A THEORETICAL FRAMEWORK FOR KENYA'S CENTRAL BANK MACROECONOMETRIC MODEL

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Abstract
The macroeconometric model was developed as a tool that would aid the Central Bank of Kenya (CBK) in forecasting key economic variables in a consistent manner. The economy is modeled in line with economic theory and structure of Kenya's economy. The monetary sector, which is the main focus of the model, is highly disaggregated to capture the various relationships that exist between the monetary sector and the real economy. The external, real and fiscal sectors are not as highly disaggregated and have been incorporated into the model to capture the transmission channels and the impact of monetary policy. This working paper gives the theoretical foundations upon which the CBK macroeconometric model is built. The theoretical framework is discussed with the aim of delineating the model's underlying logical structure and the theoretical base for its core behavioral equations.

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1. INTRODUCTION

This paper presents the theoretical framework for the Central Bank of Kenya (CBK) macroeconometric model. In addition, it highlights the theoretical base for the model's main behavioral equations. The justification for the model relates to its usefulness in aiding the policymaking process at the CBK. It is expected that the model will support the Monetary Policy Committee (MPC) and Research Department in further understanding how the economy works through the complex interactions of various economic agents. The conduct of monetary policy requires fairly accurate analyses and forecasts backed up by sound economic theory and a rationale ensuring that effective monetary policy is formulated and implemented. In this regard, the model will provide consistent short-term forecasts of key macroeconomic variables such as economic growth and inflation. In addition, the model will be helpful in evaluating the impact of various shocks and policies on the economy. The MPC may also use the model as an instrument to help in structuring its communication with the public on the rationale behind its decisions.

This paper is organized as follows. The rest of Section 1 discusses the type of macro model developed, Section 2 presents the model's logical and theoretical framework and illustrates the linkages between the monetary submodel and the other blocks of the model, Section 3 discusses the theoretical foundations of the model's behavioral equations, and Section 4 concludes.

What Type of Model?

Whereas there are different variants of economic models, two types of macroeconomic models stand out for macroeconomic policy analysis and forecasting: dynamic stochastic general equilibrium (DSGE) models and macroeconometric models. The choice between these two categories in a given situation depends on a number of factors: the purpose for which the model is needed, the level of detail and complexity required, and data availability, among other factors. The two types of models complement each other because no single model can adequately answer all economic policy questions or address all needs. DSGE models are a fairly recent development, whereas macroeconometric models date back to 1940s and 1950s, when the emphasis was on constructing quantitative models that could help to describe the dynamics observed in time series economic data.
Macroeconometric models are most suitable for forecasting and policy analysis. Unlike univariate statistical modeling, macroeconometric models provide a comprehensive structural view of the entire economy in a coherent and consistent manner with the level of detail required for policy analysis. Many central banks rely on macroeconometric models for economic projections and analysis, even in cases where they have other sets of models.\(^1\) However, macroeconometric models came under strong criticism in the 1970s following the Lucas (1976) and Sargent (1981) finding that the model coefficients were likely not invariant to policy shifts or structural changes. This was compounded by Sims’s (1980) argument about lack of clearly identified assumptions that deal with simultaneity among macro variables. Despite Lucas’s critique, many central banks continued to rely on reduced-form statistical/macro models for forecasting economic variables (Gali and Gertler 2007).

DSGE models are suitable for analyzing business cycles and the cyclical effects of monetary policy since they emphasize the dynamics of the economy. They promise major benefits for rational policymaking process. DSGE models are relatively new, having arisen in the 1980s and 1990s as a result of two types of literature that emerged in response to the shortfalls of traditional macroeconomic modeling: real business cycle (RBC) theory (Kydland and Prescott, 1982 and 1990) and New Keynesian theory. These sought to provide microeconomic foundations for the Keynesian concepts (Gali and Gertler 2007). The RBC models assume flexible prices, while the New Keynesian School assumes price rigidities. These types of macroeconomic models place greater emphasis on micro foundations and theoretical coherence.

DSGE models begin by specifying a set of economic agents (households, firms, governments), each assumed to make optimal choices, in a way that clears every market. The use of DSGE models at central banks has become widespread across the globe, especially in countries that have adopted an inflation-targeting framework. However, a number of limitations have been noted for these models. First, they tend to perform poorly in terms of quantitative predictions/projections—since they are not suited for economic forecasting. Second, DSGE models are technically more difficult to solve and analyze, and their use in several central banks, particularly in developing African countries, is still constrained by capacity. Third, they abstract from sectoral details, making it difficult to study interactions between individual agents. Fourth, given the strong assumptions (e.g., complete markets), critics have argued that they can be misleading and are unable to describe the highly nonlinear dynamics of economic fluctuations. The validity of these assumptions in poor and developing countries with less developed and incomplete markets is also questionable. Fifth, they may overstate individual rationality and foresight and the degree of homogeneity.

To overcome the challenges and shortcomings of relying on one model, the use of multiple models (i.e., a suite of models) has also become popular with central banks. Given the current forecasting needs and the needs of the MPC, the priority was to first develop a simple macroeconomic model with solid forecasting performance as part of the overall strategy for developing a suite of models for the CBK. This was further motivated by the need to develop a macroeconometric model with a systematic framework entailing a detailed monetary sector and how it relates to the other sectors of the economy. The latter is made possible by the flexibility provided by the macroeconomic type of models, in terms of their ease of application and ability to describe economic relationships based on empirical data and economic theory. To effectively execute its primary task of formulating and conducting monetary policy and its secondary role of promoting economic growth and employment, the CBK takes cognizance of the structural economic interlinkages in the economy.
2. AN OVERVIEW OF THE MODEL

The Structural Model

The structure of the model can be summarized using a flowchart, as shown in Figure 1. The figure shows the structural interlinkages of the model and captures the interaction between the various sectors of the economy (blue boxes). It also shows the interaction between markets—that is, between the financial market and other markets. The white boxes denote different domestic prices determined in the model (consumer price, exchange rate and domestic interest rates). For instance, the interaction of money demand and money supply determines domestic interest rates. Excess money demand acts as one of the triggers of monetary policy actions. The CBK follows the monetary-targeting framework, where it controls quantities (money stock) to affect the prices in the economy. Reserve money (RM) is the operating target and is under the CBK’s control. Broad money, or the money supply (M3), is the intermediate target and is related to reserve money through the money multiplier. CBK uses M3, given that it is perceived to contain all the instruments used to influence policy (currency, demand deposits, quasi-money and foreign currency deposits). Actions on the fiscal side have implications on domestic interest rates (e.g., through the price effect of crowding out or through the effects on the return to government securities relative to other assets). Aggregate demand, price level, exchange rate and interest rates determine money demand. Other markets—that is, the external sector

Figure 1. Logical Framework and Structure of the Model
and the government/fiscal sector—are represented in the bright blue boxes.

Unlike the KIPPRA–Treasury Macro Model (KTMM), which was designed with a detailed government sector to meet the government’s needs in the national budgetary and planning process, the CBK model has a more detailed monetary sector tailored to the needs of the monetary policy process.

The nominal exchange rate is determined in the foreign exchange market by the differentials between domestic and world interest rates, as well as between the domestic price and world prices. For instance, a rise in domestic interest rates relative to foreign interest rates will lead to an appreciation of the local currency. The real exchange rate follows by definition (i.e., from the nominal exchange rate, the foreign price and the domestic price level). The (real) exchange rate has an impact on the exports and imports of goods and services in the external sector. World price and foreign interest rates are exogenous (gray boxes).

At the aggregate level, the model closely follows the aggregate demand (AD)–aggregate supply (AS) framework. In the short run, income is determined in the real economy from the demand side, that is, aggregate demand. Total demand equals the sum of final consumption expenditures by households, investment (capital formation), government expenditure and exports less imports. On the supply side, it is assumed that output is produced in accordance with a constant elasticity-of-substitution production function with capital and labor as key inputs. Production leads to demand for labor and investment goods. Business enterprises produce goods and services that are sold in the domestic as well as global markets. The demand for labor and wages is determined in the labor market. There is a wage-price spiral that can lead to a vicious cycle in which business owners raise prices to protect profit margins from rising costs, while wage-earners push for higher nominal wages to catch up with rising prices to prevent real wages from falling. This may be triggered by higher aggregate demand relative to supply, or the effect of supply shocks such as international oil price hike. Consumer price is domestically determined through the interaction of AD and AS, as well as by excess money having an impact on the price of nontradables. The difference between actual and potential output is known as the output gap. Theoretically, this gap is given by the difference in output between short-run equilibrium, where AD intersects AS, and the long-run AS, which refers to the natural rate of output.

The production sector pays net taxes to the government. Households sell their labor in the labor market and receive income in form of remuneration, as well as dividend payments and transfers from the government. They pay taxes and spend their net income (disposable income) to buy goods and services. Government spending consists of government expenditure (consumption and investment) and transfers to households.

