THOMAS J. SARGENT
University of Minnesota

Rational Expectations, the Real Rate of Interest, and the Natural Rate of Unemployment

THE INTERACTION OF EXPECTED INFLATION and nominal rates of interest is a topic that has received its share of attention since Milton Friedman gave Irving Fisher's theory a prominent role in his presidential address to the American Economic Association in 1967.[1] The relationship between interest and expected inflation depends intricately on the interactions of the real and financial sectors of the economy, so that the subject of this paper lies in the domain of macroeconomic analysis. Partial equilibrium analysis won't do. Therefore, even though my main subject is the relationship between interest rates and expected inflation, there is no way to avoid such matters as the nature of the Phillips curve, the way expectations are formed, and, in some formulations, the sizes of various interest elasticities: those of the demand and supply for money and those of aggregate demand and its components.

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1. Milton Friedman, "The Role of Monetary Policy," *American Economic Review*, Vol. 58 (March 1968), pp. 1–17. One statement of Fisher's theory can be found in Irving Fisher, *The Theory of Interest* (MacMillan, 1930), pp. 399–451.

Thus, consider Irving Fisher's theory. In one interpretation, it asserts that an exogenous increase in the rate of inflation expected to persist over a given horizon will produce an equivalent jump in the nominal yield on bonds of the corresponding maturity. That assertion concerns the way the whole economy is put together; in particular, it is about the reduced form equations for nominal rates of interest. If it is to hold, various restrictions must be imposed on the parameters of the structural equations of a macroeconomic model, which in turn imply important restrictions on the reduced form equations for endogenous variables besides the interest rate—for example, aggregate income and prices, variables that properly concern policy makers more than do nominal interest rates. For example, in standard IS-LM-Phillips curve models,[2] the response of the interest rate to an exogenous shock in expected inflation, like the response to any other shock that affects aggregate demand, is distributed over time.[3] A once-and-for-all jump in expected inflation eventually leaves the real rate of interest unaltered, but in the short run drives it down and output up.[4] Only in the special case in which the LM curve is vertical, the IS curve is horizontal, or the short-run Phillips curve is vertical (price adjustments being instantaneous whenever employment threatens to deviate from full employment) does an increase in expected inflation produce an immediate, equivalent jump in the nominal interest rate.[5] These special sets of parameter values obviously impart a very monetarist or classical sort of behavior to the model.

On this interpretation of Fisher's theory, all of the parameters influencing the slopes of the IS, LM, and Phillips curves are pertinent in evaluating its adequacy. Conversely, evidence that the theory seems adequate contains indirect implications about the parameters of the macroeconomic structure, and therefore might have some clues relevant for evaluating the relative efficacy of monetary and fiscal policies.

2. See, for example, Martin Bailey, *National Income and the Price Level: A Study in Macrotheory* (McGraw-Hill, 1962), especially pp. 49–54, which contains a good exposition of Fisher's theory from the standpoint of the standard macroeconomic model.

3. This point has been made by Edward J. Kane, among others. See "The Rasche and Andersen Papers, A Comment by Edward J. Kane," *Journal of Money, Credit and Banking*, Vol. 5 (February 1973), Pt. 1, pp. 39–42.

4. Some of Keynes' views about the effect of an increase in expected inflation on interest and employment are contained in John Maynard Keynes, *The General Theory of Employment, Interest and Money* (Harcourt, Brace, 1936), pp. 141–43.

5. For example, see Thomas J. Sargent, "Anticipated Inflation and the Nominal Rate of Interest," *Quarterly Journal of Economics*, Vol. 86 (May 1972), pp. 212–25.

While the preceding statement of Fisher's theory may be of interest in highlighting its macroeconomic content, the theory can be stated in an alternative and less confining form, which probably comes closer to what modern adherents to Fisher's doctrine have in mind. This statement is less confining because its truth does not require any restrictions on the magnitudes of the slopes of the IS, LM, and short-run Phillips curves. Furthermore, it does not involve pursuing the implications of an exogenous jump in expected inflation. Instead, expectations of inflation are assumed to be endogenous to the system in a very particular way: they are assumed to be "rational" in Muth's sense[6]—which is to say that the public's expectations are not systematically worse than the predictions of economic models. This amounts to supposing that the public's expectations depend, in the proper way, on the things that economic theory says they ought to. Beyond this, the alternative statement of Fisher's theory assumes that the Phelps-Friedman hypothesis of a natural rate of unemployment is true, and thus that no (systematic) monetary or fiscal policies can produce a permanent effect on the unemployment rate.[7] Given these two hypotheses (which are related to one another, since it seems impossible to give the natural rate hypothesis a proper formal statement without invoking the hypothesis of rationality), it follows that the real rate of interest is independent of the systematic, or foreseen, part of the money supply, which therefore can influence the nominal rate only through effects on expected inflation.

