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# *Patent Renewal Data*

IN MANY COUNTRIES, including recently the United States, holders of patents must pay a renewal fee to keep their patents in force. If that fee is not paid in any one year, the patent is permanently canceled. Assuming that renewal decisions are based on economic criteria, agents will renew their patents only if the value of holding them an additional year exceeds the cost of renewal. Observations on the proportion of patents renewed at different ages, along with the relevant renewal fee schedules, will thus contain information on the distribution of the holding values of patents and on the evolution of this distribution over the life span of the patents. Since patent rights are seldom marketed, renewal data are one of the few sources of information on patent value. This paper considers how that information can be put to use.

In the first section we consider renewal models and their implications for what can (and cannot) be learned from renewal data. Renewal decisions are based on the patentees' perceptions of the value of the protection provided by the institutionally generated property rights created by the patent. In dealing with the modeling issue, therefore, we focus on the question of what the observed renewal data can tell us about the distribution of the value of patent rights and about differences in this distribution among different patents and in different institutional and economic environments. We begin by discussing the possibility of using

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patent renewal data to test for differences in the distribution of the value of patent protection, and to identify the stochastic process generating the returns to patent protection, when we are allowed to assume only mild regularity conditions on the functional forms of interest. (That is, we begin with a nonparametric analysis of the stopping model generating renewal behavior.) We provide simple nonparametric tests for the null hypotheses that (1) the distribution of the value of patent protection in one group of patents dominates that in another, and (2) there is no difference between the two distributions. The extent to which the renewal data can nonparametrically identify the form of the stochastic process generating the returns to patent protection is intricately tied up with the extent of variation in the renewal fee schedules. We show that, though with unlimited variation we can identify all aspects of this process, with limited variation we can provide information only on conditional means and not on entire conditional distributions. We conclude the section with a brief comparison of this nonparametric approach to the parametric approach taken by Pakes in 1986 and a discussion of when one approach (possibly one that integrates aspects of both of them) might be preferred.[1]

So far we have been concerned with our ability to use renewal data to analyze the value of patent protection. It is important to distinguish between the value of the protection provided by patents and the value of the ideas underlying the patents. For several reasons renewal data allow us to construct more accurate measures of the value of patented ideas than the measures obtained from the patent count indexes currently in use. The age dimension of the renewal data permits us to use separate counts of the number of patents renewed until different ages, instead of just a single count index for the total number of patents, to construct our measures of the value of patented ideas. Further, the weights used to aggregate the age-specific count indexes into a single overall measure of the value of patented ideas can be allowed to depend on the relevant renewal fee schedules.

In the second section of the paper we consider first the potential usefulness of renewal data in analyzing issues related to the value of patent protection and then their usefulness in analyzing issues related to the value of patented ideas. Both parts contain brief summaries of the

1. Ariel Pakes, "Patents as Options: Some Estimates of the Value of Holding European Patent Stocks," *Econometrica*, vol. 54 (July 1986), pp. 755–84.

existing literature, and both provide illustrative calculations of the extent to which renewal data are likely to help overcome some of the problems that arise in that literature.

The third section, which provides information on renewal data sources, contains the results of a survey questionnaire sent to more than one hundred national and regional patent offices to assess the accessibility of data on the renewals of patents under their jurisdiction. Since most patent documents give the international patent classification of the industry of the patent (the IPC code), and not the standard industrial classification (SIC) often used in empirical analysis, we also consider the concordances available to reclassify patents previously classified by IPC code into SIC codes.

The final section begins by providing a statistical procedure for integrating into the statistical analysis the classification error these concordances generate. It then presents some new empirical results from the analysis of a data set on Norwegian and Finnish renewals obtained from the respective patent offices by Margaret Simpson. Like all the newer data sets reviewed in the third section, but unlike the data used in published results to date, the Scandinavian data contain disaggregated information on the characteristics of the patentee and the type of patent. So our analysis focuses on nonparametric tests for differences in renewal behavior among groups of patents classified by their industry, their cohort (date of application), and the nationality of the patent holder. The analysis of the first section allows us to interpret our tests in terms of differences in the underlying distribution of patent values.

There are three main findings. First, though there are distinct differences in the renewal behavior of patents filed by residents of different countries (with some countries clearly dominating others), once we control for industry the differences among nationalities largely disappear, indicating that they are chiefly the result of different nationalities tending to patent in different industries. The exception here is patents taken out by residents of the granting countries. These tend to be renewed for shorter periods, possibly because lower effective application costs induce residents of the home country to apply for patents on inventions whose patent rights are less likely to be as valuable. Second, we nonetheless find distinct differences in the distribution of patent values among patents in different industries, whether or not we condition on nationality. We summarize the information on industry differences by providing an intuitive definition for the statement that one stochastic process generating patent values is better than another, and then, where possible, ordering the industries according to this definition. (The definition does not define a complete order, so that sometimes we cannot tell whether one stochastic process is preferred to another without more detailed assumptions on renewal fee schedules, discount rates, and precise functional forms.) Third, we find that distinct differences exist in the value distributions of different cohorts of patents, and that these differences support earlier evidence on there being a negative correlation between the number of patents in a cohort and the average value (or quality) of the patents in that cohort.[2] (These points remain valid whether or not we condition on industry.) This point should be kept in mind when one considers the implications of the fall in patent applications, and the almost drastic cut in patent-to-R&D ratios, observed in almost all Western countries from the late 1960s to the late 1970s.[3] In particular, the fall in patents may not indicate as strong a fall in technological opportunities as one would assume, which, in turn, makes a fall in technological opportunities a less powerful potential explanation of the observed productivity slowdown. At the end of the paper, we briefly discuss alternative economic models that could lead to the observed inverse relationship between the quantity and the quality of the patents in different cohorts.

To conclude this introduction, we provide some background information on the renewal laws, the type of information available on the documents that are the source of the renewal data, and the work that has been done using that data. After being confronted with the existence of renewal fees, an economist might first ask how they are set, or what an optimal renewal fee schedule would look like. Somewhat surprisingly this issue has been discussed very little, either by economists or, apparently, by administrators. A subset of the set of possible renewal fee schedules is the class of schedules that is zero until some statutory limit to the length of patent lives, say $L$, and "infinity" thereafter. The

2. See Mark Schankerman and Ariel Pakes, "Estimates of the Value of Patent Rights in European Countries during the Post-1950 Period," *Economic Journal*, vol. 96 (December 1986), pp. 1052–76.

3. For a summary of the data see Robert E. Evenson, "International Invention: Implications for Technology Market Analysis," in Zvi Griliches, ed., *R&D, Patents, and Productivity* (University of Chicago Press, 1984).

original work of Nordhaus and subsequent developments by Tandon analyze the issue of the optimal fee schedule within this class (or the choice of $L$) for some simple environments.[4] The only developed country we know that limits itself to this class of renewal fee schedules is Canada; and Canada is now in the process of changing its patent law to require payment of positive fees in earlier ages. The United States instituted renewal fees in laws of 1980 and 1982 (P.L. 96-517 and P.L. 97-247). Renewal fees have been a feature of patent law in most European (and many other) countries for a long time.

The U.S. government kept two objectives in mind when setting the renewal fees: to enable the patent office to cover its costs and not to impose an undue financial burden on the patentee (particularly patentees that were small businesses). The U.S. law differs from the law in most European countries in several respects. It requires payment of a renewal fee only at three times over a seventeen-year potential life span for the patent, and its fee schedule differentiates between small businesses and large corporations. Almost all other countries have an annual renewal fee, a twenty-year potential life span for renewing, and a single fee schedule regardless of the type of patent or patentee. Patent laws also differ among countries in many other ways, and various policy questions surround the issue of the appropriateness of different patent restrictions for different settings (see the literature cited in later sections).

The information listed on the patent documents that form the basis for the renewal data sets also varies somewhat among countries. The German documents are fairly typical for Europe. They include the name and address of the patentee (be it an individual or a business; some data bases also have a code for whether the patentee is a major corporation or a minor corporation by the extent of its patenting) and information on the dates of application, grant, and lapse of the patent; on whether the patent right was contested (there is a formal contesting procedure in Germany); on the IPC of the patent (a rather detailed classification based on the technical field of the patent); on the country and date of first application for a patent on the given idea; and on the other countries the patent has been taken out in and its status in those countries. Note that the German, and in fact most, renewal data contain enough detail to

4. William D. Nordhaus, *Invention, Growth, and Welfare: A Theoretical Treatment of Technological Change* (MIT Press, 1969); and Pankaj Tandon, "Optimal Patents with Compulsory Licensing," *Journal of Political Economy*, vol. 90 (June 1982), pp. 470–86.

allow researchers to use them at all the traditional levels of aggregation: firm, region, industry, and economy. (For the industry level of analysis the concordance between the IPC classes and the more traditional SIC classes is necessary.) Note also that the renewal data have the detail required to enable us to study the application and renewal behavior of the same patent in different countries.[5]

To date, published work on patent renewals has not used data sets with the rich microeconomic detail currently available. The initial 1984 paper by Pakes and Schankerman used a deterministic model of patent renewal, a model in which the returns that will be earned should the patent be kept in force decay deterministically over time, and aggregated international and intercohort differences in renewal behavior and renewal fee schedules to estimate the rate of obsolescence in the returns to patents. In 1986 Pakes extended this framework to allow patent holders to be uncertain about the sequence of returns that will accrue to the patent if it is to be kept in force.[6] This uncertainty is introduced to allow for the fact that agents often apply for patents at an early stage in the innovation process, when an agent is still exploring alternative opportunities for earning returns from the information embodied in the patented idea. Early patenting arises partly from the incentive structure created by the patent system: if the agent does not patent the information available to him, somebody else might. Reinforcing this incentive is the fact that the renewal fees in all countries studied were quite small during the early years of a patent's life.

In an uncertain world a patent holder who pays the renewal fee obtains both the current returns that accrue to the patent over the coming period and the option to pay the renewal fee and maintain the patent in force in the following period should he or she desire to do so. A patentee who acts optimally will pay the renewal fee only if the sum of the current returns plus the value of this option exceeds the renewal fee. The model assumes that the patentee values the option at the expected discounted value of future net returns (current returns minus renewal fees), taking into account that an optimal policy will be followed in each future period,

5. See Jonathan Putnam, "International Differences in the Value of Patent Protection: An Empirical Analysis of Renewals of the Same Patents in Different Countries," Ph.D. dissertation, Yale University, 1989.

6. Ariel Pakes and Mark Schankerman, "The Rate of Obsolescence of Patents, Research Gestation Lags, and the Private Rate of Return to Research Resources," in Griliches, ed., *R&D, Patents, and Productivity*, pp. 73–88; and Pakes, "Patents as Options."

and conditional on the information currently at the disposal of the agent. An optimal sequential policy for the patentee has the form of an optimal renewal (or stopping) rule, a rule determining whether to pay the renewal fee at each age. The proportion of patents that drop out at age $a$ corresponds to the proportion of patents that do not satisfy the renewal criteria at that age but did so at age $a - 1$. Pakes's model implied that the dropout proportions predicted by the model were a function of the precise value of the model's parameter vector and the renewal fee schedule. The data provided the actual proportion of dropouts. The estimation problem consisted, roughly speaking, of finding those values of the model's parameters that made the dropout proportions implied by the model as "close" as possible to those actually observed.

Pakes's estimates implied that most of the uncertainty associated with the returns to patent protection was resolved before the fifth year of the patent's life. Using this result, Schankerman and Pakes examined only renewal decisions after age five, used the simpler deterministic model, and looked for changes in the value distribution over time and correlates of these changes.[7] They also initiated the discussion on using renewal data to overcome some of the measurement problems that have made empirical analysis of the process generating technological change so difficult.

We have tried to structure this paper so that the separate sections are understandable in isolation. That is, the reader ought to be able to go directly to any of the four sections and obtain a reasonably self-contained report. Again, the first section discusses the modeling issues, the second the potential usefulness of patent renewal data as a measure of the value of both patent protection and patented ideas, and the third the availability of patent renewal data. The final section provides empirical results on differences in the stochastic process generating the value of patent protection among industries and cohorts, and between patentees of different nationalities.

## Patent Renewal Models

In this section we show what can be learned from combining alternative modeling assumptions with the renewal data. Throughout we use the general modeling framework detailed in section 2 of Pakes, "Patents

7. Schankerman and Pakes, "Estimates of the Value of Patent Rights."

as Options.'' It allows patentees to be uncertain about the sequence of returns that will accrue to the patent should it be kept in force, and assumes that renewal decisions are made to maximize the expected discounted value of net returns (returns minus renewal fees) from the patent.

To fix notation, let $V(\alpha)$ be the expected discounted value of patent protection to the patentee just before the patent's $\alpha$th renewal. If the renewal fee is not paid, the patent lapses and $V(\alpha) = 0$. If the renewal fee is paid, the patentee earns the current return to patent protection and also maintains the option to renew and keep the patent in force at age $\alpha + 1$. The value of this option equals the expected discounted value of the patent at age $\alpha + 1$ (that is, of $V[\alpha + 1]$) conditional on current information. Formally then,

$$(1) \quad V(\alpha) = \max\{0, r_\alpha + \beta E[V(\alpha + 1) / \Omega_\alpha] - c_\alpha\} \quad (\alpha = 1, \ldots, L),$$

where $L$ is the statutory limit to patent lives, $r_\alpha$ is the current return to patent protection, $E$ is the expectation operator, $\Omega_\alpha$ is the information set of the agent in the patent's $\alpha$th year, and $c_\alpha$ is the cost of renewal.[8] In the equation, $r_\alpha + \beta E[V(\alpha + 1) / \Omega_\alpha]$ is the total benefit from holding the patent: the sum of current returns and the discounted value of the option. If this expression is less than $c_\alpha$, the agent lets the patent lapse.

To complete the description of the value function, we must specify the conditional distributions of future returns and costs of renewal. Given these distributions, the solution for the sequnce $[V(\alpha)]_{\alpha=1}^{L}$ can be obtained by starting with the terminal equation, $V(L) = \max(0, r_L - c_L)$, and integrating the system in equation 1 backward recursively.

For the purpose of this discussion we assume that agents hold point expectations on the renewal fees that will be required to keep the patent in force at later ages equal to the current real renewal fee for those ages. This assumption simplifies the analysis considerably. Moreover, it can be motivated by the facts that renewal fee schedules are published data and that in all countries studied the real renewal fee at any age does not vary much with the year the patent reaches that age. As for the returns from patent protection, we assume that the stochastic process generating them is a (not necessarily stationary) Markov process. That is, the

8. It is understood that zero is an absorbing state in the stochastic process generating $[V(\alpha)]_{\alpha=l}^{L}$ (so that if the patent is not renewed at any age it will not be in force thereafter).

probability that next year's returns are greater than any given $x$ conditional on current information depends only on current returns and age. Then for each age we have the family of distributions

(2a) $$\mathbb{P}_\alpha = [P_\alpha(\cdot \,/\, r), r \in R_+],$$

where $P_\alpha(\cdot \,/\, r)$ provides the distribution of $r_{\alpha+1}$ conditional on $r_\alpha = r\,[P_\alpha(\cdot \,/\, \cdot)\colon R_+ \times R_+ \rightarrow (0,1)]$. Finally, to start off the process we need a distribution of initial returns, say $P_O(\cdot)$. So the stochastic process generating returns is an $L$-tuple, say $\mathbb{P}$, where

(2b) $$\mathbb{P} = [P_O(\cdot), \mathbb{P}_1, \ldots, \mathbb{P}_{L-1}].$$

Throughout we assume that $\mathbb{P}$ has the property that the family of conditional distributions for $r_{\alpha+1}$ given $r_\alpha$ is stochastically increasing in $r_\alpha$ [for $\alpha = 1, \ldots, L-1$]. That is, we assume that the probability that future returns are greater than any $x$ is larger the higher the current returns, so that if $r_\alpha \geq r'_\alpha$ then

(3) $$P_\alpha(r_{\alpha+1} \geq x \,/\, r_\alpha) \geq P_a(r_{a+1} \geq x \,/\, r'_\alpha)$$

for any $(x, \alpha)$.

Equations 1 through 3 provide the general framework set out in Pakes, "Patents as Options." Section 3 of that paper then makes assumptions that determine the structure of $\mathbb{P}$ up to the values of seven parameters and provides an estimator for those parameters. We come back to this parametric approach after an investigation of whether equations 1 through 3, in and of themselves, provide enough structure to answer some questions of interest.

One such question is whether these equations (perhaps with some additional regularity conditions) enable us to construct tests of the hypothesis that one group of patents has a "better" $\mathbb{P}$ than another (groups here could be differentiated by industry, date of application, patentee, and so on). We show below that it is not only possible but also quite easy to construct such tests. In the last section of the paper we use the test developed here to look for interindustry, intercohort, and international (by nationality of patentee) differences in the value distributions of postwar Norwegian and Finnish patents.

Given these results on testing, we can then ask whether the assumptions contained in the equations are sufficient to identify the precise form of the processes that govern the distribution of returns from patent protection (that is, the component functions of $\mathbb{P}$ and the value of $\beta$). Later we show that (again under mild regularity conditions) these assumptions do indeed suffice to identify both $\mathbb{P}$ and $\beta$, at least if there is enough variation in the observed renewal fee schedules. If there is not enough variation in these schedules, we can identify only certain properties of $\mathbb{P}$.

Since the renewal fee schedules show limited variation in all the data sets we are aware of, we conclude with a semi- (or partially) parametric set of assumptions that allow us to identify more characteristics of $\mathbb{P}$ when the renewal fee schedules are assumed to take on only a limited range of values. The appeal of our procedure is twofold. First, it provides information on objects of inherent interest (the expectation of future returns conditional on current information), and second, it requires only parametric assumptions on the conditional probabilities of future returns being less than the values of future costs that are actually observed in the data. (No parametric assumptions are made on the form of the conditional distribution of returns in a range greater than the observed range of renewal fees.)

The semiparametric analysis also allows us to mitigate two other problems that appear when we try to actually use the estimators suggested by the nonparametric identification analysis. First, the nonparametric analysis abstracts from the estimation problems induced by small (finite) sample sizes. The more detailed our data, the more we will want to allow for differences in $\mathbb{P}$ resulting from differences in observed characteristics, and the smaller the sample sizes for patents with given values of those characteristics will be. Second, though the nonparametric analysis can, at least in principle, allow for both unobserved differences in the initial returns among patents and differences in the Markov process generating subsequent returns associated with observed differences in the characteristics of the patent (or its economic environment), it cannot allow for (general forms of) differences in the Markov process not associated with observed characteristics. The semiparametric analysis provides a natural framework for both: integrating the effects of unobserved heterogeneity into the estimation algorithm and conserving on the number of parameters to be estimated.

To simplify the analysis, the discussion of both testing and identification will focus on the special case where $L = 2$. (A simple inductive argument can be used to extend our results to the case of any finite $L$.)

Recall from equation 1 that agents renew their patents if and only if $V(\alpha) > 0$. In a two-period model, $V(2)$ will be $r_2 - c_2$ if $r_2 > c_2$ and zero otherwise, so that a patentee will renew in the initial year if and only if

(4a) $$r_1 + \beta \int_{c_2} (r_2 - c_2) P(dr_2 / r_1) - c_1 \geq 0.$$

Note that since the Markov property (equation 2) ensures that the distribution of $r_2$ depends only on $r_1$, the left-hand side of equation 4a depends only on $r_1$ and parameters that do not vary among patents. Also since equation 3 ensures that the distribution of $r_2$ conditional on $r_1$ is "better" the larger is $r_1$, the integral in equation 4a, and hence the entire left-hand side of that equation, will be increasing in $r_1$. This ensures that there will be a unique lowest value of $r_1$ that satisfies equation 4a. Let that value be $\bar{r}_1 = \bar{r}(c_1, c_2, \beta)$. If $r_1 \geq \bar{r}_1$, a patentee will find it profitable to renew in the initial period. Summarizing then, the renewal rules in the two periods can be written as

(4b) $$V(1) > 0 \text{ if and only if } r_1 \geq \bar{r}_1,$$

and

(4c) $$V(2) > 0 \text{ if and only if } r_2 > c_2.$$

We now move on to our discussions of testing and identification. The testing subsection begins by defining what we mean when we say that one stochastic process generating the returns from patent protection is better than, or dominates, another. The definition ensures that if the process for patents of type I dominates that for patents of type II, the fraction of type I patents whose value will be larger than any given $x$ will be greater than the fraction of type II patents. We then provide two renewal curve–based tests: one for the equivalence of the stochastic process generating returns from two groups of patents, and the other for the process generating the returns from type I patents dominating that of type II patents. As might be expected, the first is a test for the equality of the renewal curves (the fraction paying the renewal at each age) generated by the patents from the two groups, and the second is a test for the null that the renewal curve for type I patents lies entirely above that for type II patents. The discussion of identification does not require any of the results from the testing subsection, so that the reader for whom this brief summary has sufficed may go on to the subsection on identification.

*Testing*

Let $\mathbb{P}^{\mathrm{I}}$ be the Markov process associated with patents of type I, and $\mathbb{P}^{\mathrm{II}}$ be the Markov process associated with patents of type II. Then we say $\mathbb{P}^{\mathrm{I}}$ is better than, or dominates, $\mathbb{P}^{\mathrm{II}}$ and write

$$\mathbb{P}^{\mathrm{I}} \geq_s \mathbb{P}^{\mathrm{II}}$$

if and only if for every $x$

$$P_0^{\mathrm{I}}(r_1 \geq x) \geq P_0^{\mathrm{II}}(r_1 \geq x), \tag{5a}$$

and for each possible triple $(x, r_2, r_1)$,

$$P^{\mathrm{I}}(r_2 > x / r_1 = r) \geq P^{\mathrm{II}}(r_2 > x / r_1 = r). \tag{5b}$$

$\mathbb{P}^{\mathrm{I}}$ stochastically dominates $\mathbb{P}^{\mathrm{II}}$ if the proportion of the initial returns that are greater than any given $x$ is larger for type I patents, or $P_0^{\mathrm{I}}(\cdot)$ stochastically dominates $P_0^{\mathrm{II}}(\cdot)$, and if for every $r_1$ the probability that $r_2$ is greater than any given $x$ is larger for type I patents, or $P^{\mathrm{I}}(\cdot / r_1)$ stochastically dominates $P^{\mathrm{II}}(\cdot / r_1)$.

