Online Appendix for Caballero, Farhi and Gourinchas (2008)

Appendix C

Derivations for Sections 2 and 3

C.1 Asset Decline and Trade Balance when $\sigma < 1$: the petrodollar effect

In this section, we provide the analysis underlying the claims in our section on global imbalances in the short run (p. 27 in the main text) regarding the case where the elasticity of substitution σ between X and Z is less than one. Formally, recall that in equilibrium $\theta W_t = \left(1 + \alpha p_t^{1-\sigma}\right) X_t$. Because $I_{t_0} = 0$, $W_{t_0^+} = V_{t_0^+}$ and therefore $V_{t_0^+} - \left(V_{t_0^-} + B_{t_0^-}\right) > 0$ when $\sigma < 1$. This increase in global asset value (when measured in units of X) mitigates the fall in the value of U-assets. In fact the change in wealth can be expressed as

$$B_{t_0^-} + V_{t_0^-}^U - V_{t_0^+}^U = B_{t_0-}\left(1 - x_{t_0}^U\right) - \frac{\alpha}{\theta} X_{t_0}^U \left(p_{t_0^+}^{1-\sigma} - p_{t_0^-}^{1-\sigma}\right). \tag{C.1}$$

The first term on the right hand side of this expression is positive and is exactly the same as in equation (20) in the main text. It represents the direct effect of the bubble-burst and also corresponds to the drop in $U's$ wealth that would occur in the benchmark no-inventory economy. The second term is negative in our economy but vanishes in the benchmark no-inventory economy. It represents the drop in the share of the X good in total consumption, and mitigates the fall in the value of U's assets. As we argued earlier, this "petrodollar" term plays a key role in limiting the extent of short-run rebalancing.

Consider now the impact effects of the growth slowdown shock. We maintain the assumption of extreme home bias so that immediately before the growth slowdown shock hits, all of U's wealth is invested in U assets. The adjustment in the trade balance is always positive:

$$TB_{t_1^+}^U - TB_{t_1^-}^U = -\theta\left(W_{t_1^+}^U - W_{t_1^-}^U\right) = \theta \mu_{t_{1-}}\left(V_{t_1^-}^U - V_{t_1^+}^U\right),$$

where $\mu_{t_{1-}} = W_{t_1^-}^U / \left(V_{t_1^-}^U + B_{t_{1-}}\right)$. The change in the value of U assets can be computed as above

$$V_{t_1^+}^U - V_{t_1^-}^U = x_{t_1}^U \frac{\alpha}{\theta}\left(p_{t_1^+}^{1-\sigma} - p_{t_1^-}^{1-\sigma}\right) X_t - x_{t_1}^U \left(p_{t_1^+} - p_{t_1^-}\right) I_{t_1}.$$

The second term is negligible if I_{t_1} is small. Since the first term is equal to 0 when $\sigma = 1$, we have that in this benchmark the impact of the growth slowdown on the trade balance is negligible. By contrast, when $\sigma < 1$, the first term is negative and, under our calibration, it swamps the second term. Thus, the trade balance improves at impact.

C.2 The Trade Balance in the short Run

In this section, we analyze the behavior of the trade balance in the short run both in our economy and in the benchmark no-inventory economy.

We can decompose this difference more finely by studying the trade balance decomposition into exports $X_t^U - \theta W_t^U / \left(1 + \alpha p_t^{1-\sigma}\right)$ and imports $\alpha p_t^{1-\sigma} \theta W_t^U / \left(1 + \alpha p_t^{1-\sigma}\right)$:

$$\begin{aligned} TB_{t_0^+}^U - TB_{t_0^-}^U &= \left(\frac{\theta W_{t_0^-}^U}{1 + \alpha p_{t_0^-}^{1-\sigma}} - \frac{\theta W_{t_0^+}^U}{1 + \alpha p_{t_0^+}^{1-\sigma}} \right) - p_{t_0^+} \left(\frac{\alpha p_{t_0^+}^{-\sigma} \theta W_{t_0^+}^U}{1 + \alpha p_{t_0^+}^{1-\sigma}} - \frac{\alpha p_{t_0^-}^{-\sigma} \theta W_{t_0^-}^U}{1 + \alpha p_{t_0^-}^{1-\sigma}} \right) \\ & -(p_{t_0^+} - p_{t_0^-}) \frac{\alpha p_{t_0^-}^{-\sigma} \theta W_{t_0^-}^U}{1 + \alpha p_{t_0^-}^{1-\sigma}}. \end{aligned}$$

