The Potential of Geometric Sequences to Foster Young Students' Ability to Generalize in Mathematics

Joan Moss
Ruth Beatty
Susan London McNab
Ontario Institute for Studies in Education
Janet Eisenband
Teachers College Columbia

Abstract

- NCTM has proposed that patterning may promote algebraic reasoning skills; however, persuasive evidence suggests that pattern work may not support a smooth transition to algebraic thinking. While students tend to be adept at extending both repeating and growing patterns to “next” positions, they display difficulty making predictions far down the sequence of growing patterns. Employing Case’s theory of mathematical development, we have been designing, implementing and assessing experimental patterning curricula in second and fourth grade classrooms in diverse urban settings. An important feature of this instructional design is the focus on the integration of visual/geometric and numeric patterns. Our goal is to explicitly link ordinal pattern positions with the number of elements in that position, and to bridge the learning gap between scalar sequence and functional relations. The components of the curricula include geometric pattern building with position cards, function machines and t charts, and activities that integrate these activities.

Abstract

An additional and important feature of this project involves Knowledge Forum® (Scardamalia & Bereiter), a web-based knowledge-building platform that provides a shared database across the research classrooms for students to post questions, contribute theories and debate ideas in the form of notes.

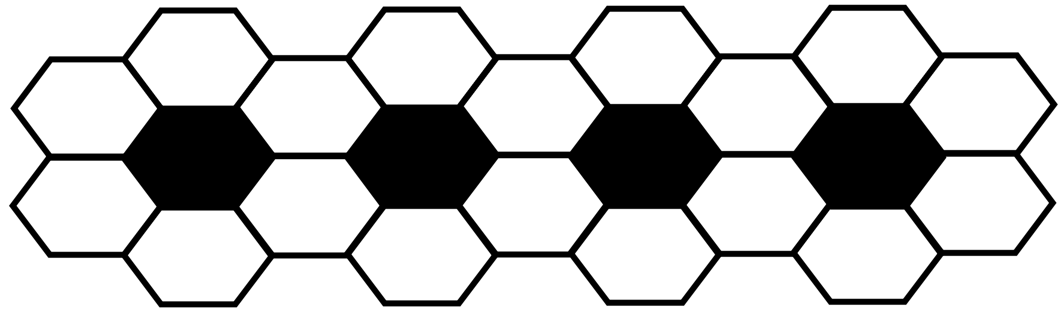
The research reported here involves two separate studies. The first study was conducted in 3 experimental second grade classrooms in New York and Toronto (n = 74), and a fourth classroom (n=20) that served as a control group. The second study was conducted in 3 fourth grade classrooms (n=72). In each of these studies, data collection and analyses involved pre- and posttest measures, analyses of field notes and of transcriptions of videotaped lessons as well as transcripts and videos of ongoing interviews with students. Quantitative analyses based on pre- and post-test measures revealed that students improved significantly from pre to post and outperformed students from control or normative groups on items that involved far generalizations and functional reasoning.

Abstract

Furthermore, in both research projects the rate of change from pre- to post-test was consistent over high, medium, and lower achievers. Qualitative analyses of the grade 2 studies revealed a number of interesting and unanticipated results that will be presented. in particular, we found that the experimental group who had not had formal instruction in multiplication either before or during the intervention, invented a series of informal strategies that allowed them to find correct answers to multiplicative function problems. In addition, these students in the experimental group were more able to apply multiplication than the students in the control group all of whom had received formal instruction in multiplication.

- Analyses of the fourth grade study indicated that these students were able to generalize not only in terms of specific data sets, but also across structurally similar problems. Further, the Knowledge Forum database revealed a trend in the students' increasing ability to generalize, to negotiate multiple rules, to use various modes of representation. Finally the data revealed students' developing use of mathematical symbolic language and increasing sophistication for offering proof and justifications for their conjectures.

Can you a build a "Times 2 plus 1" pattern?(video)



Agenda

- Early Algebra and Patterning – An Introduction affordances and difficulties
- Overview of theory and patterning research
- Grade 2 study
- Grade 4 study
- Knowledge Forum discussions
- Overall results and relation to Strands of Mathematical Proficiency
- Discussion and implications
  - Focus and priority on spatial representations of patterns
  - Algebrafied arithmetic?

Why Study Patterns?

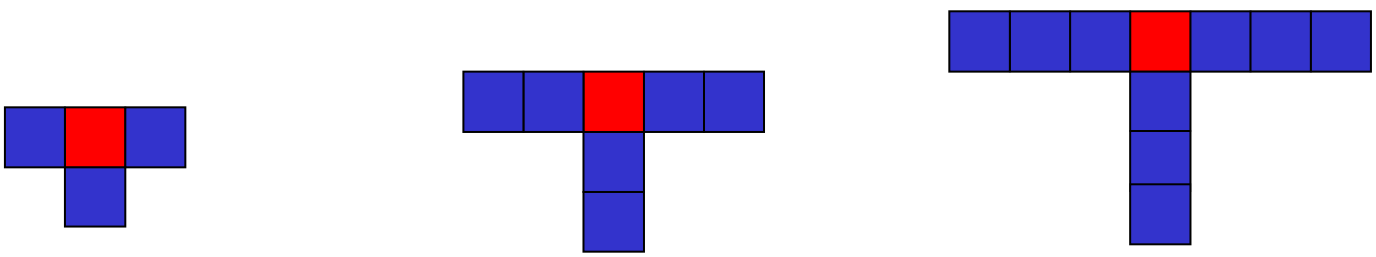
- “Patterns, Relations & Functions” part of the most recent preK-12 Algebra recommendations (NCTM 2000)
- Patterns can constitute one route toward generalizing (e.g. Ferrini-Mundy et al.; Kenny & Silver; Schliemann et al.; Smith; Kaput)
- Patterns can develop understanding of the dependent relations among quantities that underlie mathematical functions
- Young children naturally interested (Seo and Ginsburg, 2004) and older students find patterns compelling
- Spans ability levels

Patterning Expectations for Elementary Students NCTM

- Sort, classify, and order objects by size, number, and other properties
- Analyze how both repeating and growing patterns are generated
- Describe, extend, and make generalizations about geometric and numeric patterns
- Represent and analyze patterns and functions using words, tables, and graphs

How Do Children Do?

- Limited idea of patterns only as *repeating*
- “A pattern is something that goes on forever that never stops. Like if you said big little big little from this day until you were about 10 that would be a pattern.”
- Children can *extend* patterns, but have trouble *describing & generalizing -* finding elements far down the sequence



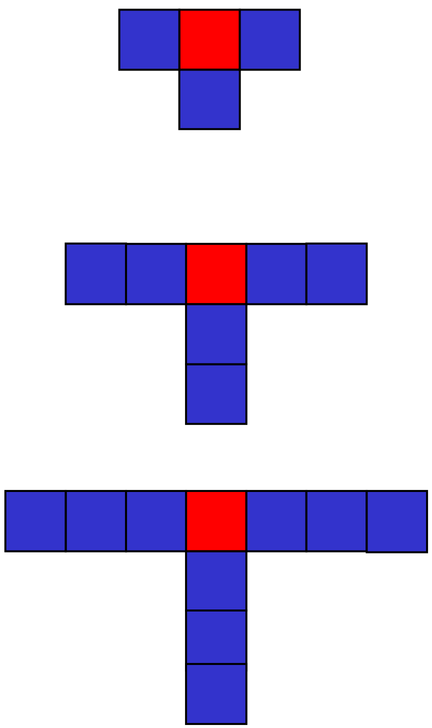
Challenges

- Tendency to find and use the *scalar,* or *recursive*, rules

1

2

3



Position	Blocks
1	4
2	7
3	10
4	13
5	16

+3

+3

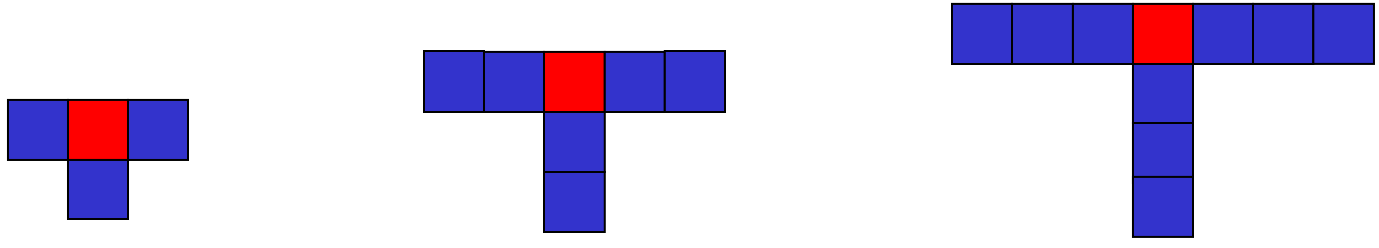
+3

+3

x 3+1

Whole object reasoning

Incorrect use of a proportional strategy to reason e.g. that if there are 10 blocks in the 3rd position then there will be 20 blocks (rather than 19) in the 6th position.



