Algebraic Reasoning: Discussion

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Basic Issues

- What?
  - Content: Mathematics as an academic discipline
  - Content: Mathematics as an applied skill
  - What is the human mind prepared to learn of the above?
- How
  - Depends critically on the what; Content X What is learned easily and what is not
  - Built in transfer (positive + negative); transfer related to prior learning
- When?
  - In terms of necessary background knowledge
  - In terms of cognitive & development influences on learning

Is the Human Mind Prepared to Learn Algebra?

- **Primary abilities** are universal and coalesce around the areas of folk psychology, folk biology, and folk physics.
  - On the basis of recent neuroimaging research, the foundation for an evolved number-counting-arithmetic system may be folk physics.
  - These abilities emerge through interaction between inherent biases and child-initiated activities, such as play and social discourse
- **Secondary abilities** are built from folk systems through mechanisms that are not fully understood but likely involve attentional control and explicit (sometimes implicit) representation of the to-be-learned information in working memory.

Schools and Organization of Instruction

- Scientific, technological, and intellectual advances were initially built upon folk domains, but corrected and extended folk knowledge
- These advances result in a **gap** between folk knowledge and the theories and knowledge base of the associated disciplines
- Schools emerge in societies in which this gap between folk knowledge and the competencies needed for successful living in the society needs to be closed
- One function of schools is to organize children's activities to allow them to acquire the competencies that close this gap

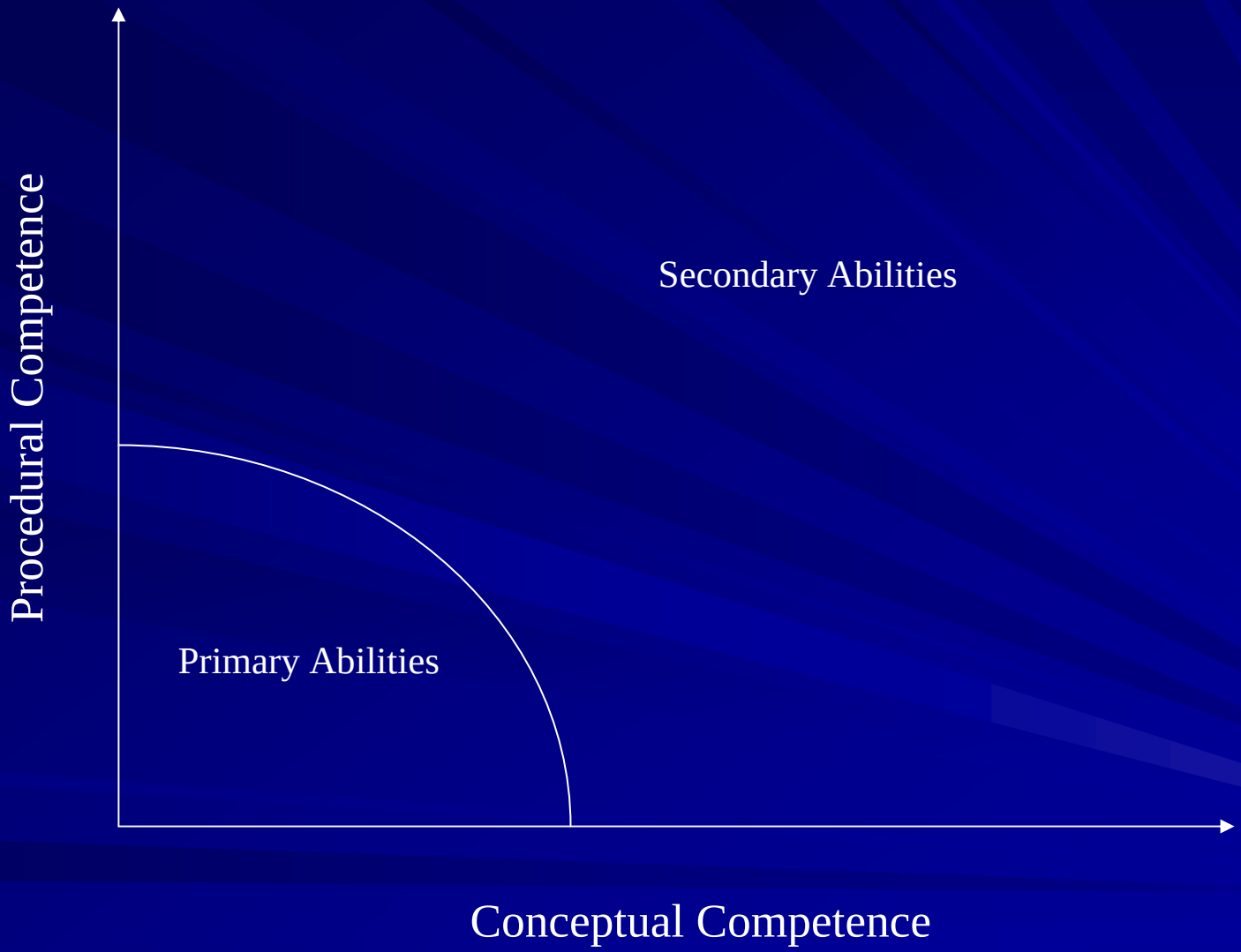
Schools and Organization of Instruction: Predictions

- Children are innately curious about and motivated to actively engage in activities that will develop folk knowledge (e.g., socializing & folk psychology)
  - These can conflict with the need to engage in activities that will lead to the mastery of academic competencies.
- The inherent cognitive systems and child-initiated activities that foster the development of primary abilities may not be sufficient for the acquisition of secondary abilities, such as reading and writing
  - the need for instruction will be a direct function of the "remoteness" of the secondary ability to the supporting primary systems

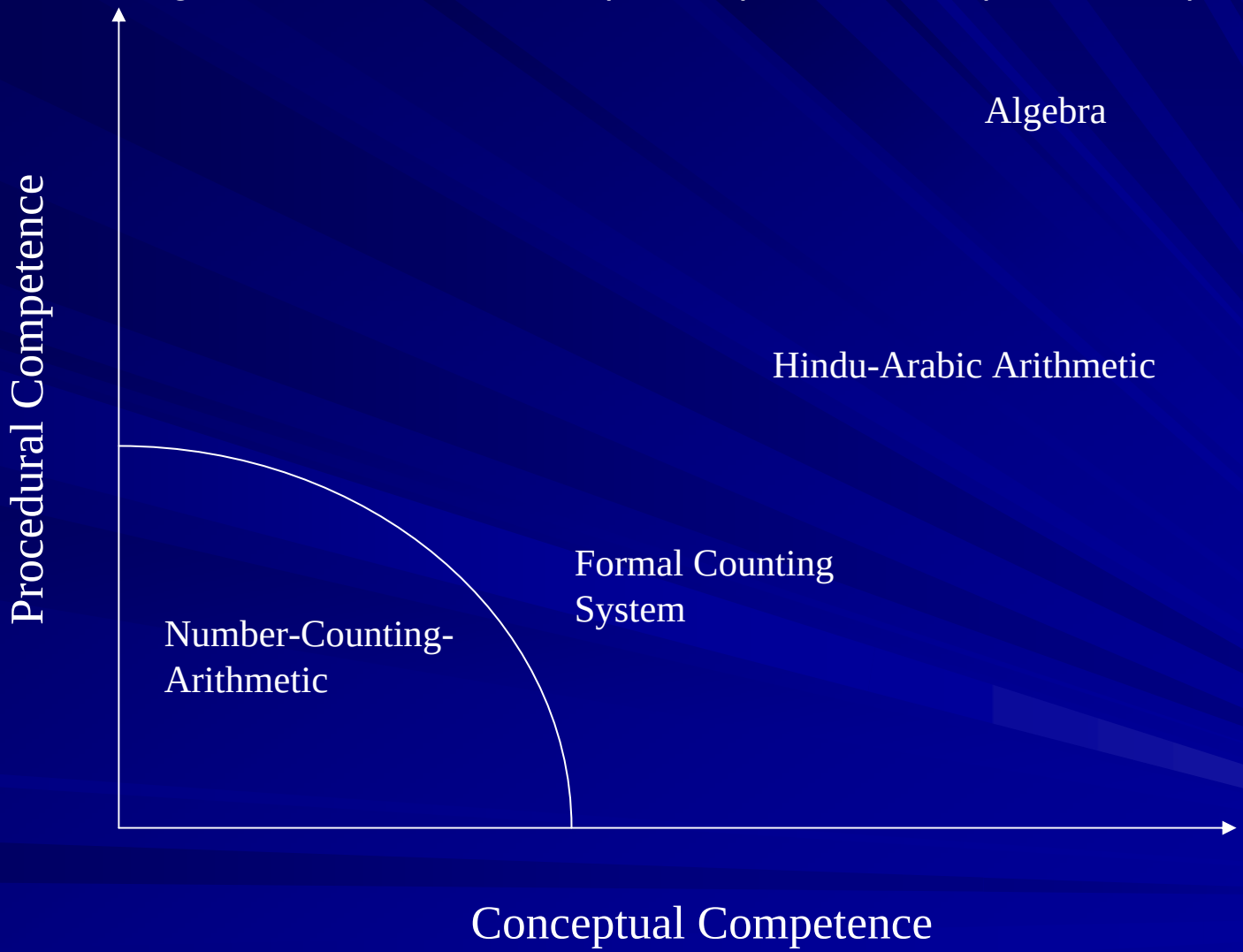
Primary Number-Counting-Arithmetic System

