the individual's pension benefits cumulated from age 65 until death may be substantially greater or smaller than that individual's lifetime payments of pension taxes.

Seen from the perspective of an individual cohort *s*, the transfer scheme embodied in the public pension system can be summarized as follows:

$$ptr(s,t) = \begin{cases} -\tau_{ss}(t)y(s,t); \quad s > i(t) \\ \Downarrow_{ss}(t)\dfrac{Y(t)}{N(t)} - \tau_{ss}(t)y(s,t); \quad s = i(t), \end{cases} \tag{36}$$

where $ptr(s,t)$ is the transfer amount at time t, $\tau_{ss}(t)$ is the pension tax rate, $\Downarrow_{ss}(t)$ is the elderly benefit rate, $y(s,t)$ is the individual cohort's labor income, and $Y(t)/N(t)$ is average labor income in the economy. Equation (36) states that younger generations ($s > i(t)$) pay pension taxes into the system and receive no benefits; older agents having reached the threshold elderly age ($s = i(t)$) are "pensioners" and receive a benefit. Since pensioners still receive modest amounts of labor income

[19] Although the elderly in this model never fully retire, they are not penalized for working in old age and start collecting pension benefits at age 65.

after they reach age 65, as noted above they still pay modest pension taxes on that income; an individual's 'net pension benefit' is the difference between the gross benefit and the pension taxes still paid on their declining labor income.

The aggregate nominal amounts of pension taxes collected and benefits paid by the government in period *t* are:

$$PT_{SS}(t) = N(t)\tau_{SS}(t)\frac{Y(t)}{N(t)} = \tau_{SS}(t)Y(t), \tag{37}$$

$$PB_{ss}(t) = Eld(t)\Downarrow_{ss}(t)\frac{Y(t)}{N(t)} = \phi(t)\Downarrow_{ss}(t)Y(t), \tag{38}$$

where $PT_{ss}(t)$ and $PB_{ss}(t)$ are the aggregate taxes and aggregate benefits, $Y(t)$ is aggregate labor income in the economy, $Eld(t)$ is the number of elderly, $N(t)$ is the number of adults, and $\phi(t)$ is the elderly dependency ratio. Equation (37) states that the total of pension taxes collected equals the pension tax on average labor income multiplied by the number of adults in the economy. Similarly, equation (38) states that total pension benefits transferred to the elderly are the product of the average benefit and the number of elderly.

For this pension transfer scheme, a "pure PAYG" status (the pension system never having a difference between tax revenues and benefit payments) requires that total taxes collected must equal total benefits paid out ($PT_{ss}(t) = PB_{ss}(t)$), in the current and all future periods. The continuously balanced status of the pension system can be written as:

$$\frac{\Downarrow_{ss}(t) - \tau_{ss}(t)}{\tau_{ss}(t)} = \frac{1-\phi(t)}{\phi(t)}. \tag{39}$$

Equation (39) shows the well-known condition that the net benefit-to-contribution ratio must equal the support ratio – the number of working-age adults (19 through 64 years of age) relative to elderly dependents (65 years and over).

When equation (39) is not satisfied, there will exist a positive or negative "financing gap" in the pension system reflecting the degree of over- or underfunding of current-period benefit payments. In many of our analyses of the interactions of demographic changes and public pension systems, we want to analyze the cases of such over- or underfunding. Accordingly, we define a variable for the financing gap (in other words, changes in the "trust fund" for the social security system):

$$PTFGAP(t) = PT_{ss}(t) - PB_{ss}(t). \tag{40}$$

If, for example, the pension trust-fund gap is negative, the deficit in the pension system will have to financed by changes elsewhere in the government's budget: increases in revenues from income taxes, cuts in government spending on goods and services, or increased government borrowing through additional issuance of government debt. The pension-system financing gap must therefore be included explicitly as part of the overall government budget identity and must be taken into account in assuring that the government budget satisfies the criterion of intertemporal consistency (discussed further in section 3).

Any imbalance between tax revenues and benefit payments in the pension system will also affect the dynamics of aggregate human wealth. Equation (32) must therefore be replaced by :

$$HW(t) = hw(t,t)b(t)N(t) + r(t)HW(t) \text{ - } [Y(t) - T(t) - PTFGAP(t) - c_j J(t)] \text{ .} \tag{41}$$

When the pension system is continually balanced so that current-period pension taxes exactly cover current-period pension benefits paid to the elderly, the *direct* effect of the pension system on aggregate human wealth nets out to zero, as can be readily seen from equation (41).

However, even for the continually balanced cases where *PTFGAP* never differs from zero, the inclusion of a public pension system in the model has important *indirect* effects on human wealth (and hence on consumption and other key macroeconomic variables) through the evolution of *hw(t,t)*, the human wealth of the newest cohort just entering adulthood and the workforce. Under the pension system here, the equation for *hw(t,t)* must be rewritten as:

$$hw(t,t) = \left[\sum_{k=1}^{3} hw_k(t,t) \right] + hw_{ss}(t,t) - hw_v(t,t)\,. \qquad (42)$$

In equation (42) the term $hw_v(t,t)$ is as before (see equation (35)). The $hw_k(t,t)$ equations are modified, however, to allow for the pension taxes that are paid on labor income:

$$hw_k(t,t) = (r(t) + p_n(t) + \alpha_k) hw_k(t,t) - a_k w(t)[1 - \tau(t) - \tau_{ss}(t)]\,; \quad \text{k. } \{1,2,3\} \qquad (43)$$

and a new component, $hw_{ss}(t,t)$, is added to reflect the real value of the stream of pension benefits that the new adult cohort expects to receive eventually after reaching the threshold elderly age (47 years in the future, $t + .$), discounted back to the present:

$$hw_{ss}(t,t) = [r(t) + p_n(t)] hw_{ss}(t,t) - [\, \mathcal{U}_{SS}(t + .\ ) \frac{Y(t + .\ )}{N(t + .\ )}] e^{-\int_t^{t+.} (r(s) + p_n(s)) ds} \qquad (44)$$

The second term in square brackets in (44) is the per-elderly gross pension benefit expected in the future at the date today's new adult expects to reach the threshold elderly age. Note that even for the pure PAYG case in which aggregate pension taxes continually equal aggregate pension benefits, the discounted stream of an *individual* new worker's future pension taxes will not necessarily be equal to the discounted stream of pension benefits that that worker expects to receive from age 65 until death. Consequently, even a continuously balanced PAYG pension system could affect individual consumption and saving behavior through its effect on individual human wealth. The effects on the human wealth, consumption, and saving of particular individuals will be influenced even more strongly when the trust-fund gap in the pension system is non-zero.

3. Using the Theoretical Framework in an Empirical Macroeconomic Model

The empirical research carried out at the Brookings Institution for this project on the global dimensions of demographic change has used a stylized two-region abridgement of the IMF staff's MULTIMOD model. This abridgement is a revised and updated version of the two-region model originally created in the mid-1990s by Bryant and Zhang (1996a, 1996b). An earlier paper in the project (Bryant and McKibbin, 2001) compared this abridgement of MULTIMOD with the MSG3 model used for the empirical research in the project at the Australian National University.[20]

The starting point for the empirical model is a set of equations describing the U.S. economy (US for short), based largely on the equations for the United States in MULTIMOD. A second hypothetical economy, labeled ZZ, is an identical, mirror image of the US. These two equal-sized and equally-open regions are carefully linked together with the balance-sheet and income-flow identities that would have to hold if the world were composed of only these two economies. A single exchange rate links the two regions' currencies and economies. The exchange rate is proximately determined by a variant of the uncovered interest-parity relationship.[21] The current-account balance and the net-foreign-asset position of the ZZ economy are exactly the negative of the current account and the net-foreign-asset position of the US economy.[22]

Each region consists of several types of economic agents: households, firms, a government, and a central bank. A single composite good is produced in each country. Output in each country is a function of capital and productivity-augmented labor. The composite goods from each country are imperfect substitutes; each country exports some of its production to the other. Imports in each

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20 For a description of the IMF staff's MULTIMOD Mark III, see Laxton-Isard-Faruqee-Prasad-Turtelboom (1998). The Mark II version of MULTIMOD is presented in Masson-Meredith-Symansky (1990).

21 Of course, the exchange rate is indirectly influenced and determined by most of the other variables in the model when asymmetric shocks cause the economies of the two countries to behave differently.

22 The label "ZZ" is deliberately awkward, to call attention to the hypothetical nature of this second, mirror-image economy. Bryant and McKibbin plan to extend the project research into more realistic multi-country empirical models. For an initial effort along these lines, see McKibbin and Nguyen (2001).

country are a function of income and relative prices. Agents in a given country are assumed to have identical preferences over foreign and domestic goods.

Demographics, age-earning profiles, and the "bottom-up" approach to modeling labor incomes were built into the empirical model by early 2001 (Bryant-McKibbin, 2001). Youth dependency was incorporated in the fall of 2001 (Bryant-Faruqee-Velculescu, 2001).

At this point in the research, three key exogenous variables determine population dynamics: a birth rate for children, a mortality rate for children, and a separate mortality rate for adults. As explained in section 2, variables such as the growth rate of the adult population and the rate at which youths enter the labor force to begin work are endogenously determined within the model.

The model presupposes that both firms and households engage in intertemporal optimization.[23] Our use of the model requires long-run evolutions of the economies that result in steady-state, balanced-growth equilibrium paths.

Agents in the model are forward-looking in their behavior. The model is solved with a software algorithm that imposes model-consistent ("rational") expectations. Hence agents are presumed to know the structure of the model and to correctly anticipate the entire future paths of the model's exogenous variables. Imposition of model-consistent expectations is the now-standard working assumption in most empirical work in macroeconomics and our use of this assumption is familiar ground. Yet the assumption is extreme. Worse, the assumption is inherently implausible for demographic shocks that begin gradually and then wane gradually over many years. In previous research we have shown how it is possible to modify the model-consistent-expectations assumption by phasing in "correct expectations" about the paths of exogenous variables with the passage of

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[23] A partial exception stems from an allowance for a fraction of consumers whose consumption is constrained by an inability to borrow. Those constrained consumers cannot smooth their consumption intertemporally. For the simulations reported in this paper, the fraction of constrained consumers is set to zero. The model permits an explicit assumption about the value of the consumers' elasticity of intertemporal substitution (EIS). For expositional simplicity, the simulation results reported in this paper assume a value for the EIS of unity.

time rather than permitting expectations to be correct immediately and fully. Interesting work has recently been done by others applying "learning" ideas to the evolution of expectations. In further work in the project, we hope to make modifications in the assumed treatment of expectations. For the time being, we report the results with the now-familiar, full model-consistent expectations.[24]

For each country in the model, policy reaction functions are specified for the central bank and the government fiscal authority. Sensible dynamic simulations of the model are not possible in the absence of such reaction functions.

Neither MULTIMOD nor our abridgement of it contain a fully explicit financial sector for each country. The model therefore does not capture the behavior of private financial intermediaries and hence does not include loans made by the intermediaries or deposit liabilities issued by them. Because of the absence of private financial intermediaries, the model cannot distinguish between narrow money (the "high-powered" liabilities of the central bank) and broad money (high-powered money plus the deposit liabilities of the private intermediaries). The monies of the two countries accordingly are only high-powered money. Demands in each country for the high-powered monies depend negatively on short-term nominal interest rates and positively on the value of aggregate outputs.

The central bank in each country is assumed to follow a policy rule that ensures long-run nominal stability of the model's behavior. In practice, the model can enforce either a targeting rule for (high-powered) money, a nominal-GNP-targeting rule, or a rule combining inflation targeting with real GNP targeting. These rules are explained and analyzed in Bryant, Hooper, and Mann

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24 An overview of the "BM2R" (Bryant Multimod 2 region) abridgement of the IMF's MULTIMOD is presented in Bryant and McKibbin (2001). Bryant is preparing a separate paper that will provide a complete and detailed description of the BM2R model and our incorporation into it of endogenous demographics and the bottom-up determination of labor income and human wealth resulting from the incorporation of age-earning profiles.

(1993).[25] In the series of papers prepared so far in this project, we have assumed for expositional simplicity that the central banks follow money-targeting rules.

Given that we are now incorporating public-pension systems in the model, it is necessary to be explicit about the "intertemporal fiscal closure rule" followed by the government fiscal authority. We need here also to clarify the alternative assumptions that can be made about the operation of the pension systems.

The government budget identity for each country, written in *nominal* rather than *real* terms and in *discrete-time* notation as opposed to the *continuous-time* notation in section 2, can be expressed as:

$$GDEF_t = G_t P_t + r_t B_{t-1} - TAXY_t(P_t) - [PT_{SS,t}(P_t) - PB_{SS,t}(P_t)]\,. \qquad (45)$$

The overall nominal deficit in the government's budget during period t, $GDEF_t$, is the sum of nominal spending on goods and services (G_tP_t) where G_t is real spending and P_t is the overall price level *plus* the nominal value of interest payments on the government debt (r_tB_{t-1}, where r_t is the interest rate during period t and B_{t-1} is the nominal stock of debt at the beginning of period t) *less* the nominal value of revenue receipts from income taxes during the period ($TAXY_t(P_t)$) and *less* the difference, if any, between the nominal amount of pension tax receipts and pension benefit transfers during the period [$PT_{SS,t}(P_t) - PB_{SS,t}(P_t) = PTFGAP_t(P_t)$].[26] Budget deficits are expressed as positive numbers, and surpluses as negative.

The model identity for the financing of a government deficit is:

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[25] In recent years, a growing literature has focused on central-bank reaction-function "rules" for monetary policy. For overviews, see for example Taylor (1999) and Svensson (1999).

[26] For expositional simplicity, the equations in section 2 are all expressed in real rather than nominal terms and in continuous rather than discrete time. The empirical model carefully distinguishes real and nominal values and endogenously determines prices in the two countries. The empirical model is written in discrete time. The notation here in section 3 reflects the empirical model and, where appropriate for expositional simplicity, uses nominal rather than real magnitudes in equations. Another minor difference in notation between sections 2 and 3 occurs with income taxes: the aggregate amount of income taxes in real terms in section 2 was denoted as *T(t)*; here in section 3, we use the notation $TAXY_t$ to denote the real value of aggregate income taxes.

$$GDEF_t = (B_t - B_{t-1}) + (M_t - M_{t-1})\,; \tag{46}$$

B_t and M_t are the nominal values of the stocks of government debt and (high-powered) money at the end of period t and B_{t-1} and M_{t-1} are the beginning-of-period nominal stocks.

In any carefully specified macroeconomic model, private agents will not hold government debt unless per capita government debt is eventually forced to grow at a rate less than the interest rate paid on the debt. To implement this constraint, our empirical model uses an intertemporal fiscal closure rule that is a variant of "debt-stock targeting." (The differences between alternative specifications of intertemporal fiscal closure rules are carefully described in Bryant and Zhang (1996a).)

The particular variant of debt-stock targeting in our model assumes that the fiscal authority sets -- or has set for it -- an exogenous path for a target ratio of nominal government debt to nominal GDP, $BRATIOT_t$. Given this exogenous target ratio, the model calculates an endogenous path for the nominal amount of target debt, BT_t , as:

$$BT_t = BRATIOT_t(NOMGDP_t) \tag{47}$$

where $NOMGDP_t$ is the simulation path of nominal gross domestic product. The fiscal authority is then assumed to focus on the gap between the target path, BT, and the actual path of government debt, B:

$$BTGAP_t = \log\left(\frac{B_t}{BT_t}\right). \tag{48}$$

Denote τ_Y as the tax rate on incomes imposed by the fiscal authority. As its method of ensuring intertemporal fiscal consistency, the authority is presumed to vary τ_Y up or down in response to non-zero values of $BTGAP$. In the language of control theory, the fiscal authority's response has both proportional and rate-of-change terms, but not an integral term. More precisely, the debt-stock targeting behavioral rule specifies that:

$$\tau_{Y,t} - \tau_{Y,t-1} = \;._1(BTGAP_t) + .\;_2(BTGAP_t - BTGAP_{t-1}) \quad . \tag{49}$$

Higher values of $._1$ and $._2$ produce faster adjustment of the income-tax rate to a level that brings *BTGAP* back close to zero; lower values permit the government's deficit and debt to deviate for a longer time before ultimately being pulled back to levels that produce intertemporal budget consistency.[27]

The equations for the revenues and benefit payments and the financing gap (if any) of the public pension system, summarized in section 2, are written here for convenience in nominal terms:

$$PT_{SS,t}(P_t) = N_t \tau_{SS,t} \frac{Y_t(P_t)}{N_t} \tag{50}$$

$$PB_{ss,t}(P_t) = Eld_t \Downarrow_{ss,t} \frac{Y_t(P_t)}{N_t} \tag{51}$$

$$PTFGAP_t(P_t) = PT_{ss,t}(P_t) - PB_{ss,t}(P_t). \tag{52}$$

In this setup for the public pension system, three variables -- the pension tax rate $\tau_{SS,t}$, the elderly benefit rate $\Downarrow_{ss,t}$, and the nominal financing gap $PTFGAP_t(P_t)$ -- control how the system operates in practice. When the model is run, any two of these three must be set as exogenous. The remaining variable is then determined endogenously.

We distinguish between three alternative ways of operating the public pension system within the model. The first alternative assumes that the trust fund gap is exogenous and equal to zero, what might be termed a "pure" or "continuously balanced" PAYG system. We set the elderly benefit rate as exogenous and constant, which means that the pension tax rate, $\tau_{SS,t}$ is the endogenous variable which must adjust period by period to keep the financing gap exactly on its

[27] Income taxes in the model are paid on both labor incomes and capital incomes. The income-tax rate appearing in the text equations applies to the sum of labor and capital income; this income-tax total is allocated to laborers and owners of capital in proportion to the shares of labor and capital in output.

exogenous path.[28] For convenience, we denote this case as **P1**. In the second alternative, which we label **P2**, we set both the pension tax rate and the elderly benefit rate as constant and exogenous; $PTFGAP_t(P_t)$ then becomes unbalanced and adjusts endogenously through time (resulting in an "unbalanced" PAYG system). As $PTFGAP_t(P_t)$ feeds into the government debt, the debt-stock targeting behavior of the fiscal authority plays a crucial role in determining how the public pension system affects the model economy. For the P2 alternative, we assume that the government does not change either its debt target path, nor the rule by which income taxes adjust to prevent deficits and debt from rising too rapidly. In the third alternative scenario, which we regard as more realistic, we allow for both an unbalanced PAYG system (the trust fund varies endogenously as in case P2) and for the fiscal authority to make adjustments to its target debt ratio. This case is labeled as **P3**.

To gain some intuition about the economic implications of each of the three options for the pension system, consider the thought experiment of a fertility decline leading to higher elderly dependency. When a pronounced rise in the elderly dependency ratio occurs in the P1 case, the pension system must pay out more pension benefits than can be financed at the former baseline pension tax rate. Because the pension trust fund cannot become unbalanced in this pure PAYG system, the pension tax rate must rise sharply to keep $PTFGAP_t(P_t)$ at zero.

If the pension system were operated according to the P2 case, on the other hand, then the pension trust fund itself would run a growing deficit because pension transfers to the larger numbers of elderly would increasingly outstrip pension tax revenues at the unchanged tax rate $\tau_{SS,t}$. As the deficit in the pension trust fund rises, it would other things being equal increase the overall budget

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[28] One could also set the pension tax rate as exogenous and constant and allow the elderly benefit rate to continuously adjust so as to keep the trust fund gap at zero. In real life PAYG systems, fixing pension taxes or fixing benefits will have quite different political and economic implications. Even in a PAYG system in which pension taxes were fixed, it seems implausible that the pension authorities would vary benefits continuously, period by period, so as to keep the pension trust fund exactly balanced. In any event, in this paper when we consider a continuously balanced PAYG system, we focus on the case where the pension tax rate is varied endogenously while the pension benefit rate is kept constant.

deficit *GDEF* and hence raise the outstanding stock of government debt. Instead, however, the income-tax-rate reaction function goes to work to raise income taxes to prevent the overall budget deficit and the debt stock from increasing too rapidly. If income taxes adjust quickly enough,[29] the dynamic behavior of many macroeconomic variables in the economy -- especially over the longer run – would be little different between the P1 and P2 cases. In the P1 case, pension taxes have to rise sharply. In the P2 case, income taxes rather than pension taxes rise. Since our model does not have differential labor-supply responses to different types of taxes, the life-cycle decisions of agents in the model about consumption and saving are not differentially influenced by whether they have to pay greater amounts of pension taxes or income taxes.[30]

For the P3 case for the operation of the pension system and the behavior of the fiscal authority, government debt is allowed, at least temporarily, to increase following the shock. The result will be permanently higher income taxes in the long run. The dynamic behavior of most macroeconomic variables in the P3 case is expected to differ substantially from their behavior in the P1 and P2 cases. We will analyze the different options for the operation of the pension system in more detail in the next section.

