

BENJAMIN M. FRIEDMAN  
*Harvard University*

DAVID I. LAIBSON  
*London School of Economics*

# *Economic Implications Of Extraordinary Movements In Stock Prices*

MOST PEOPLE AGREE that stock prices sometimes behave in strange ways. Going beyond this simple observation typically proves more difficult.

For at least the past quarter century, economists have been well aware that the variation of stock prices does not nicely match the familiar bell-shaped normal distribution.<sup>1</sup> The problem is too many extreme movements. Very large increases or decreases would always be possible even if changes in stock prices were normally distributed, but they would occur only rarely. By contrast, actual stock prices rise or fall by large percentage amounts fairly often—certainly often enough to raise serious doubts that the usual normal distribution provides a useful way to think about how they vary.

Economists and other analysts of the stock market have tended to react to this problem in either of two ways. The most common approach is simply to ignore it and go ahead to analyze changes in stock prices as

The authors are grateful to Daniel Kessler, Kenneth Kuttner, Andrew Lo, Robert Merton, Vance Roley, Neil Shephard, and James Stock for helpful discussions. Friedman's research was supported by the National Science Foundation, the General Electric Foundation, and the Harvard Program for Financial Research. Laibson's research was supported by the Financial Markets Group of the London School of Economics and the Marshall Aid Commemoration Commission.

1. Application of one of the many central limit theorems is often used as motivation for the normal distribution. Standard references that describe the nonnormality of stock returns are Mandelbrot (1963) and Fama (1965).

if they did fit the normal distribution. Whether proceeding this way is useful clearly depends on just how far the reality of stock price variation is from the normal distribution, as well as on the use to which the results of the investigation are put. The second reaction is to characterize stock prices by some alternative distribution consistent with a greater frequency of large movements than under the normal distribution.<sup>2</sup> One drawback to this approach is that it sacrifices the convenient simplicity that makes most forms of analysis based on the normal distribution so attractive in the first place. Another is that no consensus exists on how best to model the nonnormality in equity returns.

The chief contention of this paper is that extreme movements in stock prices are potentially important, both in practical stock market contexts and for understanding how the economy behaves, and that failing to take explicit account of the fact that such extraordinary movements have occurred from time to time in the past—and can occur at any time in the future—is therefore a serious omission. In particular, the paper illustrates the potential importance of very large stock price movements by two examples—one bearing on the role of the stock market (and of speculative asset markets in general) in allocating the economy's resources and one bearing on how what happens in the market for stocks (and other financial assets) influences fluctuations in macroeconomic activity.

Whether the stock market serves as an efficient mechanism for allocating scarce capital resources is a long-standing issue central to the modern private enterprise system. Prices set in the stock market determine the actual cost of new capital for firms that issue shares and, much more important for the United States, the opportunity cost of capital accumulated by firms that retain at least part of their earnings. The basic rationale for an economy's allocating capital in this way is the presumption that, both in the aggregate and at an individual firm level, the prices set in the stock market are "efficient" in the sense that they embody all available relevant information—or at least more such information than any alternative capital allocation mechanism could bring to bear.<sup>3</sup> Not surprisingly, an enormous empirical literature has developed around

2. Both Mandelbrot and Fama suggested that stock returns are well characterized by the stable Paretian distribution. Press (1967), Clark (1973), and others advocated a mixed jump-diffusion process. More recently Bollerslev, Engle, and Woolridge (1988) have modeled asset returns with an ARCH process.

3. A standard reference is Baumol (1965).

this subject, relying on a variety of procedures to test whether stock markets really are efficient in this sense.

In recent years, many such tests of market efficiency have turned on whether the returns to holding stocks exhibit volatility that changes over time, and even more important, whether changes in the volatility of stock returns are persistent in the sense that greater or lesser volatility observed at any given time implies correspondingly greater or lesser volatility for at least some interval thereafter. If shocks to volatility are persistent, then movements in the returns required to render the demand for stocks equal to the outstanding supply will also be persistent, so that equilibrium asset prices will tend to fluctuate much more dramatically than most standard models predict.<sup>4</sup> Empirical analysis presented in this paper, based on an explicit distinction between ordinary and extraordinary movements in stock prices, provides an explanation for the consistent failure of past research to find evidence of long-term persistence in the volatility of equity returns.

Hyman Minsky's "financial instability hypothesis" provides an illustration of the potential importance of extraordinary stock price movements for overall fluctuations in the economy.<sup>5</sup> Minsky has long argued not only that financial crises play a central role in causing fluctuations of real economic activity, but also that, as time passes after a financial crisis, behavior changes in such a way as to reduce the financial system's ability to withstand shocks without sustaining some kind of rupture, and hence in such a way that the likelihood of the next financial crisis increases over time. Although Minsky's hypothesis is typically stated with less than explicit grounding in the theory of economic behavior, the analysis presented in this paper shows that when the fluctuation of stock (or other asset) prices includes both an ordinary and an extraordinary component, each with about the same dimensions as have prevailed in the United States since World War II, behavior consistent with the Minsky hypothesis can follow as a result of risk-averse investors continually using the limited information available to them to assess the market's future prospects and allocate their portfolios accordingly.

Because the Minsky hypothesis is clearly about more than just how investors allocate their portfolios between stocks and other assets (at

4. See, for example, Poterba and Summers (1986).

5. See, for example, Minsky (1972, 1977).

the very least, it is about the choice of liabilities as well as assets), the connection between it and the view of extraordinary stock price changes advanced here is obviously illustrative rather than direct. The point is simply that conceptualizing risk in the way suggested in this paper—that is, as consisting of an ordinary and an extraordinary component—can readily explain behavior of the kind hypothesized by Minsky to underlie the irregular occurrence of financial crises with major negative effects on nonfinancial economic activity. In the highly simplified model used below to demonstrate this point, the risk associated with holding stocks is the only form of risk considered and hence is a metaphor for the far wider range of financial and business risks included in Minsky's rich descriptions.

The paper begins by briefly reviewing stock price movements in the United States, both since World War II and earlier, and then developing the basic representation of stock price movements in terms of an ordinary and an extraordinary component. The data presented show that the familiar finding of too many extreme price movements to fit the normal distribution emerges regardless of the period chosen. Moreover, over the postwar period these extreme movements overwhelmingly consist of market crashes, not rallies. The model introduced to represent this process uses some simplifying assumptions to identify the magnitude and timing of the movements that it is possible to regard as extraordinary. As it turns out, all of these extraordinary movements since World War II have been price declines. Further, each of the crashes pinpointed in this way is a familiar episode in market history, and, except for the one in 1987, each coincided with some independent event that potentially could have caused it. Surprisingly, the estimated magnitude of the extraordinary crash component is identical in each of these episodes.

The next section of the paper illustrates the implications of this two-part representation of stock price movements for the question of volatility persistence (and, ultimately, market efficiency). The analysis here relies in part on the familiar ARCH model developed by Robert Engle.<sup>6</sup> But it also introduces a more robust form of this model—MARCH (for “modified ARCH”)—designed specifically for this purpose. The resulting estimates shed additional light on the time series properties of equity returns, including in particular the question of persistence of volatility.

6. See Engle (1982); Engle, Lilien, and Robins (1987); and Bollerslev, Engle, and Woolridge (1988).

The last section takes up the Minsky hypothesis to illustrate the relevance of extreme movements in stock prices to fluctuations in nonfinancial economic activity. Using empirical estimates of the extraordinary component of stock price changes analogous to those presented in the first section of the paper, the analysis shows how the behavior of investors trying to allocate their portfolios as best they can, using whatever information they have available at any time, can cause the overall financial system to become more fragile with the passing of time after the most recent market crash, just as Minsky has suggested. Especially in this context, however, it is appropriate to question a central assumption maintained (purely for convenience) throughout this paper—specifically, that the ordinary and extraordinary components of stock price changes occur independently of one another. The paper concludes with a brief discussion of the interesting, though difficult to investigate, possibility that the two may be related in an important way.

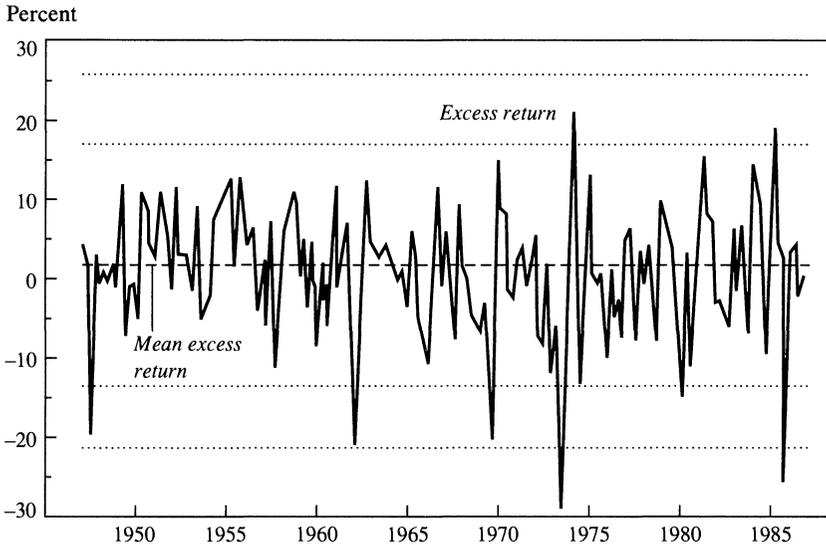
### **Stock Returns: Ordinary and Extraordinary**

Since the end of World War II, the average pretax return on stocks traded in U.S. markets has been positive in 28 years and negative in 15 years. The average rate of return during 1946–88, measured by the Standard and Poor's 500, was 12.62 percent a year. Compared with 4.75 percent a year for Treasury bills, the average excess return on stocks during 1946–88 was 7.86 percent a year.<sup>7</sup>

Figure 1 plots the excess returns on stocks over Treasury bills for 1946–88 using the quarterly time unit that is standard for most macroeconomic analyses of the postwar era. Two features of the data stand out. First, in several quarters stock prices moved far enough—either up or down—to render the total excess return very large or very small compared with the usual range of variation.<sup>8</sup> More specifically, the kurtosis of the excess returns series is 1.41, statistically significant at the

7. All returns data are computed from Ibbotson and Associates (1989).

8. Because dividends tend to move so much more smoothly than stock prices, the identification of extraordinary returns and extraordinary price changes, as implicitly maintained throughout this paper, is entirely legitimate. For the post-World War II data analyzed below, the quarterly standard deviation of the total return series for equities is 0.0770. The standard deviations of the underlying price change and dividend series are 0.0761 and 0.0036, respectively.

**Figure 1. Quarterly Excess Returns for Stocks, 1946–88<sup>a</sup>**

Source: Ibbotson and Associates (1989).

a. Dotted lines are drawn at distances of two and three standard deviations from the sample mean.

0.01 level under the maintained hypothesis that returns are independently drawn from an identical (that is unchanging) normal distribution (i.i.d.).<sup>9</sup> Hence, stock returns are leptokurtotic, meaning that the tails of the distribution have too many extreme observations to fit the normal distribution. The 22.6 percent *one-day* decline in stock prices on October 19, 1987, was unique, but from the perspective of a quarterly time frame the 1987:4 episode was merely one of several unusually large rallies or crashes.

Second, at least since World War II these unusually large movements have more often been crashes than rallies. Of the eight quarters during 1946–88 in which the excess return on stocks differed from the postwar mean (equal to 1.87 percent a quarter) by more than two standard deviations (equal to 15.65 percent a quarter), two saw positive returns and six negative. Moreover, the six negative excess returns were, on balance, somewhat more extreme than the two positive ones. The only two quarters in the entire postwar period to see excess returns more than *three* standard deviations from the mean were the crashes in 1974:3

9. Kurtosis statistics are measured in excess of three. According to this standardization a normally distributed variable has zero kurtosis.

and 1987:4. As a result, the skewness of the entire series (which would be zero if excess returns were normally distributed) is  $-0.68$ , significantly different from zero at the 0.01 level under the maintained hypothesis of i.i.d. normality.<sup>10</sup>

U.S. stock returns from before World War II likewise exhibit leptokurtosis, although not negative skewness. Table 1 summarizes the relevant data by showing the first four moments of stock returns, computed not just for the postwar period but also for various samples throughout the nineteenth and twentieth centuries. These calculations rely on excess return data for the S&P 500 for 1926–88 and total return data compiled by William Schwert for 1802–1987.<sup>11</sup>

The finding of significant leptokurtosis—too many extreme observations to fit the normal distribution—appears regardless of the period chosen. Apparently, U.S. stock returns have always been subject to occasional extraordinary movements. By contrast, the skewness of the series is positive, sometimes significantly so and sometimes not, for periods beginning before World War II.<sup>12</sup>

This historical record provides ample grounds for doubts about the standard representation of stock returns in terms of a single normally

10. Monthly data for 1946–88 likewise exhibit leptokurtosis (1.96) and negative skewness ( $-0.35$ ), where again both are statistically significant at the 0.01 level. These statistics cannot be compared directly with the quarterly statistics because of the effect of aggregation. It is interesting, however, that the leptokurtosis and skewness of the quarterly series are each greater in magnitude than what would follow from the leptokurtosis and skewness of the monthly data if months were aggregated into quarters purely at random. The contrast suggests that there is some tendency for individual months with large negative observations to bunch together.

11. See Schwert (1989). Because the variation of stock returns dominates these data, there is little difference between the properties of Schwert's total return data and those of the S&P excess return data for periods in which the two overlap.

12. The finding of significant positive skewness in the 1926–88 S&P series and the 1900–87 and 1802–1987 Schwert series is a reflection of two huge rallies during the Great Depression of the 1930s. After the October 1929 crash, the stock market fell throughout much of 1930, 1931, and the first half of 1932, but then it rallied sharply in the summer of 1932. From June 30 to September 30 the total return on the S&P was 84.7 percent (not annualized). After again declining later that year and in early 1933, the market rallied even more sharply during the famous first "100 days" of the Roosevelt administration. From March 31 to June 30 the market rose 88.6 percent (again, not annualized). These two positive returns are clearly extreme compared with the remainder of the history of U.S. stock returns. The next largest quarterly return on the S&P index during 1926–88 was 38.4 percent, in 1938:2, and the lowest return on the S&P index during this period was  $-37.9$  percent, in 1932:2 (just before that year's summer rally). In Schwert's data for 1802–1925, the largest quarterly return was 28.7 percent, in 1900:4, and the lowest was  $-19.2$  percent, in 1854:3.

