Cracking the Conundrum

Between June 29, 2004, and February 2, 2005, the Federal Open Market Committee increased the target federal funds rate by 150 basis points (bp), or 1.50 percentage points. Over the same period, the long end of the yield curve fell, with the ten-year yield declining by 70 bp and the ten-year (instantaneous) forward rate by more than 100 bp. This pronounced rotation in the yield and forward rate curves caught many by surprise. Then–Federal Reserve chairman Alan Greenspan was one. In his February 2005 testimony to Congress, he noted:1

This development contrasts with most experience, which suggests that . . . increasing short-term interest rates are normally accompanied by a rise in longer-term yields. Historically, even . . . distant forward rates have tended to rise in association with monetary policy tightening. . . . For the moment, the broadly unanticipated behavior of world bond markets remains a conundrum.

Bill Gross was another. PIMCO’s legendary bond investor advised clients in February 2004 to reduce the duration of their bond holdings, only to see

This paper was written while Jonathan Wright was on leave at the University of Pennsylvania. It began as two separate discussions of a paper by John Cochrane and Monika Piazzesi (2006). We thank them for their insights. We also thank Frank Diebold, Don Kim, Richard Sylla, and Stanley Zin for comments and advice. The views expressed are the sole responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other employees of the Federal Reserve System.

long-term bonds rally. Thirteen months later he confessed confusion over what had occurred:2

Who would have thought that the bond market could have [done so well]? Not yours truly, nor Alan Greenspan. . . . How then to explain it? . . . I must tell you that we at PIMCO have been talking about this topic for months. We, too, have been befuddled.

The Federal Reserve continued raising the federal funds target thereafter, but the ten-year forward rate continued to fall. In all, the ten-year forward rate fell 170 bp between June 2004 and June 2005 and has rebounded only about 50 bp since then.

If the word “conundrum” from a respected central banker covered red faces in the investment community, other observers had no shortage of explanations for what had happened: some linked the decline in long-term yields to a decline in expectations of long-run economic growth or inflation, others to an increase in the global supply of savings, or to an improvement in the ability of global capital markets to allocate risk efficiently, or to massive purchases of U.S. Treasury securities by Asian central banks and petroleum exporters, or to increased demand for long-term bonds on the part of pension funds, or to reduced supply of long-maturity instruments following the Treasury’s 2001 decision (later rescinded) to discontinue auctions of the thirty-year bond, or to a global decline in macroeconomic and financial market uncertainty, perhaps the result of improved procedures for monetary policy or the state of the business cycle.

In this paper we look at several of these proposed explanations, holding each up to a broad array of evidence. The paper’s first section is devoted to establishing the facts. Among those we consider key are the following: over the conundrum period, real interest rates fell along with nominal rates; long-term survey expectations of inflation, growth, and interest rates were roughly flat; volatilities and risk spreads fell across a wide range of assets; and distant-horizon forward rates on British and German government bonds fell. The next section considers various interpretations of the facts based on affine bond-pricing models, which allow us to decompose forward rates into term premiums and expected future short-term rates, and then to further parse each into real and inflation components. The models suggest that term premiums declined considerably over the period of interest. The final section considers changes in the macroeconomic environ-

ment that might help explain such changes in term premiums, including a discussion of the explanations mentioned above.

**Facts**

Here we collect a number of related facts to provide a context for the later discussion. We organize them around a series of topics.

**Cyclical Behavior of Interest Rates**

The essence of the conundrum is evident in figure 1. During the three preceding episodes of monetary policy tightening, starting in 1986, 1994, and 1999, the ten-year yield on U.S. Treasurys increased sharply along with the federal funds target. However, as figure 2 shows more clearly, during the first year of the most recent period of tightening, ten-year yields declined. The ten-year forward rate, as already noted, fell 170 bp from June 2004 to June 2005, reaching its lowest point in thirty years in the summer of 2005. The more modest decline in the ten-year yield is
essentially an average of the sharp decline in long-term forward rates and the concurrent increase in short-term rates. We focus much of our analysis on forward rates, since they capture the behavior of interest rates in a particularly clean way. Here and below, yields and forward rates are derived from prices of Treasury securities by the Nelson-Siegel-Svensson method. Data sources and descriptions are given in appendix A.

The recent behavior of forward rates is clearly different from that in the three preceding tightening episodes (figure 1), but what can one say about the cyclical behavior of forward rates more generally? Figure 3 is a scatterplot of monthly ten-year forward rates against the monthly unemployment rate, a convenient indicator of the state of the business cycle. There is a clear positive association between the two variables (the correlation coefficient is 0.64), indicating that long-term forward rates have been countercyclical. The slope of the relationship in the recent past (the gray diamonds) is not very different from what it was before, but the level appears 1½ to 2 percentage points lower.

3. As described in Gürkaynak, Sack, and Wright (2006).
4. Lehman Brothers (2007) includes a similar figure (figure 6).
Real and Nominal Interest Rates

Figure 4 compares the change in the nominal forward rate curve between the end of June 2004 and the end of June 2005, in the top panel, with the analogous change in the real forward rate curve, as measured by rates on Treasury inflation-protected securities (TIPS), in the bottom panel. Evidently the two curves experienced similar shifts, although the nominal curve shifted by a larger amount. Measured at the ten-year horizon, the nominal curve dropped by 172 bp and the real curve by a little more than half that (96 bp). The difference between the two curves reflects compensation for inflation. It is often used as a measure of inflation expectations, but it actually includes both an inflation risk premium and a TIPS liquidity premium. We will return to this later.