The three terms on the right hand side have a traditional Marshall-Lerner interpretation: The first one represents the increase in the volume of exports. The second one represents the decrease in the volume of imports times the terms of trade. These two are positive since the volume of exports rises and the volume of imports falls. The third term is negative and represents imports times the change in the terms of trade. Note that the terms of trade effect – the last term– would be absent in the benchmark no-inventory economy. However, we can show that the positive quantity effect – the first two terms – is stronger in our economy than in the benchmark economy, which means that the difference in trade rebalancing between these two economies is strictly less than the direct effect of the change in terms of trade resulting from speculation in commodities.[55]

[55] This can be verified as follows. The claim amounts to showing that

$$(p_{t_0^+} - p_{t_0^-}) \frac{\alpha p_{t_0^-}^{-\sigma} \theta W_{t_0^-}^U}{1 + \alpha p_{t_0^-}^{1-\sigma}} > \theta \frac{W_{t_0^-}^U}{B_{t_0^-} + V_{t_0^-}^U} \frac{\alpha}{\theta} X_{t_0}^U \left(p_{t_0^+}^{1-\sigma} - p_{t_0^-}^{1-\sigma} \right)$$

which can be re-arranged into

$$\frac{\frac{p_{t_0^+}}{p_{t_0^-}} \left(1 - \frac{p_{t_0^+}^{-\sigma}}{p_{t_0^-}^{-\sigma}} \right)}{\frac{p_{t_0^+}^{1-\sigma}}{p_{t_0^-}^{1-\sigma}} - 1} + 1 > \frac{1}{1 + \frac{B_{t_0^-}}{B_{t_0^-} + V_{t_0^-}} \left(1 - \frac{X_{t_0}^U}{X_t^U} \right)}.$$

C.3 The Commodity-Price-Jump Term

The drop in interest rates at $t = t_0$ when $\sigma < 1$ can be expressed as:

$$
\begin{aligned}
r_{t_0^+} - r_{t_0^-} &= -g\frac{B_{t_0^-}}{W_{t_0^-}} \\
&\quad + \left(\frac{\delta\theta\left(1 - s_{zt_0^-}\right) + \theta\left(\left(\frac{p_{t_0+}}{p_{t_0-}}\right)^{1-\sigma} - \frac{p_{t_0+}}{p_{t_0-}}\right) s_{zt_0^-}}{1 + s_{zt_0^-}\left(\sigma\left(\frac{p_{t_0+}}{p_{t_0-}}\right)^{1-\sigma} - 1\right)} \right) - \delta\theta\left(1 - s_{zt_0^-}\right) - \frac{g\left(1-\sigma\right)}{\sigma} s_{zt_0^-}
\end{aligned}
$$

where where s_{zt} is the expenditure share of commodities. The first term on the right hand side represents the 'bubble-burst' term discussed in the section on the financial crash and commodity boom (p. 19 in the main text). The second term is equivalent of the 'commodity-price-jump' term introduced in equation (15) in the main text when $sigma < 1$.

C.4 The Numeraire

Throughout we have chosen the X-good as the numeraire. This section confirms the claim in footnotes 10 and 19 in the main text that the main substantive results do not depend on this particular choice of numeraire. The price index corresponding to the composite consumption good is given by $\left(1 + \alpha p_t^{1-\sigma}\right)^{1/(1-\sigma)}$ if $\sigma \neq 1$, and $p_t^{\frac{\alpha}{1+\alpha}}$ if $\sigma = 1$. We term the corresponding numeraire the composite numeraire. We denote with a tilde the variable expressed in the composite numeraire. The interest rate in the composite numeraire $\tilde{r}_t$ can be computed from r_t as follows:

$$
\tilde{r}_t = r_t - \frac{d \log\left[\left(1 + \alpha p_t^{1-\sigma}\right)^{1/(1-\sigma)}\right]}{dt} = r_t - \frac{\alpha p_t^{-\sigma}}{1 + \alpha p_t^{1-\sigma}} \dot{p}_t
$$