**Table 1. Characteristics of Quarterly U.S. Stock Returns, Various Periods, 1802–1988**

<i>Sample</i>	<i>Mean</i>	<i>Standard deviation</i>	<i>Skewness</i>	<i>Kurtosis</i>
<i>S&amp;P excess returns<sup>a</sup></i>				
1946–88	0.0187	0.0783	–0.68 <sup>b</sup>	1.41 <sup>b</sup>
1954–88	0.0170	0.0814	–0.63 <sup>b</sup>	1.24 <sup>b</sup>
1926–88	0.0221	0.1232	2.16 <sup>b</sup>	17.13 <sup>b</sup>
<i>Schwert total returns</i>				
1801–1987	0.0220	0.0893	1.68 <sup>b</sup>	18.43 <sup>b</sup>
1900–1987	0.0278	0.1089	1.82 <sup>b</sup>	16.46 <sup>b</sup>
1802–1899	0.0167	0.0665	0.13	1.73 <sup>b</sup>

Sources: Data on S&P returns from Ibbotson and Associates (1989), exhibit 20. Schwert data from Schwert (1989).

a. Excess returns on stocks over Treasury bills.

b. Significant at 0.01 level against the null hypothesis of i.i.d. normality. No results were significant at the 0.10 or 0.50 level.

distributed random variable. An alternative that is only modestly more complex, and yet potentially consistent with observed market movements, is to represent stock returns as the sum of *two* elements: first, in each quarter a component that is normally distributed with given mean and given variance (and neither skewness nor leptokurtosis), and, second, an additional return component that is realized only on occasion, independently of the first component, and with different mean and variance.

In general, there is no reason why over time the first of these two components, the one present every quarter, may not exhibit serial correlation, time-varying volatility (with or without persistence over time), or any of the other characteristics that have been the focus of so much attention in the literature studying the time series properties of returns on stocks and other assets.<sup>13</sup> By contrast, because what makes the second component relevant in the first place is that it occurs in an explicitly irregular way over time, the underlying motivation of this two-part representation suggests that the second component be serially uncorrelated.

The implications of this two-part representation of stock returns, both for investor behavior and for consequent macroeconomic outcomes, will clearly depend on the frequency and the pattern over time—actually, the lack of pattern over time—characterizing the appearance of the

13. See, for example, Bollerslev, Engle, and Woolridge (1988); Fama and French (1988); and Poterba and Summers (1988).

second random element. Those features presumably depend in turn on the nature of whatever phenomenon is behind these occasional events. Are they the market's reflection of important but unexpected developments in the economic or the political sphere? Are they merely the bursting of market "bubbles" of the sort that Olivier Blanchard and Mark Watson, Kenneth West, and other researchers have shown can arise even when all market participants are fully "rational"?<sup>14</sup> Or, still further from any underlying fundamentals, do they result from what Robert Shiller has called "fads and fashions," pursued by investors who influence one another in ways that resemble the incidence of infectious disease epidemics?<sup>15</sup> In the absence of some firm basis for identifying the substantive content of these extraordinary events, any qualitative characterization of their occurrence over time is bound to be arbitrary.

One potentially useful device for characterizing events that occur at irregular intervals is the Poisson distribution. A Poisson random variable can take on any nonnegative integer value. The Poisson distribution is commonly used to model processes that involve a count, like the number of incoming telephone calls to a switchboard per unit of time, or the number of meteorite craters on the surface of a planet per unit of area. A characteristic of the Poisson distribution that is especially important in the specific context of this paper is that the sum of two (or more) independent Poisson variables also has the Poisson distribution. As a result, in the absence of serial dependence, the level of temporal aggregation of any time series is irrelevant. Hence, using quarterly observations as in this paper—or monthly, or annual—presents no problem, even if the true underlying process generating these observations is daily (or hourly, or second-by-second).<sup>16</sup>

14. See, for example, Blanchard and Watson (1982); and West (1987).

15. See Shiller (1984).

16. A further attraction of the Poisson distribution for the purposes of this paper is that a large body of literature has explored its implications for investor behavior and asset prices, both theoretically and empirically. Theoretical work on this subject has emphasized the crucial implications of discrete jumps in asset prices (as opposed to smooth diffusion processes) in continuous-time settings; see, for example, Merton (1971, 1976); and Cox and Ross (1976). Examples of empirical applications to equity prices include Press (1967); Clark (1973); Ball and Torous (1983); Jarrow and Rosenfeld (1984); and Akgiray and Booth (1987). Feinstone (1984, 1985), Akgiray and Booth (1988), and Tucker and Pond (1988) have applied the idea to foreign exchange rates. The specific model used here was first proposed by Press (1967), who applied the model to the returns of individual stocks from the sample 1926–60.

In general, the Poisson distribution could also characterize a process exhibiting serial dependence (the annual number of volcanic eruptions on Hawaii could be modeled as an autocorrelated Poisson process), in which case the level of temporal aggregation would no longer be flexible. The model of stock market crashes presented below, however, assumes serial independence—that is, that the number of crashes in any given quarter is independent of the number of crashes in the previous quarter. The fact that the market crashed in late 1987, for example, made a second crash in early 1988 neither more nor less likely than it would have been if the market had rallied continuously throughout 1987.

With the assumption of a serially independent Poisson distribution for the timing of the occasional component, a two-component representation of stock returns is then

$$\begin{aligned}
 (1) \quad & \bar{r}_t = \bar{\varepsilon}_t + \tilde{v}_t, \\
 (2) \quad & \bar{\varepsilon}_t \sim N(\mu, \sigma_{\varepsilon}^2), \\
 (3) \quad & \tilde{v}_t = \sum_{i=0}^{\tilde{m}_t} \tilde{v}_i, \\
 (4) \quad & \tilde{\gamma}_i \sim N(\psi, \sigma_{\gamma}^2), \\
 (5) \quad & \tilde{m}_t \sim P(\lambda),
 \end{aligned}$$

where  $\bar{r}_t$  is the excess return on stocks in time period  $t$ ;  $\bar{\varepsilon}_t$  is the once-per-period realization of a disturbance distributed normally, with mean  $\mu$  and variance  $\sigma_{\varepsilon}^2$ ;  $\tilde{v}_t$  is the sum of  $\tilde{m}_t$  realizations of  $\tilde{\gamma}_i$ , a disturbance distributed normally with mean  $\psi$  and variance  $\sigma_{\gamma}^2$ ;  $\tilde{m}_t$  is the once-per-period observation from a Poisson process with characteristic parameter  $\lambda$  (equal to the expected value of  $\tilde{m}_t$ ); and  $\sim$  indicates random variables. (Writing the means and variances of the two normal distributions without subscripts, thereby indicating constancy over time, is a simplifying assumption to be examined more carefully below.) Hence  $\bar{r}_t$ , which is observable, is the sum of  $(1 + \tilde{m}_t)$  elements, each of which is individually unobservable and each of which is distributed normally.<sup>17</sup>

Previous work with models along these lines, using high-frequency currency and equity returns, has resulted in large estimated values for

17. Thinking of the model in this way motivates its representation as a mixture of normals:  $r_t \sim N[\mu + (m_t \cdot \psi), \sigma_{\varepsilon}^2 + (m_t \cdot \sigma_{\gamma}^2)]$ , with  $m_t \sim P(\lambda)$ .

the  $\lambda$  parameter, implying that any given observed return is likely to be composed of  $\bar{\epsilon}_t$  and *many* realizations of  $\tilde{\gamma}_i$ . By contrast, the intuition of extraordinary events that occur only occasionally corresponds to the case in which  $\lambda$  is small—substantially less than unity—so that  $\bar{m}_t$  is typically zero and  $\bar{r}_t$  usually consists merely of  $\bar{\epsilon}_t$ . With  $\lambda$  small,  $\bar{m}_t = 1$  at irregular intervals—approximately once every  $1/\lambda$  periods on average—so that  $\bar{r}_t$  only rarely consists of  $\bar{\epsilon}_t$  plus a realization of  $\tilde{\gamma}_i$ .<sup>18</sup> Multiple realizations of  $\tilde{\gamma}_i$  (that is, values of  $\bar{m}_t$  greater than one) are also possible, albeit unlikely.

With  $\lambda$  small, the representation of stock returns given in equations 1–5 is potentially consistent with the pattern shown in figure 1—or, for that matter, with any of a variety of familiar intuitive characterizations involving occasional market crashes like that of October 1987. Leptokurtosis without skewness corresponds to a large variance  $\sigma_\gamma^2$  (compared with  $\sigma_\epsilon^2$ ). Negative skewness corresponds to a negative mean  $\psi$ . The combination of kurtosis and skewness that is apparent in figure 1 can result from  $\psi < 0$  and large  $\sigma_\gamma^2$ , or merely from a large enough (in absolute values)  $\psi$  even if  $\sigma_\gamma^2$  is small. Indeed, the familiar postwar notion that the market is subject to occasional crashes, without any analogous discontinuities in the upward direction, corresponds to a large negative  $\psi$  with small  $\sigma_\gamma^2$ .

Estimating the model in equations 1–5 involves maximizing the appropriate likelihood function (see the appendix) with respect to the five parameters  $\mu$ ,  $\sigma_\epsilon^2$ ,  $\psi$ ,  $\sigma_\gamma^2$ , and  $\lambda$ . Carrying out this estimation using S&P 500 quarterly excess returns spanning 1954–88 results in a “border solution” in which the estimated value of  $\sigma_\gamma^2$  (the variance of the extraordinary disturbance) is zero.<sup>19</sup> Estimating the model’s other four parameters subject to the constraint  $\sigma_\gamma^2 = 0$  leads to the following results,

18. Under the assumption of serial independence, these intervals are explicitly irregular, in that  $\bar{m}_t = 1$  is no more (or less) likely if  $\bar{m}_{t-1} = 1$  than if  $\bar{m}_{t-1} = 0$ .

19. The model is obviously undefined for  $\sigma_\gamma^2 < 0$ . Having the sample begin in 1954, as is the practice in many macroeconomic models of the United States, is a way of excluding not only the World War II and Korean War periods but also the intervening years during which monetary policy still operated under a formal wartime commitment to peg the price of U.S. government securities. The presumption is that the 1951 Treasury–Federal Reserve Accord, which freed monetary policy from this restriction, changed the behavior of asset markets in an important way. This assumption is particularly relevant in the context of the model developed below, in which market participants attempt to learn about the stochastic processes governing stock market returns by analyzing observed return data.

where  $\hat{\cdot}$  signifies a maximum likelihood estimate and the numbers in parentheses are asymptotic  $t$ -statistics:

$$\hat{\mu} = 0.0246, \quad (3.9)$$

$$\hat{\sigma}_\varepsilon^2 = 0.0048, \quad (7.7)$$

$$\hat{\psi} = -0.2333, \quad (5.0)$$

$$\hat{\lambda} = 0.0327. \quad (1.7)$$

This set of values immediately corresponds to the intuitive notion of an ordinary level of market risk, represented by an ongoing once-per-period normally distributed disturbance, punctuated by occasional market crashes. The random component realized once per period has estimated mean 2.46 percent (per quarter) and standard deviation 6.93 percent. The occasional crash element, by contrast, has estimated mean  $-23.33$  percent (again, per quarter) with zero variance. The estimated frequency of such crashes is once every 30.6 quarters — that is, about every eight years on average.

Conditioning on the estimated parameter values reported above (and with  $\sigma_\gamma^2 = 0$ ), corresponding estimates of  $\varepsilon_t$ ,  $v_t$ , and  $m_t$  in each quarter follow from maximizing the likelihood function (see appendix) in each period with respect to the (nonnegative integer) value of  $m_t$ . These conditional estimates will be denoted with  $\dagger$ 's. The estimated values  $m_t^\dagger$  are all zero with the exception of four quarters — 1962:2, 1970:2, 1974:3, and 1987:4 — in which  $m_t^\dagger$  is unity.<sup>20</sup> These four specific quarters identified as realizations of the crash process ( $m_t^\dagger = 1$ ) are all familiar historical episodes. In traditional market lore, 1962:2 corresponds to the Kennedy administration's battle with the steel industry; 1970:2 to tight monetary policy (what was regarded at the time as a "credit crunch") and the default of Penn Central; and 1974:3 to some combination of the OPEC price rise, tight monetary policy aimed at resisting the resulting inflation, the consequent deepening recession, the escalation of the Watergate scandal culminating in the resignation of President Nixon, the failure of

20. There are no estimated  $\bar{m}_t$  values greater than one.

Franklin National Bank, and, in some versions, even the U.S. invasion of Cambodia. Subsequent history may also come to associate 1987:4 with some analogous events as well. From the perspective of the data alone, reference to figure 1 makes clear that the estimation has simply picked out the four observations primarily responsible for the negative skewness and leptokurtosis of the excess return series.

With these four observations tempered by removal of the nonzero  $v_t^\dagger$  realizations, the remaining component of  $r_t$ —that is the  $\varepsilon_t^\dagger$  series defined by  $\varepsilon_t^\dagger = r_t - m_t^\dagger \cdot \hat{\psi}$ —has noticeably different statistical properties from those of  $r_t$  itself. The skewness and kurtosis that strongly indicate nonnormality of the  $r_t$  in figure 1 simply do not appear in the  $\varepsilon_t^\dagger$  series. The skewness of  $\varepsilon_t^\dagger$  is 0.06, slightly *positive* but in any case not significant at any plausible level. The kurtosis value is  $-0.28$ , slightly *negative* but again not significant at any plausible level. The contrast to the  $r_t$  series in these respects is hardly surprising, in that 1974:3 and 1987:4 are the only observations of  $r_t$  more than three standard deviations from the mean, while 1962:2 and 1970:2 are the only others more than 2.5 standard deviations from the mean. The  $\varepsilon_t^\dagger$  series, which has  $m_t^\dagger \cdot \hat{\psi} = -23.33$  removed from each of these four observations, is clearly more symmetrical and contains fewer observations in its tails.

### Does Market Volatility Persist?

This two-part representation of stock returns, which separates out the very large movements that occur only rarely, has interesting implications for several long-standing questions about the time series behavior of these returns.