**The Quadrant Approach**

Instead of using the flowchart/logical framework to summarize the macro economy, the alternative approach is the use of the four-quadrant IS–LM–BP framework (see Figure 2). The money market falls in the first quadrant, while the interaction between AD and AS is shown in the second quadrant. AD is downward sloping, while AS is upward sloping. Among other factors, the gap between the AS and AD (output gap) determines the price movement. The foreign exchange market is captured in the third
quadrant. A rise in domestic price leads to depreciation of nominal exchange rate. Given the uncovered interest parity (UIP) assumption, a policy that leads to lower interest rates (relative to foreign interest rates) depreciates the domestic currency. The fourth quadrant is about the internal and external balance. The different prices (general domestic price, interest rates and exchange rate) are the main adjusting components.

**Basic Setup**

**Quadrant I: Goods and the Money Market**

The IS curve is a combination of points where the interest rate ($r$) and output ($Y$) intersect in the goods market. The goods market is in equilibrium along those points. It is negatively sloped to indicate that higher interest rates reduce investment spending, which in turn reduces aggregate demand and income. The LM curve, conversely, shows a combina-
tion of interest rates and output derived in the money market where money demand equals money supply. Along this curve, the money market is in equilibrium. The demand for money is demand for real balances. The curve is upward sloping, implying that demand for real balances increases with increases in income when the money supply is fixed. At the intersection of the IS and LM curves, the goods and money markets are in equilibrium.

**Quadrant II: Aggregate Demand and Aggregate Supply**

The AD and AS curves intersect at a general equilibrium price level. The AD curve is derived from the IS–LM equilibrium income at different price levels. The AS curve reflects the economy’s price adjustment mechanism. The AS curve is upward sloping in the short run but vertical in the long run, as it is expected that equilibrium in the labor market remains the same at different price levels. The labor market does not respond to surprises immediately, hence the upward-sloping AS curve in the short run, due to nominal rigidities such as the sticky wage setup.

**Quadrant III: The Foreign Exchange Market**

The diagram assumes a small, open economy where a floating exchange rate regime is followed. In the foreign exchange market, the endogenous variable in Quadrant III is the exchange rate, $E$, whereas the exogenous variables are the domestic interest rate ($r^d$), the foreign interest rate ($r^f$), the expected exchange rate $E_e$, domestic prices ($P^d$) and foreign prices ($P^f$). The value of the equilibrium exchange rate is affected by domestic and foreign rates of return in the market.

**Quadrant IV: The Balance of Payments**

The internal equilibrium is achieved through $Y = C(y) + l(r) + G + X(e) - Z(p,e,Y)$ and the $Ms/p = L(r) + k(y)$. An appreciation of the exchange rate, $e$, requires a decrease in government expenditures to have an internal equilibrium. An appreciation of $e$ worsens the current account balance position but increases output ($Y$) through the multiplier effect. In order to counteract the change in $Y$, government spending has to decline. An external balance is attained through the balance-of-payments ($BOP$) equilibrium of an open economy. This may be defined by $BOP = X(p,e,Y) - Z(p,e,Y) + F(r)$. The endogenous variable in Quadrant IV is the balance of payments, and the exogenous variables are prices ($P$), the exchange rate ($E$) and income (output) ($Y$). Above the $BOP$ curve, as government spending increases, trade deficits and hence balance-of-payments deficits arise. As a result, the government must utilize its foreign reserves to preserve the exchange rate.

The CBK modeling approach closely follows the Keynesian and monetarist lines of thought, in which fiscal and monetary policy can be used to raise the level of output and employment in the economy in the short run. It contains features of the New Keynesian economics that emphasize sticky or sluggishness in wages and prices to explain why monetary policy affects production activities in the short run and the new classical models that shed light on how households and firms make decisions over time. Combining the different strengths of competing approaches is expected to yield robust results both in terms of the theoretical underpinnings and practical applications of the model.

**Monetary Policy Transmission Mechanisms**

Monetary policy transmission mechanisms describe how policy-induced changes in the nominal money stock or the short-term nominal interest rate have an impact on real variables, such as aggregate output. For instance, the MPC changes the Central Bank Rate (CBR) from time to time to signal the monetary
policy stance it is following. This is represented in Figure 3, and the different transmission channels are briefly explained here.

**Interest rate channel:** Through this channel, the policy-induced increase in the short-term interest rate (i.e., the policy rate) influences the other short-term rates as well as long-term interest rates, such as the lending rate. As a result of increased lending rates, firms cut back on their investment expenditures, as the real cost of borrowing over the horizons has increased. Households also cut spending on consumption of durables due to the higher costs of borrowing. This leads to a fall in aggregate demand and, consequently, lower inflationary pressures (see Mishkin 1995; Ireland 2005).

**Exchange rate channel:** When domestic interest rates rise relative to foreign interest rates, under the UIP assumption, equilibrium in the foreign exchange rate market would require a gradual depreciation of the domestic currency (Ireland 2005). This expected future depreciation initially requires an appreciation of the domestic currency, which makes home-produced goods more expensive relative to foreign-produced goods. Net exports fall, leading to a fall in total demand. The rise in the domestic interest rate above the foreign interest rate also attracts capital inflows, which lead to an appreciation and, depending on the magnitude of foreign direct investment, also have implications for aggregate output. The precise impact of a change in the official rate on exchange rates may, thus, be uncertain.
The credit channel works in two ways: First, the credit channel affects the bank lending channel and, second, it affects the balance sheet of households and firms. When the central bank reduces the money stock by reducing the quantity of reserve money, the bank’s reserves decline, hence reducing the amount banks have available for lending out. Conversely, the reduction in the money stock leads to a worsening of households’ and firms’ balance sheets through the fall in asset and equity prices. This reduces the net worth of the borrowers. Banks then need to screen borrowers in order to avoid adverse selection and then monitor the borrowers to reduce moral hazard. This process reduces the amount of loans given by the banks. Adverse selection and moral hazard are driven by information imperfections in the market. In both cases, the effects would reduce domestic demand, total demand and eventually output. Key assumptions are that bank loans are firms’ principal sources of funds, for which few close substitutes exist.

The changes in the policy rate may influence expectations about the future path of real economic activity and the confidence with which those expectations are held. Changes in perception are likely to affect participants in the financial markets as well as economic agents in other markets (Bank of England 1999). However, it is hard to predict the direction in which such effects work. For instance, a rise in the CBR may be interpreted to imply that the MPC assumes the economy is likely to be growing faster than previously thought, which gives a boost to expectations of future growth and enhances confidence. But the action can also be interpreted to imply the need to slow the growth in order to hit the inflation target, thus yielding expectations of low future growth and lower confidence. Some previous studies on monetary policy transmission mechanisms in Kenya have shown that the bank lending channel is stronger than all the other channels. The balance sheet and expectations channels are not modeled in the model since they are not well developed.

Details of the Monetary Sector and Its Links with the Other Sectors

In broad overview, Figure 4 combines Figure 1, the structural model, and Figure 3, the transmission mechanisms of monetary policy, to other sectors in the economy. The figure provides a more detailed breakdown of the monetary sector of the model, illustrating the central role that monetary policy instruments play and the main channels through which monetary impulses influence other factors such as credit to the private sector in the economy. As before, the other sectors are the fiscal, external and real sectors. The flowchart further shows the relationship between the growth in the money supply and overall inflation and output in the real sector. The ultimate objective of the central bank is to ensure price stability.
ments of short-term rates which eventually affect the long-term rates such as the lending rate. The long-term lending rates would affect credit to the private sector and that will lead to an increase in money supply. Credit to the private sector also affects investment demand in the real sector. Overall inflation is primarily influenced by changes in the prices of imported goods and services, productivity developments, excesses in money demand and price expectations. Wage rate fluctuations also play an important role in price determination.

The money supply is determined via the money multiplier, \( mm \), and reserve money, \( RM \), and disaggregated by sources—that is, net foreign assets (\( NFA \)), net credit to government (\( NCG \)), credit to the private sector (\( PSC \)) and other items net (\( OIN \)). Each is a summation of two components, for CBK and other depository corporations (ODC). The CBK components add up to reserve money. The \( NFA \) is determined in the external sector, that is, through the BOP accounts. \( NCG \) is determined in the government sector by the government’s borrowing requirements net of government deposits at the CBK. This is the same approach that underlies the current monetary program at the CBK. On the liabilities side, money demand is composed of currency outside bank (\( COB \)) and deposits. Deposits can be further disaggregated into demand deposits, quasi-deposits and foreign currency deposits.
3. THE THEORETICAL BASE OF THE MODELED EQUATIONS

The economy is modeled at the macro level, which is in line with the modeling approaches employed in these types of models. In accordance with the underlying theoretical framework that blends the New Keynesian and Keynesian traditions, the model allows for nominal and real inertia as well as the role of aggregate demand in determining output in the short run. In the long run, the economic variables gravitate toward their corresponding supply-side-determined steady states or equilibrium levels. The model captures the Kenyan economy as a small, open economy. The key variables are determined in the model as endogenous variables. The main sectors considered and the main core equations estimated are summarized in Table 1. The equations have been developed in line with the theoretical framework laid out in this paper. The sectors (monetary, external, real and fiscal sector) are as described from the flowcharts given as Figures 1 through 4. The following subsections discuss the theories underpinning the core equations in Table 1.