As a foundation for the empirical analysis to be discussed in the following sections, it is first necessary to develop model-consistent, steady-state baseline solutions for the evolution of the ZZ and US economies. For transparency, we assume that both economies follow identical paths and exhibit identical behavior along these steady-state baselines. Hence the baseline exchange rate is constant over time at unity and the trade balances, current-account balances, and net-foreign-asset positions in the baseline are all constant at zero.

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29 As is the case under our parameterization, which assumes rather large values for the $._1$ and $._2$ coefficients, so as to prevent major, sustained accumulations or decumulations of actual debt relative to target debt.

30 In theory, the labor-supply response to a dollar of pension taxes on labor income should differ from the labor-supply response to a dollar of income tax on combined labor and capital income.

In both economies the baseline fertility or child birth rate is assumed to have a constant value of .0232 (2.32 percent per year). The child mortality rate is constant at .01 (implying that life expectancy at birth is approximately 100 years). The adult mortality rate is constant at .0167 (implying that at age 19, individuals expect to live another 60 years) . These baseline assumptions about fertility and child mortality have the consequence that children enter the baseline adult labor force at the constant rate .0187, which in turn results in the total population growing slowly in the baseline at the constant rate .002 (0.2 percent per year).

For baseline assumptions for the pension system, we essentially implement the P1 option where the trust fund is continuously balanced at zero and the elderly benefit rate is specified exogenously. For the illustrative elderly benefit rate, we select a value of 0.32. This rate is roughly the order of magnitude of the average size of benefits relative to average wages in both the Japanese and U.S. public pension systems. With the $PTFGAP_t(P_t)$ path fixed at zero and the elderly benefit rate at 0.32, the baseline pension tax rate consistent with those assumptions is approximately 0.135. This pension tax rate also seems broadly consistent with the actual experience in the Japanese and U.S. public pension systems.[31]

Slightly different baseline solutions are needed for the model depending on what baseline assumptions are made for the level of child support and its distribution across different ages of parents (c_j and .   in the notation of section 2). For an analytical benchmark, we also construct a baseline for the case where both child support and the public-pension system are assumed completely absent (c_j = . = $\Downarrow_{ss,t} = \tau_{SS,t} = PTFGAP(P) = 0$). Regardless of the assumptions about child support and the pension system, the baseline assumptions for the demographic variables remain as described above (a steady-state growth rate for the total population of 0.2 percent per year).

To highlight the effects of introducing the pension system, we carry out the analysis in the next two sections of the paper with child support taken out of the model ($c_j = . = 0$). This procedure of ignoring youth dependency has the advantage of transparently revealing the effects of transfers to elderly dependents via the public pension system in isolation from the effects of transfers to children. When we reach section 6 of the paper, on the other hand, we bring child support back into the analysis and examine the combined effects of incorporating youth dependency and elderly dependency.

4. Effects of a Decline in Fertility: The "Global" Case

To demonstrate the properties of the model with population aging and public pension systems, we concentrate on an illustrative shock in which fertility declines sharply, stays low for an extended period, and then subsequently recovers back to its baseline rate. For simplicity and transparency, we assume that only the fertility rate changes. The shock assumes no changes from baseline in the mortality (life expectancy) of either adults or youth dependents.

We select this shock for study for its analytical value in understanding the operation of pensions in our model and because it includes essential elements of the recent and prospective demographic experience of Japan. The value of 0.2% for population growth in the initial years of the baseline, for example, is roughly consistent with recent estimates of current growth by the Japan National Institute of Population and Social Security Research (1997). The major decline in fertility in Japan of course took place in the second half of the 20th century. Projections of Japanese demographics in the first half of the 21st century indicate pronounced population aging (see Figure

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[31] Actual public pension systems in Japan and the United States, and in other countries, are of course much more complex in their provisions than the simplifying assumptions we make here.

1) and negative population growth rates. Our illustrative shock is thus motivated both by Japan's past fertility experience and by projections of its demographic future.[32]

Our illustrative shock is transitory, albeit lasting over a long period of some 100 years. Because the shock is ultimately transitory, we can readily gauge from these initial simulations whether the model is appropriately behaving in the long run as theory argues it should. Later on, in the course of further research on the pensions part of the model, we will examine other illustrative shocks. In particular, we will want to examine *permanent* as well as transitory demographic shocks. For example, we are likely to construct baseline model solutions in which the initial birth rate and population growth rates are higher than those selected for this paper; then the shock studied would postulate a large decline in the fertility rate which, after persisting for a long period, would be only partly reversed. Hence the economy would start out in a steady state with high fertility and population growth but ultimately settle into a steady state with population growth much lower (or even stationary at a zero growth rate). Such a shock would represent even more accurately the demographic experience of Japan over the entire period starting in the middle of the 20th century and running through the second half of the 21st century.[33]

Here in section 4, we assume that the illustrative shock occurs identically in both the ZZ and US regions. When a shock is identical in both regions, our model produces identical simulation paths for both economies. In effect, each economy behaves as though it were completely closed, which is of course true for the world as a whole (hence we attach a "global, closed-economy" label to the simulation). We initially focus on this symmetric, global case because it facilitates interpretation of the most basic, domestic consequences of introducing pension systems into the

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[32] To simplify the exposition for the purposes of this paper and the earlier companion paper on youth dependency, we have assumed that the baseline steady-state growth rates for productivity and the baseline steady-state inflation rates are both zero rather than some positive number.

[33] The demographic shock studied in Bryant-Faruqee-Velculescu (2001) was in part a *permanent* shock because the ultimate steady state of stationary population growth was different from the initial steady state with its positive population growth at 0.2 percent per year.

model. The global, closed-economy case will then serve as a benchmark for interpretation of an "asymmetric shock" (country-specific) case in section 5. Analysis of the asymmetric shock will highlight the importance of the openness of economies in adjusting to demographic shifts and in partially transmitting the consequences of such shifts to the rest of the world.

The demographic assumptions underlying our illustrative shock are summarized in Figures 3, 4, and 5. The child birth rate in both economies falls persistently below the baseline level of 2.32 percent to 1.64 percent over a 35 year period, then remains at that lower level for 30 years. The birth rates then gradually recover over the subsequent 35 years (Figure 3). The consequences for growth rates of the adult and total populations are shown in the bottom part of Figure 3. The illustrative shock results for some eight decades in a negative growth rate for the total population and, with a lag of 18 years, for working adults. A pronounced aging of the population starts some three decades after the onset of the decline in fertility. The ratio of elderly to the total adult population eventually rises by some 10 percentage points (Figure 4). As the shock is gradually unwound and as the fertility rate eventually recovers back to baseline, the ratio of the elderly to the total population eventually declines back to baseline.

As pointed out in our earlier papers, changes in the effective labor force -- reflecting demographic shocks and the presence of a humped-shape age-earning profile -- play a central role in conditioning the dynamic behavior of the model. When individuals first enter the labor force, they have relatively low productivity and are relatively low savers. Then as younger workers age, gain experience, and have higher productivity, they in effect ascend the left side of the hump of the economy's age-earning profile (Figure 2). Individuals reach their years of peak earnings and high savings when they are in their forties and fifties. Eventually, they start to descend the right side of the humped age-earning profile, and consequently their labor incomes and saving decline. At the later stages of life, their consumption must be increasingly financed out of (apart from pension transfers from the government) their privately accumulated financial wealth. Speaking loosely, as demographic shocks pass through the age-earning profile, the dynamic effects of the demographic movements, significant in themselves, get still further amplified.

In our illustrative shock, during the early years of the fertility decline the adult population and the effective labor force continue to increase at the baseline positive rate of population growth. Even when the total population begins to decline, the adult population and effective labor force continue to increase (Figure 5, which plots the levels of the variables). The increases in the effective labor force reflect the fact that the number of young workers, who are less productive, decline relative to the number of older, higher-productivity workers. Then as the demographic shock passes into the ranks of the highest-productivity workers, the effective labor force begins a protracted, sharp decline (at a rate even steeper than that of the adult population as a whole). The decline in the effective labor force is eventually reversed as the proportion of youth in the economy rises again and ultimately stabilizes at its baseline level. After the demographic shock has passed completely through the entire age distribution and the economic effects are fully worked out, the labor force, adult population, and total population then resume their steady growth at the baseline rate of 0.2 percent per year.[34]

Throughout the paper, we report simulation results graphically. Demographic shocks can have consequences over very long periods. To be sure that our model simulations correctly reach new long-run steady states, we typically run the simulations over a horizon of 600 years or more. Most of the paper's charts report the results over a long period of 250 years.

The time paths of variables in the charts are sometimes shown as *levels* (as, for example, in Figures 3-5 and 7-10). Most often, the curves plotted in the charts are *deviations* between the simulated path of a variable and its path in the baseline model solution. The units in which the

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[34] The initial baseline level of the *effective* labor force is some 1.6 times greater than the level of the total adult population, reflecting the calibration of the labor force with its incorporation of the age-specific relative productivities of different aged workers. The level of the effective labor force represents, in effect, the number of labor "efficiency units," not the total number of workers.

deviations of the variables are measured are specified along the vertical axes of the charts. For charts that show deviations of variables from baseline, a value of zero indicates that the variable at that point is unchanged from its baseline path.

In what follows, our charts typically report results for the illustrative shock for only two variants of the pension system, P1 and P3. We omit results for a P2 case because of the close similarity, for most macroeconomic variables of interest, between P1 and P2 simulations. The reason for the close similarity between P1 and P2 simulations has already been explained in section 3: the background operation of the income-tax-rate reaction function in P2-type simulations produces effects on the *total* of taxes collected (pension taxes plus income taxes), on the *overall* government budget and on government debt -- and hence on variables such as human wealth, consumption, and saving -- that are quite similar to the effects in P1 simulations. Major differences exist, however, between a P2 (or P1) simulation on the one hand and a P3 simulation on the other.

The most revealing analytical insights about the operation of the public pension system in the model stem from a comparison of three simulations to be shown in Figures 7-12. The illustrative shock is identically imposed in each simulation. In the simulation labeled P0, included as a benchmark for analysis, the pension system is absent altogether. The P0 curves illustrate how the fertility decline influences model variables when pension transfers to the elderly are suppressed.[35] The P1 curve in the subsequent charts is from a simulation in which, by construction, the pension trust fund must be continuously balanced (and hence the pension tax rate must be continuously adjusted).

The last of the three curves in Figures 7-12 is from a particular example of a P3 simulation. In the example shown here, the fiscal authority, for a considerable transition period as population aging puts pressure on the pension system, permits not only the pension trust fund to run a deficit

[35] Recall that youth dependency and child support are also absent from the simulations here in section 4 and in section 5.

but also allows the *overall* budget to run increasing deficits. Hence the government's debt accumulates to much higher levels in relation to the economy than permitted in the P1 and P2 cases. In effect, as the deficits in the pension system and the overall budget develop, the fiscal authority is assumed to "let things roll" in the sense of riding with the situation and not reacting to the rising ratio of its debt to the economy. But then eventually, as the deficits and debt mount to higher and higher levels, the fiscal authority is assumed to get cold feet and to acknowledge that the government budget cannot be moved into intertemporal balance and the debt ratio stabilized unless it raises some form of taxes (either pension or income taxes) or cuts some form of expenditures (pension benefits or other government spending). The specific way in which we implement this P3 example is that the fiscal authority is assumed gradually to raise its target debt ratio *BRATIOT* (set exogenously -- see equations (47), (48), and (49)) so that the ratio moves up from its baseline value of 0.35 to the level 0.70, double the baseline ratio. Once the debt ratio approaches twice the level it otherwise would have been, the fiscal authority then reevaluates the situation, refuses to allow the target debt ratio to rise further, and promptly begins to raise the income tax rate.[36]

Figure 6 illustrates these different behaviors of the pension and fiscal authorities. The curve for the P1 simulation in Figure 6 shows the deviation from baseline of the *pension* tax rate during the long period in which the population aging resulting from the fertility decline (see Figure 4) puts pressure on the pension system. During this period, the pension tax rate has to be continuously adjusted, first upwards and then eventually downwards, to keep the financing gap in the pension

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[36] We calculate the path for the incremental government debt implied by the rising path of *BRATIOT* in the P3 simulation to be roughly equal to the accumulation of the deficits in the pension trust fund that occurs in a P2-type simulation of the same illustrative shock. By focusing attention on this particular P3 example, we do not intend to suggest either that this behavior of the fiscal authority is the most likely behavior or that such behavior is desirable. Rather, we use the example as a reminder of the complexity of the options facing the fiscal and pension authorities and as an illustration of how differences in the behavior of the fiscal and pension authorities can produce quite different macroeconomic outcomes. An unsatisfactory feature of our P3 example (and indeed of P1 and P2 simulations as well) is that agents in the model correctly anticipate at the outset of the illustrative shock, well before the population aging actually occurs, exactly how the fiscal and pension authorities will behave in the future at the time that population aging puts pressure on the pension system.

system at zero. The P2 curve in Figure 6 shows deviations from baseline in the *income* tax rate during the same period, due to the background operation of the income-tax-rate reaction function. The trust fund in the pension system in the P2 simulation becomes progressively unbalanced while population aging is putting pressure on the pension system. The fiscal authority in the P2 case, however, does *not* adjust its target debt ratio or the response coefficients in its income-tax-rate reaction function. Accordingly, variation in the pension tax rate in the P1 simulation and variation in the income tax rate in the P2 simulation are very similar. As a result, the P1 and P2 paths for most macroeconomic variables are also broadly similar.

Figure 6 also plots the variation in the income tax rate that occurs with P3 behavior by the fiscal authority. As population aging initially puts pressure on the pension system and as the government's overall budget deficit is permitted to widen, the P3 incremental rise in the income tax rate is much less than that required in the P2 case. The ratio of government debt to the economy rises, with the fiscal authority permitting a doubling to occur over a period of some four decades. Once the fiscal authority halts the burgeoning deficits and rise in the debt ratio, however, the income tax rate must rise very rapidly. Moreover, it must climb to a much higher level than in the P2 simulation. When the rate can eventually fall back, it falls to a value permanently higher than in the initial baseline.

The first effects meriting emphasis of introducing a PAYG public pension system into the model are the consequences for the steady-state levels of variables.[37] These consequences can be seen by examining the left-hand and right-hand sides of charts for the real interest rate (Figure 7), the capital-output ratio (Figure 8), the per-adult value of human wealth (Figure 9), and the per-adult value of financial wealth (Figure 10). The initial (pre-shock) and ultimate (post-shock) steady-state values of these variables when pensions are suppressed (P0) are significantly different from the

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[37] Remember that the curves describe both the ZZ and US regions because the shocks, and hence the outcomes here in section 4, are identical in both countries.

levels when the pension system is included (P1 and P3). Hence the levels of the variables throughout the dynamic simulations also tend to differ significantly.

The primary explanation for these differences is that a PAYG pension scheme in the model discourages individuals somewhat from private saving for their old age. Intuitively, if the present discounted value of an individual's current and future pension taxes ("social-security contributions") will be roughly equal to the present discounted value of future pension benefits, the introduction of a public pension system will not much change the individual's human wealth. But the private saving that forward-looking individuals would have done on their own without a pension system tends to be displaced by their pension taxes under a PAYG pension scheme. If an individual were to expect future pension benefits to exceed current and future pension taxes (both being appropriately discounted to present value), then the individual's private saving would tend to fall by a still larger amount. (Conversely, if discounted pension taxes were expected to exceed discounted future pension benefits, private saving would rise above what it would be with pension taxes and pension benefits expected to be equal.) Another consideration is that the introduction of a public pension system providing assured annuities in old age reduces the precautionary motive to save when longevity is uncertain, as is true in our model.

Thus both in the initial baseline and throughout the shock simulations, lower private saving with than without a pension system leads to lower private capital accumulation, and hence to lower economy-wide output and consumption per adult. The real interest rate reflects the marginal product of capital; hence the level of the real interest rate must be higher when a pension system is present (Figure 7). Less capital accumulation with a higher marginal product of capital entails a lower capital-output ratio (Figure 8). In the initial steady-state baselines, the level of real human wealth per adult is roughly similar with and without a public pension system (Figure 9).[38] Because

[38] We assume that the pension trust fund is continually balanced in the initial steady-state baselines. Given the other assumptions used to calibrate the baselines, the present discounted value of the stream of pension taxes is barely

pension tax payments work to offset private saving, however, the level of real financial wealth per adult is substantially reduced when public pensions are incorporated (Figure 10).

Although the illustrative shock in the simulations lasts over an extended period, ultimately it is transitory. The ultimate steady-state levels of many variables in the P0 and P1 simulations thus are identical to the levels in the initial pre-shock baselines (Figures 7-10).[39] In sharp contrast, the levels of the same variables in the final steady state in the P3 simulation are significantly different from initial values. Government debt in relation to the economy in the P3 simulation is ultimately much higher. When the P3 simulation is compared with the P1 simulation in which the pension trust fund can never become unbalanced, the higher debt leads to permanently higher income taxes, and thus to a higher real interest rate (higher marginal product of capital), a lower capital-output ratio, lower human wealth per adult, and higher financial wealth per adult.

The dynamic pattern of movements in the real interest rate through time is similar regardless of whether the pension system is present (Figure 7). Cyclical movements in the real interest rate tend to reflect the cyclical movements in the effective labor force (Figure 5). During the early decades when the labor force is still rising, the real interest rate rises somewhat. Then during the period of long decline in the effective labor force, the real interest rate also declines; the effective labor force is lower relative to the capital stock, and hence the marginal product of capital must fall. When the effective labor force eventually recovers as the long period of fertility decline is reversed, the real interest rate also recovers strongly. In the longest run, after the capital stock is again higher

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smaller than the present discounted value of future benefits. The human wealth of new adult entrants in the work force -- see equations (42) through (44) -- is slightly higher with the introduction of a pension system. However, since the proportion of new adults (19-year olds) to the total adult population is quite small, the incremental contribution of new workforce entrants to aggregate human wealth is also quite small. (If the birth rate were higher in the initial steady state, this incremental contribution would arguably be somewhat larger.)

[39] The levels of many demographic and macroeconomic aggregates for the entire economy (as opposed to per-capita or per-adult variables) are *not* the same in the ultimate and initial steady-state baselines. For example, although the shock is ultimately transitory in terms of demographic *rates of growth*, the ultimate steady-state levels of the effective labor force and GDP are much lower than in the initial baseline.

relative to the labor force, the real interest rate gradually falls back from its peak level toward its ultimate steady-state value.

The dynamic behavior of private saving is implicit in the chart for real financial wealth per adult (Figure 10) but we also show an explicit chart for movements in the ratio of nominal private saving to nominal GDP (Figure 11, where the curves are now deviations of the simulation ratio from the baseline ratio). The dynamic movements in the saving ratio are broadly similar in the P0 (no pensions) and P1 (continuously balanced) cases. After an initial period of little change, saving gradually declines and stays lower for the entire period of the fertility decline. When the fertility rate eventually recovers, the private saving ratio also recovers and eventually returns back to baseline.

The dynamics in the P3 case, on the other hand, are entirely different. Note again, furthermore, that even though the economy in the P1 and P3 cases starts from the same steady state, the ultimate steady states are noticeably different. Private saving in the P3 simulation rises rapidly during the period when the financing gap in the pension system is becoming progressively negative and the fiscal authority is permitting government debt to rise sharply (Figures 10 and 11). As soon as the fiscal authority hits the brakes and permits no further increase in its target debt ratio, however, income taxes start to rise very sharply and private saving falls quickly. The private saving ratio eventually recovers back toward the initial baseline level as the demographic shock passes and income taxes can be lowered. But the eventual level of income taxes in relation to the economy is higher, the saving ratio is lower, and financial wealth per adult is higher than in the initial steady state.

The dynamic effects of the fertility decline on real adult consumption per adult can be seen in Figure 12 (the curves are again reported as levels). This variable comes closer than any other variable in the model to being an indicator of economic welfare.[40] The P0, P1, and P3 simulations all show an initial rise during the third and fourth decades of the shock, a steep and protracted fall for the next eight decades, and then a long rise back toward baseline and the ultimate steady state. The P1 simulation ultimately returns to the same steady-state value observed before the shock occurred. The P3 simulation, however, ultimately settles at a permanent steady-state value somewhat *below* that in the initial baseline.[41]

5. *Effects of An Asymmetric Fertility Decline*

The preceding examination of domestic consequences of a fertility decline as they would occur in a closed economy provides a foundation for further analysis. But our primary interest is in situations where one part of the world economy experiences different shocks and different outcomes from those occurring elsewhere. Accordingly, we leave the "global" symmetric shock behind and now focus on an "asymmetric" shock in which the fertility decline and eventual subsequent recovery occur only in the ZZ economy. This asymmetric shock vividly demonstrates the importance of the openness of economies in adjusting to demographic shifts and in partially transmitting the consequences of such shifts to the rest of the world. We believe the discussion also provides analytical insights into the macroeconomic consequences for countries, such as Japan, that experience population aging more dramatically and at a faster pace than other industrial nations.