For example, a long-standing issue in research bearing on whether the stock market is efficient, in the sense that prices always fully incorporate all available information, is whether either prices or returns are serially correlated.<sup>21</sup> Performing a simple first-order autoregression using the  $r_t$  series plotted in figure 1 for 1954–88 yields the usual extremely weak evidence of serial correlation in stock returns shown in the first row of table 2. The coefficient on the lagged return is barely significant

21. See, for example, Fama and French's (1988) return regressions, Poterba and Summers's (1988) work on mean reversion, and the references cited in these papers. The basic reference describing the movement of speculative asset prices in an efficient market is Samuelson (1965).

Table 2. Autoregressions with One or Two Lags for Excess Returns and Squared Deviations from Mean Returns<sup>a</sup>

Measure of returns	Coefficient		Returns <sub>(t-2)</sub>	Summary statistic		
	Constant	Returns <sub>(t-1)</sub>		$\bar{R}^2$	Standard error	Durbin-Watson
$r_t$	0.0142 <sup>b</sup> (2.0)	0.1420 <sup>c</sup> (1.7)	...	0.0128	0.0809	1.96
$\varepsilon_t^2$	0.0183 <sup>d</sup> (3.0)	0.2070 <sup>b</sup> (2.5)	...	0.0360	0.0681	1.96
$(r_t - \bar{r})^2$	0.0066 <sup>d</sup> (5.7)	0.0015 (0.0)	...	-0.0074	0.0118	2.00
$(\varepsilon_t^+ - \bar{\varepsilon})^2$	0.0047 <sup>d</sup> (7.0)	0.0107 (0.1)	...	-0.0072	0.0063	2.00
$(r_t - \bar{r})^2$	0.0056 <sup>d</sup> (4.4)	-0.0006 (0.0)	0.1628 <sup>c</sup> (1.9)	0.0116	0.0118	2.04
$(\varepsilon_t^+ - \bar{\varepsilon})^2$	0.0036 <sup>d</sup> (4.7)	0.0087 (0.1)	0.2406 <sup>d</sup> (2.9)	0.0436	0.0061	1.98

a. Numbers in parentheses are *t*-statistics. Sample period is 1954:1-1988:4.

b. Significant at 0.05 level.

c. Significant at 0.10 level.

d. Significant at 0.01 level.

at the 0.10 level. Higher-order autoregressions (not shown in the table) yield similar results. By contrast, analogous tests based on the ordinary component of the variation of stock returns, represented by the estimated  $\varepsilon_t^\dagger$  values from equations 1–5, provide quite different results. The estimated value of the coefficient on the lagged ordinary return element shown in the second row of table 2 is larger than that for the observed total return, and it is easily significant at the 0.05 level. Indeed, the  $t$ -statistic is just short of the critical value for the 0.01 level. Again, higher-order autoregressions (not shown) yield similar results.

A question that has received even more attention in recent stock market research is that of time-varying volatility, together with the related issue of whether “volatility shocks” themselves exhibit persistence over time—in other words, whether an interval of unusually high variance in returns is typically followed by high variance, or whether high-variance periods tend to occur purely at random. Like the serial correlation issue, the debate about volatility persistence is interesting, ultimately, because it bears on the efficiency of stock prices in reflecting available information and, consequently, on the efficiency of the stock market mechanism in allocating scarce capital. In addition, complex patterns of time-varying volatility may help to explain the finding of autocorrelated returns.

As the results reported in the third and fourth rows of table 2 show, simple first-order autoregressions of squared deviations of excess stock returns from the corresponding mean show no evidence of volatility persistence regardless of whether the variable under study is the observed return  $r_t$  or the estimated ordinary component  $\varepsilon_t^\dagger$ . Second-order autoregressions tell a different story, however. The second-order autoregression for squared deviations of  $r_t$  from its mean, reported in the fifth row, yields the more familiar modest evidence that volatility is in fact persistent. Coefficient  $\hat{\beta}_2$  is easily significant at the 0.10 level and almost so at the 0.05 level. By contrast, in the same autoregression based on squared deviations of  $\varepsilon_t^\dagger$  from its mean, reported in the last row,  $\hat{\beta}_2$  is significant at the 0.01 level. Autoregressions of order higher than two (not shown) echo these respective results. While the evidence for persistence of volatility in *observed* stock returns is weak, therefore, use of a two-part representation of returns, as in equations 1–5 above, provides a much stronger basis for inferring that there is volatility persistence in the resulting *ordinary component* of stock returns.

Naive autoregressions like those reported in table 2, however, are a crude way to investigate issues like persistence in volatility of asset returns. A more attractive approach, which models time-varying variances in an explicit way, is the autoregressive conditional heteroskedasticity (ARCH) model developed by Robert Engle and subsequently generalized along several dimensions by Engle and various coauthors. The essence of this approach as applied to the excess return on a risky asset like stocks is to combine some simple asset-pricing model, which relates the excess return to the level of risk perceived by investors, with an explicitly time-varying representation of that perceived risk, at any point in time, in terms of the observed history of market volatility up to that point.

For example, the standard one-period capital asset pricing model for a single risky asset is

$$(7) \quad \bar{r}_t = \rho \hat{\sigma}_t^2 \alpha_{t-1} + \bar{u}_t,$$

where  $\bar{r}_t$  is again the excess return on stocks compared with Treasury bills (treated here as a risk-free asset),  $\rho$  is the market-average coefficient of relative risk aversion,  $\hat{\sigma}_t^2$  is the perceived variance of the excess return (conditioned on all information then available),  $\alpha_{t-1}$  is the share of the total market portfolio consisting of the risky asset, and  $\bar{u}_t$  is a normally distributed disturbance term.<sup>22</sup> A standard “GARCH” (generalized ARCH) representation of the evolution over time of the perceived variance is in turn

$$(8) \quad \hat{\sigma}_t^2 = \kappa + \theta \cdot \hat{u}_{t-1}^2 + \phi \hat{\sigma}_{t-1}^2,$$

where  $\hat{u}_{t-1}$  is the estimated value of the disturbance in equation 7—that is, the “surprise” in the return on the risky asset—in the previous period.<sup>23</sup> Apart from the constant  $\kappa$ , the variance of the excess return

22. Although the usual capital asset pricing model includes no constant term, other researchers—for example, Bollerslev, Engle, and Woolridge (1988)—have often added one. Estimation results for models equivalent to equations 7–8, and equation 7 together with equation 9 below, never provided evidence for a constant in equation 7 at any plausible significance level. All results reported in this paper are for equation 7 as written, with no constant.

23. More general GARCH equations could include any number of lags of  $\hat{\sigma}_t^2$  and  $\hat{u}_t^2$ . Indeed, several lags would be needed to reproduce precisely the unusual pattern of volatility autocorrelations implied by the regressions discussed above. Following earlier research, however, the GARCH model considered here is constrained to represent the conditional variance as a function of only  $\sigma_{t-1}^2$  and  $\hat{u}_{t-1}^2$ .

on equities (which is conditional on the information available at the outset of each period) therefore depends on the previous period's "surprise," weighted by  $\theta$ , and on the perceived variance as of the outset of the previous period, weighted by  $\phi$ . Depending on the values of  $\theta$  and  $\phi$ , the perceived variance can move smoothly or not over time, and can exhibit strong or weak persistence, or none at all.

The first two rows of table 3 summarize the results of estimating equations 7 and 8 jointly by maximum likelihood methods, using the same quarterly time series spanning 1954–88, first for the observed excess return on stocks  $r_t$ , and then for the ordinary component  $\varepsilon_t^\dagger$  estimated above.<sup>24</sup> There are two conceptually different ways of inferring persistence of stock return volatility from the estimated parameters of equation 8, and in both cases the values shown in the second row based on the  $\varepsilon_t^\dagger$  series indicate greater persistence than the corresponding values in the first row based on  $r_t$ .

First, if equation 8 describes the actual evolution over time of the variance of  $\tilde{u}_t$  (which is equal to the conditional variance of  $r_t$ ), then the  $\hat{\sigma}_t^2$  that equation 8 determines for each period is not just the variance perceived by investors but the actual variance as well. In that case a large positive or negative realization of  $\tilde{u}_t$  results in the variance of  $\tilde{u}_{t+1}$  increasing, so that the persistence of volatility of returns depends on the sum  $(\theta + \phi)$ . In particular, when the sum  $(\theta + \phi)$  is close to one, the increase or decrease in variance resulting from any given realization of  $\tilde{u}_t$  tends to die out slowly over time, while  $(\theta + \phi)$  near zero implies that any such increase or decrease will tend to disappear quickly.<sup>25</sup> Comparison of the first and second rows in table 3 shows a modest difference in this respect, with  $(\hat{\theta} + \hat{\phi}) = 0.83$  based on  $r_t$ , as against  $(\hat{\theta} + \hat{\phi}) = 0.87$  based on  $\varepsilon_t^\dagger$ . In the aftermath of any given realization of  $\tilde{u}_t$  that increases the variance, therefore, the expectation is that after four quarters only 48 percent of that increase will remain according to the results based on  $r_t$ , as against 57 percent according to the results based on  $\varepsilon_t^\dagger$ .

24. The data for the asset shares are calculated from the Flow of Funds balance sheet data for the household sector. Maximization of the likelihood function was achieved by applying numerical derivatives to the quadratic hill-climbing algorithm from the GQOPT Microsoft Fortran library.

25. On the assumption that  $\sigma_t^2 = \hat{\sigma}_t^2$ , expected volatility can be expressed by the recursion

$$E[\sigma_{t+1}^2] = \kappa + (\theta + \phi) \cdot E[\sigma_t^2], \text{ where } s \leq t.$$

This implies that shocks to volatility decay geometrically at rate  $(\theta + \phi)$ .

**Table 3. Results for CAPM-GARCH and CAPM-MARCH Models<sup>a</sup>**

<i>Model</i>	<i>Independent variable</i>	$\hat{\rho}$	$\hat{\kappa}$	$\hat{\theta}$	$\hat{\phi}$	$\hat{a}$
GARCH	$r_t$	6.19 <sup>b</sup> (3.0)	0.0014 <sup>c</sup> (1.7)	0.3057 (1.6)	0.5287 <sup>b</sup> (4.0)	...
GARCH	$\varepsilon_t^\dagger$	10.89 <sup>b</sup> (3.9)	0.0007 (1.5)	0.2200 <sup>d</sup> (2.0)	0.6471 <sup>b</sup> (5.3)	...
MARCH	$r_t$	7.78 <sup>b</sup> (3.7)	0.0003 (0.7)	0.0084 (1.5)	0.6147 <sup>b</sup> (5.7)	67.00 (1.5)

a. Asymptotic *t*-statistics are in parentheses. Sample period is 1954–88. Capital asset price model with generalized (GARCH) and modified (MARCH) ARCH models.

$$\text{CAPM: } y_t = \rho \hat{\sigma}_t^2 \alpha_{t-1} + u_t,$$

$$\text{GARCH: } \hat{\sigma}_t^2 = \kappa + \theta(\hat{u}_{t-1}^2) + \phi \hat{\sigma}_{t-1}^2,$$

$$\text{MARCH: } \hat{\sigma}_t^2 = \kappa + \theta F[\hat{u}_{t-1}^2] + \phi \hat{\sigma}_{t-1}^2$$

$$F[\hat{u}_{t-1}^2] = \begin{cases} \sin[a \cdot \hat{u}_{t-1}^2] & \text{if } a \cdot \hat{u}_{t-1}^2 < \frac{\pi}{2} \\ 1 & \text{if } a \cdot \hat{u}_{t-1}^2 \geq \frac{\pi}{2} \end{cases}$$

- b. Significant at 0.01 level.
- c. Significant at 0.10 level.
- d. Significant at 0.05 level.

Alternatively, equation 8 could merely describe how investors form their perceptions of the variance of the risky asset’s return, without necessarily implying that the actual variance evolves in that way (or even that it changes at all). In this context equation 8 can be interpreted as the familiar “error-learning” equation, which is not necessarily optimal.<sup>26</sup> For example, if the underlying distribution of returns actually has a constant conditional variance, but investors nevertheless use equation 8 to form their forecasts, then a large realized squared “surprise”  $\hat{u}_{t-1}^2$  raises the perceived variance  $\hat{\sigma}_t^2$  but does not affect the true mathematical expectation of the future value  $\hat{u}_t^2$ .<sup>27</sup> Hence, a shock to investors’ perceived volatility will on average decay geometrically at rate  $\phi$  rather than rate  $(\theta + \phi)$ .<sup>28</sup> In this case the difference in implied

26. Muth (1960) derives the conditions under which an error-learning equation represents an optimal forecast procedure. Meiselman (1963) is a standard reference for an early application of the error-learning model to investor behavior. In most applications, the two weights (here  $\theta$  and  $\phi$ ) sum to unity, and the equation does not contain a constant term. As table 3 shows, the sum of estimated values  $\hat{\theta}$  and  $\hat{\phi}$  is close to unity, but the constant term has nonzero value.

27. This will be exactly true only if  $\hat{u}_t = r_t - E[r_t]$ .

28. On the assumption that the conditional variance  $\sigma_t^2$  is equal to some constant  $\sigma^2$ , for all  $t$ , but investors nevertheless use equation 8 to form their one-period-ahead volatility

persistence is even greater, with  $\hat{\phi} = 0.53$  in the results based on  $r_t$  as against  $\hat{\phi} = 0.65$  based on  $\varepsilon_t^\dagger$ . Four quarters after any given increase in perceived variance, due to an unusually large positive or negative realization of  $\hat{u}_t$ , the amount of that increase that will remain on average is therefore only 8 percent according to the results based on  $r_t$ , as against 18 percent according to those based on  $\varepsilon_t^\dagger$ .

Hence the implication of separating stock returns into ordinary and extraordinary components is the same regardless of whether the method used is a simple autoregression or a more sophisticated ARCH model with either of the interpretations offered above. In all cases the evidence for persistence is greater for the ordinary, every-period component than it is for the total return consisting of both the ordinary component and the occasional extraordinary shock.