Table 1. Summary of Core Sectors and Equations

<table>
<thead>
<tr>
<th>SECTOR</th>
<th>CORE EQUATIONS</th>
</tr>
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<tbody>
<tr>
<td>Money market</td>
<td><strong>MONETARY SECTOR</strong></td>
</tr>
<tr>
<td></td>
<td>At equilibrium, aggregate money demand ($M^d$) equals money supply ($M^s$). The deviation between the two, depending at what level the economy is operating, has implications on inflation and hence triggers monetary policy action. $M^d$ is determined via money multiplier ($mm$), i.e., as a product of $mm$ and reserve money ($RM$). This equals money sources ($NFA + NCG + PSC + OIN$). NFA is net foreign assets, NCG is net credit to Government, PSC is private sector credit and OIN is other items net. The transmission mechanism works through the domestic interest rates, i.e., from policy rate ($r_{CBR}$) to short-term rates (interbank rate($r_{int}$), repo/reverse repo) to long-term deposit rate ($r^D$), lending rate ($r^L$) to the real sector (private investment). The credit channel works via the private sector credit. Treasury bill rate ($TBR$), which is considered a medium-term rate, is a risk-free rate and alternative rate of return. $r^{CBR}$ is policy rate determined by the MPC. On the liabilities side (demand side), money (M3) is disaggregated into currency outside banks (COB) and Bank Reserves (BR). BR is further decomposed into its components (cash in tills, required reserves and excess reserves).</td>
</tr>
<tr>
<td></td>
<td>$M^d = mm \times RM$, where $mm$ is money multiplier</td>
</tr>
<tr>
<td></td>
<td>$M^s = NFA + NDA + OIN$</td>
</tr>
<tr>
<td></td>
<td>Where $NFA = NFA_{CBK} + NFA_{ODC}$ and $NDA = NCG + PSC + OPC$</td>
</tr>
<tr>
<td></td>
<td>$PSC = f(lending\ rate, GDP, credit\ to\ public\ sector)$</td>
</tr>
<tr>
<td></td>
<td>$M_P = f(RGDP, CPI, TBR, US\ TBR\ and\ exchange\ rate)$</td>
</tr>
<tr>
<td></td>
<td>$COB = f(RGDP, CPI, TBR)$</td>
</tr>
<tr>
<td></td>
<td>Excess reserves $= f(\text{interbank rate, TBR, government deposits})$</td>
</tr>
<tr>
<td></td>
<td>$r^D = f(r^{CBR}, \text{excess reserves, government deposits})$</td>
</tr>
<tr>
<td></td>
<td>$TBR = f(r^*, \text{price\ expectations\ (lagged\ CPI), GDP, NSE\ index})$</td>
</tr>
<tr>
<td></td>
<td>$r^* = f(TBR, \text{real GDP})$</td>
</tr>
<tr>
<td></td>
<td>Deposit rate $= f(\text{interbank rate, TBR, GDP, NSE index})$</td>
</tr>
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<td></td>
<td>The repo and reverse repo operationalize the $r^{CBR}$</td>
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</table>

A THEORETICAL FRAMEWORK FOR KENYA'S CENTRAL BANK MACROECONOMETRIC MODEL
<table>
<thead>
<tr>
<th>SECTOR</th>
<th>CORE EQUATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>BOP accounts</td>
<td><strong>EXTERNAL SECTOR</strong>&lt;br&gt;Core accounts of BOP are the current account (CA), capital and financial account (KA). Under current account, export/import of goods and export/import of services is determined separately. Export of goods ($X_{good}$) is determined based on small country assumption as a function relative price (weighted export price as a ratio of world non-fuel commodity price index) and income of Kenya’s key trading partner. Import of goods ($M_{good}$) is determined by domestic income and relative price (weighted producer price index of trading partners (source countries for imports) relative to domestic price (CPI), while import of services is a function of domestic income.&lt;br&gt;&lt;br&gt;Identities and bridge equations link the capital account to the money supply process and import and export demands to the real sector for consistency.&lt;br&gt;&lt;br&gt;Exchange rate ($e$) is one of the fundamental prices in the model. A modified PPP (i.e., domestic price ($P$), foreign price ($P^<em>$) and interest rate differential ($r - r^</em>$)) determines the exchange rate in the long run. Changes in these variables together with the current account balances (CA) capture the short run dynamics. Interest rate differentials absorb deviations from the PPP.</td>
</tr>
<tr>
<td>Foreign asset market</td>
<td>$BOP = KA + CA$&lt;br&gt;$X_{good} = f(\text{relative price, weighted income of trading partners})$&lt;br&gt;$X_{services} = f(\text{weighted income of trading partners, political stability index})$&lt;br&gt;$M_{good} = f(\text{relative price, real GDP})$&lt;br&gt;$M_{services} = f(\text{weighted GDP of trading partners, real GDP})$&lt;br&gt;$e = f(P, P^<em>, r - r^</em>, CA)$</td>
</tr>
<tr>
<td>Aggregate demand</td>
<td><strong>REAL SECTOR</strong>&lt;br&gt;Aggregate demand (AD) equals total available supply in the economy (AS).&lt;br&gt;&lt;br&gt;AD is derived as a summation of private consumption ($C$), investment demand ($I$), government expenditure ($G$) and exports ($X$) less imports ($M$). $C$, $I$, $X$ and $M$ are endogenously determined as behavioral equations. $X$ and $M$ are already determined in the external sector. Investment function is an eclectic of flexible accelerator and neoclassical theories.&lt;br&gt;&lt;br&gt;CPI/inflation is modeled as emanating from different markets/sources (i.e., money market, imported inflation, output gap).</td>
</tr>
<tr>
<td></td>
<td>$C = (\text{GDP, interest rate}(r'))$&lt;br&gt;$I = f(\text{interest rates}, r', \text{GDP}, \text{private sector credit, SC, as a share of total credit})$&lt;br&gt;$CPI = f(\text{excess money demand, imported inflation, wage inflation and output gap})$</td>
</tr>
</tbody>
</table>
### The Demand for and Supply of Money

#### The Demand for Money

The demand for money is rooted in the classical quantity theory of money, as articulated in the work of Fisher (1911b). The Fisher equation of exchange has been modified over time to become

\[ M^D = \frac{PY}{V} \tag{1} \]

Using \( k \) to represent inverse velocity, this equation becomes

\[ M^D = kPY \tag{2} \]

This equation of exchange states that the level of nominal income \((PY)\) determines the quantity of money that people demand. However, this formulation assumes that interest rates have no effect on the demand for money. It is this assumption with which Keynes (2006) disagrees in his paper, which argues that interest rates affect the demand for real balances, as specified in (3):

\[ \frac{M^d}{P} = f(Y, i) \tag{3} \]

where \( Y \) is real income and \( i \) is the nominal interest rate.

Combining the Fisher equation of exchange and Keynes's formulation—and taking into account that Kenya has a small, open economy with a floating exchange rate—the CBK model postulates money demand as a function of real GDP, the price level, nominal interest rates and the nominal exchange rate:

\[ M^d = \alpha Y + \beta P - \gamma i + \phi E \tag{4} \]

where \( M^d \) denotes nominal demand for money, \( Y \) is real GDP, \( i \) is the nominal interest rate, and \( E \) is the nominal exchange rate. The interest rates are market determined through the forces of money demand and money supply. Movements in the nominal exchange rate are assumed to affect the demand for money.

#### Money Supply

The CBK follows a monetary-targeting framework to implement monetary policy. The operating target is reserve money, and the CBK controls reserve money to influence the broad money supply through the money
multiplier process. The CBK’s balance sheet is represented by (5):

$$RM = e.NFA + NDA$$  \hspace{1cm} (5)$$

Equation 6 shows the relationship between the money supply and reserve money in (5). The money supply in the CBK model is thus defined as a function of reserve money and the money multiplier:

$$M^* = mm \times RM$$  \hspace{1cm} (6)$$

where $M^*$ is the broad money supply, $mm$ is the money multiplier, and $RM$ is reserve money.

The money multiplier can be described generally as

$$mm = \frac{1 + cc}{cc + rr}, \hspace{0.5cm} cc = \frac{cu}{d} = f(f)$$

and $rr$ is the reserve-deposit ratio, which is assumed to be exogenous.