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40 As the demographic shock moves through the populations of the economies, the levels of the adult population and the effective labor force become much smaller than they otherwise would have been. The economy-wide aggregate levels of human wealth, financial wealth, output, consumption, and the aggregate capital stock thus all decline to markedly lower levels. Such economy-wide aggregates for macroeconomic variables, however, cannot be readily used to make normative or welfare judgments about the consequences of demographic shocks. Per capita or per adult measures of macroeconomic variables are likely to be a more useful focus for normative comparisons of pre-shock and post-shock outcomes.

41 Note also that the curves for P1 and P3 in Figure 12 lie everywhere below the curve for P0. This outcome again reflects the fact that lower private saving in the model with than without a pension system leads to a higher real interest rate, lower private capital accumulation, and hence lower economy-wide output and consumption per adult.

Figure 13 summarizes the asymmetric, open-economy variant of the illustrative shock. The paths for the child birth rate and population growth rates in the ZZ economy are identical in every way with the paths in Figure 3. In the US economy, the birth rate never changes from its initial value and US population growth rates remain unchanged throughout at 0.2 percent per year. The levels of the adult populations and effective labor forces in the two economies, shown in Figure 14, are dramatically and progressively different as the illustrative shock occurs in the ZZ but not in the US economy.

In the next charts that follow, we continue the preceding practice of showing curves for a given variable generated by three simulations characterized by the labels P0 (no pension system), P1 (continuously balanced pension system), and P3 (unbalanced pension trust fund with the fiscal authority permitting a sizable increase in its target debt ratio). Because the effects on a variable of course differ between the ZZ and US economies, the charts have altogether six curves -- 3 for the variable in each of the two economies. To help differentiate the curves, we use lines with relatively thicker weight for the three simulation paths for the variable in the ZZ economy and the paths for the P3 simulations carry small diamond markers. Each of the Figures 15-23 reports results in the form of deviations of the variable from baseline.

Figure 15 plots deviations of real interest rates from baseline for the ZZ and US economies. Apart from an initial transitory jump in the first year, the real interest rates are little changed during the first two decades of the shock (before the adult population and the labor force have yet experienced the consequences of the fertility decline). The cumulating sharp fall in the ZZ effective labor force thereafter leads to a progressively larger fall in the ZZ real interest rate. The extent of the fall in the ZZ real interest rate, however, is somewhat damped because of the ZZ economy's openness to the rest of the world. Over the longer run, the real interest rate rises well above baseline and then ultimately falls back to the new steady state. The dynamic path of the ZZ real interest rate is similar to the fluctuations in the global closed-economy case and occurs for the same reasons. The real interest rate in the US shows an analogous, but much smaller, decline than that in the ZZ economy.

The disincentives for private saving associated with a public pension system discussed in section 4 of course continue to be present in the ZZ economy for the asymmetric shock. The behavior of the ZZ private saving ratio, shown in Figure 16, thus is broadly similar to the closed-economy behavior seen in Figure 11 (though again the cyclical fluctuations are somewhat damped relative to the symmetric case). In the US economy which does not experience the demographic shock, on the other hand, private saving initially rises *above* baseline (for both variants of the pension system, and even when both countries have no pension system). The opposite effects on ZZ and US private saving can be traced to developments in the exchange rate and external-sector balances, to which we now turn.

The model enforces a variant of the uncovered interest parity condition. Hence an interest differential between the two economies -- once it opens up beginning in the third decade of the shock -- puts strong pressure on the real and nominal exchange rates (Figures 17 and 18). The ZZ currency begins a sustained appreciation, first in nominal terms, then with a lag in real terms.[42] By the end of the seventh decade of the shock, both the nominal and real values of the ZZ currency have appreciated by some 16-17 percent. The real exchange value of the ZZ currency appreciates substantially further over the next several decades, reaching a peak appreciation of more than 30 percent before reversing and falling back. In the new long-run steady state, both the nominal and the real exchange rates settle at levels at least 20 percent higher than in the baseline solution.

To understand why the asymmetric illustrative shock results in a real exchange rate permanently higher in the ultimate than in the initial steady state, recall that the fertility decline is transitory in terms of demographic *rates of growth* but has permanent effects on the *levels* of

[42] An upward movement in the exchange rate in the model represents an appreciation of the ZZ currency (a depreciation of the US currency).

demographic and macroeconomic variables. In particular, while the ZZ fertility rate remains below baseline during the shock, the ZZ population and effective labor force fall further and further below baseline. The ratio of the ZZ to the US effective labor force and the ratio of the two countries' populations fall correspondingly. Once the decline in the ZZ fertility rate is reversed, eventually the ZZ and US demographic rates of growth again become equal. The ratio of the ZZ to the US effective labor force, however, remains permanently smaller (see Figure 14). In the new long-run steady state, therefore, the quantity of ZZ-produced goods is smaller than the quantity of US-produced goods. Given unchanged preferences in each economy for the two types of goods, relative prices in the world economy must change. A permanent real appreciation of the ZZ currency (an improvement in the ZZ economy's real terms of trade) is an integral part of the required change in relative prices.

The dominant cause of the appreciation of the ZZ currency, in other words, is the fertility decline itself and its pervasive effects on macroeconomic variables (see the P0 simulation in Figures 17 and 18). Introduction of a public pension system in both countries unambiguously *reduces* the degree to which the ZZ currency appreciates.[43] When the pension system is unbalanced and the ZZ fiscal authority permits a substantial rise in its target debt ratio (P3 simulation), the *downward* effect on the size of the appreciation of the ZZ currency that would otherwise occur is especially noticeable.

Large changes in exchange rates generate powerful expenditure-switching incentives between the two economies. By the fourth decade of the shock, the ZZ economy thus begins to import substantially more of the now relatively cheap goods produced in the US. ZZ exports to the US relative to baseline are inhibited by the appreciation of the ZZ currency. The ratios of real

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[43] The public pension system is introduced in both countries; because the asymmetric demographic shock occurs only in the ZZ economy, however, severe pressures on the pension system occur in the ZZ economy but much less so in the US economy. For the P3 simulation, moreover, only the ZZ fiscal authority (but not the US fiscal authority) is assumed to raise its target debt ratio.

imports to real GDP in the two economies are shown in Figure 19. After the underlying demographic variables have begun to return to their ultimate steady-state values, the ZZ import ratio has risen about 5 percentage points above its baseline value (it ultimately falls back to some 3-4 percentage points above baseline). The US import ratio reaches a level more than 3-1/2 percentage points below baseline (ultimately coming to rest around 3 percentage points below baseline). As would be expected from the differential effects on the real and nominal exchange rates, the introduction of public pension systems *reduces* the degree to which the ZZ economy increases and the US economy cuts back imports.

For the initial decades of the shock, the ZZ real trade balance relative to real GDP exhibits a modest deficit and changes little. As the shock progresses after that point, however, the expenditure-switching effects associated with the currency appreciation permit the ZZ economy to run a larger and larger *deficit* on real trade account (Figure 20). This net import of real resources from abroad provides a cushion of support to the ZZ economy that permits it to sustain a significantly higher level of consumption than would otherwise be possible.

For the short and early medium runs, the trade deficit of the ZZ economy is associated with a deficit on current account (Figure 21). (The vertical scales for Figures 20 and 21, chosen to make it easier to identify the dynamic movements of the curves, are dissimilar; the range of the vertical distance in Figure 20 is 8 percentage points whereas it is only 2 percentage points in Figure 21.) The current-account deficit relative to nominal GDP grows for several decades; the deficit ratio is larger with the pension system in the model than without. The current-account deficit ratio in the short and early medium runs is largest of all for the P3 case. By the fourth decade of the shock, the ratio of the current-account balance to nominal GDP begins a sustained move in the opposite direction, toward surplus. Some 100 years into the shock, the current-account balance in fact shows an actual surplus (greatest without pensions, and least large for the P3 case). After reaching a peak surplus value, the ZZ current-account ratio thereafter falls back toward its eventual steady-state level. (Because of the stylized nature in the model of the two-region world economy, the external-balance positions of the US economy are, of course, the mirror image of those for the ZZ economy.) When the pension system is not in the model, the ZZ current-account ratio returns to a value of essentially zero. Introduction of the pension system in the P1 variant means that the ZZ current-account ratio ultimately settles at a long-run steady state of a small surplus. The ultimate steady state for the P3 variant is a small deficit.

The net foreign asset positions of the two economies, shown relative to nominal GDPs in Figure 22, are the integral over time of the current-account imbalances. Consistent with its initial decades of current-account deficit, the ZZ economy decumulates net foreign assets over an extended period stretching into the medium run. When pensions systems are excluded from the model, the relatively early medium-run move of the current-account balance into sizable surplus causes the ratio of net foreign assets to nominal GDP eventually to become positive and settle at a moderate surplus value in the ultimate steady state. The continuously-balanced pension systems of the P1 simulation are associated with current-account deficits that are always larger (or surpluses that are always smaller) relative to the P0 simulation. Hence it takes a very much longer time for the ZZ net-foreign-asset ratio to reach a positive value. In fact, even after 250 years the net-foreign-asset ratios in the P1 simulation have not yet approached their steady-state values. For the P3 case in which the ZZ fiscal authority permits a substantial increase in the ratio of ZZ government debt to the ZZ economy, the ZZ net-foreign-asset ratio *never* improves significantly after its fall in the medium run; the ultimate steady-state outcome is a sizable net-foreign-*liability* position.

Domestic variables in the ZZ and US economies are strongly influenced by the cross-border transactions between the two economies. The openness of the economies thus crucially affects the dynamic evolution of the consequences of the ZZ fertility decline. In particular, the US economy bears some of the brunt of the large demographic shock emanating from the ZZ economy.

Alternatively stated, because of the openness of the economy, ZZ domestic variables are partly cushioned from the full impacts of the fertility decline.

The presence or absence of pensions systems -- and the method of operation of the pension systems -- can have substantially different implications for the external sectors of the two economies. The capacity of the ZZ economy over the medium run and long run to adjust to demographic shocks, furthermore, is significantly shaped by the interactions of exchange-rate and external-sector developments with the operations of the pension systems. The differences between the P0, P1, and P3 cases in Figures 17-22 have already illustrated these points.

To carry the discussion further, consider the paths for consumption in the two economies. Figure 23a plots the percentage deviations from baseline for adult consumption per adult in both economies. Despite the fact that the demographic shock occurs in the ZZ economy but not in the Us economy, ZZ per adult consumption is actually *higher* than that in the US economy throughout the initial decades of the shock. Eventually, it is also much *higher* in the new long-run steady state. In the medium run, on the other hand, ZZ per adult consumption does fall substantially below the level of US per adult consumption. Perhaps the most striking result of all is the relative values of per adult consumption in the two economies in the very long run. Long-run ZZ per adult consumption is significantly *higher* than in the initial baseline, even though the demographic shock occurs only in the ZZ economy and causes the ZZ population and ZZ aggregate output to be significantly *below* baseline. US per adult consumption in the ultimate steady state is much *lower* than in the initial baseline despite higher US population and higher US aggregate output. The ZZ fertility decline, though ultimately reversed and a *transitory* change to demographic rates of growth, causes significant *permanent* changes to the steady-state values of per-adult macroeconomic variables.

From the perspective of the ZZ economy, the cushioning effect of the openness of the economy is a matter of first-order importance. Nothing can prevent the ZZ demographic shock from having major negative effects on ZZ aggregate output and consumption. The ZZ paths for aggregate consumption and aggregate real GDP in fact fall much further below baseline than those for the US. Yet the ZZ paths for those variables are significantly *above* the paths that would be experienced in the hypothetical case where the ZZ economy is completely closed and therefore unable to cushion its shock through transactions with the rest of the world. The openness of the economy works to mitigate the size of the negative effects on the aggregates. As seen in Figure 23a, moreover, the cushioning effects are substantial.

The cushioning effects for the ZZ economy cannot be directly attributed to the evolution of the ZZ net-foreign-asset position. As seen in Figure 22, the ZZ economy does not accumulate a large positive net asset position vis-à-vis the rest of the world, permitting it to earn large amounts of net investment income from abroad. In fact, with the P1 pension system in operation, the ZZ economy is still paying rather than receiving investment income from abroad after two centuries. With the P3 pension system, the net *payments* of investment income to foreigners are large. The cushioning effects on the ZZ economy, however, are attributable to the external sector of the economy in a more indirect sense. In particular, the permanent appreciation in the real value of the ZZ currency enables the ZZ economy to enjoy a large permanent improvement in its real terms of trade with the rest of the world.

We conclude this analysis of the asymmetric shock with brief comments on the distributional consequences. Significant distributional effects can be identified, both across countries and across generations.

The counterpart of the favorable cushioning effects of openness on the ZZ economy in the presence of the ZZ-originating demographic shock is an unfavorable sharing of the shock as appraised from the perspective of the US economy. The effects on US adult consumption per adult are unambiguously adverse relative to the no-shock baseline situation. This adverse outcome obtains for the short, the medium, and the long runs and for any variant of the pension systems. The least adverse outcome for the US occurs when the ZZ fiscal authority permits its budget deficit and debt issuance to accumulate (P3 case) rather than keeping its pension trust fund balanced (P1).

The distributional effects across generations within the ZZ economy are more subtle but also significant. As noted already, the effects on ZZ adult consumption per adult relative to the no-shock baseline are an *improvement* in the shorter run, a major protracted *worsening* in the medium run, and a large permanent *improvement* in the long run (Figure 23a). But the presence or absence of a pension system, and the choice of variant for operating the pension system, makes a noticeable difference in how various cohorts of ZZ adults will appraise the outcomes.

Contrast, for example, the evolutions of the ZZ economy in the P1 and P3 cases. As the fertility decline and subsequent population aging first influence the economy, the cohorts of ZZ workers then alive and consuming will be pleased if the ZZ fiscal and pension authorities are prepared to let the budget deficit roll on, financing the needed increases in pension benefits by issuing more government debt rather than by raising taxes (whether pension taxes or income taxes). So-called Ricardian equivalence does not hold in our model. Thus individuals working today will not raise their saving (lower their consumption) in full anticipation of higher taxes that ultimately will have to be paid in the future. They recognize that they may be dead by the time tax rates will be raised. Rather, today's workers will prefer to consume part of the windfall resulting from the fiscal authority's behavior today, albeit at the expense of future generations. Cohorts and generations are not altruistically linked in our model framework, and the welfare of generations in the future thus does not influence the current generation’s utility. Adult consumption per adult will therefore be higher in the shorter and medium runs when P3 rather than P1 is the pension policy.

When economic welfare is judged from the perspective of future generations, the appraisal goes in the other direction. Future generations would prefer today's government to keep the pension system continuously balanced by raising more taxes (pension or income) rather than by issuing more debt. Future generations are the cohorts that will have to bear the "debt burden" of higher shorter-run debt issues under the P3 policy. The less favorable effects for future generations from the P3 relative to the P1 pension policy are clearly visible in Figure 23a. To show that today's working cohorts are biased in the opposite way, preferring P3 to P1, Figure 23b has the same information as in Figure 23a except that it portrays the earlier years of the simulation more clearly by including fewer years on the horizontal axis. As can be seen, for the first six decades of the shock, the P3 simulation everywhere permits a higher level of per-adult consumption than when the P1 pension policy is in operation.

6. Combining the Effects of Youth Dependency and Public Pension Systems

In our earlier paper (Bryant-Faruqee-Velculescu (2001)), we introduced youth dependency and child support into our analysis. We argued that introduction of those aspects was essential for studying the full, general-equilibrium effects of demographic changes leading to long-run population aging. In that paper, we showed that the incorporation of youth dependency and child support into macroeconomic models can generate significantly different inferences about the economic behavior of countries linked to the global economy. The differences were seen to be especially important when economies experience demographic shocks that are asymmetric and country-specific.

The earlier paper demonstrated that youth dependency has crucial implications for both the steady state and the transitional dynamics of the domestic economy. The consumption-saving behavior of individual adults who provide in-vivo transfers to children is dramatically different from the behavior of identical individuals without financial responsibilities for child support. Lower fertility rates reduce the financial burden on existing adults. Resources are accordingly freed for additional consumption and saving. That reallocation of resources radically changes the transitional dynamics and the ultimate steady state of the economy compared to what it would otherwise be in an analysis that ignores children.

The earlier paper also demonstrated that when open economies experience asymmetric shocks that differ in timing and magnitude, the effects of youth dependency on consumption and saving can cause major changes in exchange rates and external-sector variables. A fertility decline tends to induce relatively higher saving, part of which goes into increased assets held abroad. Exchange rates, interest rates, the trade balance and the current account can all be powerfully influenced by changes in youth dependency. Thus inferences about the evolution of key macroeconomic variables in an open economy -- and judgments about policy implications and economic welfare -- can be critically influenced by whether or not children and child support are explicitly introduced into the analysis.

This paper has added to our earlier work by explicitly incorporating macroeconomic effects at the older end of the age distribution and by focusing on the support of elderly dependency through public pension systems. Unlike a decline in youth dependency that frees up economic resources to working-age adults, a rise in elderly dependency places additional demands on the resources of working adults. When a public pension system is in place, younger workers must be taxed to provide the resources that are transferred to the elderly recipients of pension benefits.

Although our research combining the effects of public pension systems and youth dependency is at an early stage, we conclude this paper with some preliminary results. In what follows, we use the same illustrative shock of a protracted but eventually reversed fertility decline analyzed earlier. The most interesting and relevant results are those from the asymmetric shock. So again we postulate that the fertility decline occurs only in the ZZ economy and takes exactly the form shown in Figures 13-14.

To clarify the differential contributions of public pensions and child support, we present a series of charts in Figures 24-30 all of which take a standard form. Each chart, pertaining to a single variable, shows four different curves, corresponding to four simulations with the model. The first simulation, labeled P0_C0, excludes the pension systems and likewise excludes youth dependency. This case is straightforwardly implemented by setting all the key variables in the pension system at zero ($\Downarrow_{ss,t} = \tau_{SS,t} = PTFGAP(P) = 0$) and simultaneously setting the level of child support and the age-distribution parameter for child support at zero (c_j = . = 0). The simulation is identical to the P0 simulation presented in section 5 and is repeated here to facilitate analytical comparisons.

A second simulation, referred to as P0_C1, inserts child support into the model but keeps the pension system excluded. Our paper on youth dependency demonstrated that alternative levels of child support and alternative allocations of the burden of child support across age cohorts of parents have significantly different implications for key macroeconomic variables. In the illustrative simulations including youth dependency in this paper, we err on the conservative side by choosing a level of child support, case C1, where each child receives 36 percent of initial baseline adult consumption ($c_j = 0.36$). This level of child support is one half the level of child support suggested by Cutler, Poterba, Sheiner and Summers (1990), who calculate that child consumption is approximately 72% of adult consumption, using estimated data from Lazear and Michael (1980), to which they add expenditures on public education. The 36% level is in the middle of the range of values investigated in our earlier paper. For the parameter governing the age distribution of child support, we also choose a value in the middle of the range investigated in the earlier paper (. = 0.05). A comparison of the P0_C1 and the P0_C0 simulations indicates the differential effect of adding just child support to the model.

The third simulation shown in Figures 24-30, indicated by P3_C0, incorporates the pension systems in the model but excludes youth dependency. We choose the P3 variant of the pension system for comparison in these charts. Results from this simulation have also already been shown in Figures 15-23 and discussed in section 5; the relevant curve is repeated here in the charts that follow. As before, comparison of the P3_C0 and P0_C0 simulations reveals the differential effect of putting the pension systems in the model and assuming that the ZZ fiscal authority responds to the population aging by permitting a doubling of its target debt ratio.

Finally, we show a P3_C1 simulation that combines the P3 pension case and the C1 youth dependency case. Comparison of this combined simulation with the P3_C0 and P0_C1 simulations indicates the net consequences of integrating both transfers to children and public transfers to the elderly into the model and allowing them to interact.

Consider first the nominal exchange rate, shown in Figure 24. The thinnest curve, without a marker, is the no-pensions, no-child-support case (P0_C0). The ZZ currency appreciates strongly, as discussed before, regardless of pensions or child support. The differential effect of introducing non-zero child support (P0_C1, darker curve with circle markers) is to substantially *increase* the degree of appreciation. Introduction of public pension systems (P3_C0, lighter curve with diamond markers) unambiguously *reduces* the degree to which the ZZ currency appreciates. When both child support and pensions interact together (P3_C1, heaviest curve with square markers), the two tend to offset each other and the degree of appreciation falls in between the cases with children alone or pensions alone.