Results like those presented in tables 2 and 3 are interesting for the light that they shed on the behavior of the ordinary component of stock returns, given the prior exclusion of the extraordinary component as estimated using the model in equations 1–5. Nevertheless, because the form of equations 1–5 estimated here allows neither for serial correlation of the ordinary component  $\varepsilon_t$  nor for variation over time of its variance  $\sigma_\varepsilon^2$  (or variance  $\sigma_\gamma^2$  either, for that matter), there is a tension between the procedure used to isolate the estimated  $\hat{\varepsilon}_t$  series and the analysis to which it is then subjected. One way to resolve this tension would be to use a procedure like that developed by Christopher Sims to generalize equations 1–5 so as to allow for serial correlation and a time-varying variance.<sup>29</sup> The alternative suggested immediately below avoids the need for *prior* separation of the ordinary and extraordinary components of returns, by modifying the basic ARCH model itself to make it robust to “outlier” events. In particular, the point of such a modified model is to enable the ARCH mechanism to focus on the ordinary component of stock returns by deemphasizing the extraordinary events that appear to operate outside of the ARCH context.

---

perception  $\hat{\sigma}_t^2$ , the mathematical expectation of perceived volatility can be expressed by the recursion

$$E_s[\hat{\sigma}_{t+s}^2] = \kappa + \theta \cdot \sigma^2 + \phi \cdot E_s[\hat{\sigma}_t^2], \text{ where } s \leq t.$$

This implies that shocks to perceived volatility will decay geometrically at rate  $\phi$ .

29. See Sims (1989). In related work we are attempting to implement this alternative as well.

To motivate such a modification, it is useful to recall the basic idea underlying the separation of stock returns into ordinary and extraordinary components in the first place: namely, the repeated finding that observed returns include occasional very large shocks—not many, but too many to be consistent with the once-per-period observation of a single, normally distributed random variable. If these occasional large movements stem from something separate from the ordinary forces at work in the market all the time, then they should not affect the future volatility of the ordinary element of returns. Applying the conventional ARCH model to the ordinary component only, as above, is one way to represent this process. But doing so requires some *prior* way of distinguishing the ordinary and extraordinary components. The objective of a modified ARCH model in this context is to avoid that requirement. In particular, the modified model, to be estimated using observed returns, should disregard (or at least deemphasize) any part of a quarter's observed return that is likely to reflect an extraordinary event.

In its most general form, the modified ARCH model ("MARCH") corresponding to equation 8 is just

$$(9) \quad \hat{\sigma}_t^2 = \kappa + \theta F[\hat{u}_{t-1}^2] + \phi \hat{\sigma}_{t-1}^2,$$

where  $F[\cdot]$  can, in general, be any function that transforms the previous quarter's surprise before it affects the current quarter's conditional risk. The third row of table 3 shows the result of estimating, again using the observed quarterly return series for 1954–88, the MARCH model consisting of equations 7 and 9 where  $F[\cdot]$  is a truncated sine curve,

$$(10) \quad F[\hat{u}_{t-1}^2] = \begin{cases} \sin [a \cdot \hat{u}_{t-1}^2] & \text{if } a \cdot \hat{u}_{t-1}^2 < \frac{\pi}{2} \\ 1 & \text{if } a \cdot \hat{u}_{t-1}^2 \geq \frac{\pi}{2}, \end{cases}$$

with  $a > 0$ .<sup>30</sup> Because the GARCH model in equation 9 is just a special case of the MARCH model in equation 10, it is appropriate to apply a likelihood ratio test of GARCH against the MARCH alternative.<sup>31</sup> The

30. An alternative candidate for  $F[\cdot]$  is the truncated quadratic function. For purposes of the model presented in this paper, results based on the truncated sine function and the truncated quadratic function are indistinguishable.

31. The GARCH model can be approximated arbitrarily closely with the MARCH model by letting  $a$  go to zero and multiplying  $\theta$  by  $1/a$ .

test clearly favors MARCH over GARCH. The relevant  $\chi^2$  statistic is 4.54, easily large enough to warrant rejecting the GARCH model in favor of MARCH at the 0.05 level.

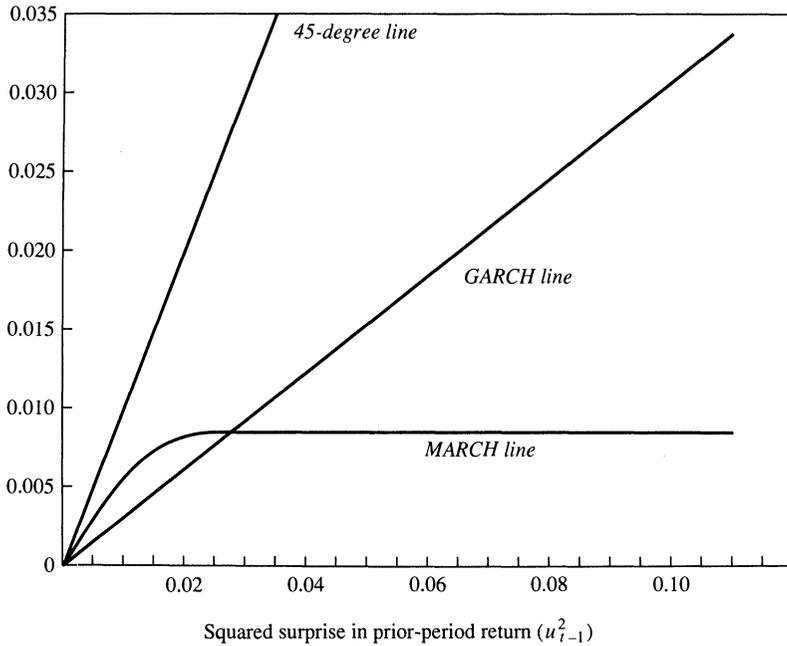
Because the effect that any observed surprise exerts on perceived risk in the MARCH model depends on the interaction of  $\theta$  and  $a$  (and on the sine function), it is difficult to compare the respective impact implied by the GARCH and MARCH models merely by inspecting the estimated parameter values shown in table 3. Figure 2 provides a visual comparison of the MARCH model with  $\theta = 0.0084$  and  $a = 67$  (from the third row of table 3) and the GARCH model with  $\theta = 0.31$  (from the first row). For each model, figure 2 plots the impact on the perceived variance  $\hat{\sigma}_t^2$  resulting from a prior-period surprise  $\hat{u}_{t-1}^2$  of any given magnitude. For purposes of comparison, the figure also includes a 45° line corresponding to the special case of a GARCH model with  $\theta = 1$ . The GARCH model line is simply a straight line with slope given by  $\hat{\theta} = 0.31$ . By contrast, the MARCH model line rises more rapidly at first, implying a “ $\theta$  equivalent” substantially greater than 0.31 for small surprises. The sine function reaches a maximum at  $\hat{u}_{t-1}^2 = 0.0234$  ( $\hat{u}_{t-1} = \pm 15.3$  percent), however, after which any marginal increment is implicitly presumed to reflect only extraordinary (nonpersistent) volatility, so that it does not affect the variance for the next period. By contrast, the GARCH model continues to weight all squared surprises, no matter how large, in a strict linear manner. For surprises greater than  $\hat{u}_{t-1}^2 = 0.0275$  ( $\hat{u}_{t-1} = \pm 16.6$  percent), the effect on  $\sigma_t^2$  is greater under the GARCH model, and thereafter the difference grows linearly.

The great majority of the residuals, for either model, lie well within the  $0 \leq \hat{u}_{t-1}^2 \leq 0.0275$  range for which the MARCH model implies *greater* impact on the conditional variance in any given period due to the surprise in the previous period. Only seven observations—the same ones that stand out in figure 1—generate  $\hat{u}_{t-1}^2 > 0.0275$ . In essence, therefore, the estimated MARCH model is doing what it is intended to do: distinguishing the extraordinary movements and removing most of their impact, and then analyzing the persistence in volatility of the remaining ordinary component.

Like the results shown in the second row for the GARCH model estimated using the  $\hat{\varepsilon}^+$  series, the MARCH model estimates—not just for  $\theta$  and  $a$ , as illustrated in figure 2, but for  $\phi$  as well—have important implications for the persistence debate. Unlike the GARCH model, the

**Figure 2. Impact of Prior-Period Surprise ( $\hat{u}_{t-1}^2$ ) on Perceived Variance of Return ( $\hat{\sigma}_t^2$ )<sup>a</sup>**

Impact on perceived variance of return ( $\sigma_t^2$ )



a. See equations for variance in table 3.

MARCH framework implies that the rate of decay of a volatility shock depends on the magnitude and sign of that shock. Numerical simulations of the two models show the extent to which the MARCH model implies *greater* persistence of positive volatility shocks of ordinary magnitude, but *less* persistence of extraordinarily large volatility shocks.<sup>32</sup> For example, in the event of a surprise that immediately raises the conditional variance 15 percent above its unconditional mean, the MARCH model implies that the current expectation of the conditional variance four quarters in the future is still 12.9 percent above the unconditional mean while under the GARCH model it is just 7.3 percent above. Similarly, after eight quarters the comparison is 7.8 percent under MARCH, as

32. Since there is no available analytic solution for the MARCH model, the data reported here were computed using Monte Carlo methods. For each value of  $\sigma_0^2$ , 1,000,000 iterations of the stochastic sequence  $(\hat{\sigma}_1^2, \hat{\sigma}_2^2, \dots, \hat{\sigma}_{20}^2)$  were generated to determine the expected conditional mean for each  $\hat{\sigma}_t^2$ ,  $t = 1, 2, \dots, 20$ , following any given assumed  $\sigma_0^2$  (that is,  $E_0[\hat{\sigma}_t^2]$ ,  $t = 1, 2, \dots, 20$ ).

against just 3.5 percent under GARCH. By contrast, in the aftermath of a surprise that immediately raises the conditional variance 400 percent above the unconditional mean, the current expectation of the conditional variance four quarters in the future is still 193.9 percent above the unconditional mean under the GARCH model but only 114.9 percent under the MARCH model. After eight quarters, the analogous comparison is 94.0 percent under GARCH, as against only 46.7 percent under MARCH.<sup>33</sup>

The MARCH model estimates therefore imply that extremely high volatility levels (like that at the time of the October 1987 crash) decay relatively quickly, while only marginally high volatility levels decay much more slowly. This interesting nonlinearity is at the heart of the motivation for the MARCH model, and it can potentially explain the consistent failure of past research to find long-term volatility persistence in stock returns.<sup>34</sup> If returns are in fact generated by a process like the MARCH model, naive volatility autoregressions will not pick up the rich nonlinear pattern of serial dependence. Moreover, since least-squares regressions tend to be dominated by outlier observations, the pattern of weak persistence that describes the large volatility shocks will overwhelm the stronger persistence of low-level shocks. In addition, this problem is not specific to the MARCH framework. Indeed, any component process in which the respective volatilities of the components have different persistence properties is subject to the same difficulty.

In sum, the results based on the new MARCH model, which transforms the observed surprises within the estimation, support the earlier findings based on the use of an independent procedure to distinguish extraordinary observations *ex ante*. Once again, effectively removing the impact of large shocks provides greater evidence that at least some movements of stock return volatility over time do exhibit relatively high persistence.

33. The interpretation pursued in this paragraph assumes  $\sigma_t^2 = \hat{\sigma}_t^2$ . Alternatively, one could assume that the conditional distribution of  $\hat{u}_t^2$  is constant for all  $t$ , but investors do not know this value and use equation 9 to form their one-period-ahead volatility perception  $\hat{\sigma}_t^2$ . This approach echoes the second interpretation proposed above for the GARCH equation, and once again this approach implies that shocks to perceived volatility will on average decay geometrically at rate  $\phi$ . Finally, because the MARCH estimate of  $\phi$  (0.61) is greater than the GARCH estimate of  $\phi$  (0.53), the interpretation proposed here implies that the MARCH model exhibits relatively greater persistence in shocks of perceived volatility.

34. See, again, for example, Poterba and Summers (1986).

## **Market Fluctuations and Macroeconomic Fluctuations**

Extraordinary movements in stock prices are interesting in more respects than for the light they shed on questions like how the risk of investing in stocks changes over time or whether the stock market is efficient. A long tradition has also associated financial crises, including sharp declines in stock prices, with subsequent declines in macroeconomic activity.<sup>35</sup> Indeed, when the U.S. stock market crashed in October 1987, much of the reaction in both popular and professional circles focused on the potential parallel to the October 1929 crash and the depression of the early 1930s.

Any theory of business fluctuations that assigns a central role to stock market crashes in bringing about economic downturns must address two questions. First, why would a decline in stock prices lead to a decline in real economic activity? And second, what causes the stock market itself to crash? Standard economic theory has had a fair amount to say on the first question, beginning with wealth effects on consumer spending and cost-of-capital effects on business investment, continuing through credit rationing effects due to loss of collateral value, and finally including the effects of breakdown in one or more parts of the market mechanism (a collapse of the banking system, for example, or the cessation of trading in stocks or other assets). By contrast, standard economics has had little to say about what causes the market to crash in the first place.

A prominent exception to the silence is the “financial instability hypothesis” advanced by Hyman Minsky. Minsky has not attempted to account for market fluctuations with any precision, of course, nor to predict future fluctuations. But he has argued that there is something systematic about the occurrence of stock market crashes and other financial crises. In particular, the central tenet of Minsky’s hypothesis is that as the most recent such crisis becomes a more distant memory, the relevant actors in the economy change their behavior so as to erode the financial system’s ability to withstand a major shock without sustaining a rupture of the kind typically associated with a severe downturn in real output and spending. For a given likelihood of such a shock’s occurring, therefore, a financial crisis—and following that, a severe decline in economic activity—becomes more likely as time passes.

35. See, for example, Kindleberger (1978).

Even a casual reading of Minsky's repeated descriptions of this idea immediately shows that the financial instability hypothesis encompasses a range of financial and economic activities far broader than merely the willingness to invest in stocks. Although both Minsky and Charles Kindleberger have emphasized the role of speculative investment as a precursor to most if not all financial crises, their writings make clear that the assets on which people have speculated at such times include not just publicly traded corporate equities but also illiquid business interests, real estate, land, and even collectibles. Further, Minsky's own work has placed more emphasis on liabilities than on assets. In most of his descriptions, the phenomenon mainly responsible for the deterioration over time of the economy's ability to withstand an adverse shock is the increasing prevalence of "speculative" and even "Ponzi" finance, in preference to "hedge" finance.<sup>36</sup> A shock that causes cash flows to be insufficient to service debts therefore leads overextended borrowers to sell assets to meet their obligations—as Minsky has often put it, "selling position in order to make position"—and hence contributes to the decline in asset values (including stock prices) that is perhaps the most visible characteristic of financial crises.