The notion that the real rate of interest is independent of the systematic part of the money supply embodies the key aspect of Fisher's theory appealed to by Friedman in his presidential address. To obtain this property for the real rate requires no assumptions about the slopes of the IS, LM, and short-run Phillips curves, for rationality and the natural unemployment rate hypothesis are sufficient to support it. From this point of view, then, the important thing is not the response of the system to an exogenous shift in expected inflation.

It is important to determine the relationship that the standard way of empirically implementing Fisher's theory bears to the preceding statement

6. John F. Muth, "Rational Expectations and the Theory of Price Movements," *Econometrica*, Vol. 29 (July 1961), pp. 315–35. Some very important implications of assuming rationality in Muth's sense in certain kinds of models of forward markets were pointed out by Paul A. Samuelson in his "Proof that Properly Anticipated Prices Fluctuate Randomly," *Industrial Management Review*, Vol. 6 (Spring 1965), pp. 41–49.

7. See Edmund S. Phelps, *Inflation Policy and Unemployment Theory: The Cost-Benefit Approach to Monetary Planning* (W. W. Norton, 1972), pp. 35–43.

of the theory. Irving Fisher and most of his followers[8] have implemented the theory by estimating a model of the form

$$r_t = \rho + \pi_t + u_t$$

$$\pi_t = \sum_{i=0}^{n} w_i \,(p_{t-i} - p_{t-i-1}),$$

where r_t is the nominal rate of interest, ρ is a constant, π_t is the unobservable expected rate of inflation, p_t is the logarithm of the price level, w_i and n are parameters, and u_t is a random error assumed to be distributed independently of past, present, and future values of p. These two equations have typically been combined to yield the equation

$$r_t = \sum_{i=0}^{n} w_i \,(p_{t-i} - p_{t-i-1}) + u_t + \rho,$$

which has been estimated by a variant of the method of least squares. The w_is have been interpreted as estimates of the distributed lags by which the public forms its expectations of inflation. (Some of Fisher's followers have added some regressors in an effort to improve his equation.[9])

Generally speaking, the results of estimating this equation have reflected poorly on the model. For data extending over very long periods of time, estimates of the w_is depict extraordinarily long distributed lags, much too long to be useful in forming predictions of inflation. Consequently, the estimated w_is do not seem to provide a plausible description of the way people form expectations of inflation—at least if they do so in an informed way.[10] For this reason, Fisher's empirical results have often been viewed with suspicion.[11]

8. For example, see William E. Gibson, "Price-Expectations Effects on Interest Rates," *Journal of Finance*, Vol. 25 (March 1970), pp. 19–34, and William P. Yohe and Denis S. Karnosky, "Interest Rates and Price Level Changes, 1952–69," Federal Reserve Bank of St. Louis, *Review*, Vol. 51 (December 1969), pp. 18–38.

9. For example, see Martin Feldstein and Otto Eckstein, "The Fundamental Determinants of the Interest Rate," *Review of Economics and Statistics*, Vol. 52 (November 1970), pp. 363–75.

10. This point has been made by Phillip Cagan in *Determinants and Effects of Changes in the Stock of Money, 1875–1960* (Columbia University Press for the National Bureau of Economic Research, 1965), pp. 252–59.

11. Nerlove has proposed comparing regressions of dependent variables (like r_t) on current and lagged proxies for psychological expectations (like p_t, p_{t-1}, . . .) with the distributed lags associated with the optimal forecast of the variables about which expectations are being formed (in this case inflation) on the basis of the regressors. See Marc Nerlove, "Distributed Lags and Unobserved Components in Economic Time Series," in

As it turns out, such negative empirical results carry no implications about the validity of the version of Fisher's theory considered here. Even if the theory is correct, there is in general no reason to expect that regressions of nominal interest rates on current and lagged rates of inflation should give distributed lag functions that could reasonably be used to form expectations of inflation. The theory cannot be tested by running regressions like Fisher's.

This paper is organized as follows. The first section describes a very simple and fairly standard macroeconomic model within which to analyze the relationship between interest and inflation. The second section takes a short detour from the main theme of the paper to analyze the interest-inflation relationship that obtains when expectations of inflation are generated by the standard "adaptive" mechanism, the usual assumption in empirical work. Here I briefly outline the restrictions on the macroeconomic structure necessary to rationalize the kind of procedure used by Fisher in his empirical work. Next comes a description of the behavior of the model embodying "rational" expectations; I show that under this assumption, the natural unemployment rate hypothesis and a version of Fisher's theory about the interest rate and expected inflation form a package. Proper empirical tests of the model are also discussed, and two of them are implemented. As it turns out, the most straightforward way to test the model is to test the natural unemployment rate hypothesis.

The argument in this paper is heavily dependent on the analysis of the natural rate hypothesis carried out by Lucas in a series of papers.[12] The

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William Fellner and others, *Ten Economic Studies in the Tradition of Irving Fisher* (John Wiley, 1967). An application of such a comparison to Fisher's equation, with the results confirming Cagan's doubts about the plausibility of long lags, is contained in my paper, "Interest Rates and Prices in the Long Run: A Study of the Gibson Paradox," *Journal of Money, Credit and Banking*, Vol. 5 (February 1973), Pt. 2, pp. 385–449.