- **Numerosity** Ability to accurately determine the quantity of a small set of items without counting. Appears to be limited to sets of four or fewer items
- **Ordinality** Understanding of *more than and less than*, and later an understanding of ordinal relationships. Early on, appears to be limited to sets of four or fewer items
- **Counting** Preverbal counting system for enumeration of three to four items. Basic principles can be mapped onto counting words
- **Arithmetic** Early sensitivity to increases (addition) and decreases (subtraction) in the quantity of small sets

What are the primary competencies that foster and hinder algebraic learning: How do humans naturally think about patterns? Will this help or hinder learning? Ken – language vs. symbols



Example: Extension of Formal Counting: Mapping of Inherent Counting/Number Knowledge onto Inherent Magnitude Estimation System: Integration of numerical and spatial representations (David, Joan)



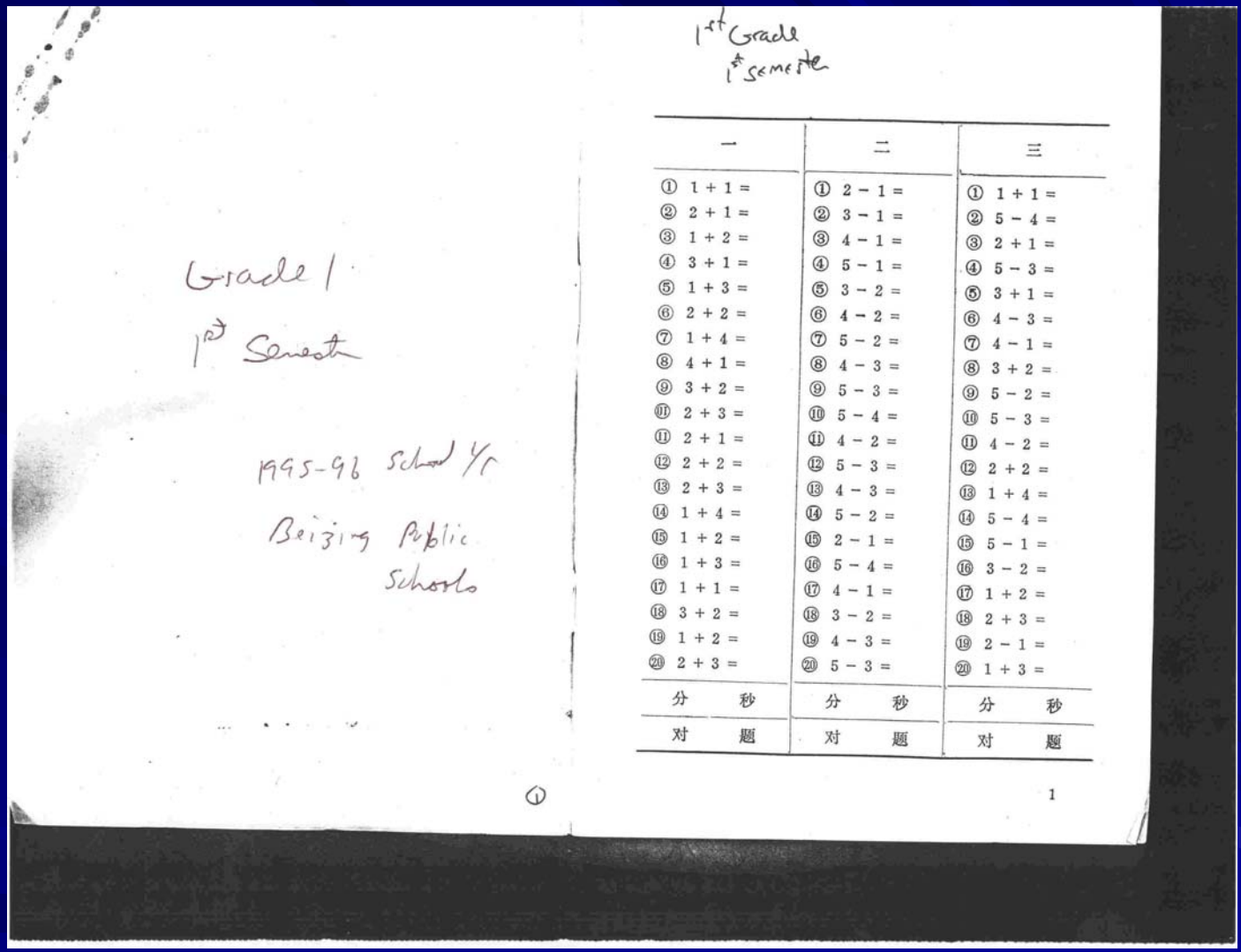
How and When: Arithmetic & Algebra

- Prediction: The more “remote” the secondary competencies from the primary foundation, the more:
  - The material must be organized by teachers, book developers, etc.
  - The period of instruction will be longer, and more preliminary preparation will be needed (e.g., learning of foundational secondary competencies); interference from prior secondary learning (Martha)
  - For many children, the material will be subjectively difficult and inherently uninteresting
  - Motivation to learn the material comes from adults’ understanding of what is needed for successful social living and preparation for technical fields, and NOT the inherent interests of children

How and When: Arithmetic & Algebra in China

- Practice Workbooks, 1st to 6th Grade
  - Is too much practice bad (negative transfer)?
  - Or, can practice be constructed in better ways?
- Extensive Practice for Speed/Accuracy: Well organized
- Pre-algebra is Integrated into Already Practiced Arithmetic

Extensive Practice for Speed/Accuracy



Grade 1
1st Semester

1995-96 School Yr
Beijing Public
Schools

1st Grade
1st Semester

一	二	三
① 1 + 1 =	① 2 − 1 =	① 1 + 1 =
② 2 + 1 =	② 3 − 1 =	② 5 − 4 =
③ 1 + 2 =	③ 4 − 1 =	③ 2 + 1 =
④ 3 + 1 =	④ 5 − 1 =	④ 5 − 3 =
⑤ 1 + 3 =	⑤ 3 − 2 =	⑤ 3 + 1 =
⑥ 2 + 2 =	⑥ 4 − 2 =	⑥ 4 − 3 =
⑦ 1 + 4 =	⑦ 5 − 2 =	⑦ 4 − 1 =
⑧ 4 + 1 =	⑧ 4 − 3 =	⑧ 3 + 2 =
⑨ 3 + 2 =	⑨ 5 − 3 =	⑨ 5 − 2 =
⑩ 2 + 3 =	⑩ 5 − 4 =	⑩ 5 − 3 =
⑪ 2 + 1 =	⑪ 4 − 2 =	⑪ 4 − 2 =
⑫ 2 + 2 =	⑫ 5 − 3 =	⑫ 2 + 2 =
⑬ 2 + 3 =	⑬ 4 − 3 =	⑬ 1 + 4 =
⑭ 1 + 4 =	⑭ 5 − 2 =	⑭ 5 − 4 =
⑮ 1 + 2 =	⑮ 2 − 1 =	⑮ 5 − 1 =
⑯ 1 + 3 =	⑯ 5 − 4 =	⑯ 3 − 2 =
⑰ 1 + 1 =	⑰ 4 − 1 =	⑰ 1 + 2 =
⑱ 3 + 2 =	⑱ 3 − 2 =	⑱ 2 + 3 =
⑲ 1 + 2 =	⑲ 4 − 3 =	⑲ 2 − 1 =
⑳ 2 + 3 =	⑳ 5 − 3 =	⑳ 1 + 3 =
分 秒	分 秒	分 秒
对 题	对 题	对 题

① 1

Grade 1
2nd Semester

(3)

1st Grade
2nd Semester

第一片	第二片	第三片
① 30 + 10 =	① 30 + 5 =	① 50 + 8 =
② 20 + 50 =	② 37 − 7 =	② 69 − 9 =
③ 20 − 10 =	③ 40 + 8 =	③ 85 − 80 =
④ 40 − 30 =	④ 80 − 40 =	④ 23 − 3 =
⑤ 10 + 20 =	⑤ 30 + 40 =	⑤ 37 − 30 =
⑥ 60 + 10 =	⑥ 48 − 8 =	⑥ 40 + 7 =
⑦ 40 + 50 =	⑦ 40 + 6 =	⑦ 74 − 4 =
⑧ 50 − 30 =	⑧ 99 − 90 =	⑧ 35 − 30 =
⑨ 70 − 60 =	⑨ 8 + 70 =	⑨ 70 + 30 =
⑩ 80 − 40 =	⑩ 75 − 5 =	⑩ 56 − 6 =
⑪ 30 + 50 =	⑪ 82 − 2 =	⑪ 97 − 90 =
⑫ 70 + 20 =	⑫ 60 + 30 =	⑫ 81 − 1 =
⑬ 90 − 60 =	⑬ 50 − 20 =	⑬ 60 + 6 =
⑭ 80 − 30 =	⑭ 39 − 9 =	⑭ 48 − 40 =
⑮ 50 − 40 =	⑮ 20 + 2 =	⑮ 77 − 7 =
⑯ 10 + 70 =	⑯ 75 − 70 =	⑯ 90 + 8 =
⑰ 80 + 20 =	⑰ 7 + 40 =	⑰ 70 + 2 =
⑱ 40 + 40 =	⑱ 60 + 40 =	⑱ 43 − 3 =
⑲ 90 − 70 =	⑲ 100 − 40 =	⑲ 87 − 80 =
⑳ 50 + 50 =	⑳ 90 − 20 =	⑳ 40 − 40 =