If $\mathbb{P}^{\mathrm{I}} \geq_s \mathbb{P}^{\mathrm{II}}$, then it can be shown that no matter the value of the vector $(c_1, c_2, \beta)$, we have for any $x$ (and $\alpha = 1$ or 2)

$$Pr^{\mathrm{I}}[V(\alpha) \geq x] \geq Pr^{\mathrm{II}}[V(\alpha) \geq x].$$

That is, the proportion of type I patents that have value greater than any $x$ is larger than the same proportion of type II patents (and this at any age). Thus if $P^{\mathrm{I}} \geq_s P^{\mathrm{II}}$, it is natural to claim that type I patents are more valuable than type II patents.

Let $\pi(\alpha)$ be the proportion of the population of patents that the model predicts to drop out at age $\alpha$, that is, the proportion predicted to pay the renewal fee at $\alpha - 1$ but not at $\alpha$. Then the test we build for the hypothesis that patents of one type are more valuable than patents of another type is based on the following proposition.

*Proposition 1 (stochastic orderings and renewal propositions).* If $P^{\mathrm{I}} \geq_s P^{\mathrm{II}}$, then no matter $(\beta, c_1, c_2)$

$$\begin{bmatrix} \pi^{\mathrm{I}}(1) \\ \pi^{\mathrm{I}}(2) + \pi^{\mathrm{I}}(1) \end{bmatrix} \leq \begin{bmatrix} \pi^{\mathrm{II}}(1) \\ \pi^{\mathrm{II}}(2) + \pi^{\mathrm{II}}(1) \end{bmatrix}.$$

The proof follows from equations 1, 3, 4, and 5.

The proposition states that if $P^{I} \geq_{s} P^{II}$, or if patents of type I are more valuable than patents of type II, then no matter the discount rate or the renewal fee schedule, a larger proportion of type I patents should be in force at every age. This is an easy proposition to test. A simple first cut would be to plot the proportion renewing of a random sample of patents of each type at each age and see if, at every age, type I patents have a larger proportion renewing than type II patents.

Building a formal statistical test is not much harder. Note first that each patent will drop out at one of the renewal ages or pay the final renewal and stay in force until the statutory limit to patent lives. This set of possible outcomes is mutually exclusive and exhaustive and therefore defines a multinomial distribution with cell probabilities

$$\pi(1), \ \pi(2), \ \text{and } 1 - \sum_{\alpha=1}^{2} \pi(\alpha).$$

Let $\pi$ be the vector of these probabilities, so that if

$$J = \begin{bmatrix} 1\,0\,0 \\ 1\,1\,0 \end{bmatrix},$$

then $J\pi$ would provide the vector of proportions that the model predicts drop out by each age. Now draw a random sample of size $n$ from the population of interest and let $f_n$ be the vector of sample proportions falling into each cell. Then $Jf_n$ provides the proportion renewing to each age. Moreover, a standard application of the central limit theorem for multinomial distributions implies that

(6a) $$\sqrt{n}\,(f_n - \pi) \sim> N(0, V),$$

so that

(6b) $$\sqrt{n}\,J(f_n - \pi) \sim> N(0, \ JVJ'),$$

where $\sim$ reads converges in distribution to, $N(\cdot\ ,\ \cdot)$ is the standard normal distribution, and $V = \text{diag}\ \pi - \pi\pi'$, where diag $x$ is a diagonal matrix with the vector $x$ on the principal diagonal.[9]

Now say we had two independent samples, one of patents of type I and one of patents of type II. A reasonable testing sequence would be to

9. Note that $V^* = \text{diag}\, f_n - f_n f_n'$ is a consistent estimate of $V$, so that we could substitute the observable $V^*$ for the unobservable $V$ in equation 6 without affecting any of the distributional or testing results.

first ask if one can assume that they are both random samples from the same $\pi$. That is, we would first ask if we could maintain the null

(7a) $$H_0^E: \pi^{\mathrm{I}} = \pi^{\mathrm{II}}.$$

Equation 6a implies that standard tests for the equality of means from independent samples taken from normal distributions with known variances can be used to produce a chi-square test for the hypothesis in equation 7a with desirable asymptotic properties. If equation 7a is rejected, we might want to go on to test whether the data are consistent with the statement that type I patents are more valuable than type II patents. Proposition 1 suggests that an implication of this is the null hypothesis that

(7b) $$H_0^S: J\pi^{\mathrm{I}} - J\pi^{\mathrm{II}} \leq 0.$$

The recent econometric literature on testing subject to inequality constraints, together with equation 6b, makes it relatively straightforward to provide a test statistic for the null in equation 7b.[10]

The testing procedures discussed thus far assume that the two groups of patents being compared face the same renewal fee schedules. Since in almost all countries studied renewal fees do not vary among patents of a given age in a given period, these tests suffice to test for differences among patents applied for at the same date. The following corollary of proposition 1 helps to test for intercohort differences in value distributions.

*Corollary 1 (stochastic ordering and renewal proportions: intercohort comparisons).* Let $C^{\mathrm{I}}$ and $C^{\mathrm{II}}$ be the renewal fee schedules faced by patents of type I and type II respectively. Then (a) if $\mathbb{P}^{\mathrm{I}} \geq_s \mathbb{P}^{\mathrm{II}}$ and $C^{\mathrm{I}} \leq C^{\mathrm{II}}$, $J\pi^{\mathrm{I}} \leq J\pi^{\mathrm{II}}$; while (b) if $C^{\mathrm{I}} \geq C^{\mathrm{II}}$, and $J\pi^{\mathrm{I}} \leq J\pi^{\mathrm{II}}$, $\mathbb{P}^{\mathrm{II}} \not\geq_s \mathbb{P}^{\mathrm{I}}$.

Part a of the corollary replaces the assumption that $C^{\mathrm{I}} = C^{\mathrm{II}}$, which is implicit in proposition 1, with the inequality $C^{\mathrm{I}} \leq C^{\mathrm{II}}$ and notes that this does not alter the conclusion of the proposition. Part b notes that if $C^{\mathrm{I}} \geq C^{\mathrm{II}}$ and we still have $J\pi^{\mathrm{I}} \leq J\pi^{\mathrm{II}}$ (larger fractions of type I patents renewing), we at least know that the value distribution of type II patents

10. For an explanation of how to build such a test and for references to the more detailed statistical literature, see Ariel Pakes and Richard Ericson, "Empirical Implications of Alternative Models of Firms Dynamics," SSRI Discussion Paper 8803 (University of Wisconsin, 1987).

does not dominate that of type I patents. (The testing procedure for the corollary is analogous to that for the original proposition.)

### *Identification*

How much can we learn about the stochastic process generating the returns from patent protection from patent renewal data? The answer is clear. Provided we are willing to make the assumptions in equations 1 through 3 and impose some mild regularity conditions, patent renewal data do, at least in principle, contain enough information to allow us to learn everything there is to know about the stochastic process generating the returns to patent protection (and can also determine the discount factor). We stress the term *in principle* here because this result emerges from an identification analysis, one that relies totally on the internal logic of the model and never asks about the extent to which existing data are rich enough to actually unravel the objects of interest.

The data can be deficient in at least two dimensions. First, existing sample sizes can be too small, which leads to imprecise estimates of the calculated objects. Second, there can be insufficient variation in the explanatory variables, which leads to an inability to estimate parts of the surface of interest. As noted earlier, the analysis of patent renewal data done to date has been on aggregate cohorts of patents, and sample size has been very large (about 20,000 to 40,000 patents per cohort). Of course, the fact that we now have patent renewal data with more detail on the underlying patent, together with testing procedures such as those just described, may move us in the direction of estimating separate stochastic processes for different types of patents, and provided no further restrictions are imposed, that will reduce the relevant sample sizes significantly (see the empirical results in our final section). The explanatory variables in our analysis are the renewal fee schedules. These are subject to government control and in most countries studied do not change much in real terms over the period of analysis. This lack of variance in the renewal fee schedules limits what can be learned about the stochastic process generating the returns from patent protection from patent renewal data, at least what can be learned without imposing further functional form restrictions. Nonetheless, we can limit the identification analysis to the logical implications of the responses to the *observed* range of renewal fee schedules and still identify certain aspects of the stochastic process generating the returns to patent protection. Moreover, if we are willing to maintain parametric assumptions on *only* that part of the conditional distributions of future returns actually swept out by the observed variation in renewal fee schedules, we can identify more aspects of the relevant stochastic process.

Let $\Pi_1 = 1 - \pi_1$, and $\Pi_2 = 1 - \pi_1 - \pi_2$, that is, $\Pi_1$ and $\Pi_2$ are the proportion of the patents that the model predicts to pay the renewal fee in year one and year two, respectively. Note that $\Pi_\alpha = \Pi_\alpha(c_1, c_2, \mathbb{P}, \beta)$, where $\mathbb{P}$ contains the distribution functions that determine the stochastic process generating the returns to patent protection. The identification question, then, is the following. If we had a free hand to vary the renewal fee schedules, that is, the couples $(c_1, c_2)$, what could we learn about $\beta$ and $\mathbb{P}$ from the $(\Pi_1, \Pi_2, c_1, c_2)$ quadruples that we generate?

To analyze this question in a context that enables us to disregard regularity conditions on the smoothness and boundedness of the relevant functions, we impose the additional assumption that the stochastic process generating the returns to patent protection is a finite-state Markov chain with a discrete initial distribution (on the points $x_1, \ldots, x_K$). That is, we assume

(8a) $$Pr(r_1 = x_k) = p_k,$$

with $\sum_k p_k = 1$, and that there exists $(y_1, \ldots, y_J)$ such that

(8b) $$Pr(r_2 = y_j / r_1 = x_k) = p_{k,j},$$

with $\sum_j p_{k,j} = 1$, for $k = 1, \ldots, K$.

Recall that we are free to choose $(c_1, c_2)$ couples. Note first that if we choose $c_2 = c^* > y_J$, there is no probability that any patent will pay the renewal in the second period. As a result, initial renewals will be paid only if initial returns are greater than the initial renewal fee (see the decision rules in equations 4a and 4b). This implies that

(9a) $$\Pi_1(c_1, c^*) = \sum_k \mathrm{I}(x_k \leq c_1) p_k,$$

while

(9b) $$\Pi_2(c_1, c^*) = 0,$$

where I $(\cdot)$ is the indicator function that takes the value of one if the logical condition inside it is satisfied, and zero elsewhere. Clearly then, if we hold $c_2 = c^*$ and vary $c_1$, equation 9a will allow us to trace out the

$K$ couples $(x_i, p_i)$ (for $i = 1, \ldots, K$), or the initial distribution we are after.

It is helpful here to introduce notation for the positive part of a function, say $f^+(x) = \max[0, f(x)]$. Then for any $c_2 \geq 0$, let $\phi_K(c_2, \beta)$ be the unique value of $c_1$ that satisfies

$$\phi_K(c_2, \beta) = x_K + \beta \Sigma (y_j - c_2)^+ p_{K,j}. \tag{10}$$

If $(c_1, c_2) = [\phi_K(c_2, \beta), c_2]$, then the stopping value for $r$ in the initial year (that is, the value of $r$ for which $r + \beta E[V(\alpha + 1) / r] = c_1$) equals $x_K$. In this case a patentee with initial returns equal to $x_K$ will be just indifferent between paying the renewal fee and dropping out (see equation 4b; for ease of exposition assume that in this situation the agent renews). Note that since both current returns and the distribution of future returns are better the higher is $x$ (see equation 3), patentees with initial returns less than $x_K$ will *not* renew. As a result, the $(\Pi_1, \Pi_2)$ couples generated from the combination $[\phi_K(c_2, \beta), c_2]$ are

$$\Pi_1 = p_K \tag{11a}$$

and

$$\Pi_2 = p_K \sum_j \mathrm{I}(y_j > c_2)\, p_{K,j}. \tag{11b}$$

It can be shown (see figure 1 below) that as we move $c_2$ from 0 to $y_J$, the values of $c_1$ given by $\phi_K(c_2, \beta)$ remain positive and finite (and hence are possible values for $c_1$). But equations 11a and 11b make it clear that if we do increase the schedule $(\phi_K(c_2, \beta), c_2)$ in this fashion, we will sweep out the entire sequence of couples $[(p_{K,j}, y_j)]_{j=1}^{J}$; or the distribution of $r_2$ conditional on $r_1 = x_K$. Note that, in practice, the way we would find $[(p_{K,j}, y_j)]_{j=1}^{J}$ is that for each fixed $c_2$ we would increase $c_1$ until $\Pi_1 = p_K$.

To go further, go back to equation 10 and replace $K$ with $K - 1$ so that $\phi_{K-1}(c_2, \beta)$ is the value of $c_1$ that would induce only those patents with initial returns equal to $x_K$ or $x_{K-1}$ to pay the renewal fee. A similar argument to that given above shows that the $(\Pi_1, \Pi_2)$ couples generated from $[\phi_{K-1}(c_2, \beta), c_2]$ are

$$\Pi_1 = p_K + p_{K-1} \tag{12}$$

$$\Pi_2 = p_K \sum_j \mathrm{I}(y_j > c_2)\, p_{K,j} + p_{K-1} \sum_j \mathrm{I}(y_j > c_2)\, p_{K-1,j}.$$

Moreover, since the previous calculations determined $p_K$, $p_{K-1}$ and the sequence $[(p_{K,j}, y_j)]_{j=1}^{J}$, this calculation suffices for $[(p_{K-1,j}, y_j)]_{j=1}^{J}$, or the distribution of $r_2$ conditional on $r_1 = x_{K-1}$. This procedure can be repeated with the obvious modifications to determine the entire family of conditional distributions, that is $[(p_{k,j}, y_j)]_{j=1}^{J}$, for $k = 1, \ldots, K$.

We have demonstrated that the quadruples $(\Pi_1, \Pi_2, c_1, c_2)$ suffice to identify both the initial distribution, or $(p_k, x_k)_{k=1}^{K}$, and the family of conditional distribution, or $(p_{kj}, y_j)_{j=1}^{J}$ for $k = 1, \ldots, K$. These, in turn, determine the Markov chain ($\mathbb{P}$) we are after. Moreover, given $\mathbb{P}$, it is easy to go back to equation 10, find the precise value of $(c_1, c_2)$, which sets $x_k + \beta \Sigma (y_j - c_2)^+ p_{k,j} - c_1 = 0$, and solve this equation for $\beta$. Since $(\beta, \mathbb{P})$ and the observed renewal fee schedules determine all renewal behavior, we have just shown that all the relevant parameters are identified, at least if we are allowed to use the information that could be generated from arbitrary choices of renewal fee schedules.

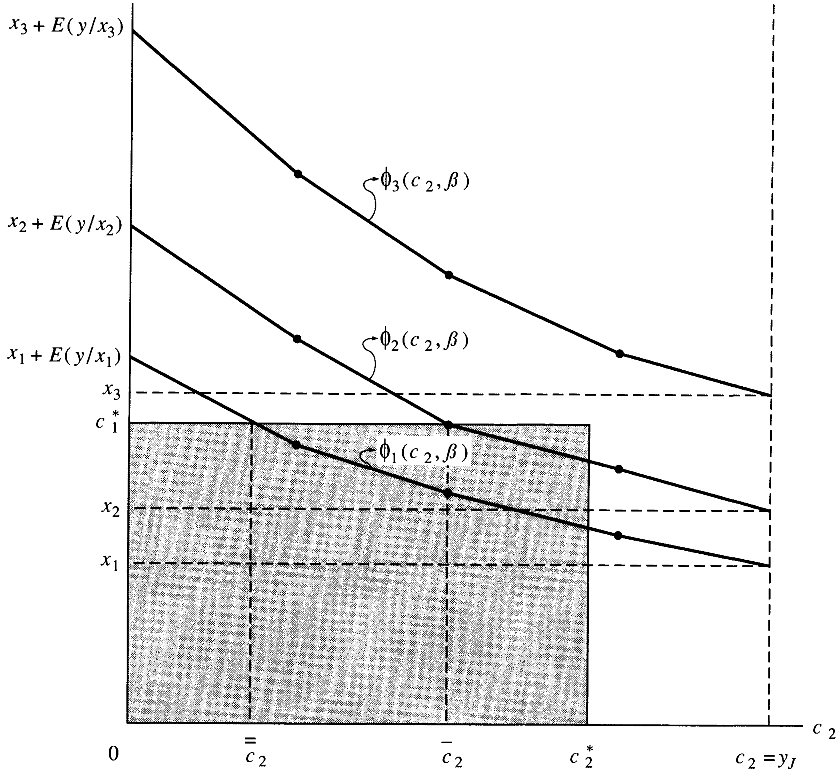
As noted at the outset, in any actual data set the range of the observed renewal fee schedules is limited. This fact raises the question which of the properties of $(\beta, \mathbb{P})$ can be identified if we are allowed to vary only $(c_1, c_2)$ in a restricted way. Note that there is a trivial answer to this question. For every $(c_1, c_2)$ we can always identify the fraction of the initial distribution with an $x$ for which $x + \beta \Sigma (y_j - c_2)^+ p_{xj} \geq c_2$, and the probability that $y \geq c_2$ conditional on $x$ satisfying this condition, by simply examining $\Pi_1 (c_1, c_2)$ and $\Pi_2(c_1, c_2)$. We want to know if we can go further than this.

Figure 1 is designed to help us analyze identification when we are allowed to use only the information obtained from a restricted subset of the possible $(c_1, c_2)$ values. For simplicity we assume that, though we do not know the possible values of $x_i$, we do know that there are only three of them. (This assumption can be relaxed at the cost of some added notation.) The curves labeled $\phi_i(c_2, \beta)$ in the figure provide the $(c_1, c_2) = [\phi_i(c_2, \beta), c_2]$ combinations that would make a patent with initial draw equal to $x_i$ just indifferent between renewing and dropping out in the initial year (see equation 10) for $i = 1, 2, 3$. To get the end points of these curves, we note that when $c_2 \geq y_J$ (the largest possible value of second-period returns), $\phi_i (c_2, \beta) = x_i$, and when $c_2 = 0$, we have $\phi_i (0, \beta) = x_i + E(y / x_i)$. The change in $\phi_i (\cdot, \beta)$, say $\Delta\phi_i$, corresponding to an increase in $c_2$ from $c_2$ to $c_2'$, say $\Delta c_2$, is given by

$$\Delta\phi_i = -\beta\Delta c_2 \sum_j \mathrm{I}(y_j > c_2')\, p_{i,j} - \beta \sum_j \mathrm{I}(c_2 < y_j \leq c_2')\, p_{ij}(y_j - c_2).$$

Note that only the first part of this term is relevant if $\Delta c_2$ does not include

**Figure 1. Nonparametric Identification**




a value of $y_j$. Hence the curve is linear with slope $\beta\Sigma I\,(y_j > c_2')\,p_{i,j}$ between each two possible $y_j$ values, and changes slope (is nondifferentiable) when it passes over a possible $y_j$ value. The identification analysis set out above is easy to picture in this figure. By setting $c_2 \geq y_J$ and increasing $c_1$, we map out $(x_i, p_i)_{i=1}^k$. Then choose $(c_1, c_2)$ couples just below the $\phi_3\,(c_2, \beta)$ and trace out $(y_j, p_{k,j})_{j=1}^J$. Finally, we get $\beta$ from the slope of the $\phi_k\,(\cdot, \beta)$ curve between any two values of $y_j$.

Now assume that instead of being able to choose $(c_1, c_2)$ combinations that fill out the entire positive orthant, we are allowed to use only the implications of values of $(c_1, c_2)$ that are members of the set

$$\mathscr{C} = [(c_1, c_2) \in \mathbb{R}_+^2 : 0 \leq c_1 \leq c_1^*, 0 \leq c_2 \leq c_2^*],$$

that is, values bounded by $(c_1^*, c_2^*)$, as in the shaded area in the figure.

Here $(c_1^*, c_2^*)$ is meant to represent the largest renewal fees observed in the data, so that our working hypothesis is that we can observe fairly complete variation of renewal fee schedules in a range below $(c_1^*, c_2^*)$, and that we have no observations on renewal fees larger than $(c_1^*, c_2^*)$. It is rather important to note that in the example drawn in the figure $c_1^* < \phi_3 (c_2^*, \beta)$, so that there is no $(c_1, c_2) \in \mathscr{C}$ large enough to induce all patents to drop out in the first year, and $c_2^* < y_J$, so that there will be some patents paying the final renewal for every $(c_1, c_2) \in \mathscr{C}$. This is the empirically relevant case.