The behavior of the exchange rate is proximately influenced, as always, by the interest differential between the two countries. The exchange value of the ZZ currency appreciates by the largest amount in the no-pensions, child-support case because cyclical fluctuations in interest rates are largest in that case. When child support is included in the model and children consume some part of the economy's output, a decline in ZZ youth dependency frees up resources that would otherwise have gone to ZZ child consumption. Accordingly there is less need to deplete the ZZ capital stock to maintain ZZ consumption; the marginal product of capital in the ZZ economy will be lower than otherwise, and hence the swings in the ZZ interest rate relative to the US interest rate are larger than would otherwise occur.

The pattern of differential effects evident in Figure 24 -- the introduction of child-support alone and pensions alone tending to push a variable in offsetting directions and the combined interaction of the two resulting in an intermediate, net outcome -- is fairly common with many of the macroeconomic variables. But interesting differences emerge as well. Figure 25 portrays the paths of ZZ real human wealth per adult in the four simulations. Introducing child support alone gives a large incremental boost to human wealth in the shorter and early medium runs but has less of a differential effect in the longer run. The introduction of pensions alone in the P3 variant has only a small differential effect in the shorter and early medium runs, but then substantially lowers per-adult human wealth in the longer run. The combination of child support and pensions together tends to track the P0_C1 case in the shorter run but then moves toward the P3_C0 case in the longer run.

We show the paths of ZZ real financial wealth per adult and the ratio of ZZ private saving to nominal GDP in Figures 26 and 27. When child support is included in the model, the resources incrementally freed up to working adults from the decline in fertility lead to substantially higher private saving and larger rises in financial wealth. As discussed earlier, the operation of the pension system with the P3 case induces higher private saving during the period when the ZZ fiscal authority permits its debt ratio to rise. The go-stop behavior of private saving in the P3_C0 case is mirrored in the P3_C1 case when youth dependency is also present, but the differential inducements to greater saving arising from the freeing up of resources from child support substantially raises the medium-run level of private saving in the combined case.

The offsetting pulls of the effects of youth dependency and elderly dependency are clearly evident in the effects on external-sector balances (Figures 28 and 29), just as they are in the effects on the exchange rate (Figure 24). Because of the freeing up of resources from child support, ZZ saving and adult consumption increase in the shorter run. Financial wealth rises significantly relative to what it would have been without the demographic shock; some of the incremental financial wealth is invested abroad earning the higher rates of interest paid on foreign assets. Hence the ZZ economy can earn a higher flow of investment income from abroad, offsetting a real trade deficit, and after several decades the ZZ economy can even run a substantial current-account surplus (P0_C1 case). The differential effects of public pensions, however, especially in the P3 variant, pushes the external-sector balances in the other direction; other things equal, the effects of the P3 pension system lead to significantly larger current-account deficits in the short and medium runs (and much smaller surpluses later on during the transitory period of surpluses). The combination of child support and pensions (P3_C1) thus produces a path for the current-balance ratio that most of the time tends to fall between the P0_C1 and P3_C0 simulations. The offsetting differential effects show up especially strongly in the evolution of the net-foreign-asset position (Figure 29) since these are the cumulation through time of the current-account imbalances.

The final chart returns again to adult consumption per adult (Figure 30). Our youth dependency paper emphasized that a fertility decline and lower youth dependency in the ZZ economy differentially permits a higher level of consumption per adult (and per capita) than would otherwise be possible. Section 5 above demonstrates that a higher level of elderly dependency supported by a public pension system differentially lowers the level of consumption per adult (which also means a lower level per capita). Thus the combination of lower youth dependency and higher elderly dependency again produce an intermediate outcome where the two opposing forces are partially offsetting.

7. Concluding Remarks

Our research demonstrates that transfers to elderly dependents through public pension systems, and alternative ways in which the pension systems are managed, can have large effects on the evolution of open economies. The discussion in section 6, together with our earlier paper on youth dependency, demonstrates that transfers to child dependents likewise can greatly influence the outcomes for open economies.

Three related conclusions stemming from our analysis of population aging merit special emphasis. First, transfers to the elderly through public pensions and transfers to children from parents often have offsetting effects on key macroeconomic variables. Second, lower child dependency ratios resulting from declines in fertility mitigate some of the negative consequences of population aging, reflected in higher elderly dependency ratios. Third, for an open economy asymmetrically experiencing fertility declines and population aging, negative consequences accompanying the demographic shifts are typically cushioned because the negative effects are shared with the rest of the world. That cushioning and sharing may not be desirable as seen from the perspective of foreigners, but it can produce sizable welfare gains for home residents.

These conclusions underscore the importance of analyzing dependency and transfers at both ends of the age distribution in an integrated way and with open-economy aspects fully taken into account. The widespread tendency to focus on the effects of population aging on pension systems and government budgets without considering the effects of fertility decline on child support and youth dependency can easily lead to incorrect inferences about the net economic effects of demographic changes on society. Ignoring the powerful macroeconomic effects working through exchange rates and cross-border transactions is also likely to provide an inaccurate assessment of the net impacts of demographic change.

Our earlier paper on youth dependency stressed that the particular way in which transfers to children are modeled can lead to different inferences about the consequences of a demographic shock. In a similar manner, we show in this paper that alternative ways of operating public pension systems and managing government debt can lead to substantially different macroeconomic outcomes, especially when the openness of economies is integrated into the analysis. For example, whether the fiscal authority in the shorter run increases taxes to prevent government debt from accumulating or alternatively allows deficits and debt to burgeon will have quite different distributional effects across generations and will have sharply contrasting implications for the long run net-foreign-asset position of the economy.

The net economic consequences of population aging for any individual nation will be importantly conditioned by idiosyncratic structural features of its economy, by the intensity and timing of its demographic changes relative to those in other countries, by the mechanisms and magnitudes of its transfers to both its children and its elderly, and by the nature and degree of its openness to the global economy. Our research has not yet progressed to the point where we can accurately gauge the net effects of population aging for particular countries. But our work has identified potentially important aspects of demographic change that have been previously neglected or understudied. And we like to believe that our approach takes significant steps toward a more unified and appropriate analytical framework.

The research so far, described in this and earlier papers, has numerous limitations. Much further work is needed to refine it. We plan refinements along several dimensions. We are confident that we can improve aspects of how we model child support, for example by linking child consumption more plausibly to adult consumption and by implementing a more realistic, hump-shaped age-distribution of child support payments among adults.[44] We plan to explore other, more

[44] These refinements are discussed in Bryant, Faruqee, and Velculescu (2001); see especially the appendix to that paper.

realistic variants of public pension arrangements that would allow for advance funding of pension benefits or that would permit pension taxes or benefits (or both) to gradually adjust over time in response to demographic developments that buffet the pension system. Much further analysis is warranted on alternative specifications of intertemporal fiscal closure rules and how they interact with public pension arrangements.

In future installments of the research, we plan to analyze a variety of permanent as well as transitory demographic shocks, in both symmetric-global and asymmetric-country-specific forms. We also want to study the differential consequences of activating other dimensions of our model not used in the simulations reported in this paper, such as allowing for positive growth in labor productivity, for some fraction of consumption that is borrowing-constrained, and for an intertemporal elasticity of substitution that is lower than unity.

In the months to come, we hope that this paper, our earlier papers, and the subsequent revisions in our research will shed new light on the complexities of demographic change. We look forward to sharing our research results with colleagues, inspiring other researchers to work on these fascinating and important issues, learning from their work, and generating new insights with them collaboratively.

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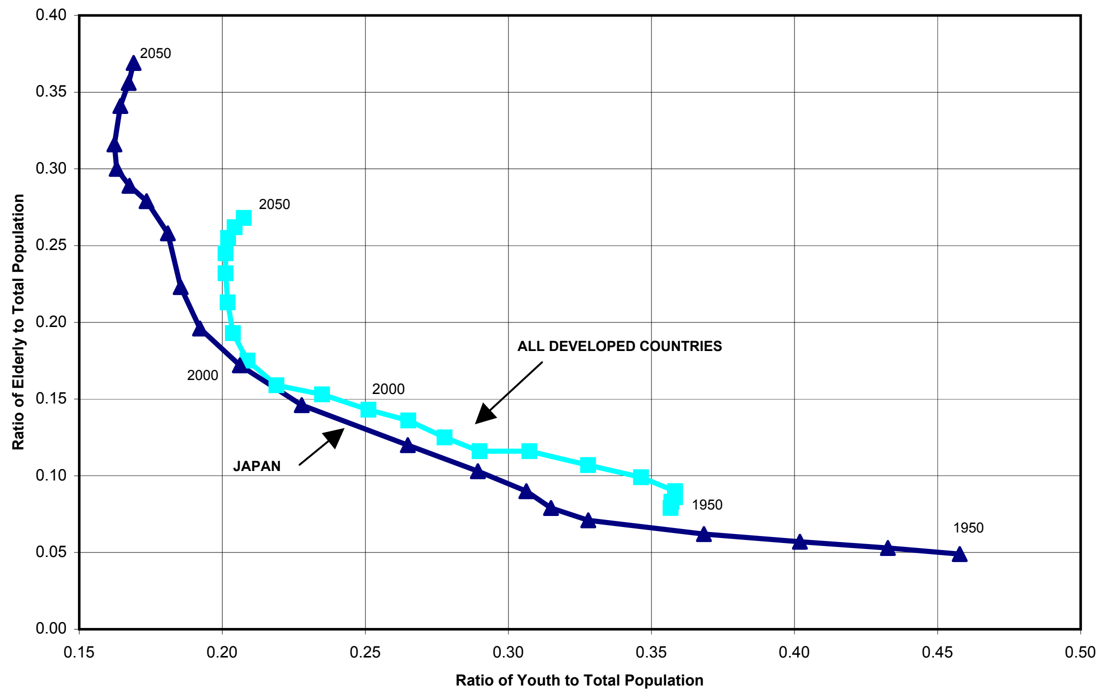
Figure 1. Evolution of Youth Dependency and Elderly Dependency: Japan versus All Developed Countries (1950-2050)



Source: United Nations Population Division. The Sex and Age Distribution of Populations, 2000 Revision.
Youth: Ages 1 through 19, Elderly: Ages 65 yeass and older.

Figure 2. Age Earnings Profiles, Japanese Data 1970-1997

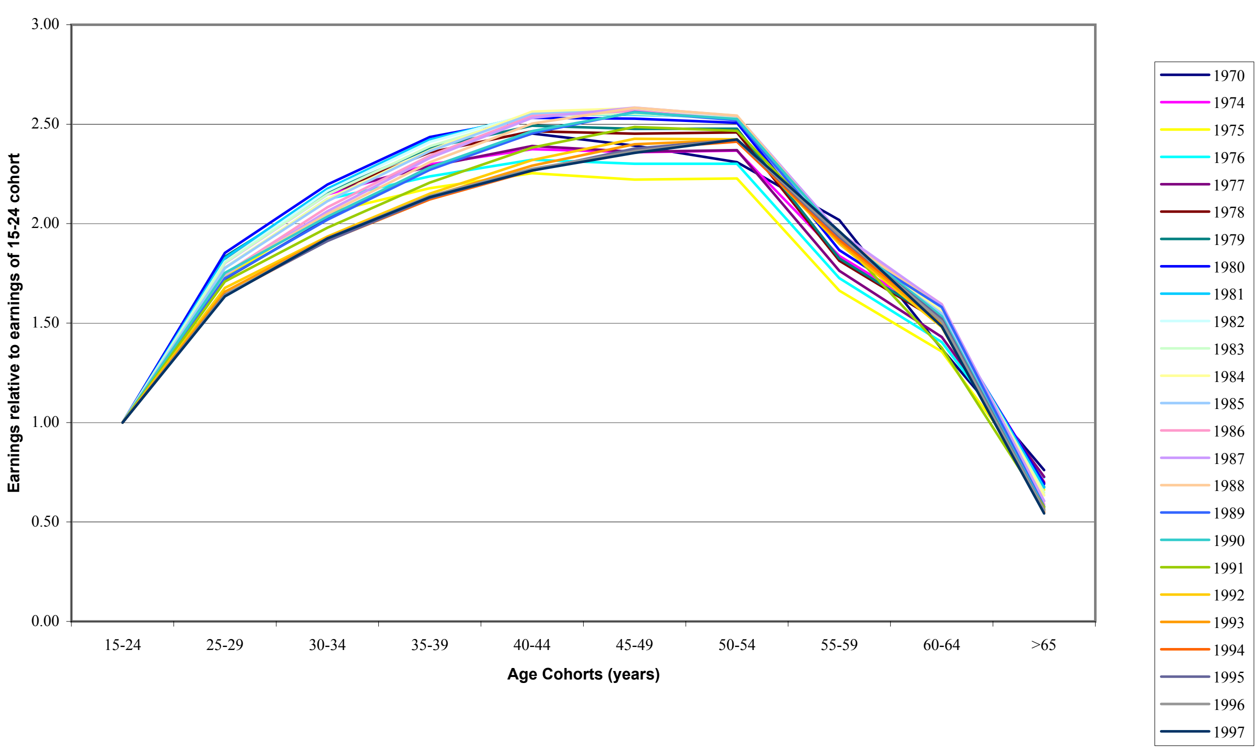


Figure 3. Illustrative Demographic Shock:
Major, Sustained Decline in Fertility Rate (eventually reversed)
"Global" (Both Regions the Same, "Closed-Economy")

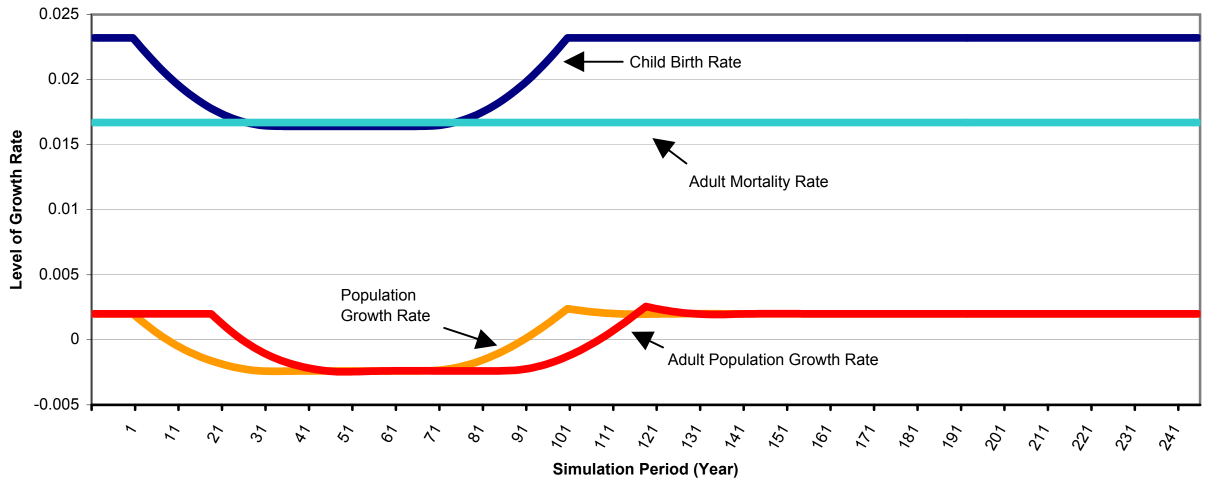


Figure 4. Elderly Dependency Ratio (Elderly as fraction of Adult Population)
"Global" (Both Regions the Same, "Closed-Economy")

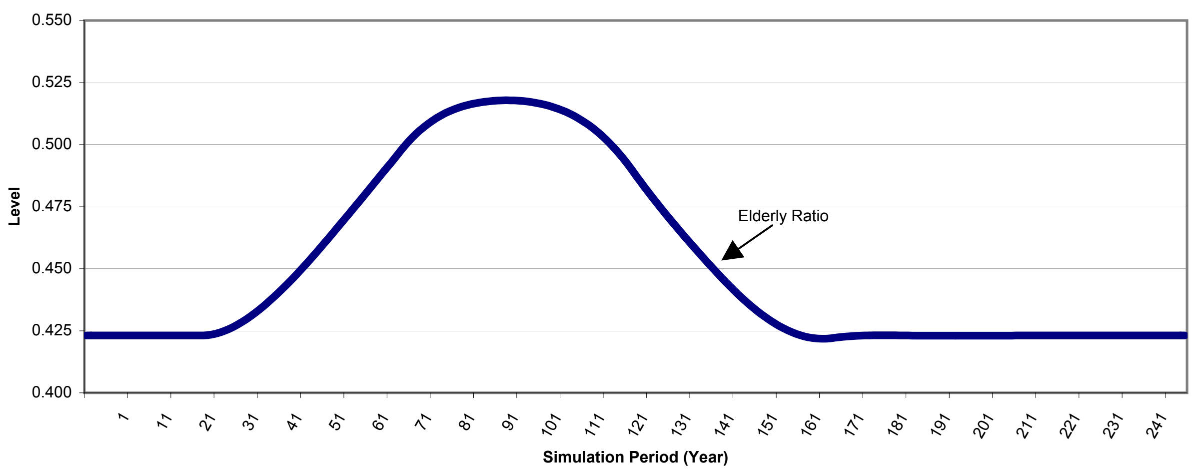


Figure 5. Adult Population, Total Population, and Effective Labor Force
Global Shock (Both Regions the Same, Closed-Economy)

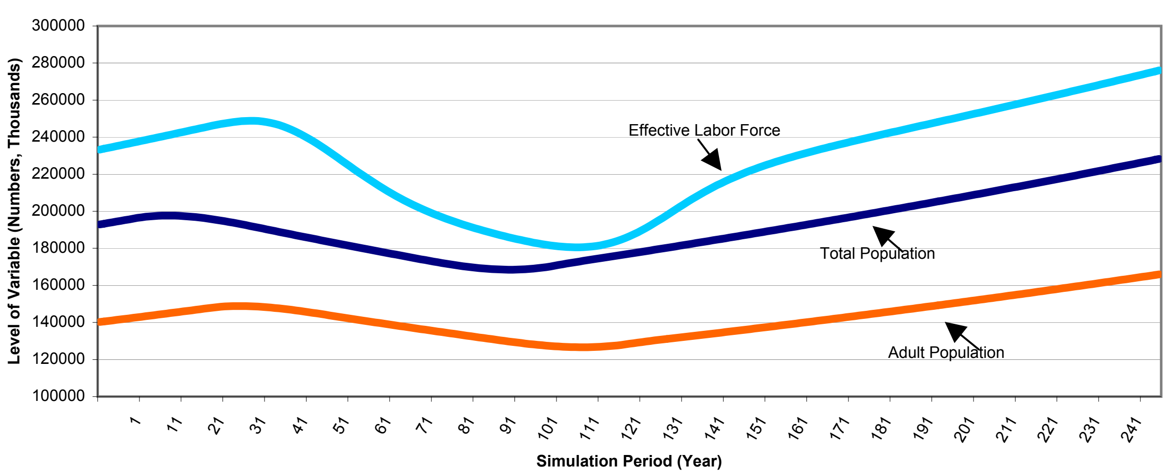


Figure 6. Endogenous Changes in Tax Rates in P1, P2, and P3 Simulations
Global Shock (Both Regions the Same, Closed-Economy)

