What is g?

(and *An Economist's Thoughts about Modeling Psychological Phenomena)*

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What I Want to do This Morning

- Talk a bit about how modeling is used in economics
- Present my thesis: Psychologists could benefit from doing more theory based modeling (though it would look different from what economists do) and that psychometricians are best positioned to bring this about
- Provide an example of how this would be beneficial from my own work on cognitive ability showing that theory based modeling can
  - make rigorous the link between assumptions and results
  - lead to surprising new insights
  - provide tests and functional forms for estimation

Why Economists Model

- Primary motivation for modeling in economics is different from other disciplines – *our models can claim to be normative even when they aren't very effective as positive models*
  - Under the assumptions that most people have a fair idea of what's good for them and will choose the best of available courses there is a strong suggestion that what people do on their own is the best that can be done (Adam Smith's invisible hand)
  - This can be proved under various assumptions with varying degrees of realism

Why Economists Model (cont.)

- What is Interesting is When the Assumptions that Imply Optimality Fail
  - Modeling exercise works out the implications of the assumptions and demonstrates potential welfare loss from failure of optimality (For example what if information on prices is costly? This can yield monopoly power to sellers thus prices too high and quantity of sales too low as a consequence)
  - More important, models allow economists to propose ways to correct the problem (In the example above, making the list of firms charging the lowest price publicly available for free restores the optimal competitive equilibrium)

What Economists Get From Modeling

- Discipline of intuition – Often things that seem to follow logically don't or require additional assumptions
- Additional insight – With surprising frequency modelers find out that the assumptions that give them the results they sought also imply other interesting phenomena
- Rigorous foundation for testing
  - Can better judge what assumptions one needs to interpret one's statistical results
  - Sometimes theory can suggest identifying restrictions and functional forms not apparent without it

Models in Economics are Heuristic – Not Necessarily Realistic

- Models, by their nature, are abstractions and simplifications of reality
- Physicists often draw conclusions from frictionless models of the movement of point masses about how real objects in an atmosphere will behave
- Similarly economists will draw conclusions from models with perfect competition and complete rationality that inform their judgments about how policy will affect a world where people aren't rational and markets aren't perfect
- *The trick is to understand whether the real world is close enough to your assumptions for the insight gained from your models to be useful. If not you need to develop new models that capture the essential features*

Psychologists Could Benefit from More Theory Based Modeling

- Much modeling in psychology begins with measurement theory rather than psychological theory (though there are good counter examples such as van der Maas et al. *Psych Rev* 2006).
- As such the measurement theory tail sometimes wags the psychological theory dog (as with g theory).
- Hope to give some examples of how more theory based modeling could be helpful using my own work on cognitive ability

- Work I'm reporting on today is an extension of my joint work with Jim Flynn in Dickens and Flynn (2001) "Heritability Estimates vs. Large Environmental Effects: The IQ Paradox Resolved" *Psychological Review,* 108, 346-369 as well as new ongoing work with Eric Turkheimer and Chris Beam.
- Jim's take on some of these issues have been published in his book *What is Intelligence? Beyond the Flynn Effect*
- A working draft of my paper "What is g?" is available on my Brookings web page (www.brookings.edu/scholars/wdickens.htm)

Why is an Economist Working on Cognitive Ability?

- A large part of my career has been about importing psychological theory into economics (for example see "The Economic Consequences of Cognitive Dissonance," *Am. Econ. Rev.* (1982) with George Akerlof)
- When *The Bell Curve* came out in 1994 I was at Brookings and was asked to brief members of the Clinton administration on its implications for policy.
- Briefing turned into an article which caught the attention of Jim Flynn, which led to a collaboration that turned into a bit of a career.

Important Issue for Policy: How Malleable is Cognitive Ability?

- Pre-school programs and adoption both produce large gains in cognitive ability for most participants, but gains in cognitive ability from preschool are mostly gone a few years after programs end.
- Most studies show no evidence of statistically significant correlation of adopted children's cognitive ability with that of adopting family by late adolescence or adulthood.
- Shared family environment explains a moderate amount of variance between young children but seems to disappear as a significant factor in explaining differences in adults leaving mainly variance explained by genetic endowment (55-80%).

Jensen's Box

- Assume M=aG+bE
  - M is measured cognitive ability
  - G is genetic endowment
  - E is environment
  - a and b are coefficients
- Then if E only explains 25% of variance b=.5 (unstated assumption cor(G,E)=0) and
  - Average black environment must be 2SD below average white environment to explain B-W IQ differences without recourse to differences in G
  - But...

