TIMSS in Perspective

Lessons Learned from IEA's Four Decades of International Mathematics Assessment

Ina V.S. Mullis and Michael O. Martin Co-Directors, TIMSS & PIRLS International Study Center Lynch School of Education, Boston College The Brookings Institution January 23, 2008





TIMSS & PIRLS International Study Center Lynch School of Education, Boston College

FIMS – The First International Mathematics Study

- Pioneering international assessment IEA's first study
- Expanding education systems need for effective measure of output in terms of student achievement
- Applying psychometric techniques
- The world as an educational laboratory





FIMS – The First International Mathematics Study (cont.)

FIMS conducted 1961-65 in 12 countries

- Two grades targeted (grade with most 13-year-olds; final grade for students studying advanced mathematics)
- Mathematics achievement in relation to school organization, mathematics curriculum and teaching, and student attitudes and backgrounds
- Showed that international comparisons of student achievement were feasible, and could provide reliable data for policy makers





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SIMS – The Second International Mathematics Study

- Major emphasis on curriculum and instruction
- SIMS mathematics curriculum model
 - Intended curriculum (curricular goals and intentions)
 - Implemented curriculum (what is taught in the classroom)
 - Achieved curriculum (what mathematics students have learned)





SIMS – The Second International Mathematics Study (cont.)

SIMS conducted 1980-82 in 20 countries

- Extensive curriculum analysis
 - Intended curriculum through expert reports
 - Implemented curriculum through "Opportunity to Learn" questionnaires
- Showed that improving mathematics education begins with the curriculum





SS & PIR

TIMSS – Third International Mathematics and Science Study

- Conducted in 1994-95, first to include both mathematics and science
- Launch of TIMSS coincided with an upsurge of interest in international studies – almost 50 countries participated
- Ambitious scope
 - Wide-ranging assessment of student achievement
 - Comprehensive curriculum analysis
 - Video study of instructional practices





S S

TIMSS – *Trends in* International Mathematics and Science Study

• TIMSS 1995 – 4th and 8th grades

- also 3rd, 7th, and 12th

- TIMSS 1999 8th grade only
- TIMSS 2003 4th and 8th grades
- TIMSS 2007 4th and 8th grades

Planning TIMSS 2011 – 4th and 8th grades





TIMSS

Major methodological contributions

- Ambitious curriculum frameworks for mathematics and science
- Extensive pool of items
- Matrix-sampling design
- Advanced psychometric scaling for analysis and reporting of student achievement
- Systematic emphasis on data quality and data comparability





Exhibit 1: Distribution of Mathematics Achievement (2)

TIMSS200)3
Grade	

Countries	Average Scale Score		М	athem	atics Ac	hievem	ent Dis	tributio	n	
Singapore	594 (5.6)	0						-		
[†] Hong Kong, SAR	575 (3.2)	0					1000		-	
Japan	565 (1.6)	0							-	
Chinese Taipei	564 (1.8)	0								
Belgium (Flemish)	551 (1.8)	0								
† Netherlands	540 (2.1)	0				1		100		
Latvia	536 (2.8)	0				_		-		8
¹ Lithuania	534 (2.8)	0				_		1000) 20
Russian Federation	532 (4.7)	0						-		MSS
[†] England	531 (3.7)	0						_		EA
Hungary	529 (3.1)	0						1000		Stuc
[†] United States	518 (2.4)	0					- E.	1000		ence
Cyprus	510 (2.4)	0				-		-		d Scie
Moldova, Rep. of	504 (4.9)					_	-	1000		and
Italy	503 (3.7)					-		1000		SOURCE: IEA's Trends in International Mathematics and Science Study (TIMSS) 2003
TIMSS Scale Avg.	500									them
† Australia	499 (3.9)					-	•	-		Mat
New Zealand	493 (2.2)	۲			1			-		ona
[†] Scotland	490 (3.3)	۲				-		-		rnat
Slovenia	479 (2.6)	۲						_		Inte
Armenia	456 (3.5)	۲			_			_		12. 12.
Norway	451 (2.3)	۲						-		Ireno
Iran, Islamic Rep. of	389 (4.2)	۲			-		-			A's
Philippines	358 (7.9)	۲					_			<u>تن</u>
Morocco	347 (5.1)	۲					_			URC
Tunisia	339 (4.7)	۲					_			8
		0	100	200	300	400	500	600	700	800
tha	age significantly higher n international average	0	-	ith	Percentile 25th	es of Perfor	mance — 75th	95th		
,	rage significantly lower	۲		050/ 0	e a Cala a se t	T		2652		
tha	n international average			95% C	onfidence li	nterval for /	Average (±	25E)		

Trends in International Mathematics and Science Study



- 13 above average, U.S.
 - 3 about average
 - 9 below average
- Wide range of achievement across and within countries

