The Margin of Error in the Florida Recount

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Since the recount in Florida commenced two days ago, the margin for Bush has decreased by over 1400 votes. Out of approximately six million votes cast, this seems like a very small number, representing a shift of little more than 2 votes per 10,000. Thus one might be inclined to think that the shift probably arose purely by chance, and that any further recounts will have statistical flukes of a similar magnitude.

We shall show, however, that under benchmark assumptions about the error-generating process, 1400 is actually a very large number. It is so large, in fact, that it strongly suggests that the errors in the original count were not purely random. This is not to say that these errors were necessarily malicious or intentional; it could be, for example, that the errors in each locality were unbiased but that more errors tended to occur in counties where Gore was ahead. But the data do suggest that the change in the vote count was not the result of purely random errors in the ballot-counting process.

The argument goes as follows. Suppose that there is a small probability p that any given ballot is "mishandled" in a given count (say due to machine error). Suppose further that the error-generating process is completely unbiased: if a given ballot is mishandled, the information on it is ignored, and a vote is recorded for Bush with probability 50% and for Gore with probability 50%. (Notice that the probability of an error is actually p/2, because a mishandled ballot might, by accident, be in agreement with the voter's actual intent.)

Let S be the net shift in votes to (or away from) Gore between the original count and the recount. For different values of p we can estimate statistically how large S is likely to be. The results are shown in the following tables:

p = 1/1000

Value	Likelihood
-160 S 160	over 95.0%
-240 S 240	over 99.0%

p = 1/100

Value	Likelihood
-506 S 506	over 95%
-760 S 760	over 99%

The likelihood that one could have gotten a shift as large as 1400 through pure random error is extremely remote in either of these cases. In fact, one would have to assume that about 1 in every 10 ballots was mishandled to get numbers that are reasonably consistent with the observed shift of 1400 votes. This is an astronomically high rate of ballot mishandling, which, if true, would by itself undermine the credibility of the outcome.

We believe that a low rate of ballot mishandling is, in fact, the more likely scenario. If so, the observed shift of 1400 votes toward Gore requires an explanation. Statistical analysis by itself cannot pinpoint the cause of the problem; it can only suggest that a problem exists.

Derivation of the tables. Assume that p = 1/1000. Out of six million ballots cast, the expected number of mishandled ballots equals 6,000. Call this subset M1. (The size of M1 will be within 6000 ± 160 with probability 95%; this range does not materially affect the calculations.)

Assume that, in a recount, 6,000 ballots are again mishandled. Call this set M2.
If some ballots are prone to mishandling, then M1 and M2 overlap substantially. The most conservative assumption, however, is that the draws leading to M1 and M2 are independent, in which case the expected overlap is only 6.

We now wish to estimate the net shift in votes between the first count and the second. Assume that in reality Gore and Bush are exactly tied with 3 million votes each. The standard error for the first mishandled set of ballots, M1, is sqrt 3000 = 55.

[Note: the number actually for Gore in M1 is a binomial random variable X with standard error = sqrt (.5 x .5 x 6000). The number of reported Gore in M1 is another binomial random variable Y with the same standard error. Since the error model is purely random, X and Y are independent, so their difference has standard error equal to sqrt 3000.]

By a similar argument, the standard error for the second mishandled set of ballots, M2, is also 55.

Assume now that M1 and M2 have small overlap. (This is the conservative assumption.) Then in the recount all votes outside M1 and M2 will be counted the same way both times. Almost all votes in M1 will be counted correctly the second time. Thus the net change for or against Gore within M1 has standard deviation 55. The votes in M2 were correctly counted the first time, and so the net shift for or against Gore in M2 also has standard deviation 55.

Since the two sets are independent, the net shift S has standard error sqrt (55 x 55 + 55 x 55) = 77.5 votes. Two standard errors is 154 votes; three standard errors is 233 votes. The numbers in the table above are rounded so as to comfortably capture the range of values that will occur with

95% probability and 99% probability. The calculations for p = 1/100 are similar except that the upper and lower bounds for S are multiplied by a factor of sqrt 10 = 3.16.