If $\max\{I_t, \dot{I}_t\} > 0$, we have

$$
\tilde{r}_t = r_t - \frac{\alpha p_t^{-\sigma}(r_t + d)p_t}{1 + \alpha p_t^{1-\sigma}} = \frac{r_t - d\alpha p_t^{1-\sigma}}{1 + \alpha p_t^{1-\sigma}}
$$

which using (10) we can rewrite as

$$
\tilde{r}_t = \theta \frac{\left(\delta + g\frac{B_t + p_t I_t}{X_t}\right) - \left(\frac{p_t Z}{X_t} - \alpha p_t^{1-\sigma}\right)}{\left(1 + \alpha\sigma p_t^{1-\sigma}\right)\left(1 + \alpha p_t^{1-\sigma}\right)} + \tilde{\epsilon}_d
$$

where $\tilde{\epsilon}_d = -\sigma d\alpha p_t^{1-\sigma} / \left(1 + \alpha\sigma p_t^{1-\sigma}\right)$. In the rest of this discussion we consider the limit case $d = 0$. Let's now investigate how this change in numeraire would modify our conclusions concerning the crash and the steady states. Note that when $\sigma = 1$ then we simply have

$$\tilde{r}_t = \frac{r_t}{1+\alpha}.$$

Hence

$$\tilde{r}_{t_0^-} - \tilde{r}_{t_0^+} = \frac{r_{t_0^-} - r_{t_0^+}}{1+\alpha} = \frac{1}{1+\alpha}\left[\frac{\theta g}{1+\alpha}\frac{\tilde{B}^U_{t_0-}}{p_{t_0^-}^{\frac{\alpha}{1+\alpha}} X_{t_0}} + \frac{\theta}{1+\alpha}\frac{Z}{X_{t_0}}\left(p_{t_0^+} - p_{t_0^-}\right)\right].$$

Let's now turn to the drop in wealth and asset value at impact. We have

$$\begin{aligned}
\frac{\theta \tilde{W}_{t_0^-}}{1+\alpha} &= p_{t_0^-}^{-\frac{\alpha}{1+\alpha}} X_{t_0} = \tilde{V}^U_{t_0^-}\frac{X_{t_0}}{X^U_{t_0}} + \tilde{B}^U_{t_0-} \\
\frac{\theta \tilde{W}_{t_0^+}}{1+\alpha} &= p_{t_0^+}^{-\frac{\alpha}{1+\alpha}} X_{t_0} = \tilde{V}^U_{t_0^+}\frac{X_{t_0}}{X^U_{t_0}}
\end{aligned}$$

Hence we obtain

$$\tilde{V}^U_{t_0^-} + \tilde{B}^U_{t_0-} - \tilde{V}^U_{t_0^+} = \tilde{B}^U_{t_0-}\left(1 - \frac{X^U_{t_0}}{X_{t_0}}\right) + \left(p_{t_0^-}^{-\frac{\alpha}{1+\alpha}} - p_{t_0^+}^{-\frac{\alpha}{1+\alpha}}\right)\frac{1}{\theta}X^U_{t_0}$$

The amount of rebalancing is given by $\widetilde{CA}^U_{t_0^+} - \widetilde{CA}^U_{t_0^-}$

$$\begin{aligned}
&\left[\tilde{r}_{t_0^-}\left(1 - \frac{\tilde{W}^U_{t_0^-}}{\tilde{V}^U_{t_0^-} + \tilde{B}^U_{t_0-}}\right) + \theta\frac{\tilde{W}^U_{t_0^-}}{\tilde{V}^U_{t_0^-} + \tilde{B}^U_{t_0-}}\right]\left(\tilde{V}^U_{t_0^-} + \tilde{B}^U_{t_0-} - \tilde{V}^U_{t_0^+}\right) - X^U_{t_0}\frac{1}{\theta}\left(p_{t_0^-}^{-\frac{\alpha}{1+\alpha}} - p_{t_0^+}^{-\frac{\alpha}{1+\alpha}}\right) \\
&+ \left(\tilde{r}_{t_0^-} - \tilde{r}_{t_0^+}\right)\left(\tilde{V}^U_{t_0^-} + \tilde{B}^U_{t_0-} - \tilde{W}^U_{t_0^-}\right)\frac{\tilde{V}^U_{t_0^+}}{\tilde{V}^U_{t_0^-} + \tilde{B}^U_{t_0-}}
\end{aligned}$$

The right comparison when we use the composite numeraire should be with an economy with exogenous commodity prices. With endogenous commodity prices, the interest rate drops more. The drop in asset value is also more pronounced because the trees pay dividend in X-goods the value of which depreciates at impact. Both effects contribute to more rebalancing with endogenous commodity prices. A counterbalancing effect is that the value of GDP goes down since GDP is composed of X goods, the value of which depreciates at impact. This last effect contributes to less rebalancing.