Exhibit 1: Distribution of Mathematics Achievement

									$\underline{\circ}$
Countries	Average Scale Score		Math	ematics Ac	hieven	ient Dis	stributi	on	
Singapore	605 (3.6)	0				_			
Morea, Rep. of	589 (2.2)	0						-	
[†] Hong Kong, SAR	586 (3.3)	0				-		_	
Chinese Taipei	585 (4.6)	0			_		-	-	
Japan	570 (2.1)	0						_	
Belgium (Flemish)	537 (2.8)	0					-		
† Netherlands	536 (3.8)	0					-		
Estonia	531 (3.0)	0			-		-		
Hungary	529 (3.2)	0					-		
Malaysia	508 (4.1)	0					-		
Latvia	508 (3.2)	0					-		
Russian Federation	508 (3.7)	0					-		
Slovak Republic	508 (3.3)	0					-		
Australia	505 (4.6)				-		and the second		
[‡] United States	504 (3.3)				-		-		
¹ Lithuania	502 (2.5)				-		-		
TIMSS Scale Avg.	500								
Sweden	499 (2.6)	-					-		
[†] Scotland	498 (3.7)				-		-		
² Israel	496 (3.4)						-		
New Zealand	494 (5.3)				in the second	-	-		
Slovenia	493 (2.2)	۲			-		-		
Italy	484 (3.2)	۲					_		
Armenia	478 (3.0)						_		
¹ Serbia	477 (2.6)				-		_		
Bulgaria	476 (4.3)				-		_		
Romania	475 (4.8)	•		-			-		
Norway	461 (2.5)	۲					-		5000
Moldova, Rep. of	460 (4.0)	•							
Cyprus	459 (1.7)				_		_		NE NE
² Macedonia, Rep. of	435 (3.5)						-		÷
Lebanon	433 (3.1)	Ť				-			
Jordan	424 (4.1)	•							io
Iran, Islamic Rep. of	411 (2.4)				- e -	1000			2
¹ Indonesia	411 (4.8)	•			-				2
Tunisia	410 (2.2)					-			at at
Egypt	406 (3.5)			and the second	•	1000			athe
Bahrain	400 (3.3)					-			2
Palestinian Nat'l Auth.	390 (3.1)	•		-		-			001007F - 1EA's Transfer in International Mathematics and Science Study (TIMSS) 2003
Chile	387 (3.3)	•							torn
1 ‡ Morocco	387 (2.5)	•							2
Philippines	378 (5.2)	Ť		-					- Por
Botswana	366 (2.6)	•		-		_			e Tra
Saudi Arabia	332 (4.6)	•		-					EA.
Ghana	276 (4.0)	•			-				ė
South Africa	264 (5.5)	•		-		_			8
	204 (3.3)	_	400	200	100	500	600	700	
		0	100 2	00 300	400	500	600	700	800

TIMSS2003

Grade

- 13 above average
 7 about average, U.S.
 25 below average
- Wide range of achievement – across and within countries
- Range between 5th and 95th percentiles approximately 270-300
- Average in Singapore same as 95th percentile in lowest countries





TIMSS & PIRLS International Study Center Lynch School of Education, Boston College

The TIMSS Advanced International Benchmark (625)

Percentage of eighth-grade students reaching the advanced benchmark in 2003

Singapore	44%
Chinese Taipei	38%
Korea, Rep. of	35%
Hong Kong SAR	35%
Japan	24%
United States	7%





Exhibit 3: Trends in Average Mathematics Achievement (2)
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Australia 499 (3.9) 4 (5.2) Cyprus 510 (2.4) 35 (4.0) • England 531 (3.7) 47 (5.0) • Hong Kong, SAR 575 (3.2) 18 (5.1) • Hungary 529 (3.1) 7 (4.8) • Iran, Islamic Rep. of 389 (4.2) 2 (6.5) • Japan 565 (1.6) -3 (2.4) • Latvia (LSS) 533 (3.1) 34 (5.5) • Netherlands 540 (2.1) -9 (3.6) • New Zealand 496 (2.1) 26 (4.9) • Norway 451 (2.3) -25 (3.8) • Scotland 490 (3.3) -3 (5.3) • Slovenia 479 (2.6) 17 (4.1) • United States 518 (2.4) 0 (3.8) •	Countries	2003 Average Scale Score	1995 to 2003 Difference
England 531 (3.7) 47 (5.0) • Hong Kong, SAR 575 (3.2) 18 (5.1) • Hungary 529 (3.1) 7 (4.8) Iran, Islamic Rep. of 389 (4.2) 2 (6.5) Japan 565 (1.6) -3 (2.4) Latvia (LSS) 533 (3.1) 34 (5.5) • Netherlands 540 (2.1) -9 (3.6) • New Zealand 496 (2.1) 26 (4.9) • Norway 451 (2.3) -25 (3.8) • Scotland 490 (3.3) -3 (5.3) Singapore 594 (5.6) 4 (7.2) Slovenia 479 (2.6) 17 (4.1) United States 518 (2.4) 0 (3.8)	Australia	499 (3.9)	4 (5.2)
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Hungary 529 (3.1) 7 (4.8) Iran, Islamic Rep. of 389 (4.2) 2 (6.5) Japan 565 (1.6) -3 (2.4) Latvia (LSS) 533 (3.1) 34 (5.5) O Netherlands 540 (2.1) -9 (3.6) • New Zealand 496 (2.1) 26 (4.9) O Norway 451 (2.3) -25 (3.8) • Scotland 490 (3.3) -3 (5.3) Singapore 594 (5.6) 4 (7.2) Slovenia 479 (2.6) 17 (4.1) O United States 518 (2.4) 0 (3.8)	England	531 (3.7)	47 (5.0)
Iran, Islamic Rep. of 389 (4.2) 2 (6.5) Japan 565 (1.6) -3 (2.4) Latvia (LSS) 533 (3.1) 34 (5.5) • Netherlands 540 (2.1) -9 (3.6) • New Zealand 496 (2.1) 26 (4.9) • Norway 451 (2.3) -25 (3.8) • Scotland 490 (3.3) -3 (5.3) Singapore 594 (5.6) 4 (7.2) Slovenia 479 (2.6) 17 (4.1) • United States 518 (2.4) 0 (3.8)	Hong Kong, SAR	575 (3.2)	18 (5.1)
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Netherlands 540 (2.1) -9 (3.6) • New Zealand 496 (2.1) 26 (4.9) • Norway 451 (2.3) -25 (3.8) • Scotland 490 (3.3) -3 (5.3) • Singapore 594 (5.6) 4 (7.2) • Slovenia 479 (2.6) 17 (4.1) • United States 518 (2.4) 0 (3.8) •	Japan	565 (1.6)	-3 (2.4)
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Norway 451 (2.3) -25 (3.8) • Scotland 490 (3.3) -3 (5.3) • Singapore 594 (5.6) 4 (7.2) Slovenia 479 (2.6) 17 (4.1) • United States 518 (2.4) 0 (3.8)	Netherlands	540 (2.1)	-9 (3.6) 💿
Scotland 490 (3.3) -3 (5.3) Singapore 594 (5.6) 4 (7.2) Slovenia 479 (2.6) 17 (4.1) United States 518 (2.4) 0 (3.8)	New Zealand	496 (2.1)	26 (4.9)
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Slovenia 479 (2.6) 17 (4.1) • United States 518 (2.4) 0 (3.8)	Scotland	490 (3.3)	-3 (5.3)
United States 518 (2.4) 0 (3.8)	Singapore	594 (5.6)	4 (7.2)
	Slovenia	479 (2.6)	17 (4.1)
TIMSS Scale Avg 500	United States	518 (2.4)	0 (3.8)
This scale Avg. 500	TIMSS Scale Avg.	500	

