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ELEMENTARY INDEXES FOR A CONSUMER PRICE INDEX

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What is the proper way to form a price index for, e.g., bananas? The U.S. Bureau of Labor Statistics (BLS) collects monthly prices for bananas at individual retail outlets and must choose the mathematical formula for aggregating them into a price index for bananas.

Carruthers, Sellwood, and Ward (1980, page 12) refer to "800 price quotations for granulated sugar" that were collected for the U.K. retail price index, and state: "the problem is how these should be combined to get an index for this item." Price indexes for bananas and sugar are among the 211 (before January, 1998, the number was 207) *elementary indexes* in the U.S. Consumer Price Index (CPI), and comparable elementary indexes exist for consumer or retail price indexes in nearly all countries.²

It has long been known that different calculation methods for elementary indexes give different results. Diewert (1995) suggests that the issue extends back at least to Fisher (1922).

¹ A portion of this paper was written while in residence at the Australian Bureau of Statistics. It has benefitted from a seminar presentation at ABS, and from helpful discussions with Keith Woolford.

² There is, regrettably, no standard nomenclature for the lowest-level price indexes that are the building blocks in a CPI system. Elementary indexes are also referred to as "basic components," "basic indexes," and "elementary aggregates," and the BLS uses the terms "strata indexes," or "item strata indexes."

Schultz (1994) computed alternative elementary price indexes from data taken from the Canadian CPI. He found enormous differences over a 5-year interval--for example, price indexes for soft drinks computed with six different index number formulas exhibited a range from 15.1 to 925.7. The highest index was over 60 times the level of the lowest, and the data were the same! Turvey (1995) cites recalculations of CPIs in Austria, Sweden, and France that showed annual differences of 2 percentage points or more from different formulas for elementary indexes. And the Boskin Commission (1996) maintained that the use of a particular index number formula for elementary indexes imparted an upward bias of approximately 0.25 percentage points annually in the overall U.S. CPI, an estimate that was based on work by Moulton (1996) comparing arithmetic and geometric mean elementary indexes.

To date, research on elementary indexes has mostly been restricted to unweighted formulas (arithmetic and geometric means) because detailed quantity data are not available *within* elementary indexes. However, scanner data containing very detailed information about store prices and quantities for very specific items are becoming available. How should they be used in the CPI? In particular, how should one would use detailed *high-frequency* price and quantity data (from periodic store “sales,” for example) in the CPI?

Thinking about high-frequency prices and quantities in elementary indexes has proceeded by analogy with the existing index number literature that concerns consumer behavior toward *low-frequency*, or trend, price and quantity change (for example, the well known “substitution bias” problem). Are straightforward parallels of this kind adequate to the task? To borrow

from the title of a paper I wrote some time ago (Triplett, 1983a), before I ask, "What is the answer?" I first want to be sure that I understand: "What is the question?"

In the first section of this paper, I discuss some very simple points about weighted and unweighted arithmetic and geometric mean elementary indexes. Some of these points have been made before (and are worth making again), but others of them have not figured in recent discussions of the elementary index problem. With data on high-frequency price and quantity changes, even the geometric mean gives answers that many would regard counter-intuitive. In the second section of the paper, I discuss some rather complicated points about constructing a (buyers') cost of living index on high-frequency data gathered from sellers, points that have been grievously neglected in recent literature on elementary price indexes.

I. Some Simple Points About Straightforward Index Number Calculations

I have already noted that Schultz found a huge range of different price indexes for soft drinks, depending on the formula used in the calculation (see Schultz, 1994, Table 1, page 100). Reviewing those Canadian CPI soft drink price data and some of the calculations will make clearer both the kinds of retail price changes that cause the elementary index number problem and why Schultz got the results he did.

Panel A of Table 1 presents an extract from Schultz's unpublished soft drink price data for one Canadian city. The extract consists of prices for one brand and one size of soft drink, from only two stores in that city, and only four months; although these are actual transaction prices collected for the Canadian CPI, I have deliberately chosen an extract that illustrates,

with the minimum amount of data, the points I want to make.³ Note that this soft drink is a homogeneous commodity--there is no commodity substitution taking place in this example.

It is quite evident that the price data in table 1 display recurring grocery store "sales" on the soft drink. The full data set shows that such sales were very common, and they are also very frequent events in U.S. grocery stores as well. Schultz referred to the pattern of prices in Table 1 as "price bouncing." I use the term "high-frequency price changes," because it is not only the reversal of price movements that characterizes this data, but also the magnitudes and frequency of the changes.

A. Unweighted Price Indexes Using the Canadian Soft Drink Data

Three unweighted (or equally weighted) calculation methods have been proposed for elementary indexes. These are displayed in panel B of Table 1.

The ratio of average prices method (RAP) is computed from average prices for "matched" observations in the two periods. The average of price relatives (APR) method uses, as the name implies, the ratios of the individual (matched) price quotations. Both are arithmetic averages. Finally, there is the geometric mean (GM) of price relatives.

Panel B of table 1 shows each of these three methods in three alternatives. The alternatives can be illustrated with the RAP method. The first line of panel B (period-to-period RAP) shows the percent change one gets from applying the RAP formula successively in months 2, 3, and 4. The next line (chained RAP) shows the results of multiplying together the

³ My choice obviously also exaggerates somewhat the index number comparisons that are based on the extracted data. The calculations in Schultz (1994), however, show that any exaggeration produced by the abstract is inconsequential, because the full data set also produces a huge outcome range from the application of different index number formulas.

period-to-period indexes of line 1 (a chain index). The final line (direct RAP) shows the result of comparing successively each of the three months (2, 3, and 4) with month 1. Note that the chained RAP and the direct RAP give the same answer in this example.

Looking at the results of the three methods, the APR gives an increase in all three periods, which is not true for either of the other alternatives. Moreover, even though the prices in the two stores are simply reversed between periods 1 and 2, the APR index rises (because the store 1 price increase is a 100 percent, whereas the store 2 price decrease is only 50 percent). And when the increases in the APR are chained together, the result is an index that rises some 80 percent, even though prices in the two stores are by month 4 back exactly to where they were in month 1. Both these results are frequently regarded as counter-intuitive (more on this in the following). Note, however, that the direct APR returns to 1.000 in month 4, which suggests that chaining is somehow a part of the problem with the APR method.