Expectations of Inflation and Growth

Figure 5 shows that there was no apparent change in forecasts of inflation, GDP growth, or short-term interest rates in 2004–05 that might account for the drop in long-term forward rates. The data summarized in
the figure are “long-range” (five- to ten-year) forecasts produced by Blue Chip Economic Indicators and consist of averages over a large number of professional forecasters. The steady drop in the forecast of long-term expected inflation over the 1990s coincided with a similar drop in the ten-year yield (compare with figure 1), but there was little or no movement in expected inflation or in expected GDP growth during 2004–05.

Figure 4. Nominal and Real U.S. Treasury Forward Rate Curves, June 2004 and June 2005

Percent a year

Nominal

Real

Sources: Federal Reserve Board and Gürkaynak, Sack, and Wright (2006).
a. Instantaneous ten-year forward rates on non-inflation-protected Treasury securities (nominal) and TIPS (real).
The inflation forecast in the figure is for the consumer price index; the analogous series for the GDP deflator (not shown) is lower throughout but moves similarly through time. The long-range GDP growth forecast increased dramatically in 2000; one might have expected such a rise to be associated with an increase in long-term forward rates, but that did not happen. Similarly, the long-range forecast of the three-month Treasury bill rate has been flat over the last three years. Given this evidence, it is hard to argue convincingly that the recent decline in long-term yields reflects expectations of lower inflation, slower growth, or lower short-term interest rates.

Credit Spreads and Volatility

One striking feature of the recent past has been the unusually narrow spreads on risky instruments. The spread between yields on Baa- and Aaa-rated corporate debt, for example, is an indication of how much the market discounts lower-rated bonds. It reflects a combination of the risk inherent in these bonds and the price applied to such risk. The top panel of figure 6...
shows that the Baa-Aaa spread declined sharply between 2004 and early 2005. The middle panel depicts a measure of interest rate risk: the implied volatility of one-year interest rate caps, which is a market-based measure of short-term volatility of a short-term interest rate (the six-month London interbank offer rate). It, too, fell sharply over the period of interest. Moreover, realized volatility—in this case, the standard deviation of daily changes in forward rates—also fell, as Richard Berner and David Miles have noted.5 The annual standard deviation of one-day changes in the ten-year instantaneous forward rate dropped from 6.8 bp in 2003 to 4.1 bp in 2006. Over the conundrum period, not only did investors expect less interest rate volatility; there was less volatility. The bottom panel of figure 6 shows that the Chicago Board of Exchange’s volatility index, whose price is tied to the short-term (thirty-day) volatility of S&P 500 stock index options, also declined between 2002 and 2005. Together this evidence indicates that financial market risk and risk premiums across a range of assets were substantially lower in 2005 than they had been two or three years earlier.

**Macroeconomic Uncertainty**

The underlying source of the decline in asset market volatility remains unclear. Federal Reserve communications have, in the last few years, given markets more forward-looking guidance on the path of monetary policy, making it more predictable. This might account for a fall in near-term interest rate uncertainty, but its broader impact on asset market volatility is less evident. Macroeconomic uncertainty, particularly at long horizons, is inherently difficult to measure, but the dispersion of long-horizon survey predictions serves as a crude proxy.

The Blue Chip surveys report a simple dispersion measure for their long-horizon survey questions: the difference between the averages of the ten highest and ten lowest responses. Figure 7 shows a time series of this measure for five- to ten-year expectations of consumer price inflation, real GDP growth, and three-month Treasury bill yields. Although somewhat noisy, all three measures have been trending noticeably lower. Note, too, that this period of reduced macroeconomic volatility came well after the start of the so-called Great Moderation, the decline in macroeconomic

David K. Backus and Jonathan H. Wright

Figure 6. Daily Credit Spreads and Interest Rate and Stock Market Volatility, 1985–2007

Baa-Aaa spread

<table>
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<tr>
<th>Percentage points</th>
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<tr>
<td>1.6</td>
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<td>1.2</td>
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<td>0.8</td>
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<td>0.4</td>
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Caps volatility

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<tr>
<td>80</td>
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<td>60</td>
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VIX

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<th>Percent</th>
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<tr>
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<td>30</td>
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<td>20</td>
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<td>10</td>
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|------|------|------|------|

Source: Datastream.

a. Difference in average yields on Baa- and Aaa-rated bonds.
b. Implied volatility of one-year interest rate caps (options on short-term interest rates), measured as the standard deviation of the logarithm of the six-month London Interbank Offered Rate.
c. Chicago Board Options Exchange Volatility Index, an indicator of the implied volatility of the S&P 500 stock index, measured as the standard deviation of the logarithm of this index.
volatility that researchers typically date to the early 1980s. Most of the decline in the dispersion of interest rate and inflation expectations occurred in the early 1990s. They then rose again in the last recession before moving back down over the conundrum period. Perhaps investors believe that macroeconomic uncertainty has declined further, or perhaps they only lately became convinced of the Great Moderation.

**Forward Rates in Germany and the United Kingdom**

The fall in forward rates since mid-2004 is not unique to the United States. Figure 8 plots ten-year forward rates for the United States, Germany, and the United Kingdom. The conundrum is evident in all three, even though they are at different stages of the business cycle. Indeed, forward rates in these countries have been unusually highly correlated over the last two or three years. Randall Kroszner observes that long-term rates have declined even in such middle-income countries as Brazil, Chile, Colombia, Indonesia, Mexico, Russia, and Thailand.\(^6\) Several of these countries now

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\(^6\) Kroszner (2006).
issue local-currency debt at modest yields at maturities at which they did not even issue debt a decade ago.

**Affine Interpretations of Forward Rates**

The recent decline in long-term forward rates, the similar decline in many measures of volatility, and the apparent absence of any change in expectations of inflation and Treasury bill rates suggest to us that smaller term premiums play a role in explaining the conundrum. We would call the evidence suggestive, however, rather than definitive. A complementary source of information is estimates of affine bond pricing models. There are many examples in the literature, not all of them the same, but a collection of models estimated by researchers at the Federal Reserve illustrates their potential to shed light on the behavior of long-term forward rates.