2003 Country average significantly higher

2003 Country average significantly lower 💿



- 6 countries improved from 1995 to 2003
- 7 stayed the same, including U.S.
- 2 countries declined





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Exhibit 3: Trends in Average Mathematics Achievement

Countries	2003 Average Scale Score	1999 to 20 Differenc		1995 to 20 Difference	
Australia	505 (4.6)			-4 (6.0)	
Belgium (Flemish)	537 (2.8)	-21 (4.3)	۲	-13 (6.5)	۲
Bulgaria	476 (4.3)	-34 (7.3)	۲	-51 (7.2)	۲
Chile	387 (3.3)	-6 (5.5)		$\diamond \diamond$	
Chinese Taipei	585 (4.6)	0 (6.1)		\diamond	
Cyprus	459 (1.7)	-17 (2.4)	۲	-8 (2.8)	۲
Hong Kong, SAR	586 (3.3)	4 (5.4)		17 (7.0)	0
Hungary	529 (3.2)	-2 (4.9)		3 (4.5)	
Indonesia	411 (4.8)	8 (6.9)		$\diamond \diamond$	
Iran, Islamic Rep. of	411 (2.4)	-11 (4.1)	۲	-7 (4.5)	
Israel	496 (3.4)	29 (5.2)	0		
Italy	484 (3.2)	4 (5.0)			
Japan	570 (2.1)	-9 (2.7)	۲	-11 (2.6)	۲
Jordan	424 (4.1)	-3 (5.4)		$\diamond \diamond$	
Korea, Rep. of	589 (2.2)	2 (2.9)		8 (2.9)	٥
Latvia (LSS)	505 (3.8)	0 (5.1)		17 (5.2)	0
Lithuania	502 (2.5)	20 (4.9)	0	30 (4.8)	0
Macedonia, Rep. of	435 (3.5)	-12 (5.5)	۲	$\diamond \diamond$	
Malaysia	508 (4.1)	-11 (6.0)		$\diamond \diamond$	
Moldova, Rep. of	460 (4.0)	-9 (5.6)		\diamond	
Netherlands	536 (3.8)	-4 (8.1)		7 (7.2)	
New Zealand	494 (5.3)	3 (7.4)		-7 (7.1)	
Norway	461 (2.5)	$\diamond \diamond$		-37 (3.3)	\odot
Philippines	378 (5.2)	33 (7.9)	0	$\diamond \diamond$	
Romania	475 (4.8)	3 (7.5)		2 (6.6)	
Russian Federation	508 (3.7)	-18 (7.0)	\odot	-16 (6.5)	\odot
Scotland	498 (3.7)	$\diamond \diamond$		4 (6.8)	
Singapore	605 (3.6)	1 (7.2)		-3 (5.4)	
Slovak Republic	508 (3.3)	-26 (5.2)	۲	-26 (4.5)	\odot
Slovenia	493 (2.2)			-2 (3.7)	
South Africa	264 (5.5)	-11 (8.8)			
Sweden	499 (2.6)	$\diamond \diamond$		-41 (5.0)	۲
Tunisia	410 (2.2)	-38 (3.3)	۲	$\diamond \diamond$	
United States	504 (3.3)	3 (5.2)		12 (5.8)	0



- 17 three-cycle trends
 - 34 two-cycle trends
- 5 had improvement 1995-2003, U.S.