Note additionally that the unweighted geometric mean (GM) gives results that are broadly similar, in both chained and direct forms, to the RAP.⁴ The index number results in table 1 have similar patterns to those published by Schultz (1994). The relationships among these unweighted indexes have by now become familiar in the price index literature.

B. Unweighted and Implicitly Weighted Laspeyres Price Indexes

In many countries, the Laspeyres index (not the cost-of-living index) is regarded as the conceptual starting point for measuring consumer prices. The consumer price index is a "pure price index," on this way of looking at it, and the Laspeyres index provides its measure.

⁴ As noted below, this is not necessarily true for a weighted geometric mean.

Additionally, the U.S. CPI has used, since 1978, a statistical estimator for elementary indexes that was derived from specifying the Laspeyres index as the population statistic to be estimated (U.S. Department of Labor, 1997). Thus, I start the analysis with a Laspeyres interpretation of the unweighted indexes.

It is well known that one can write the Laspeyres price index equivalently in quantity-weighted and expenditure-weighted forms, or:

$$(1a) \quad L_{12} = \sum P_2 Q_1 / \sum P_1 Q_1$$

$$(1b) \quad = \sum w_1 (P_2 / P_1),$$

$$\text{where } w_1 = P_1 Q_1 / \sum P_1 Q_1$$

Because RAP looks like an approximation to the quantity-weighted Laspeyres index number form (1a), and the APR looks similar to the expenditure-weighted form (1b), it has sometimes been regarded as puzzling that RAP could differ so much from APR. The explanation is that RAP and APR make different *implicit* assumptions about the unknown quantity weights in the Laspeyres index.

To see this, consider the quantity weights that, when plugged into a standard Laspeyres formula, would make (unweighted) RAP exactly the equal to the *weighted* Laspeyres price index. If both stores sold 150 units in period 1, for example, then the *weighted* Laspeyres price index would equal RAP (see table 2, line L.1). Moreover, this would be true whether the weighted Laspeyres price index were calculated in quantity-weighted or expenditure-weighted form.⁵ Thus, RAP implies the assumption that quantities are equal in the two stores.

⁵ With the price data from table 1 and the quantities from table 2 (note 1), store 1 would gain one-third of the total revenue, store 2 two-thirds of total revenue, and the reader may confirm that the expenditure-weighted Laspeyres index would equal 1.000 in month 2, just as

On the other hand, APR implies that the revenues in the two stores are equal in month 1. If store 1 sold twice the quantity as store 2 in month 1, then they would have equal revenues. With equal revenue shares, the Laspeyres price index would rise to 1.258 in month 2 (see table 2, line L.2), when calculated in expenditure-weighted form, and would thus equal APR. It would have exactly the same value when calculated in quantity-weighted form.

Thus, RAP and APR involve different assumptions about the quantities that will be sold when the soft drink goes on sale. Use of RAP as an elementary index involves the assumption that putting the soft drink on sale has no effect on quantity, so that store 1 just loses revenue when it cuts the price of the soft drink. Use of APR, on the other hand, implies that the sale price works, to an extent--quantities increase, though revenue does not.

If forced to choose between these two admittedly unsatisfactory sets of assumptions, APR seems the most reasonable, because it at least acknowledges that special promotion or sales prices are intended to increase the quantity sold.⁶ I do not mean to imply by this that the APR is the best way to measure elementary indexes, just that it seems a more reasonable implementation of the *Laspeyres* framework than does the RAP (this has not been the usual presumption). As noted below, a better inference is: For a CPI, one might not want to push the Laspeyres formula all the way down to the store level.

the RAP (the unweighted measure).

⁶ The US CPI is a probability sample, with a statistical estimator derived from the statistical structure of the index. I presume that the somewhat more reasonable implicit assumption embodied in the APR had something to do with the selection of APR as the estimator for CPI elementary indexes (though this is not the logic displayed in the BLS Handbook of Methods--U.S. Department of Labor, 1997).]

In summary, this section shows that we can find an implicitly-weighted Laspeyres formula that rationalizes each of these two unweighted indexes, that is, the APR and the RAP.

C. Unweighted and Implicitly Weighted Geometric Mean Indexes

Let us apply this same trick to the explanation of unweighted geometric mean indexes, and their comparison to unweighted arithmetic ones. That is, we use the assumed quantities and revenues that underlie the RAP and the APR (cases 1 and 2 in the notes to table 2), to calculate geometric mean indexes that are parallel to RAP and APR. We can then compare geometric and arithmetic mean indexes that are based on the same assumptions.

In doing so, we run into an anomaly. With arithmetic mean (Laspeyres) indexes, the quantity-weighted arithmetic mean (1a) gives the same result as the expenditure-weighted arithmetic mean (1b), provided the quantities and expenditure shares are consistent. With the geometric mean, this same property is not maintained, a curiosity that does not seem to have emerged in recent discussions of the use of arithmetic and geometric means for elementary price indexes.

I am introducing here a perhaps novel distinction--between a quantity-weighted geometric mean index and an expenditure-weighted geometric mean index. An expenditure-weighted geometric mean index is quite straightforward. It consists of:

$$(2) \text{GM}_{t+1,t} = \prod (P_2 / P_1)^s,$$

$$\text{where } s = P_1 Q_1 / \sum P_1 Q_1$$

In the theory of the cost of living index, equation (2) is the price index that corresponds to a Cobb-Douglas utility function. When no actual information on expenditure or revenue shares is available, one often assumes that the *share weights*, s , are equal. For example, in the data of

Table 1, one could assume that the revenue shares of the two stores were equal in the initial month ($s_1 = s_2 = 0.5$). Recall that equal revenue shares is the implicit assumption for APR.

An example of a quantity-weighted geometric mean index is an unweighted, or equally-weighted, geometric mean of price relatives. Because each price change is being weighted equally, one presumes the interpretation that the store quantities have been assumed the same. Thus, the equally-weighted, quantity-weighted geometric mean (or unweighted geometric mean) is parallel to the unweighted RAP, which makes the same assumption.

Equal quantity weights and equal expenditure weights are not equivalent assumptions for calculating either geometric or arithmetic means, as the data in tables 1 and 2 make clear. Suppose, first, the assumption that the two stores have equal quantities (Case 1 in table 2). Under this assumption, a quantity-weighted geometric mean index shows no change between month 1 and month 2 (table 2, line GM.1), which is the same result one gets by applying the identical assumption to the implicitly-weighted Laspeyres index (RAP).