In an open economy, with the $LM$ curve representing the money market equilibrium $M^d = M^s$, the interest rate moves to clear the money market; thus, nominal interest rates are a function of the money supply, real GDP, prices, the repo rate, the volume of Treasury bills and the exchange rate. In equilibrium, the demand-for-money equation is sufficient to show the dynamics of the interest rate, given that the interest rate is an inverted form of money demand:

$$i = \alpha Y + \beta P + qE - M^*$$  \hspace{1cm} (7)$$

This interest rate, further, gives us the link between the foreign exchange markets and the money market, given that the interest rate differential is one of the determinants of the exchange rate.

### The Monetary Policy Reaction Function

As early as 1898, Wicksell argued that the objective of the central bank should be to maintain price stability, which in theory could be realized if the interest rate were set such that it was equal to the economy’s natural rate of interest. Given that the natural rate was an unobservable and abstract concept, he argued that a simple policy rule that responded systematically to prices would suffice.

The CBK currently describes its monetary policy framework in terms of targets on monetary aggregates. Under the financial programming framework being used, assumptions of economic growth and inflation targets enable the desirable growth path of monetary aggregates targets on a monthly and quarterly basis. However, with the formation of the MPC, the CBK seems to be laying its emphasis on the policy instrument (the CBR) to signal the policy direction. This implies that it is also feasible to consider modeling a Taylor-type of monetary policy rule. In modeling the monetary policy reaction function, therefore, both aspects need to taken into consideration.

The approach follows Berg et al. (2010), in which the CBK implements an active monetary policy that responds both to inflationary pressure and output fluctuations. As such, the monetary policy reaction function, with the monetary aggregate as the instrument, could be specified as

$$\Delta M_t^* = \Delta M_t^* - (m - m^*)_{t-1} + (\pi_t - \pi^*) - \delta(E_t(\pi_{t+1}) - \pi^*) - \gamma(y - y^*)$$  \hspace{1cm} (8)$$

Where $\Delta M_t^*$ is the long-run equilibrium change in money base, $m$ is real money balances, $\pi$ is actual inflation in time $t$, $\pi^*$ is the target or long-term rate of inflation, $y$ is actual output and $y^*$ is potential output. $E_t$ is the expectation at time $t$, about what
the variable will be in the following period. Finally, $m^*$ is the real money balance, which ensures that $(\pi_t - \pi^*) = 0$ and $(y - y^*) = 0$.

By choosing a particular growth target for monetary aggregate in (8) above ex ante, the CBK implicitly has chosen a short-term interest rate that is mutually consistent with the chosen money target through the money demand function specified in (4). For example, to derive the implication of (8) for the relevant nominal interest rate involves solving for the nominal interest rate by substituting (8) in the money demand function specified in (3) (for a similar approach, see Berg et al. 2010). This yields (9):

$$r^p = r^* + \pi^* + \varphi_1 (E_t(\pi_{t+1}) - \pi^*) + \varphi_2 (y_t - y^*)$$  \hspace{1cm} (9)

where $r^p$ is the policy rate and $r^*$ is some notional equilibrium (natural) interest rate, and both are in nominal terms.

The reaction function in (8) and (9) above postulates a Taylor-type, forward-looking monetary policy framework that reacts to deviations of inflation from its long-run path and fluctuations of output from its full potential. The parameters in (8) could be derived from the estimation of the demand function (4) and the weights that the MPC attaches to the deviations of inflation from the target and output from the target.

**Interest Rate Modeling**

Several theories have been advanced to explain how interest rates are determined. These include the Fisher equation, Keynes’s liquidity preference, and the term structure theories, which include the expectations hypothesis, market segmentation theory and liquidity premium theory.

**The Fisher Equation**

The Fisher equation is a theory of interest rate determination whereby the nominal interest rate is equal to the real interest rate plus the expected level of inflation. This relationship is captured in the equation

$$i_t = r_t + \pi^*_t$$  \hspace{1cm} (10)

where $i_t$ is the nominal interest rate, $r_t$ is the real interest rate, and $\pi^*_t$ is the expected rate of inflation. Short-term interest rates, therefore, depend on expected inflation in the near future, while longer-term interest rates depend on expected inflation in the more distant future, if the investment was worthwhile.

**The Loanable Funds Theory**

Wicksell’s loanable funds theory implies that the equilibrium interest rate will be determined by the demand and supply conditions in the market for loanable funds. Thus, market interest rates are determined by the factors that control the supply of and demand for loanable funds. These funds are financial assets, which mainly comprise bank loans and household savings. The market for loanable funds brings together households, business firms, government and foreigners as either borrowers or savers. The intersection of the supply and demand curves generates an equilibrium interest rate that clears the market for loanable funds. At equilibrium, national savings equal domestic savings plus net foreign investment. The supply-for-loans function is upward sloping, implying that the suppliers of loanable funds will supply more as the return on their savings increases. Conversely, the demand schedule is downward sloping, implying that borrowers will demand for loans as the price for loans declines. Since borrowers seek loans to expand capital in order to produce more goods and services, they assess the cost of loanable funds (the interest rate) vis-à-vis the additional rev-
enue from the additional capital (the rate of return on capital). Hence, borrowers' demand for additional capital (loans to expand capital) will continue to grow so long as the return on the borrowed funds exceeds the interest rate on the borrowed funds, and will stop at the point where the interest rate equals the rate of return on capital. That is, a company’s demand continues to grow so long as the rate of return on its capital continues to increase.

**Keynes’s Liquidity Preference**

According to the liquidity preference theory, the market rate of interest is determined by the demand for and supply of money balances. This theory states that people demand money for transactions, as a precaution and for speculative purposes. The transaction demand and precautionary demand for money increase with income, while the speculative demand is inversely related to interest rates because of the forgone interest. The supply of money is determined by the monetary authority (i.e., the central bank), by the lending of commercial banks and by the public preference for holding cash.

This is consistent with the liquidity preference theory of the term structure of interest rates, whereby interest rates are expected to increase as the maturity profile of securities increases. This is so because the longer the maturity, the greater is the uncertainty; and therefore the premium demanded by investors to part with cash increases as the maturity profile increases. The expectation, therefore, is that forward rates should offer a premium over expected future spot rates since those who are risk-averse demand a premium for securities with longer-term maturities. A premium is offered by way of greater forward rates in order to attract investors to longer-term securities. Consequently, current interest rates reflect expected inflation rates, income (GDP) and expected money supply changes:

\[ i = f(GDP, m', \pi') \]  \hspace{1cm} (11)

**The Expectations Theory of the Term Structure**

According to expectations theory, long-term interest rates are averages of expected future short-term interest rates. Thus, a return on a two-year security should be equal to the anticipated return from investing in two consecutive one-year securities:

\[ (1 + R_2)^2 = (1 + R_1) (1 + t+1 r_1) \] \hspace{1cm} (12)

where \( R_1 \) and \( R_2 \) are, respectively, the known annualized rate on one-year and two-year securities and \( t+1 r_1 \) is one-year interest rate that is anticipated in time \( t+1 \). This can be generalized to \( n \) periods, so that an “\( n \)”-year security should offer a return that is equal to the expected return from investing in \( n \) consecutive one-year securities:

\[ (1 + R_n)^n = (1 + R_1) (1 + t+1 r_1) (1 + t+2 r_1) \cdots \cdots \cdots (1 + t+n r_1) \] \hspace{1cm} (13)

where \( t+n r_1 \) is the one-year interest rate that is anticipated as of time \( t+n \) and \( R_n \) is the known annualized rate on an “\( n \)”-year security.

**Interest Rate Modeling for the CBK Macroeconometric Model**

The interest rate equation for the CBK model incorporates all the aspects of the three theories discussed above. In addition, based on the MPC’s market survey—which indicates that lenders take into account short-term rates (interbank, Treasury bill rates) in the lending rate—the interest rate equation is modeled as
\[
i_t = \mu + \theta_1 i_{t-1} + \theta_2 \text{RGDP}_{t-1} + \theta_3 \text{CPI}_{t-1} + \theta_4 \text{Depr}_{t-1} + \\
\quad \theta_5 \text{Tbill}_r + \sum_{j} \theta_{6,j} \Delta i_{t-j} + \sum_{k} \beta_k \Delta \text{RGDP}_{t-k} + \\
\quad \sum_{l} \gamma_l \Delta \text{CPI}_{t-l} + \sum_{m} \omega_m \Delta \text{Depr}_{t-m} + \sum_{n} \alpha_n \Delta \text{Tbill}_{r,n} + \epsilon_t \quad \text{(14)}
\]

Where \( i \) is the overall weighted average lending rate of commercial banks; \( \text{RGDP} \) is real output; \( \text{CPI} \) is the consumer price index; \( \text{Tbill} \) is the average 91-day Treasury bill rate and \( \text{Depr} \) is the overall weighted average deposit rate of commercial banks.

### Inflation Determination

Inflation in open economies results from different disequilibria in many markets—specifically, the domestic money market, external/foreign markets and the labor market. Thus we recognize that inflation emanates from three main sources in the CBK model: excess money supply, foreign prices and cost push factors.