Hong Kong SAR and Korea Latvia and Lithuania





Summary

- Many lessons learned since the pioneering days
- Different countries use different approaches but an effective educational system always requires enormous effort
 - High percentages of students in school through secondary school, and taking advanced courses
 - A rigorous and progressive curriculum
 - Well-prepared teachers
 - Economic resources, facilities, and materials
 - Students ready to learn and encouraged by society





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TIMSS & PIRLS International Study Center Lynch School of Education, Boston College Lack of Focus in the Mathematics Curriculum: A Symptom or a Cause

> Presented by William H. Schmidt Education Policy Center Michigan State University

The Brooking Institution Washington, D.C.

January 23, 2008

Instructional Content Constructs

Curricular Coherence Curricular Structure

Curricular Focus Exposure Time (OTL)

Curricular Rigor Sevel of Cognitive Complexity

High Achieving Countries' Mathematics Standards

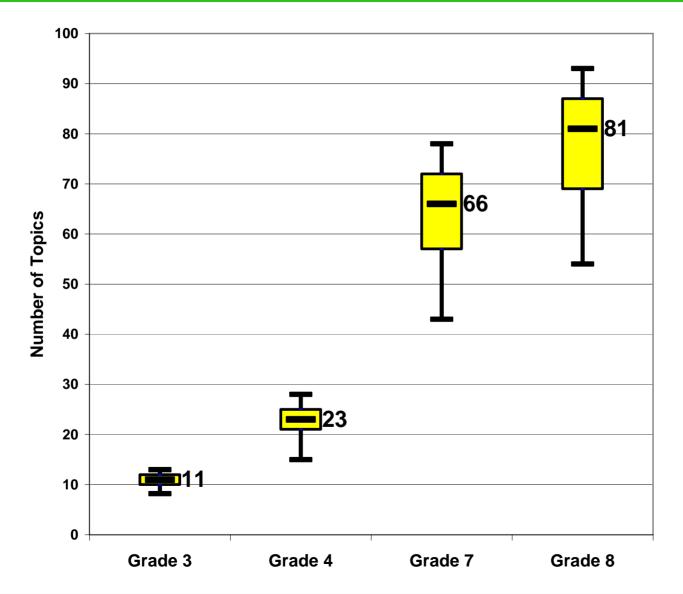
Mathematics Topics								
	Grade	Grade	Grade	Grade	Grade	Grade	Grade	Grade
Торіс	1	2	3	4	5	6	7	8
Whole Number Meaning		\bullet						
Whole Number Operations		\bullet	\bullet	\bullet				
Measurement Units		\bullet	\bullet	\bullet	\bullet	\bullet	\bullet	
Common Fractions								
Equations & Formulas								
Data Representation & Analysis			\bullet	\bullet		\bullet		
2-D Geometry: Basics								
Polygons & Circles				\bullet				\bullet
Perimeter, Area & Volume								
Rounding & Significant Figures								
Estimating Computations								
Properties of Whole Number Operations								
Estimating Quantity & Size								
Decimal Fractions								
Relationship of Common & Decimal Fractions								
Properties of Common & Decimal Fractions								
Percentages								
Proportionality Concepts						\bullet	\bullet	\bullet
Proportionality Problems								
2-D Coordinate Geometry								
Geometry: Transformations								
Negative Numbers, Integers & Their Properties								
Number Theory								\bullet
Exponents, Roots & Radicals								
Exponents & Orders of Magnitude								
Measurement Estimation & Errors								
Constructions w/ Straightedge & Compass								
3-D Geometry								
Congruence & Similarity								
Rational Numbers & Their Properties								
Patterns, Relations & Functions								
Slope & Trigonometry								

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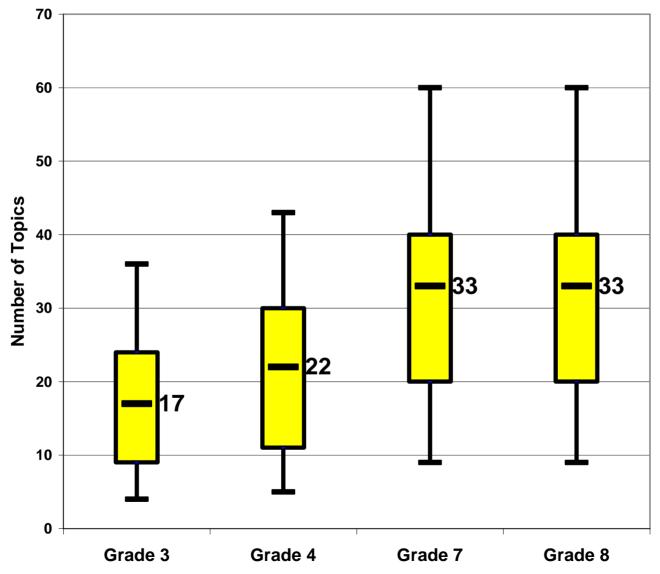
21 States' Mathematics Standards