In the context of any bond pricing model, the starting point is a common decomposition of long-term forward rates into expected future short-term rates and term premiums:

\[ f_t^{(n)} = E_t \left( r_{t+n} \right) + \theta_t^{(n)}. \]
Here \( f_t^{(n)} \) is the \( n \)-periods-ahead forward rate at date \( t \), \( r_{t+n} = f_{t+n}^{(0)} \) is the short-term rate \( n \) periods in the future, and \( tp_t^{(n)} \) is a term premium. Equation 1 is essentially a definition of the term premium; it has content only if we have a way to identify the components.

We face two hurdles in turning equation 1 into a usable decomposition. The first is the time-series variability of long-term forward rates. Stationary ergodic models imply that the variance of forward rates converges to zero as the maturity increases. Robert Shiller made this point in a setting with constant term premiums, but it applies more generally.\(^7\) It applies, for example, to most affine models. One way or another, a realistic model must incorporate a high degree of persistence somewhere, so that convergence works slowly enough that forward rates with maturities of (say) ten years still have significant volatility. A related issue is the impact of news on long-maturity forward rates. It is not uncommon for a single macroeconomic news announcement to cause large changes in forward rates for maturities of ten years or more.\(^8\) Whether this reflects changes in expectations of distant future short-term rates or term premiums remains to be seen, but it is clear that persistence is needed in one or the other.

The second hurdle is identifying the two components of equation 1. An older line of research used the expectations hypothesis, which in this context amounts to the assumption that term premiums are constant: they can vary with \( n \) but not with \( t \). In this case all of the movement in long-term forward rates is associated with changes in expected short-term rates. This not only assumes the answer to the decomposition in equation 1, but it conflicts with an enormous body of work documenting the empirical weaknesses of the expectations hypothesis.\(^9\) One issue is the predictability of long-term yields. Under the expectations hypothesis, a steep yield curve implies that short-term interest rates are expected to increase, and so long-term yields should rise. In U.S. data, however, we find the opposite, as Frederick Macaulay noted years ago:\(^10\)

The yields on [long-maturity] bonds should \textit{fall} during a period in which short-term rates are higher than the yields of the bonds and \textit{rise} during a

\(^7\) Shiller (1979).
\(^8\) See, for example, Beechey (2007), Gürkaynak, Sack, and Swanson (2005), and Lu and Wu (2006).
\(^9\) See, among many others, the survey and synthesis in Dai and Singleton (2002).
\(^10\) Macaulay (1938).
period in which short-term rates are lower. Now experience is more nearly
the opposite.

Similarly, excess returns on long-term bonds can be predicted from the
term structure of interest rates, which also contradicts the expectations
hypothesis. All of this evidence implies that term premiums vary with
time, and therefore that the expectations hypothesis is a poor guide to
interest rate dynamics.

Financial models use this predictability to estimate term premiums from
observed bond yields and perhaps other data. They have the potential to
distinguish between changes in term premiums and changes in expected
short-term rates, and they therefore point to the source of the conundrum.
The most popular of these models are affine: they generate yields and for-
ward rates that are linear functions of a vector of state variables, which in
turn follows a linear autoregressive process. We describe below some of
the properties of estimated versions of Gregory Duffee’s “essentially
affine” model, an example of the more general class of affine models char-
acterized by Darrell Duffie and Rui Kan. 11 We describe a one-factor ver-
sion in appendix B.

The Kim-Orphanides Model

Most estimated essentially affine models identify the two compo-
nents of forward rates from the predictability of yields or returns alone,
as described in the previous paragraph. Don Kim and Athanasios
Orphanides add direct information about the dynamics of interest rates
to such a three-factor model. 12 They use professional forecasts of Treas-
ury bill rates to help identify the expected future short-term rate com-
ponent of the forward rate, which allows relatively precise estimation of
the term premium through equation 1. The estimated model is stationary
but very persistent: the half-life of the most persistent factor is thirteen
years, which allows it to generate substantial movement in long-maturity
forward rates.

The results are apparent in figure 9, which plots the components of the
ten-year forward rate implied by the model. This decomposition suggests
that the decline in ten-year-ahead forward rates from June 2004 to June

2005 is nearly fully explained by a declining term premium. Expected future short-term rates edged lower over this period, but the forward term premium declined from 2½ percentage points to 1 percentage point. Since June 2005, forward rates have risen a little, but the term premium has remained at around 1 percentage point. One can likewise decompose long-term yields into components. The term premium component of the ten-year yield is estimated to have fallen from 1⅛ percentage points in June 2004 to a scant 30 bp in June 2005.

Real and Nominal Term Premiums

We can look further into the role of inflation in these developments by incorporating inflation and TIPS rates into the analysis. If one adds the dynamics of inflation to the model, it becomes possible also to price synthetic real bonds and to decompose real forward rates into a real term premium and the expected future real short-term interest rate. Such an exercise does not require TIPS: it is a real term structure model constructed from inflation and nominal interest rate data alone. Don Kim and Andrew
Ang, Geert Bekaert, and Min Wei provide two such models. Both allow for a four-way decomposition of nominal forward rates into the expected future real short-term interest rate, expected future inflation, the real term premium, and the inflation risk premium. The expected nominal rate is the sum of the expected future real short-term interest rate and expected future inflation; the nominal term premium is the sum of the real term premium and the inflation risk premium.

It seems a pity, however, to completely ignore the information in TIPS. TIPS have only been trading since 1997, and evidently they were subject to large liquidity premiums in the early years of trading, whereas the synthetic real bond that we would wish to price would have liquidity comparable to that of a nominal Treasury bond. But the liquidity of TIPS has improved, and the inflation compensation implied by TIPS is now a widely used barometer of inflation expectations, even though inflation compensation is not the same thing as expected inflation.