Another quantity we could look at is the amount of rebalancing as a fraction of GDP $\widetilde{CA}^U_{t_0^+} / \left(p_{t_0^+}^{-\frac{\alpha}{1+\alpha}} X^U_{t_0}\right) -$

$\widetilde{CA}^U_{t_0^-} / \left(p_{t_0^-}^{-\frac{\alpha}{1+\alpha}} X^U_{t_0} \right)$

$$\left[\tilde{r}_{t_0^-} \left(1 - \frac{\tilde{W}^U_{t_0^-}}{\tilde{V}^U_{t_0^-} + \tilde{B}^U_{t_0-}} \right) + \theta \frac{\tilde{W}^U_{t_0^-}}{\tilde{V}^U_{t_0^-} + \tilde{B}^U_{t_0-}} \right] \left(\frac{\tilde{V}^U_{t_0^-} + \tilde{B}^U_{t_0-}}{p_{t_0^-}^{-\frac{\alpha}{1+\alpha}} X^U_{t_0}} - \frac{\tilde{V}^U_{t_0^+}}{p_{t_0^+}^{-\frac{\alpha}{1+\alpha}} X^U_{t_0}} \right)$$
$$+ \left(\tilde{r}_{t_0^-} - \tilde{r}_{t_0^+} \right) \frac{\tilde{V}^U_{t_0^-} + \tilde{B}^U_{t_0-} - \tilde{W}^U_{t_0^-}}{p_{t_0^-}^{-\frac{\alpha}{1+\alpha}} X^U_{t_0}} \frac{\tilde{V}^U_{t_0^+}}{\tilde{V}^U_{t_0^-} + \tilde{B}^U_{t_0-}} \frac{p_{t_0^-}^{-\frac{\alpha}{1+\alpha}}}{p_{t_0^+}^{-\frac{\alpha}{1+\alpha}}}$$

which we can rewrite as

$$\left[\tilde{r}_{t_0^-} \left(1 - \frac{\tilde{W}^U_{t_0^-}}{\tilde{V}^U_{t_0^-} + \tilde{B}^U_{t_0-}} \right) + \theta \frac{\tilde{W}^U_{t_0^-}}{\tilde{V}^U_{t_0^-} + \tilde{B}^U_{t_0-}} \right] \frac{B^U_{t_0-}}{X^U_{t_0}} \left(1 - \frac{X^U_{t_0}}{X_{t_0}} \right)$$
$$+ \left(\tilde{r}_{t_0^-} - \tilde{r}_{t_0^+} \right) \left(\tilde{V}^U_{t_0^-} + \tilde{B}^U_{t_0-} - \tilde{W}^U_{t_0^-} \right) \frac{V^U_{t_0^+}}{V^U_{t_0^-} + B^U_{t_0-}}.$$

With this normalization, endogenous commodity prices only contribute to less rebalancing with $\sigma = 1$ because they lead to lower interest rates.

Let's now turn to $\sigma < 1$. In this case we have $\tilde{r}_t = \frac{r_t}{1+\alpha p_t^{1-\sigma}}$ so that

$$\tilde{r}_{t_0^-} - \tilde{r}_{t_0^+} = \frac{r_{t_0^-} - r_{t_0^+}}{1 + \alpha p_{t_0^-}^{1-\sigma}} + r_{t_0^+} \left(\frac{1}{1 + \alpha p_{t_0^-}^{1-\sigma}} - \frac{1}{1 + \alpha p_{t_0^+}^{1-\sigma}} \right).$$

Similarly, we have

$$\frac{\tilde{V}^U_{t_0^-} + \tilde{B}^U_{t_0-}}{\left(1 + \alpha p_{t_0^-}^{1-\sigma}\right)^{-1/(1-\sigma)} X^U_{t_0}} - \frac{\tilde{V}^U_{t_0^+}}{\left(1 + \alpha p_{t_0^+}^{1-\sigma}\right)^{-1/(1-\sigma)} X^U_{t_0}} = \frac{B^U_{t_0-}}{X^U_{t_0}} \left(1 - \frac{X^U_{t_0}}{X_{t_0}} \right) - \alpha \frac{1}{\theta} \left(p_{t_0^+}^{1-\sigma} - p_{t_0^-}^{1-\sigma} \right).$$