Intriguing as this hypothesis may be, and notwithstanding the richness of Minsky's descriptions, its behavioral underpinnings have remained vague. The hypothesis clearly requires that some people, or some institutions, change their actions in ways that reduce the financial system's ability to withstand shocks. But why do they do so?

With the addition of one crucial ingredient—in particular, the assumption that people learn over time, having incomplete information but using observations on past market movements to do the best job they can to discern important features of the relevant financial environment—the view developed here of stock market fluctuations as consisting of both an ordinary once-per-period component and an occasional extraordinary component can provide a behavioral motivation for the pattern of changes suggested by Minsky. Specifically, when stock returns have such a two-component form, people who learn over time in this

36. In Minsky's terminology, "hedge finance" means borrowing for purposes with a high probability of generating adequate cash flow to service the debt in all future periods; "speculative finance" means borrowing for purposes with a high probability of providing adequate cash flow to service the debt after some time though not initially, albeit with positive expected net present value; and "Ponzi finance" means borrowing for purposes having negative expected net present value.

way will become more willing to hold stocks, and correspondingly less willing to hold “safe” assets, as time passes following the most recent financial crisis. To be sure, in this stripped-down context with no risky assets other than stocks, and no liabilities at all, investors’ willingness to hold stocks can at best stand as a metaphor for the public’s willingness more generally to speculate on other assets and to assume extended liability positions. Elaborating a richer model, including other risky assets as well as liabilities, would clearly bring the analysis closer to Minsky’s statements of the financial instability hypothesis. But even the simplest model involving merely the choice between stocks and a “safe” asset is sufficient to show how the two-component structure of market risk suggested here can motivate the kind of change in behavior over time that is necessary for Minsky’s hypothesis.

An extremely simple prototype model along these lines could rest on the following four assumptions.

First, participants in the financial markets allocate their wealth between two assets, stocks and Treasury bills (taken to be free of all risk).

Second, market participants are risk averse. In each time period they choose portfolios to maximize a mean-variance utility function characterized by constant “price of variance”  $\rho/2$  (where, under appropriate assumptions about the nature of the uncertainty,  $\rho$  is equivalent to the coefficient of relative risk aversion).

Third, market participants believe that stock returns consist of both an ordinary component, distributed normally (mean  $\mu$ , variance  $\sigma_{\bar{e}}^2$ ) and realized in each period, and an extraordinary component, of magnitude  $\psi$ , realized in some but not all periods. The probability of any given period’s return containing an extraordinary element is  $p$ .<sup>37</sup>

37. This formulation is a special case of the Poisson model, equations 1–5, presented above. In particular, as the parameter  $\lambda$  goes to zero, the Poisson model is increasingly closely approximated by the model outlined here. This formulation simplifies the Poisson model, in that it precludes multiple occurrences of the extraordinary return component (none of which were estimated to have occurred in the results presented above). The simpler formulation is more tractable in the context of the portfolio allocation problem below. In addition, for some of the sample periods for which moving-sample results are presented below, the Poisson model resulted in multiple maxima of the likelihood function. The model used here is also related to, but simpler than, the Markov models proposed by Reitz (1988) and Cecchetti, Lam, and Mark (1989). It is also related to the work of Blanchard and Watson (1986), who applied a “mixture of normals” in a macroeconomic context.

Finally, market participants believe that they know the values of parameters  $\mu$  and  $\sigma_{\varepsilon}^2$  describing the component of stock returns that they observe in every period, but do not know the values of parameters  $\psi$  and  $p$  representing the magnitude and probability of occurrence of the extraordinary component. After each new observation of returns, they therefore estimate  $\psi$  and  $p$  using all available information and then base their portfolio choice for the next period on the resulting point estimates.

Whether the model that market participants believe is correct or not, it is likely that over a long enough time their estimates of these parameters will tend to converge, and after this point no basis will remain for motivating changes in their behavior from this source. Any reading of the literature of empirical economics immediately suggests, however, that economists do not study the economy as if the models that they are estimating had never changed. There is little reason to think that financial market participants do so either. The most familiar approach is to base empirical analyses on data beginning only after some distinct event believed on a priori grounds to have changed the behavior under study. Familiar examples include the founding of the Federal Reserve System, World War II, and the breakdown of the Bretton Woods system of fixed exchange rates. An alternative approach is to acknowledge implicitly that such changes occur in a more frequent and continual way, and therefore to use either a rolling sample or a continually expanding sample with discounting of past observations. Under either approach, the sample of relevant experience is finite, and in general the resulting estimated parameter values therefore change as additional observations become available.

Following much other work on U.S. financial markets, the specific change assumed here to demarcate relevant data is the Treasury-Federal Reserve Accord negotiated in 1951 and implemented in the year or so thereafter.<sup>38</sup> After the elimination of several additional quarters so as to avoid the Korean War period, the sample used to represent market participants' perceptions therefore begins with the first quarter of 1954. Estimating  $\mu$ ,  $\sigma_{\varepsilon}^2$ ,  $\psi$ , and  $p$  for the model described in the third assumption above, using maximum likelihood methods and quarterly data for 1954–88, results in the following values (asymptotic  $t$ -statistics in parentheses):

38. In particular, the relationship between stock returns and interest rates has been sharply different since the accord than it was before. See, for example, Campbell (1989).

$$\begin{aligned}
 \hat{\mu} &= 0.0247 && (4.0) \\
 \hat{\sigma}_\varepsilon^2 &= 0.0048 && (7.7) \\
 \hat{\psi} &= -0.2320 && (5.0) \\
 \hat{p} &= 0.0330. && (1.7)
 \end{aligned}$$

Because the model estimated here is closely motivated by the results for the Poisson model developed in the paper's first section, it is not surprising that these estimates are almost indistinguishable from those shown in equation 6 above.<sup>39</sup> Also, as before, the procedure for picking out extraordinary shocks yields 1962:2, 1970:2, 1974:3 and 1987:4 as the only four quarters in which a realization of the extraordinary return component is estimated to have occurred.

Given parameter values like those shown in equation 11, market participants choose the shares of stocks and Treasury bills in their portfolios so as to maximize their expected utility,

$$(12) \quad \max_{\alpha_t} \{M_t - \frac{\rho}{2} V_t\},$$

where  $\alpha_t$  is the portfolio share invested in stocks (the single risky asset), and  $M_t$  and  $V_t$  are, respectively, the perceived mean and variance of the rate of return on the entire portfolio. The time subscripts on  $M$  and  $V$  emphasize the point that these perceptions are continuously being updated as the investors' information set grows over time. Given the two-component structure of the risk associated with the excess return on stocks, the perceived mean and variance are, respectively,

$$(13) \quad M_t = \alpha_t (\hat{\mu}_t + \hat{p}_t \hat{\psi}_t),$$

$$(14) \quad V_t = \alpha_t^2 [\hat{\sigma}_{\varepsilon,t}^2 + \hat{p}_t (1 - \hat{p}_t) \hat{\psi}_t^2].$$

39. The Poisson model and the model estimated here are non-nested though they are closely related (see again footnote 37). Application of the Akaike Information Criterion suggests that the specification estimated in this section is preferable. The two models have the same number of parameters (assuming  $\sigma_\gamma^2 = 0$ ), and the likelihood of the Poisson model is marginally less than the likelihood of the model presented here.

The solution that follows from the first-order condition for equation 12, after substituting  $M_t$  and  $V_t$  from equations 13 and 14, is

$$(15) \quad \alpha_t^* = \frac{\hat{\mu}_t + \hat{p}_t \hat{\psi}_t}{\rho[\hat{\sigma}_{\epsilon,t}^2 + \hat{p}_t(1 - \hat{p}_t)\hat{\psi}_t^2]}$$

where \* indicates the optimal value.<sup>40</sup> For the values of  $\hat{\mu}$ ,  $\hat{\sigma}_{\epsilon}^2$ ,  $\hat{\psi}$ , and  $\hat{p}$  shown in equation 11, and for  $\rho = 7.8$  (chosen from the third row of table 3), the optimal portfolio allocation for a market participant who behaves as described in the four assumptions given above is  $\alpha_t^* = 0.332$ —that is, to put 33.2 percent of the portfolio into stocks and the remaining 66.8 percent into Treasury bills.

Because this optimal allocation is based on the parameter values shown in equation 11, which were estimated using data spanning 1954–88, it should properly refer to investors choosing their portfolios in the first quarter of 1989. According to the fourth assumption, market participants at any earlier time would not have known the values of  $\psi$  and  $p$  but instead would have had to estimate them using the data then available. In general, with a different information set the resulting parameter estimates would have been different. And with different parameter estimates, the optimal portfolio choice in equation 15 would also have been different.

The solid line in figure 3 shows how market participants' estimate of  $p$  (the probability of a stock market crash occurring) would have changed over time, had they been reestimating the model in the way described by the fourth assumption in each quarter beginning at the end of 1962:2.<sup>41</sup> The figure plots the estimated values of  $p$  for a series of 106 samples, each beginning in 1954:1, and ending in successive quarters from 1962:2 to 1988:4.<sup>42</sup> The variation is substantial. (By contrast, the estimated values of  $\psi$ , the magnitude of the crash if it occurs, never deviate much

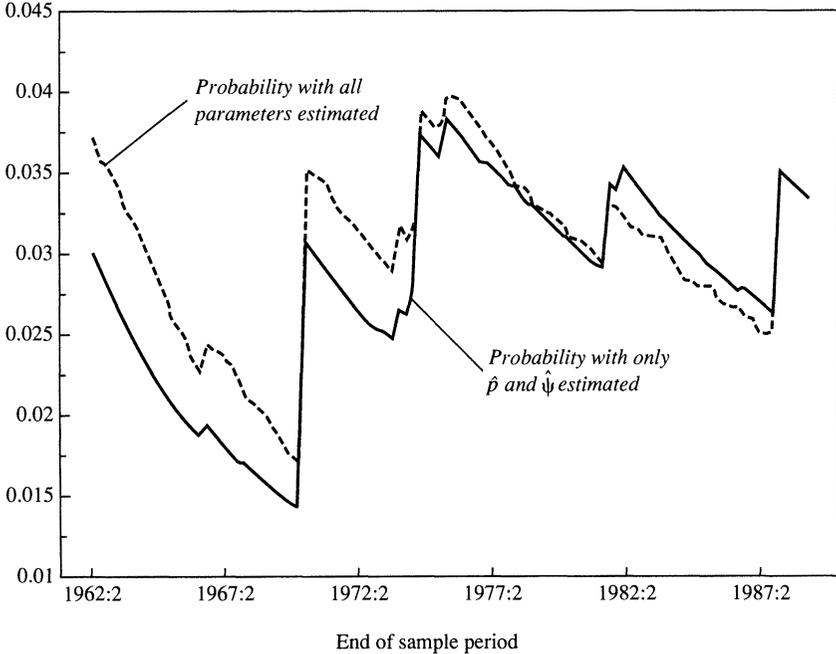
40. This solution treats the estimated values of all four parameters describing stock returns as if they were known with certainty.

41. As stated in the fourth assumption, investors undertake this estimation with the belief that they know  $\mu$  and  $\sigma_{\epsilon}^2$ . Both these parameters are therefore set to their full sample maximum likelihood values whatever sample period is used.

42. The model yielded no sensible value of  $\hat{\psi}$  for samples ending before 1962:2 (the first observation picked out as a realization of the extraordinary component in any sample ending later than 1962:1). We also experimented with an estimation procedure that discounted past observations, as in Friedman and Kuttner (1988), at a rate of 0.99 (per quarter), but the results were indistinguishable from those shown in the figure.

**Figure 3. Estimated Probability of Crash Using Expanding Sample Period\***

Maximum likelihood  
estimates of probability



a. Sample periods all begin in 1954:1.

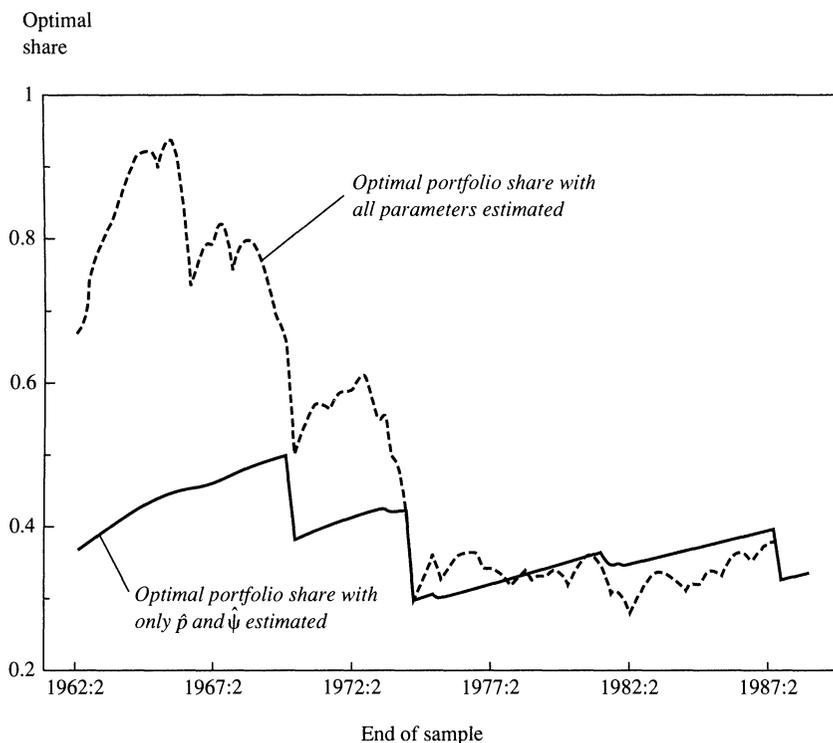
from  $-0.23$ .) The values of  $\hat{p}$  range from a minimum of 0.0144 (estimated with the sample ending in 1970:1) to a maximum of 0.0385 (estimated with the sample ending in 1975:3). Although the correspondence is not precise, the general tendency is clearly for  $\hat{p}$  to decline as time passes after a crash, and then to rise sharply in the aftermath of each new crash.