#### The Monetarist Model of Inflation

The monetarist views inflation as “always and everywhere a monetary phenomenon” (Friedman 1963). Further, macroeconomic theory proposes that growing the money supply in excess of real growth causes inflation. This is also the result from the Harberger (1963) model, which assumes “that prices adjust to excess money supply in the money market.” It is on the basis of this assumption that it is possible to invert the real money demand equation as a price equation in equation 15:

\[
P = f(M, y, i)
\]

where \( P \) is the price level, \( M \) is the nominal money supply, \( y \) is real GDP, and \( i \) is the nominal interest rate.

On the basis of the Harberger model, which makes it possible to invert the real money balances function to become a price equation, we obtain

\[
P_t = \alpha_1 M_t - \alpha_2 y_t + \alpha_3 i_t, \quad \text{(16)}
\]

#### Imported Inflation

Modeling imported inflation follows either the purchasing power parity (PPP) or the UIP condition or a combination of the two.

The theory of PPP requires that nominal exchange rates move to equalize the price of goods and services across countries. There are two versions of the PPP: the absolute version (law of one price) and the relative version. Following the relative PPP theory, inflation rate differentials between two countries or regions are offset through inverse changes in the nominal exchange rate so that purchasing power ratio between the two remains constant.

The relative PPP is formally expressed as

\[
e_t + c = p_t - p^*_t \quad \text{(17)}
\]

where \( p_t \) is the log of the domestic price level, \( p^*_t \) is the log of the foreign price level, \( e_t \) is the log of the spot nominal effective exchange rate (i.e., the Kenya shillings price of a unit of foreign currency) and \( c \) is a constant representing the permanent deviation from absolute PPP due to productivity differentials and other factors.

Since goods arbitrage is slow, PPP represents a longer-term relationship than equation 17 suggests. There are a number of factors that could drive the exchange rate temporarily away from the PPP. These include commodity prices, different interest rate structures, relative growth differentials and speculative price
movements. Whenever there is a deviation from PPP, it is expected that the exchange rate will move smoothly toward restoring relative PPP, as shown in equation 18:

$$\Delta e_{t+1} = \alpha [p_t - p^*_t - e_t - c]$$

(18)

where $0 < \alpha < 1$.

Conversely, the theory of UIP requires that if interest rates in Kenya are higher than similar interest rates in the United States, then investors will have an incentive to purchase Kenyan assets. UIP can be expressed algebraically as

$$E_t(e_{t+m}) - e_t = (r^*_m - r^*_t) + u_t$$

(19)

where $E_t = \text{is the expectation on the basis of all information available at time } t$, $r^*_m = \text{is the yield on domestic assets with maturity } m$ at time $t$, $r^*_t = \text{an equivalent foreign interest rate}$, and $u_t = \text{is the risk premium associated with holding Kenya shilling assets}$.

The available evidence shows that PPP and UIP have been rejected individually due to a failure to recognize the systematic relationship between the two conditions. Following Choy (2000) we combine PPP and UIP as follows.

Assuming rational expectations, equation 18 can be rewritten as

$$E_t(e_{t+m}) - e_t = a[p_t - p^*_t - e_t - c]$$

(20)

The exchange rate expectations also feature in the UIP condition in equation 19. If we assume that PPP forms the basis of the expectations in the UIP assumption, we can substitute equation 20 for expectations in equation 19;

$$a[p_t - p^*_t - e_t - c] = i^*_m - i^*_t + u_t$$

(21)

Rearranging,

$$e_t - p_t + p^*_t + \frac{1}{\alpha} (i^*_m - i^*_t) + k = 0$$

(22)

where $k = c + \frac{u}{\alpha}$

In the real world, the relationship in equation 22 is not deterministic. Speculative activity or commodity price movements could, for instance, lead to a sustained and significant deviation from equation 22. Thus equation 22 can be viewed as an equilibrium condition toward which prices, interest rates and the exchange rate tend to move in the long run—that is, prices, interest rates and the exchange rate are cointegrated:

$$\beta_i e_t + \beta_{i-1} p_t + \beta_{i-2} p_{t-1} + \beta_{i-3} i^*_t + \beta_{i-4} i^*_m + \beta_{i-5} i^*_t + \beta_{i-6} i^*_m + \beta_k l = I(0)$$

(23)

Equation 21 can be further relaxed in three ways to take into consideration the weak form of PPP and UIP. First, we are not concerned with meeting the actual magnitudes stated in equation 22 but rather the direction. Second, the analysis takes into consideration the small country assumption, whereby external variables are treated as weakly exogenous. Finally, crude oil prices are added as an additional variable given the fact that Kenya is a net importer of oil.

**Wages and Inflation**

There are three different levels of wage formation in Kenya: unionized, competitive and administered. This implies that the observed wage is a function of competitive, administered and bargained wages, which can be explained by different theories of unemployment.

First is the *efficiency wage* theory, in which there is a cost as well as a benefit to the firm of paying lower
wages (e.g., the model of Shapiro and Stiglitz 1984). The second are the contracting models (explicit and implicit contracts). Under implicit contracts, there is a long-term relationship between firms and workers. Firms do not hire workers afresh every period but, rather, many jobs involve long-term attachments and considerable firm-specific skills acquired by workers. This model implies that wages do not have to adjust to clear the labor market each period. Thus, workers are content to stay in their current jobs as long as the salary they expect to obtain is preferable to the outside job opportunity. This model is applicable to some labor, especially in central and local government, as well as some private firms.

The third model is the insider-outsider model, which assumes that there are two groups of potential workers. The first group is the insiders who have some connection with the firm at the time of bargaining, and thus the wage contract takes their interest into account. The second group are the outsiders who have no initial connection with the firm but who may be hired after the contract is set. This model assumes that the outsiders’ and insiders’ wages are linked.

The fourth model is the bargaining theory of wages, which holds that wages, work hours and conditions are dependent on the relative bargaining strengths of the parties involved in the agreement. The details of wage bargaining are presented by Huizinga et al. (2001) (see appendix 2).

The different bargaining structures show different relationships between an increase in the real consumption wage and the real product wage (see Flanagan, Moene and Wallerstein 1993). Under markup pricing, with increases in wages in one sector and not others, output price will increase but the impact on the consumer price will be smaller. Conversely, if bargaining is centralized so that wages rise in most of the economy, then the increase in real product wages will be similar to the increase in the consumption wage. The price wedge also depends on the extent of product market competition. If a particular industry is exposed to intense foreign competition, product prices cannot be increased as much, even though domestic wages rise. However, since the consumption basket contains imported goods and services, foreign trade and the domestic service sector will also influence the price of real consumption wages.

If a nationwide wage increase initially affects a highly exposed industry with output prices already at or above the competitive foreign trade levels, the industry has three options. First, it can reduce employment until the marginal cost equals the competitive price. Second, it can increase labor productivity. Or third, it can close down the industry with regard to labor productivity. It is important to make a distinction between a rise in labor productivity due to technological progress or an elimination of previous excess capacity.

In Kenya, there are formal collective wage bargaining agreements, and there are also administered minimum wage guidelines. The government legislates minimum wages through wage guidelines, introduced in 1973, to guide the Industrial Court in adjudicating trade disputes. The government made some changes in 1994. Wage guidelines were liberalized, allowing workers and employers greater freedom in wage negotiations for maximum compensation for rises in the cost of living. The wage guidelines give consideration to productivity and performance in addition to changes in the cost of living as reflected by the inflation rate.

The empirical equation for testing is in the form

\[ w_t - p_{c,t} - pr = a_0 + a_1(p_{c,t} - p_{p,t}) + a_2 r + a_3 + \epsilon_{t} \]  

(24)
where \( w_t - p_{c,t} - pr_t \) is the negative of the markup; \( p_{c,t} - p_{r,t} \) is the price wedge (i.e., the ratio of the CPI to the GDP deflator; \( rer_t \) is the real exchange rate; and \( r_t \) is the real interest rate. The expectation is that \( 0 \leq a_1 \leq 1 \) and measures the bargaining power of the labor unions; \( a_2 \geq 0 \) (i.e., lower markup following exchange rate appreciation; Phelps 1994); and \( a_3 \geq 0 \) (i.e., higher markup following an increase in real interest rates).

