NAEP Algebra Doesn’t Measure Up

Algebra on the Grade 8 NAEP

- There are many problems that are mathematically incorrect
- Virtually all the problems are at a low level relative to U.S.A. math standard
- But they are at an even lower level when compared to foreign problems

Grading the questions

- Of the 41 eighth grade NAEP algebra problems, 8 were incorrect and one was meaningless
- Moreover about 10 of the correct problems were just questions about vocabulary, not mathematics

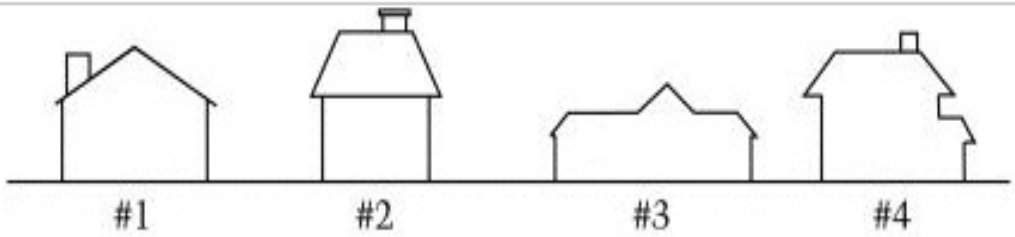
- Of the 22 grade 4 questions, four were incorrect, four others were essentially vocabulary, only one could be judged challenging, but the average level was about grade 4

8. Tetsu rides his bicycle x miles the first day, y miles the second day, and z miles the third day. Which of the following expressions represents the average number of miles per day that Tetsu travels?
Correct problem. Reasonable but not challenging at 7th to 8th grade level. More vocabulary than anything else.

A) $x + y + z$
B) xyz
C) $3(x + y + z)$
D) $3(xyz)$
E) $(x + y + z)/3$

Did you use the calculator on this question?

○ Yes ○ No



17. Allen, Bridgitte, Chaz, and Diann each live in a different house on the same side of a street. The houses and their numbers are shown above.

- Only one of the other three people lives next to Bridgitte.
- Chaz lives next to Bridgitte and next to Diann.

Which person could live in house number 2?

Solid problem. Definitely tests skills at seventh or eighth grade level that are important.

A) Allen only

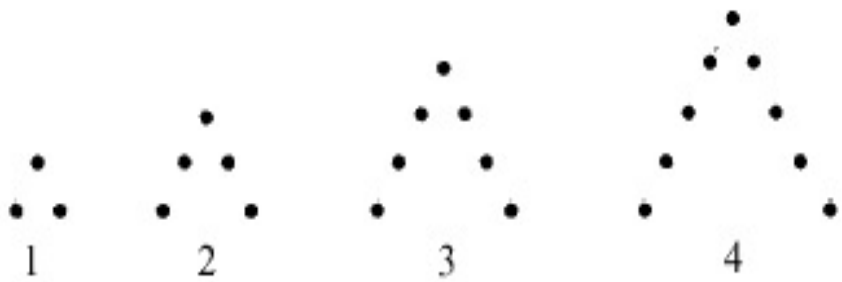
B) Chaz only

C) Diann only

D) Chaz or Diann

E) Any of these four people could live in house number 2.

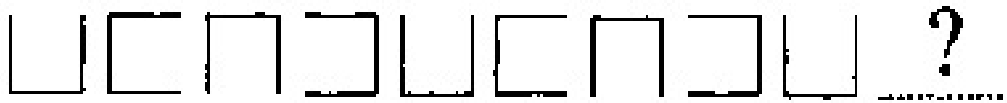
The following question refers to the following pattern of dot-figures.



35. If this pattern of dot-figures is continued, how many dots will be in the 100th figure? Another miserable question. I've seen this question in a few fifth grade exams. It is not well posed.

A) 100
B) 101
C) 199
D) 200
E) 201

Problems Common to Both Exams



For all the usual reasons this problem is not well posed. A perfectly good answer would be "George."

22. In the pattern above, which figure would be next?

A) 

B) 

C) 

D) 

Puppy's Age	Puppy's Weight
1 month	10 lbs.
2 months	15 lbs.
3 months	19 lbs.
4 months	22 lbs.
5 months	?

24. John records the weight of his puppy every month in a chart like the one shown above. If the pattern of the puppy's weight gain continues, how many pounds will the puppy weigh at 5 months?

This problem is not well posed.

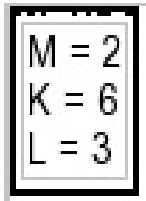
A) 30

B) 27

C) 25

D) 24

Sample Problems from Grade 4 NAEP



7. What is K + L - M? Reasonable, but low level. It is more a test of vocabulary than algebra.

A) 1
B) 5
C) 7
D) 11

20. A pattern of shapes is to be repeated many times. The figure above shows one completed pattern and the beginning of the next. What shape comes next? This problem is well posed for a change. It is probably more like second or third grade algebra, but it is legitimate.

A) 

B) 

C) 

D) 

Did you use the calculator on this question?

○ Yes ○ No

Algebra Problems from other Countries

Grade 4: Russia

Equations and Their Solutions

Problem. The left pan of a set of scales (fig. 80) contains 5 identical boxes of noodles, and the right pan contains 3 identical boxes and two 2-kg weights. The scales are balanced. How much does each box weigh?