Variation over time in the perceived probability of a stock market crash, over anything like the range shown for  $\hat{p}$  in figure 3, can plausibly account for changes in behavior that—again treating the willingness to hold stocks as a metaphor for willingness to take on risky positions more generally—correspond to what is required by Minsky's financial instability hypothesis. A risk-averse investor choosing a portfolio for the coming year will take a more exposed position when the probability that a crash will occur within the year is less than one in seventeen (based on

quarterly  $p = 0.0144$ ) than when it is better than one in seven (based on  $p = 0.0385$ ). Similarly, an investor deciding whether to acquire an illiquid asset will act differently if the chances are perceived to be even that a crash will occur within the next twelve years than if the chances are even within the next four years.

The solid line in figure 4 illustrates the dependence of investors' behavior on their perceptions of the likelihood of a crash, within the restricted context of the one-period one-risky-asset model treated explicitly above, by plotting the values of  $\alpha_t^*$  (the optimal portfolio share invested in risky assets) calculated as in equation 15 from the values of  $\hat{p}_t$ , plotted by the solid line in figure 3 and the corresponding value of  $\hat{\psi}_t$  (not plotted) together with constant values of  $\hat{\mu}$  and  $\hat{\sigma}_\varepsilon^2$  as shown in equation 11. The effect of changing perceptions of the probability of a crash is striking, with  $\alpha_t^*$  varying from a maximum of 49.8 percent (in 1970:1) to a minimum of 29.3 percent (1974:3). It is especially interesting that, although the  $\alpha_t^*$  series is derived purely from data on stock returns, it also roughly corresponds (inversely) to measures of risk assessments that are observable in debt markets. For example, the simple correlation between  $\alpha_t^*$  and the interest rate spread of Baa-rated corporate bonds over 10-year U.S. Treasury bonds is  $-0.51$ , so that on average the model estimated here suggests a decreased willingness to hold stock at times when bond market participants are demanding an increased default premium on debts of less than investment grade. This correspondence gives further support to the notion of using the perceived probability of stock market crashes to stand for perceived riskiness more generally in Minsky's richer context.

It is also possible, of course, that, contrary to the fourth assumption above, market participants do not act as if they know the values of  $\mu$  and  $\sigma_\varepsilon^2$  describing the ordinary once-per-period component of stock returns but instead estimate them in each period just as they estimate  $p$  and  $\psi$ . In that case, estimated values  $\hat{\mu}$  and  $\hat{\sigma}_\varepsilon^2$  would in general vary over time as well and there would be two further reasons for portfolio choice  $\alpha_t^*$  to vary. The broken line in figure 3 plots a series for  $p_t$  when all four parameters,  $\mu$  and  $\sigma_\varepsilon^2$  as well as  $p$  and  $\psi$ , are estimated anew each period. The broken line in figure 4 plots the corresponding series for  $\alpha_t^*$ . The effect of the sharp increases in  $p$  after each of the four observations identified as crashes is still evident, but with  $\hat{\mu}_t$  and  $\hat{\sigma}_{\varepsilon,t}^2$  also changing,  $\alpha_t^*$  is no longer close to a mirror of changing  $\hat{p}_t$ . The estimated values  $\hat{\mu}_t$ ,

**Figure 4: Estimated Optimal Portfolio Share of Stocks at End of Each Sample Period<sup>a</sup>**

a. Sample periods all begin in 1954:1.

(not plotted) decline from the mid-1960s until the mid-1980s, thereby depressing  $\alpha_t^*$ . Also, the estimated values  $\hat{\sigma}_{\epsilon,t}^2$  (not plotted) increase after the mid-1970s, depressing  $\alpha_t$  yet further. The correlation between  $\alpha_t$  calculated in this way and the Baa-Treasury interest rate spread is  $-0.70$ .

Within the limited context of the one-period one-risky-asset model, therefore, these changes over time in the behavior of risk-averse investors correspond to the central phenomenon hypothesized by Minsky. As time passes since the most recent market crash, investors tend to perceive the probability of a crash as smaller, and hence take ever more exposed positions. Given the institutional richness of Minsky's own work, it is not difficult to translate this behavior into a much broader context including, for example, business ventures in which the prospects for success hinge on whether a market crash does or does not occur, and loans against ventures in which the prospects for default similarly hinge

on whether or not a crash occurs. In each case, the key result is that a risk-averse investor is more willing to enter into any given risky transaction as the perceived probability of a crash is lower, and that with a limited sample of observations the perceived probability declines as time passes since the most recent crash.

It is worth pointing out that, as is familiar in such models, this result includes a form of internal contradiction. In particular, as the *perceived* crash probability  $\hat{p}_t$  declines, the resulting *true* underlying probability of a crash is actually rising. Lower  $\hat{p}_t$  leads investors to take increasingly extended and exposed positions, and so the system as a whole becomes increasingly susceptible to a financial crisis in the event of an adverse shock. The contradiction is that investors, acting only on the basis of  $\hat{p}_t$  estimated from observed prior returns, do not recognize this increasing systemic fragility; if they did they would choose different, more conservative portfolios. As in other formulations of the Minsky hypothesis, therefore, some element of myopia is a crucial ingredient here as well.<sup>43</sup> Another way of thinking about this myopia is to recognize that investors are basing their decisions on a model in which returns are temporally uncorrelated, but the resulting behavior of these investors tends to generate unforeseen cycles of speculation, crashes, and retrenchment. One implication of this pattern is that crash episodes will actually be negatively autocorrelated, a result that could explain the repeated empirical finding that stock prices are mean-reverting.<sup>44</sup>

### Concluding Thoughts

The empirical evidence presented in this paper supports several conclusions about the usefulness of a two-component representation of stock returns. First, the behavior of quarterly stock returns in the United States since World War II is consistent with a representation consisting of two random components, an ordinary component realized in each quarter and an extraordinary crash component realized only at infrequent

43. As Tobin (1989) put it in commenting on Minsky's own formulation, "Rational expectations adherents will doubtless object that the alleged cycle would vanish as soon as borrowers and lenders understood it." The existence of such myopic traders has been proposed in several recent papers. See, for example, De Long and others (1988).

44. See again Poterba and Summers (1988); and Fama and French (1988).

and irregular intervals. Second, viewing stock returns in this way sheds new light on familiar questions about the time series properties of returns. Specifically, the evidence for both serial correlation and persistence of volatility is greater for the ordinary component of returns than for observed returns consisting of both ordinary and extraordinary components. Third, risk-averse market participants, allocating their portfolios and attempting to learn about the magnitude and frequency of market crashes from evidence that they have in hand, could plausibly behave in such a way as to give rise to the macroeconomic phenomenon posited by Minsky's financial instability hypothesis.

One consistent assumption underlying these results bears closer attention, however. In particular, the conceptual apparatus used throughout this paper to divide observed stock returns into ordinary and extraordinary components simply assumes that the two occur independently of one another—that is, that a crash episode is equally likely to occur whether there is a positive or a negative realization of the ordinary random element, or, if there is a negative realization of this element, equally likely whether it is large or small. In the absence of a unique interpretation of these occasional crashes—something we explicitly do not attempt to offer here—it is difficult either to support or to reject this assumption out of hand. Because it is central to the analysis carried out throughout the paper, however, it is worth questioning closely.

For example, it is certainly not implausible that any given shock that *may* cause a market crash—a major adverse political development, a run of bad economic news, a Shiller-style epidemic of negative psychology, or whatever—is more likely *actually to do so* if stock prices are already falling for other reasons. Similarly, under the Minsky hypothesis the likelihood that an extraordinary adverse shock will interact with an increasingly fragile financial structure in such a way as to produce a crisis could well be greater in the presence of a negative realization of the economy's ordinary random processes. Nor need a relationship along any of these lines be simple or linear. It is entirely plausible that what makes the market vulnerable to a crash when such a shock occurs is not just a negative realization of the ordinary component of returns but a *large* negative realization (greater than, say, one standard deviation).

To be sure, simple examination of the empirical results presented in this paper offers no particular support for this idea. The estimated

realizations of the ordinary component of stock returns in the four specific quarters identified as crash episodes include two positives (1962:2 and 1970:2) and two negatives (1974:3 and 1987:4). But this result is hardly surprising because of the difficulty of statistically distinguishing crash episodes from large negative realizations of the ordinary component when the two covary. A more general model, explicitly including a procedure for resolving this identification problem, would be complicated to implement empirically, but not impossible. Alternatively, individual case studies exploiting nonprice data—for example, business failure and debt default rates—could provide a way to unravel the effect of the ordinary component and the extraordinary component during crash episodes. Especially in conjunction with a well-developed substantive view about what kinds of shocks represent “crash potential,” research along either of these two lines could prove instructive.

## APPENDIX

### *Maximizing the Likelihood Function*

MAXIMUM likelihood estimates of  $\mu$ ,  $\sigma_{\xi}^2$ ,  $\psi$ ,  $\sigma_{\gamma}^2$ , and  $\lambda$  from equations 1–5 are given by

$$\operatorname{argmax}_{\mu, \sigma_{\xi}^2, \psi, \sigma_{\gamma}^2, \lambda} \left\{ \sum_{i=1}^T \left[ -\lambda + \ln \left( \frac{1}{\sqrt{2\pi}} \right) + \ln \sum_{m=0}^{\infty} \left( \frac{\lambda^m}{m!} \cdot \left\{ \hat{\sigma}_{\xi}^2 + m\hat{\sigma}_{\gamma}^2 \right\}^{-1/2} \cdot \exp \left\{ \frac{-(r_i - \mu - m\psi)^2}{2 \cdot (\sigma_{\xi}^2 + m\sigma_{\gamma}^2)} \right\} \right) \right] \right\},$$

where  $T$  is sample size and the bracketed term is the log-likelihood function. To estimate values for  $m_t$ , the period  $t$  likelihood function is

maximized with respect to  $m_t$ , conditional on the maximum likelihood estimates  $\hat{\mu}$ ,  $\hat{\sigma}_\xi^2$ ,  $\hat{\psi}$ ,  $\hat{\sigma}_\gamma^2$ , and  $\hat{\lambda}$ ,

$$\begin{aligned} & \operatorname{argmax}_{m_t} \left\{ -\hat{\lambda} + \ln \left( \frac{1}{\sqrt{2\pi}} \right) + \right. \\ & \quad \left. \ln \left[ \frac{\hat{\lambda}^{m_t}}{m_t!} \cdot \left( \hat{\sigma}_\xi^2 + m_t \hat{\sigma}_\gamma^2 \right)^{-1/2} \cdot \exp \left( \frac{-(r_t - \hat{\mu} - m_t \hat{\psi})^2}{2 \cdot (\hat{\sigma}_\xi^2 + m_t \hat{\sigma}_\gamma^2)} \right) \right] \right\} \\ & = \operatorname{argmax}_{m_t} \left[ \ln \left( \frac{\hat{\lambda}^{m_t}}{m_t!} \right) - \frac{1}{2} \cdot \ln \left( \hat{\sigma}_\xi^2 + m_t \hat{\sigma}_\gamma^2 \right) - \left( \frac{(r_t - \hat{\mu} - m_t \hat{\psi})^2}{2 \cdot (\hat{\sigma}_\xi^2 + m_t \hat{\sigma}_\gamma^2)} \right) \right], \end{aligned}$$

where  $m_t$  is constrained to take on nonnegative integer values.

## *Comments and Discussion*

**Hyman P. Minsky:** As Benjamin M. Friedman and David I. Laibson recognize, their paper, which initially focuses on stock prices and offers a model to explain their excessive volatility, has a relevance that extends beyond the behavior of stock prices. In particular, their paper instructs us on how to investigate complex processes that have some components whose impact is felt quickly and others whose impact is delayed while variables cumulate—that is, as the environment changes. One way the Friedman-Laibson insight for the explanation of excessive volatility of stock prices can be extended is by transforming it into an explanation of the historical pattern of mild and deep business cycles. To do so, it is necessary to specify what cumulates, why such cumulation takes place, and how such cumulation changes the environment so that deeper cycles can be triggered. Once such cumulative processes are identified, it is possible to specify both what happens during the deeper cycles and the economic relations that can contain such deeper cycles.

Friedman and Laibson observe that the historical volatility in stock prices is too great to be ascribed to processes of the type that give rise to a “normal” distribution. Because the prices of stocks that are traded on exchanges can be adapted to be used as proxies for the prices of real capital assets, that observation can be extended to support the proposition that the volatility in the market valuation not only of financial assets but also of capital assets as collected in firms is too great to be ascribed to random errors. What is needed is a construct that accounts for the excessive richness of the tails of the distribution.

Friedman and Laibson provide such a construct. They posit that two processes generate stock prices and, by the extension they draw in the section titled “Market Fluctuations and Macroeconomic Fluctuations,”

the business cycles of experience. One is a random error process that would generate a nice bell-shaped distribution if it were the only process operating.<sup>1</sup>

The other is a Poisson process that kicks in from time to time with a large displacement. Friedman and Laibson identify large negative displacements of stock prices as crashes.

Friedman and Laibson tie their work into what they call “Hyman Minsky’s ‘financial instability hypothesis,’ ” which holds that cumulative changes in the financial structure—mainly in the liabilities used to finance positions in assets but also in the assets and liabilities acceptable in portfolios—take place over a run of good times. As a result, an originally robust financial structure, one characterized by hedge finance (as in 1946), is transformed into a fragile one, one characterized by substantial speculative and Ponzi finance (much as we see today). The hypothesis grew out of my efforts to explain the pattern of mild and severe recessions-depressions noted by Joseph Schumpeter, Milton Friedman and Anna Schwartz, and Moses Abramovitz.<sup>2</sup> My work linked the difference between mild and serious recessions to the robustness or fragility of the financial structure and the large reactions to small proximate stimuli that take place in fragile structures. I argued that the behavior of profit-seeking units over a run of good times transforms the financial structure from being robust to being fragile, so that crises, financial disturbances, and debt deflations, which characterize a deep depression cycle, can take place.<sup>3</sup>

I appreciate that Friedman and Laibson link their detailed, innovative, and sophisticated work to ideas I put forth in various forms over the past thirty years. I want to take the opportunity afforded by their paper to enlarge upon what I now, long after the initial labeling of a particular interpretation of experience and literature as the financial instability hypothesis, mean by the financial instability hypothesis. In particular, I want to examine whether the hypothesis was advanced, as Friedman and Laibson put it, “with less than explicit grounding in the theory of economic behavior” and to address the question posed by Friedman

1. This might well be a Frisch-Slutsky process like the one with which Milton Friedman and Robert Lucas worked. See Frisch (1933); Slutsky (1937); Friedman (1968); Lucas (1972).