12. Robert E. Lucas, Jr., and Leonard A. Rapping, "Real Wages, Employment, and Inflation," *Journal of Political Economy*, Vol. 77 (September/October 1969), pp. 721–54; Lucas, "Econometric Testing of the Natural Rate Hypothesis," in Otto Eckstein (ed.), *The Econometrics of Price Determination Conference*, sponsored by the Board of Governors of the Federal Reserve System and Social Science Research Council (FRB, 1972); Lucas, "Expectations and the Neutrality of Money," *Journal of Economic Theory*, Vol. 4 (April 1973), pp. 103–24; Lucas, "Some International Evidence on Output-Inflation Tradeoffs," *American Economic Review*, Vol. 63 (June 1973), pp. 326–34; Lucas, "Econometric Policy Evaluation: A Critique," *Journal of Money, Credit and Banking* (forthcoming).

proposition that the real interest rate is independent of the systematic part of the money supply, given both rationality and the natural rate hypothesis, follows quite directly from Lucas' work. In important ways, the structure of the argument in this paper resembles that of Friedman's presidential address, in which the close connection between the hypothesis of a natural rate of unemployment and Fisher's theory of the real rate of interest was brought out.

A Simple Macroeconomic Model

I assume a macroeconomic structure that can be described by the following equations:

(1) *Aggregate supply schedule*

$$y_t = k_t + \gamma(p_t - {}_tp^*_{t-1}) + U_t, \qquad \gamma > 0;$$

(2) *Aggregate demand schedule, or IS curve*

$$y_t = k_t + c[r_t - ({}_{t+1}p^*_t - p_t)] + dZ_t + \epsilon_t, \qquad c < 0;$$

(3) *Portfolio balance schedule*

$$m_t = p_t + y_t + br_t + \eta_t, \qquad b \leq 0.$$

Here y_t, p_t, and m_t are the natural logarithms of real national income, the price level, and the exogenous money supply, respectively; r_t is the nominal rate of interest itself (not its logarithm), while Z_t is a vector of exogenous variables. The parameters c, γ, and b are assumed to be scalars, while d is a vector conformable to Z_t.[13] The variables U_t, ϵ_t, and η_t are mutually uncorrelated, normally distributed random variables. They may be serially correlated. The variable ${}_{t+1}p^*_t$ is the public's psychological expectation as of time t of the logarithm of the price level expected to prevail at time $t + 1$. The variable k_t is a measure of "normal" productive capacity, such as the logarithm of the stock of labor or of capital or some linear combination of the two; it is assumed to be exogenous.

13. All of the results carry through if c and b are assumed to be polynomials in the lag operator, so that the equations in which they appear involve distributed lags. Also, almost all of them carry through if the random terms are permitted to be correlated across equations. The only exceptions occur where the assumption that they are uncorrelated is used to rationalize a version of Fisher's equation under "adaptive" expectations.

Equation (1) is an aggregate supply schedule relating the deviation of output from normal productive capacity directly to the gap between the current price level and the public's prior expectation of it. Unexpected rises in the price level thus boost aggregate supply, because suppliers mistakenly interpret surprise increases in the aggregate price level as increases in the relative prices of the labor or goods they are supplying. This mistake occurs because suppliers receive information about the prices of their own goods faster than they receive information about the aggregate price level. This is the kind of aggregate supply schedule that Lucas and Rapping have used to explain the inverse correlation between observed inflation and unemployment depicted by the Phillips curve.[14]

Equation (2) is an aggregate demand or IS schedule showing that the deviation of aggregate demand from capacity is inversely related to the real rate of interest, which, in turn, equals the nominal rate r_t minus the rate of inflation expected by the public, ${}_{t+1}p_t^* - p_t$. The rate r_t is assumed to be the yield to maturity on a one-period bond. Aggregate demand also depends on a vector of exogenous variables, Z_t, which includes government expenditures and tax rates.[15]

Equation (3) summarizes the condition for portfolio balance. Owners of bonds and equities (which are assumed to be viewed as perfect substitutes for one another) are satisfied with the division of their portfolios between money, on the one hand, and bonds and equities, on the other, when equation (3) is satisfied. Equation (3) posits that the demand for real balances depends directly on real income and inversely on the nominal rate of interest.