Whole Object:
Tables & Chairs (video)

Difficulties with Patterns

- Route from perceiving patterns to finding useful rules and algebraic representations is very difficult (Kieran, Noss et al, 1997)
- Patterning problems become “arithmetic exercises” with tables: little understanding of relationships implied in underlying structure
- Difficulty with geometric representations of patterns
- Students lack rigour and commitment to justifications
  - “Once students selected a rule for a pattern, they persisted in their claims even when finding a counter example to their hypotheses”. (Cooper and Sakane 1986)

Recommendations

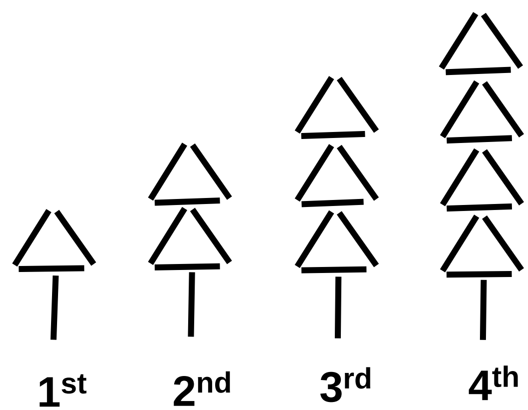
- Noss & Hoyles (1999) difficulties due to “disconnections” between actions, the result or output, and the students' expressions of rules. Suggests focus on integration of geometric: Rethink hierarchical prioritizing of numbers as route to abstraction
- Lee (1996) suggests that perceptual agility is being able to abandon pattern rules that do not prove useful
- Mason (1996) suggest visualization and manipulation of the geometric pattern facilitates construction of the rule
- Hand over the responsibility to the students

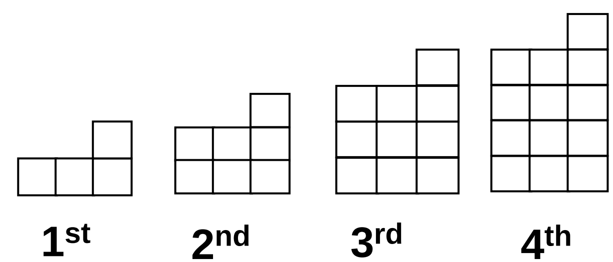
In a recent survey that we conducted with students from grades 2-5 we found that students had a great deal of difficulty with all kinds of patterns….

Normative Patterning Study

- 97 interviews with students in grades 2-5 (53 male, 44 female)
- Each interview consisted of five patterning questions including different mathematical functions: 3x+1, 2x+2, 3x+1, 5x+1 and 3x
- Each function was presented in one of five formats:

A	B
1	4
3	10
4	13
6	19
9	
41	
n	





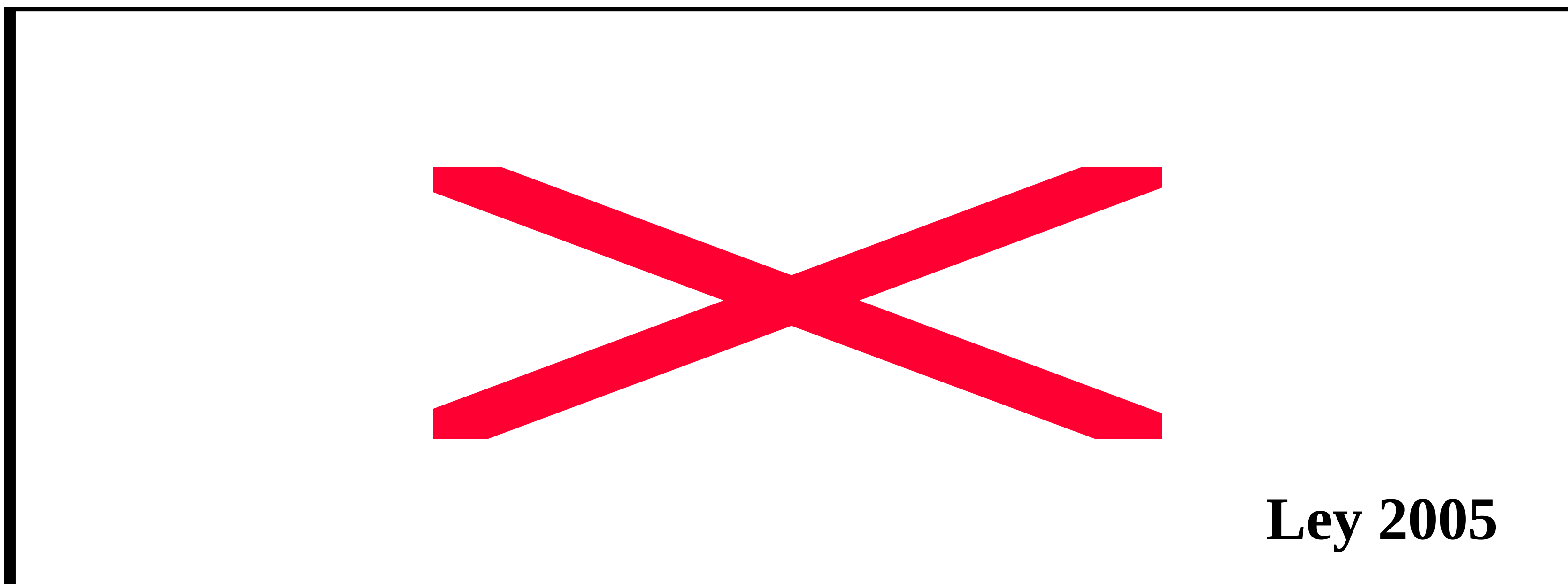
4,7,10, 13, ___, ___, …

Jane really wants a brand new scooter. She has only $1.00 saved up. She decides she's going to save money she earns from babysitting. She gets a job babysitting for an hour each day. After babysitting on the first day, she puts the money she earned from babysitting in her piggy bank along with the $1.00 she already had saved. Now, altogether, she has $4.00 in her piggy bank. After babysitting for a second day, she now has $7.00 altogether in her piggy bank. After the third day of babysitting, Jane now has $10.00 altogether in her piggybank.

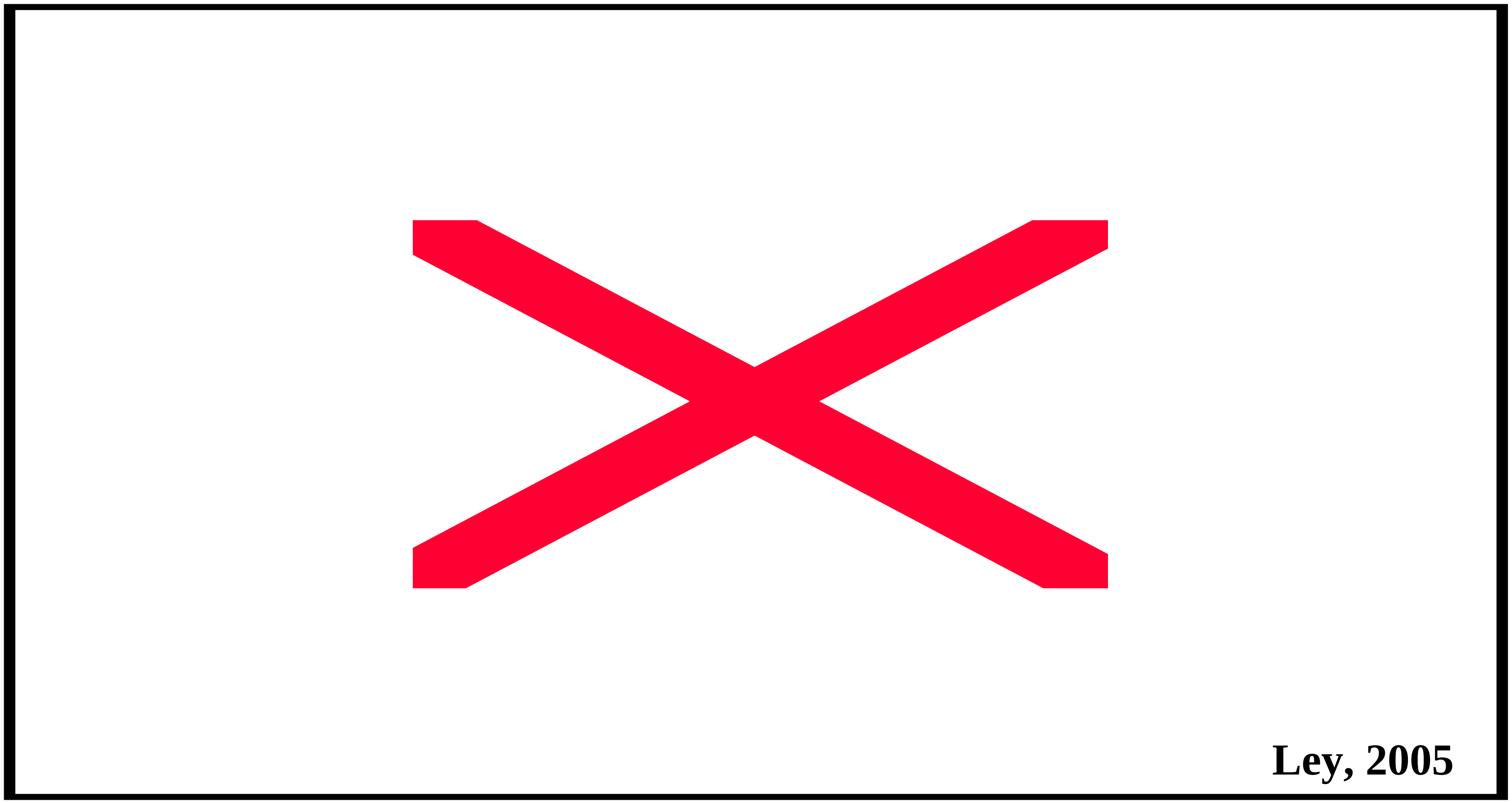
Ley 2005

Accuracy Percentages for all Formats and all Grades on the 5th, 9th and 41st positions

- N=97 (grades 2-5)



Overall (all grades combined) frequency of strategy use in each of the five formats



Our Pilot Research

Goals:To have children engage with geometric and numeric patterns in ways that could

- Forge connections between different pattern representations
- Illuminate mathematical structures underlying patterns
- Provide natural situations for mathematical problem solving, discussion, reflection and knowledge building
- **Our study:** Design, implement and assess pattern activities in grades 2 and 4 classrooms

Case Theory and the Instructional Design

- The theoretical framework follows the work on mathematics learning and the design for instruction of Case and colleagues, i.e., Griffin and Case's work on whole number learning (e.g.