Equal quantities, however, imply quite different revenue shares in month 1 (Case 1, table 2): Equal quantities imply that the store with the lowest price has a revenue share of only one-third. Applying these implied revenue shares as weights, and again using the prices in table 1, results in an expenditure-weighted geometric mean price index that falls (table 2, line GM.2). The reason, of course, is that the expenditure-weighted geometric mean index, under the equal quantities assumption, gives a small weight to the price that rises (store 1 has only one-third of the weight) and a large weight to the price that falls (store 2).

Now consider the case where the expenditure, or revenue, shares of the two stores are assumed equal in month 1 (Case 2 in table 2). In this case, the expenditure-weighted

geometric mean index (GM.4) shows no change. But the quantity-weighted geometric mean index (GM.3) shows a sharp increase, to 1.264. Under the equal expenditures assumption, the quantity-weighted geometric mean rises because the store that has the soft drink on sale in month 1 sells two-thirds of the total quantities, and the other store only one-third (table 2, note 2)--the quantity-weighted geometric mean puts a heavy weight on the price that rises and a small weight on the price that falls. Note that this is almost the same result (to the second decimal place) we get with the Laspeyres index when we make the identical assumption, namely that the two stores had equal expenditure (revenue) shares--see the APR calculation in table 2, line L.2. And the reason is the same: When one assumes that store revenues are equal in month 1, quantity-weighted arithmetic and geometric mean indexes both give a high weight to price increases and a low weight to price decreases.

The foregoing exercise suggests there are four different ways to calculate an implicitly-weighted geometric mean price index. Several implications emerge.

First, many price index studies have compared or calculated or discussed unweighted (that is, equally quantity-weighted) geometric means (GM.1) and equally share-weighted geometric means (GM.4), without apparently noticing that these two calculations involve inconsistent assumptions. It is well known that an unweighted (equally quantity-weighted) geometric mean is always equal to an expenditure-share weighted geometric mean with equal shares. But equal quantity weights and equal expenditure weights are very different assumptions. One cannot assume simultaneously that the quantities are equal and that the revenue shares are equal.

Second, under the assumption that expenditures in the stores are equal, a quantity-weighted geometric mean (GM.3) and a quantity-weighted Laspeyres give almost the same answer, namely, a sharply rising price index between months 1 and 2, even though the prices in the two months are simply reversed between the two stores. The quantity-weighted geometric mean when store expenditures are equal (GM.2) looks remarkably like the APR, which is the Laspeyres index when store expenditure shares are equal (compare lines L.2 and GM.3 in table 2).

Third, if we calculate an expenditure-weighted geometric mean with equal quantity assumptions (recall that equal quantities is the assumption that rationalizes the RAP procedure), the geometric mean index does not remain constant, but falls sharply to 0.791. Treating the unweighted geometric mean and the RAP as equally valid, as in the European harmonized CPI, involves inconsistent assumptions about the unobserved weights.

One might argue that "crossed" forms of geometric mean indexes (GM.2 and GM.3) are not legitimate versions of the geometric mean. I do not see why not, if the purpose is simply to review properties of alternative index number formulations. They may not be *desirable* forms of the geometric mean for price index purposes, but that is another matter.

In summary, parallel assumptions about the expenditure and quantity shares may give similar indexes, even though the functional forms of the indexes differ. For unweighted indexes, it is the implicit assumptions about the weights, as well as the assumed functional form for the price index, that determines the differences in the values of the indexes.

In particular, I conclude that the "bad" properties of the APR are actually shared by the geometric mean--if we make parallel assumptions about expenditure shares. That is not

necessarily an argument in favor of the APR. It is simply an assertion that some properties of the alternatives to the APR have not been fully explored.

D. An Extension: Store Weights, Sales Prices, and Weighted Index Numbers

At the initial meeting of the Ottawa Group (International Working Group, 1994), fairly strong assertions were made that if we just had actual store quantities, the elementary index problem could be solved. Scanner data on store quantities, linked with prices, are becoming available for price index research and are being explored in a number of countries' statistical offices. For high-frequency price changes, they will undoubtedly show that neither of the two alternative implicit assumptions about store quantities that are implied by RAP and APR is realistic, because under neither of those assumptions is the sale very effective. In this section, I simulate weighted index numbers under more plausible assumptions for the quantities that are sold when soft drinks are put on sale. Weighted index numbers produce results that are even "worse" (on the usual way of looking at it) than the problematic APR.

In table 3, I present hypothetical store quantities and revenues, similar to what scanner data may show, corresponding to Schultz' Canadian CPI soft drink prices. The quantities incorporate two assumptions.

First, I assume that when the soft drink goes on sale, the sale works. Large increases in quantities occur, so store revenue increases at the sale price, rather than remaining constant, as the implicit assumption in the APR method (or falling, for RAP).

Second, I assume (following Diewert, 1995) that consumers have preferences over stores. Thus, store 1 sells more than store 2 at the sale price, and it also sells more at the regular price (refer to the hypothetical quantities in table 3), perhaps because store 1 has a

better location or has some other "non-price" advantage. When store 1 conducts a "half-off" sale, and store 2 remains at the "regular" price, store 1 earns 75% of the total revenue; when the situation is reversed, store 2 only gains 55% of the total revenue.

It is not surprising that the weighted Laspeyres index in table 3 rises more sharply in period 2 than does the unweighted Laspeyres in table 1: The weight applied to store 1's price increase is larger than the implicit assumption for either RAP or APR. The Laspeyres index systematically gives a high weight to the price increase when the soft drink goes off sale, and it systematically gives a low weight to the price decrease when the soft drink goes from the regular to the sale price. Conversely, the Paasche index gives a low weight to price increases and a high weight to price decreases, resulting in a sharply falling price index. If the elementary index problem has something to do with counter-intuitive behavior of the APR in period 2, use of store weights will make matters, from this perspective, worse.

Because these weighting effects are systematic, the chained Laspeyres more than doubles over these four months, and the chained Paasche falls by more than half. Note, however, that direct versions of both Paasche and Laspeyres price indexes return to their initial levels when the initial prices (month 1) are restored in month 4. As with the unweighted APR, this implies that the problem with the Laspeyres is connected with chaining (Schultz, 1994; Triplett, 1983b, note 6). The biases, if they are bias, in the direct Laspeyres and Paasche indexes are confined to the "improbable" or "counter-intuitive" results in month 2.