A repetition of the argument given above shows that if we generate $(c_1, c_2)$ couples by holding $c_2 = c_2^*$ and increasing $c_1$ from 0 to $c_1^*$, the resulting $(\Pi_1, \Pi_2)$ couples will suffice to identify

$$(13) \qquad [p_i, Pr(y \geq c_2^* / x_i)]_{i=1}^3.$$

That is, for each $i$ we will be able to identify the probability of the initial draw $(p_i)$, and the conditional (on $i$) probability that the second-year draw exceeds $c_2^*$ (note that $x_i$ itself is not identified). Now lower $c_2$ to be a value contained in the interval $(\bar{c}_2, c_2^*)$ and repeat the process. An analogous argument shows that this identifies $[Pr(y \geq c_2 / x_i)]_{i=1}^3$. Given equation 13, then, we have $[Pr\,(c_2^* \geq y \geq c_2 / x_i)]_{i=1}^3$. Moreover, since we can take a value of $c_2$ arbitrarily close to $c_2^*$, a value of $c_2'$ arbitrarily close to $c_2$, and so on, it is easy to see that we can identify the densities between $c_2^*$ and $\bar{c}_2$, or

$$(14) \qquad [p\,(y = c_2 / x_i) : \bar{c} \leq c_2 < c_2^*]_{i=1}^3.$$

Now move into the $(c_1, c_2)$ range where $\bar{\bar{c}}_2 \leq c_2 < \bar{c}_2$, and $c_1 \leq c_1^*$. In this range there is no longer a value of $(c_1, c_2)$ that causes the $i = 2$ patents to drop out; hence all we can identify is

$$(15) \qquad [p\,(y = c_2 / x_1), \sum_{i=2}^{3} p\,(y = c_2 / x_i)\, p_{i,j} : \bar{\bar{c}}_2 \leq c_2 \leq \bar{c}_2].$$

That is, we can find only the probability that $y = c_2$ conditional on $i$ is 2 or 3. Finally, when $c_2 \in (0, \bar{\bar{c}}_2)$ and $c_1 \leq c_1^*$, there is never a value of $(c_1, c_2)$ that induces any of the initial draws to drop out, and all we can identify is

$$(16) \qquad [\sum_{i=1}^{3} p\,(y = c_2 / x_i)\, p_{i,j} : 0 \leq c_2 \leq \bar{\bar{c}}_2].$$

The probabilities listed in equations 13 through 16 contain all the distinct aspects of $\mathbb{P}$ that we can learn by unraveling the $(\Pi_1, \Pi_2)$ combinations obtained by varying $(c_1, c_2)$ over its possible values in $\mathscr{C}$. It does not use, however, the additional information on $\mathbb{P}$ that our model implies is contained in initial-year renewal behavior. That is, since we know which patents drop out and which do not, equation 4 tells us that we know whether, for each $(c_1, c_2) \in \mathscr{C}$ and for each $x_i$,

$$x_i + \beta \Sigma (y_j - c_2)^+ p_{ij} - c_1 \gtreqless 0.$$

Rearranging terms, this equation can be rewritten as

$$(17) \quad x_i + \beta Pr(y > c_2^* / x_i) E[(y - c_2^*) / y \geq c_2^*, x_i] \gtreqless \psi_i(c_1, c_2, \beta),$$

where

$$\psi_i(c_1, c_2, \beta) = c_1 - \beta \Sigma (y - c_2)^+ \mathrm{I}(y \leq c_2^*) p_{i,j} - (c_2^* - c_2) Pr(y \geq c_2^* / x_i).$$

The right-hand side of equation 17, that is $\psi_i(\cdot)$, is a function only of $(c_1, c_2, \beta)$ and objects that can be calculated from equations 13 through 16. The left-hand side does not depend on $(c_1, c_2)$ at all. The first point to note then is that equation 17 enables us to identify $\beta$. To see this, consider two values of $(c_1, c_2)$ that are on the same $c_1 = \phi_i(c_2, \beta)$ curve. For each of these values type $i$ patents are indifferent to renewing, so equation 17 holds with equality. Since the left-hand side of this equation is independent of $(c_1, c_2)$, this means that the right-hand side must have the same value for the alternative combinations of $(c_1, c_2)$—and this determines $\beta$.

Given $\beta$, equation 17, when combined with equations 13 through 16, provides information on whether one *fixed* linear combination of $x_i$ and $E[(y - c_2^*) / y \geq c_2^*, x_i]$ is larger than alternative known numbers. Indeed, since there exist $(c_1, c_2)$ values in $\mathscr{C}$ that make patents with $i = 1$ or $2$ just indifferent to renewing—that is, $\phi_1(c_2, \beta)$ and $\phi_2(c_2, \beta)$ intersect $\mathscr{C}$—we can calculate the precise values of $\psi_i$ such that

$$(18a) \quad x_i + \beta Pr(y > c_2^* / x_i) E[(y - c_2^*) / y \geq c_2^*, x_i] = \psi_i,$$

for $i = 1, 2$, whereas for $i = 3$ we know only that

$$(18b) \quad x_3 + \beta Pr(y > c_2^* / x_3) E[(y - c_2^*) / y \geq c_2^*, x_3] > \psi_3(c_1^*, c_2^*).$$

Gathering results on the identification status of the model when we restrict $(c_1, c_2) \in \mathscr{C}$, we note that $\beta$ is identified and that for $i = 1, 2$ we can identify $p_i$, the (almost complete) information on $Pr\,(y < c_2 / x_i)$ for $c_2 < c_2^*$ given in equations 13 through 15, and that a known linear combination of $x_i$ and $E(y - c_2^* / y \geq c_2^*, x_i)$ equals a known constant. A similar statement applies when $i = 3$ except that here we know only that the linear combination is greater than $\psi_3(c_1^*, c_2^*)$. This information suffices to map out most of the conditional distributions for $y$ at low $y$ values and to obtain bounds on both $x_i$ and $E(y / x_i)$.

To go further, we need more detailed assumptions. Here we briefly sketch out a set of assumptions we think ought to provide a good starting point. These assumptions do not allow us to identify all of $\mathbb{P}$, but then the data simply do not contain detailed information on the conditional probabilities of $y$ at high $y$ values. We do, however, close in on the couples $[x_i, E(y - c_2^* / y \geq c_2^*, x_i)]$, and this at a cost of a set of assumptions we believe to be minimal.

Note that equation 18 implies that the problem we have in identifying $E(y - c_2^* / y \geq c_2^*, x_i)$ lies not in a lack of detailed information on the structure of the conditional probabilities for $y$ values greater than $c_2^*$, but rather in our inability to separate out the effect of the possibility of large future returns from that of large initial returns on initial renewal behavior. That is, any additional information that allows us to identify the $(x_i)$ will, by virtue of equation 18 and equations 13 through 16, allow us to identify $[E(y / x_i)]_{i=1}^{2}$. One way of identifying the $(x_i)$ is to provide a parametric probability model for the *lower part* of the distribution of $y$ given $x$ (leaving the upper part of that distribution unrestricted), and then use the nonparametric information in equations 13 through 16, together with the assumptions on the parametric family, to back out information on the $(x_i)$. The $(x_i)$ and equation 18 can then be used to obtain information on $E(y / x_i)$. Note that this procedure makes parametric assumptions only on the portion of the conditional density of $y$ whose domain is actually swept out by the observed variation in the second-period renewal fees. Thus, provided we use a tight enough model, we will be able to use the data to check whether a particular parametric family seems appropriate for the problem at hand.

More precisely, assume that for some $\theta \in \Theta$,

$$Pr(y < c/x_i) = P(y < c / x, \theta), \text{ for } c \leq c_2^*, \text{ and } i = 1, 2, 3. \tag{19}$$

What we would like to do is to use this equation and the information in the data on low $y$ value events (the nonparametric information in equations 13 through 16, for example) to identify both $\theta$ and the $(x_i)$. Given the $(x_i)$, equations 18a and 18b would imply that

$$(20) \qquad E[(y - c_2^*) / y \geq c_2^*, x_i] = (\psi_i - x_i) / \beta Pr(y \geq c_2^* / x_i), \quad \text{for } i = 1 \text{ or } 2, \text{ and}$$
$$E[(y - c_2^*) / y \geq c_2^*, x_3] > (\psi_3 - x_3) / \beta Pr(y \geq c_2^* / x_3).$$

Equations 19 and 20 together would allow us to identify $E(y / x_i)$ for $i = 1$ or 2 and provide a bound for $E(y / x_3)$, without making any but the mildest of assumptions on the conditional probabilities of $y$ for $y$ values greater than the range of $c_2$ values in the data. (We do need the assumption that appropriate expectations exist.)

A possible problem with this procedure is that the parametric families that might seem appropriate for equation 19 may not allow us to identify both $\theta$ and the $(x_i)$ from the information in the data on the low $y$-value events. An example in which both $\theta$ and the $(x_i)$ are identified is when we can approximate the distribution of $y$ conditional on $x_i$ in the region $y < c_2^*$, as;

$$(21) \qquad y = \delta x_i, \text{ and } \log \delta \sim N(0, \sigma_i^2), \text{ for } i = 1, 2, 3.$$

In this equation the conditional probabilities of low $y$-value events are given by a model with a lognormal stochastic decay coefficient, but the form of the conditional probabilities at high $y$ values are left unrestricted. Here $\theta = (\sigma_i)$, so what we identify is the couples $(\sigma_i, x_i)$. To see how the possibility of underidentification arises, relax the assumptions in equation 21 to

$$(22) \qquad y = \delta D x_i, \text{ with } \log \delta \sim N(0, \sigma_i^2), \text{ for } i = 1, 2, 3.$$

Given only this equation, the probabilities of low $y$-value events generated from $[D, (x_i), (\sigma_i)]$ are identical to the probabilities generated from $[D', (x_i'), (\sigma_i)]$ provided $x_i' = D x_i / D'$. As a result, equation 22 identifies $(x_i)$ only up to a factor proportionality; that is, it identifies the couples $(Dx_i, \sigma_i)$. In such cases we can, provided $x_1 \geq 1$, substitute $Dx_i/Dx_1$ for $x_i$ in equation 20 and still obtain an upper board for $E(y - c_2^* / y \geq c_2^*, x_i)$.

Of course, these are only a few of the many possible suggestions for parameterizing the form of the conditional distributions at low $y$ values. The point we would like to emphasize is that such assumptions usually result in aspects of the model being overidentified relative to the $(\Pi_1, \Pi_2)$ predictions generated by the observed variation in the renewal fee schedules. Thus the assumptions are, at least in some dimension, testable, without making any but the mildest of assumptions on the conditional distributions of $y$ in the range that the observed renewal fees never sweeps out.

The parametric assumption in equation 19 also serves to mitigate other problems that arise when we try to translate the identification analysis into a tool that enables us to learn from the data. At best, the identification analysis suggests consistent estimators for different aspects of the model; it says nothing about the likely precision of those estimators. As noted earlier, we now have data sets with information on detailed characteristics of both the patents within them and the economies that generate the values of those patents. This fact, together with tests like those described earlier, is likely to move research toward investigating differences in the returns to patent protection between patents with different characteristics. If the characteristics we would like to differentiate among took on only a finite set of values (for example, industries, sampled time periods), the nonparametric procedure for investigating differences among patents would involve a separate analysis for each distinct set of observed characteristics. That would leave us with estimators based on much smaller numbers of patents, and we may begin to worry about the trade-off between a consistent, but highly variant, nonparametric estimator and a (possibly marginally) inconsistent, but much less variant, parametric estimator (especially if we could use either the current data or previous research to suggest appropriate parametric families).

A similar, though technically more complicated, problem would arise if the differentiating characteristics took on a continuum of values. Then we would look for a smoothed estimator of the response of the distribution functions in $\mathbb{P}$ to changes in the values of that characteristic, and the smoothing procedure itself would place its own restrictions on the family of functions being estimated. The semiparametric analysis in equations 19 and 20 allows us to summarize differences in the returns process associated with differences in the economy's or the patent's characteristics, in terms of differences in $\theta$ and in (the function) $E[(y - c_2^*) / y \geq c_2^*, x]$—restrictions that can reduce the dimensionality of the estimation problem significantly. It also suggests a method for allowing for heterogeneity in the Markov transition probabilities that is not associated with observable differences among patents. (Recall that we have already allowed for unobservable differences in initial returns through the distribution $P_0(\cdot)$.) That is, if we are worried that differences in observable characteristics are not rich enough to differentiate among patents perceived to have very different transition probabilities, we could allow both $\theta$ and $E[(y - c_2^*) / y \geq c_2^*, x]$ to be functions of unobserved as well as observed characteristics, and integrate a parametric family for the distribution of those unobserved characteristics into the estimation algorithm.

Thus far we have ignored the computational problems that would have to be surmounted to obtain our semiparametric estimator. These are of two kinds. The first is in calculating the stopping value for the returns (the $\bar{r}$'s) conditional on all sources of heterogeneity, the renewal fee schedules, and alternative possible values of the parameter vector. The second is in finding the sum (or the integral) determining the fraction of patents with returns greater than the stopping value in a given year, conditional on the observed sources of heterogeneity, the renewal fee schedule, and the parameter vector. Both these problem are currently surmountable but do require some additional work.[11]

## How Can Renewal Data Be of Help?

As noted in the introduction, we have divided the discussion of this section into two parts: renewal data and research on the value of patent protection and renewal data and research on the value of patented ideas. Each subsection provides a brief review of issues, a short summary of the results from related work, and a discussion of what renewal data

11. Conditional on being able to calculate the stopping values, a simulation estimator similar to the estimator introduced by Pakes and subsequently generalized and analyzed by McFadden, and Pakes and Pollard, can be used to evaluate the required integrals. Pakes, "Patents as Options"; Ariel Pakes and David Pollard, "Simulation and the Asymptotics of Optimization Estimators," *Econometrica,* forthcoming; and Dan McFadden, "A Method of Simulated Moments for Estimation of Multinomial Probits without Numerical Integration," *Econometrica,* forthcoming. The degree of difficulty involved in calculating the stopping values depends on the functional forms assumed for the transition probabilities. Often artificial intelligence programs capable of performing symbolic algebraic manipulations (integration) will be of help here. See Pakes, "Patents as Options."

might have to say about the problems at hand. We have not tried to be comprehensive, and we apologize for the many omissions likely to have been made.

*Renewal Data and the Value of Patent Protection*

Patents are an institutionally created property right designed to enhance the ability of inventors to appropriate the returns from their inventions, thereby increasing the incentive to engage in innovative activity. So to evaluate the efficiency of the patent system (and how that may vary in different economic and institutional environments), we need a measure of patentees' perceptions of the value of the benefits that patent protection provides. Since patent rights are seldom marketed, there is no direct measure of their value. The indirect measures of patent values that do exist have been obtained by analyzing the relation between patent counts and alternative measures of the total value of the patenting unit.[12] When carefully interpreted, these measures almost always end up being estimates of the distribution of the values of the underlying ideas being patented rather than of the value of the protection the patent confers on those ideas.[13] Since currently available evidence shows that the value of patent protection is usually not the main determinant of the values of patented ideas, and that the relative importance of patent protection in determining that value can vary greatly by the type of patent and patentee, estimates of the distribution of the value of patented ideas may not contain much information on the value of patent protection.

Patent renewal decisions are associated with particular patents and, most important, are determined solely by the patentee's perceptions of the value patent protection provides. But the renewal data contain only a rough gradation of the value of patent protection; that is, they tell us only whether the value that would result from holding the patent over an additional year exceeds the cost of renewal. As noted earlier, the value of holding the patent consists of the sum of the value of the current-year

12. See the articles, and literature cited, in Griliches, ed., *R&D, Patents, and Productivity*.

13. See, for example, Henry G. Grabowski and John M. Vernon, "Studies on Drug Substitution, Patent Policy and Innovation in the Pharmaceutical Industry," final report for the NSF Grant no. PRA-79-17524, Duke University, May 1983; and Ariel Pakes, "On Patents, R&D, and the Stock Market Rate of Return," *Journal of Political Economy*, vol. 93 (April 1985), pp. 390–409.

returns to patent protection and the value of the option of renewing in the next year should the patentee desire to do so. As a result, to be able to use renewal data to make inferences on the distribution of returns to patent protection, we need assumptions on how agents value the option to renew in subsequent years and on the nature of the stochastic process generating the returns from patent protection. In the previous section we found that even if only mild regularity conditions were imposed on that process, use of the discrete information on whether a patent was renewed at alternative ages, together with the relevant renewal fee schedules, could still yield a good deal of information on the value of patent protection. This section focuses on how that information can be put to use.

One caveat is in order before proceeding. Renewal data can shed light only on the returns resulting from renewing patents that have already been applied for. The option to patent, even if not exercised, and the process of filing an application, even if the patent is never renewed, can be valuable in themselves (since they can both deter competitors and guarantee that patents on similar substances will not be obtained in the future). These aspects of the value of patent protection will not be captured by our estimates of the returns earned by renewing patents already applied for; thus our estimates should be interpreted as a lower bound to the total value of the protection the patent system provides. Note that if we were to integrate an empirical analysis of the application decision per se, and the possibilities for such an analysis are enhanced by the fact we now have information on applications of the same patent in different countries,[14] we could also analyze the value of applying for a patent, but that is an alternative we do not pursue here.

We first consider issues related to international differences in patent laws. Here the debate seems to hinge on two separate and often contradictory notions of what a "fair" international patent system would look like. Fairness seems to be defined in one of two ways: by a system in which the international flow of benefits from patent protection do not accrue disproportionately to nationals of a given country, or by a system in which the institutions and laws governing the protection provided a patent do not vary among countries. (These institutions should be defined broadly enough to include detection probabilities and punishments for

14. See Putnam, "International Differences in the Value of Patent Protection."

noncompliance.) The first definition of fairness has been used by policymakers in developing countries (countries whose nationals seldom take out patents in the developed world but whose governments frequently issue a large number of patents to nationals of developed countries) to argue for discontinuing patent protection, or at least for changing the type of protection granted.[15] At the same time, the substantial differences that do exist among the patent laws in different countries have led to claims (particularly by the U.S. business community) that the current situation is unfair.[16] As a result, securing more stringent intellectual property rights has become both a major trade issue of the 1980s and a possible arena for GATT-type discipline.[17]

It is not our purpose to take a stand on these issues. Indeed, given the lack of empirical information on the extent to which nationals of different countries benefit from the patent system, and on the effect of the characteristics of the institutions associated with patenting on the value of patent protection, an informed discussion of the alternatives would be difficult. But renewal data should prove helpful. Because they contain detailed information on the nationality of the patentee, we should be able to use them to analyze the international flow of returns from the patent system. Furthermore, the large observed variation in the characteristics of the patent system, together with the fact that we can follow the renewal behavior of the same patent in different countries, should

15. See Evenson, "International Invention," and the literature cited there; and Mary Ellen Mogee, "International Issues in Intellectual Property Rights."

16. These differences range from specifically precluding the patentability of any substance in a particular field (pharmaceuticals seems to be the main, but not the only, victim of this kind of clause), to an assortment of less drastic measures such as compulsory licensing requirements; requirements for the invention to be worked within a (sometimes unreasonably short) period of time; administrative practices forcing claims to be applied narrowly (thereby allowing others to obtain patents for minor variants of the original patented products); substantive differences in procedural formalities (for example, first-to-innovate versus first-to-file priority systems); differences in the stringency of the criteria used to ascertain whether applications should be granted; and differences in the renewal fee schedules (including differences in the statutory limit to the length of patent lives).

17. See Mogee, "International Issues in Intellectual Property Rights"; U.S. International Trade Commission, *Foreign Protection of Intellectual Property Rights and the Effect on U.S. Industry and Trade,* Report to the United States Trade Representative, Investigation 332–245, under section 332(g) of the Tariff Act of 1930 (Washington, February 1988); and Richard Stern, "Intellectual Property," in J. Michael Finger and Andrzej Olechowski, eds., *The Uruguay Round: A Handbook on the Multilateral Trade Negotiations* (Washington: World Bank, 1987), pp. 198–206.

allow us to use renewal data to unravel aspects of the relation between the characteristics of patent laws and the returns to patent protection.

A good deal of policy debate also surrounds domestic policy legislation. Some is in response to the international issues just discussed. Other questions have arisen because of the emergence of technologies which are thought to produce intellectual property that needs to be protected by new legislation (such as computer programs, semiconductor chip designs, and the natural substances produced by the biotechnology industry). Finally, there are the traditional questions about whether patent protection is more important for certain types of patentees (particularly individuals and small businesses), for certain industries, and given the type of industry and of patentee, in different institutional frameworks.[18] To appreciate the importance of these issues, one must keep in mind several facts: patents exist to rectify natural market imperfections that may be more important in one setting than in another; the advantages gained by "protected" innovations are thought to be a leading source of change in market power, growth, and the structure of wealth; and the government frequently intervenes to change the institutions that govern patentees' abilities to appropriate the benefits from invention (besides changes in the patent system, changes in regulatory requirements can have a great effect).

Much of what we currently know about the value of patent protection comes from the survey evidence analyzed in Taylor and Silberston, Mansfield, Schwartz, and Wagner, Mansfield, and Levin and others.[19] Though the studies used different survey instruments, some of their more qualitative results were fairly consistent.[20] They all showed that the importance of patent protection varied greatly among industries:

18. See, for example, the discussion in F. M. Scherer, *Industrial Market Structure and Economic Performance,* 2d ed. (Chicago: Rand McNally, 1980).

19. C. T. Taylor and Z. A. Silberston, *The Economic Impact of the Patent System: A Study of the British Experience* (Cambridge University Press, 1973); Edwin Mansfield, Mark Schwartz, and Samuel Wagner, "Imitation Costs and Patents: An Empirical Study," *Economic Journal,* vol. 91 (December 1981), pp. 907–18; Edwin Mansfield, "Patents and Innovation: An Empirical Study," *Management Science,* vol. 32 (February 1986), pp. 173–81; and Richard C. Levin and others, "Appropriating the Returns from Industrial Research and Development," *Brookings Papers on Economic Activity: Special Issue on Microeconomics 3:1987,* pp. 783–831.

20. For a review see Wesley M. Cohen and Richard C. Levin, "Empirical Studies of Innovation and Market Structure," in Richard Schmalensee and Robert Willig, eds., *Handbook of Industrial Organization* (North-Holland Publishers, forthcoming).

pharmaceuticals consistently, and usually other chemically related industries, came out at the top of the list, with electronics near the bottom and mechanical industries near the middle (a ranking consistent with most observers' views on the technologically and legally determined degree of difficulty encountered when trying to "invent around" a given patent). Further, patent protection seemed to be more important for product than for process innovations.[21] The studies agree less about differences in the importance of patent protection by characteristics of the patentee (conditional on industry or not), about changes in the value of patent protection brought about by changes in the institutional setting (though Grabowski and Vernon do consider the effect of changes in Food and Drug Administration requirements on the drug industry),[22] or about changes in the value of patent protection over time (as will be discussed).