- Griffin & Case, 1997; Griffin, Case & Siegler, 1994), Moss and Case's work on rational number (e.g., Moss & Case, 1999; Moss 2000, 2004), and Kalchman and Case (1998) on mathematical functions in the middle grades.

- A central tenet of instructional design of Case et al is the focus on the development of spatial/visual schemes. Not only is instruction designed to extend students' prior *numerical* understandings, but also to help students integrate their numerical and *visual* knowledge, (Kalchman; Moss & Case, 2001). The theoretical proposal is that the merging of the *numerical* and *visual* provides a new set of powerful insights that may underpin not only the early learning of a new domain but also subsequent learning as well.

Theoretical Framework

- Case’s theory offered a model that paralleled our interest in linking geometric and numeric patterns

- For patterning curriculum:
  - Children have experience looking at, and finding the next position for geometric patterns
  - Children familiar with finding next number in series as well as counting by 2’s, 5’s and 10’s
  - Design instruction to integrate these abilities:
- First, instruction focused on both geometric patterns and numeric patterns independently. Next, lessons included activities, special props, and tools designed to foster a bridging and integration of students’ visual and numeric understandings. Finally, activities were created that allowed opportunities for students to move back and forth across these two types of representations of patterns.

Second Grade Study

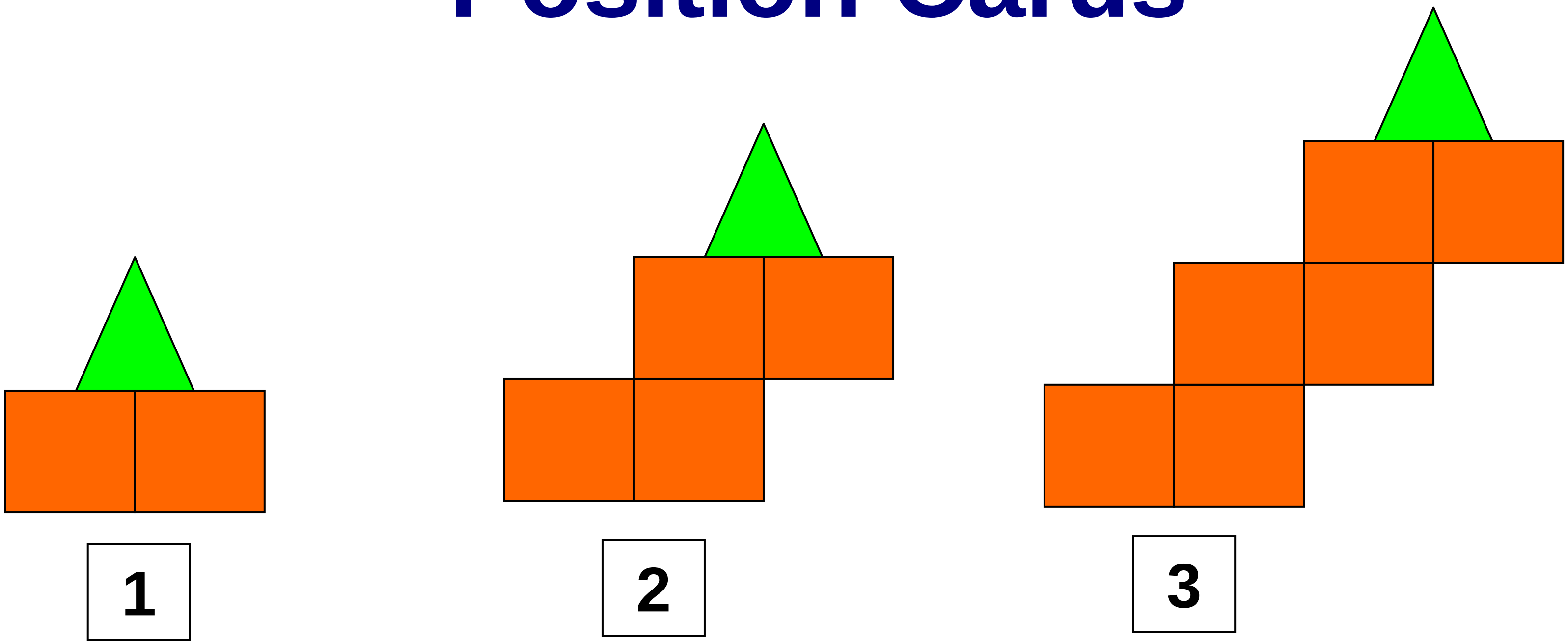
Method

- 3 classes of grade 2 n=78
- Control class n=22
- Pre and post interview measures in all 4 classes
- Intervention taught over 3 months included 20 lessons

The Lesson Sequence

- Geometric patterns and position cards
  - Multiplicative and composite functions
- Building Sidewalk
- Function Machine
  - Introduce t-tables
- Walkathon

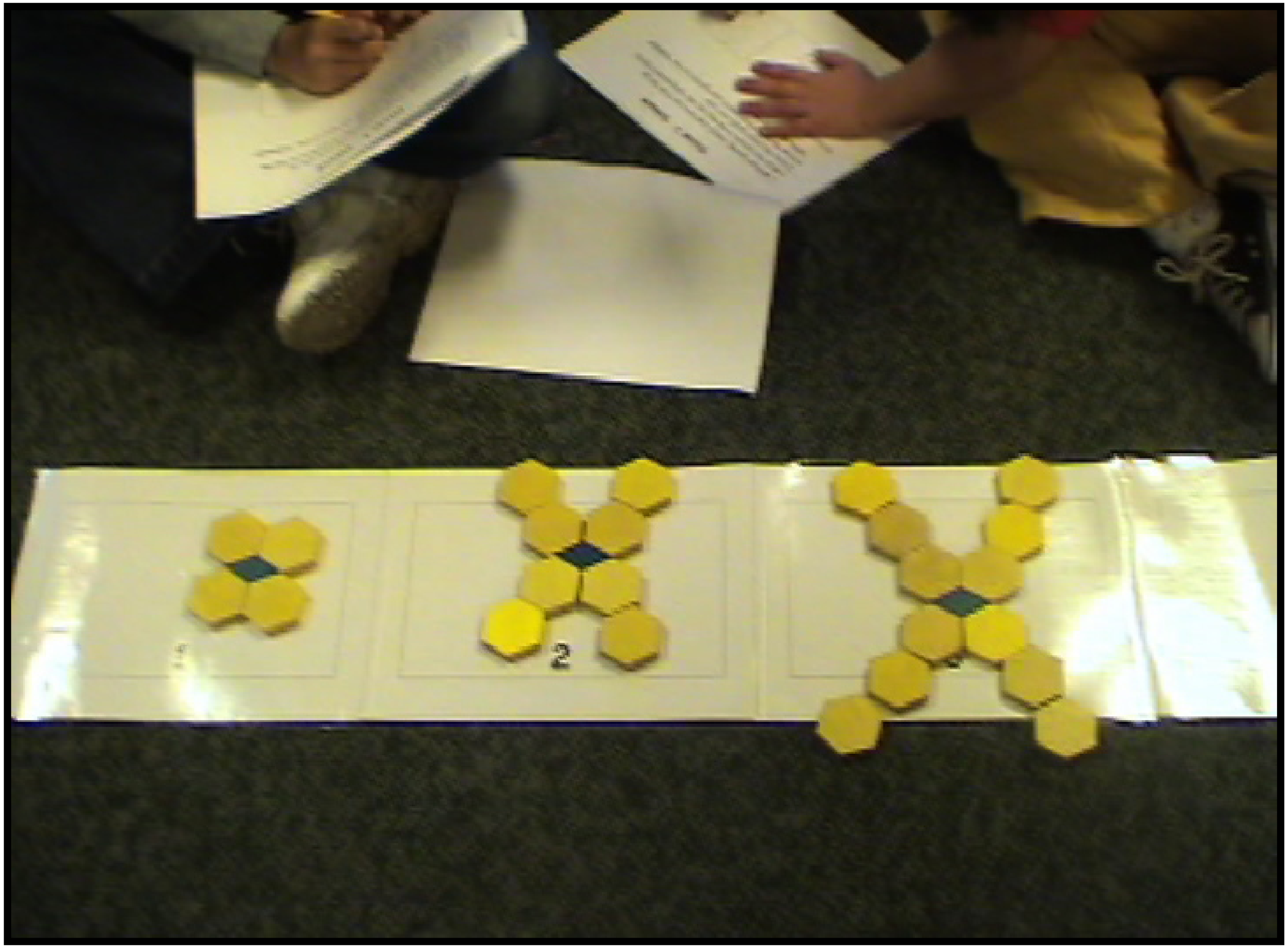
Geometric Patterns and Position Cards



- Multiplicative function – number x 2
- Composite function – number x 2+1

Geometric Composite Functions

Building Sidewalk



Function Machine

Walkathon

Kalchman 2005

Data Collection

- Lessons were videotaped and transcribed for further data analysis
- Field notes were taken
- Classroom artifacts collected
- Ongoing interviewing of students
- Case’s number knowledge test administered pre instruction to determine developmental level
- Pre and Post interviews with all students

Quantitative Analyses

Students assigned to one of three levels (low, medium, high) based on Case Number Knowledge Task

- Prepost interview consisting of 10 patterning problems and sub items administered as pre and post-test

- Categorized test items into combined variables showing ability to solve, near/next predictions Far predictions, Explicit expressions of rules.