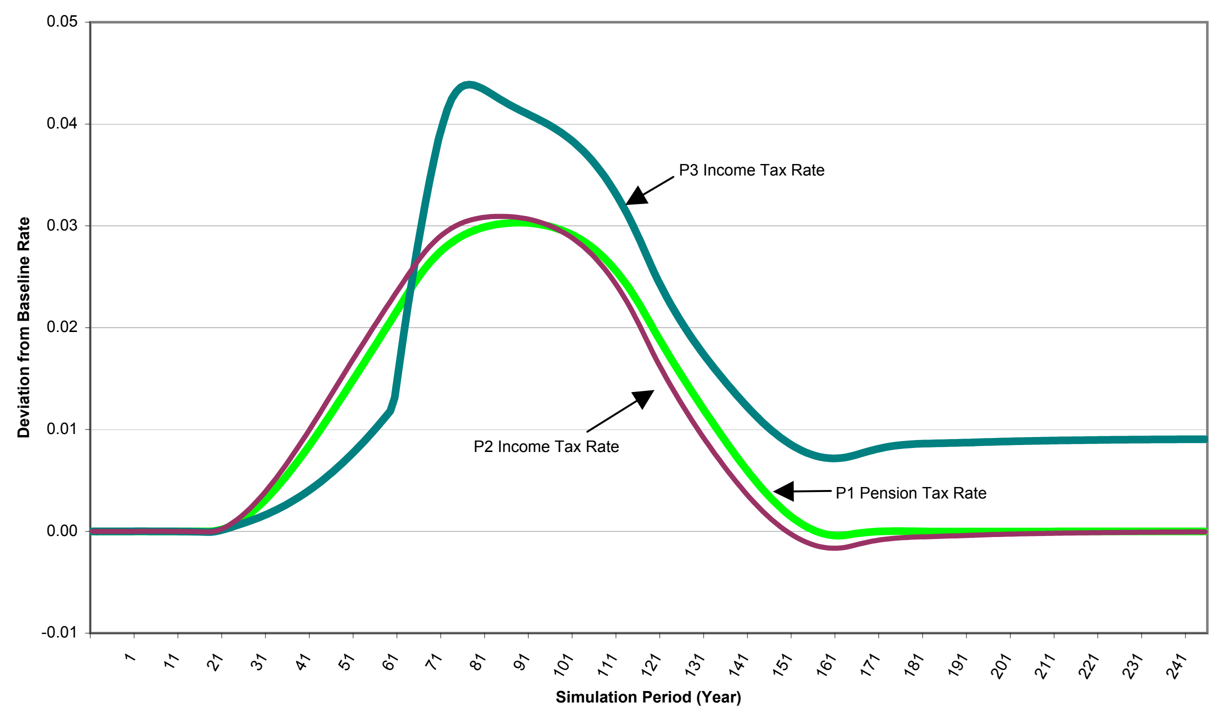


Figure 7. Real Short-Term Interest Rate (Level)
Symmetric Global Shock ("Closed-Economy," Both Regions the Same)

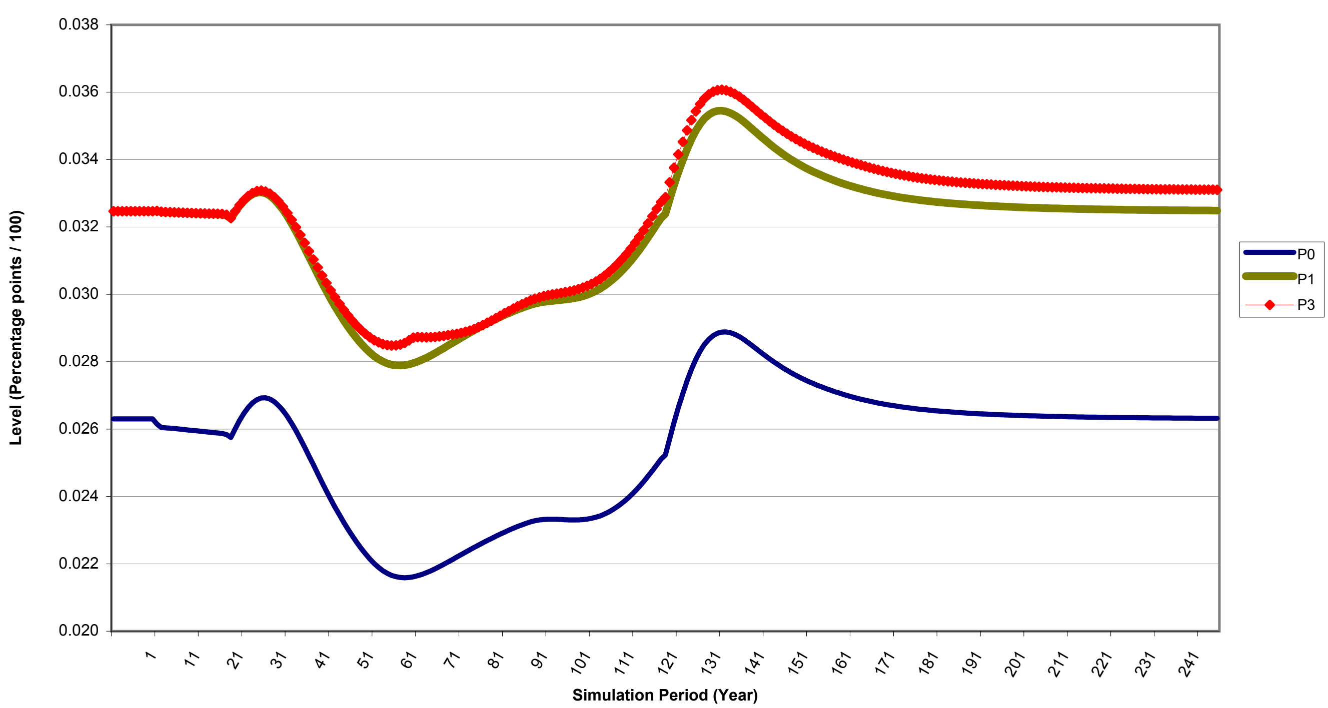


Figure 8. Capital-Output Ratio (Level)
Symmetric Global Shock ("Closed-Economy," Both Regions the Same)

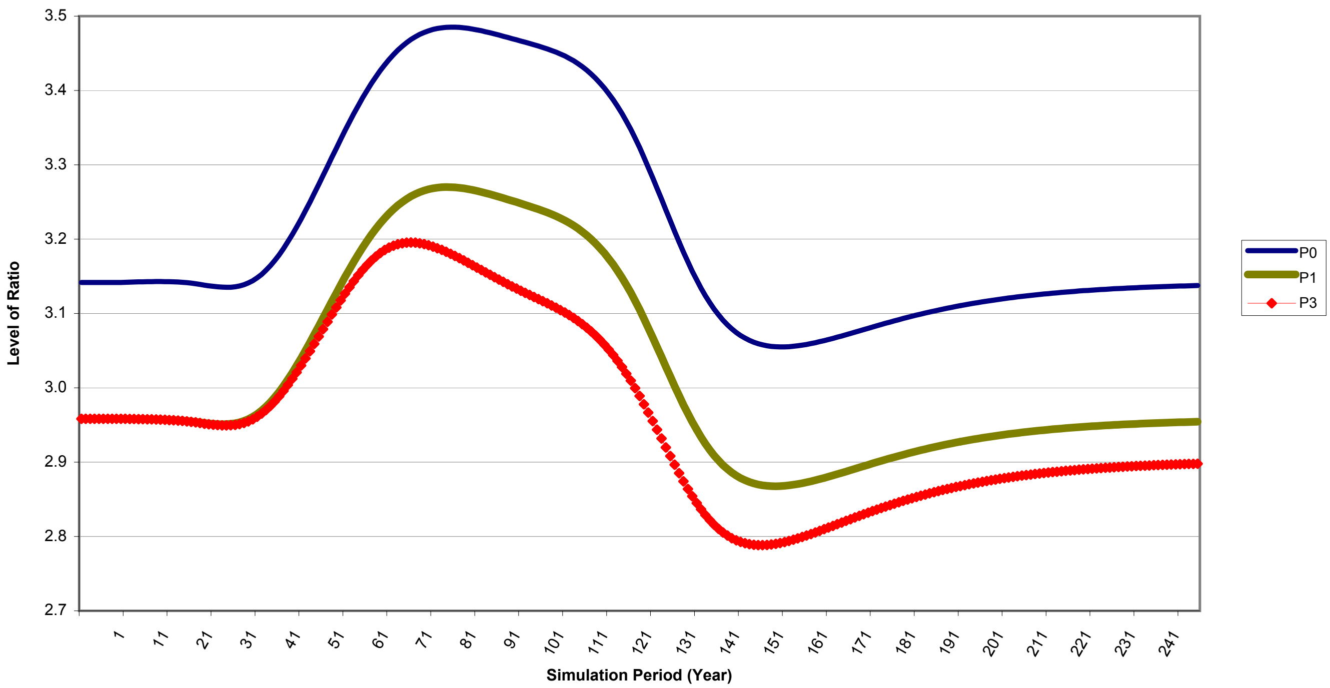


Figure 9. Real Human Wealth Per Adult (Level)
Symmetric Global Shock ("Closed-Economy," Both Regions the Same)

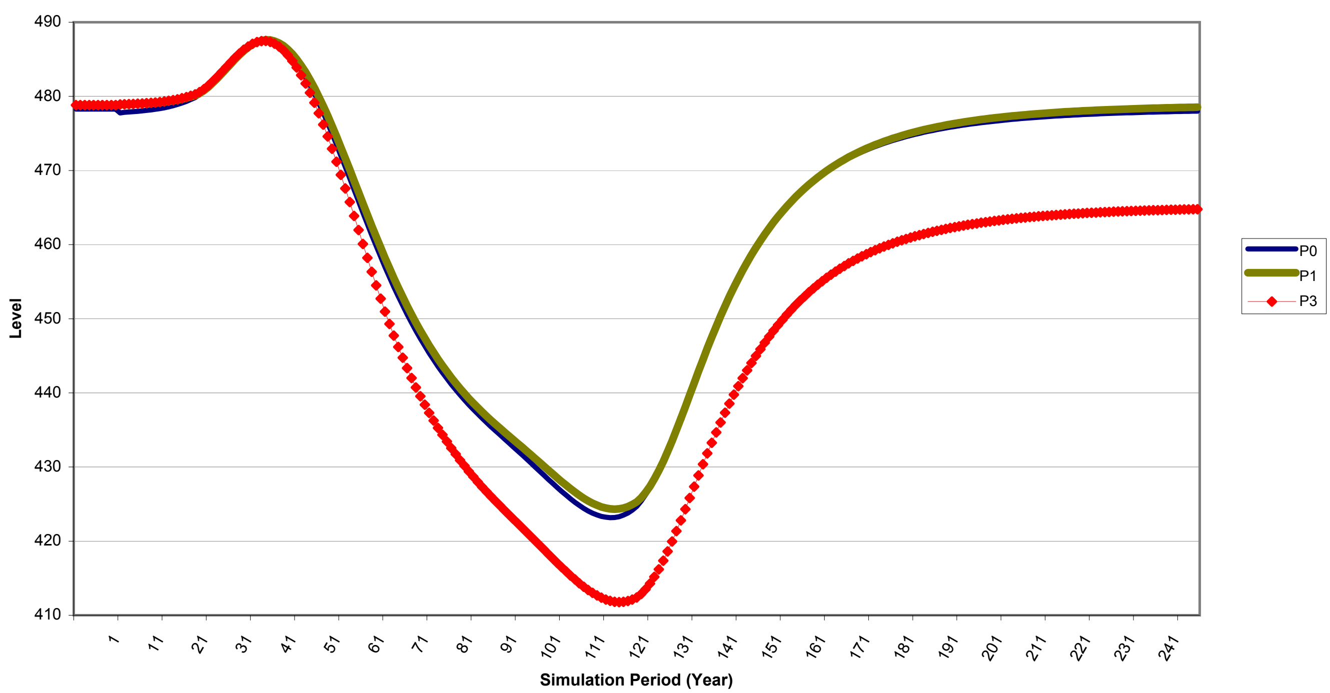


Figure 10. Real Financial Wealth Per Adult (Level)
Symmetric Global Shock ("Closed-Economy," Both Regions the Same)

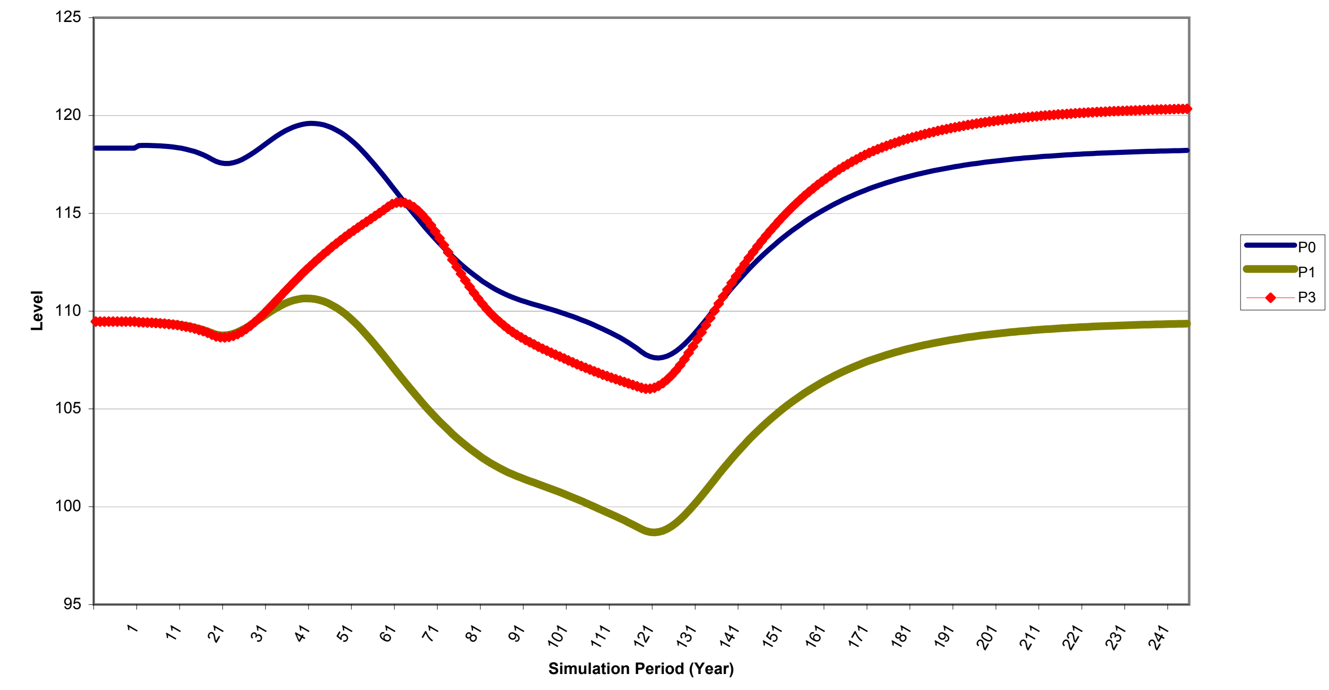


Figure 11. Ratio of Nominal Private Savings to Nominal GDP
Symmetric Global Shock ("Closed-Economy," Both Regions the Same)

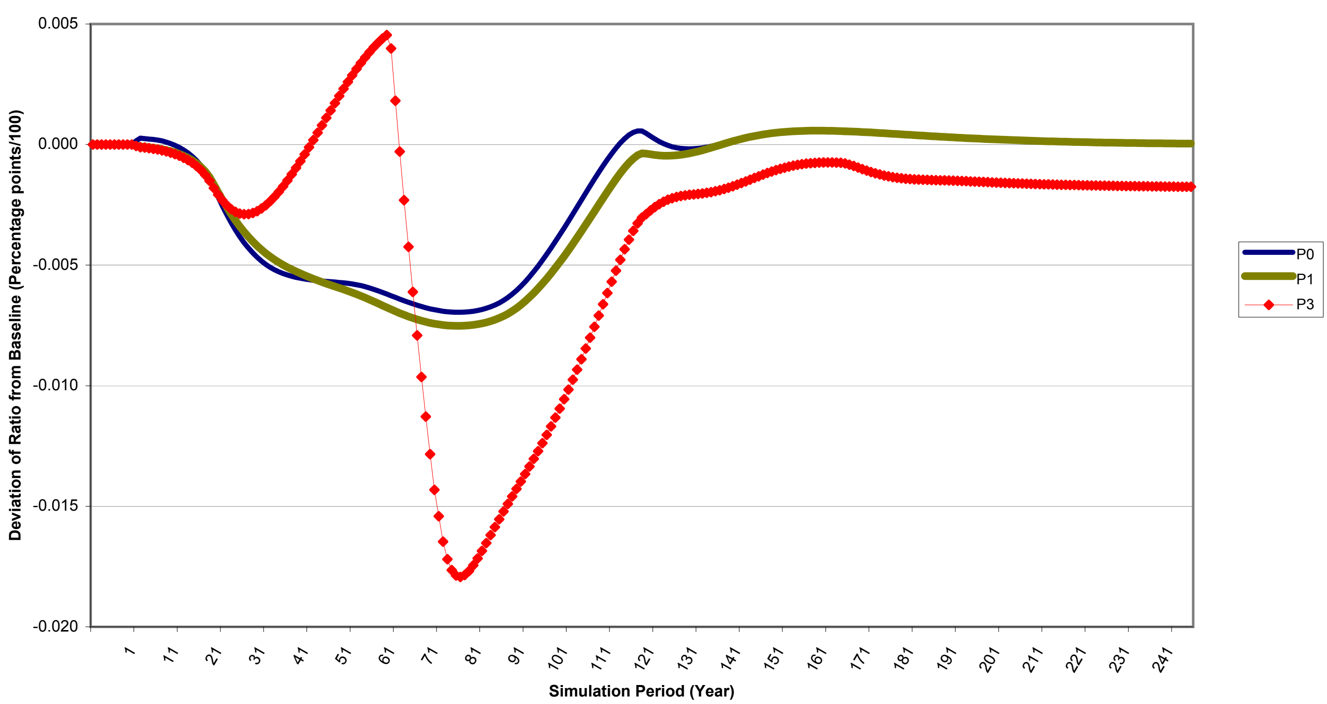
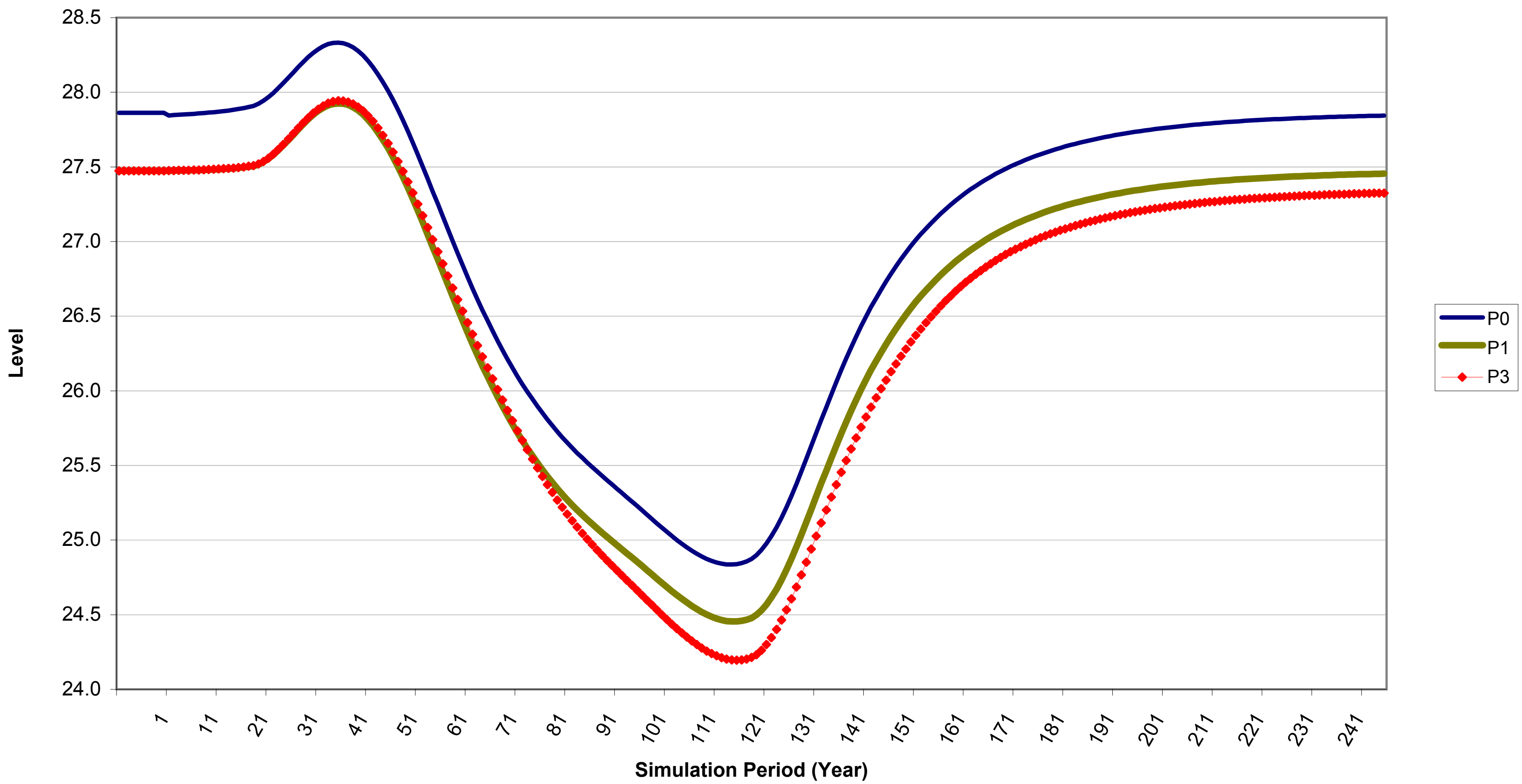