### The Consumption Function

In this model, the consumption function is derived from an intertemporal choice model in which a representative consumer faces the following utility function:

\[
U = U(C_0, C_1, \ldots, C_T)
\]

Assuming a logarithmic utility function suggests that at time \( T \), consumption will be

\[
U(C) = \ln C_0 + \frac{\ln C_1}{1+\rho} + \ldots \ldots + \frac{\ln C_T}{(1+\rho)^T} \tag{25}
\]

Given a level of income in each period, the budget constraint becomes

\[
Y_t + \frac{Y_t}{1+i} + \ldots + \frac{Y_t}{(1+i)^T} - C_t - \frac{C_t}{(1+i)} - \ldots - \frac{C_t}{(1+i)^T} = 0 \tag{26}
\]

In compact form, this can be written as

\[
\sum_{i=0}^{T} \frac{Y_t}{(1+i)^i} - \sum_{i=0}^{T} \frac{C_t}{(1+i)^i} = 0 \tag{27}
\]

The intertemporal problem becomes

\[
\text{Max} \sum \frac{\ln C_t}{(1+\rho)^t} \text{ subject to } \sum \frac{Y_t}{(1+i)^i} - \sum \frac{C_t}{(1+i)^i} = 0 \tag{28}
\]

We form a Lagrangian expression as

\[
L = \sum \frac{\ln C_t}{(1+\rho)^t} + \lambda \left[ \sum \frac{Y_t}{(1+i)^i} - \sum \frac{C_t}{(1+i)^i} \right] \tag{29}
\]

The first-order conditions for optimization are as follows:

\[
\frac{\partial L}{\partial C_0} = \frac{1}{C_0} - \lambda = 0 \quad \frac{\partial L}{\partial C_1} = \frac{1}{(1+\rho)C_1} - \frac{\lambda}{(1+i)^1} = 0
\]

\[
\frac{\partial L}{\partial C_{t-1}} = \frac{1}{(1+\rho)^{t-1}C_{t-1}} - \frac{\lambda}{(1+i)^t} = 0
\]

\[
\frac{\partial L}{\partial \lambda} = \sum \frac{Y_t}{(1+i)^i} - \sum \frac{C_t}{(1+i)^i} = 0 \tag{30}
\]

At two consecutive periods, say period \( t \) and period \( t+1 \), we can solve for \( \lambda \) from the first two equations to obtain

\[
\frac{C_{t+1}}{C_t} = \left( \frac{1+i}{1+\rho} \right) \quad \text{or} \quad C_{t+1} = \left( \frac{1+i}{1+\rho} \right) C_t \tag{31}
\]

If \( i = \rho \), then

\[
C_{t+1} = C_t \tag{32}
\]

This equation says that present consumption is purely a function of immediate past consumption, as shown by Friedman (1957). Combining this result with the budget constraint \( C_t + \frac{C_{t+1}}{1+i} = Y_t + \frac{Y_{t+1}}{1+i} \) yields the following results:

\[
C_{t+1} = C_t \left( \frac{1+i}{1+\rho} \right) Y_t + \frac{1}{(2+\rho)} Y_{t+1} \tag{33}
\]

This can be generalized as

\[
c = f(y, i, z) \tag{34}
\]

where \( y \) refers to current and future incomes, \( i \) is the interest rate and \( z \) is other factors that affect consumption.
The Private Investment Function

Several theories have been advanced to explain investment behavior. These include the accelerator principle, the neoclassical theory—of which Dale W. Jorgenson’s user cost model is central—Keynesian theory, Tobin’s $q$ theory and irreversible investment models. This section primarily focuses on the neoclassical theory of Jorgenson’s user cost.

The neoclassical theory on investment attributed to Jorgenson emphasizes the role of factors that determine a firm’s profitability given its production function parameters. The key generalization of the Jorgenson’s neoclassical model is that demand is a key variable that influences the desired capital stock for a typical firm, and that demand for capital is positively related to output but inversely related to the user cost of capital. The optimal investment conditions for a typical firm can be derived assuming various functional forms of the production functions, such as constant returns to scale, Cobb-Douglas and constant elasticity of substitution (CES).

Let $Q$, $K$ and $L$ respectively represent output, capital and labor; and let $p$, $r$ and $w$ respectively represent the prices of output, capital and labor. Given a linearly homogeneous production function

$$Q = f(K, L)$$

the profit function can be derived as total revenue minus costs:

$$\pi = pQ - rK - wL$$

A typical firm tries to maximize its profits (37), subject to the conditions that its labor and capital inputs are in line with conditions set by the production function (36). Setting the Lagrange condition and solving for the price of capital $r$ and the wage rate $w$ yields

$$H = pQ - rK - wL - \lambda (Q - f(K, L))$$

The first-order conditions for profit maximization require that

$$\frac{\partial H}{\partial L} = -w + \lambda \frac{\partial Q}{\partial L} = 0$$

$$\frac{\partial H}{\partial K} = -r + \lambda \frac{\partial Q}{\partial K} = 0$$

Equations 39 and 40 can be solved to obtain the marginal productivity conditions whereby both capital and labor are utilized up to the point where their marginal product equals their respective prices:

$$\frac{\partial Q}{\partial K} = \frac{p}{r}, \text{ solving for } r \text{ gives } p \frac{\partial Q}{\partial K} = r$$

$$\frac{\partial Q}{\partial L} = \frac{w}{p}, \text{ solving for } w \text{ gives } p \frac{\partial Q}{\partial L} = w$$

Hence, investors will use labor input up to the point where the return from the last unit of labor—its marginal product—equals the real price of labor. Similarly, capital is used up to the point where its marginal product equals the real price of capital. If the marginal product of capital exceeds the rental cost of capital, then it is profitable for firms to invest in more capital.

More specifically, the CES production function can be used in place of the general specification in equation 36 to show that the steady-state capital stock ($K^*$) depends on the output and the user cost of capital. Let $Q$, $K$ and $L$ respectively represent output, capital and labor, and with $\alpha$, $\delta$ and $\rho$ as constant parameters, then the CES production function takes the form
\[
Q = \alpha \left[ \delta K^{-\rho} + (1 - \delta)L^{-\rho} \right]^{1/\rho}
\] (43)

Let \( u \) represent \[ \delta K^{-\rho} + (1 - \delta)L^{-\rho} \]

Then, \[
\frac{\partial Q}{\partial K} = -\frac{1}{\rho} \left( \frac{\alpha^{\rho+1}}{\alpha^\rho} \right)^{1/\rho} \delta \frac{K^{-\rho}}{K^{\rho+1}} = \frac{r}{\rho}
\] (44)

But then \( \alpha \) can be expressed as \( \frac{\alpha^{\rho+1}}{\alpha^\rho} \), and therefore (44) can be reformulated as

\[
\frac{\partial Q}{\partial K} = -\frac{1}{\rho} \left( \frac{\alpha^{\rho+1}}{\alpha^\rho} \right)^{1/\rho} \delta \frac{K^{-\rho}}{K^{\rho+1}} = \frac{r}{\rho}
\] (45)

\[
= \frac{\delta}{\alpha^\rho} \left( \frac{Q}{K} \right)^{(\rho+1)} = \frac{r}{\rho}
\] (46)

\[
= \frac{\delta}{\alpha^\rho} \left( \frac{Q}{K} \right)^{(\rho+1)} = \frac{r}{\rho}
\] (47)

Given that the elasticity of substitution between capital and labor in the CES production function is given by \( \frac{1}{1+\rho} \), letting \( x = \frac{1}{1+\rho} \) and solving (47) for the steady-state capital stock gives

\[
K^* = \frac{\delta}{\alpha^\rho} \left( \frac{Q}{r^x} \right)
\] (48)

Differencing both sides of the equation gives the investment equation

\[
I_t = \frac{\delta}{\alpha^\rho} \Delta \left( \frac{Q}{r^x} \right)
\] (49)

If we assume, as is the case with Geda and Ndung’u (2001), that there is a delivery lag in investment so that current investment encompasses investment orders made in previous periods, then (49) can be reformulated to include lagged investments:

\[
I_t = \sum_{i=0}^{n} \beta_i \Delta \left( \frac{Q}{r^x} \right)_{t-i}
\] (50)

This illustration has highlighted the role of output (GDP) and the user cost of capital (lending rate) as key variables for the investment equation.

The modeling of determinants of private investment takes an eclectic approach, as follows:

\[
I_t = \mu + \theta_1 I_{t-1} + \theta_2 RGDPI_{t-1} + \theta_3 CPI_{t-1} + \theta_4 r_{t-1} + \sum \beta_i M_{t-i} + \sum \beta_j \Delta RGDPI_{t-j} + \sum \gamma_k \Delta CPI_{t-k} + \sum \omega_l \Delta r_{t-l} + \epsilon_t
\] (51)

where \( I_t \) is real private investment and \( I_{t-i} \) is included to capture the effects of previous investments, since most macroeconomic variables exhibit a high degree of persistence; \( RGDPI \) is real output; \( CPI \) is the consumer price index, and changes in the CPI represent the rate of inflation which is a proxy for uncertainty; and \( r \) is an interest rate to capture the user cost of capital. The estimated final private investment equation is a function of lagged GDP, the interest rate (lending rate) and private sector credit.