To complete the model requires an hypothesis explaining the formation of the public's expectations of the price level. Here the behavior of the model will be analyzed under two such hypotheses: first, with one particular kind of ad hoc, extrapolative expectations, consistent with the formula-

14. Lucas and Rapping, "Real Wages, Employment, and Inflation."

15. The results would apply if c and d were polynomials in the lag operator; choosing those polynomials appropriately would be equivalent to putting lagged ys and ks in the aggregate demand schedule. For these results, an important thing about equation (2) is that it excludes as arguments both the money supply and the price level, apart from the latter's appearance as part of the real rate of interest. This amounts to ruling out direct real balance effects on aggregate demand. It also amounts to ignoring the expected rate of real capital gains on cash holdings as a component of the disposable income terms that belong in the expenditures schedules that underlie equation (2). Ignoring these things is usual in macroeconometric work.

tion adopted in almost all empirical work on the Fisher relationship; and subsequently with the assumption that the public's expectations are "rational."

The Interest-Inflation Relationship under "Adaptive" Expectations

To equations (1), (2), and (3) I temporarily add the hypothesis

$$(4) \qquad {}_{t+1}p_t^* = \sum_{i=0}^{\infty} v_i\, p_{t-i} \equiv v*p_t,$$

where the v_is are a set of parameters. Equation (4) is an example of the so-called "adaptive" expectations hypothesis proposed by Cagan and Friedman.[16] Given the exogenous variables m_t, k_t, and Z_t and the random terms U_t, ϵ_t, and η_t, equations (1) through (4) form a system that is capable of determining y_t, p_t, r_t, and ${}_{t+1}p_t^*$.

To obtain a version of the equation estimated by Fisher, substitute the expectation hypothesis (4) into the aggregate demand schedule (2), and solve for the nominal rate of interest:

$$(5) \qquad r_t = v*p_t - p_t + c^{-1}(y_t - k_t) - c^{-1}\, dZ_t - c^{-1}\, \epsilon_t.$$

This equation has a disturbance term, $-c^{-1}\,\epsilon_t$, which is simply a linear function of the disturbance in the aggregate demand schedule, and so is in general correlated both with p and with $y - k$. Because of this correlation, single-equation methods like least squares ought not to be expected to provide reliable estimates of the parameters of (5). In general, random shocks to aggregate demand affect r, p, and $y - k$, contributing to the existence of a relationship between r and p quite apart from any effects of expected inflation on the interest rate. This influence poisons the data from the point of view of extracting estimates of the parameters of (5) by single-equation methods.

However, some restrictions can be placed on the parameters of the model so as to make p and $y - k$ independent of current and lagged ϵs, thus rationalizing the statistical procedures used by Fisher and his followers. In

16. Phillip Cagan, "The Monetary Dynamics of Hyperinflation," in Milton Friedman (ed.), *Studies in the Quantity Theory of Money* (University of Chicago Press, 1956); Milton Friedman, *A Theory of the Consumption Function* (Princeton University Press for the National Bureau of Economic Research, 1957).

particular, suppose that in the portfolio balance schedule, $b = 0$, so that the demand for money is independent of the nominal rate of interest. It is also essential that k_t, the measure of productive capacity, be exogenous and not dependent on current or past values of either the nominal or the real rate of interest. This requirement amounts to ruling out effects of the real rate of interest on the rate of formation of productive capacity. Given that $b = 0$, nominal aggregate output is determined by the portfolio balance schedule (3), which can be arranged to read

$$p_t + y_t = m_t - \eta_t. \tag{6}$$

The division of nominal output between real output and the price level is then determined by the aggregate supply schedule (1) and the expectations generator (4):

$$y_t - k_t = \gamma p_t - \gamma v{*}p_{t-1} + U_t. \tag{7}$$

Equations (6) and (7) jointly determine p and y, so that aggregate demand plays no role in affecting either p or $y - k$; that is, the LM curve is vertical, so that shifts in the IS curve have no effects on output. The interest rate bears the full burden of equilibrating the system when shocks to aggregate demand occur. In such a system, ϵ is uncorrelated with both p and $y - k$, so that application of least squares to (5) can be expected to produce statistically consistent estimates. Note that if k depends on lagged values of the real rate of interest, it also depends on lagged values of ϵ. But then serial correlation of the ϵs implies that least squares estimates of (5) are not consistent, even if $b = 0$. Hence, k_t must be assumed independent of lagged real rates of interest in order to rationalize least squares estimation of equation (5).

But the problem is more than a simple matter of statistical technique. Unless $b = 0$, a jump in expected inflation is not fully reflected immediately in the nominal rate of interest. To see this, let $({}_{t+1}p_t^* - p_t)$ in equation (2) and ${}_tp_{t-1}^*$ in equation (1) both be exogenous, thus abandoning (4). Then an exogenous jump in $({}_{t+1}p_t^* - p_t)$ has the readily apparent effect of shifting the IS curve upward in the r, $(y - k)$ plane by exactly the amount of the shift. Unless $b = 0$, making the LM curve vertical, the upward shift in the IS curve increases r, but by less than the increase in $({}_{t+1}p_t^* - p_t)$; $y - k$ also increases. How much of the adjustment to a jump in expected inflation is borne by the nominal interest rate and how much by real output depends on the slopes of the IS curve, the LM curve, and the short-run Phillips curve. The nominal interest rate bears more of the burden of adjustment the steeper is the LM curve, the flatter is the IS curve, and the more responsive are prices to output in the short-run Phillips curve—that is, the steeper is the short-run Phillips curve.[17]