Motivated by these considerations, Stefania D’Amico, Kim, and Wei augment a real term structure model with TIPS. The model is estimated over the period since 1990. TIPS are treated as missing data before 1999, which is straightforward to handle via the Kalman filter. Also, the yields on TIPS are assumed to equal the sum of yields on the synthetic liquid real bonds and a TIPS liquidity premium. The properties of their specification are illustrated in figure 10, which plots the four components of the ten-year forward rate. The estimates suggest that the real term premium has fallen sharply since June 2004, and the inflation risk premium by a lesser amount. On the other hand, expected future real short-term interest rates have been flat, and expected future inflation has actually risen modestly.

**Accounting**

All in all, given the time variation in term premiums, the facts about risk spreads and volatilities (figure 6), survey expectations (figure 5), and the Kim-Orphanides estimates of the affine model (figure 9), we think that a decline in term premiums is likely to be the principal explanation for the decline in long-horizon forward rates from June 2004 to June 2005. But we

14. D’Amico, Kim, and Wei (2007). We refer the reader to that paper for more details on the specification.
cannot be sure. What we can be sure of is the arithmetic identity that forward rates are the sum of expected future rates and term premiums. If term premiums did not decline, then the long-run expectation of the federal funds rate must have fallen by about 1.7 percentage points. Indeed, there are economic forces that may have pushed expected equilibrium real interest rates down. For example, an increase in the global supply of savings could represent a permanent leftward shift in the IS curve, resulting in lower expected future real short-term interest rates. Or investors may have scaled back their expectations about productivity growth amid weak business investment. Still, during the period in question, economic growth was robust, inflation was drifting upward, and a 1.7-percentage-point decline in expected future short-term interest rates would seem surprising. But arithmetically this is the only alternative explanation to a fall in the term premium. In a similar calculation, on average over the last twenty years, the spread between ten-year forward rates and the federal funds rate has been 2 percentage points, which is an estimate of the average term premium over that period. The ten-year forward rate now stands around 5 percent. This means that either the federal funds rate ten years hence is expected to
be close to 3 percent, or the term premium is below its average over the past twenty years.

**Integrating Macroeconomics and Finance**

Affine models suggest that term premiums have fallen in the recent past, but the absence of macroeconomic structure in these models makes it difficult to say why. Did Federal Reserve policy change? Did macroeconomic risk fall? Are any of the ideas mentioned in the introduction persuasive? We fill the gap with a combination of speculation and casual empiricism.

The first issue is the cyclical behavior of the term premium. A wide range of evidence on asset returns suggests that risk premiums in general are countercyclical, smaller in booms than in recessions.\textsuperscript{15} Since the recent period has been an expansion, one would indeed expect term premiums to have declined. And this helps explain the otherwise puzzling correlation between unemployment and the distant-horizon forward rate shown in figure 3.

A secular decline in asset price volatility could also account for lower risk premiums in general, and lower term premiums in particular.\textsuperscript{16} That in turn raises the question of why asset price volatility fell. But as figure 7 showed, judging from surveys, the dispersion of agents’ long-run macroeconomic expectations, and especially their long-run inflation expectations, has fallen, which is at least suggestive of a decline in uncertainty about long-run inflation rates. This could in turn owe to more credible and transparent monetary policy, in which future interest rate moves are indicated well in advance. The last decade has seen a global trend toward central banks becoming more independent of political influences, providing more explicit information to the public about their decisions (including publishing forecasts), and adopting some form of inflation targeting. Also, greater integration of financial markets may reduce the potential short-term gain from any one country adopting more inflationary policies, which in turn would make a commitment to a low and stable level of inflation

\textsuperscript{15} Cochrane and Piazzesi (2005, 2006) and Ludvigson and Ng (2006) are good examples for bond returns.

\textsuperscript{16} As discussed by Rudebusch, Swanson, and Wu (2006).
more credible. All this might help account for a global decline in inflation uncertainty and hence in inflation risk premiums.

Some evidence for this interpretation is the especially sharp decline in forward rates in the United Kingdom around 1997, when the Bank of England was granted operational independence, as can be seen in figure 8. Arguably, in the last few years investors in the United States and abroad have become persuaded that inflation has been conquered forever—or at least for the relevant time horizon—and so no longer demand large inflation risk premiums. In the United States the Volcker disinflation took place a long time ago, and it is indeed a little hard to see why only in the last three years investors should suddenly have become confident that inflation would remain contained indefinitely. Still, reduced inflation uncertainty amid changes in central bank policy seems to be high on the list of plausible explanations for the global decline in distant-horizon forward rates. Uncertainty about economic growth might also have declined, for a variety of reasons: monetary policy, financial innovation, or inventory management.

Some analysts have suggested that a fall in term premiums may owe to increased demand for longer-duration securities stemming from the prospect that corporate pension reform might encourage pension funds to better match the durations of their assets and liabilities. This seems a plausible story in the United Kingdom, where pension funds must satisfy strict duration-matching requirements. The real yield on fifty-year inflation-indexed government bonds in the United Kingdom fell below 50 basis points in January 2006, which is consistent with special demand from pension funds for this unusually long duration security. Still, the explanation does not make as much sense in the United States, where corporate pension reform has proceeded more slowly.