In Contrast...

- Large secular gains affect people at all points in life (The Flynn Effect)
- Gains of 1SD or more a generation have been documented
- Almost certainly environmental in origin

Elements of a Theory

- We know from many sources that both genetic endowment and environmental influences affect cognitive ability
- It has also been suggested (often by advocates of the hereditarian view) that cognitive ability affects environment
- So what if we allow ability and environment to have reciprocal effects in a dynamic model (ie add the equation $E_t = c\ M_{t-1} + e_t$ where e_t is a stochastic innovation

Dickens and Flynn 2001

- Worked out the implications of a dynamic stochastic model where there were reciprocal effects between environment and ability.
- Multiplier effects M=>E=>M=>E... blow up the effects of both genetic endowment and persistent environmental causes (differences in G or in the mean of e)
- Environment can be puny or powerful
  - If most environmental effects are transient then they fail to gain the full advantage of the multiplier and may explain little variance in the cross section
  - But long lasting (or permanent) environmental differences can gain the full benefit of the multiplier and have very large impacts on cognitive ability
- As people get older and get more control over their environment the effects of family background fade and genetic endowment becomes more powerful

What Was Missed by Not Modeling

- Both environmentalists and hereditarians had understood part of this but
- While hereditarians pointed out that G=>M=>E
  - could explain why h^2 would get larger with age and c^2 would get smaller with age
  - and that a correlation between E and M didn't mean E=>M
  - they missed the point that reciprocal effects between E and M meant that small persistent differences in exogenous environmental advantage between blacks and whites could get blown up into large differences in ability (that despite high h^2 environment could matter a lot)!
- Some environmentalists understood that reciprocal effects could inflate environmental effects but missed that for the model to be stable (and fit the facts) effects of environmental advantages that were not maintained (pre-school, adoption) would tend to fade

But Are Secular Gains Real Gains in Ability?

- Several papers have now shown that the pattern of gains across time on subtests are not purely g gains
  - gains are much less highly correlated (and in some cases negatively correlated) with g loadings of subtests than (for example) the black-white gap (method of correlated vectors)
  - can easily reject the hypothesis that a gain in g alone explains Flynn gains (Wicherts et al. 2004)
- This has led some hereditarians (for example Rushton and Jensen 2009) to suggest that secular gains are not evidence of the malleability of cognitive ability
- But would we expect the pattern of subtest gains to reflect the pattern of g loadings?
- *What is g?*

A (very) Brief History of g

- Spearman notes correlation of scores on wide range of tests and attributes it to largely inherited differences in a single innate ability
- Thurstone proposes multiple abilities
- Modern compromise (Carroll 1993) hierarchy of abilities with (fluid) g at the top

About g

- More highly g loaded sub-tests
  - have scores that are more highly heritable
  - have stronger correlation with physiological correlates of cognitive ability
  - show the largest black-white gap
- A person's g score is a good predictor of many important life outcomes
- These facts have been interpreted by some to indicate a large role for genetic endowment in individual as well as black-white differences in g
- Some argue that nearly all the ability of IQ tests to explain life outcomes is due to g

A Model of the Factor Structure of Measured Abilities

$$(1) \quad m_i = Ag_i + Ve_i$$

- *m* -- vector of measured abilities for person *i*
- *g* -- vector of measures of genetic endowment
- *e* -- vector of environmental influences
- *A* -- matrix of parameters relating genetic endowment to measured abilities
- *V* -- matrix of parameters relating environmental influences to measured abilities

If We Assume That g and e Are Independent Sources of Variation

- We have the standard behavior genetics decomposition of variance equation (without making the normal distinction between shared and non-shared environment).
- Further we can write down a model that can give a pretty good account of nearly all the facts about g

What (at least some) "g men" probably have in mind (with three abilities)

$$\begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} A_{1g} & A_{11} & 0 & 0 \\ A_{2g} & 0 & A_{22} & 0 \\ A_{3g} & 0 & 0 & A_{33} \end{bmatrix} \begin{bmatrix} "g" \\ g_1 \\ g_2 \\ g_3 \end{bmatrix} + \begin{bmatrix} V_{11} & 0 & 0 \\ 0 & V_{22} & 0 \\ 0 & 0 & V_{33} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$$

- Think of *m* s as 1st or 2nd order factor scores
- A_{1g}-A_{3g} are g loadings
  - They will be highly correlated with subtest heritabilities if A_{11}-A_{33} aren't very big.
  - If there is a black-white difference in average "g" then black-white differences will be strongly correlated with g loadings (and heritabilities if A_{11}-A_{33} aren't very big).

g model (continued)

- If g is something like innate information processing ability its not hard to imagine that it would be a good predictor of school and life outcomes.
- If secular gains aren't biggest on the most g loaded tests they aren't g and perhaps aren't important.