State A	State F	State K	State P	State U	State D	State S
Topics G1 G2 G3 G4 G5 G6 G7 G8 Whole Number Meaning	Topics G1 G2 G3 G4 G5 G6 G7 G8 Whole Number Meaning	Topics G1 G2 G3 G4 G5 G6 G7 G8 Whole Number Meaning • • • • •	Topics G1 G2 G3 G4 G5 G6 G7 G8 Whole Number Meaning • • • • • •	Topics G1 G2 G3 G4 G5 G6 G7 G8 Whole Number Meaning • • • •	Topics G1 G2 G3 G4 G5 G6 G7 G8 Whole Number Meaning • • • • • • • •	Topics G1 G2 G3 G4 G5 G6 G7 G8 Whole Number Meaning
Whole Number Operations • • • • • • •	Whole Number Operations	Whole Number Operations	Whole Number Operations	Whole Number Operations	Whole Number Operations	Whole Number Operations
Measurement Units	Measurement Units Common Fractions	Measurement Units Common Fractions	Measurement Units •	Measurement Units	Measurement Units Common Fractions	Measurement Units Common Fractions
Equations & Formulas • • • • • • • •	Equations & Formulas • • • •	Equations & Formulas • • • • • • •	Equations & Formulas • • • • • • • •	Equations & Formulas • • • • • • • •	Equations & Formulas • • • • • • •	Equations & Formulas • • • • • •
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Polygons & Circles • • • • • • •	Polygans & Circles	Polygons & Circles	Polygons & Circles • • • • • • •	Polygons & Circles	Polygons & Circles • • • • • • •	Polygons & Circles • • • • • • •
Perimeter, Area & Volume Rounding & Significant Figures	Perimeter, Area & Volume • • • • • • •	Perimeter, Area & Volume Perimeter, Area & Volume Rounding & Significant Figures	Perimeter, Area & Volume Perimeter, Area & Volume Rounding & Significant Figures	Perimeter, Area & Volume Perimeter, Area & Volume Rounding & Significant Figur	Perimeter, Area & Volume Perimeter, Area & Volume Bounding & Significant Figures	Perimeter, Area & Volume Rounding & Significant Figures
Estimating Computations • • • • • • • •	Estimating Computations	Estimating Computations	Estimating Computations	Estimating Computations	Estimating Computations • • • • • • • •	Estimating Computations
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Percentages • • • •	Percentages • • • •	Percentages • • • • •	Percentages • • •	Percentages • • • • • • •	Percentages • • • •	Percentages • • • • • • •
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2-D Coordinate Geometry	2-D Coordinate Geometry • • • • • • • • •	2-D Coordinate Geometry	2-D Coordinate Geometry	2-D Coordinate Geometry • • • • • • • •	2-D Coordinate Geometry	2-D Coordinate Geometry • • • • • • • • •
Geometry: Transformations	Geometry: Transformations Geometry: Transformations Negative Numbers, Integers & Their Properties Geometry: Transformations Geometry: Transformati Geometry: Transformations Geometry: Transformations Geo	Geometry: Transformations • • • • • • • • • • • • • • • • • • •	Geometry: Transformations Geometry: Transformations Geometry: Transformations Geometry: Geomet	Geometry: Transformations Geometry: Transformations Negative Numbers. Integers Output Description Output De	Geometry: Transformations	Geometry: Transformations Geometry: Transformations Negative Numbers. Integers & Their Pro Geometry: Transformations Geometr
Number Theory	Number Theory	Number Theory	Number Theory	Number Theory	Number Theory	Number Theory
Exponents, Roots & Radicals	Exponents, Roots & Radicals Exponents & Orders of Magnitude	Exponents, Roots & Radicals	Exponents, Roots & Radicals • • • • • • • • • • • • • • • • • • •	Exponents, Roots & Radicals	Exponents, Roots & Radicals Exponents & Orders of Magnitude	Exponents, Roots & Radicals
Measurement Estimation & Errors • • • • • • • •	A Draes of Magnitude	Measurement Estimation & E	Measurement Estimation & E	Measurement Estimation & E	A Draes of Magnitude	Measurement Estimation & E
Constructions w/ Straightedge & Compass 3-D Geometry	Constructions w/ Straightedge & Compass 3-D Geometry	Constructions w/ Straightedge & Compass 3-D Geometry	Constructions w/ Straightedg	Constructions w/ Straightedge & Compass 3-D Geometry	Constructions w/ Straightedge & Compass	Constructions w/ Straightedg
Congruence & Similarity	Congruence & Similarity	Congruence & Similarity	Congruence & Similarity	S-D Geometry Congruence & Similarity	Congruence & Similarity	3-D Geometry Congruence & Similarity Congruence & Similarity Congrue
Rational Numbers & Their Properties	Rational Numbers & Their Properties	Rational Numbers & Their Properties	Rational Numbers & Their Properties	Rational Numbers & Their Properties	Rational Numbers & Their Properties	Rational Numbers & Their Properties
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Number of Topics Aligned with Ideal Scenario



Number of Topics Not Aligned (Introduced Prior to Ideal Scenario)

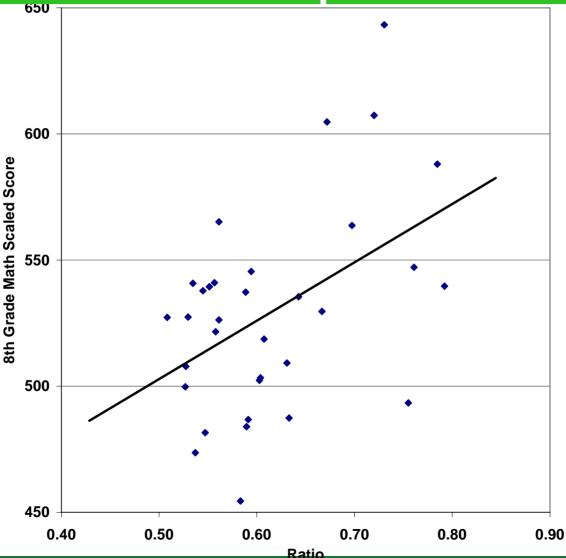


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Alignment with Ideal Scenario

