By How Much Does GDP Rise If the Government Buys More Output?

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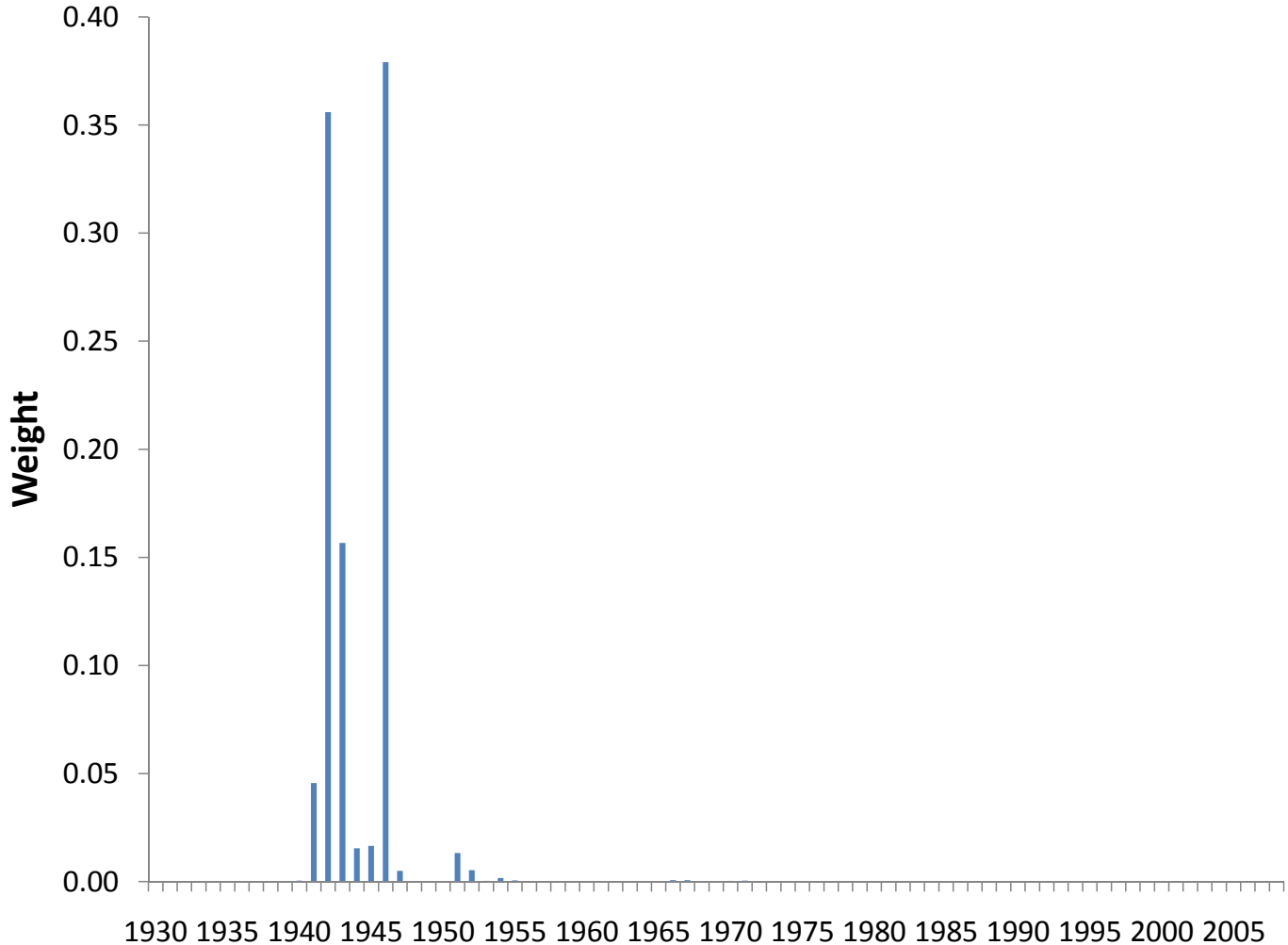
Estimates of Output and Consumption Multipliers for Military Spending