Notice that the weighted geometric mean exhibits the same "counter-intuitive" increase in month 2 as the Laspeyres (though the geometric mean rises less than the weighted Laspeyres). And the chained geometric mean index still winds up substantially higher when the

initial prices are restored in month 4, though the increase (69 percent) in the chained geometric mean index is lower than in the chained Laspeyres (177 percent),.

These indexes rise in period 2 because the Schultz (1994) price data pertain to sales prices, and the quantities in table 3 reflect the assumption that sales prices work--sales prices increase revenue while the sale is in effect. *If one takes the store weights in period 1 to calculate a price index between month 1 and month 2, it does not matter which formula one uses--the price index will rise, and it will rise again if store weights for month 2 are used in a price index between months 2 and 3. Both the Laspeyres and the weighted geometric mean in some sense overweight price increases coming off sale and underweight price decreases when the soft drink goes on sale.*⁷ And for both the weighted Laspeyres and the weighted geometric mean, chained index numbers do not return to their initial levels when the initial prices are restored (month 4).

Thus, the "counter-intuitive" behavior of the Laspeyres index, whether in APR or in the form of an index weighted by store revenue, is matched by "counter-intuitive" behavior of the geometric mean index *when it is weighted similarly*. Many of the past comparisons have been performed on geometric mean and arithmetic mean indexes that are not weighted similarly. The only case in which the expenditure-weighted geometric mean index does not give this

⁷ With the prices and quantities in the example, one might characterize the weighted geometric mean (which has a unitary substitution elasticity) as having too little substitution in it--note that the Fisher index, which is superlative, does not rise so sharply. Note that the Fisher rises in month 2 because of the specification that consumers have preferences between the two stores (see section I.E).

"counter-intuitive" result is when we assume "counter-intuitive" shares, namely, that sale prices in stores do not increase store revenue (as GM.4, in table 2).⁸

Notice that the direct unit value, direct geometric mean, and direct Fisher all give "intuitively" reasonable results by month 3. But then so does the direct Laspeyres. All four indexes show that prices have fallen between month 1 and month 3, and the differences among them are not particularly large.⁹

What does one make of all this? First, the form of the elementary index number calculation may be less the villain than it has been portrayed in some of the recent literature. And chaining of index numbers in the face of high-frequency price changes makes the problem in elementary indexes more severe, whether arithmetic or geometric mean (Schultz, 1994, emphasized the chaining problem for arithmetic means).

Second, all of the indexes of table 3 are implausible, in some sense or other. We need to ask ourselves why this is so.

One might contend that the assumed quantities in the example imply more substitution than permitted by the weighted geometric mean index--the weighted geometric and the

⁸ Keith Woolford (1994) and also Paul Armknecht (1996) have noted that in the long run using equal quantities or equal revenues for the two stores (that is, ignoring the quantities associated with special sales) might produce a sensible result. But if that is the case, this implies something like the direct APR. As Table 1 shows, the APR gives "intuitively acceptable" results for any period that does not involve the direct return from the sales price to a "normal" price. I return to this

⁹ Obviously, the quantities in this example are hypothetical, so the precise results depend on the assumed responsiveness of store sales to sale prices, and to the assumed differences in responses between the two stores. The example has not in any way been contrived to obtain the results shown here, but I have not conducted sensitivity tests for alternative assumed quantities.

weighted Laspeyres indexes both contain substitution biases of the ordinary kind. The solution is to compute the monthly changes with a superlative index number that does not constrain the amount of substitution (in this case, substitution across retail outlets). This contention says, in effect: The classic substitution paradigm in the cost of living index is adequate, what is wrong in this example is the form of the index number.

This contention is not unreasonable. Possibly the behavior of consumers who switch between stores in response to sales prices is adequately modelled by some form of index number that was originally derived from a commodity substitution paradigm. Nevertheless, I do not think this obviates consideration of the topics discussed in section II of the present paper.

More troubling, however, is what one should do with scanner data in the CPI, if the contention is correct. If the geometric mean does not solve the elementary index number problem because there is too much substitution at the detailed level, can we hope to do Fisher indexes with scanner quantities at the elementary index level? And notice that even the Fisher index rises in period 2, behavior that has been considered unacceptable in the APR. If that price increase is acceptable in the Fisher, what then of the logic that originally made the APR unacceptable in the same circumstance? There is a dilemma here that suggests that the elementary index number problem cannot fully be resolved by some mathematical rule for combining store prices.

Another response might be: The example, being partly abstracted and partly invented, overstates the amount of price change and quantity change that will actually be observed in an elementary index. Most actual elementary indexes do not consist of homogeneous

commodities like bananas (even the soft drink index has lots of heterogeneity in it). The effects of sales prices will be muted in the aggregate data for an elementary index by more slowly moving trend price and quantity change (low-frequency changes), so the problem of sales prices will not be forced on us in such stark terms.

There is something in this contention as well. But it is still important to think through how we handle high-frequency price and quantity changes. The conceptual model that underlies elementary indexes must be satisfactory not only for an elementary index that is affected by lower level commodity substitution, but also for the price index for, e.g., bananas, where there is by definition no commodity substitution. The variability of banana prices does not fit the commodity substitution paradigm.

The example suggest the question: Should we think about elementary indexes merely as ordinary price indexes with weights driven "all the way down" to the store level? I think the answer is no, or at least partly no. This is the topic of Section II of the paper. But I explore first a digression on "seasoning" prices.

E. BLS "Seasoning" of Prices

The BLS changed its calculation method for elementary indexes in 1995, putting into effect a procedure that it called "seasoning" of prices. When price collection was newly initiated in a retail outlet, BLS did not use the price in calculating the index for several months after initiation (prices were "seasoned" before they were used).

An interpretation of seasoning can be drawn from the numerical example. The anomolous result always occurs in the first monthly index (month 1 to month 2). "Seasoning"

these soft drink prices by discarding the first monthly price change eliminates the anomaly.

That is true even with the APR method: The direct APR between months 1 and 3 falls, and the price does not change between months 1 and 4. It is likewise true for all the weighted index numbers.