Country	Total Number of Topics in Curriculum	Number of Topics Not Aligned (Introduced Prior to Ideal Scenario)	Number of Topics Aligned with Ideal Scenario	Number of Topics Not Aligned (Introduced Following the Ideal Scenario)
Argentina	81	3	54	24
Australia	136	32	75	29
Belgium (Fl)	155	37	87	31
Canada	181	58	92	31
Cyprus	162	49	87	26
Czech Republic	119	32	83	4
France	156	40	85	31
Germany	117	33	74	10
Hong Kong	79	9	62	8
Hungary	158	49	93	16
Iceland	137	32	81	24
Japan	128	39	86	3
Korea	125	27	90	8
Netherlands	97	14	54	29
Norway	159	33	96	30
Portugal	144	48	84	12
Singapore	115	11	84	21
Spain	109	23	69	17
USA	186	63	98	25

8th Grade Math Scaled Score vs. Ratio of No. of Aligned topics to the Total No. of Topics in Curriculum



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Table 1: Regression Analyses relatingAchievement to Coherence and Focus Measure

Model	Grade 3	Grade 4	Grade 7	Grade 8
Predictor	Estimate (StdError, p)	Estimate (StdError, p)	Estimate (StdError, p)	Estimate (StdError, p)
Number of Topics Aligned with Ideal Scenario	9.94 (7.38 , 0.196)	7.48 (4.04 , 0.081)	3.57 (1.48 , 0.023)	2.83 (1.12 , 0.017)
Total Number of Topics in Curriculum	-2.42 (1.13 , 0.047)	-2.58 (1.05 , 0.025)	-1.64 (0.57 , 0.008)	-1.39 (0.45 , 0.004)
Model Fit				
R-Square	0.2191	0.2574	0.2205	0.2490
Residual Mean Sq.	1652.9	1489.0	1476.0	1331.4
<i>p</i> <	0.1222	0.0687	0.0270	0.0136
No. of Countries	20	21	32	33
Standardized Coeffici	ent			
Number of Topics Aligned with Ideal Scenario	0.4374	0.7331	0.8481	0.8240
Total Number of Topics in Curriculum	-0.6969	-0.9703	-1.0044	-1.0204

Table 2: Regression Analyses relating Achievement to Coherence and Specific Focus Measure

Model	Grade 3	Grade 4	Grade 7	Grade 8
Predictor	Estimate (StdError, p)	Estimate (StdError, p)	Estimate (StdError, p)	Estimate (StdError, p)
Number of Topics Aligned with Ideal Scenario	7.52 (6.57 , 0.268)	4.91 (3.19 , 0.141)	1.84 (1.06 , 0.094)	1.66 (0.87 , 0.068)
Number of Topics Not Aligned (Introduced Prior to Ideal Scenario)	-2.42 (1.13 , 0.047)	-2.58 (1.05 , 0.025)	-1.79 (0.71 , 0.017)	-1.97 (0.70 , 0.009)
Model Fit				
R-Square	0.2191	0.2574	0.1826	0.2122
Residual Mean Sq.	1652.9	1489.0	1547.9	1396.6
<i>p</i> <	0.1222	0.0687	0.0538	0.0279
No. of Countries	20	21	32	33
Standardized Coeffici	ent			
Number of Topics Aligned with Ideal Scenario	0.3309	0.4807	0.4382	0.4825
Number of Topics Not Aligned (Introduced Prior to Ideal Scenario)	-0.6209	-0.7646	-0.6405	-0.7151



Lessons Learned: What International Assessments Tell Us about Math Achievement January 23, 2008

U.S. Algebra Performance in an International Context

Jeremy Kilpatrick, University of Georgia

Coauthors:

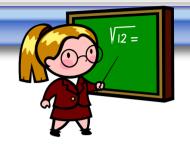


Vilma Mesa, University of Michigan

Finbarr Sloane, Arizona State University

U.S. school algebra

- Separate high school courses
- Designed for the college bound



- Arose from college entrance requirements as algebra moved from the college to the high school
- Beginning algebra moving to Grade 8 and earlier
- Began as generalized arithmetic; has kept much of that character
- Slowly becoming more practical and functions based

Performance in TIMSS 2003

- U.S. 4th graders' performance in algebra, measured by score points, was somewhat better than it was on all mathematics items taken together and better than the average for 4th graders in other countries
- U.S. 8th graders' performance in algebra was above its average on all items and above that of 8th graders in other countries

TIMSS 2003: Performance of U.S. fourth graders

	Number of items	U.S. average scaled score	26-country average scaled score
Number	63	516	495
Patterns, equations, & relationships	22	524	495
Measurement	33	500	495
Geometry	24	518	495
Data	17	549	495
Total	161	518	495

TIMSS 2003: Performance of U.S. eighth graders

	Number of items	U.S. average scaled score	26-country average scaled score
Number	57	508	466
Algebra	47	510	466
Measurement	31	495	466
Geometry	31	472	466
Data	28	527	466
Total	194	504	466

Item characteristics

- Content
- Representation
- Cognitive demand



Content

- Patterns
- Informal algebra
- Equations
- Inequalities
- Functions
- Algebraic reasoning
- Algebraic manipulation



Representation

- Numerical
- Verbal
- Graphical
- Symbolic
- Pictorial

- $N \rightarrow V$
- $V \rightarrow G$
- $S \rightarrow N$
- $P \rightarrow P$
- etc.



