Appendixes

1.1. Appendix A: Data

Fernald (2014) Quarterly Growth-Accounting Data (for Figure 1 and Table 2). Annual data run from 1948 through 2015, though our sample ends in 2014 because of the data for the output adjustments. Current vintage data are available at http://www.frbsf.org/economic-research/economists/john-fernald/ under “A Quarterly, Utilization-Adjusted Series on Total Factor Productivity”, “Supplement (+)”: quarterly_tfp.xls. The vintage used for this paper was February 4, 2016. The dataset includes quarterly growth-accounting measures for the business-sector, including output, hours worked, labor quality (or composition), capital input, and total factor productivity.

Output is a geometric average from the income and expenditures sides. Hence, labor-productivity growth in Figure 1 differs slightly from the BLS Productivity and Cost data, which use the expenditure-side. Capital input is a user-cost-weighted aggregate of capital input growth of disaggregated types of equipment, software, intellectual property, and inventories that are available quarterly, as well as land (interpolated from annual BLS estimates).

The chart below shows growth-accounting contributors to business-sector labor productivity:

![Contributions to growth in U.S. output per hour chart](chart)

Source: Fernald (2014a). Quarterly; samples end in Q4 of years shown except 1973 (ends Q1). Capital deepening is contribution of capital relative to quality-adjusted hours. Total factor productivity is measured as a residual.

“Normal” productivity growth has varied substantially over the post-war period. Before 1973 and from 1995-2003, labor productivity rose at above 3 percent per year. In between, its growth
rate averaged only about 1-1/2 percent per year. The slowdown in the early 2000s is statistically significant and predated the Great Recession.\(^1\)

In the four years prior to the Great Recession (2003-2007), labor productivity rose at only about a 1-1/2 percent pace. Its growth rate then rebounded modestly during and (especially) immediately after the Great Recession (2007-2010). Yet, during the five years from the end of 2010 to the end of 2015, growth has been markedly lower.

The shaded regions of each bar show the contribution of standard growth-accounting components: labor “quality” (or composition), capturing changes in the educational attainment and experience of workers; capital deepening, or capital per quality-adjusted hour; and TFP, measured as a residual.\(^2\) The contributions of labor quality and capital deepening have varied somewhat over time, but the broad patterns in labor productivity largely track TFP growth.

According to the growth accounting, the weak performance in the final bar reflects capital “shallowing”—automation has been running in reverse. Mechanically, hiring has been extremely fast, with hours worked rising at about a 2 percent annual pace. In contrast, capital services have accelerated more slowly than hiring. To some extent, this reflects an unwinding of the strong pace of capital deepening during and immediately after the Great Recession: Employment fell in the recession, leaving firms with plentiful capacity. In addition, labor quality added less than during the recession, when low-skilled workers lost jobs.

For alternative capital simulations discussed in the paper, we adjust deflators and real investment quantities for information processing and software. The simulations use published nominal values of nonresidential gross private domestic investment for computers and peripheral equipment, communications, and software (NIPA Table 1.5.5). The alternative deflators are then used to calculate alternative real investment series, which are then accumulated via perpetual inventory methods into real capital stock measures by assets. They are then aggregated with user costs into an alternative Tornquist index of real capital input.

The alternative capital deflators also imply differences in industry and sectoral TFP. Investment TFP growth is faster, but non-investment TFP growth is slower (because capital growth is faster). To understand whether the modifications in Section II change the broadbased nature of the TFP slowdown, we use relative prices to decompose aggregate TFP into growth of “investment” TFP, $\text{TPF}_{\text{Invest}}$, and “other” (non-investment, mainly consumption) TFP, $\text{TPF}_{\text{Other}}$. Suppose the two “sectors” have the same production functions (other than a multiplicative constant) and face the same factor prices. These are strong assumptions, but the literature on investment-specific technical change shows that it yields a sharp result: $\Delta \ln \text{TPF}_{\text{Invest}} - \Delta \ln \text{TPF}_{\text{Other}}$. This equation captures the intuition that the main reason why prices of consumption and other non-investment goods, $P_{\text{Other}}$, has been rising relative to the price of investment goods, $P_{\text{Invest}}$, is the fast pace of technical progress in computers and other capital goods.

When we apply this decomposition using the counterfactual liberal deflators from Section II.D, it does not change the broadbased nature of the TFP slowdown. (Results are not shown but are available on request.) Investment TFP growth is systematically stronger throughout history, whereas other (non-investment) TFP growth is systematically weaker. But after 2004, both

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1 Formal break tests justify the dates shown by the first three bars (see Fernald, 2015).
2 This standard measure of TFP does not adjust for cyclical effects on factor utilization. The Fernald (2014) dataset does include a model-based measure of factor utilization. Utilization adjustments turn out to make little difference in the subperiods shown here. Utilization had largely (though perhaps not entirely) reversed its sharp recessionary declines by the end of 2010. Of course, specific industries could be different.
investment and other TFP growth slow sharply, and by magnitudes that are similar to the base case that corresponds to Table 2, column (0).