2. Schumpeter (1939); Friedman and Schwartz (1963); Abramovitz (1959).

3. Minsky (1964)

and Laibson about “the relevant actors in the economy” who “change their behavior so as to erode the financial system’s ability to withstand a major shock without sustaining a rupture of the kind typically associated with a severe downturn in real output and spending.”

I used the phrase “the financial instability hypothesis” to describe a deviant interpretation of Keynes’s *General Theory* that I advanced in a book, *John Maynard Keynes*.<sup>4</sup> I characterized Keynes’s *General Theory* as advancing an investment theory of business cycles and a financial theory of investment. I also hypothesized that Keynes was familiar with Irving Fisher’s “Debt Deflation Theory of Great Depressions” and that some of the special results of the *General Theory* dealt with the effect of a debt deflation upon objective conditions facing businessmen and bankers and the way they view the world.<sup>5</sup> As a result, a debt deflation or even a less dramatic financial crisis affects the investment, financing, and employment decisions businessmen and bankers make.

The two price levels of a capitalist economy that are relevant to the financial instability view are the price level of capital assets and the price level of labor or, equivalently, of current output. In the famous rebuttal to Professor Viner, and in other post-*General Theory* arguments, Keynes identified liquidity preference as the determinant of the price level of capital assets, what I have usually called  $P_k$ .<sup>6</sup> Investment is determined by the gap between the price level of capital and financial assets,  $P_k$ , and the price level of investment output,  $P_i$ , along with financing conditions that integrate internal financing with the attitude toward risktaking of the proximate borrowers and lenders at the time investment, asset acquisition, and financing decisions are made.<sup>7</sup>

Keynes argued in chapter 17 of the *General Theory* that the return from holding any asset can be treated as being determined by three factors:  $q$ , the yield of the asset;  $c$ , the carrying costs of the asset; and  $l$ , the liquidity premium. I stretched Keynes’s argument to include the cash payment commitments that are embodied in the contracts used to finance positions in financial and capital assets in the carrying costs, the

4. Minsky (1975).

5. Fisher (1933).

6. Viner (1936); Keynes (1937); Keynes (1946). Viner identified Keynes’s liquidity preference as a demand for money relation with the interest rate as an argument. Keynes emphatically rejected that interpretation.

7. Minsky (1975, chap. 5).

$c$ 's. This made the  $q$ 's and the  $c$ 's cash flows. The  $q$ 's were expected gross profits and the  $c$ 's were contractual payment commitments that reflected market conditions and expectations that ruled when the contracts were signed. To be more specific, the  $c$ 's due at any time were the result of earlier negotiations between businessmen and bankers. They embody the expectations about profits and financing conditions that these agents held when the contracts that determine today's payments were negotiated. Financial commitments, along with the economy's inherited capital assets, labor force, and rules that guide institutional behavior, are legacies of the past that limit what can be done in the present.

The  $l$  return that assets earn is subjective. It represents the value of being insured against contingencies that can make a unit unable to purchase, hire, or fulfill payment commitments. Money is an asset that derives its value from its ability to discharge financial commitments and from the ability to purchase and to maneuver that it bestows upon those who hold it. The price of a unit of money is always  $l$ , and the money prices of assets that yield mainly  $q - c$  rise and fall as the quantity of assets that yield mainly  $l$  rise and fall and as the subjective value put on  $l$  falls and rises. Keynes is interpreted as proposing a quantity-and-quality-of-money explanation of asset prices.<sup>8</sup>

This Keynes  $q$ ,  $c$ , and  $l$  construct yields the prices of individual capital and financial assets as well as capital assets collected in bundles as firms. The market prices of firms at every date place values on intangibles, such as market position or power, and reflect the auras of optimism or pessimism about the future that are assigned to firms, industries, and economies.<sup>9</sup>

The Keynes model of a capitalist economy is driven by both objective developments and subjective expectations. The value of liquidity, in the form of the holding of a stock of money or of assets that are taken to be readily transformed into money, depends upon the adequacy and the reliability of the cash flows from income generation that are expected to

8. This is wedded to a money wage cost explanation of output prices. Ferri and Minsky (1984).

9. The pricing of individual firms and assets presents no particular conceptual problems. The derivation of the index number, the price level of capital assets,  $P_k$ , is fraught with conceptual difficulties. However, there is always the Dow Jones and the Standard and Poor indexes to fall back on.

be available to fulfill each period's current payment commitments and the expected performance of the markets in which units exchange assets that are held in a portfolio for money. The value of liquidity depends on the expected performance of aggregate cash flows (profits) and the expected likelihood that financial markets may be disrupted.

The contracts that are closed on any day reflect both the extent to which current and recent  $q$ 's were available to fulfill the  $c$ 's, for both the economy as a whole and the particular units that are negotiating contracts, and what the model of the economy that helps form the expectations of the negotiating units tells them about the future of the  $q$ 's and the  $c$ 's. The principal actors in creating financial contracts are bankers and businessmen: the analytical core of the financial instability hypothesis is a model of banker-businessman negotiations. The banker-businessman negotiations that lead to the financing of investment activity are the proximate determinants of income, profits, and employment. Each participant in such negotiations has private information as well as its own market power.<sup>10</sup>

Because the financial instability hypothesis was formulated before the current fashion of formally reducing aggregate behavior to stylized unit behavior took hold, it was not reduced to a formal model based on representative agents with asymmetric information. The task of a "modern" modeling of the phenomena that are critical to the financial instability hypothesis remains undone. The emphasis on the value of  $l$  and expected  $q$ 's as determinants of the price level of capital assets means that the model of the economy used by the relevant agents (businessmen, bankers, and managers of money) in forming their expectations is of vital importance.

The financial instability hypothesis assumes that the models of system performance that help form the expectations of businessmen and bankers are affected by the recent performance of the economy and by agents' knowledge of its more remote past. The critical agents are unsure how the economy will perform, because they are unsure of the effect of recent institutional and environmental changes. As a result, businessmen, bankers, and managers of money may markedly—and unpredictably—change their behavior in response to small changes in system behavior,

10. William Janeway, an investment banker, stated what I call Janeway's first law: "Entrepreneurs lie." A banker's cliché is "I've never seen a pro forma I didn't like."

if the changes affect their belief in, or the structure of, the model of the economy they use to form expectations.

Agents know that there have been financial crises and deep depressions in the past. Legislative and administrative changes have taken place since the last crisis and depression, in part as a reaction to crises and depression. In addition, market-driven institutional and usage changes have taken place. Furthermore the structure of financing relations undergoes systematic changes as success breeds optimism about future success. The model that guides expectation formation is more volatile than the constructs that rely on the decay of the impact of a previous crisis or depression or on some universally valid model of system behavior. Furthermore, as Keynes noted, changes in the model that underlies expectation formation need not proceed at the same pace for different agents or classes of agents.

Every agent has three sources of liquidity: cash flows from operations (gross profits for business, wages for households, taxes for governments), contract realizations, and portfolio adjustments. The importance of liquidity in the form of monetary and marketable assets diminishes whenever the felt assurance of units (bankers and businessmen, mainly) of the cash flow from operations and from contract fulfillment increases. The success of policy in preventing any sharp and sustained drop in gross business profits over the postwar period has decreased the importance of liquidity in the form of asset holdings. This decline in the subjective yield of liquidity from assets has led to increases in the prices of assets that are valued mainly for the cash they are expected to yield and increases in the payment commitments that income flows are deemed capable of sustaining. The diminished importance attached to portfolio liquidity has helped sustain business investment and consumer debt-financed spending during recent financial traumas that in other circumstances may well have disrupted income flows.

Two views—first, that sustaining aggregate business profits is the key variable for successful stabilization policy and, second, that the composition of aggregate demand rather than any intrinsic productivity of capital-determined profits—are joined to the  $q, c, l$  view of asset values in the financial instability hypothesis. The Kalecki perspective on the national accounts, which emphasizes income distribution and in particular the way in which profits are related to investment and the government deficit, is a fruitful way to approach public policy issues in a world where

the success or failure to validate debt in each period is a significant determinant of the behavior of the economy.

The emphasis in Friedman and Laibson's empirical work is on the choice of assets for a portfolio. I suggest that a shift of research emphasis to the liabilities used to finance positions in assets is warranted. The same considerations—the erosion of portfolio conservatism, agents' unsureness of the significance of novel usages and institutions, beliefs that this is a new era, and the other factors that lead to the bidding up of equity prices by the representative household and its agents—apply to the decisionmakers in both ordinary business firms (the proximate owners of the economy's capital assets) and the complex of financial institutions that are the main proximate owners of the liabilities issued by ordinary firms.

One reason for shifting to an argument based upon liability structures is that the ruling pattern, of cash in from operating in the economy and cash out committed by liability structures, determines the vulnerability of the financial system to disruptive movements, the vulnerability of the economy to deep depressions, and the need for intervention by central banks and governments to contain crises and depressions. Furthermore, the argument about the pattern of income receipts and contractual payment commitments for business firms can be extended to include households, domestic government debt, and international financial arrangements.

The language I use—hedge, speculative, and Ponzi—to describe financial structures has put some off. In a hedge financial structure, the expected “cash flows in” exceed the “cash payment commitments” on the account of both principal and interest as far ahead as a reasonable person looks. A hedge financing unit is likely to have a high ratio of equity to debt. In speculative or rollover financing, the net income portion of gross cash flow exceeds the interest payments committed, but the cash flows are insufficient to meet the payments commitments on principal. Banks are speculative financing units, as is any firm that finances holdings of long-lived assets with short-term debt. Such organizations speculate that refinancing will be available on reasonable terms and are vulnerable to disruptions in financial markets. Ponzi finance—and I have been criticized for using the name of a Boston swindler for what is a not uncommon and often legitimate business practice—takes place when cash flows are not sufficient to pay the interest due on debt

and the interest is folded into the principal owed. If Ponzi finance is not used to finance long-gestation investments, then it amounts to decreasing the equity account even as indebtedness increases. Ponzi finance has a natural termination point when equity goes negative, but all too often creative accounting obscures this transformation.

The “Minsky” hypothesis can be stated in terms of the hedge, speculative, and Ponzi characterization of financing postures. Over a period of good times liability structures change so that the weight of hedge financing units decreases and the weight of speculative and Ponzi financing units increases. Note that any change toward a conservative view of what constitutes an apt liability structure for holding capital assets will put pressure on firms that are in speculative and Ponzi financing postures to use their cash flows to clean up their balance sheets: to use retained earnings to retire debt rather than as the basis for leveraged investment. In addition speculative and Ponzi debtors may be constrained to sell assets to improve their balance sheets. Such making of position by selling position can well lead to a fall in the price of assets being offered. As a result a smaller amount of cash than the books indicated will be generated. If the process is not aborted by the Federal Reserve or some similar agency the price level of assets can fall sharply. This can lead to a broad erosion of mark-to-market net worths and to a decline in the ability to finance investment. As a result investment falls and so will aggregate business profits.<sup>11</sup>

In a big-government capitalism, the impact on profits of a decline in investment is offset by an increase in the government deficit, which is a plus for business profits. Once business profits are sustained, the collapse scenario of asset prices that characterizes a deep depression will not be acted out. Modeling liability structures and integrating such structures with asset pricing is a key to understanding the dynamics of intensely financial capitalist economies.

In their closing remarks Friedman and Laibson note that there is a contradiction in the Minsky hypothesis in that even as the agents themselves view a deep depression or financial crisis as being less likely, the objective portfolio postures tend to make a depression or crisis more

11. In a small-government capitalism with a central bank constrained by rules, this dynamic could lead to serious depressions. It is worth recalling that the Federal Reserve was constrained by rules about gold reserves and the special place of discounted paper during the great collapse of 1929–33.

likely.<sup>12</sup> This apparent paradox, which I believe I usually noted, is resolved by pointing out that the interval over which debt is built up, thus making the objective conditions more favorable to a crisis, is long enough for substantial changes in institutions to have occurred. In addition, claims that more is known now than earlier and that policy is wiser now than in the past gain credence and affect expectations about system performance. Expectation formation takes into account that “The world has changed” and that “They won’t let it happen,” even though agents are not sure who “they” are and what “they” will do. Even as agents note the unfavorable objective circumstances, their significance for today is discounted.<sup>13</sup>

To return to Friedman and Laibson’s comments about the Minsky hypothesis, the financial instability model focuses on the behavior of the proximate agents, businessmen and bankers, who determine investment activity. The model does not reduce the agents of the economy to some ultimate units such as households that aim to maximize the present value of consumption flows.

In the financial instability hypothesis the cumulating process that transforms a system that is virtually immune to deep and serious depressions into one that is susceptible to such depressions results from decisions made by businessmen who invest and finance positions in capital assets, by bankers (commercial and investment) who arrange financing and take positions in assets, and by money managers who have views about the appropriate liability structure for financing positions in capital and financial assets and investment. Each unit in these classes of agents bases its decisions on current constraints—legacies from the past that are more or less constraining—and expectations of the future—mainly expectations about profits and the way financial markets will function. The model of the economy that guides expectation formation recognizes that serious depressions have occurred. Furthermore, agents are not sure that their model has got the economy quite right. As a result a sharp change in the model used in expectation formation can be induced by events.

According to the financial instability hypothesis, the relevant agents are rational and calculating, but they recognize that the world in which

12. Abba Lerner accurately characterized my view as “Stability is destabilizing.”

13. Giordano (1989).

they live is irrational or at least not fully rational. Agents recognize that the model of the economy they use cannot explain the evolution of the economy through time nor predict the impact of novel institutions. It is the uncertain knowledge underlying the model used to form expectations that makes it possible for large repercussions to follow from small events. An occasional downside displacement, such as Friedman and Laibson model as a Poisson distribution, becomes a systemic or endogenous event when it takes place as a result of heavily indebted liability structures and when the model of the economy held by agents changes in response to such a displacement so as to amplify the initial displacement.