The External Account

Export Functions

Exports are usually modeled in a supply or demand framework. External agents/buyers make decisions on exports based on the relative prices of commodities and external demand. Thus, a basic export demand function would be determined by the relative price and external demand. Relative prices are computed as the ratio of a country’s export prices to prices in the world market. In some cases, it is taken to represent the export price of the home country relative to the price of competitors or the average price in international markets. Often, the real exchange rate is taken as a measure of relative prices. The growth of external demand is captured by the weighted average of real economic activity (GDP) for the country’s main export markets. The basic export
supply specification is formulated on the basis of the assumption that producers base their production decisions on, one, domestic capacity and, two, prices, which determine the relative profitability of producing for the export market vis-à-vis producing other competing goods.

The three measures that have been primarily used as proxies for domestic capacity are Trend of gross domestic product, \( y^* \), a sectoral production index, or capacity utilization index, normally defined as deviations from trend output.

Measures of price effects, which include the real exchange rate (RER) as an indicator of the relative profitability of producing tradable versus nontradables goods; and the ratio of export prices to the prices of other tradable goods, as an indicator of the profitability of exporting specific export goods relative to other traded goods. Relative profitability in this case is measured by the ratio of export prices to an aggregate price index for the entire economy (e.g., the GDP deflator). A trend term \( t \) is often included to reflect long-run changes that affect the supply of exports.

However, estimating the export demand and supply function of a small, open economy involves an identification issue (because of the non-availability of data for export demand and export supply). Many researchers prefer to estimate an export demand function in practice as data for such a function are readily available (see Abeysinghe and Choy 2005). We follow the common approach by estimating an export function that includes both demand- and supply-side constraints given that this is more meaningful and intuitive for a small, open economy (see Kapur 1983).

Following the literature, the demand function for exports can be expressed as follows:

\[
X_t = f(p^*/p^w, y^w, z) 
\]  

Where \( X_t \) is export demand, \( p^*/p^w \) is relative prices (price of exports as a ratio of world average export prices) and \( y^w \) is the world income effect. \( Z \) captures domestic production capacity. At the estimation stage, various proxies for supply-side or production constraints are considered, including the investment/GDP ratio.

**The Import Function**

Following the literature, the major drivers of a country’s imports are income and relative prices (Bahmani-Oskooee 1998). It is anticipated that a rise in import prices relative to the domestic price level hurts import demand, leading to negative import price elasticity. An increase in domestic income will stimulate imports; hence the positive income elasticity. However, if the rise in domestic income is as a result of an increase in the manufacture of import-substitute goods, imports may actually decline, thereby resulting in negative income elasticity.

According to conventional demand theory, consumers maximize utility subject to a budget constraint; thus, the demand for imports is expressed as (Narayan and Narayan 2005):

\[
Z_t = f(y, p^d, p^m) 
\]  

where the demand for real imports is a function of domestic income (\( y \)), prices of domestic goods and services or cross prices (\( P^d \)), and prices of imports or own prices (\( P^m \)): “From microeconomic theory demand functions are considered to be homogenous of degree zero in prices and money income” (Deaton and Muellbauer 1980). This rules out the presence of the money illusion; that is, if one was to multiply all prices and money income by a positive number, the quantity demanded will remain the same.

Additional variables have been conceived in the literature to explain the importing of an item or a group
of items (see Maiti 1986). These include the real exchange rate and variables reflecting the effects of foreign exchange scarcities and/or import restrictions. Foreign exchange reserves finance gaps between imports and foreign exchange receipts (export earnings in a particular period) or smooth out the volume of imports over time. Its coefficient in the import equation is expected to be positive. The real exchange rate enters the equation as a measure of export competitiveness. The average tariff (total import tariffs as a ratio of total tax revenue) is another variable considered in some studies. In this regard, therefore, the significance of foreign exchange reserves and the real exchange rate will be empirically tested.

**Exchange Rates**

Frankel’s (1979) formulation of Dornbusch’s (1976) monetary model of exchange rates with sticky prices (i.e., sticky price monetary model of exchange rates) is often used as a workhorse model. In this model, the nominal output prices are assumed to be sticky—that is, they adjust slowly over time. Conversely, the asset markets clear continuously in response to new information or changes in expectations. The model thus adopts the principle of UIP, but PPP need not hold:

\[ s_t = \delta(m - m^*) - \phi(y - y^*) + \alpha(r - r^*) + \beta(\pi - \pi^*) + u \]

(54)

where \( s \) is the exchange rate expressed in units of the home currency per foreign currency, \( m \) and \( m^* \) are respectively the domestic and foreign money supplies, and \( y \) and \( y^* \) are the real incomes of, respectively, domestic and foreign countries. The variables are in natural logs; \( r \) and \( r^* \) and \( \pi \) and \( \pi^* \) are domestic and foreign rates of interest and inflation, respectively; and \( \alpha \) is hypothesized negative and \( \beta \) is hypothesized positive. The money stock and income coefficients are hypothesized to be positive and negative, respectively. The negative interest coefficient (\( \alpha \)) holds as long as the prices are sticky; otherwise, if prices are entirely flexible, either inflation or interest rates would enter positively. There is perfect capital mobility and substitutability between home and foreign bonds. The domestic economy is small for the rest of the world, with the assumption that the foreign interest rate is exogenous.

Due to the sluggish adjustment in the price level, a purely nominal shock or disturbance can cause short-run deviations from PPP and nominal exchange rate overshooting (Marrinan 1989). The impact of a monetary expansion would lead to a depreciation of the spot rate and one that exceeds long-run depreciation, for it is only under these circumstances that the public will anticipate an appreciation and thus be compensated for the reduced interest on the domestic assets (Dornbusch 1988). The UIP condition and demand-for-money equation form the basis of the overshooting model.

Assuming that PPP holds, equation 55 is the fundamental equation of the exchange rate for the flexible price monetary model:

\[ s_t = \delta(m_t - m^*_t) + \lambda(i_t - i^*_t) - \phi(y_t - y^*_t) \]

(55)

For simplicity, \( \lambda = \lambda^* \) and \( \theta = \theta^* \). The nominal exchange rate is expressed in terms of current relative money supplies and factors affecting money demands. Since PPP holds, an increase in the money supply causes an increase in prices as domestic agents spend more on goods and services. Moreover, increases in income, by raising the demand for money, lead to an exchange rate (appreciation). Conversely, a rise in the domestic interest rate leads to exchange rate depreciation. This contrasts with the sticky price monetary model.
because of the fact that, in the flexible price model, the effects of income and interest rates only affect the exchange rate via their impact on the demand for money (MacDonald 1988). Additionally, unlike in the sticky-price Dornbusch model, money supply increases do not lead to an exchange rate overshooting or undershooting. However, for the PPP assumption to hold, it requires very strong preconditions and hence, in practice, significant violations of the PPP hypothesis have been revealed.

Another strand of the literature takes into account portfolio considerations and regards the exchange rate as the relative price of nominal assets. The portfolio balance approach focuses on the link between the balance of payments and adjustments in asset stocks. Since their emergence in the late 1960s, these models have undergone several changes. Based on early drafts by Messe and Rogoff (1983a, 1983b), a specification incorporating cumulative trade account balances became widely adapted. Clearly, the classification of this specification as a portfolio balance model can be misleading. For Hooper and Morton (1982), the cumulative current account (adjusted for cumulative official intervention flows) appears as a risk premium term, and changes in the expected long-run real exchange rate are captured by including a measure of the nontransitory unexpected change in the current account.

Messe and Rogoff (1983a, 1993b), conversely, interpreted the cumulative trade balance or current account terms as variables that allowed for changes in the long-run real exchange rate, rather than as variables that allowed for the existence of a risk premium. This interpretation is supported by the view that cumulative current account imbalances redistribute wealth internationally, with effects on a country's levels of expenditures, incomes, and current account imbalances, and, accordingly, with implications for the level of the real exchange rate that are consistent with the long-run current account balance (Isard 1995). However, different specifications use different measures of external balance.

The analysis given above leads to a combination of the monetary/portfolio hybrid model (i.e., a sticky-price hybrid model), which can be specified as

\[ s_i = \delta(m_i - m_i^*) + \phi(y_i - y_i^*) + \alpha(i_i - i_i^*) + \varphi(\pi_{it}^e - \pi_{it}^e) + \sigma TB + e_i \]  

(56)

Applied studies in Kenya have also tended to adopt the sticky price monetary model approach by using the UIP specification. For example, Ndung'u (2000) and Ndung'u and Ngugi (1999) use a modified UIP, in which the exchange rate is specified as a function of price differentials and real interest rate differentials. They circumvent the PPP notion by arguing that interest differentials will absorb deviations from PPP. They also argue that the observed domestic interest rate partly reflects monetary policy action. Consequently, the formulation for the exchange rate equation need not include money or domestic output, based on the argument that these correlations are embodied in the price level adjustment mechanism (also see Obstfeld and Rogoff 1996). The equation given above can, therefore, be modified accordingly at the estimation stage.