In summary, useful estimates of the parameters of Fisher's equation (5) can be expected only where both $b = 0$ and k_t is independent of current and past real rates of interest. The first restriction is extremely "monetarist" in character, implying a "quantity theory" world. Many economists would have little faith in the correctness of these restrictions, making estimation of (5) an endeavor of questionable value from their point of view. But at least there exists a set of restrictions on the economic structure that makes (5) a sensible equation to estimate. As far as I can determine, no set of restrictions on the parameters of a standard Keynesian model, like the one formed by equations (1) through (4), can be used to rationalize some of the equations fitted in the literature on price expectations and the interest rate.[18]

Behavior of the Model under Rational Expectations

The implementation of Fisher's theory described in the preceding section is subject to two severe limitations. First, its appropriateness depends on the adequacy of some very tight restrictions on the slopes of the LM curve,

17. On this see Bailey, *National Income and the Price Level*, pp. 49–54, and Sargent, "Anticipated Inflation and the Nominal Rate of Interest."

18. For example, Robert J. Gordon has regressed a nominal interest rate on current and past inflation and current and past velocity (that is, the nominal income-money ratio), interpreting the coefficients on current and lagged inflation as estimates of the weights that people use in forming price expectations. I know of no way of interpreting such an equation either as a structural equation or as a reduced form equation, at least within the class of Keynesian macroeconomic models of which the simple model here is a member. See Robert J. Gordon, "The Recent Acceleration of Inflation and Its Lessons for the Future," *Brookings Papers on Economic Activity* (1:1970), pp. 8–47. (This document is referred to hereafter as *BPEA*, followed by the date.) Also see Gordon's "Discussion" in *Econometrics of Price Determination*. The point being made here is developed in greater detail in my "The Fundamental Determinants of the Interest Rate: A Comment," *Review of Economics and Statistics*, Vol. 55 (August 1973), pp. 391–93. It should be noted that Gordon has estimated a much improved equation in his "Inflation in Recession and Recovery," *BPEA* (1:1971), pp. 145–47. That equation can be regarded as the reduced form for the interest rate, on the assumption that prices are exogenous.

the IS curve, and the short-run Phillips curve. Second, equation (4) has often been criticized as an excessively naive theory of expectations, since it fails to incorporate the possibility that people form expectations about the price level by using information other than current and lagged prices. One tractable way of meeting this second criticism is to hypothesize that the expectations of the public are rational in the sense of Muth and Samuelson,[19] and are thus equivalent with the optimal predictions of economic and statistical theory. For purposes of the analysis here, this hypothesis would involve assuming that the public (a) knows the true reduced form for the price level, (b) knows the probability distributions or rules governing the evolution of the exogenous variables, and (c) combines this information to form optimal (least squares) forecasts of the price level. Two reasons might be given for entertaining the hypothesis that expectations are rational. First, it makes concrete and operational the appealing notion that people use information besides past prices in forming their forecasts of the price level. Second, in certain instances it has been possible to test the hypothesis empirically by using the test proposed by Samuelson, and the hypothesis has fared pretty well when tested on data on stock prices, commodities prices, and interest rates.[20]

When (4) is replaced with the assumption that expectations are rational, the system formed by equations (1), (2), and (3) implies a version of Fisher's theory in which the real rate of interest is statistically independent of the systematic part of the money supply, so that foreseen changes in the money supply affect the nominal rate of interest only to the extent that they alter the expected rate of inflation. This result holds regardless of the magnitudes of the slopes of the IS, LM, and short-run Phillips curves. (In fact, for the model to possess an equilibrium, b must be strictly less than zero.) In this section, I propose to show that the invariance of the real rate of interest with respect to the systematic part of the money supply requires

19. Muth, "Rational Expectations and the Theory of Price Movements," and Samuelson, "Proof that Properly Anticipated Prices Fluctuate Randomly."

20. See ibid. The evidence is reviewed by Eugene F. Fama in "Efficient Capital Markets: A Review of Theory and Empirical Work," *Journal of Finance*, Vol. 25 (May 1970), No. 2, pp. 383–417. Evidence that the hypothesis of rational expectations can be combined with the expectations theory of the term structure to produce a workable explanation of the term structure is presented in Robert J. Shiller, "Rational Expectations and the Structure of Interest Rates" (Ph.D. thesis, Massachusetts Institute of Technology, 1972). Also see Franco Modigliani and Shiller, "Inflation, Rational Expectations and the Term Structure of Interest Rates," *Economica*, N.S., Vol. 40 (February 1973), pp. 12–43.

only (a) the assumption of an aggregate supply schedule like (1), and (b) the assumption that expectations are rational.