Results Quantitative

- Control group higher on number knowledge level not significant difference when 3 experimental groups combined. However significantly stronger than Experimental Group 3

- Two groups statistically the same on the 4 categories at pre-test

- Experimental group and control group same rate of improving on near generalizations. however experimental outperformed control group on far generalizations, and explicit reasoning.

- Post-test revealed all ability levels in experimental classrooms improved significantly, no interaction of group by time, thus pointing to the potential of curriculum

- Experimental Group 1 (20) Functional/explicit category: improved by nearly 2 standard deviations and significantly outperformed control group (f- 31.877 $p < .000$)

Overview of Qualitative Results

- Able to build geometric patterns based on algebraic representations and recognize functions from geometric patterns including 2-step composite functions

- Used syncopated language to express functions

  The experimental group received no formal instruction in multiplication showed a understanding multiplication and its applications through functional understanding and used a variety of invented strategies to find solutions

- Function curriculum elicited discussion of mathematical structural understanding in particular an understanding of zero

Overview of Qualitative Results

- Difficulties with continuous movement between representations

- Difficulty with composite function challenge

Recognize Function from Geometric "Cube Sticker Problem" (video)

Constructing understanding of multiplication …..

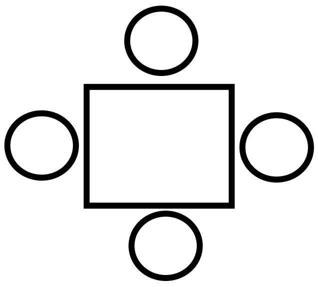
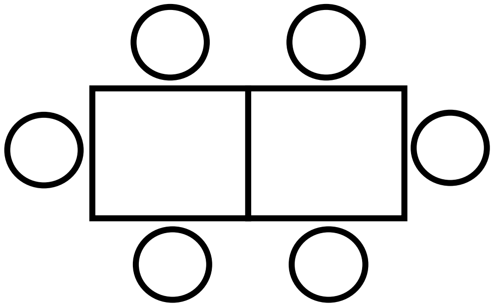
Constructed Understandings of and Informal Strategies for Multiplication

- **3 x 10**: **Skip counting/repeated addition**
  - "I used my fingers to count by 3s, and got to the 10^{th} time."
- **x 2: Doubling**
  - "Number plus itself is the same as times 2"
- **x 3**: **Tripling**
  - "Double the number, then add 1 more"
- **Constructing arrays**
- **4 x 10**: **Combination strategy**
  - "One groups of 4, 2 groups of 4… I looked at 5 groups of 4 and doubled it for 10."

Doubling “timesing by 2” the Function Machine

- It (the function machine) can help you learn some answers in math…. So in one of them it’s like doubling the number… you would learn how to figure it out quickly and you could learn your times tables because you’re kind of learning that there’s a times table in math like when you’re doubling you’re just timesing by 2”

Doubling the “Tables and Chairs Problem”

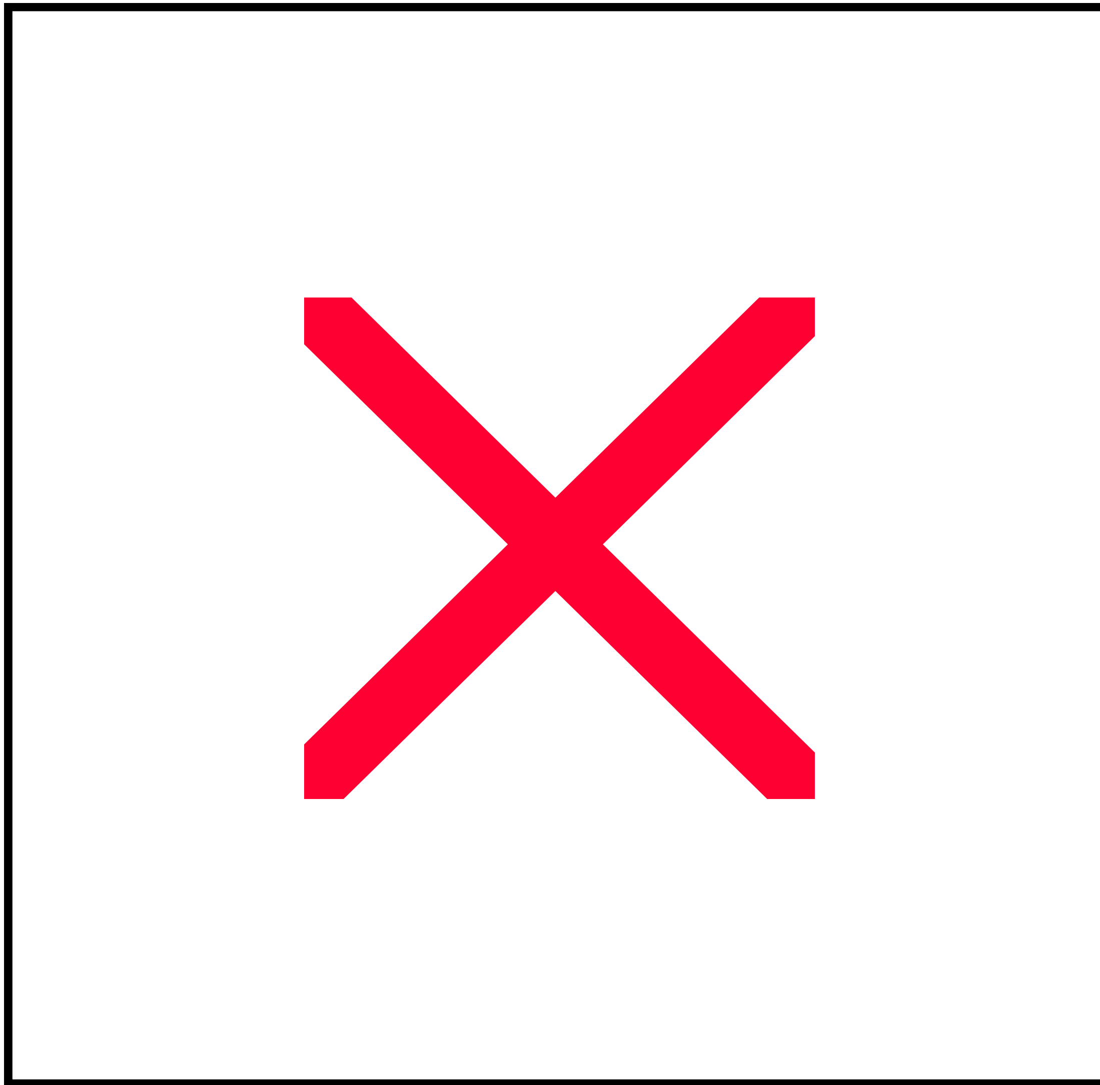
 

- R. How did you figure out how many chairs would be needed in the square table?
- C. Okay, you always add 2, because it’s 4, then plus 2. Cause the next would be 6 cause it’s only adding 2. So the 10^{th} one would be 22 so an easier way than adding 2 is just doing 2x2 and then plus 2.

Constructed Multiplication: 2n + n (video)

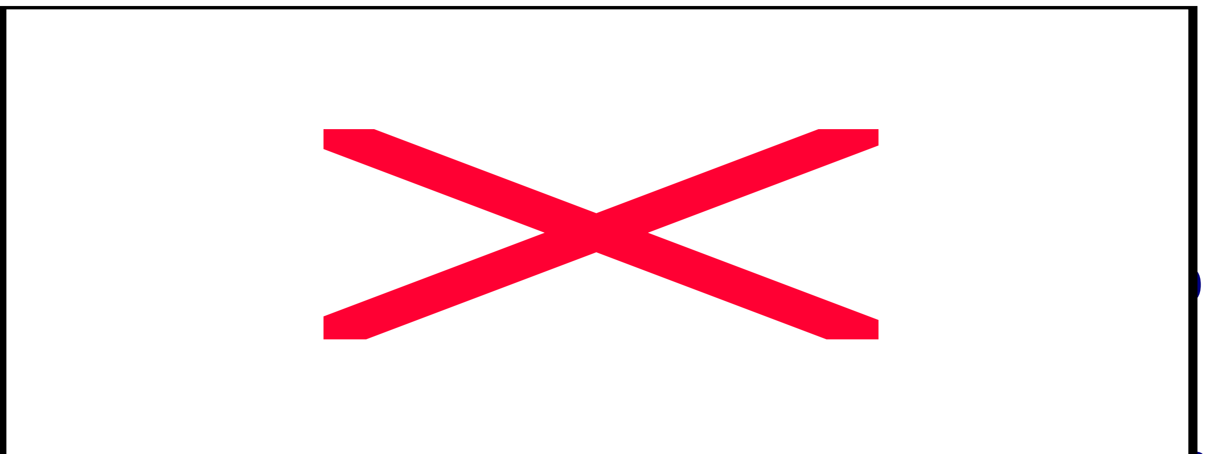
Constructed Multiplication: Describing an Array (video)

Multiplication Item
Output X 5



st	Post test
8%	Exp 80%
50%	Cont 45%

Multiplication Item
2n+2



test	Post test
- 0%	Exp - 55%
Cont -10%	Cont- 10%

What is a rule used in the table to get the numbers in column B from the numbers in column A?

How do you know?

Discussion of zero in multiplication …..

Times 3 plus 2 with 0 position

Geometric Pattern with Position Zero (video)

“number times zero plus five”

tricky function machine rule

- C: Times 0 always makes it to 0 cause it’s like 0 groups of 100.
- J: And then what’s the plus 5?
- C: And then you go to 5 so all the output numbers were 5.
- J: So did anyone even come close to figuring out your rule for the function machine?
- C: At the end we gave them really, really easy hints and then finally Brian said, “ Oh minus the number plus 5. And that was kind of like it, but not quite. And we told him that was another rule that worked but not the one that we thought of.