Period	*GDP multiplier*	*Consumption multiplier*
1930-2008	0.55 (0.08)	-0.05 (0.03)
1948-2008	0.47 (0.28)	-0.12 (0.10)
1960-2008	0.13 (0.65)	-0.09 (0.29)
1939-1948	0.53 (0.07)	-0.05 (0.02)
1949-1955	0.48 (0.56)	-0.18 (0.05)
1939-1944	0.36 (0.10)	-0.11 (0.03)
1945-1949	0.39 (0.08)	-0.04 (0.05)

Weights Implicit in Regression Estimates of Output and Consumption Multipliers



Estimates of Multipliers from Vector Autoregressions

	Variable Y=GDP, C=Con-sumption	*Multipliers and standard errors (in parentheses)*			*Source*
		Impact	*After 4 quarters*	*After 8 quarters*	
Blanchard-Perotti, stochastic trend	Y	0.90	0.55	0.65	Table IV, p. 1347
		(0.30)	(0.60)	(1.20)	
Galí, *et al.*	Y	0.41	0.31	0.68	Table 1, p. 233
		(0.18)	(0.54)	(0.72)	
	C	0.07	0.11	0.49	
		(0.05)	(0.14)	(0.21)	
Perotti	Y	0.70	1.00	1.20	Figure 3, p. 43
		(0.20)	(0.50)	(0.50)	
	C	0.10	0.30	0.40	
		(0.05)	(0.20)	(0.25)	

Consumption and labor supply

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$$wc^{-1/\sigma} = \gamma h^{1/\psi}$$

Simple static model

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$$(y-g)^{-1/\sigma} = \frac{\gamma}{\alpha} y^{\frac{1+1/\psi}{\alpha}-1}$$

Output multiplier

Normalize so $y = 1$, wlog

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$$m_y = \frac{dy}{dg} = \frac{\alpha}{\alpha + \sigma(1-g)(1-\alpha+1/\psi)}$$

Properties of the static model

$\alpha \leq 1$ and $\psi > 0$, so the *output multiplier cannot exceed one.*

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Multiplier is close to one if

- ψ large *and* α close to one, OR
- σ close to zero, OR
- g close to one

Consumption multiplier

The consumption multiplier is never positive.

Parameter values

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$g = 0.2$

Implied multipliers

Output multiplier is about 0.4, at the low end of the range of empirical findings

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Consumption multiplier is -0.6, out of line with all of the empirical evidence

Boosting the multipliers

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Negative response of investment to government purchases—dynamic model

Markup ratio declining with output

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Condition for output multiplier above one: $\omega > \frac{1-\alpha+1/\psi}{\alpha}$

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With $\omega = 0.5$, the output multiplier is 0.5 and the consumption multiplier is -0.5.

Employment rate and hours

Framework in my 2009 JPE paper

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Frisch labor supply function $h(w)$ with elasticity 0.7

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Employment rate function $n(w)$ with elasticity 1.2

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Elasticity of total hours of work $h(w)n(w)$ is 1.9

Implications

Output multiplier becomes 0.8 and the consumption multiplier -0.2, an important step toward realism

Complementarity

$$\frac{c^{1-1/\sigma}}{1-1/\sigma} - \chi c^{1-1/\sigma} h^{1+1/\psi} - \gamma \frac{h^{1+1/\psi}}{1+1/\psi}$$

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Parameter values: $\sigma = 0.4$, $\psi = 1.54$, $\chi = 0.334$ and $\gamma = 1.1$

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Frisch elasticities:

- Own-price elasticity of consumption: -0.5
- Wage elasticity of hours of work: 1.9
- Elasticity of consumption with respect to wage: 0.4

Implications

With the negative of the elasticity of the markup, ω, at 0.5, the output multiplier is 0.97 and the consumption multiplier is -0.03, figures easily consistent with the empirical evidence.