There are problems with pushing the survey approach much further. Surveys are costly, large sections of survey questionnaires are subjective, and available survey designs select out and analyze nonrandom samples from the patent population of interest. Survey answers not denominated in dollar values have been and will continue to be hard to compare across respondents, and accurate dollar-denominated answers are difficult to obtain. Because the existing samples are nonrandom, it is hard to use the current survey results to inform discussion on many policy issues, and it is difficult to see how future survey results can overcome the selection problems. For instance, all the studies sample only large firms; they do not provide any information on the importance of patent protection to small or medium-sized businesses, or to individuals (groups that might be expected to gain disproportionately from patent protection). Finally, the high degree of variance and skew that we believe characterizes the distribution of patent values may well imply

21. The measure of "important" varied greatly among these studies. Levin and others use a subjective seven point scale on the importance of patent protection relative to the importance of other means of appropriating the benefits from innovations; Taylor and Silberston ask about the fraction of R&D that is dependent on patent protection; Mansfield and others and Mansfield ask about the fraction of innovations that would not have been introduced without patent protection; and Mansfield and others attempt to estimate the increase in the imitation costs that result from patent protection.

22. Henry G. Grabowski and John Vernon, "Longer Patents for Lower Imitation Barriers: The 1984 Drug Act," *American Economic Review*, vol. 76 (May 1986, *Papers and Proceedings, 1985*), pp. 195–98.

that the summary statistics which can be estimated from surveys do not provide an accurate enough description of the return distribution for the purposes at hand.[23] To take an extreme example, a finding that only 0.5 percent of a firm's inventions was dependent on patent protection would not mean that patent protection was unimportant to the firm if that same 0.5 percent accounted for 99.5 percent of the total returns to the firm's R&D program.

Using renewal data to examine these issues has potential advantages because one can follow the renewal behavior of large random samples of patents, and because renewal behavior reflects the patentee's perceptions of the dollar value of patent protection. But analysts have only limited experience with such data. The early results of Pakes, and those that followed by Schankerman and Pakes, make particular functional form assumptions thought to be consistent with the currently available information on the returns to patents. They then use those assumptions, along with the behavioral assumption of expected discounted value maximization, to recover the entire process generating the returns to patent protection.[24] Because those early results are on aggregate cohorts of patents, they cannot provide the disaggregated detail needed for the analysis of most of the issues introduced above. As noted earlier, currently available patent renewal data sets are much richer in this regard. Still a brief review of some of the early results is worthwhile.

The empirical results from Pakes, "Patents as Options," showed that patents are applied for at an early stage in the inventive process, when there is still uncertainty about both the returns that will be earned from holding the patents and those that will accrue to the patented ideas. Gradually the patentees uncover more information about the value of their patents. Most turn out to be of little value, but the rare winner justifies the investments made in developing them. The average value of a patent right is estimated to be small, about $6,000 for patent applications

23. See, in particular, F. M. Scherer, "Corporate Inventive Output, Profits, and Growth," *Journal of Political Economy*, vol. 73 (June 1965), pp. 290–97; Scherer, "Firm Size, Market Structure, Opportunity, and the Output of Patented Inventions," *American Economic Review*, vol. 55 (December 1965), pp. 1097–1125; Grabowski and Vernon, "Studies on Drug Substitution"; and Barkev S. Sanders, Joseph Rossman, and L. James Harris, "The Economic Impact of Patents," *Patent, Trademark and Copyright Journal of Research and Education*, vol. 2 (September 1958), pp. 340–62.

24. Pakes, "Patents as Options"; and Schankerman and Pakes, "Estimates of the Value of Patent Rights."

in France and about $7,000 in the United Kingdom. In Germany, where only about 35 percent of all patent applications are granted, the average value of a patent right among grants was estimated at $16,200. The distribution of these values, however, is dispersed and skewed. One percent of patent applications in France and the United Kingdom had values above $65,000; in Germany 1 percent of patents granted had values above $118,000. Moreover, half of all the estimated value of patents rights accrues to between 5 and 10 percent of all patents. Since about 35,000 patents were applied for annually in France and the United Kingdom, and about 60,000 in Germany, these figures imply that the annual flow of the returns from patent protection is between 11 and 16 percent of the annual flow of R&D expenditures of the business enterprises in each of the three countries.[25]

Accordingly, patent protection per se is not the chief means by which firms appropriate the returns from their R&D investments, at least in the aggregate. This result is consistent with the survey evidence cited earlier (though again because of the underlying variance and skewness in the distribution of the value of patent protection, together with the interindustry variance in the number of patents, patent protection may be the chief means of appropriating the returns from R&D in some industries or for some types of patentees). Nevertheless, one should remember that patent rights are an institutionally created property right. That is, they are a policy instrument designed, in large part, to provide incentives to engage in R&D. Compared with other institutionally created incentives (such as tax breaks), an 11 to 16 percent increase in returns does not seem small. Of course, to judge the effectiveness of this incentive, one would need an estimate of the R&D response to the increase in returns, and then a way to compare the benefits from that response, plus whatever benefits there are from publicizing the content of the patent, with the costs of patent protection.

A final reason for using renewal data is to shed more empirical light on other aspects of the process generating technological change. Indeed, Pakes and Schankerman's original 1984 study of international differences in renewal behavior was largely motivated by the desire to get a more direct measure of the rate of obsolescence for the returns from profit-producing, knowledge-generating processes. The obsolescence rate they

25. Pakes, "Patents as Options," pp. 777–78.

estimated (25 percent) was then used to correct for the possible biases in production function estimates of the rate of return to research caused by applying traditional estimates of the rate of deterioration in physical capital (5–10 percent) to the construction of knowledge stocks. Similarly, Schankerman and Pakes later provided one set of estimates of their renewal model that analyzed the reduced form relationship between intercohort differences in the mean value of patent protection and differences in both GNP levels and the stock of patents in force.[26] The analysis was done separately for cohorts in the United Kingdom and for cohorts in Germany; in both cases GNP had a large positive significant effect on the mean value, whereas the stock of patents in force had a large significant negative effect. These results provide empirical support for the importance of demand inducement mechanisms and endogenous obsolescence processes on determining the appropriable returns to patents. Of course, what we really need is more detailed analysis of precisely how the demand inducement and obsolescence processes work. That will require further modeling and data matching efforts.

### *Renewal Data and the Value of Patented Ideas*

The literature on using patent count indexes as a measure of technical change has a history that predates the current resurgence of interest in patent data and includes several debates on the potential value of such an endeavor.[27] Using patent counts has several advantages: they are a more direct result of inventive activity than the other indicators of performance available, such as profits, productivity, the stock market value of the firm, and sales of new products, and more important, patent data are available for an unusually long time in a detailed breakdown (by both patentee and product class).[28] There have been, however, at least

26. Pakes and Schankerman, "Rate of Obsolescence of Knowledge"; and Schankerman and Pakes, "Estimates of the Value of Patent Rights."

27. See Simon Kuznets, "Inventive Activity: Problems of Definition and Measurement," in *The Rate and Direction of Inventive Activity: Economic and Social Factors,* a conference of the Universities–National Bureau Committee for Economic Research and the Committee on Economic Growth (Princeton University Press, 1962); the exchange between Kuznets, Sanders, and Schmookler in that volume; Taylor and Silberston, *Economic Impact of the Patent System*; and Jacob Schmookler, "Economic Sources of Inventive Activity," *Journal of Economic History,* vol. 22 (March 1962), pp. 1–10.

28. See the next section in this paper and Office of Technology Assessment and Forecast, *Reports* 1–9 (Department of Commerce, 1973–79).

two problems with using the patent data. First, though patent counts were available in principle, they were inaccessible in practice. The recent computerization of the data sets of many national patent offices (including the United States') has decreased the cost of accessing the patent data dramatically and is clearly a major factor behind the current resurgence in the use of patent count indexes.

The second, more basic problem with using patent count indexes was a concern with their quality. Patents vary greatly in both their private and social values and not all new innovations are patented, which makes the interpretation of differences in the number of patents hazardous. It is helpful to break this problem down into smaller pieces. The most one can expect from the patent count data are indexes of the value of patented output. There remains, therefore, the question of the relation between the value of patented output and the value of inventive output. To study this relation requires keeping track of the situations in which it is legally possible to obtain a patent and then asking when, conditional on legal feasibility, patentees will perceive that the value of the protection gained from patenting exceeds any detrimental effect of publishing the information contained in the patent.[29] Though there are few hard facts here, our reading of the literature suggests that, given legal feasibility, the main cause of differences in the "extent of patenting" is the technological characteristics of the invention. Whatever the reason, patents have traditionally provided greater protection for inventions that use certain kinds of technologies.

Two problems arise in using patent count indexes to measure the value of patented output. First, the average value of patented inventions may differ among the groups of patents being compared; if so, differences in the number of patents among groups will not be proportional to differences in their value (even in expectation). Second, as noted, both small-sample case study and large-sample econometric evidence indicate that the distribution of the value of patented ideas is dispersed and highly skewed. Thus, even if differences in the expected values of the count

29. For analyses of these issues in different institutional environments, see Suzanne Scotchmer and Jerry Green, "Novelty and Disclosure in Patent Law," Discussion Paper 1388 (Harvard Institute of Economic Research, June 1988); and Ignatius Horstmann, Glenn M. MacDonald, and Alan Slivinski, "Patents as Information Transfer Mechanisms: To Patent or (Maybe) Not to Patent," *Journal of Political Economy,* vol. 93 (October 1985), pp. 837–58.

indexes were, say, proportional to differences in the expected value of the patented ideas they represent, the variance about that expectation is likely to be large. Put simply, patent counts are a very noisy measure of the value of patented output.

In sum, though the evidence suggests that the large observed differences in patent applications between firms, regions, patent fields, and sometimes longer periods of time are highly correlated with differences in measures of inventive inputs (and therefore presumably with perceived opportunities for developing innovative outputs), the smaller differences, say, in the patents applied for by given firms over time seem to be dominated by noise.[30] These results often justify using patent count indexes as a proxy for a measure of inventive inputs in the many situations when R&D expenditure data are not available. (In the United States, R&D data are usually not available by product field or geographic area, for smaller business concerns, or for most large business enterprises before 1972.) Indeed, creative use of differences in patent counts in these areas has just begun and seems to have proved fruitful.[31] However, attempts to use patent count indexes to measure smaller changes in patented output, or to use them together with R&D data to examine detailed aspects of the relationships between inventive inputs and inventive outputs, have been hampered by the noise, or the measurement error, in those indexes.

To what extent can patent renewal data enable us to construct more accurate indexes of the value of patented output? The additional information provided by the renewal data consists of renewal fee schedules and a partition of the patents into $L + 1$ subgroups: one for each possible dropout age, plus a final group that paid the renewal until the statutory limit to patent lives (or $L$). Let $[n(a)]_{a=1}^{L+1}$ designate the number of patents

30. See Zvi Griliches, Ariel Pakes, and Bronwyn H. Hall, "The Value of Patents as Indicators of Inventive Activity," in Partha Dasgupta and Paul Stoneman, eds., *Economic Policy and Technological Performance* (Cambridge University Press, 1987), pp. 97–121; and the summary of results in Pakes, "On Patents, R&D, and the Stock Market."

31. See, for example, Adam B. Jaffe, "Technological Opportunity and Spillovers of R&D: Evidence from Firms' Patents, Profits, and Market Value," *American Economic Review,* vol. 76 (December 1986), pp. 984–1001; Jaffe, "Academic Research with Real Effects," Harvard Institute of Economic Research, Discussion Paper, October 1988; and Kenneth L. Sokoloff, "Inventive Activity in Early Industrial America: Evidence from Patent Records, 1790–1846," *Journal of Economic History,* vol. 48 (December 1988), pp. 813–50.

in each group. Then an obvious choice for a renewal based index of the value of patented output would be the weighted average $VP$, where

$$(23) \qquad VP = \lambda \sum_a w(a)n(a),$$

and $[w(a)]_{a=1}^{L+1}$ is a sequence of positive weights summing to one, so that $w(a)/w(a')$ provides the ratio of the average value of patents that are renewed until age $a'$ to those that are renewed until age $a$.

To use this kind of index of the value of patented output, we need estimates of the $[w(a)]_{a=1}^{L+1}$. There are several ways to go here. One is to estimate the $[w(a)]$ from the relationship between some measure of the output from innovative activity (say, stock market values, profits, or productivity) and separate count indexes for the number of patents applied for that subsequently were renewed precisely $a$ years (for $a = 1, \ldots, L + 1$). A second procedure for obtaining the $[w(a)]_{a=1}^{L+1}$ is to assume a relationship between the $\lambda w(a)$ and the average value of patent protection for patents renewed until age $a$, say $\lambda^p w^p(a)$, and then use a renewal model to obtain estimates of the $\lambda^p w^p(a)$. For example, if it were assumed that for constant $\kappa$

$$(24) \qquad \lambda w(a) = \kappa \lambda^p w^p(a)$$

for all $a$, then $\kappa$ must equal $\lambda/\lambda^p$, so that $w^p(a) = w(a)$. Thus the estimates of $\lambda^p w^p(a)$ allow us to obtain estimates of $[w(a)]_{a=1}^{L+1}$ and use them to analyze differences in the $VP$ index up to a scalar (a scalar that could, perhaps, be estimated in subsequent analysis).

Though this latter procedure requires some strong assumptions, it was the only one we could illustrate with existing empirical results. The top part of table 1 provides the weights implied by Pakes's estimates of the value of patent protection for post–World War II cohorts of patents in France and Germany, and the assumption in equation 24.[32] The bottom part of the table gives some summary statistics describing the data and characterizing the results.

Briefly the data contain at least partial information on cohorts applied for in most of the 1950s, all the 1960s, and at least the early 1970s. During this period the first renewal fee was due at age two in France and three

32. Note, therefore, that for these weights to be correct we require also all those assumptions Pakes used in "Patents as Options" to obtain his estimates of the value of patent protection.

**Table 1. Value Weights by Age from a Patent Renewal Model, and Summary Statistics, France and West Germany**

| Age | Weights: France | Weights: West Germany | Age | Weights: France | Weights: West Germany |
|---|---|---|---|---|---|
| 2 | 0.000 | ... | 12 | 0.014 | 0.034 |
| 3 | 0.000 | 0.000 | 13 | 0.019 | 0.046 |
| 4 | 0.000 | 0.001 | 14 | 0.029 | 0.056 |
| 5 | 0.001 | 0.001 | 15 | 0.043 | 0.075 |
| 6 | 0.001 | 0.003 | 16 | 0.058 | 0.100 |
| 7 | 0.002 | 0.005 | 17 | 0.078 | 0.125 |
| 8 | 0.003 | 0.008 | 18 | 0.100 | 0.149 |
| 9 | 0.005 | 0.012 | 19 | 0.123 | ... |
| 10 | 0.007 | 0.017 | 20 | 0.152 | ... |
| 11 | 0.010 | 0.024 | $L + 1$ | 0.358 | 0.344 |

| Item | Summary statistics: France | Summary statistics: Germany |
|---|---|---|
| First renewal age (year) | 2 | 3 |
| Last renewal age (year) | 20 | 18 |
| Ratio of grants to application | 0.93 | 0.35 |
| Data source | Application | Grants |
| Fraction paying last renewal (average over cohorts) | 0.07 | 0.11 |
| Fraction with weights ≤ 0.005 | 0.60 | 0.19 |

Source: Parameter estimates and data are from Ariel Pakes, "Patents as Options: Some Estimates of the Value of Holding European Patent Stocks," *Econometrica*, vol. 54 (July 1986), p. 767, table 1. Required auxiliary calculations use the average of the renewal fee schedules and the average of the dropout proportions at each age.

in Germany, and the statutory limit to patent lives was twenty years in France and eighteen in Germany (it is now twenty years in both countries). In France we follow all the patents applied for in a cohort (even though about 7 percent of these were never granted), but in Germany we follow only those patents that were eventually granted (usually only about 35 percent of the patents applied for in Germany).[33]

The estimated weights imply that patents dropping out before age five have almost no value in either country. This group typically includes about one-third of all French patents but only about 5 percent of German patents. The *VP* index will assign those patents a zero weight and discard them. Indeed, the two-thirds of the patents that drop out before age eleven in France could be discarded, as well as the 27 percent of those that drop out before age eight in Germany. Between those ages and *L*

33. Pakes, "Patents as Options," pp. 767–77.

simple proportional increments in the weights of about 35 percent per age seem to provide a reasonable fit to the data (actually the increments are slightly higher in the earlier ages and slightly lower in the later ages). Patents that pay the renewal fee for all possible ages (those in group $L + 1$) need a distinctly larger coefficient than would be predicted by this procedure; that is, $w(L + 1)/w(L) \approx 2.35$ in both countries.

There are several points to note about these estimates. The weights for the various age groups are very different. Since the renewal fees sweep out more than 90 percent of the patents, this is precisely what would be expected from a value distribution with a large variance and skew. The finding that a larger proportion of French patents are nearly worthless probably reflects the facts that the German data are for grants and that the granting procedure seems particularly stringent in Germany. That is, the patent examiners in Germany eliminate many of the less valuable patents that would otherwise drop out in the early ages. The more rapid rate of increase in the German weights after age five is likely to reflect the fact that the renewal fees are very small and comparable in both countries until that age ($50 to $100) but increase significantly faster in Germany thereafter. By age $L$ the renewal fees in Germany are between $1,500 and $2,000; those in France are only about $500. Thus the German fees at the later ages cut out patents that are relatively more valuable. Similarly, $w(L + 1)/w(L)$ is so large because of the open-endedness of the $L + 1^{\text{th}}$ group; it contains all those patents that had values greater than the renewal fee in every year. For all other groups we know that for at least one year the value was less than the renewal fee.

Having obtained $[w(a)]$, we can estimate the difference in the value of patented output between patents in, say, groups I and II by obtaining the number of the patents applied for that subsequently dropped out at age $a$ for each, $[n(a,\text{I})]$ and $[n(a,\text{II})]$, and then calculating

$$\lambda^{-1}[VP(\text{I}) - VP(\text{II})] = \sum_{a=1}^{L+1} w(a)[n(a,\text{I}) - n(a, \text{II})]. \tag{25}$$

For this calculation to give an accurate measure of the difference in the value of patented output up to the scalar $\lambda$, we require the $[w(a)]$, that is, the ratio of mean values for the patents that drop out at age $a$ ($a = 1, \ldots, L + 1$), to be constant across groups. For concreteness we discuss this assumption given equation 24, that is, given the hypothesis that the mean values of the patented ideas in the age subgroups are proportional to the mean value of patent protection in those subgroups. Analogous issues would arise if we did not maintain equation 24.

Given equation 24, the weights will be a function of the renewal fee schedules and the characteristics of the stochastic process generating the returns to patent protection. In almost all countries the renewal fees depend only on the year in which the fees are paid (a notable exception is the United States, but even so it has only two possible fee schedules). Thus no differences in fees occur if all patents studied are from the same cohort. The fee schedules do change somewhat over time, but in most countries we have studied there has not been much intertemporal variation in these schedules in real terms.[34] Further, changes in the renewal fees schedules are no problem provided we are willing to specify (or estimate) the form of the stochastic process generating the returns to patent protection, since that process implies weights that are easy to calculate (actually to simulate) for each different value of the renewal fee schedule (that is how we obtained the weights for table 1).

If there are differences in the stochastic process generating returns, then, except in special cases, there will be differences in the $[w(a)]$. The special cases correspond to instances in which the distribution of values among patents in each age cell stays constant and only the proportions of the cohorts falling into the alternative cells change. For small enough cell intervals this assumption ought not to be too bad, since then no allocation of the cell's patents to different locations within the cell will be able to alter the cell mean radically. Indeed, under mild regulatory conditions on the stochastic process generating returns, we should be able to obtain bounds for the within-cell weights (the bounds will, of course, be a function of the renewal fee schedule). Based on the research and experimentation we have done to date, our feeling is that $w(1), \ldots$ to $w(L)$ has not much room to wander, but that $w(L + 1)$ could vary substantially across groups (because of the open-endedness of the $L + 1$ group).

34. For example, Schankerman and Pakes, "Estimates of the Value of Patent Rights," partition the variances in the real renewal fees between the 1950s and the late 1970s in the United Kingdom, in France, and in Germany into a between-age and a within-age component. They find that in no country was the within-age over time component more than 14 percent of the total variance.