Challenges and difficulties….

Difficulty With Continuous Movement Between Representations

- Students had little difficulty moving from numeric representations of functions to geometric contexts and could interpret geometric patterns to express rules.
- Some students however had difficulty generating input and output numbers based on rules derived from geometric patterns, and did not seem to rely on their original geometric understanding

Times 3 + 1? or Plus 3 +1?

Confusion ! (video)

Difficulty With Composite Function Challenge

- Some students tended to ignore the constant when attempting far generalizations
- Some students confused the numbers in composite functions i.e. for a function of “n times 2+3” they might mistakenly use “n times 3+2” or “n +3 times 2”

Fourth Grade Study

Method

- Two classes: inner city (n=50) and private school (n=22)

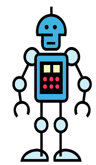
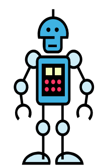
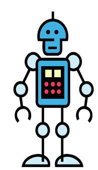
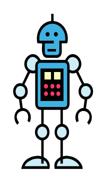
- Pre and Post Test

- 18 lessons taught over five weeks
- Three classes connected electronically on Knowledge Forum

Lesson Sequence

- "Guess My Rule"
- Building geometric patterns from rules, deciphering rules from geometric patterns
- Word Problems
- Classrooms linked via Knowledge Forum to collaboratively solve complex generalizing problems (four linear, two quadratic)
  - Cube Sticker Problem
  - Trapezoid Table
  - Perimeter Problem
  - Triangle Dot
  - Handshake Problem
  - Pattern Kingdom Problem

Guess My Rule

Input		Output
6	→  →	50
3	→  →	26
7	→  →	58
0	→  →	2

My rule is:

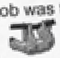
Output = Input x8+2

Geometric Patterns With Position Cards

Word Problems

The Footprints of Mystery Island

Thursica and Dave were visiting a very remote island, just off the coast of Ecuador in South America. They had been told that there were many strange creatures living on the island. Their Uncle Fliphead was a scientist studying the rare wildlife, and he had asked Thursica and Dave for their help.

Their first job was to measure the height of all the Giant 6-toed Sloths living on the island.  Uncle Fliphead had managed to measure three of the Sloths, but he needed to complete his data collection.

Here are the measurements of the Sloths he found so far:

Sloth Foot Size (cm)	Sloth Height (cm)
44	220
68	340
22	110

Uncle Fliphead told them that there were 4 more Sloths on the island, and that he needed to figure out how tall they were.

He sent Thursica and Dave out to collect data.

"Wow, how are we going to be able to measure such huge creatures?" asked Dave.

"I don't think I want to run into any Giant Sloths – what if they're vicious?" said Thursica. Dave thought she had a point – he didn't want to run into any vicious Giant Sloths either.

How could Dave and Thursica measure Sloths without having to get near them?

Can you write a rule that Dave and Thursica could use to figure out the height of the Giant Sloths?

Thursica walked along and found a six-toed footprint. She measured it – it was **28 cm** long.

Moon Bat Chart

Age (in years)	Height (in cm)
21	128
34	206
12	74

1. How many cm does a Moon Bat grow each year?

2. How tall is a Moon Bat when it is first born?

3. How tall would a Moon Bat be if it reached the age of 100?

Knowledge Forum and Knowledge Building Principles

Bereiter and Scardamalia

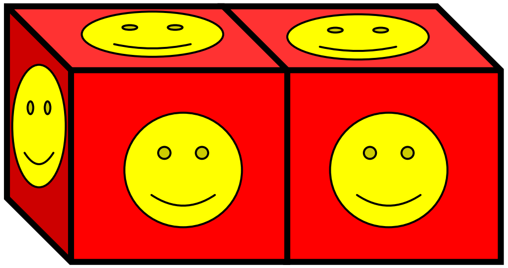
- All ideas are treated as *improvable*
- All contributions incorporated into a process of *idea refinement* (*Democratization of Ideas*)
- Group assumes the ownership and improvement of ideas ("*Epistemic Agency*")

We speculated that Knowledge Forum and knowledge building could provide an authentic situation for students' conceptual understanding of, and communication about the mathematical ideas underlying patterns and functions

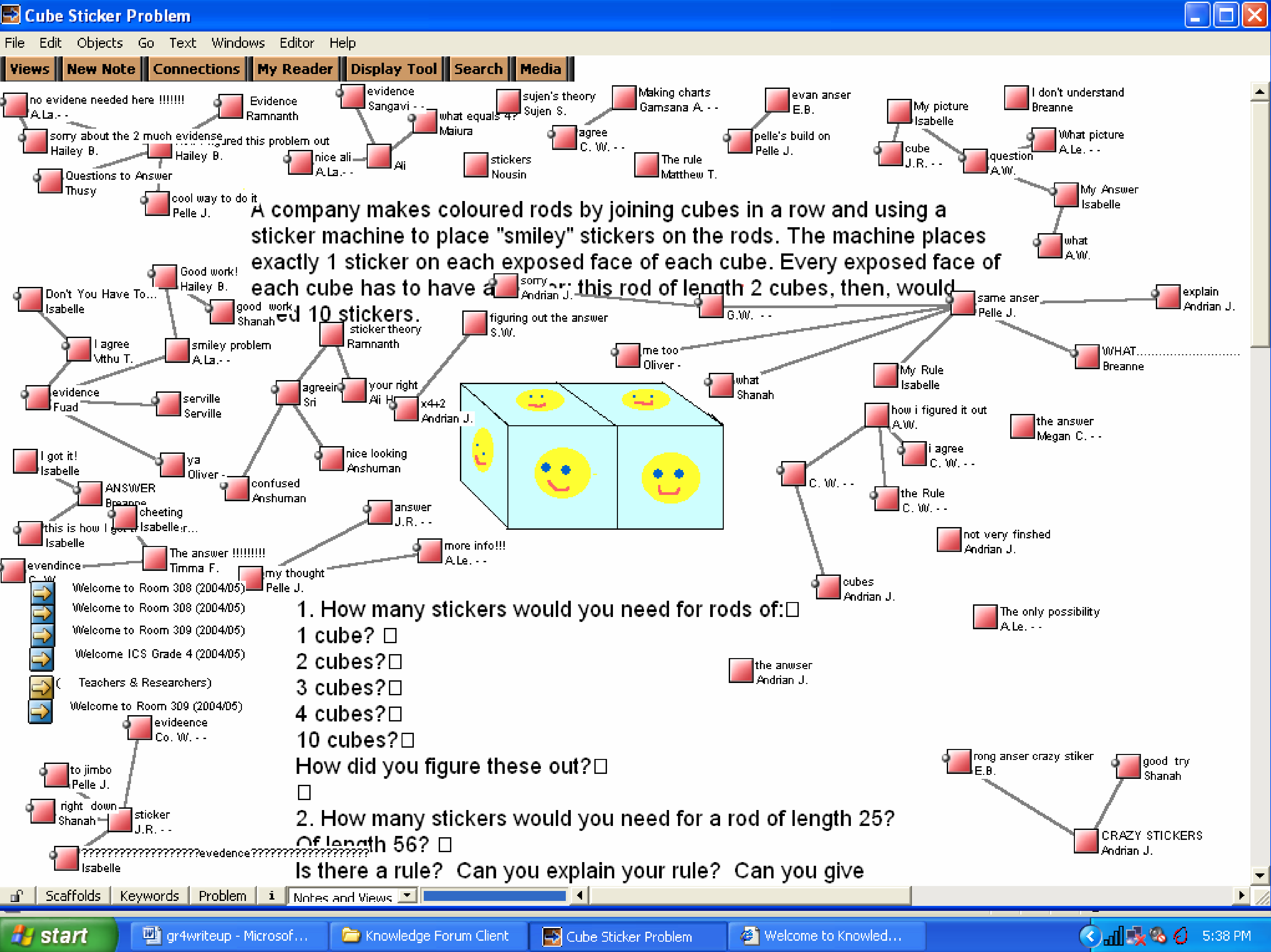
All contributions to the Knowledge Forum database were exclusively from the students. Discussions on Knowledge Forum began with the cube sticker problem…

Cube Sticker

- A company makes coloured rods by joining cubes in a row and using a sticker machine to put “smiley” stickers on the rods. The machine places exactly 1 sticker on each exposed face of each cube. Every exposed face of each cube has to have a sticker. This rod of length 2 (2 cubes) would need 10 stickers.



- How many stickers would you need for:
- A rod of 3 cubes
- A rod of 4 cubes
- A rod of 10 cubes
- A rod of 22 cubes
- A rod of 56 cubes
- What is the rule?