The weighted index numbers suggest an alternative way to interpret price seasoning.¹⁰ The BLS uses probability sampling in the CPI. One can re-interpret the revenue shares in table 3 as selection probability weights, rather than as index number weights in the usual sense. On this interpretation, price increases (as the soft drink goes off sale) will always have a higher selection probability than price decreases. The price index is biased, on this interpretation, because weights from month 1 overweight store 1 and underweight store 2, relative to their average revenue shares, *and* the sale produces a *necessary* correlation between the weights and price increases in the following period. Seasoning breaks the correlation. Because there is no correlation between subsequent price changes and month 1 weights, there is no subsequent bias in the seasoned index (compare any of the indexes for months 3 and 4).

The BLS has interpreted seasoning as applying only to its APR estimator, and not to its new geometric mean estimator for elementary indexes. The above interpretation of seasoning, however, is not restricted to the APR. It applies as well to the geometric mean, and in fact to all the weighted index numbers, including the superlative index. Further exploration of price seasoning may be warranted.

¹⁰ The BLS explanation for why seasoning works involves implicit base-period quantities in the expenditure-weighted form of the Laspeyres formula. With the change to geometric means in the CPI, the BLS will end the practice of seasoning prices.

II. The Interpretation of Elementary Indexes in the Cost-of-Living Index

Like most economists, I believe that the theory of the cost-of-living index (COL) provides the appropriate framework for guiding construction of the CPI. The present section considers “sales prices” in CPI elementary indexes from the point of view of the COL.

The COL is not currently accepted by very many of the statistical agencies that compile CPIs. Indeed, the theoretical approach to price indexes is probably now somewhat less accepted around the world than it was before the Boskin Commission (1996) report.

Nevertheless, one needs a theory to provide a framework for thinking (indeed, that is what a theory is). The theory is useful even in cases when its implications are, as I will argue they are in the present case, in some degree inchoate. Alternative approaches to consumer price indexes (Laspeyres “pure price” index, acquisitions price index, and so forth) do not provide a well-developed conceptual framework for thinking about the CPI, and for decision-making.

A. Commodity Substitution and Elementary Indexes

In most COL discussion, the elementary index number problem has been viewed as “lower level substitution” (the Boskin Commission’s term). For example, bananas were once an exotic fruit in the U.S.; the long-run decline in banana prices relative to domestically-produced fruits (apples, cherries, plums) has resulted in substantial commodity substitution toward bananas in fruit consumption. A fruit price index that fails to take this commodity substitution into account will be upward-biased, for reasons familiar from the “upper level” substitution case (see Pollak, 1989, for a classic statement of the theory of the COL).

Substitution can also proceed within the elementary indexes. For example, within the soft drink price index there may be substitution across different kinds of soft drinks--perhaps sugar sweetened soft drinks rise in price relative to sugar free soft drinks. That is traditional commodity substitution at a lower level from what has been estimated in previous studies (Braithwait, 1980, or Aizcorbe and Jackman, 1993). Within the elementary index for apples, perhaps the price of one variety of apples falls relative to another variety, as has often been asserted recently; if this actually occurs, then there may be substitution bias within the apples index.

An emerging body of research suggests that the geometric mean estimator for elementary indexes adjusts for "lower level" substitution bias fairly well. Because lower level commodity substitution is adequately discussed elsewhere, it is not necessary to consider it here.¹¹

Instead, I want to ask a question that has not been asked very much: Is classic commodity substitution in response to relative price changes *all* that is going on within the elementary indexes?

In the U.S., for example, bananas are as close to a homogeneous commodity as one can find (but not in Australia and other countries where different varieties of bananas are marketed, at different prices). BLS materials indicate great variability in banana prices, but this price

¹¹ The BLS report on geometric mean elementary indexes (U.S. Department of Labor, forthcoming) contains an excellent discussion of the commodity substitution paradigm in the context of relative price movements within elementary indexes, and of the geometric mean estimator as a potential solution to "lower level substitution bias" in elementary indexes. Various ambiguities of the theory when applied at this level are also explored there (for example, the difficulty that "subindexes" of the COL require separability of the consumer's cost function).

variance cannot correspond to commodity substitution, which is by definition impossible. A geometric mean index cannot, therefore, be adjusting for lower level consumer substitution in the case of bananas; if the geometric mean improves the measurement, it must be because it is correcting for some other measurement problem.

Schultz' soft drink dataset refers to a single soft drink. The price variance observed in it has little to do with the classic commodity substitution paradigm (or variants thereof, including the case where consumers have preferences over stores and commodities). In addition to traditional commodity substitution, an elementary index must also account for variability in banana prices and for "sales prices" such as are embodied in Schultz' soft drink data.

Another way of putting it is: What is the question for which the geometric mean is (or is not) the answer to the elementary index problem? In nearly all of the discussion to date, the question is taken to be, solely, commodity substitution, "one level down" from the more aggregative empirical substitution bias estimates of, say, Manser and McDonald (1988). In addition to "lower level" commodity substitution, I contend that the theory also needs to confront consumers' economic behavior toward periodic "sales"--consumer shopping, search, inventory and related behavior. In this, my views are closest to those of Pollak (1998). These views are not consistent with the current "mainstream" view of the elementary index problem, but I contend that the mainstream view, as represented, say, by the Boskin report, is incomplete--not wrong, but incomplete.

Section II.B develops the idea. It is not possible to do very much about incorporating consumer shopping, search and inventory behaviors into the CPI or into a cost of living index at this time (and I am sure that some users and perhaps some statistical agencies would object

that it might not be appropriate to do so, even if possible). However, before accepting the geometric mean as the “answer” (or the complete answer), it is useful to ask whether the existing analyses of the elementary index number problem have addressed all (or reasonably close to all) of the problem.

B. A Statement of the CPI "Sales Price" Problem

Table 4 repeats again the extract from unpublished data obtained from Schultz (1994), which have already been described in section I.A. As noted, the data display recurring grocery store "sales" on this soft drink. How should we think about consumer behavior toward sales prices in the COL?

Sales prices (and quantities) pose three issues that are not part of the standard commodity substitution paradigm that motivates much of the literature on the cost of living index: sellers' versus buyers' data, periodicity questions, and the distinction between consumption costs and acquisition costs.

We want to compute an aggregate cost of living index across consumers. However, we construct the CPI with data collected from retail outlets. We collect the data from the seller, not the buyer, but we would like to infer buyers' behaviors in calculating the index, even though we do not have direct information about that. In what sense can we infer the relevant behaviors of buyers by observing changes in store prices, quantities, and shares? In particular, what can we infer from high-frequency changes in prices and quantities collected from stores?