But then how are huge secular gains possible at all?

- If heritability of IQ scores in adulthood are 60 to 80% as most studies suggest (because IQ is mostly g) and model above holds it would take changes in exogenous environment of at least three or more standard deviations to explain changes. What could have changed that much?
- The simple linear model of g has a very hard time accommodating large secular changes in measured IQ scores

An Aside on Environmental g

- Note that if in equation 1 there was an environmental input (say e_1) that affected all the *m* s then that too would produce positive correlations across all the *m* s.
- If there was no other source of correlation the *V* coefficients of the common environmental influence would be strongly correlated with g loadings and if black-white difference was due to difference in this environmental factor then g loadings would be correlated with white-black gap on each factor.
- But, *g* loadings and white-black gap would likely be *negatively* correlated with the heritability of the factor scores.

An Alternative View (A Basketball Analogy)

- Suppose there was no correlation between physical attributes that made one a good basketball player (height, speed, agility, coordination, etc.)
- Scores of very young children who had never played basketball or other sports on tests of basketball skills (shooting, dribbling, passing, etc.) would show little or no correlation across skills
- *But the skills of older children would…*

An Alternative View (continued)

- Those who are taller (or quicker, or have better hand-eye coordination) will be more likely to be good at basketball and more likely to play it more.
- Those who play basketball more will improve *all* skills.
- They will become particularly good at the skills that are used most and most important to success at basketball.
  - These will be the most "basketball g" loaded skills
  - They are the most correlated with the underlying cause of correlation – *practice*
- This can give rise to all the g phenomena described above