Cube Sticker Problem
File Edit Objects Go Text Windows Editor Help
Views New Note Connections My Reader Display Tool Search Media
A company makes coloured rods by joining cubes in a row and using a sticker machine to place "smiley" stickers on the rods. The machine places exactly 1 sticker on each exposed face of each cube. Every exposed face of each cube has to have a sticker. For this rod of length 2 cubes, then, would need 10 stickers.
1. How many stickers would you need for rods of:
1 cube?
2 cubes?
3 cubes?
4 cubes?
10 cubes?
How did you figure these out?
2. How many stickers would you need for a rod of length 25?
Of length 56?
Is there a rule? Can you explain your rule? Can you give
no evidene needed here !!!!!! A.La.
Evidence Ramnanth
evidence Sangavi
what equals 4? Maiura
sujen's theory Sujen S.
Making charts Gamsana A.
evan anser E.B.
My picture Isabelle
I don't understand Breanne
sorry about the 2 much evidense Hailey B.
I figured this problem out Hailey B.
nice ali A.La.
Ali
stickers Nousin
agree C. W.
The rule Matthew T.
pelle's build on Pelle J.
cube J.R.
question A.W.
What picture A.Le.
Questions to Answer Thusy
cool way to do it Pelle J.
My Answer Isabelle
what A.W.
Good work! Hailey B.
Don't You Have To... Isabelle
good work Shanah
sorry Andrian J.
G.W.
same anser Pelle J.
explain Andrian J.
figuring out the answer S.W.
sticker theory Ramnanth
I agree Vithu T.
smiley problem A.La.
me too Oliver
WHAT. Breanne
agreeing Sri
your right Ali H.
x4+2 Andrian J.
what Shanah
My Rule Isabelle
evidence Fuad
serville Serville
how i figured it out A.W.
the answer Megan C.
i agree C. W.
I got it! Isabelle
ya Oliver
nice looking Anshuman
C. W.
the Rule C. W.
ANSWER Breanne
confused Anshuman
cheeting Isabelle
this is how I got it Isabelle
answer J.R.
not very finshed Andrian J.
The answer !!!!!!!! Timma F.
more info!!! A.Le.
evendince C. W.
my thought Pelle J.
cubes Andrian J.
Welcome to Room 308 (2004/05)
Welcome to Room 308 (2004/05)
Welcome to Room 309 (2004/05)
Welcome ICS Grade 4 (2004/05)
(Teachers & Researchers)
Welcome to Room 309 (2004/05)
The only possibility A.Le.
the anwser Andrian J.
evideence Co. W.
to jimbo Pelle J.
rong anser crazy stiker E.B.
good try Shanah
right down Shanah
sticker J.R.
CRAZY STICKERS Andrian J.
?????????????????evedence????????????????? Isabelle
Scaffolds Keywords Problem Notes and Views
start gr4writeup - Microsof... Knowledge Forum Client Cube Sticker Problem Welcome to Knowled... 5:38 PM

Cube Sticker

Making Charts – *G*

My theory is that the rule is times 4+2. I found out the rule right after I made a chart because in the second column there was a pattern going down and the rule was plus 4. Then I thought that the rule was times 4 but when I tried it it didn’t work. So I looked at the chart again and I looked across and I saw that 2 was missing. Then I knew that 2 is constant. Then I got the rule times 4 plus 2.

Figuring Out the Answer – *SW*

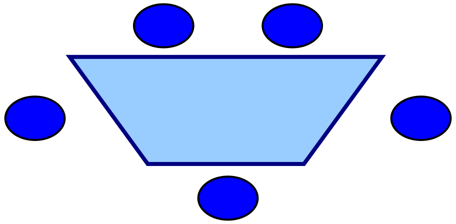
How I figured it out was I built the first few rods and then tried to think what the rule was…and then I thought there were 4 sides and one top and one bottom, and so it would be x4+2. And for the next number of rods instead of building each one I did x4+2.

The next discussion on the Trapezoid Table problem illustrates students' increasing focus on the importance of context as a way of supporting their conjectures…

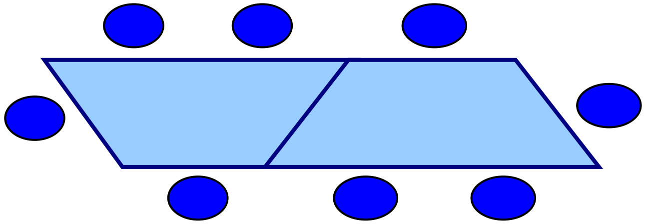
Trapezoid Tables

Mrs. Chen decided she would place the chairs around each table so that 2 chairs will go on the long side of the trapezoid and one chair on every other side of the table.

This way 5 students can sit around 1 table.



Then she found that she could join 2 tables like this:



Now 8 students can sit around 2 tables.
How many students can sit around 3 tables joined this way?
How many students can sit around 56 tables?
What is the rule? How did you figure it out? Can you give evidence?

Trapezoid Table Discussion

Close guess – *Anshuman*

number of trapezoid	number of chairs
1	5
2	8

is that the rule is the output is equal to the input x the input -1+5.I got the number 1 because when a trapezoid is adding one seat is being cancelled.My evidence is that it works for 1 like 1x1=1-1=0+5=5.it works for 2x2=4-1=3+5=8.So for three i think there should be 13 chairs . But when drew and checked there were 11 chairs . So my theory was wrong.

Trapezoid Table Discussion

Your rule doesn’t work – *Shatabdy Z*

Anshuman you are wrong !!Because when I tried your rule it only worked for the picture below the problem's discription.the real answers are:for the first one it's eight for three it's 11 and for the fifty sixth one it's 170.And I do have proof I did on a piece of paper and I was right!!!!!!!!!!!

I don’t think so – *Fuad*

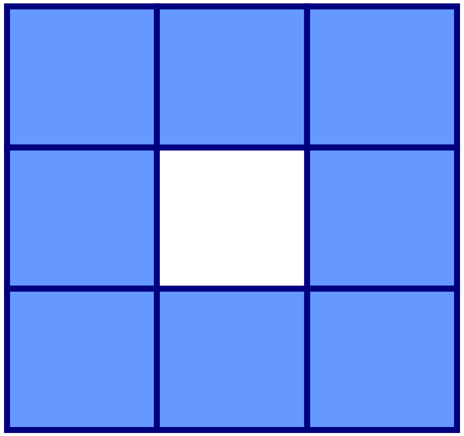
could you check your T- chart because for the first it is 5 for the seccond it is 8 for the third it is 11.

Supporting your theory – *Nousin*

I agree with you because I think Anshuman's rule is wrong too. I also did it on a peice of paper and found out the right rule which is not the rule you found. But there are many other rules but none of them are the one that you think. Your rule only applies in some of the patterns not in all of the patterns meaning that thats not the rule because it doesn't apply in all of them. So I think you should come up witha different rule.

Discussions of Steele’s “Perimeter Problem” reveal student’s negotiation of multiple rules…

Perimeter Problem



- This is a 3x3 grid of squares with only the outside edge shaded.
- If you had a 5x5 grid of squares where only the outside edge of squares is shaded, how many squares would be shaded?
- If you had a 17x17 grid of squares with only the outside edge of squares shaded, how many squares would be shaded?
- If you had a grid of 100 number of squares, how many would be shaded?
- Is there a rule? How did you figure it out? Can you give evidence?

Perimeter Discussion

Eureka! *A.W.*

for the 5x5 question you do 5x5=25 the square of 25 is 5 and you minus two from the square and square that then minus it from your original number and you have your answer!

First i drew the five by five grid and there was nine in the middle to take away - 3x3=9

so then i figured out a 6x6 square was 36 and i know that inside there would be a 4x4 square to take away so the difference between 6 and 4 is 2 - so it was 36-16=20

then 17x17=289 the square root is 17

then minus 2 from 17 which is 15 (because before there was 2 difference like between 6 and 4) and then 15x15=225 then i minused 225 from 289 and got 64

nxn=nsquared -(n-2)squared -

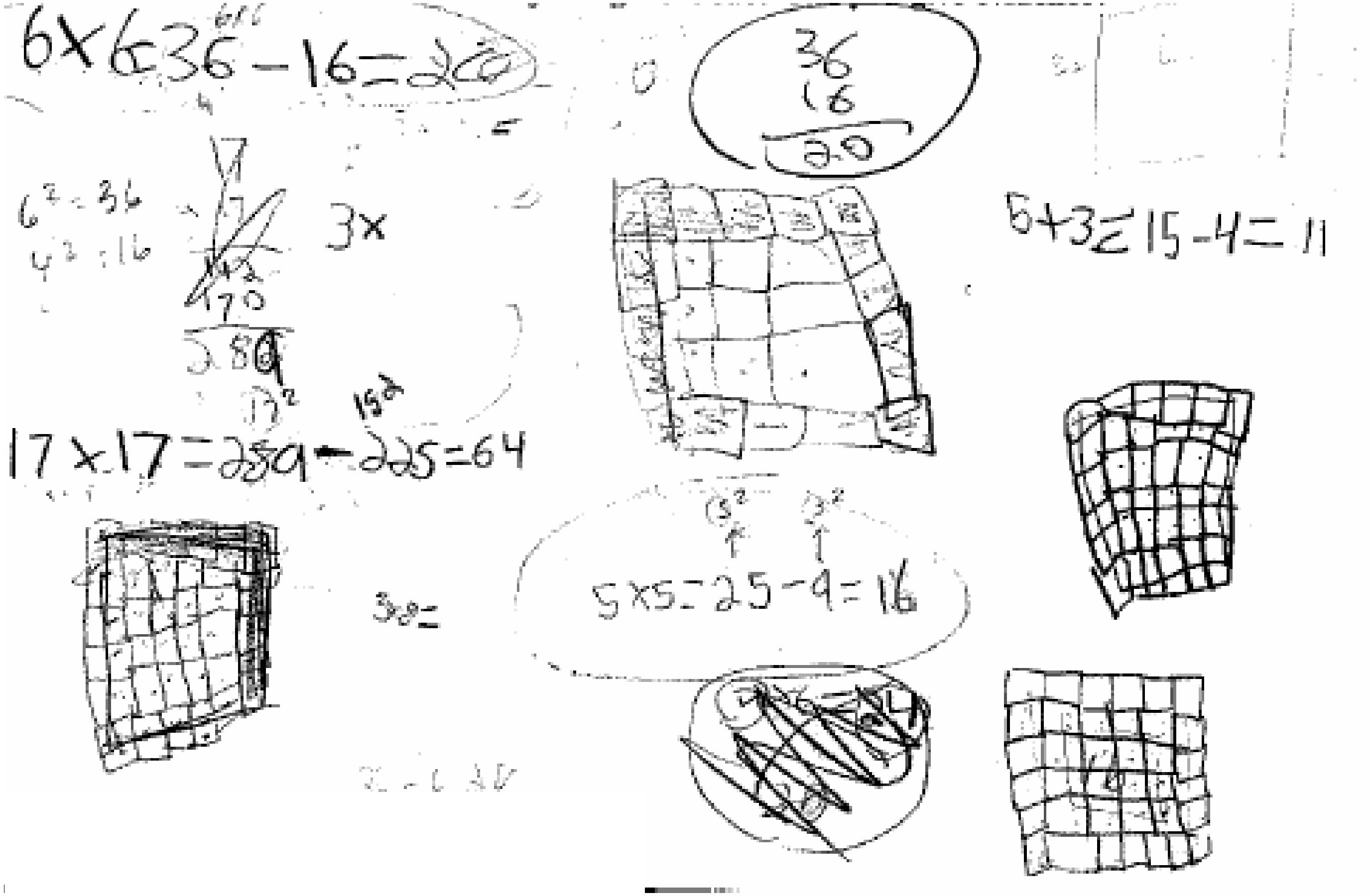
so minus (n-2)squared from nsquared

for the 100 question you have to figure out what x what equals 100.