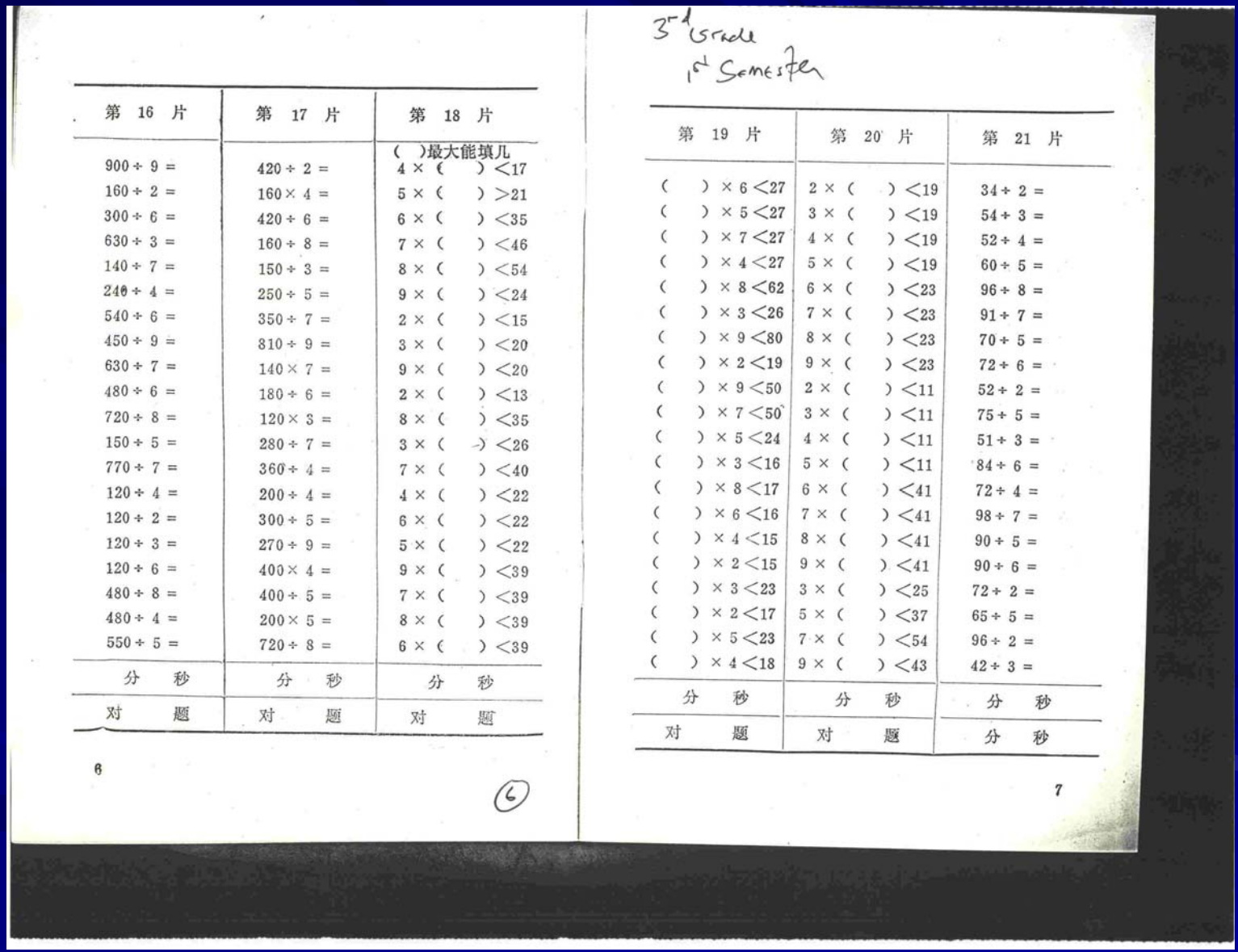
五十八	五十九	六十
① 20 ÷ 3 =	① 13 ÷ 3 =	① 2 × 3 × 5 =
② 13 ÷ 4 =	② 25 ÷ 7 =	② 4 × 2 × 7 =
③ 34 ÷ 6 =	③ 24 ÷ 4 =	③ 3 × 3 × 4 =
④ 53 ÷ 7 =	④ 23 ÷ 3 =	④ 1 × 8 × 6 =
⑤ 61 ÷ 8 =	⑤ 34 ÷ 7 =	⑤ 3 × 2 × 3 =
⑥ 29 ÷ 3 =	⑥ 47 ÷ 6 =	⑥ 7 × 1 × 5 =
⑦ 19 ÷ 2 =	⑦ 35 ÷ 5 =	⑦ 2 × 2 × 6 =
⑧ 38 ÷ 4 =	⑧ 37 ÷ 8 =	⑧ 6 × 1 × 9 =
⑨ 51 ÷ 6 =	⑨ 67 ÷ 9 =	⑨ 2 × 4 × 3 =
⑩ 71 ÷ 8 =	⑩ 35 ÷ 8 =	⑩ 5 × 1 × 8 =
⑪ 41 ÷ 6 =	⑪ 11 ÷ 2 =	⑪ 36 ÷ 4 ÷ 3 =
⑫ 62 ÷ 7 =	⑫ 25 ÷ 3 =	⑫ 64 ÷ 8 ÷ 2 =
⑬ 57 ÷ 9 =	⑬ 64 ÷ 8 =	⑬ 48 ÷ 8 ÷ 3 =
⑭ 28 ÷ 3 =	⑭ 39 ÷ 5 =	⑭ 32 ÷ 4 ÷ 4 =
⑮ 23 ÷ 7 =	⑮ 44 ÷ 8 =	⑮ 45 ÷ 9 ÷ 5 =
⑯ 20 ÷ 6 =	⑯ 29 ÷ 3 =	⑯ 48 ÷ 8 ÷ 6 =
⑰ 59 ÷ 6 =	⑰ 16 ÷ 4 =	⑰ 18 ÷ 3 ÷ 3 =
⑱ 48 ÷ 5 =	⑱ 78 ÷ 9 =	⑱ 72 ÷ 8 ÷ 3 =
⑲ 44 ÷ 5 =	⑲ 56 ÷ 8 =	⑲ 49 ÷ 7 ÷ 7 =
⑳ 26 ÷ 8 =	⑳ 49 ÷ 6 =	⑳ 24 ÷ 3 ÷ 4 =
分 秒	分 秒	分 秒
对 题	对 题	对 题

(5)

2nd Grade
2nd semester

六十一	六十二	六十三
① 7 ÷ 7 ÷ 1 =	① 1 × 4 + 15 =	① 60 − (57 − 50) =
② 72 ÷ 9 × 6 =	② 20 − 36 ÷ 4 =	② 3 × 2 × 7 =
③ 42 ÷ 6 × 5 =	③ 9 + 6 × 7 =	③ 9 ÷ (3 × 3) =
④ 3 × 8 ÷ 4 =	④ 6 ÷ 6 ÷ 3 =	④ 4 × 6 ÷ 3 =
⑤ 4 × 9 ÷ 6 =	⑤ 63 − 4 × 5 =	⑤ 70 − 35 ÷ 7 =
⑥ 18 ÷ 2 × 7 =	⑥ 18 + 56 ÷ 8 =	⑥ 8 + 5 − 10 =
⑦ 42 ÷ 7 × 3 =	⑦ 2 × 2 × 2 =	⑦ (45 − 15) ÷ 5 =
⑧ 3 × 4 ÷ 2 =	⑧ 8 × 3 + 8 =	⑧ 56 ÷ 8 × 6 =
⑨ 1 × 8 ÷ 4 =	⑨ 25 ÷ 5 + 17 =	⑨ 24 ÷ (54 ÷ 9) =
⑩ 32 ÷ 4 ÷ 4 =	⑩ 37 − 48 ÷ 8 =	⑩ 70 − 34 − 16 =
⑪ 64 ÷ 8 ÷ 2 =	⑪ 49 ÷ 7 + 7 =	⑪ 72 ÷ 8 − 9 =
⑫ 6 × 6 ÷ 9 =	⑫ 8 × 5 − 25 =	⑫ 45 ÷ 5 − 26 =
⑬ 2 × 9 ÷ 3 =	⑬ 3 × 7 ÷ 3 =	⑬ 4 × (2 + 2) =
⑭ 6 × 2 ÷ 4 =	⑭ 3 × 8 + 6 =	⑭ 81 ÷ 9 ÷ 3 =
⑮ 56 ÷ 7 ÷ 8 =	⑮ 54 ÷ 9 ÷ 2 =	⑮ 20 + 4 × 8 =
⑯ 16 ÷ 2 ÷ 4 =	⑯ 36 + 5 × 4 =	⑯ 24 + 6 + 37 =
⑰ 4 × 4 ÷ 2 =	⑰ 3 × 4 ÷ 6 =	⑰ (15 − 7) × 8 =
⑱ 18 ÷ 3 × 5 =	⑱ 50 + 4 × 9 =	⑱ 2 × 9 ÷ 1 =
⑲ 4 × 3 ÷ 6 =	⑲ 72 ÷ 9 + 4 =	⑲ 48 ÷ (76 − 70) =
⑳ 6 × 4 ÷ 3 =	⑳ 12 + 12 ÷ 6 =	⑳ 3 × 9 + 8 =
分 秒	分 秒	分 秒
对 题	对 题	对 题