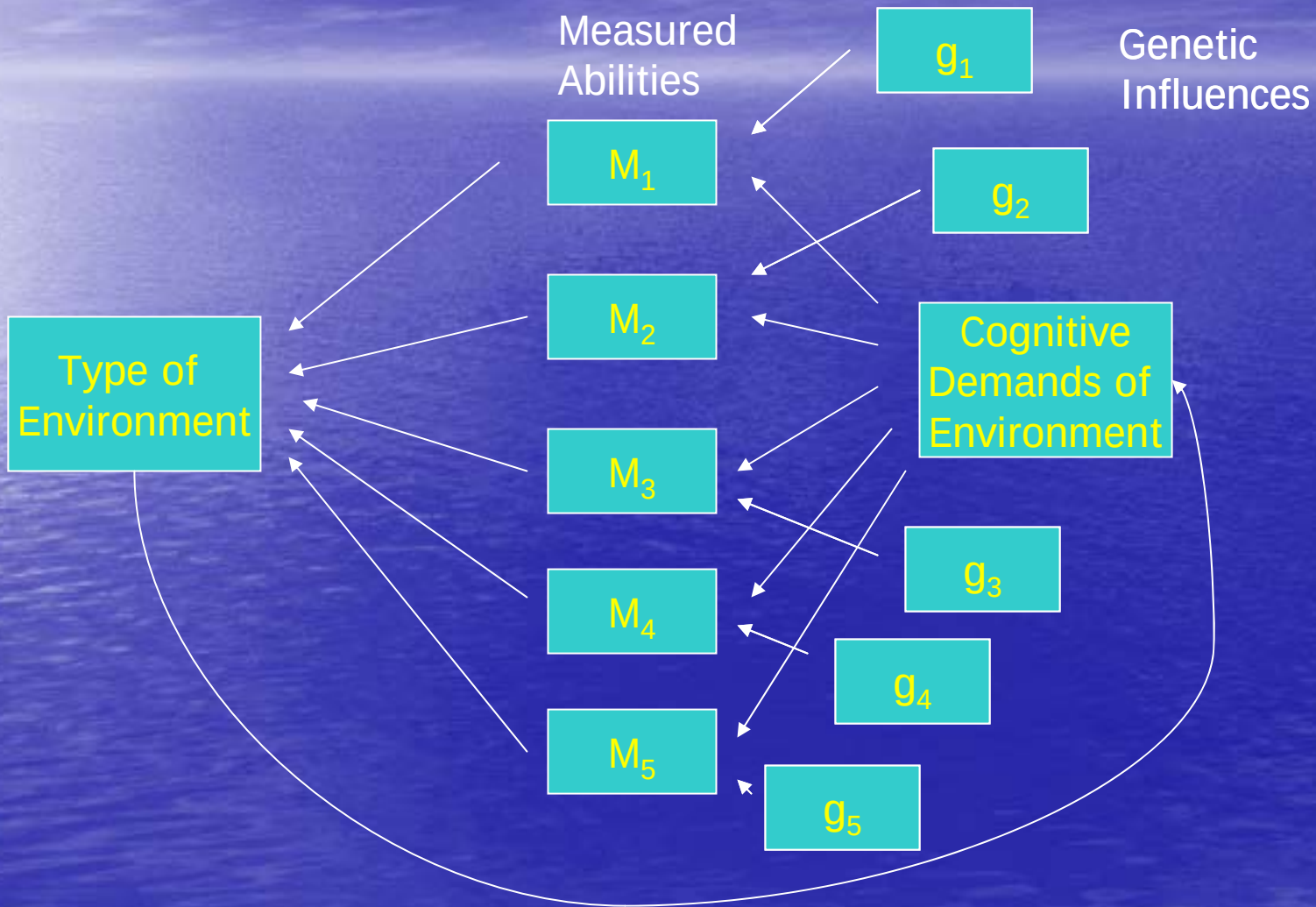
Explaining Facts About g

- Basketball g would be a very good predictor of success at basketball
- Any physiological advantage that made you good at basketball would lead to you practicing more improving all your skills, but particularly those most used and most important to basketball (the most g loaded).
- If a group was discriminated against in access to basketball teams or pick-up games all their skills would suffer, but particularly the skills most used and most important to success.

Secular Gains in the Alternative Model

- Now suppose that a decision was made to make all basketball games last twice as long and to make it illegal for players to return to the court after being taken off for any reason (like soccer).
- All skills might improve if people play more because of the longer games
- But players endurance would improve disproportionately so their scores on tests of endurance would be out of proportion to other gains
- *Thus no reason for gains to be biggest for most g loaded abilities, but they would be substantive – particularly in the new environment.*

Schematic of the Alternative View Reciprocal Effects



Are genetic influences completely independent as I propose to model them?

- To some degree?
  - Lots of evidence that different types of mental activity are highly concentrated in different parts of the brain
  - No biological characteristic explains much IQ variance
- But many physiologically local processes (like working memory) probably affect several (if not all) abilities as defined by a first order factor analysis

Determination of Environment

$$(2) \quad e_i = Bm_i + Wz_i$$

- z -- vector of environmental shocks (exogenous environment not correlated with g_i)
- B -- matrix of parameters relating abilities to environment
- W -- matrix of parameters relating shocks to (endogenous) environment

Solving Equations 1 and 2 for m

- Equilibrium value of m

$$m_i = Ag_i + VWz_i + VBm_i = (I - VB)^{-1}\left[Ag_i + VWz_i\right]$$

(*I* is the identity matrix, ie. *Im=m*)

- Note that
  - if *g* and *z* (shocks to e) are uncorrelated
  - and *B* = 0 (no effects of ability on environment)
  - then we are back to the standard BG decomposition of variance model.

Now Suppose ...

- We simplify equation 1 so that

$$m = \alpha g + \nu e$$

- Here *m*, *g* and *e* are still vectors, but α and ν are now scalars. Thus each element of m is affected only by the corresponding element of *g* and *e*. (eg.for three abilities)

$$\begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \alpha \begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix} + \nu \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$$

There is no "*g*" in equation 1 anymore so we are going to have to get it from equation 2 – the equation that describes the environmental feedback mechanism

Finding *g* in the environment

- Suppose that there is a single dimension to environments that is relevant to the formation of the full range of cognitive abilities – call it cognitive complexity.
- Suppose that the complexity of one's environment increases with all one's abilities and is given by *c'm* where *m* is the vector of abilities and *c* is a vector that weights the importance of each ability for the complexity of environment.
- Further suppose that the more complex one's environment is the better is one's environment for developing each ability, so that e*=fc'm=Bm* where f is a vector describing how much better being in a cognitively demanding environment is for the development of each skill.