Another possible explanation is that Asian central banks are buying up Treasury securities, driving the term premium down.\textsuperscript{17} This does not seem satisfying as a complete explanation for the conundrum because, as shown earlier, forward rates in Germany and the United Kingdom exhibited swings similar to those in the United States. But U.S. Treasurys and the bonds of other industrial-country governments may be close substitutes, so that purchases of Treasurys may drive foreign term premiums down. One recent natural experiment strongly suggests that Asian central bank

\textsuperscript{17} See Bernanke, Reinhart, and Sack (2004); Warnock and Warnock (2006).
purchases are part of the story. On July 21, 2005, the Peoples’ Bank of China announced that it was modestly revaluing the renminbi and adopting a new exchange rate regime. The announcement came at 7 a.m. Eastern time and was immediately understood by market participants as implying that China would not need to buy as many Treasurys to hold the value of the renminbi down as had previously been thought. At the time of the announcement, the yield of the on-the-run ten-year Treasury note jumped up about 7 bp. To be sure, 7 bp represents a tiny share of the conundrum, but the revaluation of the renminbi was not large, and so the episode suggests that foreign central bank purchases of Treasury securities have at least played some role. Capital flows from petroleum-exporting countries could have a similar effect.

Some analysts have pointed to demographics as a possible explanation for falling term premiums, as substantial cohorts of the populations of industrialized economies approach retirement. In this explanation, aging populations in industrialized countries are expected to shift their holdings away from risky assets such as equities and toward bonds and other assets perceived to be relatively safe, and these shifts may place some downward pressure on term premiums (and upward pressure on equity risk premiums). However, demographics are slow-moving and predictable, and it can hardly be claimed that there has been a substantial or unexpected shift in demographics since June 2004.

Finally, changes in term premiums need not have any fundamental explanation but could instead be a consequence of irrational behavior on the part of investors. Arguably, the choppy behavior of ten-year forward rates is hard to make sense of in terms of either expected short-term interest rates or risk compensation from any rational equilibrium asset pricing model.

A Regression Model

The affine term structure models introduced in the previous section use a set of latent factors as the state vector. This has the advantage of fitting the data well but the disadvantage of lacking a direct economic interpretation. To get some sense of the ability of some of the economic explanations discussed above to account for the conundrum, we regressed the ten-year forward nominal term premiums as estimated by the model of Kim and
Orphanides on the unemployment rate (to capture the cyclical patterns in term premiums) and the dispersion of Blue Chip long-horizon survey measures of inflation (to proxy the impact of inflation uncertainty on term premiums). The regression was run at the same frequency as the Blue Chip long-horizon survey. The results are as follows:18

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<tr>
<td>Constant</td>
<td>−0.83</td>
</tr>
<tr>
<td>Coefficient on unemployement</td>
<td>0.42</td>
</tr>
<tr>
<td>Coefficient on Blue Chip long-term inflation dispersion measure</td>
<td>0.78</td>
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Greater dispersion of inflation expectations and higher unemployment are both associated with larger term premiums, and both coefficients are statistically significant. The actual and fitted term premiums from this regression are plotted in figure 11. The model is able to explain a good bit of the decline in term premiums since 1990, and even some of the fall in the last

18. The survey is published twice a year, for a total of forty observations from March 1987 to October 2006. Both estimated slope coefficients are statistically significant at the 1 percent level, using Newey-West standard errors.
three years, as the dispersion of inflation expectations has fallen even as the economy has been expanding. Only some of the decline in term premiums since June 2004 can be explained by this model, but it suggests that these factors may be at least part of the explanation.

Conclusion

We think the evidence points to a declining term premium as the primary source of the recent fall in long-term forward rates. This interpretation is broadly consistent with observed changes in risk spreads, interest rate and stock market volatility, the dispersion in long-range forecasts, and estimates of (some) affine bond-pricing models. In contrast, long-range forecasts of inflation, growth, and short-term interest rates provide little reason to believe that they might account for the same decline in long-term forward rates. In this sense we follow a long line of work in suggesting that the expectations hypothesis intuition, based on constant term premiums, is likely to be misleading not only in this case but more generally.

The next step, in our view, should be to develop models in which macroeconomic policy and behavior can be tied more directly to the properties of interest rates. One might want to know, for example, whether changes in the volatility of output or in the nature or communication of monetary policy had an impact on the behavior of long-term rates in the recent past. Neither hypothesis can be tested with existing models. Ang and Monika Piazzesi show how macroeconomic variables influence the behavior of bond yields, but this and related work has little in the way of macroeconomic structure. Bekaert, Seonghoon Cho, and Antonio Moreno, as well as Michael Gallmeyer, Burton Hollifield, and Stanley Zin, have made progress on setting up affine term structure models in which the short-term interest rate is related to inflation and output growth by a monetary policy rule, while inflation and output growth are in turn determined within a structural New Keynesian model. Perhaps this will lead to further advances that allow the next attack on this issue to bring the macroeconomic contributions in line with the financial contributions—to the benefit of both.

APPENDIX A

Data Sources and Definitions

U.S. interest rates in this paper are estimated from quoted prices of Treasury securities. Estimates of nominal yields and forward rates are constructed from a smooth Nelson-Siegel-Svensson curve as described by Gürkaynak, Sack, and Wright (2006). The data are available on the Federal Reserve website. A similar curve (estimated by Federal Reserve staff) is used to estimate yields and forward rates from TIPS. Throughout, yields are understood to be continuously compounded rates on zero-coupon bonds. Forward rates are instantaneous.

U.K. and German forward rate data were provided by the Bank of England and the Deutsche Bundesbank, respectively, and are available on their public websites. In the case of the German data, forward rates are constructed from Nelson-Siegel-Svensson parameters provided by the Bundesbank. Other macroeconomic and financial data are taken from Datastream.

Term premium estimates were calculated by Federal Reserve staff using the models of Kim and Orphanides (2005) and D’Amico, Kim, and Wei (2007). Those using the Kim and Orphanides model are updated quarterly, and daily estimates from 1990 on are available to the public at federalreserve.gov/pubs/feds/2005/200533/feds200533.xls.