To close the model formed by equations (1), (2), and (3), I now posit that expectations about the logarithm of the price level are rational. This amounts to requiring that

$$(8) \qquad {}_{t+1}p_t^* = \underset{t}{E}p_{t+1},$$

where $\underset{t}{E}p_{t+1}$ is the conditional mathematical expectation of p_{t+1} formed using the model and information about the exogenous and endogenous variables available as of time t. Equation (8) asserts equality between the psychological expectation ${}_{t+1}p_t^*$ and the objective conditional expectation $\underset{t}{E}p_{t+1}$.

To complete the model under (8), I must specify the behavior of the exogenous variables and random terms that condition the expectation in (8). I assume that the money supply is governed by the linear feedback rule

$$(9) \quad m_{t+1} = \sum_{i=0}^{\infty} w_i\, m_{t-i} + \sum_{i=0}^{\infty} v_i^1\, \epsilon_{t-i} + \sum_{i=0}^{\infty} v_i^2\, U_{t-i} + \sum_{i=0}^{\infty} v_i^3\, \eta_{t-i} + \sum_{i=0}^{\infty} v_i^4\, k_{t-i} + \sum_{i=0}^{\infty} v_i^5\, Z_{t-i} + \xi_{mt+1},$$

where the w_is and v_i^js are parameters and ξ_{mt} is a normally distributed, serially uncorrelated random variable with mean zero; ξ_{mt+1} satisfies $E(\xi_{mt+1} \mid m_t, m_{t-1}, \ldots, \epsilon_t, \epsilon_{t-1}, \ldots, Z_t, \ldots) = 0$ and represents the random part of the money supply that cannot be predicted on the basis of past variables. This part might well result from deliberate policy-making decisions, but simply cannot be predicted on the basis of information about the state of the economy. The remaining, systematic, part of the money supply, which in (9) is represented by distributed lags in all of the disturbances and exogenous variables appearing in the model, can be predicted perfectly, given the values of all current and lagged exogenous variables and disturbances. Since each endogenous variable is a linear combination of the exogenous variables and the disturbances, any sort of linear feedback from the exogenous and endogenous variables to the money supply can be represented by (9). Thus, one justification for assuming (9) is that it is a very general rule capable of encompassing feedback from, for example, prices, output, and the interest rate to the money supply. Furthermore, for a linear model with known coefficients and a quadratic loss function, feedback rules of the form (9) with $\xi_{mt+1} = 0$ are known to be optimal.[21]

The random terms ϵ_t, U_t, and η_t, and the exogenous variables Z_t and k_t are each governed by an autoregressive process

$$
\begin{aligned}
\epsilon_{t+1} &= \rho_\epsilon^* \epsilon_t + \xi_{\epsilon t+1} \\
U_{t+1} &= \rho_U^* U_t + \xi_{Ut+1} \\
\eta_{t+1} &= \rho_\eta^* \eta_t + \xi_{\eta t+1} \\
k_{t+1} &= \rho_k^* k_t + \xi_{kt+1} \\
Z_{t+1} &= \rho_Z^* Z_t + \xi_{Zt+1},
\end{aligned}
\tag{10}
$$

where $\rho_\epsilon^* \, \epsilon_t \equiv \sum_{i=0}^{\infty} \rho_{\epsilon_i} \, \epsilon_{t-i}$, and so on. Here the ξs are mutually uncorrelated, serially uncorrelated, normally distributed random variables with means zero.

The public is assumed to know, or at least to have estimated, the parameters of (9) and (10). Where required, it uses this knowledge to calculate the pertinent expectations or least squares forecasts. Then, given the system formed by equations (1), (2), (3), (8), (9), and (10), the equilibrium price level can be written as a function of current and past m, k, Z, ϵ, η, and U:

$$
\begin{aligned}
p_t = R(&m_t, m_{t-1}, \ldots, k_t, k_{t-1}, \ldots, Z_t, Z_{t-1}, \ldots, \\
&\epsilon_t, \epsilon_{t-1}, \ldots, \eta_t, \eta_{t-1}, \ldots, U_t, U_{t-1}, \ldots),
\end{aligned}
\tag{11}
$$

which is the reduced form for the price level. This reduced form equation builds in the fact that p_t is influenced by $\underset{t}{E}\, p_{t+1}$. But p_{t+1} will be influenced by $\underset{t+1}{E}\, p_{t+2}$, so that $\underset{t}{E}\, p_{t+1}$ will depend on $\underset{t}{E}\, p_{t+2}$, and so on, and this must be taken into account under rationality. Appendix A, where R is calculated explicitly, shows how forecasts of next period's price are forced, through this dependence, to take into account forecasts of the values of the exogenous variables influencing the price level in all subsequent periods.[22] In

21. Except for the fact that I have added the stochastic term ξ_{mt}, this is an example of the kind of linear feedback rule studied by Gregory C. Chow, "Optimal Stochastic Control of Linear Economic Systems," *Journal of Money, Credit and Banking*, Vol. 2 (August 1970), pp. 291–302.