Missing Addend: Prep for Introduction of *X*



第 16 片	第 17 片	第 18 片
		(　)最大能填几
900 ÷ 9 =	420 ÷ 2 =	4 × (　) <17
160 ÷ 2 =	160 × 4 =	5 × (　) >21
300 ÷ 6 =	420 ÷ 6 =	6 × (　) <35
630 ÷ 3 =	160 ÷ 8 =	7 × (　) <46
140 ÷ 7 =	150 ÷ 3 =	8 × (　) <54
240 ÷ 4 =	250 ÷ 5 =	9 × (　) <24
540 ÷ 6 =	350 ÷ 7 =	2 × (　) <15
450 ÷ 9 =	810 ÷ 9 =	3 × (　) <20
630 ÷ 7 =	140 × 7 =	9 × (　) <20
480 ÷ 6 =	180 ÷ 6 =	2 × (　) <13
720 ÷ 8 =	120 × 3 =	8 × (　) <35
150 ÷ 5 =	280 ÷ 7 =	3 × (　) <26
770 ÷ 7 =	360 ÷ 4 =	7 × (　) <40
120 ÷ 4 =	200 ÷ 4 =	4 × (　) <22
120 ÷ 2 =	300 ÷ 5 =	6 × (　) <22
120 ÷ 3 =	270 ÷ 9 =	5 × (　) <22
120 ÷ 6 =	400 × 4 =	9 × (　) <39
480 ÷ 8 =	400 ÷ 5 =	7 × (　) <39
480 ÷ 4 =	200 × 5 =	8 × (　) <39
550 ÷ 5 =	720 ÷ 8 =	6 × (　) <39
分　秒	分　秒	分　秒
对　题	对　题	对　题

6

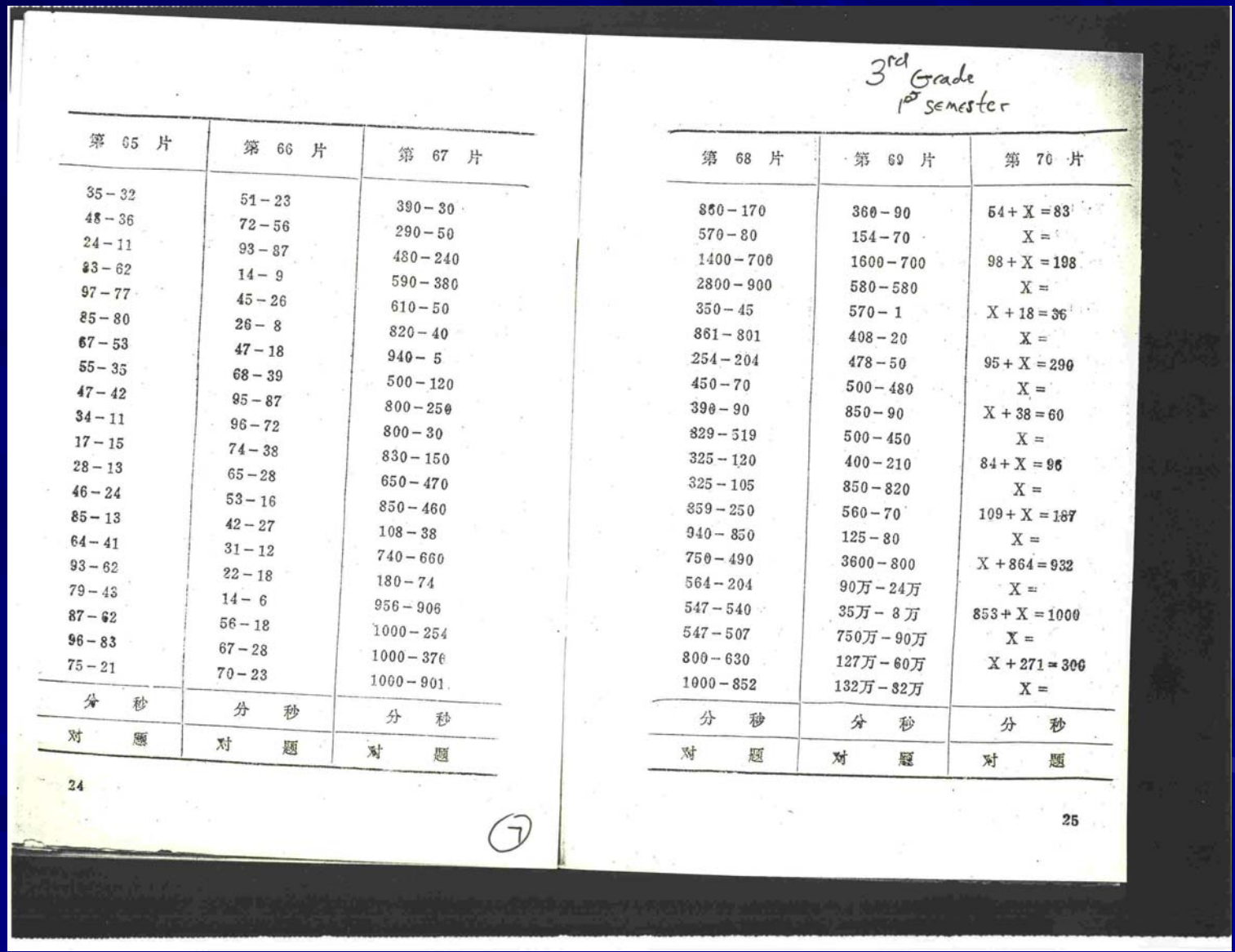
(6)

3rd Grade
1st Semester

第 19 片	第 20 片	第 21 片
(　) × 6 <27	2 × (　) <19	34 ÷ 2 =
(　) × 5 <27	3 × (　) <19	54 ÷ 3 =
(　) × 7 <27	4 × (　) <19	52 ÷ 4 =
(　) × 4 <27	5 × (　) <19	60 ÷ 5 =
(　) × 8 <62	6 × (　) <23	96 ÷ 8 =
(　) × 3 <26	7 × (　) <23	91 ÷ 7 =
(　) × 9 <80	8 × (　) <23	70 ÷ 5 =
(　) × 2 <19	9 × (　) <23	72 ÷ 6 =
(　) × 9 <50	2 × (　) <11	52 ÷ 2 =
(　) × 7 <50	3 × (　) <11	75 ÷ 5 =
(　) × 5 <24	4 × (　) <11	51 ÷ 3 =
(　) × 3 <16	5 × (　) <11	84 ÷ 6 =
(　) × 8 <17	6 × (　) <41	72 ÷ 4 =
(　) × 6 <16	7 × (　) <41	98 ÷ 7 =
(　) × 4 <15	8 × (　) <41	90 ÷ 5 =
(　) × 2 <15	9 × (　) <41	90 ÷ 6 =
(　) × 3 <23	3 × (　) <25	72 ÷ 2 =
(　) × 2 <17	5 × (　) <37	65 ÷ 5 =
(　) × 5 <23	7 × (　) <54	96 ÷ 2 =
(　) × 4 <18	9 × (　) <43	42 ÷ 3 =
分　秒	分　秒	分　秒
对　题	对　题	分　秒