Another consideration is periodicity. The cost of consumption belongs in a cost of living index, and the flow of consumption for a period is the relevant quantity. The shorter the period, the more likely that the timing of acquisitions differs from the timing of consumption.

This is a well-known property of durable goods, but it is also relevant to consumer behavior toward sales, because short-term acquisition rates will not match consumption rates and it is consumption rates that matter for a cost of living index. In particular, short-term acquisition rates will probably fluctuate much more than consumption rates, and this is the more likely when high-frequency price changes fluctuate more than trend changes in relative prices.

A third way to think about the problem is to ask whether relative consumption costs are adequately described by relative acquisition costs at the time of acquisition. This can be taken a step further by asking whether the full change in acquisition costs (and therefore also in consumption costs) is measured by the difference between sale and nonsale prices for the same product in different stores.

None of the above--buyers' compared with sellers' prices and quantities, periodicity questions, and the acquisition-consumption costs distinction--is a part of the standard COL theory. One reason is the level of abstraction in the theory, as it has been developed so far, which really concerns adjustments to new equilibrium consumption proportions when trend relative prices change, not adjustments to transitory price fluctuations, such as grocery store sales.¹² A second reason is the high level of aggregation in the data normally used for empirical estimates of COL indexes (the second reason reinforces the first). The higher the level of aggregation and the longer the time period, the less concern we need have for the factors mentioned in the preceding three paragraphs.

¹² This is not the same thing as saying that the theory cannot be developed in these directions. That it is at present inchoate implies only that it does not at present speak to these matters, not that it will not or can not find a voice in the future.

For most kinds of price index measurement problems, including most cases of commodity substitution, the store quantities give us relevant information about consumer behavior (even if the price-quantity outcome incorporates supply effects as well as demand effects). Additionally, the longer the period, the less we are concerned--obviously--with periodicity questions. Finally, even though we should always be concerned about trends in the nonacquisition costs of consumption, we will nearly always neglect long-run changes in them, partly on the (not always justifiable) presumption that such changes, where present, are swamped by relative acquisition cost changes.

In the longer run, then, and for higher levels of aggregation (forming a price index for fruit from indexes for bananas and apples), we take consumer substitution in response to relative prices as the major part of consumer behavior that is relevant to the cost of living index problem. And the corollary is: The relative price movements that we see in the indexes for apples and bananas are those that are appropriate to the consumer commodity substitution paradigm.

For interpretation of the data collected for micro-level prices, such as the prices that go into the price index for bananas or for soft drinks, all of the three issues must be considered. Much recent analysis of the elementary index problem has said, in effect, that the economic behavior that is relevant for *between* elementary indexes questions (commodity substitution between apples and bananas) is the same as the economic behavior that is relevant for *within* elementary indexes questions (how do we form an index for bananas?). Commodity substitution at the lower level is indeed the same behavior as substitution at the higher level, but there is economic behavior that exists at the lower level that does not have to be

considered in aggregating across elementary price indexes. Consumer behavior toward sales and sales prices involves considerably more than the classical commodity substitution paradigm, even when that paradigm is modified by adding consumer preferences across retail outlets.

C. Illustrative COL's for Individual Consumer Units

Panel B of table 4 represents various kinds of consumer behavior toward the soft drink prices Schultz recorded from Canadian grocery stores.

The first behavioral pattern I have labeled "habit purchaser"--this consumer doesn't switch stores and just pays whatever price prevails at the favored store. This is the consumer type for whom the price relatives observed in the two stores provide the entire information about purchase prices. Actual consumption may or may not vary with purchase price (it does not matter for our purposes because no substitution is taking place within the sample, for the habit purchasers).

The second class of customer I have labeled the "shopper." This consumer always switches stores to acquire the soft drink at the lowest prevailing price. This consumer's cost of living index does not look much like the prices paid by the habit purchasers, even if habit purchasers are aggregated in some way. The shopper experiences no price change between periods (1) and (2), for example, when the two habit purchasers experience doubling and halving of price.

Shoppers incur search and switching costs, so I have inserted a variant with an assumed constant cost for switching (line 2b). For this case, the lowest prevailing price is not always the optimal purchase, and does not always represent the cost of consumption. This is an

obvious conclusion, and does not require much in the way of a sophisticated search-switching rule (there is a small literature on search costs in the cost of living index, contributions by Anglin and Baye, 1987, and a section in Reinsdorf, 1993).¹³

Finally, it appears from this Canadian data set that this soft drink is nearly always on sale, somewhere (the full data set makes that even more apparent). Consumers certainly know this, and soft drinks are readily stored. Thus, we have the inventory/shopper, the buyer who is price sensitive in acquisition periodicity (and perhaps also in consumption periodicity), but simply stocks up when the soft drink is on sale.¹⁴ The inventory/shopper does not buy at the nonsale price, so the nonsale price is not relevant for acquisitions, and this consumer has an undefined acquisition cost for periods 2 and 4. The consumer incurs both storage costs and implicit interest costs (though small), so it is not exactly clear what the right consumption cost is for periods 2 and 4.¹⁵ In any case, the consumption cost is not given by any of the acquisition cost information in the table. The inventory/shopper's price-opportunity cost locus does not look like those of the habit purchaser, or of the shopper.

One should also consider all sorts of other variants, such as the person who switches between stores and who inventories the soft drink to avoid paying the regular price, and who

¹³ I consider below whether switching between stores in response to sales prices can be modelled satisfactorily under the rubric "outlet substitution."

¹⁴ There might be small purchases at the nonsale price, if unexpected guests turn up, and so forth.

¹⁵ One might argue on opportunity cost grounds that the acquisition prices in periods 2 and 4 are the right prices to charge oneself for consumption of inventories. But then, why does the shopper inventory the product? Should we instead treat the saving from buying on sale as implicit income from inventory holdings? Below, I take inventory costs, minus interest and storage, as an estimate of consumption cost in periods when no purchase is made.

also considers search and switching costs. Obviously, this realistic case is a very complicated one for which we have little real information on consumption costs, and for which a comprehensive treatment in a cost of living index is a matter for a rather subtle and sophisticated research project. So far as we know, it cannot be handled by some operational rule for combining store prices in the CPI.

Of all these varying consumer behaviors, constructing a geometric mean on store prices appears relevant only to a consumer with the following characteristics: consumption quantities vary with price, the consumer varies store quantities by switching stores but does not switch stores completely (continues to buy something from the high priced store), doesn't consider search costs, and doesn't inventory purchases. This is not a particularly appealing model, for any group of consumers.