Finding *g* (continued)

- Since those abilities that are used most intensely in the cognitively demanding environment are also likely to be most important for determining who is in the environment I'll assume the weights that determine the demands of the environment, and therefore the effects of being in such an environment on the further development of one's skills, are proportional to the impact of complexity on each type of environment or $\beta f = c$.
- Finally, I'll assume that environment for each ability is affected only by its own unique shock (W is a diagonal matrix) and that the weights on the shocks in equation 2 (the Ws) are roughly equal or negatively correlated with f.

With these assumptions it can be proved that...

- The correlation matrix for abilities will have all positive elements
- The first principal component will be positively correlated with all skills
- The loadings of the abilities on the first principal component will have the same rank order as their importance in determining the complexity of the environment (*c*).
- The rank ordering of the heritability of each ability will be the same as the rank ordering of its *g* loading.
- If there is a genetically driven physiological trait that affects only one cognitive ability it will be correlated with all cognitive abilities and the rank order of its correlation with those abilities (except the one it directly affects) will be the same as the rank ordering of its *g* loading.

Proofs (continued)

Finally, if one population is discriminated against in access to cognitively demanding environments so that the complexity of individual's environments is *c'm-d*, then

- Their average score on tests of all abilities will be lower than those with *d*=0.
- The gap between their average score on tests of each ability and the scores of those for whom *d*=0 will have the same rank order as the ability's *g* loading and heritability.

In addition...

- An individual's score on the first principal component will be a very good predictor of the cognitive complexity of that person's environment. If that is correlated with important life outcomes (like educational attainment and income) then the "*g*" score will be a good predictor of those outcomes.
- If the impact of cognitively demanding environments increase on certain abilities
  - there will be an increase in average cognitive ability,
  - the increased ability will lead to further increases in the cognitive complexity of people's environments (the increase is substantive),
  - but the rank order of increases need not be the same as the g loadings or heritabilities.

Theory of Reciprocal Effects Fits Lots of Facts but Are There New Predictions that can be Tested?

- I've suggested a grand longitudinal experiment that would allow one to estimate such a model, but until someone gives me 5-10 million dollars to conduct it ...
- Recently Eric Turkheimer and I began work on testing one important part of the model – the transience of environmental effects
- Most discussions of environmental effects in psychology focus on the cumulative effect of the formative experiences of childhood
- It is therefore surprising that once we control for genetic similarity there is little to no role for being raised in the same family for most traits (notably including cognitive ability)

What About Non-Shared (with other family members) Environment?

- Even there theories have focused on effects of peers in childhood or attempts by siblings to differentiate themselves
- If true then shared environmental effects ought to be very stable over time
- Alternatively, reciprocal effects theory suggests that effects should die out and simple version of the theory suggests exponential decay

A Model with Reciprocal Effects Allowing for Permanent and Transient Shocks

- In work in progress I've shown that time series on identical twin differences can be modeled and that the model can be fit using either ML or MoM.
- Estimation yields three parameters that can be interpreted as the percent of the variance in cognitive ability due to transient environmental effects (identified by their tendency for exponential decay over time), permanent (or very long lasting) environmental effects, and measurement error (uncorrelated over time).
- Eric Turkheimer and I have now estimated this model using the SATSA data

Results

- Typical for several different methods of handling missing data
  - 80% or more of environmental variance is permanent
  - Nearly all of the rest is measurement error
  - Very little, if any, role for exponential decay
- BUMMER! (Did I mention that one of the "advantages" of tightly linking your theory to your empirical work by modeling is that you can be proved decisively wrong?)

So Where Do These Long Lasting Effects Originate?