7

Introduction of *X*, in context of well learned addition



3rd Grade
1st semester

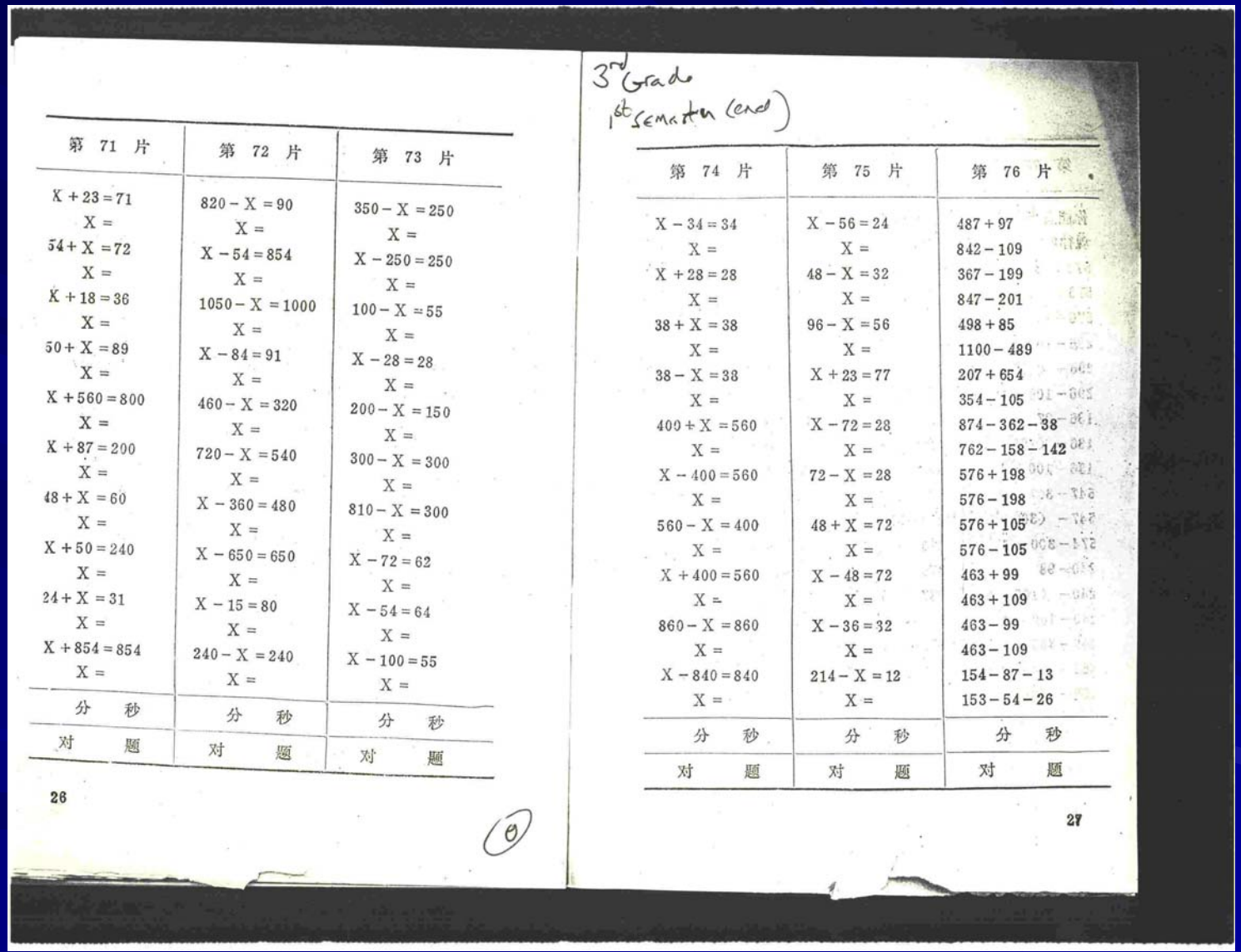
第 65 片	第 66 片	第 67 片
35 − 32	51 − 23	390 − 30
48 − 36	72 − 56	290 − 50
24 − 11	93 − 87	480 − 240
83 − 62	14 − 9	590 − 380
97 − 77	45 − 26	610 − 50
85 − 80	26 − 8	820 − 40
67 − 53	47 − 18	940 − 5
55 − 35	68 − 39	500 − 120
47 − 42	95 − 87	800 − 250
34 − 11	96 − 72	800 − 30
17 − 15	74 − 38	830 − 150
28 − 13	65 − 28	650 − 470
46 − 24	53 − 16	850 − 460
85 − 13	42 − 27	108 − 38
64 − 41	31 − 12	740 − 660
93 − 62	22 − 18	180 − 74
79 − 43	14 − 6	956 − 906
87 − 62	56 − 18	1000 − 254
96 − 83	67 − 28	1000 − 376
75 − 21	70 − 23	1000 − 901
分 秒	分 秒	分 秒
对 题	对 题	对 题

24

第 68 片	第 69 片	第 70 片
850 − 170	360 − 90	54 + X = 83
570 − 80	154 − 70	X =
1400 − 700	1600 − 700	98 + X = 198
2800 − 900	580 − 580	X =
350 − 45	570 − 1	X + 18 = 36
861 − 801	408 − 20	X =
254 − 204	478 − 50	95 + X = 290
450 − 70	500 − 480	X =
390 − 90	850 − 90	X + 38 = 60
829 − 519	500 − 450	X =
325 − 120	400 − 210	84 + X = 96
325 − 105	850 − 820	X =
859 − 250	560 − 70	109 + X = 187
940 − 850	125 − 80	X =
750 − 490	3600 − 800	X + 864 = 932
564 − 204	90万 − 24万	X =
547 − 540	35万 − 8万	853 + X = 1000
547 − 507	750万 − 90万	X =
800 − 630	127万 − 60万	X + 271 = 300
1000 − 852	132万 − 82万	X =
分 秒	分 秒	分 秒
对 题	对 题	对 题

25

Expand to Subtraction



第 71 片	第 72 片	第 73 片
X + 23 = 71	820 − X = 90	350 − X = 250
X =	X =	X =
54 + X = 72	X − 54 = 854	X − 250 = 250
X =	X =	X =
X + 18 = 36	1050 − X = 1000	100 − X = 55
X =	X =	X =
50 + X = 89	X − 84 = 91	X − 28 = 28
X =	X =	X =
X + 560 = 800	460 − X = 320	200 − X = 150
X =	X =	X =
X + 87 = 200	720 − X = 540	300 − X = 300
X =	X =	X =
48 + X = 60	X − 360 = 480	810 − X = 300
X =	X =	X =
X + 50 = 240	X − 650 = 650	X − 72 = 62
X =	X =	X =
24 + X = 31	X − 15 = 80	X − 54 = 64
X =	X =	X =
X + 854 = 854	240 − X = 240	X − 100 = 55
X =	X =	X =
分 秒	分 秒	分 秒
对 题	对 题	对 题

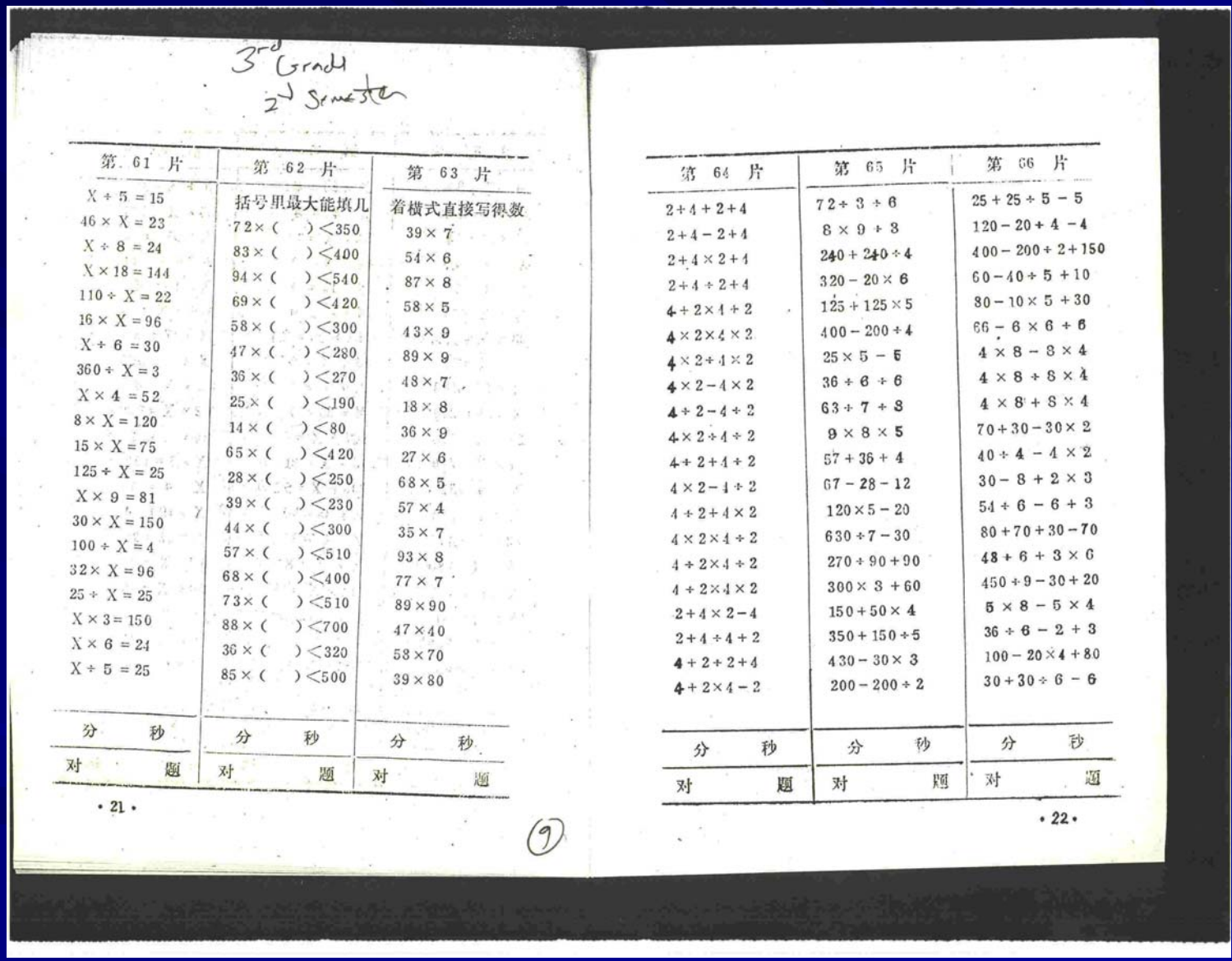
26

3rd Grade
1st Semester (end)