Figure 12. Real Adult Consumption per Adult (Level)

Symmetric Global Shock ("Closed-Economy," Both Regions the Same)



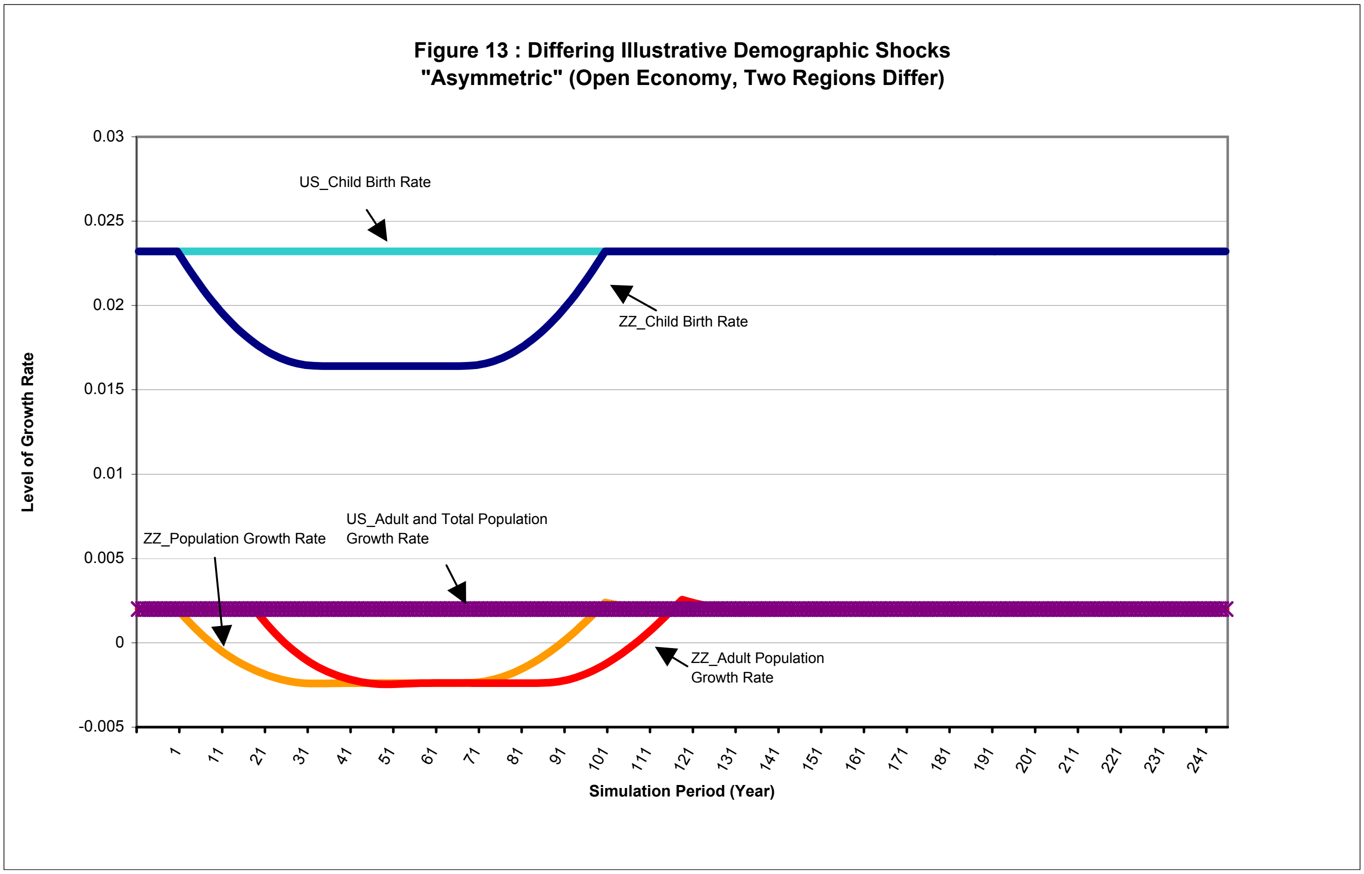


Figure 13 : Differing Illustrative Demographic Shocks
"Asymmetric" (Open Economy, Two Regions Differ)
US_Child Birth Rate
ZZ_Child Birth Rate
ZZ_Population Growth Rate
US_Adult and Total Population
Growth Rate
ZZ_Adult Population
Growth Rate
Level of Growth Rate
Simulation Period (Year)

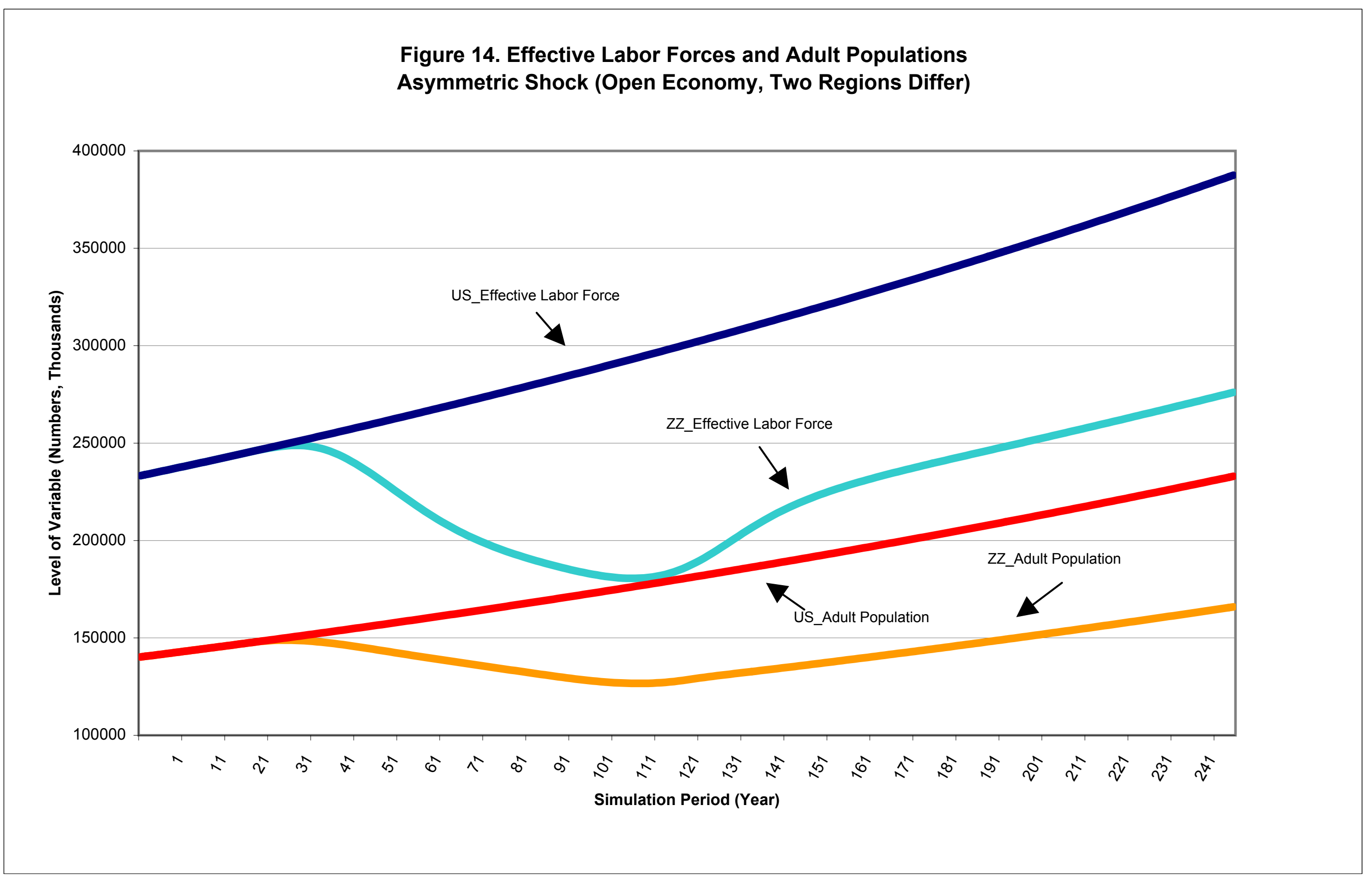


Figure 14. Effective Labor Forces and Adult Populations
Asymmetric Shock (Open Economy, Two Regions Differ)
US_Effective Labor Force
ZZ_Effective Labor Force
ZZ_Adult Population
US_Adult Population
Level of Variable (Numbers, Thousands)
Simulation Period (Year)

Figure 15. Real Short-Term Interest Rate
Asymmetric Shock (Open-Economy, Two Regions Differ)

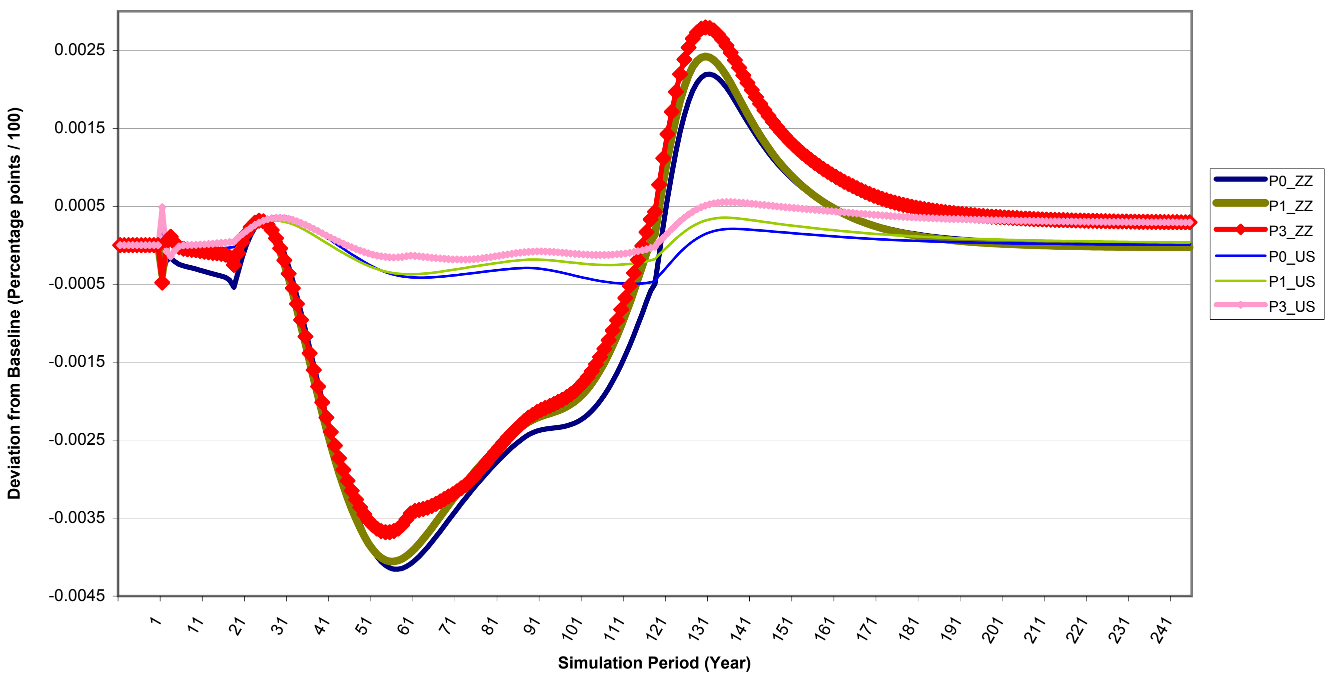


Figure 16. Ratio of Nominal Private Savings to Nominal GDP
Asymmetric Shock (Open-Economy, Two Regions Differ)

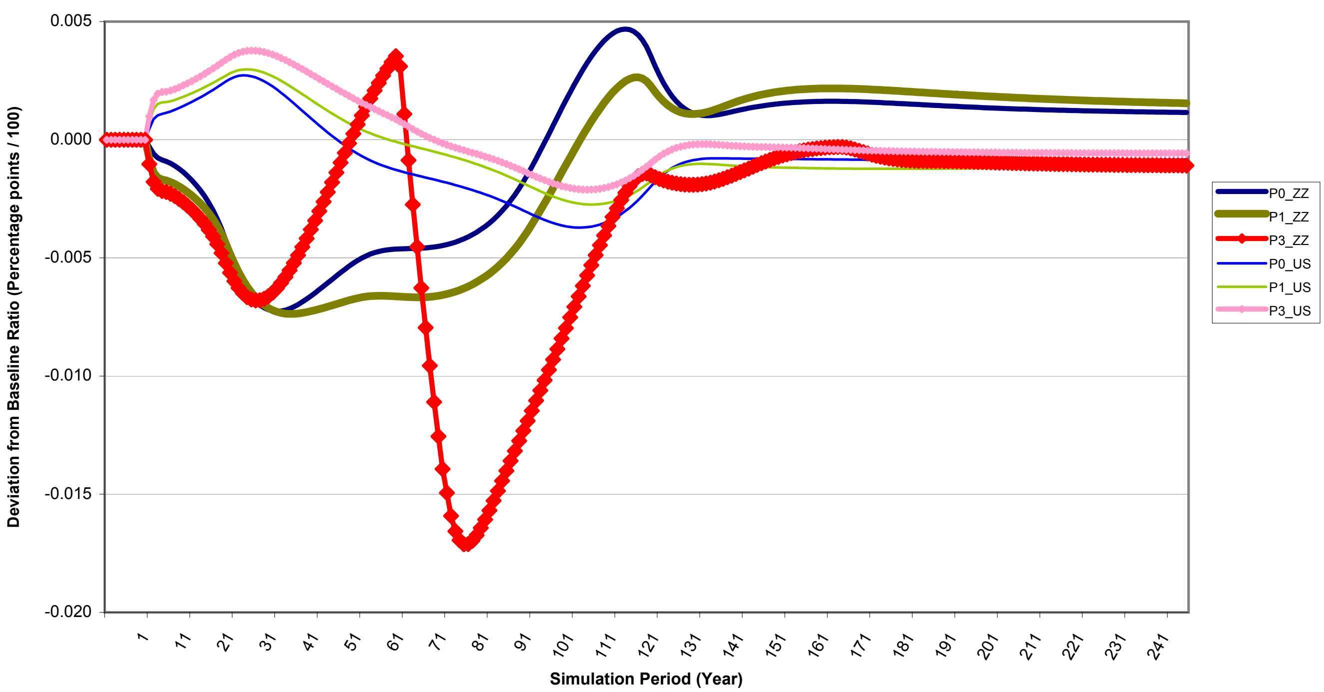


Figure 17. Nominal Exchange Rate
Asymmetric Shock (Open-Economy, Two Regions Differ)

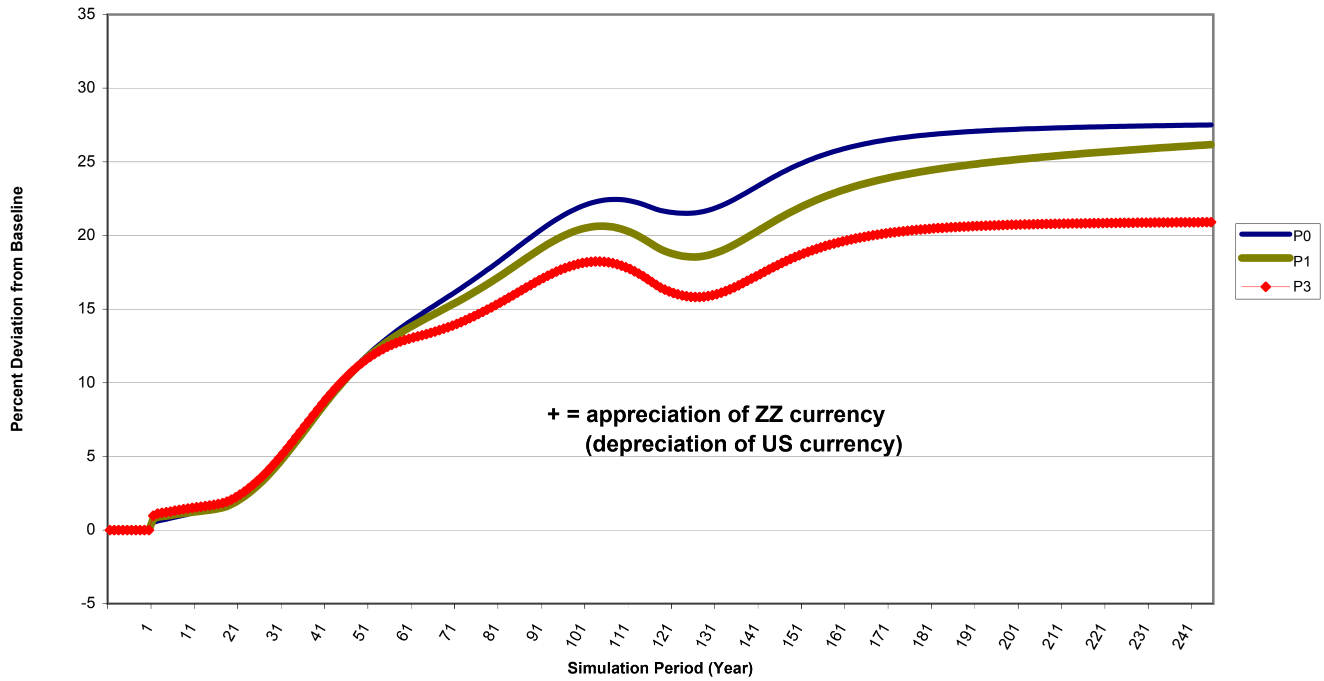


Figure 18. Real Exchange Rate
Asymmetric Shock (Open-Economy, Two Regions Differ)

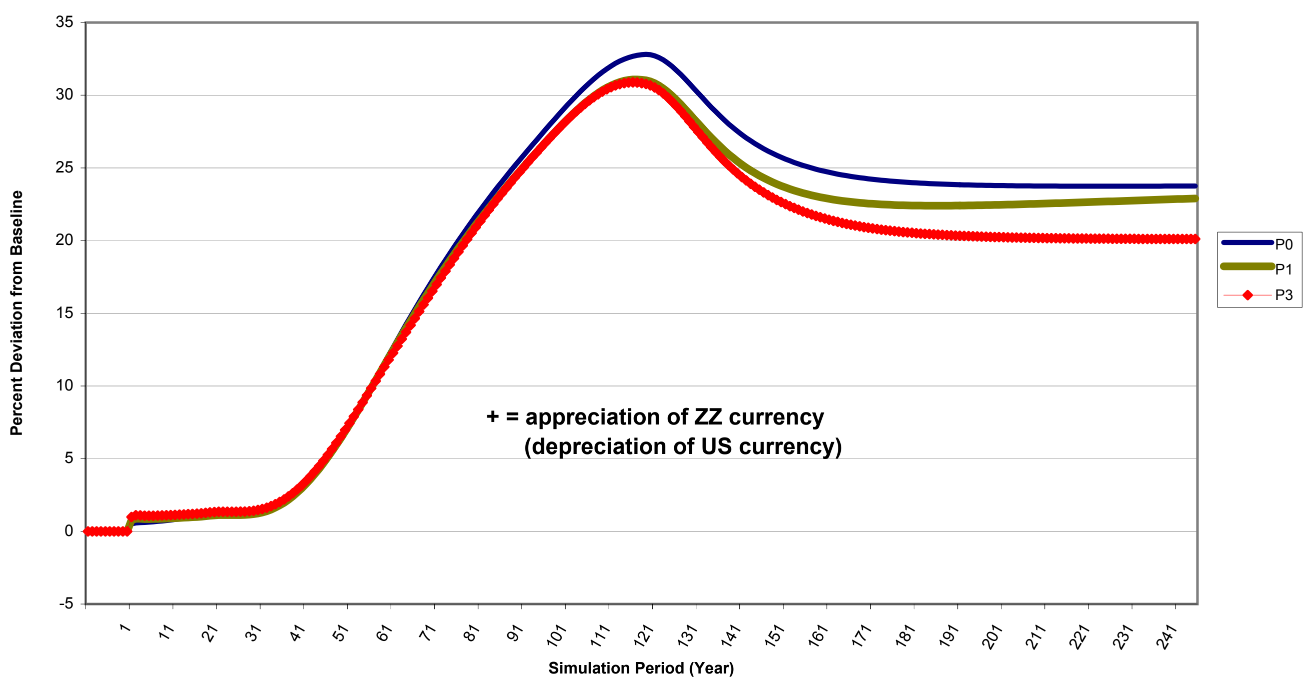


Figure 19. Ratio of Real Imports to Real GDP
Asymmetric Shock (Open-Economy, Two Regions Differ)