For a cost of living index, we want to aggregate across the individual indexes for this group of consumers. A "democratic" cost of living index would give each consumer equal weight. Though a simulation has not a great amount of value, for completeness (and to save the curious reader the trouble), panel C of table 4 aggregates the price experience of the first 5 consumer types in panel B, assuming they are equally distributed in the population. To account for the changes in sales assumed in table 3, a much heavier weight should have been placed on the shopper and the inventory/shopper. The consumption cost for the inventory/shopper is set equal to the last acquisition cost for periods in which consumption takes place out of inventories ("last in, first out" inventory valuation). For what it is worth, the aggregate COL does not look very much like any of the store-weighted index numbers, and

does not track the weighted geometric mean index, but there is really nothing compelling in a simulation.

The geometric mean has not been proposed as a method for aggregating across consumers. It is instead a method for allowing for commodity substitution. There is no method currently available, and little relevant data for building into the price index the behavior that I am considering in the present section.

Pollak (1998) has summarized the situation admirably: "... a satisfactory resolution of the item selection/basic components index problems requires a theoretical framework that recognizes that consumers do not face a single price for each good but instead face a distribution of prices. Furthermore, consumers recognize that they face a distribution of prices and their (search) behaviour reflects this recognition. The appropriate procedure for constructing the CPI under these assumptions is quite different from the appropriate procedure under the assumption that the "law of one price" holds (i.e., that the price distribution facing each consumer collapses to a single point)".

This is a price index research agenda for the future.

D. Additional Points: Outlet Substitution and Nonmarket Costs

Outlet substitution is a measurement problem in elementary indexes. The Boskin Commission (1996) put outlet substitution bias in the U.S. CPI at 0.25 percent, based on a single U.S. study (Reinsdorf, 1993). Outlet substitution has usually meant the gradual displacement of high cost retail stores by lower cost ones, or by those with a better

price-service mix.¹⁶ This is, of course, long-period price change, it is not high-frequency price change that is recorded in "sales" on particular brands and sizes of soft drink.

One might reinterpret outlet substitution bias to encompass high-frequency price variation, such as the Canadian soft drink data. As Diewert (1995) has suggested, one can add preferences over stores to the usual utility function's preferences over goods and services; then, changes in relative prices among stores lead to changes in patronage of retail outlets, which can be handled in a COL in a manner parallel to commodity substitution. The geometric mean, or the weighted geometric mean (tables 2 and 3) could then be interpreted as correcting for both commodity and outlet substitution.

I am not convinced by this argument. It merely side-steps the basic consumer shopping behavior problem. Perhaps the geometric mean will take care of this high-frequency problem, giving an aggregation of the individual consumer COL's in table 4. Simulations cannot say anything about that, it requires data--scanner data would help, but the information needed goes beyond the store prices and quantities found in existing scanner data, to incorporate information on consumer shopping and actual consumption, and perhaps time use. There is very little information of this type at present.

A second possible criticism of the above: In the overall cost of living index, search behaviour and so forth are nonmarket costs. Nonmarket costs are excluded from the cost of

¹⁶ The study by Reinsdorf (1993) that is sometimes cited for "lower level" *commodity* substitution in the U.S. CPI was interpreted by its author as providing evidence for *outlet* substitution--long-run displacement of conventional retail outlets by discount and other low-cost retailers. Reinsdorf has a useful section on search in a cost-of-living index, but his empirical work was directed toward low-frequency or trend price changes among retail outlet types, not high-frequency price changes, or sales prices, within the elementary indexes.

living variant to which the CPI is an approximation (the market cost of consuming at the base period's standard of living). Even though some economists might want a wider definition, and I have some sympathy with that, it is also important to limit for practical purposes the concept to which, pragmatically, the CPI provides an approximation.

But at the micro level, the level at which we are observing those "sales" prices and trying to understand what they signify, one cannot ignore search, shopping, and inventory behaviors. If those behaviors are ignored, then by default those Canadian soft drink sales prices must correspond to classical commodity substitution in consumption. Ignoring shopping, search, and inventory behaviour, in other words, *imposes* a behavioral explanation on those high frequency relative price changes which makes very little sense.

We do not know whether geometric mean elementary price indexes are better, with respect to consumer shopping, search, and inventory behaviour, than arithmetic ones. It might be true that the geometric mean picks up some of this behaviour and adjusts for it in some manner in constructing the elementary price indexes. It might, that is, but we do not know, and it would be well that users of the CPI be alerted to this potential problem.

Table 1

A Simple Numerical Example of the CPI "Sales Prices" Problems

A. The Data

Observation Period (Month)	(1)	(2)	(3)	(4)
Actual Price Observations, Soft Drink, Canadian City ¹				
Store 1	49¢	99¢	33¢	49¢
Store 2	99¢	49¢	99¢	99¢

B. Unweighted (or Equally Weighted) CPI Basic Component Calculations

Ratio of Average Prices Method (RAP):

$$I_{t,t+1} = (\Sigma(1/n)P_{t+1} / \Sigma(1/n)P_t)$$

Period-to-Period RAP: $I_{t,t+1}$	1.000	1.000	.892	1.121
Chained RAP: $I_{1t} = I_{12} \times I_{23} \times \dots \times I_{t-1,t}$	1.000	1.000	.892	1.000
Direct RAP: $I_{1t} = \Sigma(1/n)P_t / \Sigma(1/n)P_1$	1.000	1.000	.892	1.000

Average of Price Ratios Method (APR):

$$I_{t,t+1} = \Sigma(1/n)(P_{t+1}/P_t)$$

Period -to-Period APR: $I_{t,t+1}$	1.000	1.258	1.177	1.242
Chained APR: $I_{1t} = I_{12} \times I_{23} \times \dots \times I_{t-1,t}$	1.000	1.258	1.480	1.839
Direct APR: $I_{1t} = \Sigma(1/n)(P_t/P_1)$	1.000	1.258	0.837	1.000

Geometric Mean Method (GM)

$$I_{t,t+1} = \Pi(P_{t+1}/P_t)^{1/n}$$

Period-to-Period GM: $I_{t,t+1}$	1.000	1.000	0.821	1.219
Chained GM: $I_{1t} = I_{12} \times I_{23} \times \dots \times I_{t-1,t}$	1.000	1.000	0.821	1.001
Direct GM: $I_{1t} = \Pi(P_t/P_1)^{1/n}$	1.000	1.000	0.821	1.000