第 74 片	第 75 片	第 76 片
X − 34 = 34	X − 56 = 24	487 + 97
X =	X =	842 − 109
X + 28 = 28	48 − X = 32	367 − 199
X =	X =	847 − 201
38 + X = 38	96 − X = 56	498 + 85
X =	X =	1100 − 489
38 − X = 38	X + 23 = 77	207 + 654
X =	X =	354 − 105
400 + X = 560	X − 72 = 28	874 − 362 − 38
X =	X =	762 − 158 − 142
X − 400 = 560	72 − X = 28	576 + 198
X =	X =	576 − 198
560 − X = 400	48 + X = 72	576 + 105
X =	X =	576 − 105
X + 400 = 560	X − 48 = 72	463 + 99
X =	X =	463 + 109
860 − X = 860	X − 36 = 32	463 − 99
X =	X =	463 − 109
X − 840 = 840	214 − X = 12	154 − 87 − 13
X =	X =	153 − 54 − 26
分 秒	分 秒	分 秒
对 题	对 题	对 题

27

Expand to Multiplication



3rd Grade
2nd Semester

第 61 片	第 62 片	第 63 片
X ÷ 5 = 15	括号里最大能填几	着横式直接写得数
46 × X = 23	72 × ( ) < 350	39 × 7
X ÷ 8 = 24	83 × ( ) < 400	54 × 6
X × 18 = 144	94 × ( ) < 540	87 × 8
110 ÷ X = 22	69 × ( ) < 420	58 × 5
16 × X = 96	58 × ( ) < 300	43 × 9
X ÷ 6 = 30	47 × ( ) < 280	89 × 9
360 ÷ X = 3	36 × ( ) < 270	48 × 7
X × 4 = 52	25 × ( ) < 190	18 × 8
8 × X = 120	14 × ( ) < 80	36 × 9
15 × X = 75	65 × ( ) < 420	27 × 6
125 ÷ X = 25	28 × ( ) < 250	68 × 5
X × 9 = 81	39 × ( ) < 230	57 × 4
30 × X = 150	44 × ( ) < 300	35 × 7
100 ÷ X = 4	57 × ( ) < 510	93 × 8
32 × X = 96	68 × ( ) < 400	77 × 7
25 ÷ X = 25	73 × ( ) < 510	89 × 90
X × 3 = 150	88 × ( ) < 700	47 × 40
X × 6 = 24	36 × ( ) < 320	58 × 70
X ÷ 5 = 25	85 × ( ) < 500	39 × 80
分 秒	分 秒	分 秒
对 题	对 题	对 题

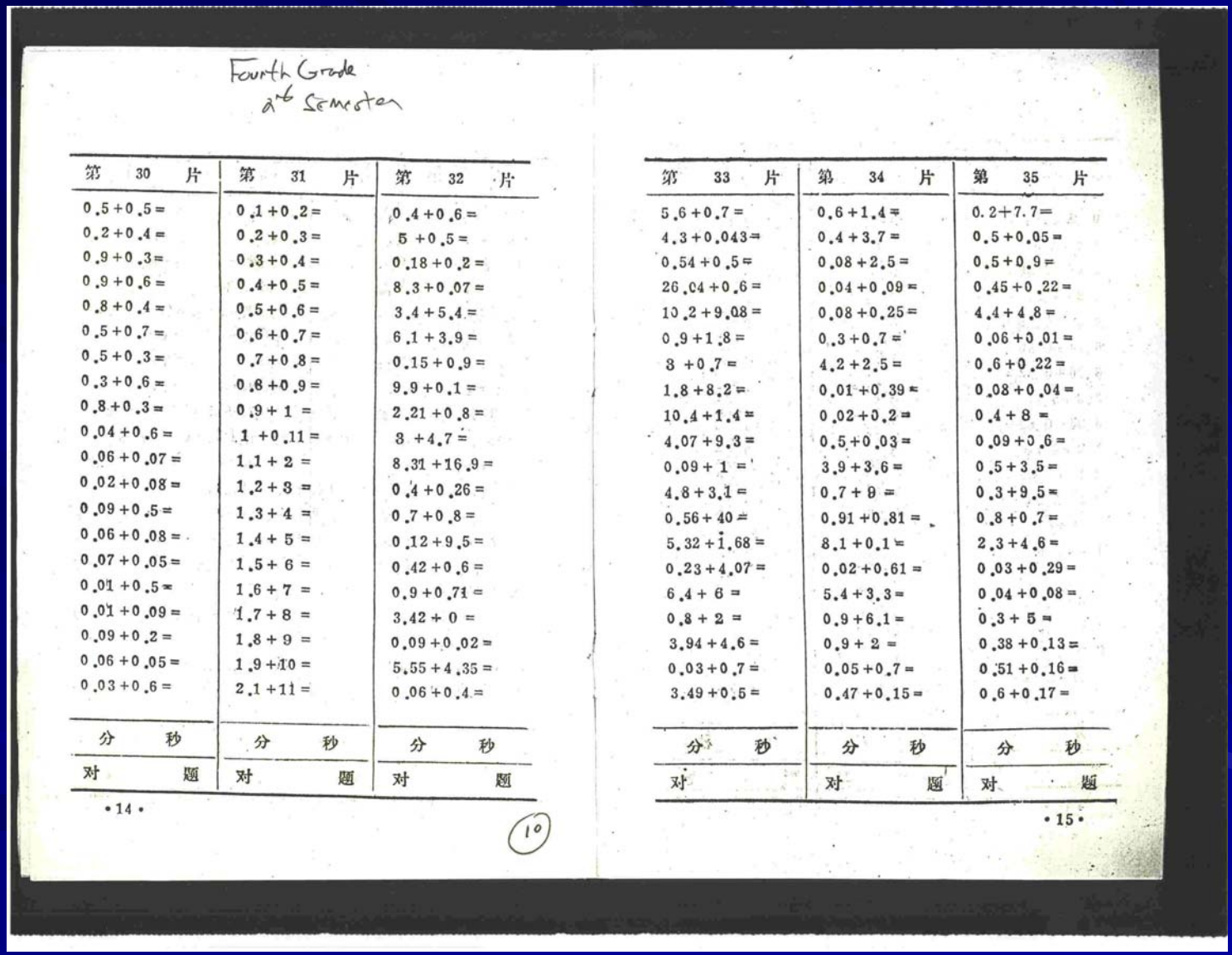
• 21 •

(9)

第 64 片	第 65 片	第 66 片
2 + 4 + 2 + 4	72 ÷ 3 ÷ 6	25 + 25 ÷ 5 − 5
2 + 4 − 2 + 4	8 × 9 ÷ 3	120 − 20 + 4 − 4
2 + 4 × 2 + 4	240 + 240 ÷ 4	400 − 200 ÷ 2 + 150
2 + 4 ÷ 2 + 4	320 − 20 × 6	60 − 40 ÷ 5 + 10
4 + 2 × 4 + 2	125 + 125 × 5	80 − 10 × 5 + 30
4 × 2 × 4 × 2	400 − 200 ÷ 4	66 − 6 × 6 + 6
4 × 2 ÷ 4 × 2	25 × 5 − 5	4 × 8 − 8 × 4
4 × 2 − 4 × 2	36 ÷ 6 ÷ 6	4 × 8 ÷ 8 × 4
4 ÷ 2 − 4 ÷ 2	63 ÷ 7 ÷ 3	4 × 8 + 8 × 4
4 × 2 ÷ 4 ÷ 2	9 × 8 × 5	70 + 30 − 30 × 2
4 + 2 + 4 ÷ 2	57 + 36 + 4	40 ÷ 4 − 4 × 2
4 × 2 − 4 ÷ 2	67 − 28 − 12	30 − 8 + 2 × 3
4 ÷ 2 + 4 × 2	120 × 5 − 20	54 ÷ 6 − 6 + 3
4 × 2 × 4 ÷ 2	630 ÷ 7 − 30	80 + 70 + 30 − 70
4 + 2 × 4 ÷ 2	270 ÷ 90 + 90	48 ÷ 6 + 3 × 6
4 + 2 × 4 × 2	300 × 3 + 60	450 ÷ 9 − 30 + 20
2 + 4 × 2 − 4	150 + 50 × 4	5 × 8 − 5 × 4
2 + 4 ÷ 4 + 2	350 + 150 ÷ 5	36 ÷ 6 − 2 + 3
4 + 2 + 2 + 4	430 − 30 × 3	100 − 20 × 4 + 80
4 + 2 × 4 − 2	200 − 200 ÷ 2	30 + 30 ÷ 6 − 6
分 秒	分 秒	分 秒
对 题	对 题	对 题