Let us write an equation based on the problem. Let the weight of one box of noodles equal x kg. The weight of 5 such boxes will then be $5 \cdot x$ kg, and the weight of three boxes will be $3 \cdot x$ kg. The right pan contains a total of $3 \cdot x + 4$ kg, and the left pan contains $5 \cdot x$ kg. Since the scales are balanced, we need to find those values of the variable for which the following equality is true:

$$5 \cdot x = 3 \cdot x + 4$$

We have written an equation. In order to solve it, let us remove 3 boxes of noodles from each of the pans. Then 2 boxes will remain on the left pan and two 2-kg weights will remain on the right pan. The scales will still be balanced. It is clear that the weight of each box of noodles is 2 kg. So the equality $5 \cdot x = 3 \cdot x + 4$ is true only for $x = 2$.

An equality with a variable is called an **equation.** Each value of the variable for which a true equality results is called a **root of the equation.**

Solving and equation means finding all its roots, that is, the set of its roots.

The equation $5 \cdot x = 3 \cdot x + 4$ has only one root—2.

The equation $(x - 1) \cdot (3 - x) = 0$ has 2 roots—1 and 3, since the equalities $(1 - 1) \cdot (3 - 1) = 0$ and $(3 - 1) \cdot (3 - 3) = 0$ are both true. This equation has no other roots. The set of its roots is $\{1; 3\}$.

The equation $2 + x = x + 2$ has an infinite number of roots, since for any value of the variable x the result is a true equality. The set of its roots consists of all numbers.

The equation $7 + x = x$ has no root. The set of its roots is empty.

262. Is 6 a root of the equation $36 - a = 24 + a$?

263. Try replacing the numbers 0, 1, 2, and 3 in the equation $y + y = y \cdot y$. Are any of these numbers roots of the equation?

267. Do the following equations have at least one root?

a) $x+16=12$;
b) $x+12=16$;
c) $x+2=4$;
d) $x \cdot x+3=0$?

268. The perimeter of an octagon is 24 cm (fig. 81). Write an equation and solve it. What does the root of this equation signify?

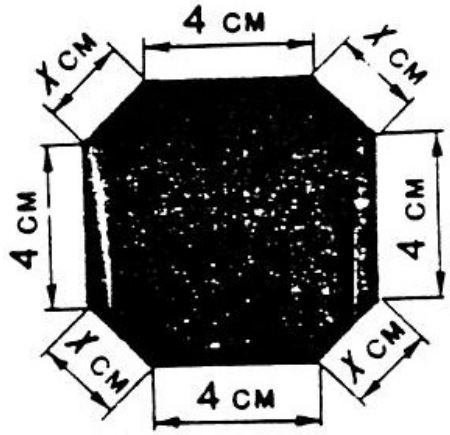


Fig. 81

269. Write an equation based on figure 82 and find the weight of one bag of flour.

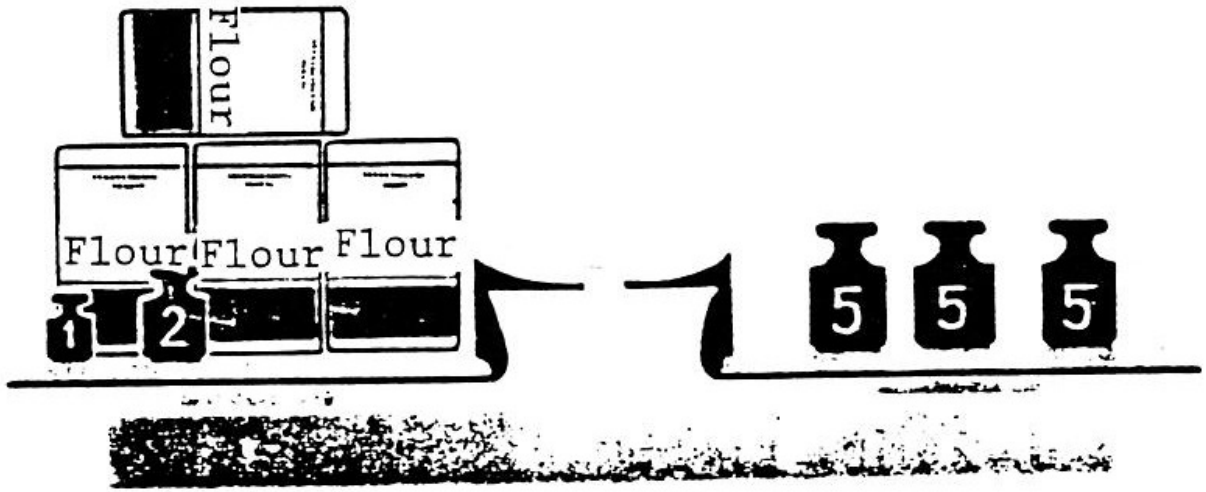


Fig. 82

With That Background

Some Problems from a Japanese Sixth Grade Exam

Here is one of the early problems. It is allocated 2 minutes.

> Jenny wanted to purchase 2 dozen pencils and a pen. Those items cost \$8.45 and she did not have enough money. So, she decided to purchase 8 fewer pencils and paid \$6.05.
>
> How much was a pen?

another 2 minute problem of a somewhat different kind.

There are 3 numbers represented by A, B, and C. The average of A and C is 9. The average of B and C is 18. When A is divided by C the quotient is 2.

What is B?

The following problem is given 5 minutes.

Thirty students took a test. The results are shown in the diagram. The average score was 3.2.

Score	1	2	3	4	5
Students	2	?	?	0	4

How many students got 3 points?

Like it or not – the world is flat

- Our expectations on our national report card do not begin to match up to foreign expectations.
- Moreover, there is no expectation that things will change for at least 5 years.
- We are not doing what we should for our children, and the risks are huge.