APPENDIX B

Affine Models

Modern asset pricing theory works as follows: in an arbitrage-free setting, there exists a pricing kernel \( m_{t+1} \) that relates the market price \( v_t \) at date \( t \) of a claim with value \( c_{t+1} \) at date \( t + 1 \):

\[
E_t \left( m_{t+1} c_{t+1} \right).
\]

(B.1)

Bond pricing is a particularly elegant application. If \( b_t^{(n)} \) is the price at date \( t \) of one unit (a dollar, say) paid at \( t + n \), bond prices can be computed recursively from

\[
b_t^{(n+1)} = E_t \left( m_{t+1} b_{t+1}^{(n)} \right),
\]

starting with \( b_t^{(0)} = 1 \).
Affine models put enough structure on the pricing kernel that prices are log-linear functions of a state vector $z$:

$$-\log b^{(n)}_t = A_n + B^n_n z_t,$$

for some parameters $\{A_n, B^n_n\}$. In essentially affine models, the link between forward rates and the dynamics of $z$ is particularly transparent. Consider a one-dimensional example:

$$-\log m_{t+1} = \left[ \lambda(z_t) \right]^2/2 + z_t + \lambda(z_t) \varepsilon_{t+1},$$

where

$$\lambda(z_t) = \lambda_n + \lambda_i z_t,$$

and

$$z_{t+1} = (1 - \varphi) \theta + \varphi z_t + \sigma \varepsilon_{t+1}.$$

Here $z$ is a (scalar) state variable, $\varepsilon$ is an i.i.d. normal random variable with zero mean and unit variance $\sigma > 0$, and $0 < \varphi < 1$. The last equation describes the dynamic behavior of the state variable: it follows a linear process with the autoregressive parameter $\varphi$. The “price of risk” $\lambda(z)$ governs risk premiums; in particular, $\lambda_i$ controls the sensitivity of the term premium to movements in $z$.

The key to the solution is that the model is linear in the right places. Equation B.2 and our guess (equation B.3) imply that the coefficients satisfy

$$A_{n+1} = A_n + B_n (1 - \varphi) \theta + B_n \sigma \lambda_n - (B_n \sigma)^2/2$$

$$B_{n+1} = 1 + (\varphi + \sigma \lambda_i) B_n$$

starting with $A_0 = B_0 = 0$. If we set $\varphi^* \equiv \varphi + \sigma \lambda_i$, the second can be solved directly:

$$B_n = \left[ 1 - (\varphi^*)^n \right]/(1 - \varphi^*).$$

The first we compute recursively.

The beauty of this solution lies in the difference between $\varphi$ and $\varphi^*$. The former represents the dynamics of the state variable, the latter the “risk-neutral” (or risk-adjusted) dynamics that are built into long-term forward...
rates and term premiums. If $\lambda_i = 0$, the two are the same and we are back in the world of the expectations hypothesis: term premiums are constant and all the variation in forward rates comes from expected future short-term rates. In general, however, they are different, and their difference is what generates time-varying term premiums. To be specific, let us define one-period (as opposed to instantaneous) forward rates by

\[
\log\left(\frac{b_i^{(n)}}{b_i^{(n+1)}}\right) = f_i^{(n)}.
\]

Then

\[
f_i^{(n)} = (A_{n+1} - A_i) + (B_{n+1} - B_z)z_i = (A_{n+1} - A_z) + (\varphi^*)^n z_i,
\]

and $r_i \equiv f_i^{(0)} = z$ (the short-term rate is $z$).

In words: the sensitivity of forward rates comes from the risk-neutral dynamics. The expected future short-term rate, in contrast, follows from the true dynamics, represented here by $\varphi$:

\[
E_i(r_{n+1}) = (1 - \varphi^*)\theta + \varphi^* z_i.
\]

The term premium is therefore

\[
\text{tp}_i^{(n)} = \text{constant} + \left[ (\varphi^*)^n - \varphi^* \right] z_i,
\]

which evidently depends on the difference between the two kinds of dynamics. The term premium component is more sensitive to movements in $z$ (hence more variable) if

\[
\left[ (\varphi^*)^n - \varphi^* \right] > \varphi^n.
\]

This holds for large $n$ if $\varphi^* > \varphi$.\]
Comment and Discussion

Glenn D. Rudebusch: This paper by David Backus and Jonathan Wright examines a timely topic of interest to macroeconomists, financial economists, and the general public of long-term savers and investors. They investigate the recent episode of continuing low long-term interest rates—a behavior that appears to some to be a “conundrum” given that short-term rates worldwide have been rising. For example, in the United States, while the Federal Reserve raised the federal funds rate from 1 percent in June 2004 to 5 1/4 percent in December 2006, the rate on ten-year U.S. Treasury notes actually edged down, on balance, from 4.7 percent to 4.6 percent. This directional divergence between short- and long-term rates is at odds with historical precedent and appears even more unusual given other economic developments at the time, such as a solid economic expansion, a falling unemployment rate, rising energy prices, and a deteriorating federal fiscal situation, all of which have been associated in the past with higher long-term interest rates rather than lower.

Of course, determining whether recent long-term interest rate movements truly represent a puzzle requires a theoretical framework that takes into account the various factors that affect long-term rates. The paper takes a joint macro-finance perspective on this problem, which, as much recent research suggests,1 is a promising strategy that can capture two broad sets of determinants of long-term rates. In particular, from a macroeconomic perspective, the short-term interest rate is a policy instrument under the direct control of the central bank, which adjusts that rate to achieve its macroeconomic stabilization goals. Therefore financial market participants’ understanding of central bank behavior, along with their views of

1. See, for example, Diebold, Piazzesi, and Rudebusch (2005).
the future direction of the economy, will be an important element in forming their expectations of future short-term rates, which, in turn, will be key in pricing longer-term bonds. For example, the widespread view over the past few years that the Federal Reserve had inflation pretty well in hand has undoubtedly helped hold down long-term bond rates. In addition, a finance perspective, which stresses the importance for bond pricing of investor perceptions of risk, is also likely to be a crucial element in assessing whether there is any bond rate conundrum. Indeed, many have suggested that a reduction in the risk premium is responsible for recent low bond rates. Such a reduction may be attributable to changes in the amount of risk or to changes in the pricing of that risk, and numerous factors have been suggested that could have induced such changes.