• 22 •

Introduction of Decimals



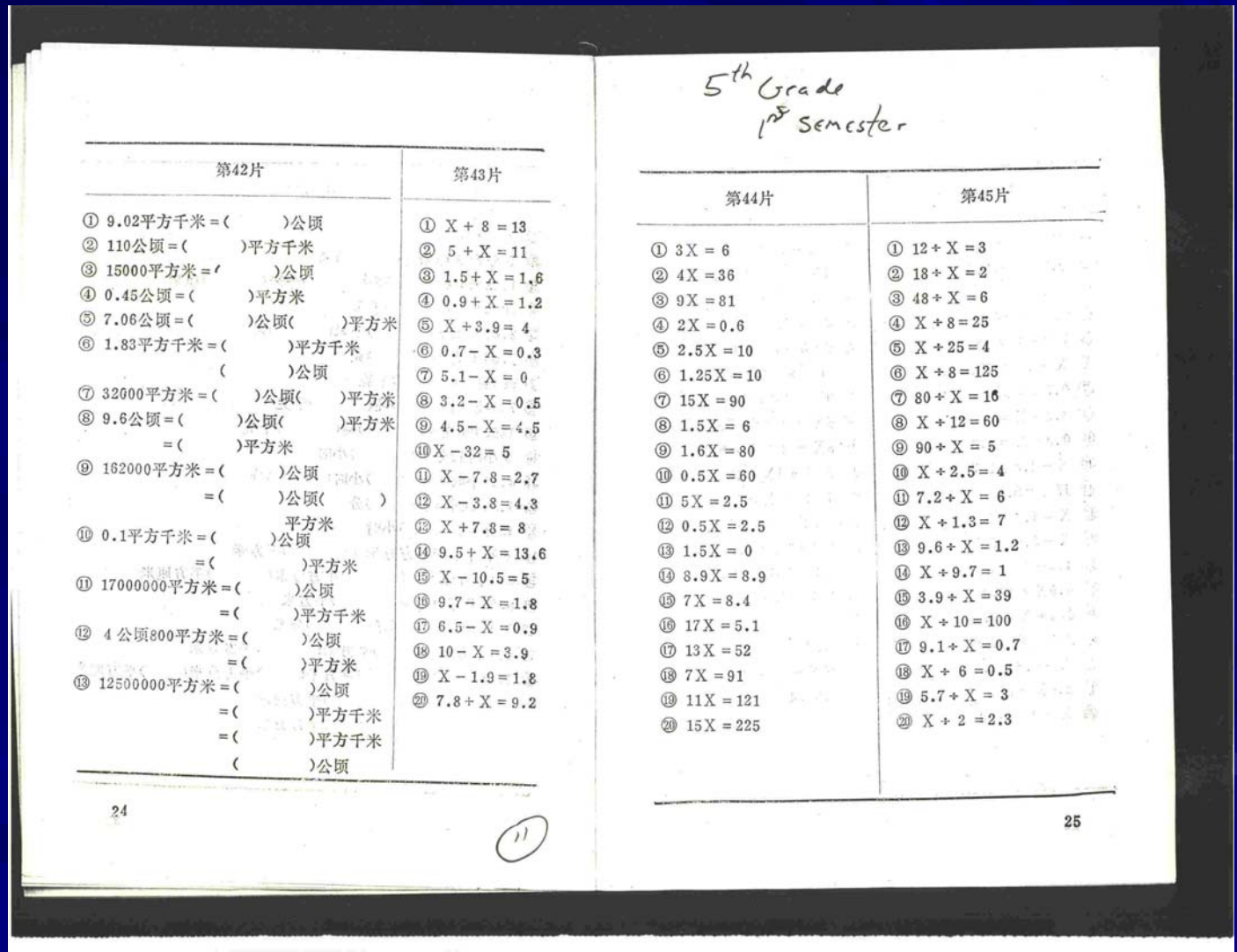
Fourth Grade
2nd Semester

第 30 片	第 31 片	第 32 片
0.5+0.5=	0.1+0.2=	0.4+0.6=
0.2+0.4=	0.2+0.3=	5+0.5=
0.9+0.3=	0.3+0.4=	0.18+0.2=
0.9+0.6=	0.4+0.5=	8.3+0.07=
0.8+0.4=	0.5+0.6=	3.4+5.4=
0.5+0.7=	0.6+0.7=	6.1+3.9=
0.5+0.3=	0.7+0.8=	0.15+0.9=
0.3+0.6=	0.8+0.9=	9.9+0.1=
0.8+0.3=	0.9+1=	2.21+0.8=
0.04+0.6=	1+0.11=	3+4.7=
0.06+0.07=	1.1+2=	8.31+16.9=
0.02+0.08=	1.2+3=	0.4+0.26=
0.09+0.5=	1.3+4=	0.7+0.8=
0.06+0.08=	1.4+5=	0.12+9.5=
0.07+0.05=	1.5+6=	0.42+0.6=
0.01+0.5=	1.6+7=	0.9+0.71=
0.01+0.09=	1.7+8=	3.42+0=
0.09+0.2=	1.8+9=	0.09+0.02=
0.06+0.05=	1.9+10=	5.55+4.35=
0.03+0.6=	2.1+11=	0.06+0.4=
分 秒	分 秒	分 秒
对 题	对 题	对 题

10

第 33 片	第 34 片	第 35 片
5.6+0.7=	0.6+1.4=	0.2+7.7=
4.3+0.043=	0.4+3.7=	0.5+0.05=
0.54+0.5=	0.08+2.5=	0.5+0.9=
26.04+0.6=	0.04+0.09=	0.45+0.22=
10.2+9.08=	0.08+0.25=	4.4+4.8=
0.9+1.8=	0.3+0.7=	0.06+0.01=
3+0.7=	4.2+2.5=	0.6+0.22=
1.8+8.2=	0.01+0.39=	0.08+0.04=
10.4+1.4=	0.02+0.2=	0.4+8=
4.07+9.3=	0.5+0.03=	0.09+0.6=
0.09+1=	3.9+3.6=	0.5+3.5=
4.8+3.1=	0.7+9=	0.3+9.5=
0.56+40=	0.91+0.81=	0.8+0.7=
5.32+1.68=	8.1+0.1=	2.3+4.6=
0.23+4.07=	0.02+0.61=	0.03+0.29=
6.4+6=	5.4+3.3=	0.04+0.08=
0.8+2=	0.9+6.1=	0.3+5=
3.94+4.6=	0.9+2=	0.38+0.13=
0.03+0.7=	0.05+0.7=	0.51+0.16=
3.49+0.5=	0.47+0.15=	0.6+0.17=
分 秒	分 秒	分 秒
对	对 题	对 题

Expand *X* to Decimals



5th Grade
1st Semester

第42片

① 9.02平方千米 = (　　)公顷
② 110公顷 = (　　)平方千米
③ 15000平方米 = (　　)公顷
④ 0.45公顷 = (　　)平方米
⑤ 7.06公顷 = (　　)公顷(　　)平方米
⑥ 1.83平方千米 = (　　)平方千米
(　　)公顷
⑦ 32000平方米 = (　　)公顷(　　)平方米
⑧ 9.6公顷 = (　　)公顷(　　)平方米
= (　　)平方米
⑨ 162000平方米 = (　　)公顷
= (　　)公顷(　　)平方米
⑩ 0.1平方千米 = (　　)公顷
= (　　)平方米
⑪ 17000000平方米 = (　　)公顷
= (　　)平方千米
⑫ 4公顷800平方米 = (　　)公顷
= (　　)平方米
⑬ 12500000平方米 = (　　)公顷
= (　　)平方千米
= (　　)平方千米
(　　)公顷

第43片

① X + 8 = 13
② 5 + X = 11
③ 1.5 + X = 1.6
④ 0.9 + X = 1.2
⑤ X + 3.9 = 4
⑥ 0.7 − X = 0.3
⑦ 5.1 − X = 0
⑧ 3.2 − X = 0.5
⑨ 4.5 − X = 4.5
⑩ X − 32 = 5
⑪ X − 7.8 = 2.7
⑫ X − 3.8 = 4.3
⑬ X + 7.8 = 8
⑭ 9.5 + X = 13.6
⑮ X − 10.5 = 5
⑯ 9.7 − X = 1.8
⑰ 6.5 − X = 0.9
⑱ 10 − X = 3.9
⑲ X − 1.9 = 1.8
⑳ 7.8 + X = 9.2

第44片

① 3X = 6
② 4X = 36
③ 9X = 81
④ 2X = 0.6
⑤ 2.5X = 10
⑥ 1.25X = 10
⑦ 15X = 90
⑧ 1.5X = 6
⑨ 1.6X = 80
⑩ 0.5X = 60
⑪ 5X = 2.5
⑫ 0.5X = 2.5
⑬ 1.5X = 0
⑭ 8.9X = 8.9
⑮ 7X = 8.4
⑯ 17X = 5.1
⑰ 13X = 52
⑱ 7X = 91
⑲ 11X = 121
⑳ 15X = 225