100x100=10000 and you keep going from there you minus two from the 100 (wich is 98) and 98x98=9604 and 10000-9604=396 so the answer to that is 396

Perimeter Discussion

Eureka! *A.W.*



Perimeter Discussion

Another rule – *Sri*

I have another rule for you and it is the output x4-4.In the rule it is x4 because there is 4 sides in a square.It is -4 because when you multiply 4 you are repeating the corners twice so you –4

Same Rule – *Rithu C.*

that is the same way I got my answer Sri, the number on one side of the grid x4-4. I know thats true because I noticed in my drawing that the sides were timing by 4 and the corner's squares were minusing by 4 too.

2 rules – *A.W.*

but there might be two rules because we got the same answer for both so i think there is more then 1 way to figure the problem out

Perimeter Discussion

Both right? - *Gamsana A*

I agree with you and disagree with you because you've got the answer but in a complicated way. I disagree with you because there's an easier way than taking the square of 25 ,subtracting 2 from it and square that and then subtract that from your original answer. I also disagree with you because the 100 problem you did was wrong. I'm not saying you're wrong without proof because I tried working out the problem and when I did that I figured out that the rule was times 4-4. So if I use that rule with 100 the answer is 396. I got the rule times 4 -4 because a square has 4 sides and you don't count the corners twice. I agree with you because for the first few questions you got it right. I think you might have the rule

but it just didn't work for 100. Can you try it again,please?

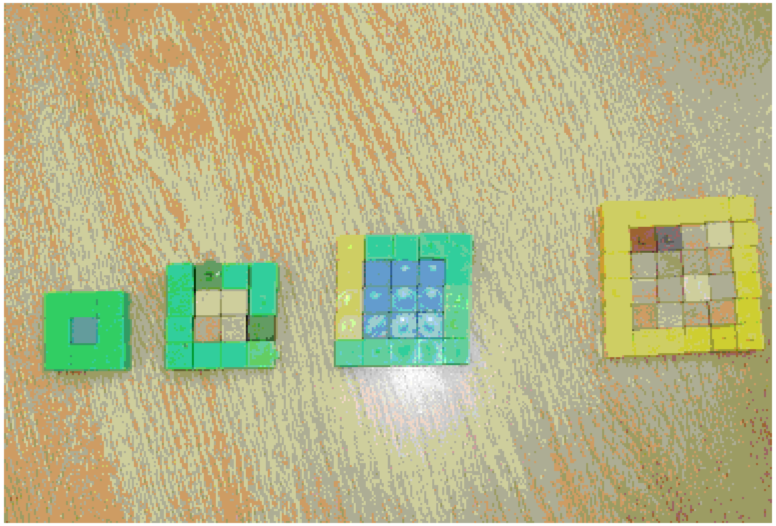
Thanks!! *A.W.*

I tried doing it again and got the same answer as you! Thanks!

Perimeter Discussion

The perimeter with squares – *Thusy*

My other theory is that the output is equal to the input x input - unshaded squares. Also it works for every one so that's how I know that it is one of the rules. I tried this rule because there were different rules so I tried different types of rules but none of them worked. Then Lavanya and I built some squares with cubes and saw it has to do with the unshaded squares. Then we subtracted the unshaded squares and tried a different rule and the rule was that the inputxinput-unshaded squares = shaded squares. This my stargety. My stregety is by building a model and thinking about the problem. Also doing it by steps.



Handshake Problem

- Imagine that the Maple Leafs won the Stanley Cup and you are at a huge party with everyone in Toronto to celebrate.
- Everyone starts to shake hands with other people who are there.
- If 2 people shake hands, there is 1 handshake.
- If 3 people are in a group and they each shake hands with the other people in the group, there are 3 handshakes.
- If 4 people are in a group and they each shake hands with the other people in the group, there are 6 handshakes.
- How many handshakes would there be if there were 10 people in the group?
- How many handshakes would there be if there were 100 people in the group?
- Can you use a rule to help you figure this out? How did you figure it out?

Handshake Discussion

my theory - *Anita*

My theory is the first person who shakes hands with the all the other people. So if there's 10 people, the first person would shake 9 people's hands. Then the next person would shake 8 people's hands. And it goes on and on and on. The last person shakes 0 hands.

So for a 100, the first person would have 99 handshakes. And you go on and on and add all the number together and that will be the answer.

to get the answer - *E.S.*

I agree with you. I think that a faster way to get the answer to a question like that is to take the number of people , so say there were 8 people , you would go 8x8-1 divided by two, because each person gets one handshake less than the person befor , so it's sort of like -1.

Handshake Discussion

The rule – *Jessica*

i am going to try your theory with threes 3x3-1 divided by 2=4 but we know the answer is 3. how come I got four? (Please answer).

Your question is answered – *E.S.*

You got 4 because I made a mistake. It's actuly 3x 3-1 wich is eczactly like 3x2 divided by 2. That equles 3. I only did -1 so you'd know how I got 2 .

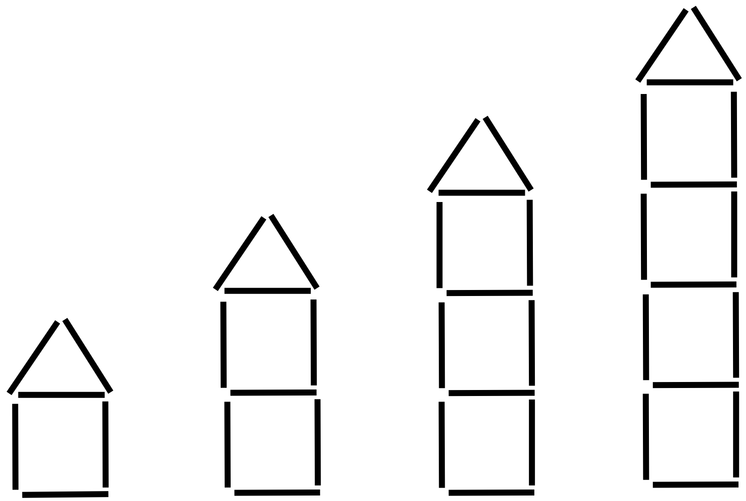
Explanation of E.S.'s note – *Jonathan*

The number of people –1x the number of people take 50% of that number and that is the awnser. 10 people – 1 = 9. Then 9 x 10 (because there are 10 people) divide by 2 (90 divided by 2 = 45) and 45 is the answer.

Quantitative Analyses and Results

- Analyses
- Five items with sub-items administered as pre and post-test (total test 10 variables)
- Students assigned to one of three levels (low, medium, high) based on teacher ratings and report cards
- Categorized variables Near, Far predictions, Recursive, Explicit expressions of rules
- Results
- Quantitative analyses based on pre- and post-test measures revealed that students improved significantly, achieving and effect size of 3.2 standard deviations
- This improvement was consistent over high medium and low achievers.

Pre Post Test Items



1st 2nd 3rd 4th

How many toothpick would it take to make the next house? (Next)

Pre 59%
Post 92%

How many would it take to make the the 11th tower? (Near)

Pre 26%
Post 80%

How many would you need to make the 41st tower? (Far)

Pre 2%
Post 77%

Pre Post Test Items

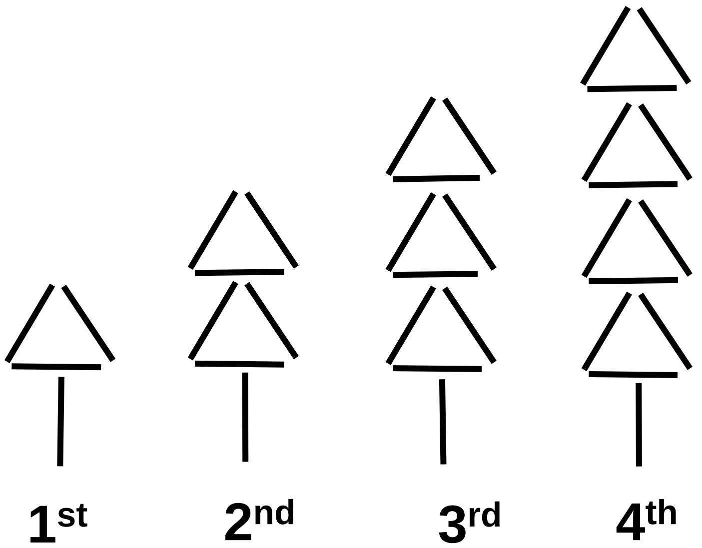
Column A	Column B
3	8
4	10
6	14
10	22

What is a rule used in the table to get the numbers in column B from the numbers in column A?