The Balance of Payments

The balance-of-payments identity in foreign currency is given by

\[ BOP = KA + CA \]  

(57)
Where \( BOP \) is the balance of payments, \( KA \) is capital account in U.S. dollars and \( Z \) is the current account balance in U.S. dollars. The capital account equation shows that the net foreign flow is used to finance the current account deficit (\( CA \)) or the fiscal deficit (\( B_g \)):

\[
KA = f_k(Z_x, B_g)
\]  

(58)

\[
CA_x = X_x - Z_x
\]  

(59)

This section concludes theoretical postulates for the core behavioral equations of the CBK model.
4. CONCLUSION

The use of macroeconometric models is becoming increasingly popular in many central banks. Macro models play a key role, particularly in the area of policy analysis and the provision of consistent macroeconomic forecasts. However, different types of models are designed to address different questions, since no one model can answer all questions. Consequently, a number of central banks employ a suite of models to help them with different aspects of the monetary policymaking process. Given the needs of the CBK with respect to the monetary policymaking process, its first priority was to develop a macroeconometric model that could be used to consistently forecast macroeconomic variables, in addition to conducting policy analysis by way of simulations. The development of the CBK macro model is the first step toward developing a suite of models at CBK, including more sophisticated economic models such as DSGE.

This paper has sought to describe the theoretical and logical framework of the CBK macro model. The model structure has been discussed, and the various key sectors of the economy and their interlinkages have been explained. Further, the impact of monetary policy on the various sectors of the economy has been elaborated through the various transmission channels identified as crucial in the Kenyan economy. The theories behind the core model equations have been discussed based on their suitability and relevance to the Kenyan economy.
APPENDIX 1: AN EXAMPLE OF A POLICY SIMULATION IN THE QUADRANTS (FROM FIGURE 2)

Policy simulations can be undertaken in the postulated quadrants. For example, suppose there was an increase in the money supply in the economy; this will shift the $LM$ curve downward to $LM_1$, directly reducing the interest rates from $r_0$ to $r_1$. The reduced interest rates leads to increased investment, while at the same time the increased money supply leads to an increase in the consumption of goods and services, with both effects increasing aggregate demand ($AD$). Output increases from $Y_0$ to $Y_1$, and $AD_0$ shifts outwards to $AD_1$, ceteris paribus. The domestic prices increase from $P_0$ to $P_1$ as the demand for goods increases, with corresponding supply held constant.

In the money market, the effect of the increase in the money supply is reflected in lower interest rates and in the increase in prices in the real sector. In an open economy, the lower interest rates relative to the world interest rate lead to capital outflow, thereby depreciating the domestic currency. At the same time, the increase in the money supply, while raising prices, also depreciates the price of the domestic currency vis-à-vis foreign currency, moving the exchange rate from $E_0$ to $E_1$. The depreciation makes exports competitive while discouraging imports and in turn leads to an increase in net exports. The $BOP$ shifts to reflect a depreciation of the home currency and an increase in net exports. A decrease in the money supply would have the exact opposite effect.
APPENDIX 2: BARGAINING THEORY OF WAGES: AN EXTENSION

The key issue under wage bargaining is the struggle over the markup, where expectations of future variables play a significant role (Layard and Nickell 1992). Labor unions strive to maximize their share of the productivity increase, defined in log form as

\[ pr_t = y_t - l_t \]  \hspace{1cm} (A1)

where \( y_t \) is the log of the output and \( l_t \) is the log of total employment.

The individual employers or their union (e.g., the Federation of Kenyan Employers) attempt to maximize the markup on unit costs, defined in log form as

\[ \text{markup}_t = pr_t + p_{y,t} - w_t \]  \hspace{1cm} (A2)

where \( p_{y,t} \) is the log of the price for output (e.g., the GDP deflator) and \( w_t \) is the log of wage costs.

The markup is assumed to be a function of the real exchange rate, \( rer_t^e \), and the expected real interest rate is \( r_t^e \):

\[ \text{markup}_t = f(re_t^e, r_t^e) + \epsilon_t \]  \hspace{1cm} (A3)

The real exchange rate is defined as

\[ rer_t^e = e_t + p_t^f - p_{y,t} \]  \hspace{1cm} (A4)

where \( e_t \) is the nominal exchange rate and \( p_t^f \) is the foreign price level.

As pointed out by Phelps (1994), \( f_{rer_t^e} > 0 \) since there is a lower markup following real appreciation and \( f_{r_t^e} > 0 \), i.e., a higher markup following an increase in real interest rates.

The labor unions try to maximize the purchasing power of their members by increasing the real consumption wage, \( w_t - p_{e,t} \), conditional on labor productivity; \( p_{y,t} \), the expected effect on unemployment, \( U_t^e \); and the expected inflation rate, \( \Delta p_t^e \):

\[ pr_t + p_{y,t} - w_t = g(U_t^e, \Delta p_t^e) + \epsilon_{z,t} \]  \hspace{1cm} (A5)

where \( g_{U_t^e} < 0 \), i.e., high expected unemployment causes labor unions to demand less real consumption wage; and \( g_{\Delta p_t^e} > 0 \), i.e., higher expected inflation causes labor unions to demand higher real consumption wage. The aggregate wage relation for real product wage is given as

\[ w_t - p_{e,t} = \alpha_0 - \theta(p_{y,t} - p_{e,t}) + pr_t + h(U_t^e, \Delta p_t^e, rer_t^e, r_t^e) + \epsilon_{z,t} \]  \hspace{1cm} (A6)

where \( \theta \) is the bargaining power of the labor unions. The aggregate wage relation for real consumption wage is given as

\[ w_t - p_{e,t} = \alpha_0 - (1 - \theta)(p_{e,t} - p_{y,t}) + pr_t + h(U_t^e, \Delta p_t^e, rer_t^e, r_t^e) + \epsilon_{z,t} \]  \hspace{1cm} (A7)

where \( p_{e,t} - p_{y,t} \) is the price wedge.
REFERENCES AND BIBLIOGRAPHY


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ENDNOTES

1. For a long time, the Bank of England’s macroeconometric model was the main tool for providing projections of GDP growth and inflation (Bank of England 2000).

2. The exchange rate has an impact on foreign currency deposits, which are part of the M3–broad money supply.

3. KTMM is the first comprehensive macroeconometric model for Kenya. It was developed in consultation with the Treasury.

4. Output can be defined in terms of absence of nominal rigidities (natural output) or in terms of absence of nominal rigidities and departures from competitive markets (potential output).

5. An elaborate description of the IS–LM–BP framework can be found in any advanced standard macroeconomics textbook.

6. The basic framework of this approach is through

\[ Y = C + I + G + NX \] (IS curve)

\[ \frac{Mr}{P} = m(r, y) \approx l(r) + k(y) \] (LM curve)

\[ BOP = CA + KA \] (BP curve)

7. In the long run, monetary policy determines the nominal or money values of goods and services, i.e., the general price level. In the short run, monetary policy has an effect on real activity.

8. The reaction functions are not included because they are to be estimated and incorporated in the model at a later stage. Further, all behavioral equations are described in details in a separate paper, giving the estimation procedures, relevant empirical literature and results for each final estimated equation presented in table 1.

9. Another way of representing the money demand function is in its disaggregated form. The components of money demand are \( COB \), \( DD \), \( TSD \) and \( FCD \). These components are determined by various factors that explain the aggregate demand for the money function explained in equation 4.

10. The \( NFA \) component of the Reserve Money in U.S. dollars is assumed to be a function of the available foreign exchange supply, and the balance of payments in U.S. dollars, \( nfa = f(BOP, F) \) where \( F \) is the supply of foreign exchange and \( BOP \) is the balance of payments. For simplicity, we assume \( NDA \) to be fully exogenous, but alternatively it can assumed to be affected by the CBK’s open market operations and change in the government cash balances with CBK.

11. The money supply may also be represented in its components—i.e. \( NFA \), \( NDA \) and \( OIN \)—that add up to the money supply—\( M^d \).


13. The money multiplier can also be expressed in much detail as follows

\[
\text{mm}\text{M} = \frac{M^3}{RM} = \frac{C + DD + ODCBK + Qmbanks + QmBFI.s + NBFIbanks + FCDs}{C + CR + ER + CIT} \\
\text{mm}\text{M} = \text{cr} \times \text{err} + \text{ddcr} + \text{qmbcr} + \text{qmcrfcr} - \text{nbfibrankcr} + \text{fdcr}
\]

where cash outside bank \( COB \) is \( C \), demand deposits are \( DD \), other deposits at central bank are \( ODCBK \), quasi-money deposits are \( Qm-banks \), quasi-money nonbank financial intermediaries are \( QmBFI.s \), nonbank financial intermediaries’ deposits are \( NBFIbanks \), foreign currency deposits are \( FCDs \), total deposits are \( TD \), cash ratio is \( CR \), excess ratio is \( ER \), cash in till is \( CIT \), time and savings deposits are \( TSD \), net foreign assets are \( NFA \), net domestic assets are \( NDA \), and other items net are \( OIN \).

14. Identification issues need not arise in this case.