¹ Extract from unpublished data from Schultz, 1994

Table 2

Implicit Price Indexes Corresponding to Unweighted Calculations in Table 1

		<u>Month 1</u>	<u>Month 2</u>
<u>Implicit Laspeyres Indexes</u>			
Laspeyres ($L_{t,t+1} = \frac{\sum P_{t+1}Q_t}{\sum P_tQ_t} = \sum w_t(P_{t+1}/P_t)$)			
L.1:	RAP: Store 1 Quantity = Store 2 Quantity ¹	1.000	1.000
L.2:	APR: Store 1 Expenditure = Store 2 Expenditure ²	1.000	1.258
<u>Implicit Geometric Mean Indexes</u>			
Implicit Quantity - Weighted Geometric Mean Index (GM(Q))			
$GM(Q)_{t,t+1} = \Pi(P_{t+1}/P_t)^{q/\sum q}$			
Implicit Expenditure - Weighted Geometric Mean Index (GM(E))			
$GM(E)_{t,t+1} = \Pi(P_{t+1}/P_t)^s$			
$(s = q_t p_t / \sum q_t p_t)$			
Assumption 1 (see Note 1): Equal store quantities			
GM.1:	GM(Q)	1.000	1.000
GM.2:	GM(E)	1.000	0.789
Assumption 2 (see Note 2): Equal store revenues			
GM.3:	GM(Q)	1.000	1.268
GM.4:	GM(E)	1.000	1.000
<u>Notes:</u>	Assumed and implied quantities and shares for evaluating the price indexes		
		<u>Store 1</u>	<u>Store 2</u>
¹ Case 1	Assumed quantities	150	150
	Implied expenditures	\$73.50	\$148.50
	(shares)	.331	.669
² Case 2	Implied quantities	200	99
	Assumed expenditures	\$98.00	\$98.01
	(shares)	.50	.50

Table 3

A Further Numerical Example of the CPI "Sales Prices" Problem

A. The Data

Observation Period (Month)	(1)	(2)	(3)	(4)
Actual Price Observations ¹				
Store 1	49¢	99¢	33¢	49¢
Store 2	99¢	49¢	99¢	99¢
Hypothetical Quantities				
Store 1	300	100	350	300
Store 2	50	250	50	50
Hypothetical Revenue (Price x Quantity)				
Store 1	\$147.00	\$99.00	\$115.50	\$147.00
Store 2	\$49.50	\$122.50	\$49.50	\$49.50
Hypothetical Revenue Shares (w_t)				
Store 1	0.75	0.45	0.70	0.75
Store 2	0.25	0.55	0.30	0.25

B. Weighted CPI Basic Component Calculations

Laspeyres: $L_{t,t+1} = \sum w_t (P_{t+1}/P_t)$	1.000	1.636	1.266	1.339
Chained Laspeyres: $I_{1t} = I_{12} \times I_{23} \times \dots \times I_{t-1,t}$	1.000	1.636	2.071	2.774
Direct Laspeyres: $I_{1t} = \sum w_1 (P_t/P_1)$	1.000	1.636	0.756	1.000
Paasche: $R_{t,t+1} = [\sum w_{t+1} (P_t/P_{t+1})]^{-1}$	1.000	0.747	0.445	1.323
Chained Paasche: $I_{1t} = I_{12} \times I_{23} \times \dots \times I_{t-1,t}$	1.000	0.747	0.332	0.440
Direct Paasche: $I_{1t} = [\sum w_t (P_t/P_t)]^{-1}$	1.000	0.747	0.747	1.000
Fisher: $F_{t,t+1} = [L_{t,t+1} \times R_{t,t+1}]^{1/2}$	1.000	1.105	0.750	1.331
Chained Fisher: $I_{1t} = I_{12} \times I_{23} \times \dots \times I_{t-1,t}$	1.000	1.105	0.830	1.104
Direct Fisher: $I_{1t} = [L_{1t} \times R_{1t}]^{1/2}$	1.000	1.105	0.751	1.000
Weighted Geometric Mean: $\Pi (P_{t+1}/P_t)^s$	1.000	1.418	0.903	1.319
Chained GM: $I_{1t} = I_{12} \times I_{23} \times \dots \times I_{t-1,t}$	1.000	1.418	1.280	1.688
Direct GM: $I_{1t} = \Pi (P_t/P_1)^s$	1.000	1.418	0.744	1.000
Unit Value				
($UV_t = \sum P_t Q_t / \sum Q_t$); $UV_{t,t+1} = UV_{t+1} / UV_t$	1.000	1.127	0.652	1.361
Direct Unit Value ($I_{1t} = UV_t / UV_1$)	1.000	1.127	0.735	1.000

¹ Extract from unpublished data from Schultz, 1994

Table 4
Price Changes Faced by Different Consumer Types

A. The Data

Observation Period (Month)	(1)	(2)	(3)	(4)
Actual Price Observations, Soft Drink, Canadian City ¹				
Store 1	49¢	99¢	33¢	49¢
Store 2	99¢	49¢	99¢	99¢

B. Price Index Relatives for Different Consumer Types

	(1)	(2)	(3)	(4)
<u>Quantity Invariant</u>	(Initial Price)			
1a The Habit Purchaser (Store 1)	49¢	2.02(99/49)	0.33(33/99)	1.48(49/33)
1b The Habit Purchaser (Store 2)	99¢	0.49(49/99)	2.02(99/49)	1.00(99/99)
2a The Shopper	49¢	1.00(49/49)	0.67(33/49)	1.48(49/33)
2b The Shopper, with assumed 15¢ search costs	49¢	1.31(64/49)	0.98(48/49)	1.94(64/33)

Quantities Vary, Consumption Doesn't

3 The Inventory/Shopper (Store 1)	49¢	0 (no purchase)	0.67(33/49)	0 (no purchase)
3a The Inventory/Shopper, who also considers search costs	[can't be calculated from data extract in the table]			

Consumption Varies

4 Consumption quantities vary, and consumer varies store quantities, but doesn't switch completely, doesn't consider search costs, and doesn't inventory purchases	[geometric mean over store prices]			
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C. Democratic COL

COL _{t,t+1}	1.00	1.16	0.94	1.38
Chained COL	1.00	1.16	1.09	1.51

¹ Extract from unpublished data from Schultz, 1994

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