If we take as given that the assumptions underlying the weights in table 1 are valid for all groups studied, how much can the availability of renewal data, and hence of renewal-based indexes of the value of patent protection, ameliorate the problem of the noise in (or the variance in the value associated with) intergroup differences in patent count indexes? The answer depends on the fraction of the samples that fall into different age groups and the within-age cell variance in value. More precisely, a standard analysis of variance argument allows us to eliminate the between-age part of that variance. To generate some illustrative numbers, we again went back to Pakes's results (in "Patents and Options") and used his parameter estimates, the mean of the renewal fee schedules, and the mean of the fraction of the alternative cohorts that fell into the various age groups in that data to calculate the within-age and the between-age component to the variance in patent values. The results implied that the within-component of the variance was less than 42 percent of the between-component in France and less than 45 percent in Germany. So the variance in value of the renewal-based index would be about 30 percent of the variance in value of the count index in both countries. If this figure was close to correct, using renewal-based indexes could lead to a substantial improvement in our ability to measure the value of patented output, though whether it will be "substantial enough" depends on the purposes at hand.[35]

Schankerman and Pakes also investigated differences in the mean value of patents by the only dimension in which their data could be disaggregated: the date of application, or cohort, of the patent. Since they estimated a renewal model with cohort specific parameters, their implicit weights are allowed to differ over cohorts. (They also reported on some simple nonparametric tests for shifts in the value distribution over cohorts with results that are consistent with the parametric results.) The results were striking. They showed a distinct increase in mean

35. It should be noted that most of both the between and the within component of the variance is a result of the contribution of the last age group, so these calculations are likely to be sensitive to that part of the distribution which is probably most difficult to estimate (and indeed, that part which seems to differ most by estimation techniques; compare the weights in table 1 to the weights obtained in Schankerman and Pakes, "Estimates of the Value of Patent Rights"). Also, the recent increase in the statutory length of patent lives in Germany to twenty years, and the increase in the renewal fees in France, should both decrease the within-age to between-age variance ratios in our calculations, as might our ability to disaggregate by industry of patent and by type of patentee.

values starting between the middle to late 1960s and continuing through the end of their sample period (1975) in all three countries studied (France, Germany, and the United Kingdom).[36] That is almost exactly the same period for which we observe a distinct fall in patent applications in all three countries. Indeed, as noted and documented by Evenson, the fall in patent applications in that period was true across almost all countries for which data are available.[37] Since inventive input (as measured by the quantity of R&D expenditures or the employment of scientists and engineers) increased over the period, the decline in patents per unit of inventive input was very dramatic. By 1975 it had fallen to between 56 and 63 percent of its 1968 value in the three countries studied by Schankerman and Pakes. Many hypotheses advanced to explain this phenomenon associated it with different reasons for a decline in the value of technological opportunities.[38] The renewal data–based indexes show that the large falls in patent applications in these countries coincided with large (and largely offsetting) changes in the mean value of patents, so that the perception of a fall in the value of patented output left by the patent count indexes is to a great extent an illusion caused by problems in the measuring device—problems that could be partly corrected by using renewal-based indexes.

Schankerman and Pakes also found that between 1955 and 1965 there was actually an increase in patent applications and a slight decrease in the mean value of patents in a cohort (at least in two of the three countries studied). Thus perhaps the most general characterization of the Schankerman-Pakes results is a *negative correlation* between the number and the mean value (or the quantity and quality) of patents in a cohort. Later in the paper we provide some simple renewal-based nonparametric tests for changes in the distribution of patent values in Norway between 1962 and 1979 and in Finland between 1969 and 1979. The results for Norway roughly mimic the results obtained by Schankerman and Pakes. That is, patent applications increase from 4,000 to 5,000 from the early to the late 1960s, then decrease to about 4,300 by the late 1970s. The value distribution, in contrast, seems to have been fairly stable until 1968 and

36. Schankerman and Pakes, "Estimates of the Value of Patent Rights."

37. Evenson, "International Invention."

38. See Evenson, "International Invention"; and Zvi Griliches, "Introduction," in Griliches, ed., *R&D, Patents, and Productivity*.

then to have been pushed significantly outward thereafter. Finland was one of the few countries for which the number of patent applications increased between 1969 (about 3,500) and the late 1970s (about 4,100). It is also the only country studied for which there is no evidence of an increase in the mean values of the patents in a cohort over this period. The negative correlation between quality and quantity seems to be quite pervasive, which leads us to conclude the empirical section of this paper with a discussion of the factors that might generate it.

## Data Requirements

In this section we provide the information needed to access patent renewal data sets and to assign patents a standard industrial classification (SIC) code. The laws governing renewal and granting procedures differ somewhat between countries, and these differences can affect the way one analyzes the data. The World Intellectual Property Organization (WIPO) in Vienna puts out publications that summarize many of the procedural differences among countries.[39] More detailed information can usually be obtained from the brochures of the respective patent offices. WIPO publishes a *Directory of National and Regional Patent Offices* with the relevant addresses.

There are at least three sources of patent renewal data: the patent offices themselves, the International Patent Documentation Center (INPADOC) files, and individuals who have gathered and organized particular renewal data bases in the past. The INPADOC files contain only renewal status information on patents applied for after 1978 and therefore are not yet as relevant a data source as they will be later. A few national patent offices (notably Germany and France) enter into contracts with private data-base companies that market their renewal data. For the most part, however, renewal data will still have to be obtained directly from the respective patent offices.

To get an overview of the accessibility and contents of the data bases at the national patent offices, we sent out a questionnaire to more than

39. Committee of Experts on the Harmonization of Certain Provisions in Laws for the Protection of Inventions, *Duration of Patents; Maintenance Fees; Provisional Protection of Applicant: Prior Users' Rights,* Memorandum of the International Bureau (Geneva: World Intellectual Property Organization, May 1988).

one hundred patent offices listed in WIPO's directory. To date we have received replies from about 40 percent of them. Table 2 summarizes the major pieces of information we obtained from these replies. The table groups countries by region: North America and Western Europe, Eastern Europe, Asia and the Pacific, Central and South America, and Africa and the Middle East. In general, the data for North America and Western Europe are better than for the other regions.

The first column of the table shows whether the country's patent law requires renewal fees, and if so, a rough breakdown on how long such fees have been in existence. Out of the forty-two patent offices that replied, thirty-three have had renewal fee requirements for more than ten years, and at least half have had renewal fee requirements for more than forty years. For the European countries we have also listed the U.S. dollar equivalent of the maximal renewal fee in the current renewal fee schedule (when we had access to that schedule). The highest fees are in Germany and Austria (just under $2,000), and most are between $500 and $1,000. In all countries studied to date, these schedules sweep out at least 90 percent of aggregate patent cohorts; that is, under 10 percent of all patents taken out in a given year survive until the statutory limit to patent lives.

The next three columns of the table give information on whether there are manual patent records (and if so, from which date) and computerized patent records (and if so, from which date). For the latter, one column shows patents *applied for* as of the date specified; the other column shows patents still *in force* at the date specified. The table indicates that at least thirty patent offices keep manual records and at least nineteen keep computerized records. We also asked whether it would be possible to access the manual and/or computerized records, and if so whether there would be any charge for doing so (the next two columns). All the patent offices with manual records (except Switzerland) permit access to those records, and almost two-thirds of them do not charge for this access. At least nine of the patent offices with computerized records allow access to these records and about four of them do so at no cost. The situation vis-à-vis the existence and accessibility of computerized records is constantly improving, so many more renewal data bases are likely to be computerized in the near future.

Almost all the patent records contain information on the priority date, country, application number, and name and nationality of original

**Table 2. Summary of the Results of the Survey of Patent Offices in Five Regions**

Fees in U.S. dollars unless otherwise noted

| Country | Time fees have been required (years) | Maximum fee | Patent records available manually since when | Patent records available on computers: Filed after | Patent records available on computers: Alive after | Priority information available | Transfer of property information available | Per patent fee for manual retrieval | Per patent fee for computer retrieval | IPC code available |
|---|---|---|---|---|---|---|---|---|---|---|
| *Western Europe and North America* | | | | | | | | | | |
| Austria | >40 | 1,870 | 1950 | n.k.[a] | n.k. | Yes | Yes | 0 | 0 | Yes |
| Belgium | >40 | 240 | 1966 | n.a. | 1973 | Yes[b] | Yes[b] | Negotiable | 80/hr. | Yes[b] |
| Canada[c] | n.a. | n.a. | n.a. | n.a. | n.a. | Yes | Yes | n.a. | n.a. | Yes |
| Denmark | >40 | n.k. | 1894 | 1982 | 1987 | Yes | Yes | 0 | 0 | Yes |
| Finland | >40 | 900 | n.k. | n.a. | n.a. | Yes | Yes | 0 | n.k. | Yes |
| France | >40 | 495 | n.a. | 1969 | n.a. | Yes | Yes | n.a. | 115/hr. | Yes |
| Great Britain | >40 | 585 | 1945 | 1978[d] | n.a. | Yes | Yes | 3.55 | n.a. | Yes |
| Ireland | >40 | n.k. | 1929 | 1988 | n.a. | Yes | Yes | 10/hr. | n.a. | n.a. |
| Netherlands | >40 | 990 | 1912 | 1965 | 1967 | Yes | Yes | 0 | 0 | Yes |
| Norway | >40 | 650 | n.k. | n.a. | n.a. | Yes | Yes | 0 | n.k. | Yes |
| Portugal | >40 | n.k. | 1896 | n.a. | 1980 | Yes | Yes | 0 | n.k. | Yes |
| Spain | 10–20 | n.k. | n.k. | n.k. | n.k. | Yes | Yes | n.k. | n.k. | Yes |
| Sweden | >40 | n.k. | 1945 | n.a. | 1975 | Yes | Yes | 0 | n.k. | Yes |
| Switzerland[e] | >40 | 517 | 1888 | n.a. | 1978 | Yes | Yes | 3.33[f] | 3.33 | Yes |
| United States[g] | <10 | 1,200 | 1836 | 1967 | n.a. | Yes | Yes | 0 | n.k. | Yes |
| West Germany | >40 | 1,833 | 1950 | n.a. | 1981 | Yes | Yes | 0 | 0.55 | Yes |
| *Eastern Europe* | | | | | | | | | | |
| Bulgaria | 15 | n.k. | 1948 | 1948 | n.a. | Yes | Yes | Lv43/hr. | n.k. | Yes |
| East Germany | 30–40 | n.k. | 1951 | n.k. | n.k. | Yes | Yes | 5.50 | n.k. | Yes |
| Hungary | >40 | n.k. | n.k. | n.a. | n.a. | Yes | Yes | 0 | n.a. | Yes |
| Poland[h] | n.k. | n.k. | 1945 | n.a. | n.a. | Yes | Yes | 0 | n.a. | Yes |
| Yugoslavia[i] | n.k. | n.k. | 1951 | 1970[j] | n.a. | Yes | Yes | 0.50 | n.k. | Yes |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | *Asia and Pacific* | | | | | |
| Australia | >40 | n.k. | 1904 | 1979 | 1988 | Yes | Yes | 0 | n.k. | n.a. |
| India | >40 | n.k. | 1912 | n.a. | n.a. | Yes | Yes | RS5 | n.a. | n.a. |
| Japan | >40 | n.k. | 1943 | n.k. | 1978 | Yes | Yes | n.k. | n.k. | n.k. |
| Malaysia | 10–20 | n.k. | 1953 | n.a. | n.a. | Yes | No | $M5 | n.a. | n.a. |
| New Zealand[k] | 40 | n.k. | 1945 | n.a. | n.a. | Yes | n.k. | n.a. | 3 | Yes |
| Sri Lanka | 10–20 | n.k. | 1869 | n.a. | n.a. | Yes | Yes | RS10 | n.a. | Yes |
| Taiwan | 30–40 | n.k. | 1950 | 1950 | n.a. | No | Yes | n.k. | n.k. | Yes |
| | | | | | *Central and South America* | | | | | |
| Argentina[l] | n.a. | n.a. | n.k. | 1983 | n.a. | Yes | No | n.k. | n.k. | Yes |
| Brazil | 10–20 | n.k. | n.k. | 1972 | n.a. | Yes | Yes | n.k. | n.k. | Yes |
| Chile[m] | n.k. | n.k. | 1970 | n.a. | n.a. | No | No | 0 | n.a. | Yes |
| Colombia[n] | <10 | n.k. | n.a. | n.a. | n.a. | No | Yes | n.a. | n.a. | n.a. |
| Honduras | 10–20 | n.k. | 1968 | n.a. | n.a. | Yes | Yes | 0 | n.a. | Yes |
| Uruguay | 10–20 | n.k. | 1970 | n.a. | 1970 | No | Yes | 0 | 0 | Yes |
| | | | | | *Africa and Middle East* | | | | | |
| Cameroon | 10–20 | n.k. | n.k. | n.k. | n.k. | Yes | Yes | 70 | n.k. | Yes |
| Egypt | 10–20 | n.k. | 1950 | n.a. | n.a. | Yes | Yes | £E5 | n.a. | Yes |
| Israel[o] | <10 | n.k. | 1930 | n.a. | n.a. | Yes | No | 0 | n.a. | n.a. |
| Morocco | >40 | n.k. | n.k. | n.k. | n.k. | Yes | n.k. | n.a. | n.a. | n.a. |
| Rwanda[p] | n.k. | n.k. | 1965 | n.a. | n.a. | Yes | Yes | 0 | n.a. | n.a. |
| Saudi Arabia | 10–20 | n.k. | n.k. | n.a. | n.a. | Yes | Yes | n.k. | n.a. | Yes |
| South Africa | 10–20 | n.k. | 1950 | n.a. | n.a. | Yes | Yes | 0.50 | n.a. | Yes |
| Tanzania | >40 | n.k. | 1932 | n.a. | n.a. | No | Yes | 1 | n.a. | n.a. |

n.a. Not available, either because data do not exist or because access to them is restricted.
n.k. Not known.
a. Data are computerized, but starting date not known.
b. Available only for computerized data.
c. Renewal fees instituted in 1987.
d. Available mid-1989.
e. Researchers not permitted either manual or computerized access to data.
f. Access available only through the patent office.
g. Renewal fees instituted in 1982.
h. Fees paid for years 1–5, 6–8, and annually thereafter.
i. Fees paid at years 1, 4, 7, 10, and 13.
j. Renewal information not computerized.
k. Fees paid at years 4, 7, 10, and 13.
l. Renewal fees not levied.
m. Fees paid at years 5 and 10, with renewal possible at year 15 under some circumstances.
n. Patents may be renewed at age 5 for 5 more years.
o. Fees paid at years 6, 10, 14, and 18.
p. All fees paid at time of application.

patentee for every patent recorded. This information allows the researcher to construct a "patent family" consisting of the different national patents generated from the same invention. Almost all the records also contain information on changes of ownership whenever that occurs. More than three-quarters of the patent offices assign an international patent classification (IPC) code to each patent recorded. A list of addresses to which one may write to get the more detailed information needed to access the data sets of each country in table 2 can be obtained from Ariel Pakes.

The IPC, a very detailed classification with more than ten thousand separate patent classes, is the means by which most countries organize their patent information into technologically related fields. It was developed in 1955 to supplant the national patent classifications then in existence and facilitate the work of patent examiners. The current classification is the fourth version of the IPC, which is meant to change every five years to keep pace with technological developments.

For many research purposes we will have to have an (at least probabilistic) assignment of patents to the more familiar SIC classes of industries. This problem has troubled both government offices and researchers in the past, and several "concordances" between patent classifications and the SIC have been built. The U.S. Patent Office assigns both its own and an IPC classification to each of its patents and has built a concordance between the U.S. patent classification and the SIC.[40] But this concordance does not permit us to work with patents classified by the IPC and has several other problems.[41] Valkonen reclassified a sample of Finnish patents from the 1980s by SIC industry of origin, and then derived the implicit multinominal probabilities for patents from IPC "*i*" originating in SIC "*j*" (this is the definition of the matrix of assignment probabilities we are after).[42] The Canadian Patent Office has been assigning up to three industries of manufacture and three

40. Office of Technology Assessment and Forecast, "Review and Assessment of the OTAF Concordance Between the U.S. Patent Classification and the Standard Industrial Classification Systems: Final Report," U.S. Department of Commerce, Patent and Trademark Office, January 1985.

41. For a discussion see F. M. Scherer, "The Office of Technology Assessment and Forecast Concordance as a Means of Identifying Industry Technology Origins," *World Patent Information,* vol. 4, no. 1 (1982), pp. 12–17.

42. Pekka Valkonen, "Patenttitilastojen Hyödyntämisestä," Teknillinen Korkeakoulu, Täydennyskoulutuskeskus, Informaatiopalvelun kurssi (Otaniemi, 1985).

industries of use to all patents applied for since 1978. Kortum and Putnam have used the implicit relationship between the SIC and IPC classifications generated in that way to build concordances both from IPC to industry of use and from IPC to industry of origin.[43]

## An Empirical Analysis of Intercohort, Interindustry, and International Differences in the Value of Patent Protection

As shown earlier in the paper, the null hypothesis that the stochastic process generating the returns from patent protection does not differ between different groups of patents implies that the renewal proportions generated from the two groups do not differ; the null hypothesis that the stochastic process generating the returns from patent protection is better in one group of patents than in another implies that the renewal curve generated by the patents in the first group lies above that in the second. In this section we use those results, along with observations on the renewal behavior of patents taken out in Finland and in Norway, to analyze international (by nationality of patentee), intercohort, and interindustry differences in the value of patent protection.

### *Analytic Procedure*

The entire analysis is done separately for the patents in Finland and Norway. In each country each patent is placed in only one group. The group specifies the international patent classification of the industry of the patent (indexed by a $q = 1, \ldots, Q$), the nationality of the patentee (indexed by an $n = 1, \ldots N$), and the year in which the patent was applied for (or its cohort, indexed by a $c = 1, \ldots, C$). We use the observations on the proportions renewing in the alternative groups, together with the test developed earlier in the paper, to investigate differences in the value of patent protection by cohort, nationality, and the standard industrial classification industry of the patent (the SIC industries are indexed by a $j = 1, \ldots, J$). If all patents belonging to any given IPC also belonged to the same SIC, we could simply aggregate

43. Sam Kortum and Jonathan Putnam, "Techniques to Estimate the Output of Patents Across Industries," Yale University, forthcoming.

all IPCs in the given SIC, obtain groups of patents defined by their SIC-nationality-cohort triple, and use these groups directly in our tests.

Unfortunately the data are not that benevolent. As noted in the previous section, at least two concordances have been built between the IPC and the SIC. The more detailed one is based on a classification of Canadian patents; the less detailed one on a classification of Finnish patents.[44] Because we thought the Finnish concordance was more likely to be appropriate for classifying Finnish and Norwegian patents, we decided to use it for our analysis. But even a cursory look at either concordance reveals that all the patents in a given IPC do not belong in the same SIC. We therefore had to modify the testing and estimation procedure to account for the fact that we do not have perfect knowledge of the SIC industries of the patents in our sample.

Turning to the observed IPC-nationality-cohort groups, we know that each member of each group will drop out in only one of the possible dropout ages. Thus, provided the renewal behaviors of the members of a group are independent of one another, the vector of the group's observed proportions dropping out at the alternative ages, say $p^g$, has a multinomial distribution in the true theoretic proportions $\pi^g$, and sample size $n_g$. Consequently, as $n_g$ grows large, the central limit theorem ensures that

$$n_g(p^g - \pi^g) \sim > N(0, \text{diag}\, \pi^g - \pi^g \pi^{g\prime}). \tag{26}$$

Again, the group, or $g$ index in the equation, runs over particular values of nationality-cohort-IPC triples, or $g = (n, c, q)$, for some $n \epsilon (1, \ldots, N)$, $c \epsilon (1, \ldots, C)$, and $q \epsilon (1, \ldots, Q)$. Our maintained hypothesis is that the probability of a patent dropping out at different ages is determined by a value of its nationality-cohort-SIC triple, or by an $(n, c, j)$ for some $n \epsilon (1, \ldots, N)$, $c \epsilon (1, \ldots, C)$, and $j = (1, \ldots, J)$. Our problem, then, reduces to expressing $\pi^g = \pi^{n,c,q}$ in terms of the $\pi^{n,c,j}$ we are interested in estimating.

The concordances contain the information that allows us to accomplish this task. They provide the probability that a patent in IPC $q$ is generated by SIC $j$, or $b(q, j)$ for $q = 1, \ldots, Q$, and $j = 1, \ldots, J$. Letting

44. Kortum and Putnam, "Techniques to Estimate the Output of Patents"; and Valkonen, "Patenttitilastojen Hyödyntämisestä."

$$X(a, i) = \begin{bmatrix} 1 \text{ if patent } i \text{ drops out in year } a \\ 0 \text{ elsewhere} \end{bmatrix},$$

and noting that the $a$th component of $\pi^{n,c,q}$, say $\pi^{n,c,q}(a)$, is just the probability that a randomly drawn patent from group $(n, c, q)$ drops out at age $a$, we have

$$\begin{aligned} (27) \qquad \pi^{n,c,q}(a) &= E[X(a, i) / i \epsilon (n,c,q)] \\ &= E_j E[X(a, i) / i \epsilon (n,c,q,j)] \\ &= E_j \pi^{n,c,j}(\mathrm{a}) \\ &= \sum_{j=1}^{J} b(q,j)\, \pi^{n,c,j}(a). \end{aligned}$$

So $\pi^{n,c,q}(a)$ is just a weighted average of the $\pi^{n,c,j}$, the weights being given by the probabilities that a patent in IPC $q$ is generated from industry $j$ (that is, by a column of the concordance matrix).

Equations 26 and 27 form the basis for our estimation and testing procedures. More precisely the $\pi^{n,c,j}$ are estimated by substituting equation 27 into equation 26 and minimizing

$$(28) \qquad X^2_{IN} = \sum_g \sum_a \frac{n^g (p^g(a) - \pi^g(a))^2}{\pi^g(a)},$$

with respect to the $\pi^{n,c,j}$.

The $\hat{\pi}^{n,c,j}$ obtained in this fashion are the minimum chi-square estimators for this problem (they are consistent and asymptotically efficient estimators), and the minimized value of $X^2_{IN}$ distributes under the null that a patent's dropout probabilities are fully determined by its nationality-cohort-SIC triple, as a $\chi^2$ deviate with degrees of freedom equal to the difference in the number of independent cells and the number of parameters being estimated.

Tests of equality among subsets of the estimated coefficients are almost as easy. To test a constraint on the $\pi^{n,c,j}$, we simply use equations 27 and 28 to estimate twice, once imposing the constraint on the $\pi^{n,c,j}$ and once not. Letting $\hat{\pi}^g_2$ be the constrained estimates and $\hat{\pi}^g_1$ be the unconstrained estimates then, under the null that the constraints are indeed appropriate, the statistic

$$(29) \qquad X^2_D = \sum_g \sum_a n_g \frac{[\hat{\pi}_1{}^g(a) - \hat{\pi}_2{}^g(a)]^2}{\hat{\pi}_2{}^g(a)}$$

will distribute as a $\chi^2$ deviate with degrees of freedom equal to the difference in the number of degrees of freedom in the initial two runs.