And we can compute $\widetilde{CA}^U_{t_0^+} / \left(\left(1 + \alpha p_{t_0^+}^{1-\sigma}\right)^{-1/(1-\sigma)} X^U_{t_0} \right) - \widetilde{CA}^U_{t_0^-} / \left(\left(1 + \alpha p_{t_0^-}^{1-\sigma}\right)^{-1/(1-\sigma)} X^U_{t_0} \right)$

$$\left[\tilde{r}_{t_0^-} \left(1 - \frac{\tilde{W}^U_{t_0^-}}{\tilde{V}^U_{t_0^-} + \tilde{B}^U_{t_0-}} \right) + \theta \frac{\tilde{W}^U_{t_0^-}}{\tilde{V}^U_{t_0^-} + \tilde{B}^U_{t_0-}} \right] \left(\frac{\tilde{V}^U_{t_0^-} + \tilde{B}^U_{t_0-}}{\left(1 + \alpha p_{t_0^-}^{1-\sigma}\right)^{-1/(1-\sigma)} X^U_{t_0}} - \frac{\tilde{V}^U_{t_0^+}}{\left(1 + \alpha p_{t_0^+}^{1-\sigma}\right)^{-1/(1-\sigma)} X^U_{t_0}} \right)$$
$$+ \left(\tilde{r}_{t_0^-} - \tilde{r}_{t_0^+} \right) \left(\frac{\tilde{V}^U_{t_0^-} + \tilde{B}^U_{t_0-} - \tilde{W}^U_{t_0^-}}{\left(1 + \alpha p_{t_0^-}^{1-\sigma}\right)^{-1/(1-\sigma)} X^U_{t_0}} \right) \frac{\tilde{V}^U_{t_0^+}}{\tilde{V}^U_{t_0^-} + \tilde{B}^U_{t_0-}} \frac{\left(1 + \alpha p_{t_0^-}^{1-\sigma}\right)^{-1/(1-\sigma)}}{\left(1 + \alpha p_{t_0^+}^{1-\sigma}\right)^{-1/(1-\sigma)}}$$

Hence when net positions are small compared to gross positions endogenous commodity prices can lead to less rebalancing. All the qualitative conclusions of the paper remain unchanged.

Appendix D

Derivations for Section 4

In this appendix, we provide the analysis underlying the calibration and the dynamic system (p. 36 in the main text).

D.1 The bubble economy: $t < t_0$.

We start the economy in the bubble equilibrium with $\sigma < 1$, for $t < t_0$. The economy is characterized by the following equations:

$$\begin{aligned}
\dot{B}_t &= r_t B_t \\
r_t &= \frac{\delta\theta + \theta g B_t / X_t + \alpha \hat{q}^{1-\sigma} X_t^{1/\sigma - 1} g\left(1/\sigma - 1\right)}{1 + \alpha \hat{q}^{1-\sigma} X_t^{1/\sigma - 1}}
\end{aligned}$$

where $\hat{q} \equiv p_t / X_t^{1/\sigma} = (\alpha/Z)^{1/\sigma}$ is constant. This is a differential system in B_t with a forcing term X_t. It requires a terminal condition. To find this terminal condition, we need to characterize the path of the economy, in the event that the bubble does not collapse. Observe that $p_t = \hat{q} X_t^{1/\sigma}$, hence the share of consumption expenditures on the Z-good, $s_{zt} = \alpha p_t^{1-\sigma} / \left(1 + \alpha p_t^{1-\sigma}\right)$, is increasing without bounds. The transition to $\sigma = 1$ must occur at some time T_1 such that $\alpha \hat{q}^{1-\sigma} X_{T_1}^{1/\sigma - 1} = \alpha'$. From T_1 onwards, the elasticity of substitution is equal to one. The bubble economy with $\sigma = 1$ reaches its steady state instantly with:

$$\begin{aligned}
r_t &= g\,; \quad q_t = \alpha'/Z \\
\frac{W_t}{X_t} &= \frac{1+\alpha'}{\theta}\,; \quad \frac{V_t}{X_t} = \frac{V_t^U}{X_t^U} = \frac{\delta}{g} \\
B_t &= \left[\frac{1+\alpha'}{\theta} - \frac{\delta}{g}\right] X_t
\end{aligned} \tag{D.1}$$

This provides the terminal condition for the value of the bubble at time T_{1^-} : $B_{T_{1^-}} = \left[\frac{(1+\alpha')}{\theta} - \frac{\delta}{g}\right] X_{T_1}$. Solving backwards from $t = T_1$, we can then characterize the entire path $\left\{B_t, W_t, V_t, V_t^U, p_t\right\}$ that is expected to occur in the absence of a collapse of the bubble.