Pre 9%

Post 82%

Comparative Test Items



How many toothpick would it take to make the next tree? (Next)

Exp 90%
norm 100%

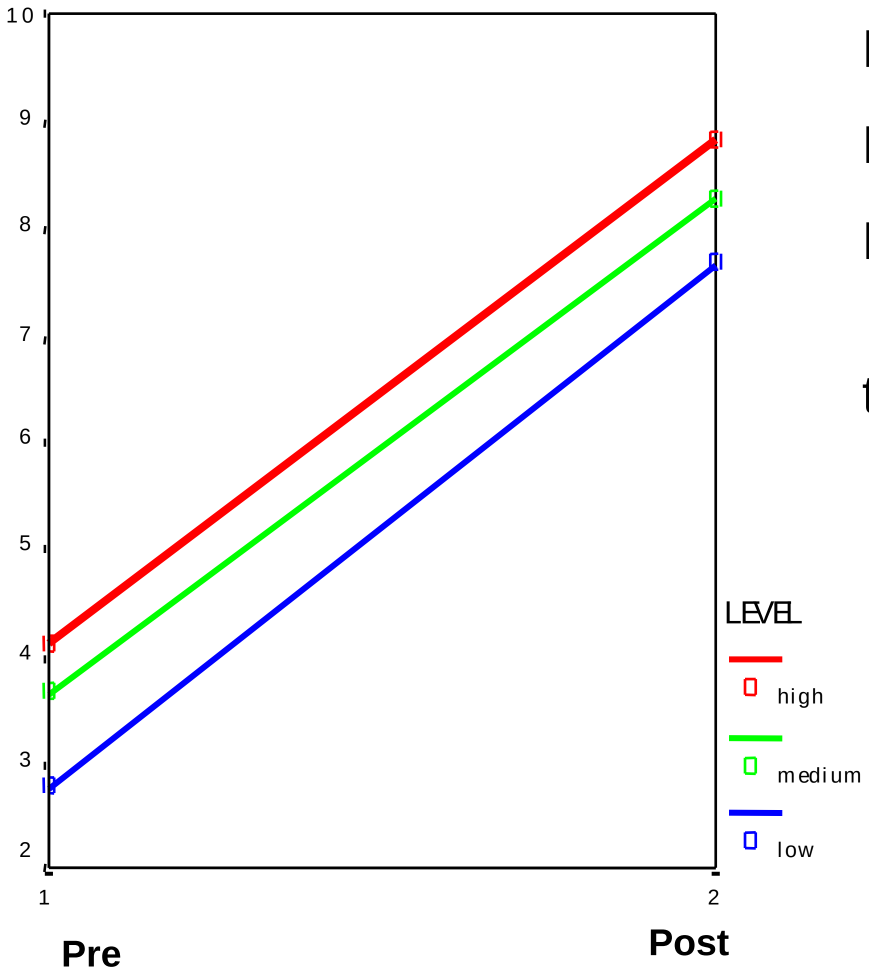
How many would it take to make the 11th tree? (Near)

Exp 86%
Norm 60%

How many would you need to make the 41st tree? (Far)

Exp 88%
Norm 11%

Results: Pre/Post Test



Pre Test Mean 3.50 (1.67)

Post Test Mean 8.23 (2.30)

Effect Size 2.38

$t = 14.70$ $p = .000$

Achievements

Children gained the ability to:

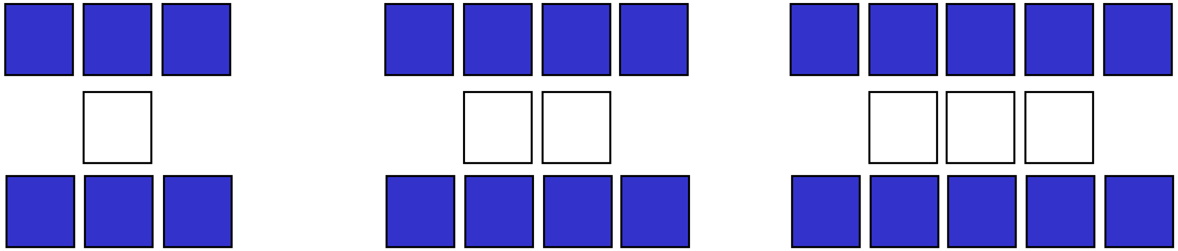
- Generalize - find functional rules for patterns, relationships between patterns, and relationships across different representations
- Perceive patterns and relationships in geometric structure which eludes most children
- Move *beyond recursive* thinking to understand the *relationships between the two variables* in the problem
- Demonstrate a disposition for rigour and developed an understanding of the need for providing evidence and justification

Achievements

- Provide explanations beyond simple substitution of numbers
- Focus on the role and meaning of the variables in the problems, using drawings, words, syncopated language (Sfard), and formal symbols
- Abandon proportional explanations based on “whole object” reasoning

Difficulty: Interpreting complex patterns

- 25 post interviews revealed that some students still had problems finding functional rules from complex geometric patterns.



Difficulty: Composite Functions

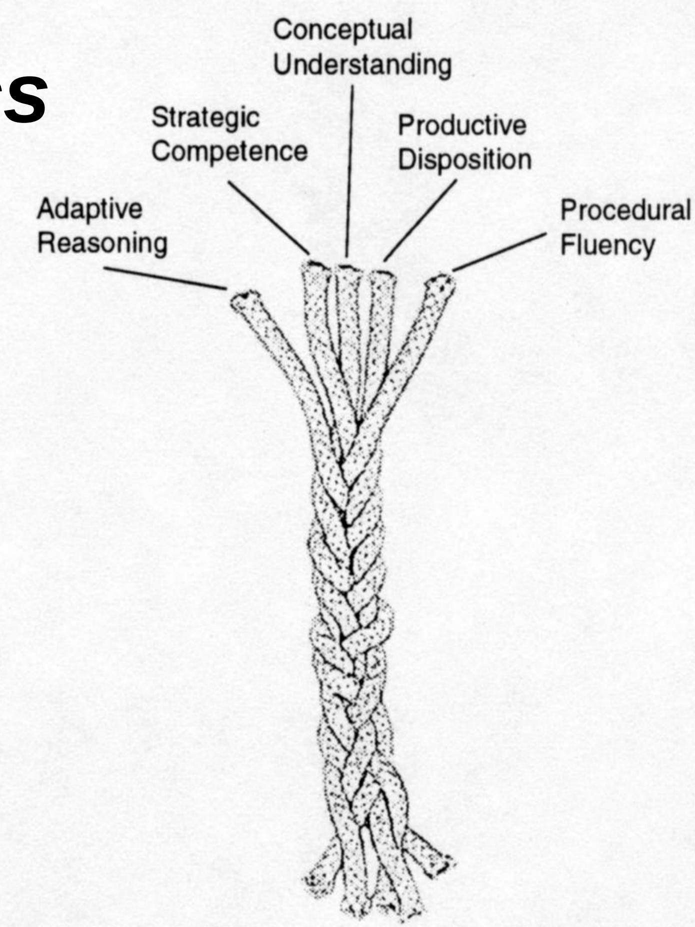
- A number of students who tended to rely primarily on numeric representations had difficulty understanding composite functions within the context of the geometric and/or narrative generalizing problems

Patterning and Mathematical Proficiency

Adding it up: Helping Children Learn Mathematics

National Research Council 2003 Report

- Conceptual Understanding
- Strategic Comprehension
- Adaptive Reasoning
- Procedural Fluency
- Productive Disposition



Intertwined Strands of Proficiency

Strands of Proficiency

- **Conceptual Understanding** - *comprehension of concepts, operations and relations*
  - Made connections between various representations of functional relationships (tables, geometric patterns and word problems) – used this knowledge to solve complex generalizing problems
  - Revealed in context-supported explanations, and ability to solve new and unfamiliar problems

Strands of Proficiency

- **Strategic Comprehension** – *ability to represent and solve mathematical problems*
  - Flexibility in terms of representing problems, and devising novel solutions
  - Ability to perceive structural relationship within problems, and apply previously successful solution strategies to solve unknown problems

Strands of Proficiency

- **Adaptive Reasoning –** *capacity for logical thought, reflection, explanation, and justification*
  - Disposition to provide justification, and explanation of conjectures (demand for evidence)
  - Multiple representations to demonstrate understanding (function tables, models, drawings, language, syncopated language, symbolic notation)
  - Refinement of initial conjectures based on new evidence

Strands of Proficiency

- **Procedural Fluency** – *carrying out procedures flexibly, accurately, efficiently*
  - Skill in carrying out procedures (both mathematical computations, and generalizing strategies) flexibly, accurately and appropriately gained through context oriented exercises requiring accurate computation
  - Efficient use of both recursive and explicit problem solving strategies

Strands of Proficiency

Productive Disposition **--** *Habitual inclination to see mathematics as sensible, coupled with belief in diligence and one's own efficacy*

- It was different – in math you don't use "my theory" and get to write it down and then get new information to help you develop your theory – in math you don't draw much – in real math you don't ask people to help you find what the rule is (grade 4 student)

- Um it's because you just have to use your brain. patterns are fun and math is hard so its like, it's always making somebody have a level of their own and things like that. So the fun part is like the patterning because the patterning just about everybody in our class knows. But it's math, so it's kind of hard and then the patterning evens everything out so it's hard for everybody. (grade 2 student)

Discussion