Recall that we are also interested in testing the null that the renewal curve for one group of patents lies above that for another, since this was our test for the distribution of patent values in the first group being better than (stochastically dominating) the second. The renewal curve for group $(n,c,j)$ lies above the curve for $(n,c,j)'$ if and only if

$$\sum_{a=1}^{\alpha} \pi^{(n,c,j)}(a) - \sum_{a=1}^{\alpha} \pi^{(n,c,j)'}(a) \leq 0, \quad \text{for } \alpha = 1, \ldots, L.$$

Let $\pi$ be the vector that strings together the dropout probabilities for all the nationality-cohort-SIC groups. Then the test for the $L$ inequality constraints that determine whether the renewal curve for one group of patents lies above that for another can be written as

$$J\pi \leq 0, \tag{30}$$

for some $L$ row matrix $J$. A program for performing such tests was developed in a paper by Pakes and Ericson, which also provided a short description of the logic underlying the inequality tests and more detailed references to the relevant statistical and econometric literature.[45] The test requires a consistent estimator of the variance-covariance matrix of $J$ times the unconstrained estimate of $\pi$. Our appendix gives a formula for this matrix that makes the matrix itself easy to manipulate on almost any personal computer.

At times we found it useful to go further than the formal testing procedures and actually plot and compare the renewal curves for different groups. When we used the three-way nationality-industry-cohort classification, however, the number of patents in each group was too small, and the number of groups too large, for the pictorial comparison to be helpful. To mitigate this problem, we often present figures that aggregate over the cohort dimension of the data. That is, we estimate probabilities separately for each cohort but then present figures based on the simple average of the estimated dropout probabilities over cohorts. Since most cohorts studied do not reach their later ages by 1987, the renewal proportions shown in the figures for these ages are based on a

45. Pakes and Ericson, "Empirical Implications of Alternative Models of Firms Dynamics."

**Table 3. Characteristics of the Patent Data for Finland and Norway**

| *Characteristic* | *Finland* | *Norway* |
|---|---|---|
| Range of years | 1969–87 | 1962–87 |
| Maximum observed patent age | 18 | 18[a] |
| Mean number of granted patents per cohort | 1,557 | 2,321 |
| Approximate ratio of grants to applications for cohorts | | |
| 1962–67 | . . . | 0.47 |
| 1968–73[b] | 0.39 | 0.47 |
| 1974–79 | 0.41 | 0.44 |
| Number of cohorts | 9 | 16 |
| Number of nationality groups[c] | 16 | 16 |
| Number of IPC subclasses | 615 | 615 |
| Number of ISIC industries[d] | 20 | 20 |
| Fraction of granted patents held by residents of granting country | 0.225 | 0.106 |

a. In 1973 a change to the Norwegian patent laws extended the statutory limit to patent lives for patents applied for during or after 1968 to twenty years. The limit to patent lives for patents applied for before 1968 was seventeen years.

b. For Finland, this is the ratio of grants to applications for cohorts 1969–73.

c. The sixteen nationality classifications used were Finland, Norway, Sweden, Denmark, West Germany, Switzerland, the Netherlands, France, Great Britain, other Western Europe, Eastern Europe and the Soviet Union, the United States, Japan, Canada, Australia and New Zealand, and all other countries.

d. The twenty industry classifications used were (with corresponding international standard industrial classification codes in parentheses): food and kindred products (31); textiles, apparel, and leather industries (32); lumber, wood and paper products (33 and 34); chemicals and allied products, except drugs and medicines (352, except 3522); drugs and medicines (3522); petroleum refining and extraction (354); rubber and plastic products (355 and 356); stone, clay, and glass products (36); primary metals (37); fabricated metal products (381); machinery, except agricultural equipment and office and computing equipment (382, except 3822 and 3825); agricultural equipment (3822); office, computing, and accounting equipment (3825); electrical equipment, except communication (383, except 3832); communication equipment (3832); motor vehicles and transport equipment, except aircraft (384, except 3845); aircraft and aerospace (3845); professional and scientific equipment (385); other manufacturing industries (39); construction and sanitation (5000 and 9200).

much smaller number of cohorts than the renewal proportions for the earlier ages. (The renewal proportion for age nineteen is, for example, based on only a single cohort in Finland.) As a result, the renewal curves we plot can actually increase from age to age, and the last few renewal proportions are likely to be more variant than the earlier ones.

### *The Data*

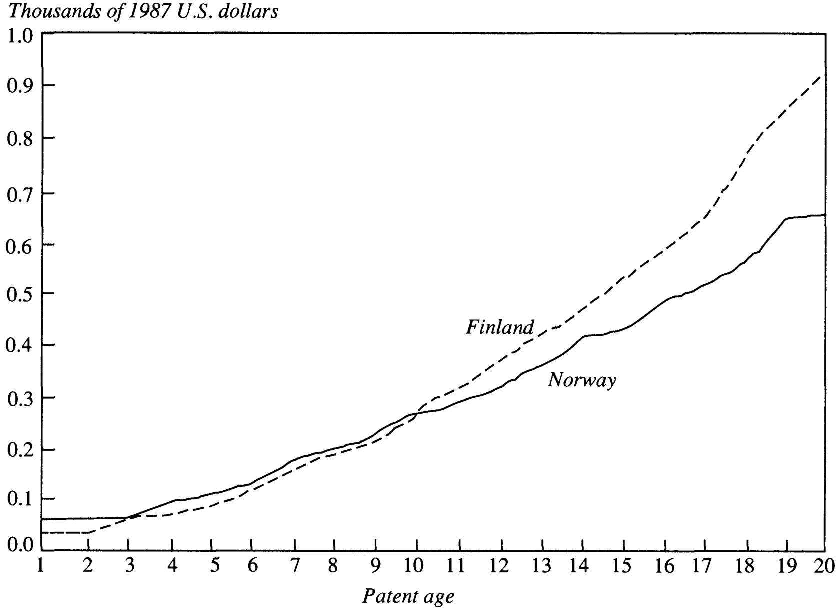
Table 3 shows some characteristics of the data, which contain information on patent renewals in Norway between 1962 and 1987 and in Finland between 1969 and 1987. Since these countries do not require payment of an annual renewal fee until after the patent is granted, we studied the value of patent protection among the population of patents granted. About 40 percent of all applications are granted in Finland and about 45 percent in Norway (for comparison, the grant rate in France and the United Kingdom during this period was much larger, 80 to 90 percent; in Germany it was only about one-third).

Because grant dates differ among patents from the same nationality-cohort-IPC triple, our nonparametric estimation and testing procedure should ideally add a grant date dimension to our definition of a group. Our early analysis did so, but we found that the number of groups we generated became cumbersome (increasing by a factor of 10), and that the results hardly differed from the simple case where we studied renewal behavior only after age ten (an age by which almost all patent grant decisions had been made) and did not distinguish patents by their grant date. In what follows, then, we ignore the information on the grant date of the patent and limit ourselves to analyzing renewal behavior after age ten. That has the effect of limiting us to nine cohorts of Finnish data and sixteen cohorts of Norwegian data. About 1,550 patents are granted per Finnish cohort and about 2,300 per Norwegian cohort. Thus we have studied about 14,000 Finnish patent grants and about 37,000 Norwegian patent grants. Note that more than 77 percent of the Finnish patents and more than 88 percent of the Norwegian patents are registered to residents of foreign countries.

The IPC classification we used has 615 categories, and we divided the data into 16 nationalities (see table 3) so the total number of possible patent groups is 88,560 ($615 \times 16 \times 9$) in Finland and 157,440 in Norway. (These are the numbers that are raised by a factor of 10 by adding the grant date dimension to the data.) We have nonzero patents in 7,037 of these groups in Finland and in 18,202 in Norway. Our model maintains that renewal probabilities are determined by the SIC-nationality-cohort triple of the patent. We used a classification involving twenty SIC industries, so there are a maximum of 2,400 vectors of multinomial probabilities to estimate from the Finnish data and a maximum of 4,500 from the Norwegian data.

Figure 2 shows the average of the renewal fee schedules over the period covered by our sample. As in all such schedules we have seen, they increase over age, starting at about \$50 and reaching about \$950 in Finland and about \$650 in Norway. These are lower than the fee schedules in Germany, Austria, and the Netherlands but somewhat higher than those in France and Great Britain.

**Figure 2. Average Patent Renewal Fees, Finland and Norway**

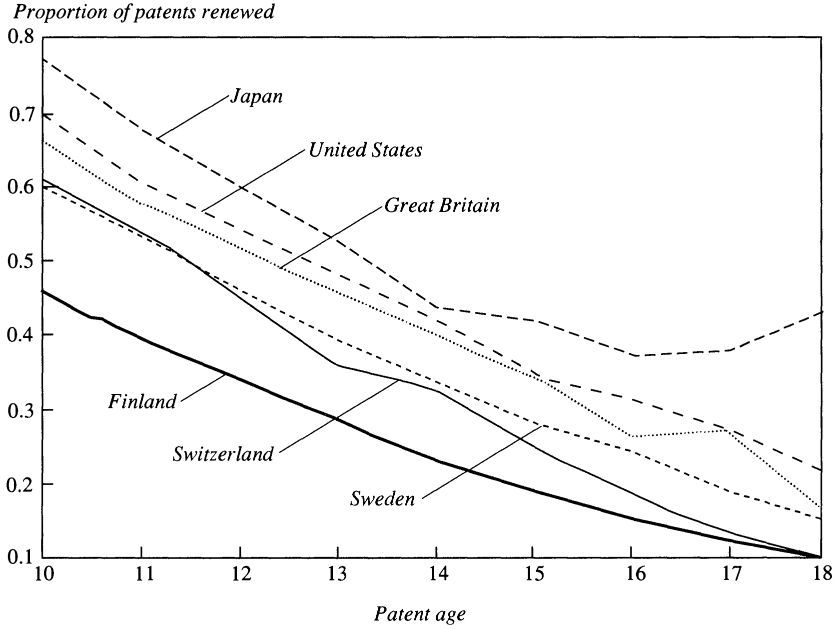



### *The Nationality and Industry Dimensions*

We began with an omnibus goodness-of-fit test for whether the SIC industry-nationality-cohort breakdown, in conjunction with the IPC-SIC concordance matrix, is rich enough to account for the variation in renewal proportions among the IPC-nationality-cohort groups in the data. This consisted of testing whether, when we substitute the estimated $\pi^{n,c,j}$ into equation 27 and the result into equation 28, the observed value of the $X^2_{IN}$ deviate is statistically significant.[46] The test statistic was obtained separately for each possible cohort, and there was no cohort in either country for which its observed value was significant. When we summed over the cohorts to obtain an aggregate test statistic, it was also insignificant in both countries. Given our sample sizes, then, the joint hypothesis that dropout probabilities are determined by the nationality-

46. For computational tractability these tests, and those in tables 4 and 5, only distinguished between four possible age cells for each industry-cohort-nationality triple: a cell for dropping out before age ten, a cell for ten to fifteen, a cell for fifteen to seventeen, and a cell for eighteen.

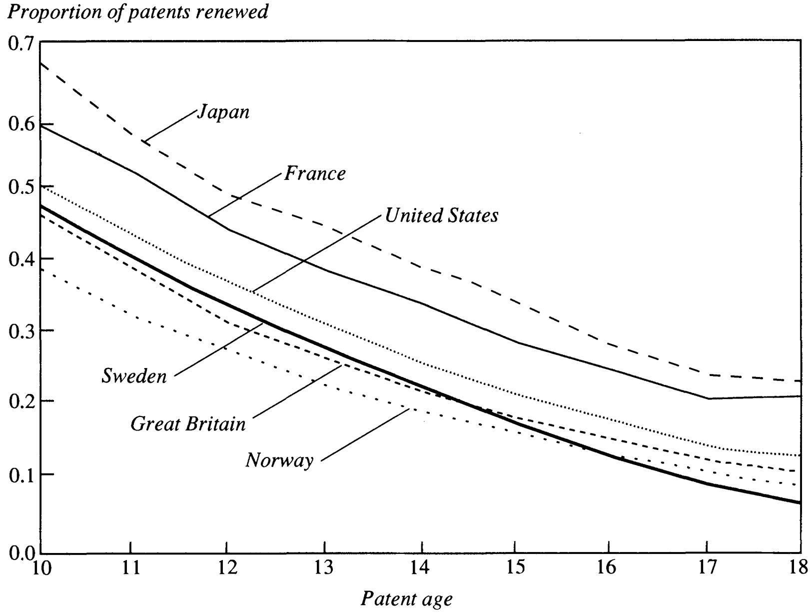
**Figure 3. Differences in Patent Renewal Curves by Nationality of the Patentee, Finland**




cohort-SIC triple of the patent, and that the IPC-SIC concordance provides the probabilities that a patent in a given IPC stems from the alternative SIC industries, cannot be rejected by the data.

Next we considered tests of whether renewal behavior differed by the nationality of the patentee. As a prelude to the full analysis, we noted that if we do not distinguish at all among industries—that is, condition only on cohort and nationality—the test of the null that nationality differences are not related to dropout probabilities is clearly rejected in each of the cohorts in both data sets. (Aggregated over cohorts, the test statistics, divided by their degrees of freedom, were 2.88 in Finland and 2.61 in Norway, with degrees of freedom equal to 300 and 600, respectively.) Indeed, simple renewal curve plots reveal a definite ordering of the value distributions by nationality of patentee (figures 3 and 4). Looking first at the patents taken out in Finland (figure 3), we find that the Japanese renewal curve dominates the U.S. curve (which is similar to the curves for France and the Netherlands); the U.S. curve seems to

**Figure 4. Differences in Patent Renewal Curves by Nationality of the Patentee, Norway**




dominate the British curve (which is similar to the curves for Norway, Denmark, and "Other Europe"); the British curve dominates the Swedish curve, which dominates Switzerland (which is similar to the Eastern European, Canadian, and West German curves), which in turn dominates Finland. The main difference between these results and those generated by the patents taken out in Norway (figure 4) is that the Norwegian and Finnish curves switch positions in the two partial orderings, so that Norwegian patents have the lowest renewal curve in Norway (as do Finnish patents in Finland). The Finnish and Norwegian curves are the lowest curves in their respective countries probably because the effective cost of applying for a patent is lower for domestic than for foreign patentees. Thus domestic patents that are less likely to be valuable are more likely to be applied for. The rest of the observed partial ordering of nationalities does not have an obvious interpretation.

Tables 4 and 5 provide formal test statistics for various sets of equality constraints for each of the cohorts studied in Finland and Norway. The

**Table 4. Equality Tests for Finland, Cohorts 1969–77**[a]

| | *No nationality differences* | | *No SIC differences* | |
|---|---|---|---|---|
| *Cohort (number of patents)* | *All data (1)* | *Only patents registered to foreigners (2)* | *All data (3)* | *Only patents registered to foreigners (4)* |
| 1977(1,757) | | | | |
| $\chi^2$/d.f. | 1.01 | 0.92 | 1.12 | 1.23 |
| d.f. | 208 | 189 | 212 | 1.94 |
| $p$-value | 0.438 | 0.77 | 0.097 | 0.016 |
| 1976(1,631) | | | | |
| $\chi^2$/d.f. | 1.15 | 1.05 | 1.25 | 1.24 |
| d.f. | 410 | 369 | 418 | 379 |
| $p$-value | 0.018 | 0.240 | 0 | 0 |
| 1975(1,581) | | | | |
| $\chi^2$/d.f. | 1.07 | 1.06 | 1.18 | 1.18 |
| d.f. | 410 | 372 | 418 | 382 |
| $p$-value | 0.157 | 0.205 | 0.005 | 0.008 |
| 1974(1,577) | | | | |
| $\chi^2$/d.f. | 1.09 | 1.04 | 1.19 | 1.334 |
| d.f. | 410 | 372 | 418 | 394 |
| $p$-value | 0.096 | 0.088 | 0.004 | 0 |
| 1973(1,657) | | | | |
| $\chi^2$/d.f. | 1.07 | 0.961 | 1.24 | 1.351 |
| d.f. | 405 | 365 | 413 | 390 |
| $p$-value | 0.158 | 0.695 | 0.001 | 0 |
| 1972(1,506) | | | | |
| $\chi^2$/d.f. | 1.29 | 1.13 | 1.36 | 1.86 |
| d.f. | 392 | 354 | 400 | 364 |
| $p$-value | 0.000 | 0.086 | 0.000 | 0 |
| 1971(1,466) | | | | |
| $\chi^2$/d.f. | 1.18 | 1.06 | 1.25 | 1.32 |
| d.f. | 604 | 530 | 616 | 548 |
| $p$-value | 0.001 | 0.171 | 0 | 0 |
| 1970(1,404) | | | | |
| $\chi^2$/d.f. | 1.04 | 0.980 | 1.17 | 1.17 |
| d.f. | 581 | 522 | 593 | 537 |
| $p$-value | 0.230 | 0.619 | 0.003 | 0.005 |
| 1969(1,462) | | | | |
| $\chi^2$/d.f. | 1.86 | 1.31 | 1.48 | 1.56 |
| d.f. | 568 | 507 | 580 | 518 |
| $p$-value | 0 | 0 | 0 | 0 |
| All cohort aggregate (14,041) | | | | |
| $\chi^2$/d.f. | 1.224 | 1.070 | 1.261 | 1.362 |
| d.f. | 3,988 | 3,579 | 4,068 | 3,706 |
| $p$-value | 0 | .002 | 0 | 0 |

a. Maintained hypothesis: separate vectors of patent renewal proportions for each cohort–nationality–SIC industry triple. The table shows the observed value of the $\chi^2$ deviate divided by its degrees of freedom for the relevant test, the degrees of freedom of the test, and the $p$-value for the observed test statistic.

first two columns maintain the hypothesis that dropout probabilities are determined by a patent's nationality-cohort-SIC industry triple and then test the null hypothesis that

(31) $H_o^N$: $\pi^{n,c,j} = \pi^{c,j}$, for $n \epsilon (1, \ldots, N), j \epsilon (1, \ldots J)$, and $c \epsilon (1, \ldots, C)$,

or that, conditional on a patent's SIC industry and cohort, differences in the nationality of the patentee are not related to differences in renewal behavior. Column 1 reports the observed value of the test statistic when the null is applied to all nationality groups; column 2 shows the observed values when we allow patents that belong to patentees who are residents of the granting country to have different renewal proportions than those of patentees who are foreign residents (though we do not allow for any distinction among residents of different foreign countries). The difference between the two columns, then, provides a $\chi^2$ test for the null that the foreign domestic breakdown is also irrelevant (with degrees of freedom equal to the difference in the number of degrees of freedom in the two columns).

Columns 3 and 4 use the same maintained hypothesis but change the null to

(32) $H_o^I : \pi^{n,c,j}, = \pi^{n,c}$, for $n \epsilon (1, \ldots, N), j \epsilon (1, \ldots, J)$, and $c \epsilon (1, \ldots, C)$,

or that, conditional on a patent's nationality and cohort, differences in the SIC industry of the patent are not related to differences in renewal behavior. Column 3 shows the observed value of the test statistics when all patents are used in the analysis; column 4 shows them when only patents registered to foreign residents are used.