In today's world large governments effectively prevent a collapse of profits and central banks intervene to assure that during situations of potential crisis not only banks but also other units that may otherwise be forced to make position by trying to sell position are refinanced. These two sets of interventions have successfully contained the aggregate reactions to the sometimes serious financial crises of the past decades.<sup>14</sup>

The combination of a financial environment that evolves and expectations that change rapidly has been behind the deep depressions of history. As we look back on the 1980s we may at first glance see a long expansion after 1982, but we should also see the regular central bank interventions (I include the refinancing of the savings and loans as a central bank intervention) and the government deficits that underwrote aggregate profits. This combination has to date contained the impact of the financial crisis and rapid changes in asset values such as the stock market crashes of 1987 and 1989. Nevertheless, it is clear that the processes that made for deep depressions in capitalist economies, which Friedman and Laibson help us understand, are alive and well: only their effect has been contained.

### **General Discussion**

A number of panelists criticized specific features of the authors' model, including the statistical process assumed to govern stock returns and the way investors are assumed to use historical data in making

14. This is the main policy theme of Minsky (1986).

portfolio decisions. Christopher Sims argued that the model makes too sharp a distinction between big shocks and normal shocks. As a result the authors fail to identify three other shocks in the postwar period, one negative and two positive, that would be characterized as extraordinary shocks under a more flexible parameterization. Steven Durlauf reasoned that the authors should have allowed for the possibility of correlation between the normal and the extraordinary component of stock prices and then tested for the null hypothesis of no correlation. Charles Holt suggested the possibility of explaining the special features of the distribution of stock returns by making use of recent work in chaos theory. Sims observed that historical data do not do a good job of discriminating among the wide variety of statistical models that have been advocated by different investigators. Hence, he argued, the rational expectations assumption that there is a true stochastic process, and that everyone knows it, is not sensible. He suggested that research be directed toward models in which market participants are not sure of the underlying stochastic process and have different views about it.

Robert Hall noted that the authors assume that an investor's estimation of the underlying parameters of the stochastic returns process is made separately from his decision about optimal portfolio shares. He suggested assuming that investors integrate their estimation and decision problems. In the two-step process, investors use the estimated parameters as if they were known with certainty when they make their investment decisions. A one-step procedure would take account of the uncertainty in the parameter estimates. Sims thought that such a Bayesian procedure might alter radically predictions about investor behavior. Because the extraordinary shocks are so rare and uncertainty about the probability of their occurring so high, investors might act very "conservatively" and not make big shifts in their portfolios after one of the infrequent large shocks to market prices. This would be qualitatively different behavior from that predicted by the authors' model. William Poole commented that the nonnormality of the distribution of stock prices reinforces the importance of diversification in portfolios. With "fat-tailed" distributions, the gains from diversification, both among stocks and between stocks and other assets, are even greater than in the case of normally distributed returns.

George Akerlof directed attention to the paper's economic model of investor behavior, and questioned the assumption that expected returns

in the bond market are constant and therefore unaffected by a stock market crash. The increased estimate of risk after a crash would result in portfolio shifts toward bonds and therefore to further declines in stock prices as aftershocks. This is contrary to what happened in the crash of October 1987, which was followed by stock price increases instead. Benjamin Friedman responded that the model could be modified to include debt securities bearing default premiums. Historically, after a stock market crash precipitated by extensive defaults, the default premiums become large for a while and eventually fall to normal levels. This process could actually generate subsequent stock price changes as “aftershocks.” In the same vein, Matthew Shapiro thought more attention should be paid to the equilibrium requirement that the demand for and supplies of stocks be equal. He noted that in the authors’ model, stock is demanded as a proportion of wealth, and is affected by changes in expected return following a crash. After a decline in stock prices, the share of stocks in wealth has decreased, so investors will try to rebalance their portfolios. These movements in demand are certainly important for determining stock prices, but they are excluded from the model.

A number of comments were made about the implications of the paper for bubble theories of the stock market. If the market rides on positive bubbles that occasionally burst, the model predicts periods of zero autocorrelation of returns, with large negative deviations at infrequent intervals. Hence, Durlauf interpreted the positive serial correlation in the normal component of stock prices as evidence against bubbles. George von Furstenberg, by contrast, suggested that the paper helped rationalize bubbles. The estimated probability that the stock market will crash tends to decline as time passes after a crash. The decline in probability of the bubble’s bursting combined with an increasing size of the bubble is consistent with constant rationally expected returns.

Several panelists were disappointed that the paper did not have more to say about the causes of large and abrupt movements in the market, which are treated simply as random events in the basic model. Friedman said that a specific explanation of big market movements, especially in the context of the Minsky hypothesis, would entail examining other economic variables such as accumulated liabilities together with stock prices. It is hard to identify specific events that cause collapses, but the Minsky hypothesis is that collapses happen only when there is an excessive accumulation of liabilities. The intended treatment in this

paper is more general, allowing for collapses due to excessive liability accumulation or to “bursting bubbles” or to Shiller-type “epidemics”; but there is clearly a price to be paid for that generality. James Duesenberry argued for the importance of looking at “fundamental” variables in addition to looking at the market’s own behavior, suggesting that changes in investors’ views about future inflation and interest rates and earnings are central to an explanation of market movements. Shapiro contrasted the authors’ description of crashes with Fischer Black’s view of the recent crash. According to Black, the crash was caused by a flight to safety—a sudden decline in the demand for risky assets—that caused the change in stock prices, and not the other way around. The price level and the volatility of returns should be treated simultaneously.

## References

- Abramovitz, Moses. 1959. *Employment, Growth and Price Level Hearings, Part 2: Historical and Comparative Rates of Production, Productivity and Prices*, pp. 411–66. Hearings before the Joint Economic Committee. 86 Cong. 1 sess.
- Akgiray, Vedat, and G. Geoffrey Booth. 1987. “Compound Distribution Models of Stock Returns: An Empirical Comparison.” *Journal of Financial Research* 10:269–80.
- . 1988. “Mixed Diffusion-Jump Process Modeling of Exchange Rate Movements.” *Review of Economics and Statistics* 70:631–37.
- Baumol, William J. 1965. *The Stock Market and Economic Efficiency*. New York: Fordham University Press.
- Ball, Clifford A., and Walter N. Torous. 1983. “A Simplified Jump Process for Common Stock Returns.” *Journal of Financial and Quantitative Analysis* 18:53–65.
- Blanchard, Olivier J., and Mark W. Watson. 1982. “Bubbles, Rational Expectations, and Financial Markets.” In *Crises in the Economic and Financial Structure*, edited by Paul Wachtel. Lexington: Lexington Books.
- . 1986. “Are Business Cycles All Alike?” In *The American Business Cycle: Continuity and Change*, edited by Robert J. Gordon. Chicago: University of Chicago Press.
- Bollerslev, Tim P., Robert F. Engle, and Jeffrey M. Woolridge. 1988. “A Capital Asset Pricing Model with Time-varying Covariances.” *Journal of Political Economy* 96:116–31.
- Campbell, John Y. 1989. “Measuring the Persistence of Expected Returns.” Cambridge, Mass.: National Bureau of Economic Research.
- Cecchetti, Stephen G., Pok-sang Lam, and Nelson C. Mark. 1988. “Mean Reversion in Equilibrium Asset Prices.” Working Paper 2762. Cambridge, Mass.: National Bureau of Economic Research.
- Clark, Peter K. 1973. “A Subordinated Stochastic Process Model with Finite Variance for Speculative Prices.” *Econometrica* 41:135–55.
- Cox, John C., and Stephen A. Ross. 1976. “The Valuation of Options for Alternative Stochastic Processes.” *Journal of Financial Economics* 3:145–66.
- De Long, J. Bradford, and others. 1987. “Economic Consequences of Noise Traders.” Working Paper 2395. Cambridge, Mass.: National Bureau of Economic Research.
- Ederington, Louis H. 1986. “Mean-Variance as an Approximation to Expected Utility Maximization.” Washington University.
- Engle, Robert F. 1982. “Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation.” *Econometrica* 50:987–1008.

- Engle, Robert F., David M. Lilen, and Russell P. Robins. 1987. "Estimating Time Varying Risk Premia in the Term Structure: The ARCH-M Model." *Econometrica* 55:391-407.
- Fama, Eugene F. 1965. "The Behavior of Stock Market Prices." *Journal of Business* 38:34-105.
- Fama, Eugene F., and Kenneth R. French. 1988. "Permanent and Temporary Components of Stock Prices." *Journal of Political Economy* 96:246-73.
- Fama, Eugene F., and James McBeth. 1973. "Risk, Return, and Equilibrium: Empirical Tests." *Journal of Political Economy* 81:607-36.
- Feinstone, Lauren J. 1984. "Efficiency and Normality in the Futures Market in Foreign Exchange." University of Rochester.
- . 1985. "Minute by Minute: Efficiency, Normality, and Randomness in Intradaily Asset Prices." University of Rochester.
- Ferri, Piero, and Hyman P. Minsky. 1984. "Prices, Employment and Profits." *Journal of Post-Keynesian Economics* 6:489-99.
- Fisher, Irving. 1933. "The Debt Deflation Theory of Great Depressions." *Econometrica* 1:337-57.
- Friedman, Benjamin M., and Kenneth N. Kuttner. 1988. "Time-Varying Risk Perceptions and the Pricing of Risky Assets." Working Paper 2694. Cambridge, Mass.: National Bureau of Economic Research.
- Friedman, Milton. 1968. "The Role of Monetary Policy." *American Economic Review* 58:1-17.
- Friedman, Milton, and Anna Jacobson Schwartz. 1963. *A Monetary History of the United States*. Princeton: Princeton University Press.
- Frisch, Ragnar. 1933. "Propagation Problems and Impulse Problems in Dynamic Economics." In *Economic Essays in Honour of Gustav Cassel*. London: George Allan and Unwin, Ltd.
- Giordano, Ralph. 1989. *Debt without Disaster*. Economic Research, Financial Market Perspectives, Goldman Sachs.
- Ibbotson and Associates. 1989. *Stocks, Bonds, Bills, and Inflation. 1989 Yearbook*. Chicago.
- Jarrow, Robert A., and Eric R. Rosenfeld. 1984. "Jump Risks and the Intertemporal Capital Asset Pricing Model." *Journal of Business* 57:337-51.
- Keynes, John Maynard. 1937. "The General Theory of Employment." *Quarterly Journal of Economics* 51:209-23.
- . 1946. "The Theory of the Rate of Interest." *Readings in the Theory of Income Distribution*. Philadelphia: Blakiston.
- Kindleberger, Charles Poor. 1978. *Manias, Panics and Crashes: A History of Financial Crises*. New York: Basic Books.
- Lucas, Robert E., Jr. 1972. "Expectations and the Neutrality of Money." *Journal of Economic Theory* 4:103-24.
- Mandelbrot, Benoit. 1963. "The Variation of Certain Speculative Prices." *Journal of Business* 36:394-419.

- Meiselman, David. 1963. *The Term Structure of Interest Rates*. Chicago: University of Chicago Press.
- Merton, Robert C. 1971. "Optimum Consumption and Portfolio Rules in a Continuous-Time Model." *Journal of Economic Theory* 3:373-413.
- . 1976. "Option Pricing When Underlying Stock Returns are Discontinuous." *Journal of Financial Economics* 3:125-44.
- Minsky, Hyman P. 1977. "The Financial Instability Hypothesis: An Interpretation of Keynes and an Alternative to 'Standard' Theory." *Nebraska Journal of Economics and Business* 16:5-16.
- . 1972. "Financial Stability Revisited: The Economics of Disaster." *Reappraisal of the Discount Mechanism*, vol. 3. Washington: Board of Governors of the Federal Reserve System.
- . 1975. *John Maynard Keynes*. New York: Columbia University Press.
- . 1964. "Longer Waves in Financial Relations: Financial Factors in the More Severe Depressions." *American Economic Review Papers and Proceedings*, 1963 54:324-32.
- . 1986. *Stabilizing an Unstable Economy*. New Haven: Yale University Press.
- Muth, John F. 1960. "Optimal Properties of Exponentially Weighted Forecasts." *Journal of the American Statistical Association* 55:299-321.
- Nelson, Daniel B. 1989. "Conditional Heteroskedasticity in Asset Returns: A New Approach." University of Chicago.
- Pickard, David K., Peter J. Kempthorne, and Arshad R. Zakaria. 1986. "Inference for Jump Diffusion Stock Prices." *Proceedings of the Business and Economics Statistics Section of the American Statistical Association*.
- Poterba, James M., and Lawrence H. Summers. 1988. "Mean Reversion in Stock Prices: Evidence and Implications." *Journal of Financial Economics* 22:27-59.
- . 1986. "The Persistence of Volatility and Stock Market Fluctuations." *American Economic Review* 76:1142-51.
- Press, S. James. 1967. "A Compound Events Model for Security Prices." *Journal of Business* 40:317-35.
- Reitz, Thomas A. 1988. "The Equity Risk Premium: A Solution." *Journal of Monetary Economics* 22:117-31.
- Samuelson, Paul A. 1965. "Proof That Properly Anticipated Prices Fluctuate Randomly." *Industrial Management Review* 6:41-49.
- Schumpeter, Joseph Alois. 1939. *Business Cycles: A Theoretical, Historical, and Statistical Analysis of the Capitalist Process*. New York: McGraw-Hill.
- Schwert, G. William. 1989. "Indexes of United States Stock Prices from 1802 to 1987." Working Paper 2985. Cambridge, Mass.: National Bureau of Economic Research.
- Shiller, Robert J. 1984. "Stock Prices and Social Dynamics." *BPEA*, 2:1984, 457-98.

- Sims, Christopher A. 1989. "A Nine Variable Probabilistic Macroeconomic Forecasting Model." Discussion Paper 14. Minneapolis: Institute for Empirical Macroeconomics.
- Slutsky, E. 1937. "The Summation of Random Causes as the Source of Cyclic Processes." *Econometrica*.
- Tobin, James. 1989. Review of *Stabilizing an Unstable Economy* by Hyman Minsky. *Journal of Economic Literature* 27:105–08.
- Tucker, Alan L., and Lallan Pond. 1988. "The Probability Distribution of Foreign Exchange Price Changes: Tests of Candidate Processes." *Review of Economics and Statistics* 70:638–47.
- Viner, Jacob. 1936. "Mr. Keynes on the Causes of Unemployment." *Quarterly Journal of Economics* 51:147–67.
- West, Kenneth D. 1987. "A Specification Test for Speculative Bubbles." *Quarterly Journal of Economics* 102:553–80.