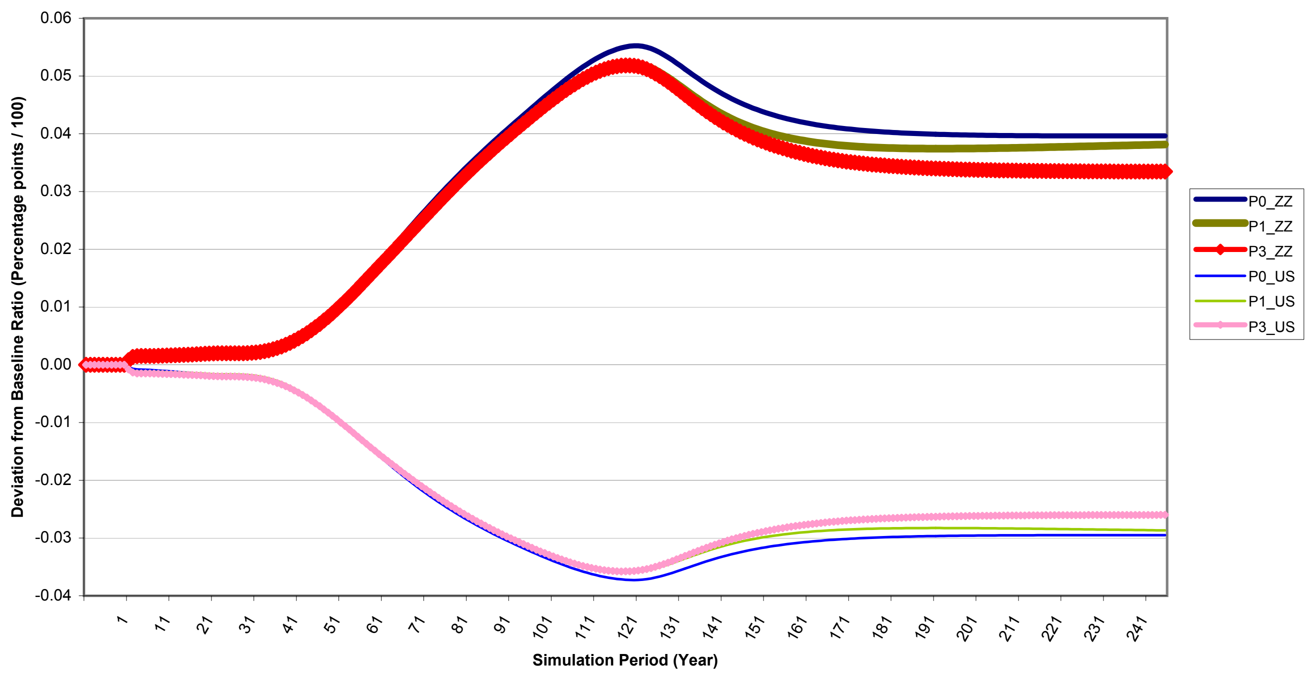
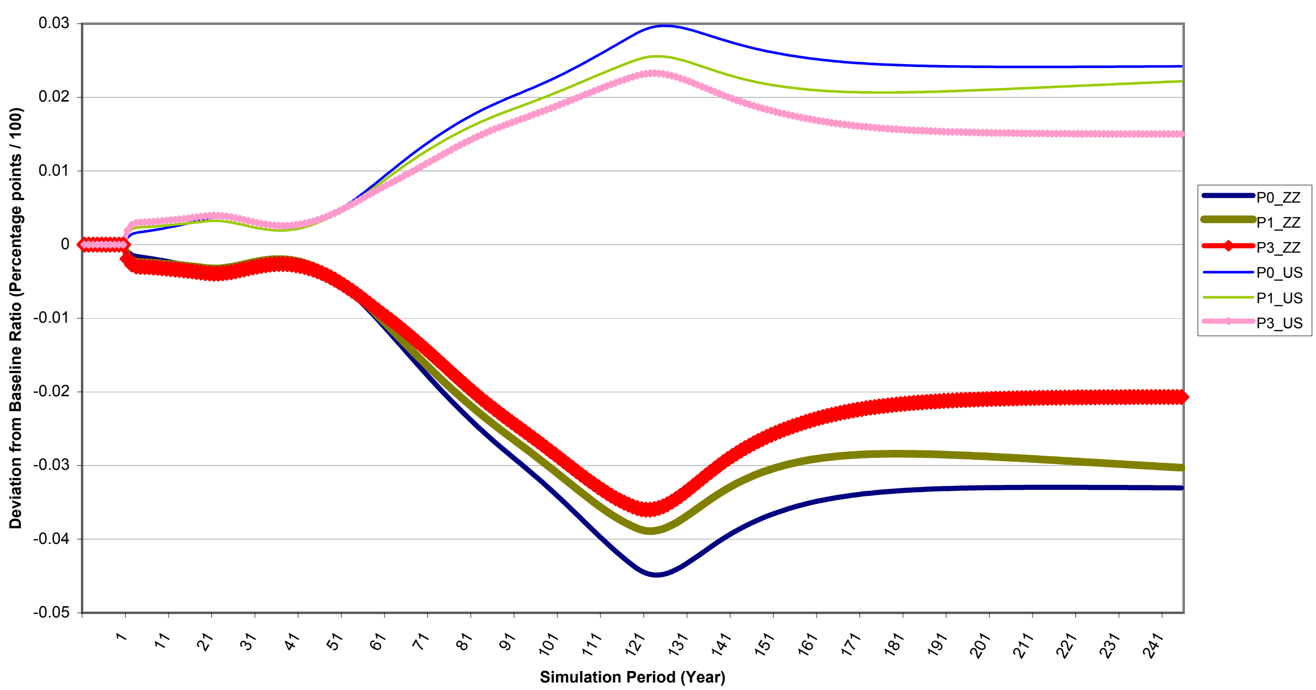


Figure 20. Ratio of Real Trade Balance to Real GDP
Asymmetric Shock (Open-Economy, Two Regions Differ)



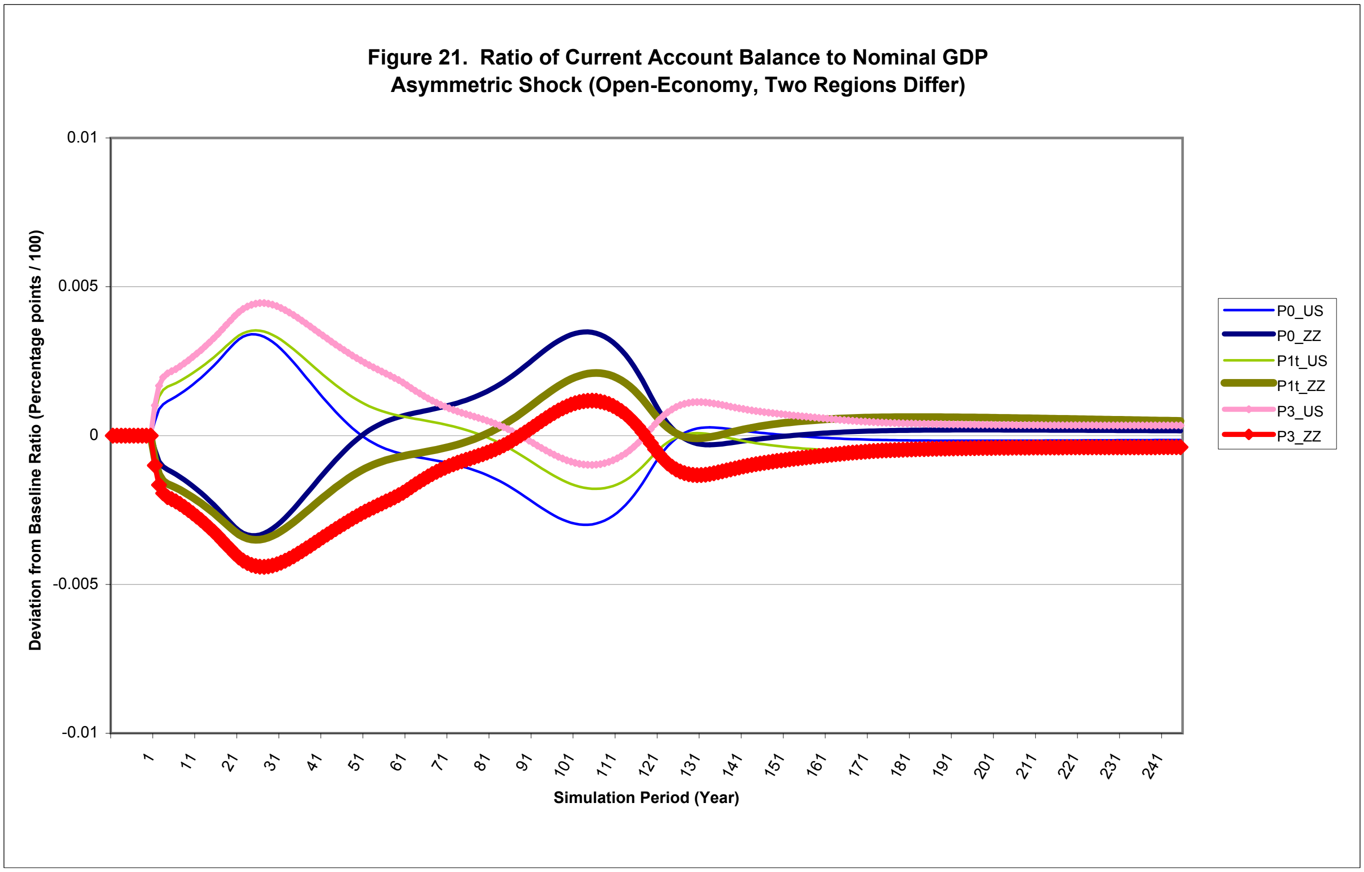


Figure 21. Ratio of Current Account Balance to Nominal GDP
Asymmetric Shock (Open-Economy, Two Regions Differ)
Deviation from Baseline Ratio (Percentage points / 100)
0.01
0.005
0
-0.005
-0.01
1
11
21
31
41
51
61
71
81
91
101
111
121
131
141
151
161
171
181
191
201
211
221
231
241
Simulation Period (Year)
P0_US
P0_ZZ
P1t_US
P1t_ZZ
P3_US
P3_ZZ

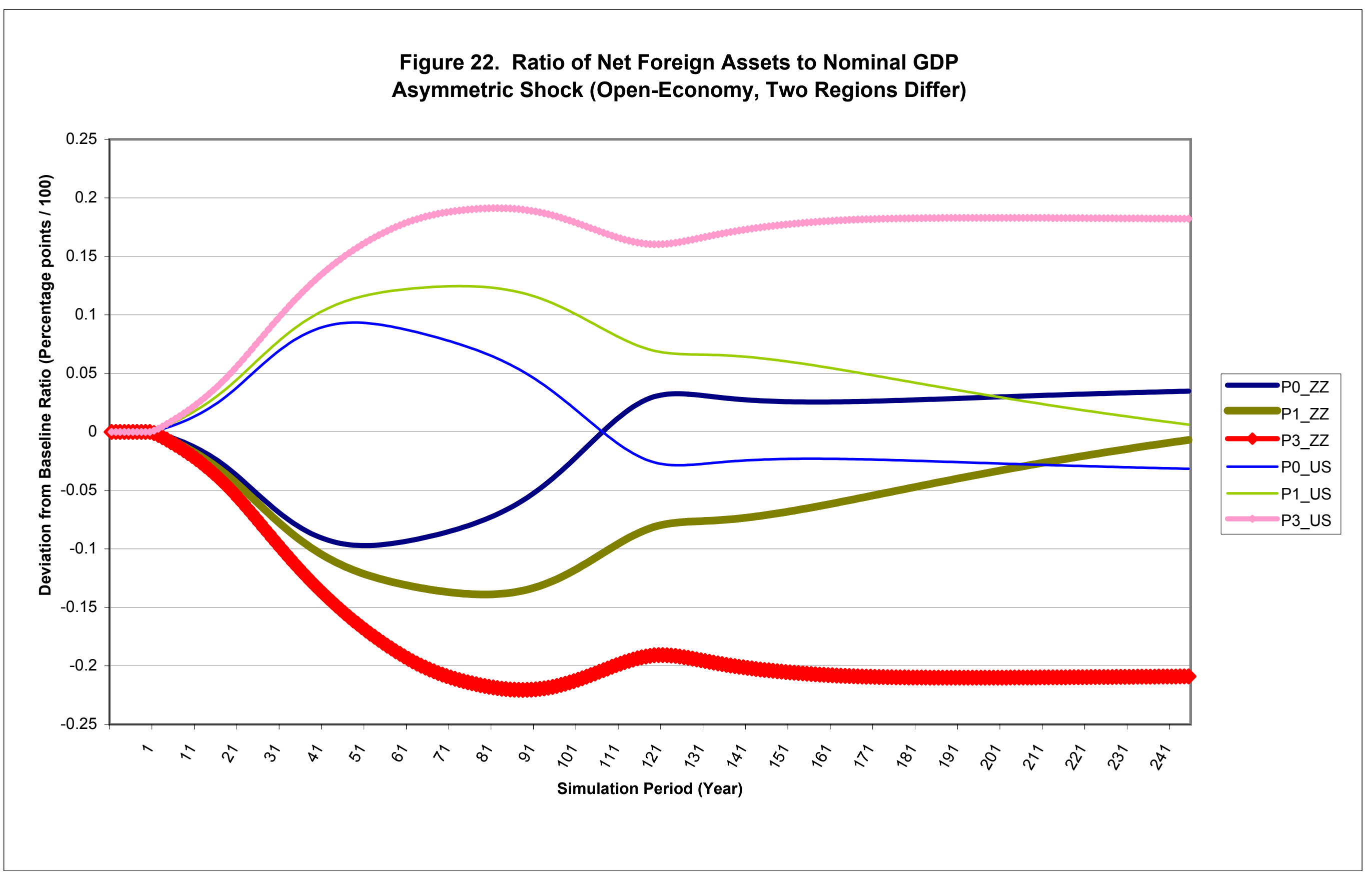


Figure 22. Ratio of Net Foreign Assets to Nominal GDP
Asymmetric Shock (Open-Economy, Two Regions Differ)
Deviation from Baseline Ratio (Percentage points / 100)
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
1
11
21
31
41
51
61
71
81
91
101
111
121
131
141
151
161
171
181
191
201
211
221
231
241
Simulation Period (Year)
P0_ZZ
P1_ZZ
P3_ZZ
P0_US
P1_US
P3_US

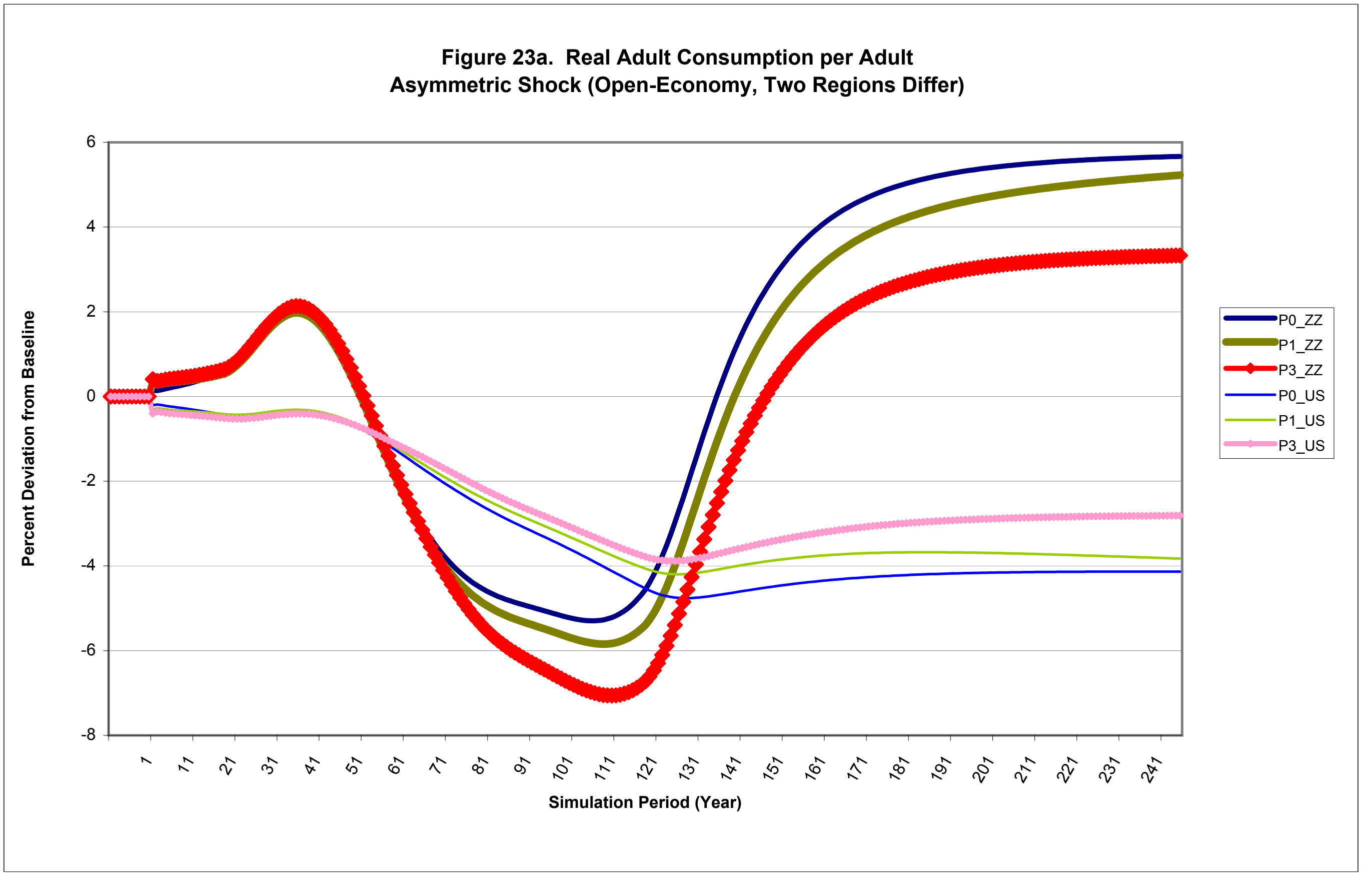


Figure 23a. Real Adult Consumption per Adult
Asymmetric Shock (Open-Economy, Two Regions Differ)
Percent Deviation from Baseline
6
4
2
0
-2
-4
-6
-8
1
11
21
31
41
51
61
71
81
91
101
111
121
131
141
151
161
171
181
191
201
211
221
231
241
Simulation Period (Year)
P0_ZZ
P1_ZZ
P3_ZZ
P0_US
P1_US
P3_US