When we compare columns 1 and 2, we find we reject the null that there are no differences in the renewal behavior of foreign and domestic patents in seven or eight of the nine cohorts in Finland and in seven to ten of the sixteen Norwegian cohorts. (The precise number depends on whether one accepts at $p$-values that are traditionally considered marginal.) Summing over cohorts, the observed values of the aggregate $\chi^2$ deviates for the domestic-foreign difference divided by their degrees of freedom were 2.58 (degrees of freedom of 409) in Finland and 2.16 (degrees of freedom of 731) in Norway—both clearly significant. These results, together with the domestic curves in figures 2 and 3 and the reasons we have to expect to find lower valued domestic patents, have persuaded us to focus the rest of the discussion on patents granted to

**Table 5. Equality Tests for Norway, Cohorts 1962–77**[a]

| Cohort (number of patents) | No nationality differences: All data (1) | No nationality differences: Only patents registered to foreigners (2) | No SIC differences: All data (3) | No SIC differences: Only patents registered to foreigners (4) |
|---|---|---|---|---|
| 1977(2,068) | | | | |
| $\chi^2$/d.f. | 1.25 | 1.19 | 1.31 | 1.44 |
| d.f. | 200 | 211 | 212 | 216 |
| *p*-value | 0.006 | 0.027 | 0.097 | 0 |
| 1976(1,992) | | | | |
| $\chi^2$/d.f. | 1.08 | 1.08 | 1.24 | 1.36 |
| d.f. | 444 | 406 | 452 | 416 |
| *p*-value | 0.111 | 0.127 | 0 | 0 |
| 1975(2,004) | | | | |
| $\chi^2$/d.f. | 1.11 | 1.11 | 1.18 | 1.23 |
| d.f. | 455 | 418 | 463 | 428 |
| *p*-value | 0.049 | 0.114 | 0.005 | 0 |
| 1974(2,138) | | | | |
| $\chi^2$/d.f. | 1.18 | 1.19 | 1.26 | 1.34 |
| d.f. | 452 | 418 | 460 | 428 |
| *p*-value | 0.004 | 0.004 | 0 | 0 |
| 1973(2,385) | | | | |
| $\chi^2$/d.f. | 1.19 | 1.16 | 1.34 | 1.34 |
| d.f. | 464 | 430 | 472 | 440 |
| *p*-value | 0.003 | 0.011 | 0 | 0 |
| 1972(2,364) | | | | |
| $\chi^2$/d.f. | 1.06 | 1.01 | 1.28 | 1.31 |
| d.f. | 471 | 433 | 479 | 443 |
| *p*-value | 0.438 | 0.447 | 0 | 0 |
| 1971(2,447) | | | | |
| $\chi^2$/d.f. | 1.23 | 1.17 | 1.40 | 1.56 |
| d.f. | 688 | 634 | 700 | 650 |
| *p*-value | 0 | 0.001 | 0 | 0 |
| 1970(2,343) | | | | |
| $\chi^2$/d.f. | 1.21 | 1.18 | 1.36 | 1.40 |
| d.f. | 667 | 615 | 679 | 630 |
| *p*-value | 0.001 | 0.002 | 0 | 0 |
| 1969(2,433) | | | | |
| $\chi^2$/d.f. | 1.24 | 1.22 | 1.38 | 1.45 |
| d.f. | 686 | 636 | 698 | 645 |
| *p*-value | 0 | 0 | 0 | 0 |
| 1968(2,567) | | | | |
| $\chi^2$/d.f. | 1.49 | 1.23 | 1.30 | 1.41 |
| d.f. | 690 | 631 | 702 | 646 |
| *p*-value | 0 | 0 | 0 | 0 |

**Table 5 (continued)**

| Cohort (number of patents) | *No nationality differences* All data (1) | *No nationality differences* Only patents registered to foreigners (2) | *No SIC differences* All data (3) | *No SIC differences* Only patents registered to foreigners (4) |
|---|---|---|---|---|
| 1967(2,469) | | | | |
| $\chi^2$/d.f. | 1.14 | 1.14 | 1.27 | 1.31 |
| d.f. | 698 | 644 | 710 | 659 |
| $p$-value | 0.006 | 0.009 | 0 | 0 |
| 1966(2,481) | | | | |
| $\chi^2$/d.f. | 1.31 | 1.28 | 1.57 | 1.63 |
| d.f. | 679 | 622 | 691 | 637 |
| $p$-value | 0 | 0 | 0 | 0 |
| 1965(2,412) | | | | |
| $\chi^2$/d.f. | 1.24 | 1.26 | 1.56 | 1.17 |
| d.f. | 664 | 607 | 676 | 622 |
| $p$-value | 0 | 0 | 0 | 0 |
| 1964(2,537) | | | | |
| $\chi^2$/d.f. | 1.16 | 1.16 | 1.58 | 1.71 |
| d.f. | 672 | 602 | 684 | 620 |
| $p$-value | 0.003 | 0.004 | 0 | 0 |
| 1963(2,356) | | | | |
| $\chi^2$/d.f. | 1.86 | 1.21 | 1.72 | 1.74 |
| d.f. | 628 | 577 | 640 | 586 |
| $p$-value | 0 | 0 | 0 | 0 |
| 1962(2,110) | | | | |
| $\chi^2$/d.f. | 1.21 | 1.21 | 1.66 | 1.78 |
| d.f. | 628 | 571 | 640 | 586 |
| $p$-value | 0 | 0 | 0 | 0 |
| All cohort aggregate (37,133) | | | | |
| $\chi^2$/d.f. | 1.26 | 1.18 | 1.23 | 1.50 |
| d.f. | 9,186 | 8,455 | 9,358 | 8,652 |
| $p$-value | 0 | 0.002 | 0 | 0 |

a. Maintained hypothesis: separate vectors of patent renewal proportions for each cohort–nationality–industry triple. The table shows the observed value of the $\chi^2$ deviate divided by its degrees of freedom for the relevant test, the degrees of freedom of the test, and the $p$-value for the observed test statistics.

foreign residents only (though, as it works out, there would be very little difference if we gave results instead for the entire sample of grants).

Column 2 shows that when we consider only patents taken out by foreign residents, and then ask whether conditional on industry and cohort there are any further differences in renewal proportions associated with the nationality of the patentee, we can accept the null that there are no such differences in eight of the nine Finnish cohorts and in three to seven of the Norwegian cohorts. When we sum over cohorts, the values of the aggregate test statistics for nationality differences are significant, a finding that might have been expected given the size of the data sets and the fact there are several thousand degrees of freedom for each test. But the test statistics themselves are not very large, at least compared with the aggregate test statistics for the other equality tests analyzed in this paper. (The observed values of the $\chi^2$ deviates normalized by their degrees of freedom were 1.07 in Finland and 1.18 for the larger Norwegian sample.) We conclude that most, if not all, of the effect of nationality on renewal proportions disappears once we condition on industry.[47] It is likely, then, that the unconditional nationality differences noted above are largely a result of two facts: different nationalities tend to patent disproportionately in different industries, and, as we now will show, distinct differences in renewal behavior are associated with patents in different SIC industries.

Columns 3 and 4 test if, conditional on the patent's cohort and nationality, differences in its SIC industry are associated with differences in renewal behavior. This test had observed values that were significant in all the Norwegian cohorts and in all except (possibly) one of the Finnish cohorts (the exception being the last cohort, for which we observe renewal behavior only in the first few ages). The values of the aggregate test statistics are 1.36 and 1.50 in Finland and Norway, both significant. Clearly, then, there are interindustry differences in renewal behavior. To see if these differences are a result of renewal behavior in only one or two "deviant" industries, we did separate tests for equality among alternative pairs and triples of industries. The only two aggregated test statistics that showed even marginal acceptance (they were significant at a 5 but not a 1 percent level in Finland and had $p$-values of about 0.005 in Norway) were the one for aggregating professional and scientific equipment with electrical equipment except communication, and the one for aggregating agricultural equipment with motor vehicles and transportation equipment and aircraft. In the rest of this paper we

47. Indeed, some further analysis not reported here indicated that what nationality differences we do find are generated by the results from one or two industries, leading us to wonder whether even these effects are largely a result of too high a level of aggregation for part of our industry classification.

aggregate each of these groups into a single industry and proceed with the resulting seventeen-industry classification.

Recall that if the distribution of the value of patent protection generated by patents in one industry is better than (or dominates) the distribution of values generated by those in a second industry, then the renewal curve of the first industry will lie entirely above that of the second. We now investigate whether the interindustry differences in renewal curves reported above generate an ordering for those curves. We did both a pictorial and a formal statistical analysis of this issue. The pictorial analysis relied on the average (over cohorts) of the industry-specific renewal curves estimated for each cohort; the statistical analysis relied on the inequality tests described earlier. Roughly, we could distinguish between five (slightly different) groups of industries in each of the countries (that is, the industries within these groups could not be ordered). The groups are listed in table 6, together with our point estimates of the number of patents in each of the component SIC industries. Figures 5 and 6 plot one industry from each of the five groups in Finland and in Norway, respectively. The industry chosen from each group was the industry with the largest estimated number of patents in that group. This left one of the industries from each of the figures not in the other (machinery was not in the Finnish figure, fabricated metals was not in the Norwegian figure), so we added curves for the omitted industries.

The orderings for the two countries are fairly similar. In Norway, drugs and medicines (pharmaceuticals) and lumber, wood, and paper dominate the rest of the industries. They are followed by a group consisting of machinery, the other chemically related industries, food products, and primary metals. The distinction between these first two groups is lost in the smaller Finnish data set, as seen in figure 6, where the drugs and medicine curve intersects the machinery curve. Also, the relative order of primary metals is different in the two countries. Next in the Finnish ordering is a group of electronic industries (professional, scientific, and electrical equipment, and communication equipment). In Norway the electronic industries are not noticeably different from fabricated metals, or stone, clay, and glass, but they are in Finland. In both countries the last two groups consist of a heavy industry grouping (farm, motor, and air, and construction) and a low-tech grouping. Neither petroleum refining and extraction nor computing and office equipment

**Table 6. A Partial Ordering of Interindustry Differences in Patent Renewal Curves, Finland and Norway**

*Finland, 1969–77*

| *Industry* | *Number of patents*[a] |
|---|---|
| *Group 1* | |
| Drugs and medicines | 1,887 |
| Lumber, wood, and paper | 692 |
| Food and kindred products | 281 |
| Rubber and plastic products | 636 |
| Machinery | 1,681 |
| Chemicals and allied products | 1,001 |
| *Group 2* | |
| Professional, scientific, and electrical equipment | 886 |
| Communication equipment | 270 |
| *Group 3* | |
| Primary metals | 312 |
| Fabricated metals | 1,000 |
| Stone, clay, and glass | 425 |
| *Group 4* | |
| Farm, motor, and air[b] | 473 |
| Construction and sanitation | 347 |
| *Group 5* | |
| Textiles, apparel, and leather | 194 |
| Other | 290 |

*Norway, 1962–77*

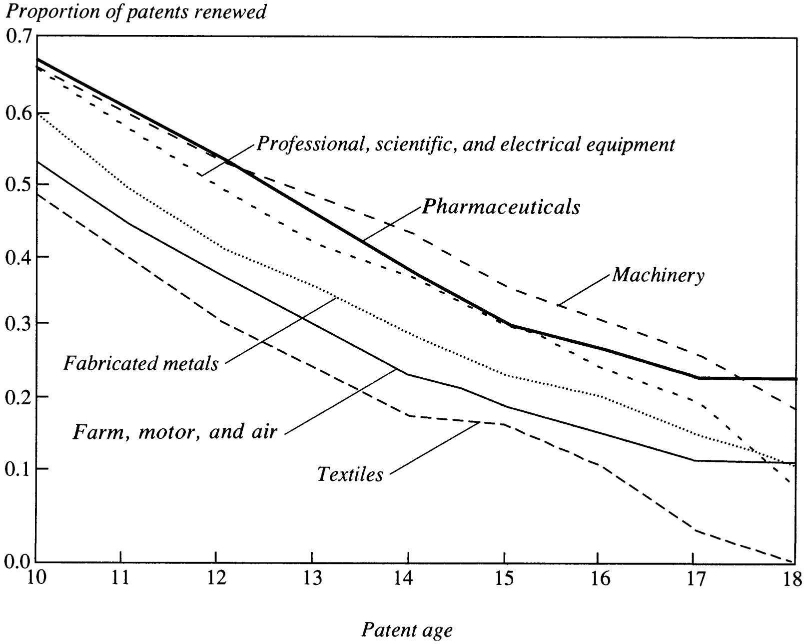
| *Industry* | *Number of patents*[a] |
|---|---|
| *Group 1* | |
| Drugs and medicines | 4,098 |
| Lumber, wood, and paper | 1,112 |
| *Group 2* | |
| Machinery | 4,863 |
| Food and kindred products | 758 |
| Rubber and plastic products | 1,220 |
| Chemicals and allied products | 3,125 |
| Primary metals | 1,198 |
| *Group 3* | |
| Professional, scientific, and electrical equipment | 3,558 |
| Communication equipment | 1,214 |
| Fabricated metals | 3,338 |
| Stone, clay, and glass | 1,307 |
| *Group 4* | |
| Farm, motor, and air[b] | 1,815 |
| Construction and sanitation | 1,224 |
| *Group 5* | |
| Textiles, apparel, and leather | 927 |
| Other | 657 |

a. Point estimates of number of patents summed over all the cohorts of the data.
b. Agricultural equipment, motor vehicles and transportation equipment, and aircraft.

is included in this ordering. Computing was excluded because its estimated number of patents was much lower than the number in the other industries (47 in Finland, and 225 in Norway); petroleum was excluded because its renewal curve cut across those of most other industries, so that it did not conveniently fit into the ordering given in table 6.

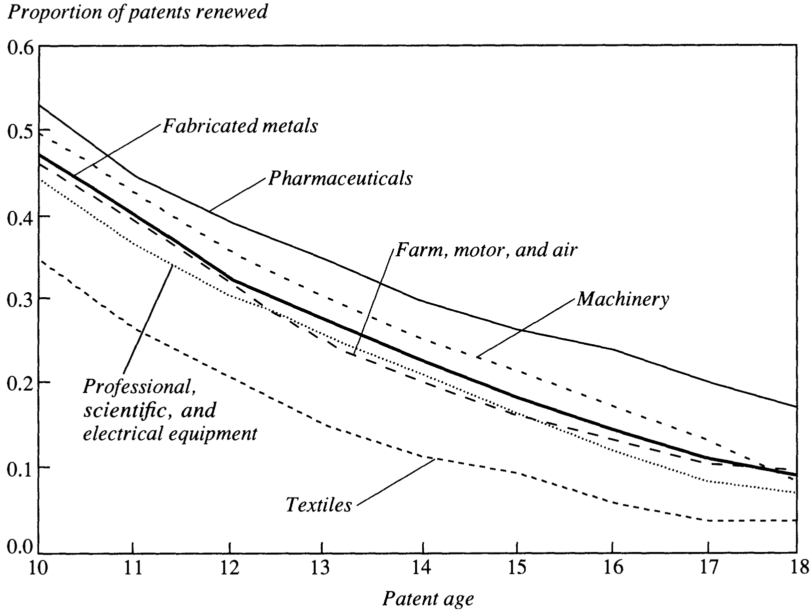
Generally, then, these results show that a patent from a chemical industry will typically be more valuable than one from a mechanical industry, and the latter will tend to be more valuable then a patent from an electronic industry, which will on average be more valuable then

**Figure 5. Industry Differences in Patent Renewal Curves, Finland**




heavy industry and low-tech patents. (But note that food and kindred products, and lumber, wood, and paper have to be put in the highest groupings for this ordering.) To go further than this ordering of value distributions and make statements about interindustry differences in the total value of patent protection, we must be able to combine the information on differences in value distributions with interindustry differences in numbers of patents. Indeed, even more information is required before we can compare our results with those on the "importance" of patents. To obtain an index of how important patents are to a given industry, we would have to divide any index of the total value of patent rights for that industry by a measure of the total returns generated from knowledge-producing activities in that industry (or perhaps of the cost of the inputs expended in those activities). Given all these problems, we find it surprising that the ordering presented in table 6 is reasonably consistent with (though more detailed than) the ordering implicit in the survey evidence on interindustry differences on the importance of patent protection reviewed earlier in the paper. Lumber, wood, and paper and

**Figure 6. Industry Differences in Patent Renewal Curves, Norway**




food and kindred products are probably the outliers in this respect, since they are not usually considered to be industries for which patent protection is important. It may, then, be worth investigating whether the placement of these two industries in the ordering has something to do with particular characteristics of the Finnish and Norwegian economies.

We find these empirical results encouraging. They show that, even assuming only the weak regularity conditions underlying our tests, patent renewal data contain enough information to enable us to provide reasonably clear-cut answers to many of the questions on nationality and industry differences in the distribution of patent values that we had set out to investigate.

## Intercohort Differences

As we discussed earlier, patent applications declined markedly in almost all Western countries between the late 1960s and the late 1970s. Because this decline was accompanied by an increase in the resources

**Table 7. Average Patent Applications and Grants over Four-Year Periods, 1964–77, Norway and Finland**

| *Country* | *Period* | *Applications* | *Grants* |
|---|---|---|---|
| Norway | 1964–67 | 4,946 | 2,475 |
| | 1969–72 | 5,007 | 2,397 |
| | 1974–77 | 4,521[a] | 2,051 |
| Finland | 1969–72 | 3,706 | 1,460 |
| | 1974–77 | 3,787 | 1,636 |

a. Average over the three-year period 1974–76.

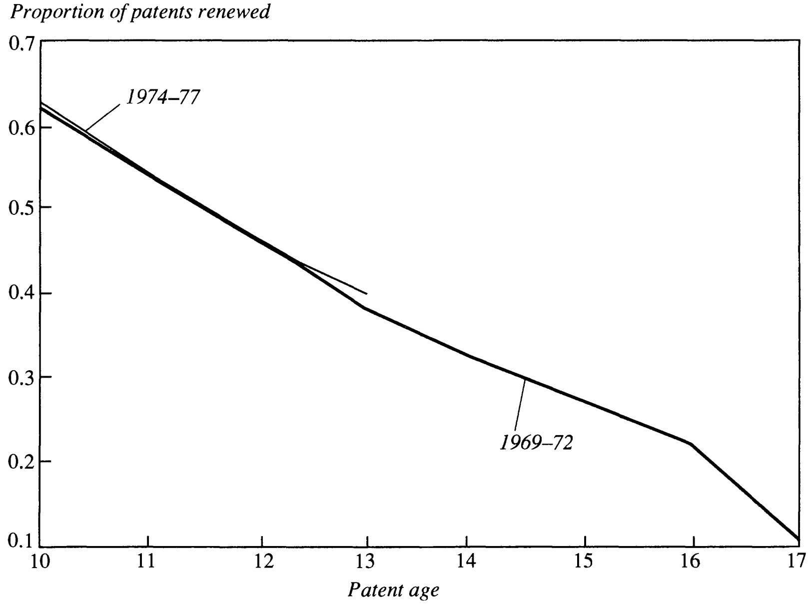
devoted to research, the fall in patent-to-R&D ratios over this period is particularly striking. It has lead to speculation about a decline in the technological productivity of research (or in technological opportunities), a decline that is used to explain part of the observed productivity slowdown. Schankerman and Pakes provided estimates of the distribution of the value of patent rights disaggregated by cohort and found that this distribution shifted to the right over this period in the three countries they studied.[48] This shift upward in the patent-value distribution partly counteracted the effect of the fall in patent numbers and generated a series on patent values per unit of R&D that declined only mildly, if at all, over the decade. This leads one to question both the extent of any fall in "technological opportunities" and the import of this particular explanation of the productivity slowdown. It also raises the more basic issue of understanding the mechanism underlying the negative relationship between quantity and average value.

In this section we look for intercohort differences in renewal curves among Finnish and Norwegian patents. Norwegian patent count data exhibit similar patterns to those just noted: patenting in Norway fell about 20 percent between 1970 and 1980. But the data for Finland are noticeably different. The number of Finnish patents increased by about 15 percent over this period. Unfortunately, more than half the increase occurred between 1977 and 1980, a subperiod not covered by our analysis.

In both countries the real renewal fee schedule changed little during the decade. To keep matters simple, therefore, we ignore any change and focus on differences in renewal curves among groups of patents aggregated over four-year intervals. Table 7 shows the average number

48. Schankerman and Pakes, "Estimates of the Value of Patent Rights."

**Figure 7. Intercohort Differences in Patent Renewal Curves, Finland, Selected Periods, 1969–77**

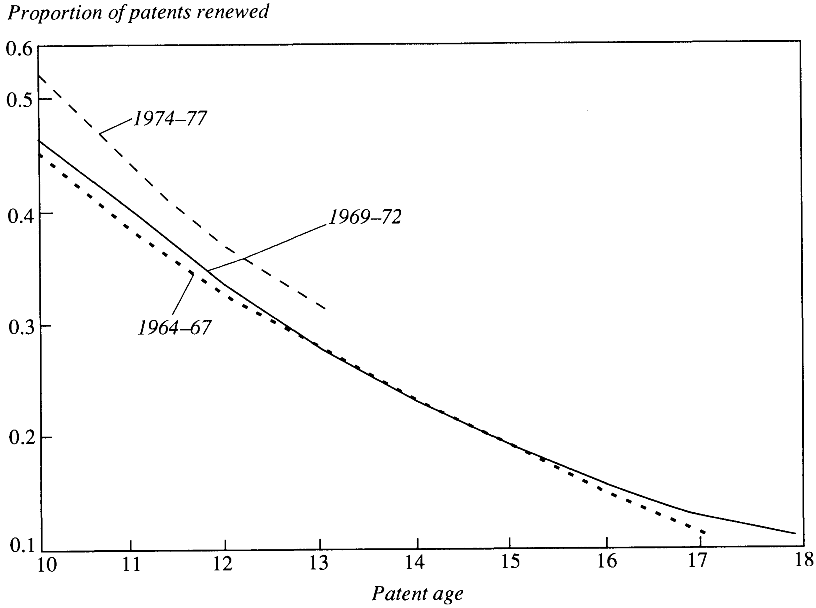



of applications and grants during each of these intervals in the two countries,[49] while figures 7 and 8 show the renewal curves for the various cohort groups in Finland and Norway, respectively.

In Norway there is very little difference in the number of patents (either applied for or granted) between the first two cohort-groups, but there is a distinct fall in both between the 1969–72 group and the 1974–77 group (a pattern similar to that for the other Western countries). The renewal curves for the first two subperiods are almost indistinguishable, but the curve for the 1974–77 period (at least what we see of it) lies sharply above those of the other two (figure 8). The distribution of patent values over the 1970s in Norway shows a rightward shift similar to the

49. The 1968 cohort in Norway, and the 1973 cohort in both countries, were deviant in that 1968 contained less, and 1973 contained more, patents than in either of their adjacent cohorts. Since these two cohorts were both also in the middle of our time subdivision, we thought it better to drop them from the analysis than to arbitrarily assign them to either the earlier or the later groups.

**Figure 8. Intercohort Differences in Patent Renewal Curves, Norway, Selected Periods, 1964–77**




one picked up by the different analytic procedures of Schankerman and Pakes for the countries they studied.

In Finland the number of patents (either applied for or granted) increased slightly between the 1969–72 and the 1974–77 groups (table 7). And Finland, unlike the other countries studied, shows no evidence of its renewal curve shifting out over the decade (figure 7). That is, the one country for which we observe an increase in patent applications over the 1970s is the one country for which we do not find evidence of a shift outward in the value distribution.

We have obtained similar results when we define cohorts to include only patents registered to foreigners and when we do the intercohort analysis by industry. Moreover, the results were even sharper in an earlier version of this paper in which we ignored the fact that a small fraction of the patents are not granted by age eight and used all the cohorts through 1979 in the analysis.

The observed falls in patent counts over the 1970s in most countries, and the simultaneous estimated improvements in the value distributions, may well result from a more general set of relationships that induce a negative correlation between the number of patents in a cohort and their average value. To conclude, we briefly discuss a few of the possible economic models that would generate such a relationship.

There are at least two kinds of explanation for these findings. One assumes that the underlying quantities of recently invented patentable ideas and the distribution of their values do not change (or change smoothly) over time, but that changes in the rules and administrative procedures of the patent offices induce different selections of them to apply for patents in different periods. The other kind of explanation posits (or derives) an economic mechanism that generates a negative relationship between the quantity and the distribution of the values of the patentable ideas produced in any given period.