Cognitive demand

		Content dimension	
		Rich	
	Quadrant II: Tasks tend to measure specific content in particular settings	Quadrant I: Tasks tend to be complex, open-ended, and require extensive mobilization of information	
Process dimension	Constrained	Open	
	Quadrant III: Tasks tend to assess factual knowledge	Quadrant IV: Tasks tend to be lean in content and assess creativity	
		Lean	

Seven systems

- Members of the Organisation for Economic Cooperation and Development (OECD)
- Participated in TIMSS 1995, 1999, 2003
- 1995 & 1999: Australia, Canada, Czech Republic, Hungary, the Netherlands, New Zealand, the United States
- 2003: Australia, Ontario, Quebec, Hungary, the Netherlands, New Zealand, the United States
- All but the U.S. and New Zealand were "A" countries in TIMSS 1995 (Valverde & Schmidt, 2000)

Relative high performance

1999 Item P09 (Grade 8)

Algebraic manipulation, $S \rightarrow S$, Quadrant III

Which of these expressions is equivalent to $n \ge n \ge n$ for all values of n?

A. $\frac{n}{3}$ B. n + 3C. 3nD. n^3

U.S. percent correct:85%Next highest percent correct (Czech Republic):81%

Relative high performance items

- 3 in 1995, 1 in 1999, 7 in 2003
- 6 at Grade 4, 5 at Grade 8
- Tend to involve transition from arithmetic thinking to algebraic thinking
 - Algebraic manipulation
 - Patterns
 - Informal algebra
- Do not involve pictorial or graphical representation
- Most located in Quadrant III (content lean and process constrained)

Relative low performance

Item V04a (Grade 8) 1999

Patterns, $P \rightarrow N$, Quadrant III

The figures show four sets consisting of circles.

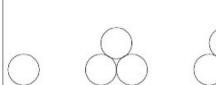


Figure 2

Figure 1



Figure 3

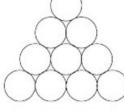


Figure 4

a) Complete the table below. First, fill in how many circles make up Figure 4. Then, find the number of circles that would be needed for the 5th figure if the sequence of figures is extended.

Figure	Number of circles	
1	1	
2	3	
3	6	
4		
5		

U.S. percent correct (lowest): Hungary percent correct (2nd lowest): 73% 77%

Relative low performance items

- 8 in 1995, 5 in 1999, 4 in 2003
- All at eighth grade
- Most involve pattern generalization
- Most involve numerical or pictorial representations (no graphical)
- Most are in either Quadrant II (rich content, process constrained) or III (lean content, process constrained)
- Items are generally difficult for students in all selected countries

Observations about U.S. performance

- Fourth graders do relatively well in interpreting a rule, engaging in transitive reasoning, translating from words to symbols, and extending numerical patterns
- Twelfth graders can interpret a function graph, but their performance otherwise is relatively weak
- Eighth graders can understand exponential notation, interpret simple algebraic expressions, reason about sequences, and solve an equation for one variable in terms of another
- Their performance is relatively weak, however, in interpreting symbols in an equation, completing tables, finding sums of series, manipulating expressions, generalizing patterns, and solving word problems

Final observations

- AIR study (Alan Ginsburg et al., 2005):
 - U.S. performance below the average for 12 countries at high and low levels of rigor for TIMSS items and at high and low levels of difficulty for TIMSS and PISA items
 - Countries: Australia, Belgium, Hong Kong, Hungary, Italy, Japan, Latvia, Netherlands, New Zealand, Norway, the Russian Federation, and the United States
- Our results indicate, however, that although U.S. students' performance may be low in general, it is not uniformly low
- The most and greatest differences in algebra performance between the U.S. and other TIMSS countries are at eighth grade
- U.S. eighth graders do better in algebra than might be expected given their limited exposure to it

Final observations



- AIR study also found that U.S. eighth-grade teachers far more likely (66% to 36% for other 11 countries) to say they relate mathematics to students' daily lives in most lessons
- TIMSS video studies show that U.S. eighth-grade teachers use few high complexity problems and spend 30% of lessons on review
- Algebra is of limited use if it is understood as generalized arithmetic only
- If students are to use algebra, they need to be proficient in functional thinking

Lessons Learned

A Response



Francis (Skip) Fennell, President National Council of Teachers of Mathematics

> & Professor of Education McDaniel College Westminster, MD



NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS TIMSS in Perspective: Lessons Learned from IEA's Four Decades of International Mathematics Assessments

Ina V. S. Mullis & Michael O. Martin



Teacher Experience & Preparation

- ...little evidence of a direct relationship between teacher training and student achievement.
- Requirements for being certified to teach mathematics:



Requirements	47 countries, 8 th grade only
Degree	33
Practicum	33
Exam	28
Probationary Period	23
Induction	11

Instruction

 The three predominant activities at both 4th and 8th grade (accounting for 60% of class time) were teacher lecture, teacher-guided student practice, and students working on problems on their own.