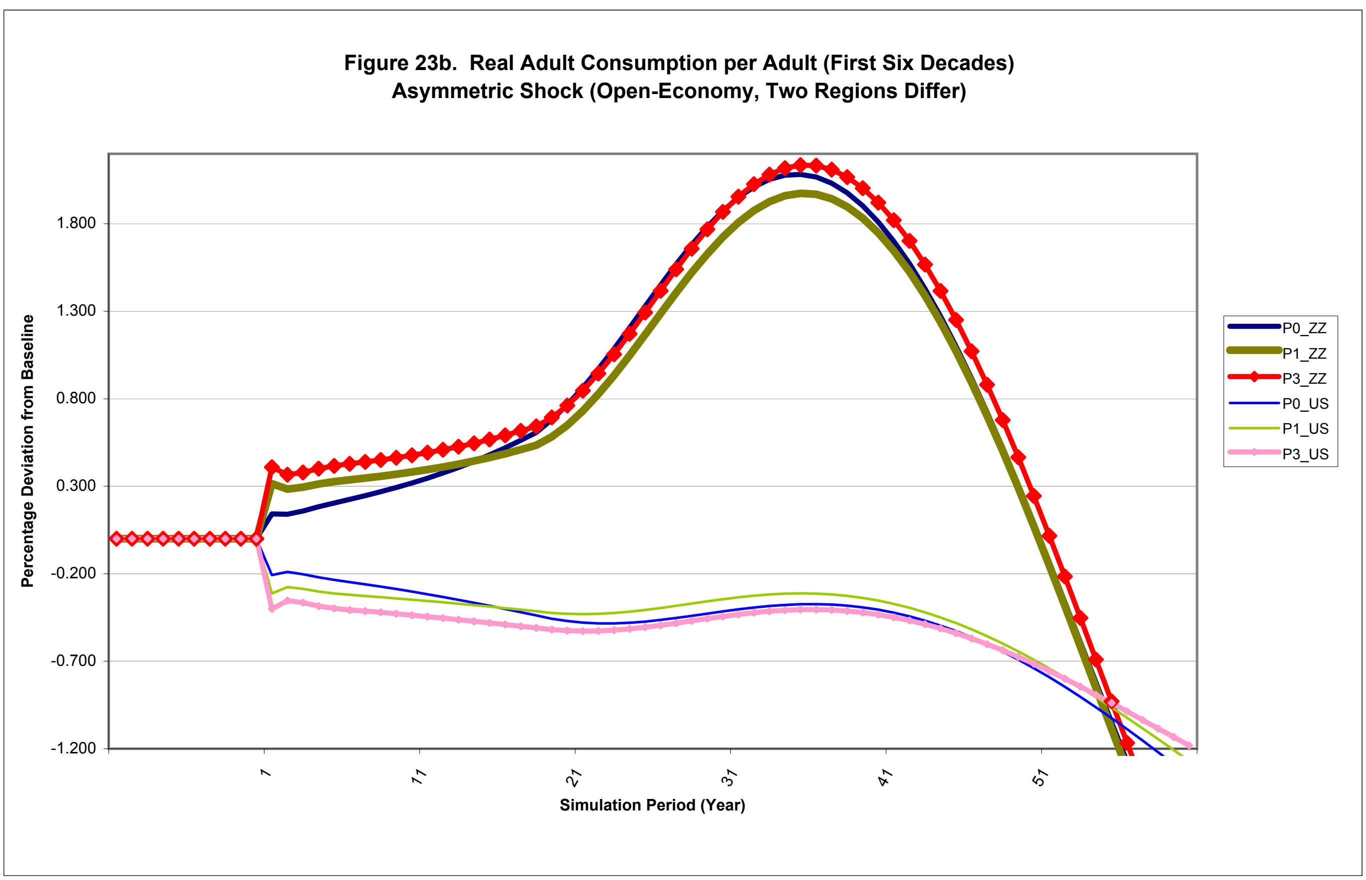


Figure 23b. Real Adult Consumption per Adult (First Six Decades)
Asymmetric Shock (Open-Economy, Two Regions Differ)
Percentage Deviation from Baseline
1.800
1.300
0.800
0.300
-0.200
-0.700
-1.200
1
11
21
31
41
51
Simulation Period (Year)
P0_ZZ
P1_ZZ
P3_ZZ
P0_US
P1_US
P3_US

Figure 24. Nominal Exchange Rate
Pensions and Youth Dependency Excluded and Included
Asymmetric Shock (Open-Economy, Two Regions Differ)

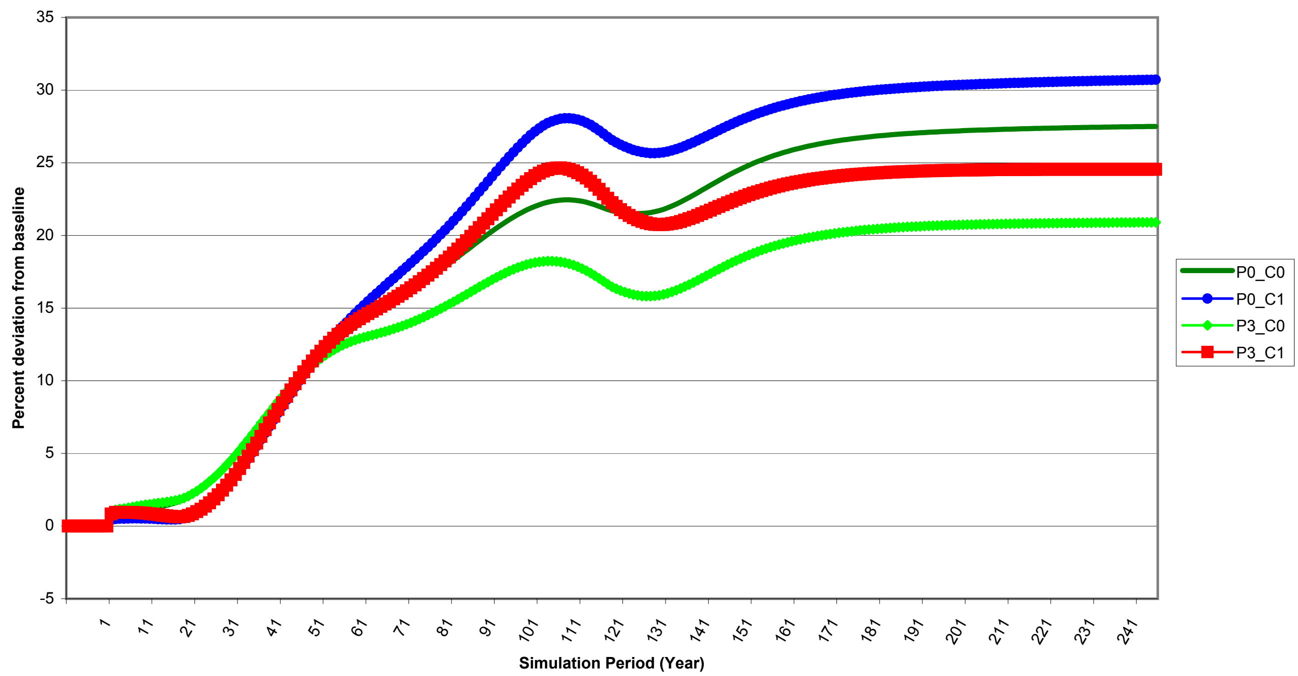


Figure 25. ZZ Real Human Wealth Per Adult
Pensions and Youth Dependency Excluded and Included
Asymmetric Shock (Open-Economy, Two Regions Differ)

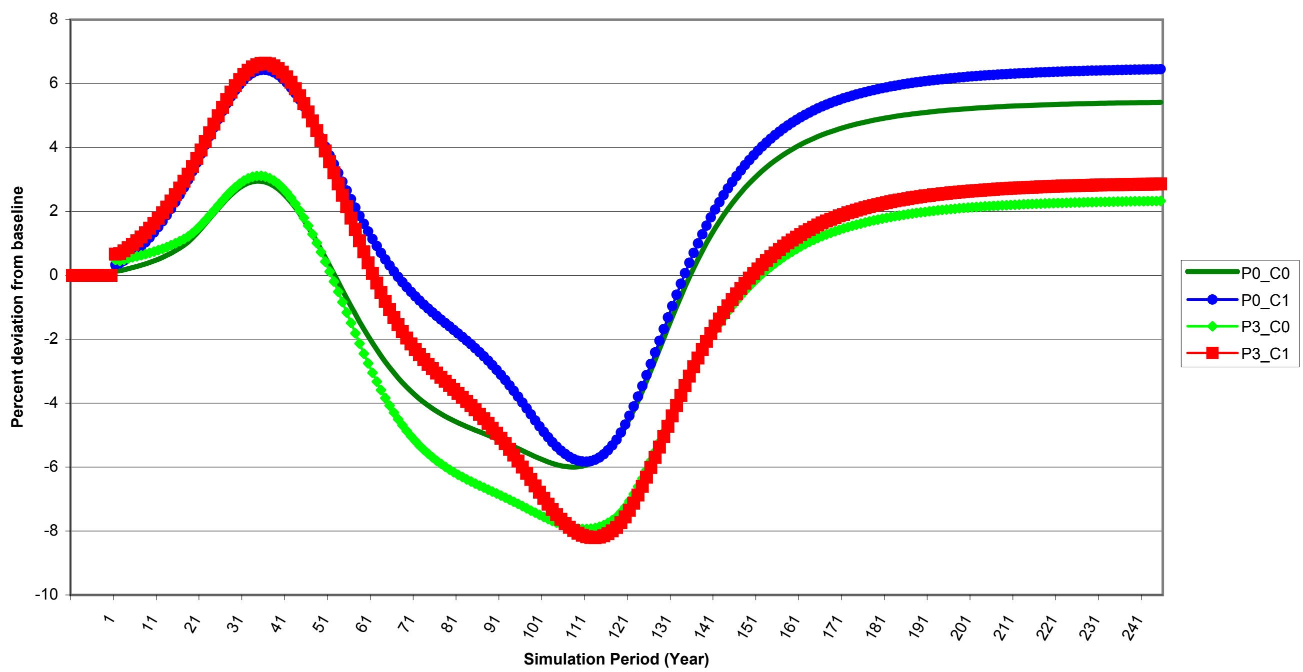


**Figure 26. ZZ Real Financial Wealth Per Adult
Pensions and Youth Dependency Excluded and Included
Asymmetric Shock (Open-Economy, Two Regions Differ)**

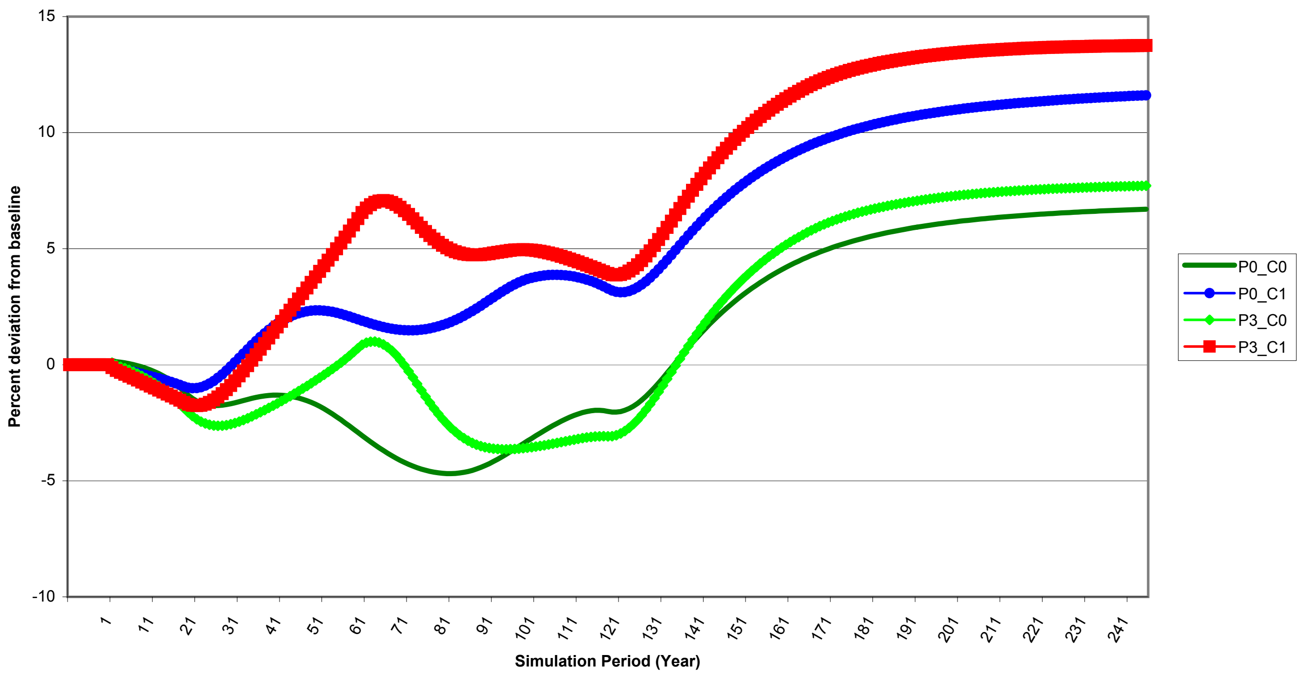


**Figure 27. ZZ Ratio of Nominal Private Savings to Nominal GDP
Pensions and Youth Dependency Excluded and Included
Asymmetric Shock (Open-Economy, Two Regions Differ)**

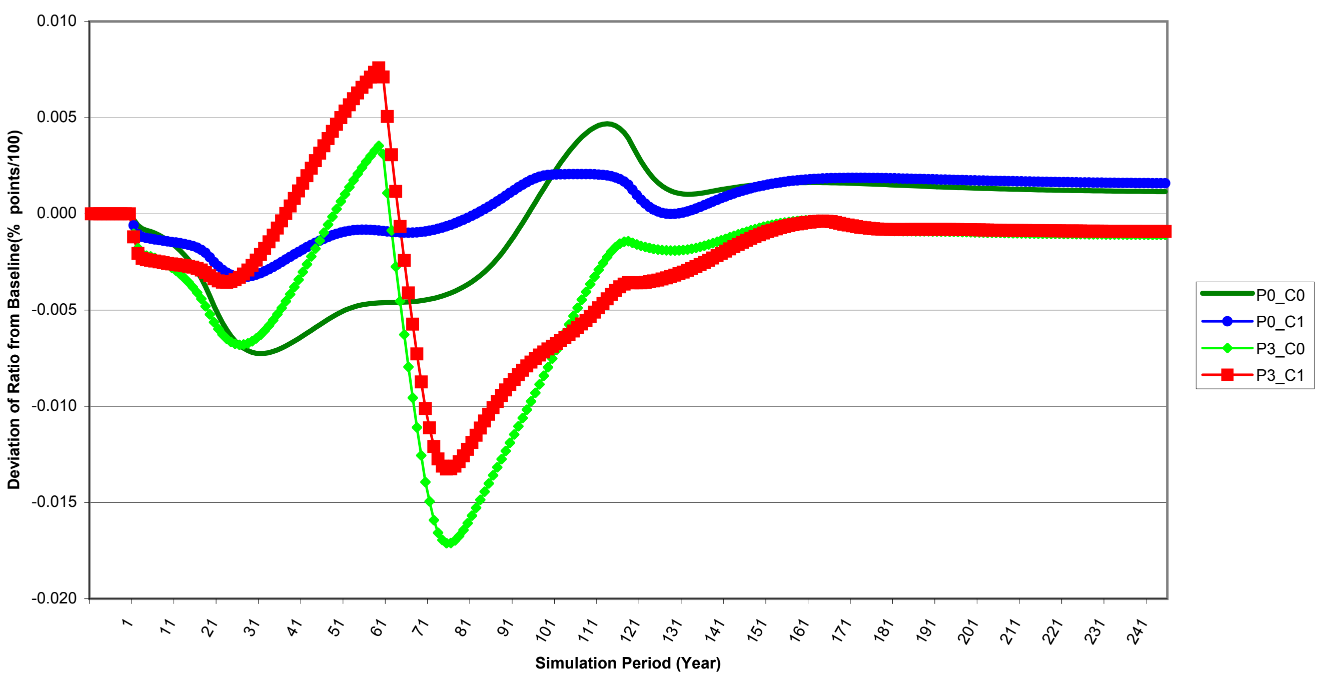


Figure 28. ZZ Ratio of Current Account Balance to Nominal GDP Pensions and Youth Dependency Excluded and Included Asymmetric Shock (Open-Economy, Two Regions Differ)

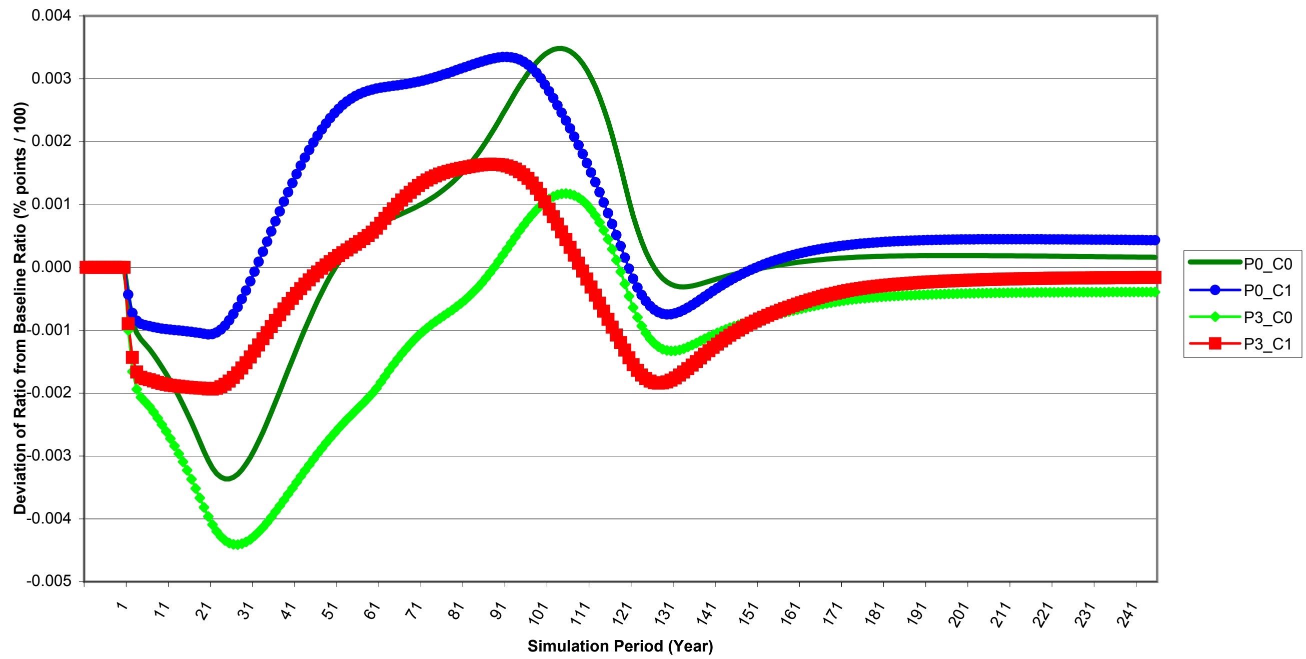


Figure 29. ZZ Ratio of Net Foreign Assets to Nominal GDP Pensions and Youth Dependency Excluded and Included Asymmetric Shock (Open-Economy, Two Regions Differ)

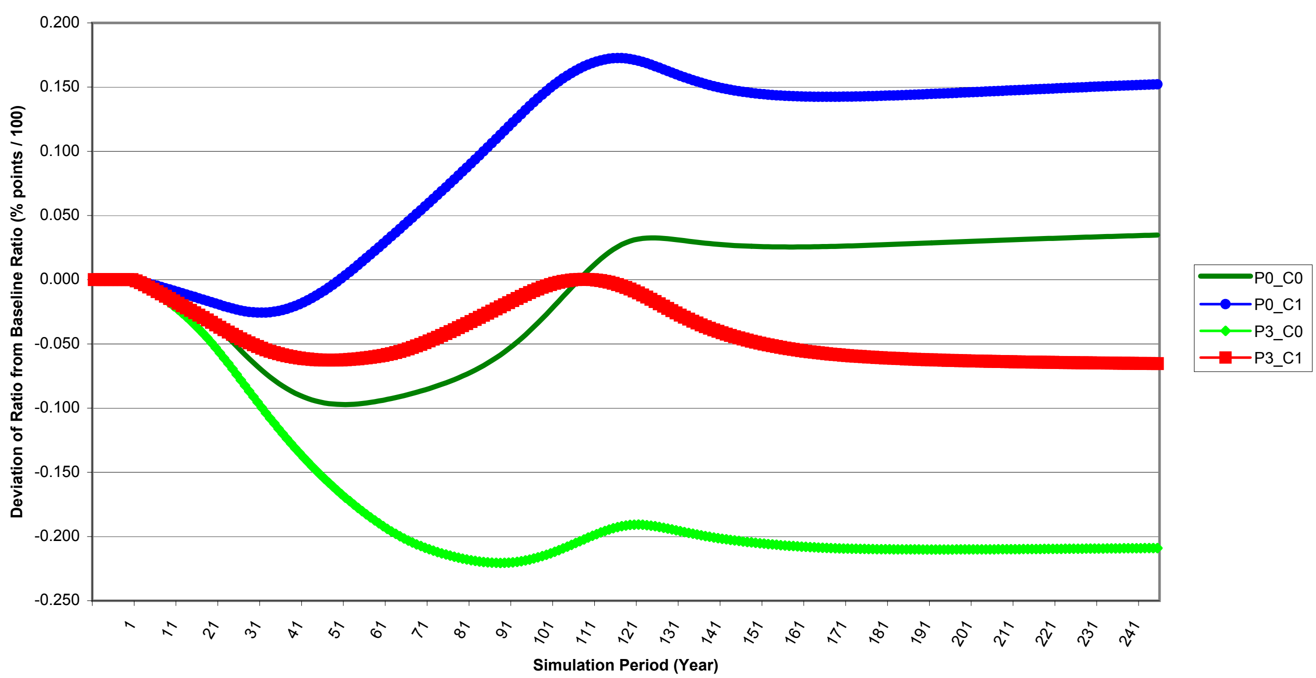


Figure 30. ZZ Real Adult Consumption Per Adult
Pensions and Youth Dependency Excluded and Included
Asymmetric Shock (Open-Economy, Two Regions Differ)