22. Such an equilibrium is calculated for a nonstochastic model by R. E. Hall in "The Dynamic Effects of Fiscal Policy in an Economy with Foresight," *Review of Economic Studies*, Vol. 38 (April 1971), pp. 229–44. For a linear, stochastic model, an example of such an equilibrium is calculated by Thomas J. Sargent and Neil Wallace in "Rational Expectations and the Dynamics of Hyperinflation," *International Economic Review*, Vol. 14 (June 1973), pp. 328–50. Also see Lucas, "Econometric Testing of the Natural Rate Hypothesis."

forming these expectations, individuals consider the money supply rule (9) and the autoregressions for the disturbances and exogenous variables (10). The parameters of equations (9) and (10) are thereby built into the reduced form R of (11). Consequently, the parameters of the reduced form R depend on *both* the structural parameters of the model and the parameters of the monetary rule (9) and the autoregressions (10). The parameters of (11) will thus not be invariant with respect to systematic changes in the money supply rule that have either been publicly announced or in effect long enough for the public to detect them.[23]

The reduced form equation (11) can be combined with the money supply rule (9) and the laws governing the random terms and exogenous variables (10) to yield the probability distribution of p_{t+1}, conditional on data observed up through time t:

$$
\begin{aligned}
&\mathit{Prob}\,(p_{t+1} < F \mid m_t = m_0, m_{t-1} = m_1, \ldots, k_t = k_0, k_{t-1} = k_1, \ldots,\\
&\qquad Z_t = Z_0, Z_{t-1} = Z_1, \ldots, \epsilon_t = \epsilon_0, \epsilon_{t-1} = \epsilon_1, \ldots,\\
(12)\quad &\qquad U_t = U_0, U_{t-1} = U_1, \ldots, \eta_t = \eta_0, \eta_{t-1} = \eta_1, \ldots)\\
&= H(F, m_0, m_1, \ldots, k_0, k_1, \ldots, Z_0, Z_1, \ldots,\\
&\qquad\qquad \epsilon_0, \epsilon_1, \ldots, U_0, U_1, \ldots, \eta_0, \eta_1, \ldots).
\end{aligned}
$$

The conditional expectation in (8) is evaluated with respect to (12):

$$
\begin{aligned}
(13)\quad & {}_{t+1}p_t^*\\
&= E\,(p_{t+1} \mid m_t, m_{t-1}, \ldots, k_t, k_{t-1}, \ldots, Z_t, Z_{t-1}, \ldots, \epsilon_t, \ldots, U_t, \ldots, \eta_t)\\
&= \int_{-\infty}^{\infty} F\, dH\,(F \mid m_t, m_{t-1}, \ldots, k_t, k_{t-1}, \ldots, Z_t, Z_{t-1}, \ldots, \epsilon_t, \ldots, U_t, \ldots, \eta_t).
\end{aligned}
$$

For convenience, let θ_t denote the set of variables upon which the expectation (13) is conditioned, so that

$$
(13') \qquad {}_{t+1}p_t^* = E(p_{t+1} \mid \theta_t),
$$

where θ_t includes all observations on m, k, Z, ϵ, U, and η dated t and earlier.

It is now easy to show that the system is described by two intimately related propositions that reflect central aspects of the monetarist point of view. First, a natural rate of output exists in the sense that the deviation of output from its normal level is statistically independent of the systematic parts of monetary and fiscal policies; that is, widely known changes

23. The implications for the theory of economic policy of this characteristic of models with rational expectations are carefully drawn out by Lucas, "Econometric Policy Evaluation."

in the ws and vs of equation (9) and in the ρ_Zs of equation (10) have no effects on the expected value of $(y - k)$. Second, the real rate of interest is independent of the systematic part of the money supply; that is, alterations in the ws and vs of the feedback rule (9) have no effects on the expected value of the real rate. (Random movements in the money supply, represented by ξ_{mt}, do have effects on both aggregate supply and the real rate of interest.)

The first of these propositions, which is due to Lucas,[24] follows from a simple and well-known property that, under rationality, characterizes the prediction error that appears in the aggregate supply schedule (1). Using (13), the prediction error is

$$p_t - E(p_t \mid \theta_{t-1}).$$

The regression of the prediction error on θ_{t-1} is

$$E\{[p_t - E(p_t \mid \theta_{t-1})] \mid \theta_{t-1}\} = E(p_t \mid \theta_{t-1}) - E(p_t \mid \theta_{t-1}) = 0,$$

which shows that the prediction error is independent of all elements of θ_{t-1}. Substituting this result into the conditional expectation of equation (1) gives

$$E((y_t - k_t) \mid \theta_{t-1}) = E(U_t \mid \theta_{t-1}) = E(U_t \mid U_{t-1}, U_{t-2}, \ldots). \tag{14}$$

Since U_t depends only on lagged Us, equation (14) shows that $y - k$ is independent of all components of θ_{t-1} except lagged values of U. That part of the current money supply (or the fiscal policy variables in Z_t) that can be expressed as a linear combination of the elements of θ_{t-1} (that is, the "systematic" part of policy) therefore has no effect on the expected value of $y_t - k_t$, regardless of the parameters of that linear combination.