**Intangibles:** Data are from Corrado and Jäger (2015), which in turn updates U.S. estimates from Corrado et al. (2009) and Corrado et al. (2012). Carol Corrado provided these unpublished data on nominal intangible investments from 1977-2014 (via email on February 12, 2016). To convert the data to real values, we deflate with the business sector deflator. For initial capital stock values, we calculate investment growth rates ($g$) for the first 10 years and then use the “steady state” formula that $K_0 = I_0 / (\delta + g)$, where $I_0$ is the initial real investment value and $\delta$ is the depreciation rate. The depreciation rates are from Corrado et al. (2009). For non-national-account intangibles, we aggregate the intangible capital stocks into a Tornquist index using estimated user costs, assuming a constant real interest rate of 5 percent per year.

To aggregate intangible output with the Fernald quarterly TFP dataset, we use a Tornquist index. The weights are nominal business-sector output and nominal intangible spending. Similarly, we aggregate capital input with national-accounting measures with the new intangibles as a Tornquist index.

We also recalculate factor shares. Capital’s share rises and labor’s share ($s_L$) falls. Intuitively, payments to labor do not change but nominal output is larger. Algebraically, the adjustment is $s_L^{\text{New}} = s_L \frac{PY}{(PY + \text{Intan})}$, where $PY$ is measured business-sector factor costs and $\text{Intan}$ is nominal intangible spending.

**Appendix B: Adjusting output**

For the simulations in Section II.D, we adjust output (business sector real GDP) as well as capital (described in Appendix A). This appendix shows that the main adjustment involves adding the Domar-weighted (i.e., industry nominal gross-output relative to aggregate value added) price adjustment to domestic gross output growth.

We start with the Tornquist approximation to the chained Fisher index of value added. From the national accounting identity, the change in aggregate value added growth is:

$$dv = \sum_i w_i dv_i$$

$dv_i$ is the log change in variable $j$. $w_i$ is the value-added share of industry $i$. Industry value-added growth, using the Tornquist (Divisia) formula, is:

$$dv_i = \frac{\left( dy_i - s_{m_{1,i}}dn_{1,i} - s_{m_{2,i}}dn_{2,i} \right)}{\left( 1 - s_{N_{1,i}} - s_{N_{2,i}} \right)}.$$
In this expression, \( dy_i \) is growth (log change) in gross output. \( dn_{1,i} \) is growth in an intermediate input (such as semiconductors) where we might want to adjust the price/quantity. \( dn_{2,i} \) is the growth of intermediate inputs that are not affected by our adjustments. \( s_{N,j} \) are the respective intermediate-input shares of the two types of intermediates.

Adjusting deflators implies new measures of output and of the first intermediate. The new growth rate is

\[
dv_i^{New} = \left( dy_i^{New} - s_{N1,i} dn_{1,i}^{New} - s_{N2,i} dn_{2,i} \right) / \left( 1 - s_{N1,i} - s_{N2,i} \right).
\]

Thus, the adjustment to industry value added is:

\[
dv_i^{New} - dv_i = \frac{dy_i^{New} - dy_i}{\left( 1 - s_{N1,i} - s_{N2,i} \right)} - s_{N1,i} \left( \frac{dn_{1,i}^{New} - dn_{1,i}}{\left( 1 - s_{N1,i} - s_{N2,i} \right)} \right)
\]

Thus, if there are no changes in input prices/quantities, the change in value-added is a “grossed up” version of the change in gross output. Conceptually, it is also necessary to adjust off the appropriately share-weighted change in intermediate-input prices/quantities as well. However, if the adjusted intermediate input (which we will take to be semiconductors) is domestically produced, we get a positive output adjustment for that industry, but then an offsetting adjustment for using industries. Therefore, we can ignore the second term—it is already captured by the adjustment below for semiconductors.

To see this effect, first note that \( w_i / (1 - s_{N1,i} - s_{N2,i}) = PY_i / PV \), i.e., the Domar weight (nominal gross output relative to nominal aggregate value added). Therefore, the weight on the “output adjustment” is just the Domar weight. It follows that the second weight, on the intermediate input adjustment, is \( s_{N1,i} PY_i / PV = P_{N,i} N_i / PV \).

Second, consider what happens when we adjust semiconductor prices. We add aggregate value added growth in (domestic) semiconductors, but then subtract the effect of domestic and foreign semiconductors used as intermediate inputs. Some algebra shows:

\[
\sum_i w_i dy_i = \frac{P_i^{DY}}{PV} \left( dy_{S,New}^{D} - dy_{S}^{D} \right) - \sum_i \frac{P_i^{D} N_{S,i}^{D}}{PV} \left( dy_{S,New}^{D} - dy_{S}^{D} \right) - \sum_i \frac{P_i^{F} N_{S,i}^{F}}{PV} \left( dy_{S,New}^{F} - dy_{S}^{F} \right)
\]

\[
= \left( dy_{S,New}^{D} - dy_{S}^{D} \right) \left( \frac{P_i^{DY}}{PV} - \sum_i \frac{P_i^{D} N_{S,i}^{D}}{PV} \right) - \left( dy_{S,New}^{F} - dy_{S}^{F} \right) \sum_i \frac{P_i^{F} N_{S,i}^{F}}{PV}
\]

The first term is the adjustment from domestic output (superscript \( D \)) multiplied by the nominal value of semiconductor exports relative to value added. The second effect is the adjustment to imported output (\( F \)), multiplied by the value of semiconductor imports to value added. In a closed economy, where exports and imports are zero, this effect disappears.

In sum, for final products, we Domar-weight the adjustment to prices/quantities. For semiconductors, we use an export weight for domestic production, then subtract off an import-weighted “foreign” adjustment.

We obtained annual values of domestic production from Board of Governors’ databases.