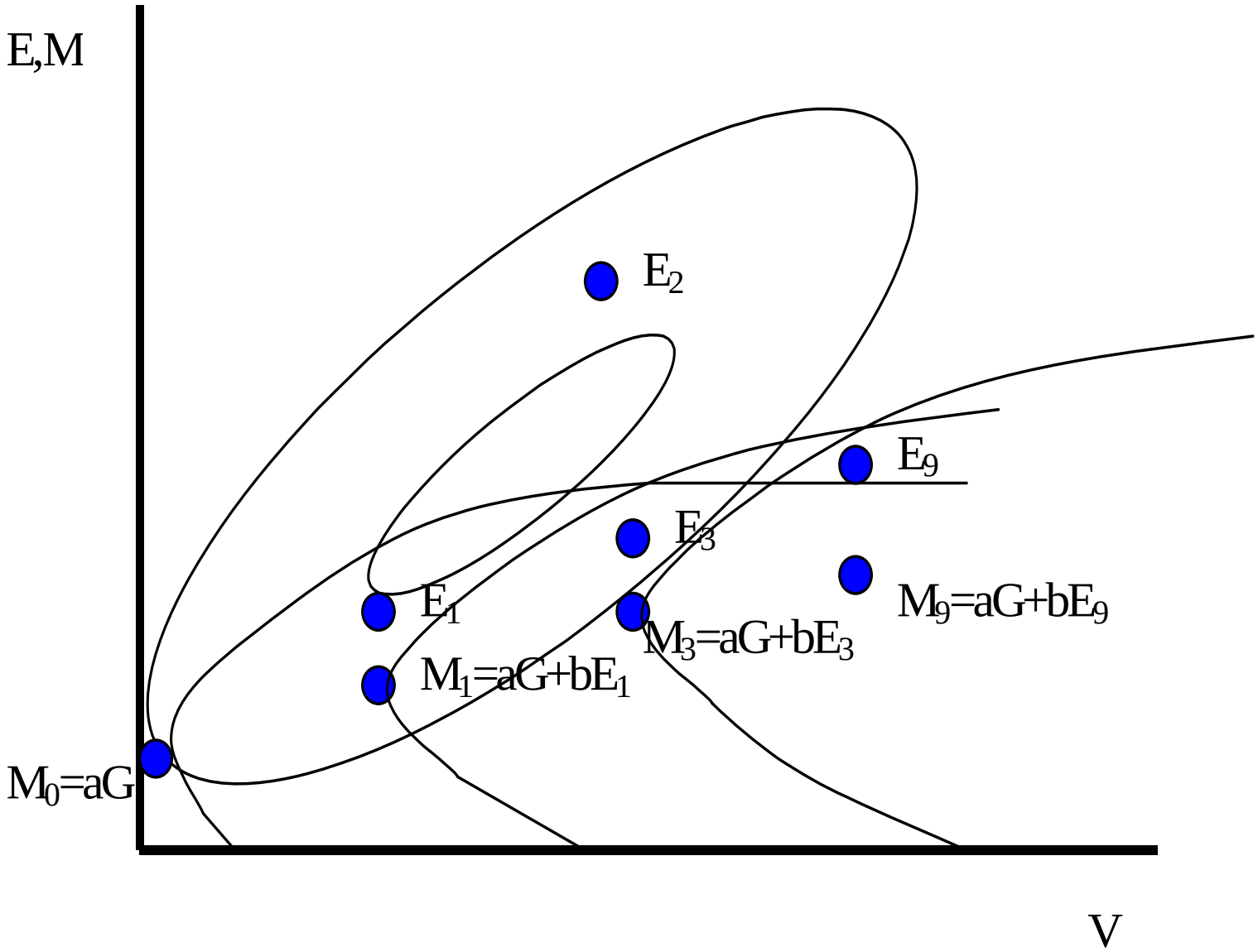
- Not in childhood it seems
  - When we estimate the same model on Danish data for children 5-18 years of age nearly all of the identical twin differences are attributed to “measurement error” (ie the correlation between twin differences over time is essentially zero)
  - Measurements are taken several years apart or more so we might find more correlation (and exponential decay!) if we looked over shorter time intervals
- Recent study shows that correlation of MZ twin differences between ages 18 and 45 are moderate suggesting that the “permanent effects” we see in older adults may be emerging during this time

Looks Like We Need a New Model

- Suppose that in each period a person gets the opportunity to change to a new environment
  - New environment has two components
    - Its cognitive demands
    - Its non-cognitive "value" (how much fun it is or how much long run value it has)
  - Those aspects are stochastic and drawn from a some sort of distribution
- Having an environment that is too cognitively demanding or not cognitively demanding enough causes "disutility" (economese for makes you less happy)
- Person chooses new environment if the value of the environment minus the disutility from cognitive mismatch is greater for the new environment than the old one
- Non-shared environment in this model reflects the degree of cognitive mismatch

Preliminary Results From New Model

- Children start off with low ability and switch environments frequently as their ability increases (thus low NSE correlations in children)
- As young adults find themselves in better and better environments the rate at which they change environments slows down as it becomes less and less likely that they will get a new environment that dominates their old environment
- Eventually changes in environment become very rare and the luck of the draw in the last few environments becomes ones non-shared environmental effects for life
- *Suggests the existence of a "Social Critical Period" in late adolescence or early adulthood that might be more important than physiological critical periods in early life.*


E,M
E2
E9
E3
E1
M9=aG+bE9
M3=aG+bE3
M1=aG+bE1
M0=aG
V