This global system is consistent with any initial net foreign asset position at $t = t_0$. Assume that we want to start the economy with an external debt $NA_{t_0}^U = \eta X_{t_0}^U$. Under the assumption that the bubble is initially located in U, this implies that U's savings are equal to $W_{t_0}^U = V_{t_0}^U + B_{t_0} + \eta X_{t_0}^U$. One can then solve for the path of domestic savings from the asset accumulation equation. In turn, this pins down net foreign assets $NA_t^U = W_t^U - V_t^U - B_t$, the current account $CA_t^U = \dot{W}_t^U - \dot{V}_t^U - \dot{B}_t$ and the trade balance $TB_t^U = X_t^U - \theta W_t^U$ in all previous periods.

D.2 Collapse of the bubble: the short run.

Consider now what happens at time $t = t_0$ when the bubble collapses. As long as $\sigma < 1$, the economy satisfies the following equations:

$$\begin{aligned}
\dot{I}_t &= Z - \alpha \hat{q}_t^{-\sigma} \\
\dot{\hat{q}}_t &= \left(r_t - \frac{g}{\sigma}\right) \hat{q}_t \\
r_t &= \frac{X_t^{1-1/\sigma}\theta\delta + \theta g \hat{q}_t I_t - \theta \hat{q}_t \left(Z - \alpha \hat{q}_t^{-\sigma}\right)}{X_t^{1-1/\sigma} + \alpha\sigma \hat{q}_t^{1-\sigma}}
\end{aligned}$$

This is a dynamic system in I and $\hat{q}$ with a forcing term X_t. We have one initial condition: $I_{t_0} = 0$, by assumption. We need a terminal condition on $\hat{q}_t$. To find it, consider what happens to the share of commodities in expenditures, $\alpha p_t^{1-\sigma} / \left(1 + \alpha p_t^{1-\sigma}\right)$ over time. From the second equation above, the growth rate of $p_t^{1-\sigma} = \hat{q}_t^{1-\sigma} X_t^{1/\sigma - 1}$ is $(1-\sigma)(r_t - g/\sigma) + g(1/\sigma - 1) = (1-\sigma) r_t$, which must be positive eventually (since the interest rate converges to g/σ). Hence the expenditure share must eventually reach $\bar{s}_z$, at which point the elasticity of substitution becomes unity. Let's denote T_2 the time at which this happens, and $\hat{q}_{T_{2-}}$, $I_{T_{2-}}$ the values of the system at that time. Note that $\hat{q}_{T_2}$ and T_2 are also linked by $\alpha \hat{q}_{T_2}^{1-\sigma} X_{T_2}^{1/\sigma - 1} = \alpha'$. Thus, we can parameterize potential equilibrium paths by T_2.

D.3 Collapse of the bubble: the long run.

When $t \geq T_2$, the economy is now in the unitary elasticity inventory model described in the previous section. The system follows a saddle path dynamics with:

$$\begin{aligned}
\dot{I}_t &= Z - \alpha' q_t^{-1} \\
\dot{q}_t &= (r_t - g)\, q_t \\
r_t &= \theta \frac{\delta + \alpha' - q_t (Z - g I_t)}{1 + \alpha'}
\end{aligned}$$

where $q_t = p_t / X_t$. The boundary conditions for that system are $I_{T_2} = I_{T_{2-}}$ and $\lim_{t\to\infty} q_t = \alpha'/Z$. Solving the saddle-path dynamics provides the unique initial value $q_{T_{2+}}$ that is consistent with the equilibrium (see figure 9 in the main text).

Lastly, continuity of the price at T_2 requires that we select T_2 such that $\hat{q}_{T_{2^-}} = q_{T_{2^+}} X_{T_2}^{(1-1/\sigma)}$. This completes the characterization of the economy.