The paper identifies a declining term premium, and in particular a declining inflation risk premium, as the proximate source of the recent fall in long-term interest rates. This seems reasonable, but in some sense one can interpret the authors’ analysis as showing that there has been no conundrum. The estimated term structure models in the paper seem to fit the recent episode as well as the earlier sample (their figures 9 and 10); that is, the recent episode is not so puzzling that it requires a shift in model coefficients or produces unusually large residuals. The analysis in the paper has essentially defined the “conundrum” away. Of course, such a conclusion would require that the authors had the correct model of interest rates and the term premium, which is far from certain. It seems useful to examine other term structure representations in order to evaluate the robustness of the authors’ conclusions; therefore my figure 1 plots five different measures, taken from the literature, of the term premium in the zero-coupon nominal ten-year U.S. Treasury yield:

—VAR measure: This is obtained from a standard, three-variable, macroeconomic vector autoregression (VAR), comprising four lags each of the unemployment rate, quarterly inflation in the consumer price index, and the three-month Treasury bill rate. This VAR can be used in each quarter to forecast the short-term rate over the next ten years, which, after averaging, provides one estimate of the risk-neutral ten-year rate. The difference between the observed ten-year rate and that risk-neutral rate provides an estimate of the term premium.

2. These measures are described in detail in Rudebusch, Sack, and Swanson (2007).
—Bernanke-Reinhart-Sack measure: A potential shortcoming of using a VAR to estimate the term premium is that it does not impose consistency between the yield curve at a given point in time and the VAR’s projected time path of yields over time. Such pricing consistency is imposed in the model of Ben Bernanke, Vincent Reinhart, and Brian Sack (BRS), which attaches a no-arbitrage model of the term structure to a VAR and provides an estimate of the term premium.

—Rudebusch-Wu measure: No-arbitrage restrictions can also be imposed on top of a New Keynesian macroeconomic model, as in the model of Rudebusch and Tao Wu (RW), which provides another estimate of the term premium.

—Kim-Wright measure: Don Kim and Wright estimate the term premium using a standard, no-arbitrage, dynamic latent factor model from finance (with no macroeconomic structure underlying the factors). In models of this kind, risk-neutral yields and the term premium are determined

by latent factors that are themselves linear functions of the observed bond yield data. This measure is the closest to the versions considered in the present paper.

—Cochrane-Piazzesi measure: John Cochrane and Monika Piazzesi analyze one-year-holding-period excess returns for a range of Treasury securities. Their primary finding is that a single factor—a particular combination of current forward rates—predicts a considerable portion of these excess returns for Treasury securities. These, in turn, together with the one-year risk-free rate, imply an expected set of zero-coupon yields one year ahead (since the only way to generate expected returns on zero-coupon securities is through changes in yield). By iterating forward, one can compute the expected excess return for each of the next ten years, thereby yielding a measure of the term premium on the ten-year security.

All five of these measures of the term premium show declines over the past few years and are generally consistent with the conclusions of Backus and Wright. However, these various measures also illustrate the considerable uncertainty that should be attached to any measure of the term premium. This caveat is worthy of elaboration; therefore I next consider in detail the implications of the results of the BRS and RW models for the recent behavior of long-term rates.

My figure 2 shows the ten-year zero-coupon U.S. Treasury yield from 1984 through 2006 together with the BRS model decomposition of that yield. The risk-neutral rate implied by the BRS model is the model’s estimated yield on a riskless ten-year zero-coupon bond, the implied ten-year Treasury yield is the model’s estimated yield on the same bond after accounting for risk, and the implied term premium is the difference between the two. The BRS model does not match the data perfectly, and so the model’s residuals—the difference between the model predictions taking into account risk and the data—are graphed in figure 3. Despite the model’s excellent fit to the data overall, the recent period of low ten-year yields is one episode that the model notably fails to fit. From mid-2004 through the end of 2006, the model overestimates the ten-year Treasury yield by around 50 basis points on average. Figures 4 and 5 present the analogous pair of graphs for the ten-year bond yield decomposition implied by the RW model. Again the fit of the model to the data is excel-

7. This discussion is based on the analysis of Rudebusch, Swanson, and Wu (2006).
Figure 2. Decomposition of the Ten-Year Treasury Yield, 1984–2006: Bernanke-Reinhart-Sack Model

Figure 3. Unexplained Portion of the Ten-Year Treasury Yield, 1984–2006: Bernanke-Reinhart-Sack Model

Source: Rudebusch, Swanson, and Wu (2006).

Figure 4. Decomposition of the Ten-Year Treasury Yield, 1988–2006: Rudebusch-Wu Model

Percentage points

Source: Rudebusch, Swanson, and Wu (2006).


Figure 5. Unexplained Portion of the Ten-Year Treasury Yield, 1988–2006: Rudebusch-Wu Model

Basis points

Source: Rudebusch, Swanson, and Wu (2006).
lent, which is all the more remarkable given that the RW model was not optimized to fit the ten-year yield at all (indeed, the five-year yield is the longest maturity used in the estimation, and the estimation sample ends in 2000).