第45片

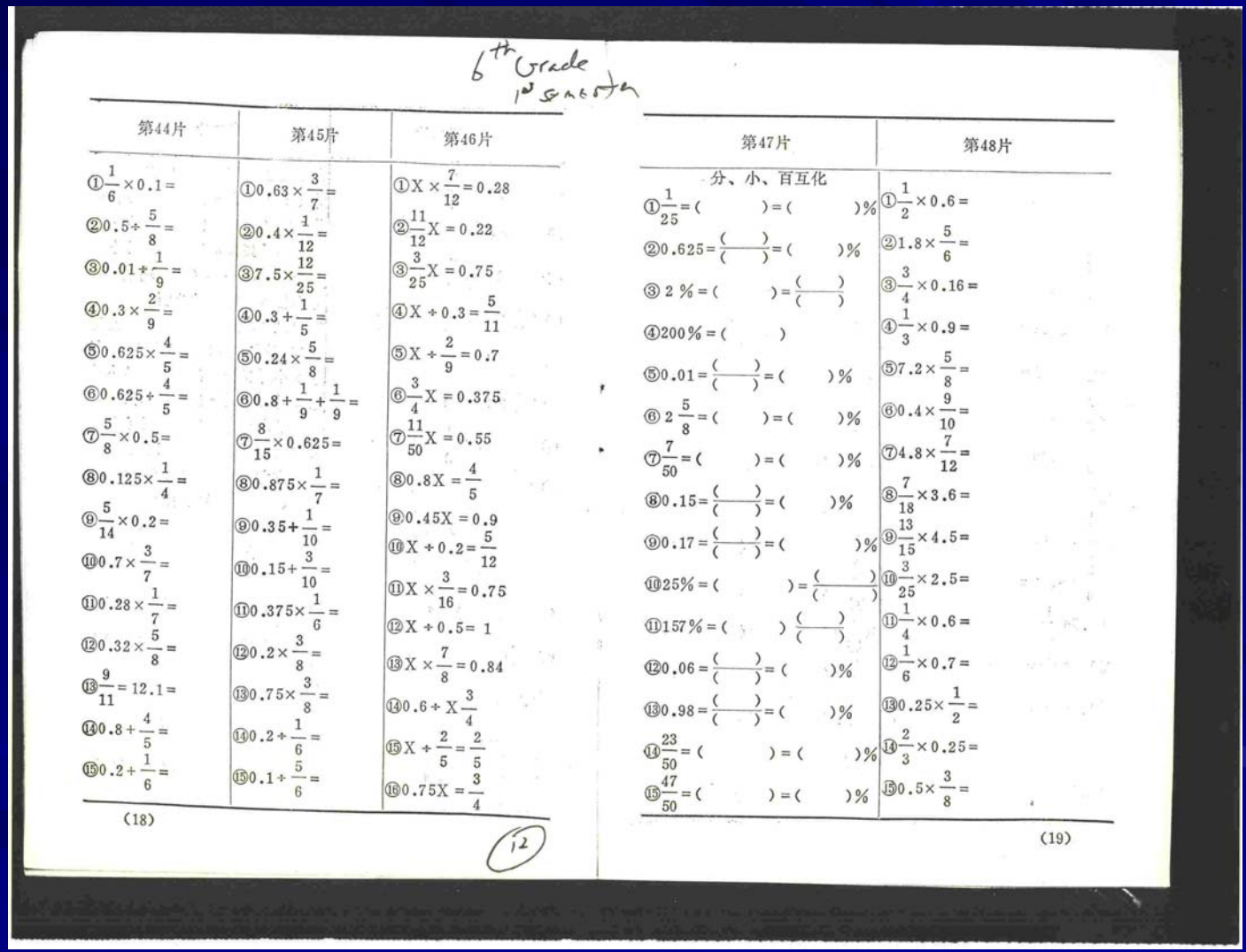
① 12 ÷ X = 3
② 18 ÷ X = 2
③ 48 ÷ X = 6
④ X ÷ 8 = 25
⑤ X ÷ 25 = 4
⑥ X ÷ 8 = 125
⑦ 80 ÷ X = 16
⑧ X ÷ 12 = 60
⑨ 90 ÷ X = 5
⑩ X ÷ 2.5 = 4
⑪ 7.2 ÷ X = 6
⑫ X ÷ 1.3 = 7
⑬ 9.6 ÷ X = 1.2
⑭ X ÷ 9.7 = 1
⑮ 3.9 ÷ X = 39
⑯ X ÷ 10 = 100
⑰ 9.1 ÷ X = 0.7
⑱ X ÷ 6 = 0.5
⑲ 5.7 ÷ X = 3
⑳ X ÷ 2 = 2.3

24

(11)

25

X in More Complex Expression



6th Grade
1st Semester

第44片

① $\frac{1}{6}\times 0.1=$

② $0.5\div\frac{5}{8}=$

③ $0.01\div\frac{1}{9}=$

④ $0.3\times\frac{2}{9}=$

⑤ $0.625\times\frac{4}{5}=$

⑥ $0.625\div\frac{4}{5}=$

⑦ $\frac{5}{8}\times 0.5=$

⑧ $0.125\times\frac{1}{4}=$

⑨ $\frac{5}{14}\times 0.2=$

⑩ $0.7\times\frac{3}{7}=$

⑪ $0.28\times\frac{1}{7}=$

⑫ $0.32\times\frac{5}{8}=$

⑬ $\frac{9}{11}=12.1=$

⑭ $0.8\div\frac{4}{5}=$

⑮ $0.2\div\frac{1}{6}=$

第45片

① $0.63\times\frac{3}{7}=$

② $0.4\times\frac{1}{12}=$

③ $7.5\times\frac{12}{25}=$

④ $0.3\div\frac{1}{5}=$

⑤ $0.24\times\frac{5}{8}=$

⑥ $0.8\div\frac{1}{9}+\frac{1}{9}=$

⑦ $\frac{8}{15}\times 0.625=$

⑧ $0.875\times\frac{1}{7}=$

⑨ $0.35+\frac{1}{10}=$

⑩ $0.15+\frac{3}{10}=$

⑪ $0.375\times\frac{1}{6}=$

⑫ $0.2\times\frac{3}{8}=$

⑬ $0.75\times\frac{3}{8}=$

⑭ $0.2\div\frac{1}{6}=$

⑮ $0.1\div\frac{5}{6}=$

第46片

① $X\times\frac{7}{12}=0.28$

② $\frac{11}{12}X=0.22$

③ $\frac{3}{25}X=0.75$

④ $X\div 0.3=\frac{5}{11}$

⑤ $X\div\frac{2}{9}=0.7$

⑥ $\frac{3}{4}X=0.375$

⑦ $\frac{11}{50}X=0.55$

⑧ $0.8X=\frac{4}{5}$

⑨ $0.45X=0.9$

⑩ $X\div 0.2=\frac{5}{12}$

⑪ $X\times\frac{3}{16}=0.75$

⑫ $X\div 0.5=1$

⑬ $X\times\frac{7}{8}=0.84$

⑭ $0.6\div X\frac{3}{4}$

⑮ $X\div\frac{2}{5}=\frac{2}{5}$

⑯ $0.75X=\frac{3}{4}$

(18)

12

第47片

分、小、百互化

① $\frac{1}{25}=(\quad)=(\quad)\%$

② $0.625=\frac{(\quad)}{(\quad)}=(\quad)\%$

③ $2\%=(\quad)=\frac{(\quad)}{(\quad)}$

④ $200\%=(\quad)$

⑤ $0.01=\frac{(\quad)}{(\quad)}=(\quad)\%$

⑥ $2\frac{5}{8}=(\quad)=(\quad)\%$

⑦ $\frac{7}{50}=(\quad)=(\quad)\%$

⑧ $0.15=\frac{(\quad)}{(\quad)}=(\quad)\%$

⑨ $0.17=\frac{(\quad)}{(\quad)}=(\quad)\%$

⑩ $25\%=(\quad)=\frac{(\quad)}{(\quad)}$

⑪ $157\%=(\quad)\ \frac{(\quad)}{(\quad)}$

⑫ $0.06=\frac{(\quad)}{(\quad)}=(\quad)\%$

⑬ $0.98=\frac{(\quad)}{(\quad)}=(\quad)\%$

⑭ $\frac{23}{50}=(\quad)=(\quad)\%$

⑮ $\frac{47}{50}=(\quad)=(\quad)\%$

第48片

① $\frac{1}{2}\times 0.6=$

② $1.8\times\frac{5}{6}=$

③ $\frac{3}{4}\times 0.16=$

④ $\frac{1}{3}\times 0.9=$

⑤ $7.2\times\frac{5}{8}=$

⑥ $0.4\times\frac{9}{10}=$

⑦ $4.8\times\frac{7}{12}=$

⑧ $\frac{7}{18}\times 3.6=$

⑨ $\frac{13}{15}\times 4.5=$

⑩ $\frac{3}{25}\times 2.5=$

⑪ $\frac{1}{4}\times 0.6=$

⑫ $\frac{1}{6}\times 0.7=$

⑬ $0.25\times\frac{1}{2}=$

⑭ $\frac{2}{3}\times 0.25=$

⑮ $0.5\times\frac{3}{8}=$

(19)

Conclusion

- WHAT

- Algebra is a form of human knowledge that has gone beyond folk knowledge;
  - Content domains reflect recent human intellectual history
  - And, very recent need for all children to learn some algebra

  - Questions: Core content: Algebra as discipline or applied skill?

- HOW

- Martha's talk examines interference from prior secondary learning and how this information interferes with symbolic understanding of algebra

  - Remote from primary knowledge; Layering and integrating various types of secondary symbols
  - Children's natural use of symbols is more about social communication; language and simple visual representations (David). This is their implicit assumption about these – use in other ways may need to be explicit

Conclusion

- Joan & Ken's talks examine attempts to fuse learning preferences evolved for primary learning for algebraic learning
  - How do primary representations help & hinder this learning? Ken's word prob
  - Built in potential for negative transfer?

- vs. Chinese method of more direct integration with arithmetic learning

  - Are either of these sufficient? Are the different approaches differently helpful for procedural vs. conceptual learning?

- Broader How:

  - Do attempts to make learning democratic, authentic etc. help or hinder algebra learning?
  - Assume children are not motivated to learn algebra

  - Greater potential to influence prior secondary instruction and reduce interference and provide sufficient background knowledge
  - Know less about help or hindrance from primary knowledge; What systems are used and how do the demands of algebra differ from evolved function?

Conclusion

- WHEN

- For secondary domains: Children's "natural" cognitive development or "readiness" may not be relevant in the same way it might be for primary learning (e.g., language)
  - Cannot make assumptions about too early

- Chinese workbooks suggest American children can be introduced to pre-algebra and in a more systematic way earlier than is currently the case