Loveless Algebra

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NAEP Algebra Items
4th & 8th Grades

1. Are the items mathematically sound?

2. Are the items assessing students' knowledge of algebra? Or something else (pre-algebra or algebra readiness)?

3. If students do well on these items, would that be strong evidence that they know algebra?

4. Are important algebra topics and skills not covered by these items?

Mathematical Soundness

An item

(a) Is clearly and precisely formulated, accessible at the intended grade, and free of non-purposeful ambiguity.

(b) Treats substantial mathematical knowledge and/or skills, with due regard for the need of a wide range of levels of challenge in the ensemble of items, to afford discrimination in results.

(c) If contextualized, has a sensible context, one that is taken seriously, and one that does not obscure the intended mathematical focus of the item.

The *ensemble* of items

(d) mathematically represents, in a reasonably proportioned and comprehensive way, the mathematical landscape of the domain (in this case, Algebra) that it putatively covers.

Conditions (a) & (b) met
Some minor exceptions to (c)

Example: G8, #24 (= G4, #12):

A table is given showing the weight w = w(m) of a puppy after m months, starting with w(1) = 10, and giving w(m) for m = 1,2,3,4. It asks for w(5). Inspection of successive increases shows that w(m+1) – w(m) = 6 - m, whence w(5) = w(4) + 2. So far, so good. But, if this model persists, the puppy starts to lose weight after 6 months, and, after 13 months, passes into anti-matter!

Validity: Questions 2, 3, 4:

#'s 2 & 4 correspond to condition (d): *Mathematical*

#3 is also *psychometric*

(Cognitive interviews?)

G4, #14 presents the first few powers of 2 – 2,4,8,16,32 – and asks, if this sequence were to be continued, whether the number 375 might appear as a value. The presumed intention of the item is to test for knowledge about even numbers, and the recognition that the odd number 375 could not appear among the even numbers 2^n. On the other hand, the student might well continue multiplying by 2, to get 2^n for n = 6,7,8,9, with $2^8 = 256$ and $2^9 = 512$. Since the numbers visibly increase, passing and missing 375, 375 cannot occur. This perfectly correct mathematical reasoning, leading to the correct answer, implies no knowledge of even numbers, and demonstrates a computational and investigatory skill perhaps not considered particularly algebraic.

What is algebra?

Traditional: Theory of algebraic (polynomial) equations:

variables, # equations, degrees

School algebra: degrees ≤ 2, (degree 1 = linear algebra) small # variables and equations. A few general items: quadratic formula, binomial theorem, factorization of $x^n - y^n$, geometric series. Symbolic fluency.

Current versions: Functions & patterns. Modeling data. Mainly linear (occasional quadratic) and exponential functions in repertoire.

Other possible topics

- algebraic identities
- sums of powers (geometric series)
- rules of exponentiation
- simplification of numerical expressions involving square roots, primes and factorization
- ….

Example: Algebraically simplify the radical expression

$$6\sqrt{54} - 3\sqrt{80} + 4\sqrt{45} - 9\sqrt{24}$$

(Use of a calculator might well not detect that this equals zero)

Problem for my engineering & CS linear algebra students (who are virtuoso arithmeticians)

Find a quadratic polynomial f(t) such that

f(-1) = 1 f(0) = 1 f(1) = 3

Knowing arithmetic does not assure algebraic fluency, even when the arithmetic of the problem is easy & whole number

Available documents

- A brief paper on the NAEP items

- An annotation of the NAEP-tables summarizing general characteristics of the items

- A table of the items using a finer grained code for mathematical characteristics of the items

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