Dynamic model with investment

$$q_t = \kappa \frac{k_t - k_{t-1}}{k_{t-1}} + 1$$

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$$k_t + \frac{\kappa}{2}\frac{(k_t - k_{t-1})^2}{k_{t-1}} = (1 - \delta)k_{t-1} + y_t - c_t - g_t.$$

Discounting and Euler equation

$$m_{t,t+1} = \beta \frac{c_{t+1}^{-1/\sigma}}{c_t^{-1/\sigma}} \frac{1 - \chi(1 - 1/\sigma) h_{t+1}^{1+1/\psi}}{1 - \chi(1 - 1/\sigma) h_t^{1+1/\psi}}$$

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$$(1 + r_{t+1}) m_{t,t+1} = 1$$

Path of government purchases

$$g_t = \bar{g} + g\phi^t.$$

Adjustment cost

I take the parameter κ to be 8 at a quarterly rate, corresponding to 2 at an annual rate, a representative value from the literature on this subject.

Implications

Output multiplier is 0.9 and consumption multiplier is zero, in line with the evidence.

Parameter values

Case	*Parameters*					
	σ, consumption curvature	*ψ, labor supply elasticity*	*χ, complementarity*	*γ, labor weight*	*κ, capital adjustment cost*	*ω, markup response*
Base	0.4	1.54	0.334	1.103	8	0.7
Constant markup	0.4	1.54	0.334	1.103	8	0
No adjustment cost	0.4	1.54	0.334	1.103	0	0.7
No complementarity	0.5	1.9	0	1.102	8	0.7
Less elastic labor supply	0.4	0.5	0.334	0.617	8	0.7

Impulse responses

	Government purchases impulse responses					
	Impact		*4 quarters later*		*8 quarters later*	
Case	*Output*	*Consump-tion*	*Output*	*Consump-tion*	*Output*	*Consump-tion*
Base	0.98	-0.03	0.68	-0.02	0.48	-0.01
Constant markup	0.60	-0.16	0.41	-0.12	0.28	-0.10
No adjustment cost	0.98	-0.03	0.69	-0.02	0.48	-0.01
No complementarity	0.92	-0.15	0.65	-0.10	0.46	-0.07
Less elastic labor supply	0.40	-0.25	0.24	-0.21	0.13	-0.18

Conclusion

The key to a substantial effect of government purchases on output is a combination of a markup that declines when output expands and elastic labor supply, possibly associated with a sticky wage.

Zero bound on nominal interest rate

At the zero bound, the output multiplier rises from 0.95 to 1.72 and the consumption multiplier from -0.07 to 0.26.

Markup response in an NK model

Price persistence, θ	*Output multiplier*	*Consumption multiplier*	*Elasticity of markup ratio,* ω
0.6	0.60	-0.21	0.06
0.7	0.62	-0.20	0.13
0.8	0.68	-0.18	0.29
0.9	1.02	-0.04	0.84
0.95	1.56	0.18	1.22