It is easy to see how an increase in patent office requirements could cause potential patentees not to patent patentable ideas that are only marginally profitable, thereby inducing a negative correlation between the observed quantities and the value distributions. Indeed, if the fraction of patents that have very low values is as large as is generally thought, small increases in application costs or decreases in granting probabilities could generate large falls in patent applications, falls resulting entirely from less patenting of patents perceived to be of low value. Table 7 shows that the ratio of grants to applications moved in the appropriate direction for this argument in both Finland and Norway, so changes in the stringency of the granting requirements may underlie part of the movements in the observed renewal curves. Recall, however, that the curves in figures 7 and 8 are for renewals of granted patents. For this argument to underlie the observed results, an increase in the stringency of the granting criteria must have caused a proportionately larger decrease in the number of patents dropping out than in the number of patents granted. Though that may be true for the numbers dropping out in the initial renewal years, we think it much less likely after age ten. (A more detailed investigation of this possibility is definitely warranted.)

Several different economic arguments could explain a negative relationship between the distribution of values and the quantities of patents. The simplest textbook explanation is probably the most difficult to test empirically. The hypothesis would involve changes in research expenditures moving agents along a schedule of research projects of diminishing expected values. The explanation becomes complicated because it is unrealistic to assume that the same profitability schedule appears anew in every period, irrespective of the number both of projects developed in the previous periods and of technological developments, and because to be appropriate a model that allows for shifts in the schedule should also treat research expenditures as endogenous. Indeed, a more detailed explanation would probably set up a dynamic model that generates a bivariate stochastic process for research expenditures and patentable output and then fit that model to the data. Such a task is complex, but given the importance attributed to technology shocks by economic historians that deal with technological change, and given the role of such shocks in real business cycle and sectoral shift theories, it may be worth pursuing.

An alternative economic argument for generating the observed negative correlation between quantities and qualities starts out just as simply. Assume there is a frontier of projects able to generate patentable results: some are less risky but likely to be less valuable if developed, and some are both more risky and more valuable. Then changes in any factor that would move us from place to place on the frontier would tend to generate the observed negative correlation. One possible factor is an increase in the number of agents attempting to develop new patentable products for a given market, due, say, to an increased ability of agents from other countries to supply (or an internationalization of) that market. If the return function for successful development had any of the winner-take-all character usually attributed to markets for patentable products, then it will be highly convex (in the extreme case it will be the maximum of zero if you are not "better" than all your competitors, and the value if the given product is the only product marketed). This will push each agent toward more risky projects; indeed, the larger the number of agents attempting to develop the product, the more risky the projects chosen ought to be.

There are undoubtedly alternative possible explanations for the observed phenomena. But the real challenge here is not to provide possible explanations, but to determine the extent to which any of them can explain the empirical findings, a much more difficult task.

## Appendix: The Variance-Covariance Matrix for the Inequality Tests

This appendix derives the variance-covariance matrix needed for the inequality tests, or the tests for stochastic dominance, we used in the last section of the paper. We are careful to provide a formula for this matrix that never has to perform inversions of matrices of dimension greater than the maximum of $J$ and $L$ (the number of SIC industries and the number of renewal years), so the required calculations should be easy to do on almost any personal computer.

We begin with notation. It is to be understood that all $\pi$'s and $p$'s in what follows are estimated parameters. (We have omitted the traditional circumflex for ease of notation.)

$\pi' = (\pi^{1\prime}, \ldots, \pi^{2\prime})$, where $\pi^l$ is the $J$-vector of dropout probabilities for the SIC industries at age $l = 1, \ldots, L$.

$B$ = the $I \times J$ concordance matrix that provides the probabilities that a patent from IPC $i$ originates in industry $j$.

$p' = (p^{1\prime}, \ldots, p^{L\prime}, p^{L+1\prime})]$, where $p^l$ is the $I$ vector of dropout .probabilities that the model predicts for the IPC industries at age $l = 1, \ldots, L, L + 1$. $p^l = B\pi^l$ for $l = 1, \ldots, L$, and $p^{L+1} = i_I - \sum_{l=1}^{L} p^l$, where here, and below $i_x$, is an $x$-vector all of whose elements are unity.

$W_l$ = an $I$-dimensional diagonal matrix with $\sqrt{n(i)}/\sqrt{p(l,i)}$ as diagonal elements, for $i = 1, \ldots, I$, and $l = 1, \ldots, L + 1$.

$W$ = diag $(W_l)$ is an $(L + 1)I$ block diagonal matrix with $W_l$ as the diagonal blocks.

Now defining

$$T = \begin{bmatrix} -I_L \otimes B \\ i_L' \otimes B \end{bmatrix},$$

where $I_x$ is an identity matrix of dimension $x$, and $\otimes$ is the Kronecker product operator. Standard arguments show that

$$V(\pi) = (T' W' WT)^{-1}$$

is a consistent estimation of Var $(\pi)$. Straightforward matrix manipulations show that

$$T' W' WT = \text{diag}\,(B' W_l B) + i_L i_L' \otimes B' W_{L+1} B,$$

where diag $B'W_lB$ is a block diagonal matrix with $L$ blocks consisting of the $B'W_lB$, for $l = 1, \ldots, L$.

Now let

$D_l = (B'\ W_lB)^{-1}$ for $l = 1, \ldots, L$,

$$Q = B'W_{L+1}B, \text{ and } F = (I + \sum_{l=1}^{L} D_lQ)^{-1}.$$

Then it is easy to verify that

$$(T'\ W'\ WT)^{-1} = (M_{ij}),$$

where

$$M_{ii} = (I - D_iBF)D_i$$

$$M_{ij} = -D_iBFD_j.$$

The vector of $L$-inequality constraints we are after is of the form $C\pi \leq 0$, where

$$C = \begin{bmatrix} 1, 0, & \ldots, 0 \\ 1, 1, 0, & \ldots, 0 \\ 1, 1, & \ldots, 1 \end{bmatrix} \otimes c'$$

and $c$ is a $J$-vector, one of whose elements is unity and one is minus unity. The variance-covariance matrix that we need for the test of the inequality constraints is then

$$CV(\pi)\,C' = (v_{ij}) = V,$$

where

$$v_{ij} = c'\left(\sum_{q=1}^{j} \sum_{p=1}^{i} M_{p,q}\right) c.$$

# *Comments and Discussion*

**Kenneth Judd:** This paper is a continuation of some very interesting work on patents. An earlier paper by Pakes laid the basis for this work using aggregate data.[1] Having a disaggregated data set across industries makes it possible to address some new questions and test some new hypotheses.

In the paper Pakes and Simpson do two things. First, they take Pakes's earlier dynamic programming estimation techniques and calculate a weighted average value of patents, a more reasonable thing to use, I believe, than patent counts. Second, they report on the industry-, nationality-, and cohort-specific dropout rates.

The authors are concerned with calculating the value of a patent. Although they take a significant step in that direction in this paper, some theoretical issues must be kept in mind. In their approach, it is assumed that when the patent holder decides whether or not to renew the patent, he believes that the value of the patent is exogenous. In particular, the patent holder does not think his decisions or his renewal policy will affect those returns. This exogeneity assumption strikes me as being odd in this context, because the purpose of a patent is to affect the behavior of one's rivals—in particular, to keep them out of the market. The fact that a firm's rivals know that it has a patent or has the option to renew the patent will change their behavior. Of course, changing their behavior should affect the profitability of having that patent today and in the future.

The assumed exogeneity, therefore, seems to be incorrect. Let me give a simple example illustrating the kind of bias it induces. Suppose

1. Ariel Pakes, "Patents as Options: Some Estimates of the Value of Holding European Patent Stocks," *Econometrica*, vol. 54 (July 1986), pp. 755–84.

that a rival and I are in a patent race, but we are at a stage before anything could be patented, since nothing is sufficiently new or well developed. Furthermore, suppose that my rival is a bit ahead of me. If there is no patent system, then it is quite possible that I am going to let him be the first to introduce a product, but I will stay just behind him and then imitate when I can. This will result in a duopoly market structure.

But if, on the other hand, he has the option at some point in the future of patenting something and if he is going to maintain that lead and get to that patentable stage before I do, then why would I stay in the race? I should get out immediately before I waste my money following him and then getting scooped. The mere fact that he has the potential to patent something in the future means that now, before we are at a patentable stage, I should get out of the game.

But when he gets to that patentable stage, the fact that I left and am out of the game means that he does not have to worry so much about me. Therefore, he may, for other reasons, decide not to patent the product at all. In this example, the patent system has substantial value, which would not be discerned by the analysis in this paper.

There is therefore a downward bias in these value estimations because of strategic responses. This is the part of the value of a patent system that is not going to be picked up in these calculations as far as I can tell.

Thinking about the strategic component affects some of the paper's interpretations, particularly when we look across industries. Maybe it is true in the pharmaceutical industries that firms will renew all patents, because it is easy for a rival to react to an unanticipated nonrenewal by quickly entering the market. But in some other industries there may be barriers to entry that would make it difficult for rivals to enter in response to an unanticipated nonrenewal. If so, patent holders need not be so careful about renewing these patents.

Hence these strategic considerations could be important in understanding the results across industries. Thinking about strategic market structure issues might help us to understand why different industries have different dropout behaviors.

Also, since this endogeneity exists, these strategic considerations make it difficult to answer questions about whether the benefits of patent protection accrue disproportionately across various industries or across various nationalities. Those differences in behavior may not reflect differences in the intrinsic values of the patent system across industries but just differences in the market structure and, therefore, in the behavior of the firms.

On the other hand, I have to admit that to incorporate this sort of endogeneity into such a model empirically would be extremely difficult. But it is an issue that we should keep in mind when trying to interpret these results.

More on the empirical side. I was a bit troubled by the fact that Pakes takes, as given and precise, the costs of renewal. The cost of renewing a patent is presumably not only the renewal fee but also the cost of remembering that you have to renew and getting a secretary to type up the form or the cost of somebody deciding whether or not to renew the patent. Since these renewal fees are often only several hundred dollars, these other kinds of bureaucracy costs could be nontrivial relative to the renewal fee.

Indeed, at one point in the paper the authors refer to such differences in effective renewal fees. Since they are aware that a renewal fee may be a muddy estimate of the true costs of renewing, it would be nice in computing these values to see how sensitive these results are. I think this is an important context in which to worry about the sensitivity, since these exercises focus on dropout behavior, which is basically behavior of people who have nearly worthless patents, and to estimate the value of things that are not worthless but are, in fact, very valuable.

One technical aspect, also, came to mind as I looked through this and the earlier papers. It might be unreasonable to do some of these simulation kinds of exercises because of the enormous cost of some of the simulation estimators. I think an alternative technique could be used here. In Pakes's *Econometrica* paper, if I understand correctly, the reason one cannot get an analytic maximum likelihood function is that one cannot integrate the log normal density function. But if one replaces that by the same polynomial that any computer uses when evaluating the log normal density function, everything would integrate out and one would get an analytic maximum likelihood function. This would make this estimation, I suspect, much more efficient. With greater efficiency this approach could do much more sensitivity analysis and explicitly include things like firm and industry effects as opposed to just doing the nonparametric tests for equality across industries.

In summary, I found the paper to be a natural extension of a very interesting area of work, where I think further development is possible and valuable.

**Edwin Mansfield:** Recent years have seen a notable increase in the amount of attention devoted by economists to patent statistics and the patent system. Studies have been carried out to try to determine the effects of patents on the amount spent on research and development and on the rate of innovation.[1] Investigations have been made of the importance of patents in promoting the appropriability of new technology.[2] Patent statistics and patent counts have been analyzed in an attempt to obtain measures of the rate of invention that can be related to other relevant variables such as R&D, productivity change, and the market value of firms.[3] Though progress has been slow, and many central questions remain largely unanswered, our knowledge of the economic effects of the patent system and of the uses and limitations of patent statistics has improved significantly.

In this paper Ariel Pakes and Margaret Simpson are concerned with the estimation of the distribution of returns earned from holding patents. Following earlier work by Pakes and Mark Schankerman, they make these estimates based on observations of the proportion of different cohorts of patents that are renewed at alternative ages, together with the relevant renewal fee schedules. The results, which pertain to Finland and Norway, lead them to the following conclusions. First, whereas there are substantial international differences in renewal curves, these differences tend to disappear when the data are disaggregated by industry. Second, the value of a patent seems to be highest in pharmaceuticals and other chemical-related industries, second highest in the mechanical industries, third highest in the electronic industries, and lowest in the low-tech industries. Third, this ranking is essentially the same as the one obtained in my own work and in that of Levin and others (cited in notes 1 and 2). Fourth, there seems to be a negative correlation between the number and the mean value of patents in a cohort; that is, quality seems to be inversely related to quantity.

1. For the effects on research and development, see C. Taylor and Z. A. Silberston, *The Economic Impact of the Patent System: A Study of the British Experience* (Cambridge University Press, 1973). For the effects on the rate of innovation, see Edwin Mansfield, "Patents and Innovation: An Empirical Study, *Management Science,* vol. 32 (February 1986), pp. 173–81.

2. Edwin Mansfield, Mark Schwartz, and Samuel Wagner, "Imitation Costs and Patents: An Empirical Study," *Economic Journal,* vol. 91 (December 1981), pp. 907–18; and Richard C. Levin and others, "Appropriating the Returns from Industrial Research and Development," *BPEA, 3:1987,* pp. 783–831.

3. Zvi Griliches, ed., *R&D, Patents, and Productivity* (University of Chicago Press, 1984).

At the outset, it is important to recognize that Pakes and Simpson estimate the value of *patent protection* to the patent holder, not the value of the *patented invention* either to the patent holder or to society as a whole. Clearly, an invention may be very valuable even though the patent on it, because it is very weak, is close to worthless. Thus their paper is not designed to evaluate the usefulness of patent counts as measures of the rate of invention, though the authors do touch on this topic. Their conclusion in this regard is that "patent counts are a very noisy measure of the value of patented output." I agree with them, in part because the average value of a patent is likely to vary greatly from firm to firm, from industry to industry, and over time.

But that is only part of the story. The proportion of patentable inventions that are patented also varies considerably. For example, in the pharmaceutical, oil, and machinery industries, more than 80 percent of patentable inventions are patented, whereas in the primary metals and automobile industries, only about 60 percent are patented.[4] In general, as one would expect, the percentage patented tends to be higher in industries where patent protection is regarded by firms as relatively important. Thus the fact that patented inventive output varies considerably as a percentage of total inventive output is another important reason why patent counts may be a misleading measure of total inventive output.

However, contrary to the view of some observers, there is no evidence that the well-known drop in the annual number of patents granted to U.S. inventors from the late 1960s and early 1970s to the early 1980s was due to a decline in the percentage of patentable inventions that were patented. Contradicting those who argue that firms have become disillusioned with the patent system, the available data show essentially no difference between 1965–69 and 1980–82 in the percentage of inventions patented. Even in the electrical equipment industry, often cited as a place where the propensity to patent has declined, more firms reported an increase than a decrease.[5]

Pakes and Simpson find, as one would expect, that most patents have little or no economic value. For example, in an earlier paper Pakes estimated that 75 percent of patents had a value under $3,731 in France,

4. Mansfield, "Patents and Innovation," p. 177.
5. Mansfield, "Patents and Innovation."

under $7,948 in the United Kingdom, and under $19,576 in Germany.[6] These results are similar in many respects to those of Sanders, Rossman, and Harris, and Grabowski and Vernon.[7] The value of patents is very skewed; a relatively small number of patents account for a large proportion of the total value. Note once again that the value of patent protection, not the value of the patented inventions, is what is being estimated. I would expect that the value of patented inventions would also be highly skewed, but, as noted, Pakes and Simpson are not able to address this question.

Since Pakes and Simpson have no direct data about the extent to which the value of a patent exceeds the renewal fee, their results depend on different assumptions, some of which can be questioned, though they are interesting first approximations. In presenting their results, Pakes and Simpson note repeatedly, sometimes with surprise, that their results are almost identical to those obtained from survey data. I interpret this as strong evidence that their findings are in the right ballpark. Because there is little or no check on the validity or predictive power of many econometric models based on off-the-shelf data in this area, it takes considerable faith to believe they are any more accurate than the surveys. Clearly, both types of analysis are useful. In my opinion, economists, unlike psychologists and natural and biological scientists, are far too inclined to shun the collection of their own data.

Besides summarizing their existing work, Pakes and Simpson suggest that patent renewal data can be used to examine various public policy issues regarding the patent system. Though that may be true, I wonder whether such data can tell us how many additional inventions and innovations are stimulated by the patent system, and how much they are worth, privately and socially. While this is not the only interesting policy issue in this area, it seems to me it is the central one, and I do not see how it can be addressed by using patent renewal data.

As Pakes and Simpson point out, the United States is now putting a great deal of emphasis on the protection of intellectual property rights.

6. Ariel Pakes, "Patents as Options: Some Estimates of the Value of Holding European Patent Stocks," *Econometrica*, vol. 54 (July 1986), pp. 755–84.

7. Barkev S. Sanders, Joseph Rossman, and L. James Harris, "The Economic Impact of Patents," *Patent, Trademark, and Copyright Journal of Research and Education*, vol. 11 (September 1958), pp. 340–62; and Henry G. Grabowski and John M. Vernon, "Studies of Drug Substitution, Patent Policy and Innovation in the Pharmaceutical Industry," final report to the National Science Foundation, Grant PRA-79-17524, Duke University, 1983.

Responding to significant losses from what they regard as inadequate protection of such rights, particularly in developing countries, American officials have increased their concern. For example, the Trade and Tariff Act of 1984 specifies that Section 301 now covers intellectual property-related trade practices; beneficiaries of the generalized system of preferences have been informed that their protection of American intellectual property rights will be scrutinized in deciding benefits under this program; and a discussion of intellectual property rights was put on the agenda of the GATT talks.

In many developing countries, such as Mexico, Brazil, Thailand, and Indonesia, patents are not granted for chemical and drug products. Although it is sometimes possible to patent chemical processes, doing so may be of limited use to the patent holder, since rivals frequently can invent around a particular manufacturing process. Moreover, it is notoriously difficult to prove that a rival is using a particular patented process. Further, if the product is imported, the local court may lack jurisdiction over the foreign manufacturer. Of course, it is easy to see why so many of the conflicts between the developed and developing countries over patent protection have focused on the pharmaceutical and chemical industries. As mentioned earlier, these are the industries in which patent protection is particularly important.[8]

When countries like Korea, Brazil, and Taiwan become increasingly industrialized, and as their industries become more innovative, their attitudes toward patents are likely to change. According to some observers, establishing stronger intellectual property rights would help to promote indigenous technological and innovative activities in the developing countries even though that is only one of many factors involved. In many industries, both in the United States and abroad, there is reason to believe that too little, from society's viewpoint, is being spent on developing new technology. How much this situation is aggravated by the lack of patent protection in the third world is not known, and research is badly needed in that area. Even crude estimates would be useful.

In conclusion, Pakes and Simpson have produced an interesting and useful paper. Pakes and his associates (including Mark Schankerman) deserve credit for devising a promising technique to analyze patent

8. Edwin Mansfield, "Intellectual Property Rights, Technological Change, and Economic Growth," in Mark A. Bloomfield and Charls E. Walker, *Intellectual Property Rights and Capital Formation in the Next Decade* (University Press of America, 1988).

renewal data. This paper is another application of that technique, the data being drawn from Finland and Norway. References scattered through the paper indicate that more applications are to come.

**General Discussion**

Although the patent renewal rate data sets explored by Pakes and Simpson, and by Pakes in other research, were viewed as providing important information about the value of patent protection, several participants echoed Mansfield's concern that the value of the patent is not necessarily strongly correlated with the social or private value of the invention itself. And the data are limited even as indicators of the value of the patent protection. Mike Scherer, for example, cautioned against treating early dropouts as indicative of low value. In a field in which technology is moving very rapidly, he noted, an invention could have a very high value at the time it is introduced and still be obsolete in a few years. Similarly, Richard Levin pointed out that there may be institutional reasons, only indirectly related to value, why renewal rates might be especially high for pharmaceuticals in the early years after the first patent application. One reason is the long regulatory delays between the time a drug is developed and the time it can be marketed.

Cliff Winston and Ed Mansfield pointed out aspects of the value of patent protection that the renewal data cannot help to measure. For example, Winston noted, the original decision to take out a patent on an idea involves a cost—the disclosure of the idea—that is obviously not incurred again upon renewal.

Nonetheless, all the discussants agreed that the information in the renewal data is a marked improvement over patent counts alone for making inferences about the value of inventions and the returns to investment in research and development. And Levin stressed that it is precisely the value of the patent protection itself that is of interest for devising and evaluating patent rules and setting patent-related international trade policies.

Peter Reiss and Scherer both emphasized the importance of knowing more about what is in the truncated tails of the value distributions estimated through the dynamic programming procedure used by the authors. Reiss suggested that it might be worthwhile to assume a functional form for the distribution despite the problems in doing so. A precise functional form would make it possible to derive information about depreciation rates for intellectual capital, and help to sort out the proportion of the value of a patent attributable to the current value of the protected idea and the proportion attributable to the renewal option.

Pakes agreed that such information would be extremely valuable, but he reiterated that the technical problems involved in assuming a functional form are fairly severe. The substantive problem is to use available information to intuit what is in the upper tail, he argued. Assuming a functional form, in effect, assumes a large part of that problem away. Scherer suggested that it would be possible to pursue information about the upper tail of the value distribution directly. Inventions in the truncated upper tail are those that are renewed for the full, legally allowed length of time. Because the number of such inventions is rather small, he argued, it would not be too costly to collect survey information on these inventions. And the patent renewal data sets provide an excellent starting point for such a research effort by identifying these inventions and providing quite a bit of information about them and their creators.