Home Support for Learning

 TIMSS 2003 data make it clear that higher levels of parents' education are associated with higher 8th grade student achievement in mathematics in almost all countries.



Attitudes and Values

- At both grades (TIMSS 2003) students had higher mathematics achievement, on average, if they were more self-confident in learning mathematics and placed higher value on the subject.
- Nevertheless...some research has indicated that the students following a demanding curriculum may have higher achievement, but little enthusiasm for the subject matter.



The Happiness Factor!!

Lack of Focus in the Mathematics Curriculum: Symptom or Cause?

William H. Schmidt & Richard T. Houang



A Mile Wide and an Inch Deep

• 1997

- U.S. curriculum typically covered more topics at each grade level than did any other country participating in TIMSS
- True of state standards, textbooks, and the NCTM Standards
- Little focus, resulting in a surface or skim level of coverage



Coherence

- Coherence articulated over time as a sequence of topics and expectations which deepen and reach closure.
- Lack of focus (larger number of topics) is the symptom, lack of coherence is the problem.

CTM NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS p. 65-66

Focus

 A decrease of fifty in the number of intended topic and grade combinations would predict an increase in achievement of almost three-fourths of a standard deviation. This is especially relevant to the U.S., ... a decrease of fifty combinations would bring it more into line with Japan and Korea.



Focus

Country	Topics	Before Ideal	Aligned with	After Ideal
U.S.	186	63	98	25
CFP	111	34	72	5

Focus & Coherence

 For a country to have a high mean level of performance, it must have a high degree of focus as well as coherence.

• The amount of "clutter" created by covering too many topics too early or before their time from a mathematical point of view must be kept small.



And now...

 The results of these analyses do not bode well for the United States. They predict poor levels of performance, especially in the middle grades. In fact, poor performance in the middle grades has been seen repeatedly, as recently as 2003.



Gr. 6-8	Before Ideal	After Ideal
NCTM Standards	10	30
CFPs	5	4

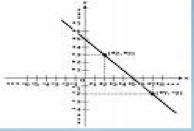
U.S. Algebra Performance in an International Context

Jeremy Kilpatrick, Vilma Mesa, & Finbarr Sloane

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

History of U.S. Algebra Courses

 Not until after the Civil War, however, was geometry required for entrance. The order in which these mathematical subjects...continues to shape, the college preparatory courses offered in secondary school, ...



 In 2000, 26% of 8th graders reported taking algebra – this has grown to ~40%, with ~5% of 7th graders taking algebra.



Findings

- U.S. students do not do well on items that involve the extension of a pattern if the item requires that they explicitly produce, describe, or represent a relationship rather than simply find the next few terms in a sequence.
- U.S. students do relatively poorly in setting up an equation to model a real situation.
- U.S. 8th grade teachers spend considerable time reviewing topics already taught; almost 30% of their lessons are devoted entirely to review.

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Thoughts

- U.S. students appear to need many more opportunities to engage in functional thinking with complex problems, and in particular, in functional thinking as it related to realistic situations.
- Algebra is of limited use if it is understood as generalized arithmetic only.



Hmm...

- The U.S. is not the only country in which 8th grade teachers could be giving greater attention to functions, but it is one in which too many people have assumed for too long that most students cannot learn, use, or value algebra.
- Implications math panel charge; NAEP and Algebra



What can TIMSS Surveys Tell Us about Mathematics Reforms in the United States during the 1990s?

Laura S. Hamilton & Jose Felipe Martinez



Is it "approaches" or mathematics?

- There are signs of a partial cease fire...
- ... growing acceptance of more traditional approaches on the part of NCTM.
- NMP...includes individuals who have been associated with both sides of the debate. Early indications...greater degree of consensus than has been obtained in the past.

CTM NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

p. 128, 129

Now what?

 Overall, the evidence of changes in U.S. teachers' use of reform practices is very limited and can only be interpreted as suggestive, among other reasons because of the limited number of practices compared.

Limitations in survey use and analysis

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Examining Educational Technology and Achievement through Latent Variable Modeling

Elena C. Papanastasiou & Efi Paparistodemou

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Calculator Use & Mathematics

• The advent of calculator technology has influenced the teaching of mathematics...



 The graphic calculator's value lies in its intrinsic use of variables in its operations and in its multiline display that allows the user to see, reflect upon, and react with several sequential inputs or outputs.

CTM NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Results

- Children from stronger educational backgrounds are advantaged in school while children from less educated backgrounds are disadvantaged.
- To be effective, educational technology has to be used appropriately.
- The inverse relationship between technology use and student achievement found in this study is in contrast to other studies that tend to praise the use of technology and showcase its positive results.

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- Activities associated with lower levels of achievement in mathematics:
 - Use of the internet
 - Use of software or online games
 - Using the computer to write reports
 - Using the computer to analyze data
 - Using the computer as a resource

Literacy use not helpful to learning mathematics

p. 222



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Technology Use & Achievement

 "There is plenty of evidence to indicate a positive relationship between technology and student achievement."

• Where is it? Particularly related to use of the graphic calculator?

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Threads

 Need for focus and coherence – across and within (e.g algebra)
 Lack of penetration of reform
 Need for research



Thank you!

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