Measuring the markup elasticity

$$\text{Marginal cost} = \frac{w}{\partial Y/\partial L}$$

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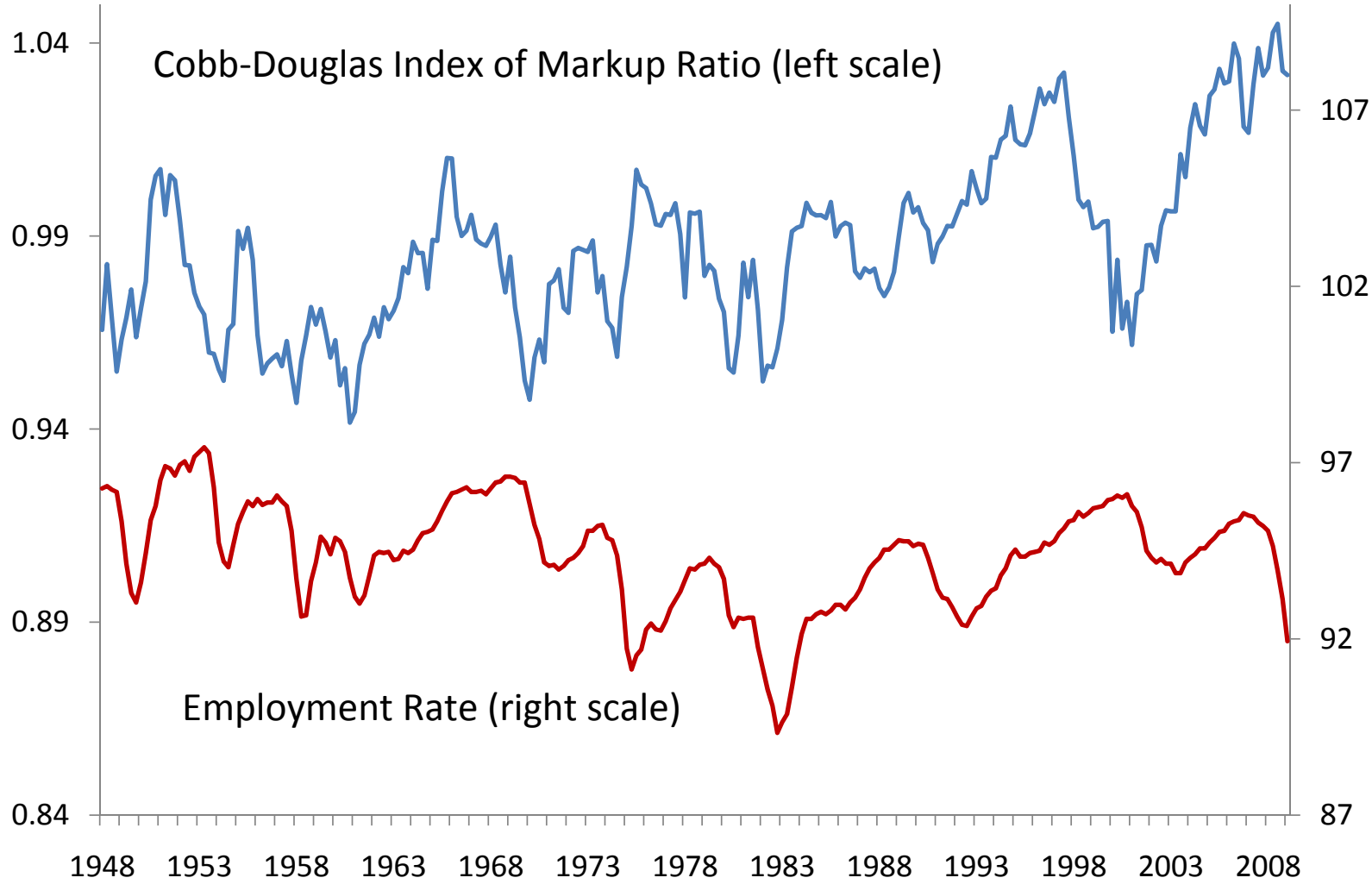
$$= \frac{\alpha}{s}$$

Cobb-Douglas Index of Markup Ratio and Employment Rate, 1948-2009



Effects of Stimulus Measure of February 2009 and of an Alternative

	2009	2010	2011	Sum
Stimulus purchases, fiscal year	34.8	110.7	76.3	221.8
Stimulus purchases, calendar year	62.5	102.1	57.2	221.8
GDP	13,700	14,043	14,604	
Stimulus as a percent of GDP	0.46	0.73	0.39	1.57
Effect on GDP, percent	1.10	1.28	0.70	3.08
Front-loaded stimulus as a percent of GDP	0.71	0.50	0.35	1.56
Effect on GDP, percent	1.35	0.94	0.62	2.90