The two models’ implied decompositions into the expected short-term rate and the term premium are very different. In the RW model the term premium is relatively constant over 1988–2006, hovering around the 2 percent level, with little high-frequency variation but a notable cyclical movement. Furthermore, the RW model attributes most of the variation in the ten-year yield over time to changes in the expected future path of short-term rates. By contrast, the BRS model attributes most of the high-frequency variation in the ten-year yield to changes in the term premium component, with the risk-neutral component generally trending smoothly downward. The differences between the estimates of the ten-year term premium in these two models largely reflect different assumptions about the long-run persistence of movements in the nominal short-term rate. The BRS model is based on estimates of a macroeconomic VAR that is specified in levels. The smoothly downward-trending risk-neutral rate from the BRS model is essentially a VAR projection of the future path of the short-term rate that reverts to its sample mean fairly quickly. In the RW model the future path of short-term interest rates is instead affected greatly by highly persistent changes in the perceived value of the central bank’s target for inflation, which allows significant variation in the risk-neutral ten-year yield. Obviously, then, estimates of the ten-year term premium are very sensitive to assumptions about the long-run properties of the short-term rate. However, such long-run properties are difficult to determine, and so, once such specification uncertainty is accounted for, the confidence intervals associated with any term premium estimate are quite substantial.

Still, despite their differences, the BRS and RW models are largely in agreement that there is, in fact, a conundrum. Specifically, the recent level of long-term bond yields is substantially lower, on the order of 30 to 80 basis points, than can be explained by either of these models. Of course, documenting the conundrum, especially as a sequence of large residuals of the same sign, is only one step toward explaining it. Rudebusch, Eric Swanson, and Wu examined several popular explanations for the conun-
Figure 6. S&P 500 Volatility Index (VIX) and Ten-Year Term Premium, 2007

Source: Federal Reserve Board and Chicago Board Options Exchange.

drum by regressing the BRS and RW macro-finance models’ residuals on various proxies for uncertainty or volatility, and they found that most of the conundrum remained unexplained. Backus and Wright have to take a somewhat different strategy because their finance yield-curve representations are so flexible that the residuals in fitting yields are apparently minuscule. Therefore they employ a two-step strategy, regressing an estimate of the term premium on macroeconomic variables, and it is only in this final regression that a bond yield conundrum can be said to emerge. Specifically, the large recent residuals from their second-stage regression, which are apparent in their figure 11, suggest that if the authors have the correct term structure model, then the conundrum is an unusually low term premium relative to macroeconomic fundamentals.

It is important to stress that even if the explanatory power of these second-stage regressions had been higher, we are still some distance away from a model that integrates the various explanations of recent low long-term rates into an underlying asset pricing model. Only such a unified

one-step formulation can provide a complete and compelling accounting of the recent episode. To give some sense of how much we do not understand, figure 6 plots daily data on the Kim-Wright term premium along with the VIX measure of implied volatility from options on the S&P 500 index as a measure of uncertainty in the stock market. Backus and Wright note that the decline in the VIX measure of financial market risk is broadly consistent with the fall in the term premium. Unfortunately, as the daily data show, this correlation does not always hold. On February 27 of this year, when financial markets were shaken by surprising news on durable goods orders and drops in Chinese equity prices, the implied volatility jumped, but the term premium fell. Of course, one could speculate that although financial risk had clearly risen, the price attached to that risk fell even more as funds flowed into the bond market in a “flight to quality.” However, such speculation shows that much remains unknown about movements in the term premium and in bond rates more generally.

**General discussion:** Benjamin Friedman called attention to the decomposition, in the authors’ figure 10, of the nominal forward rate into expected inflation and an inflation risk premium as well as the expected future real interest rate and a real term premium. He found it striking that the estimate of expected inflation had gone up rather than down with the advent of the present Federal Reserve chairman, noting that it was inconsistent with the common explanation of the long-term yield conundrum, namely, that market participants find credible the assurances that inflation is going to remain in check over the long term. The rise in expected long-run inflation should be particularly surprising to those economists who believe that an explicit inflation target should keep inflation expectations in check. To Friedman this evidence, which suggests that market participants are not convinced that inflation will remain low, made the current low level of nominal long-term rates even more puzzling.

Gregory Mankiw remarked that the paper did not so much crack the forward rate conundrum as recharacterize it as a term premium conundrum. He suggested that the next step is to explain the behavior of such premiums. He also suggested that if they are to be interpreted as risk premiums, they should be related to some measure of risk. He reminded the panel of a Brookings paper he had presented twenty-one years ago, in which he had tried but failed to find a relationship between term premiums and the risks captured in second moments.
Mankiw was also surprised by the authors’ finding of an empirical relationship between forward rates and dispersion in expectations. He cautioned against interpreting this as evidence of a risk premium, noting that dispersion is a measure of disagreement, not a measure of uncertainty. For example, all would agree that the expected outcome of a roll of dice is seven even though the actual outcome is quite uncertain, ranging from two to twelve. Disagreement must have something to do with varying information sets or different interpretations of similar information. Unfortunately, economics lacks good models of disagreement.

Richard Cooper regarded the label “term premium” as misleading and drew parallels to the panel’s discussion of the paper by Oliner, Sichel, and Stiroh in this volume, where some had criticized what they saw as the reification of the productivity residual. The label “term premium” simply denotes the unexplained residual from the estimation and should be referred to as such.

Eswar Prasad suggested looking at asset markets other than the market for U.S. Treasury securities to learn about the importance of Asian central banks’ behavior in holding down long-term rates. He noted that the Chinese authorities are putting similar amounts of money into agency bonds, such as Ginnie Maes, as into Treasury bonds. Although there is no good measure of term premiums for agency bonds, Prasad believed it could be informative to compare the behavior of those markets with that of the Treasury market. He observed further that if one assumes U.S. and foreign industrial bonds to be close substitutes, any future announcement of a revaluation of China’s currency should have had effects on those markets as well. An asymmetric response of term premiums on U.S. and foreign bond markets to the announcement that the Chinese central bank intends to diversify away from dollars into other currencies would provide further evidence of the importance of foreign bank behavior to U.S. interest rates.
References