The second proposition—that the real rate of interest is independent of the systematic part of the money supply rule—stands and falls with Lucas' natural rate proposition.[25] Solving equation (2) for the nominal rate of interest gives

24. Lucas, "Econometric Testing of the Natural Rate Hypothesis"; "Expectations and the Neutrality of Money"; and "Econometric Policy Evaluation."

25. The result requires that both m and p be excluded from the aggregate demand schedule, except for the latter's appearance as part of the term $\underset{t}{E}p_{t+1} - p_t$. As mentioned in note 15, this seems to be a standard specification in macroeconometric models. It is, however, well known that including a real balance effect in the aggregate demand schedule modifies Fisher's theory in a static, full employment context. See Robert Mundell, "Inflation and Real Interest," *Journal of Political Economy*, Vol. 71 (June 1963), pp.

(15) $$r_t = c^{-1}(y_t - k_t) - \frac{d}{c} Z_t + E(p_{t+1} \mid \theta_t) - p_t - c^{-1} \epsilon_t.$$

Taking expectations in (15) conditional on θ_{t-1} and substituting from (14) gives

(16) $$E\{[r_t - E(p_{t+1} \mid \theta_t) + p_t] \mid \theta_{t-1}\} = -\frac{d}{c} E(Z_t \mid \theta_{t-1}) + c^{-1} E(U_t \mid \theta_{t-1}) - c^{-1} E(\epsilon_t \mid \theta_{t-1}).$$

Equation (16) states that the real rate of interest is correlated with elements of θ_{t-1} only to the extent that they help predict subsequent values of the random variables U_t and ϵ_t and subsequent fiscal policy—that is, the variables in Z_t. Of course, U_t depends only on lagged Us, while ϵ_t depends only on lagged ϵs. The real rate of interest is therefore a function of the systematic parts of fiscal policy, but is independent of the parameters that determine the systematic part of the money supply. In this system changes in the money supply at t that can be foreseen as of time $t - 1$ leave the real interest rate at t unchanged. It follows that the systematic part of the money supply affects the nominal rate of interest only to the extent that it influences the expected rate of inflation. The only part of the money supply at t that affects the real rate at t is the random component ξ_{mt}.

RESULTS OF CHANGING ASPECTS OF THE MODEL

These two propositions will characterize models much more complicated than the one used here so long as expectations are assumed to be rational and aggregate supply is governed by an equation like (1).[26] For example,

280–83. The expected rate of inflation can be viewed as the rate of tax on real balances. Where $m_t - p_t$ appears in the aggregate demand schedule—either alone, as in the real balance effect, or multiplied by minus the expected rate of inflation, as implied by some definitions of disposable income—changes in the expected rate of inflation bring about changes in the real rate of interest, just as do changes in the other tax rates included in Z_t.

26. The behavior of the model under rational expectations would not be substantially altered if the aggregate supply hypothesis were expanded to be

$$y_t - k_t = \gamma\left[p_t - \underset{t-1}{E}p_t - \left(\frac{1}{n}\sum_{j=1}^{n} \underset{t}{E}p_{t+j} - \frac{1}{n}\sum_{j=1}^{n} \underset{t-1}{E}p_{t+j}\right)\right] + U_t,$$

which states that aggregate supply responds to the "surprise" component of this period's price level minus the amount by which an average of expectations of prices in n future periods is revised as a result of new information received this period. The above equation embodies the notion that aggregate supply responds to the part of the prediction error $p_t - \underset{t-1}{E} p_t$ that is viewed as transitory. The argument in the above equation still possesses the two propositions would continue to hold if the assumption of exogenous productive capacity k_t is abandoned and instead k_t is assumed to depend on past values of output and the real rate of interest. This specification would permit growth in capacity to be influenced by capital accumulation, which in turn could be governed by a version of the distributed lag accelerator.

For another modification that would leave the two propositions intact, (1) might be replaced with the alternative aggregate supply schedule

$$(1') \qquad y_t - k_t = \gamma(p_t - Ep_t \mid \theta_{t-1}) + \sum_{i=1}^{q} \lambda_i (y_{t-i} - k_{t-i}) + U_t,$$

which application of the Koyck-Jorgenson transformation shows to be equivalent to

$$(1'') \qquad y_t - k_t = \gamma \sum_{i=0}^{\infty} \phi_i [p_{t-i} - E(p_{t-i} \mid \theta_{t-i-1})] + \sum_{i=0}^{\infty} \phi_i U_{t-i},$$

where the ϕ_is are functions of the λ_is. According to (1′), deviations of aggregate supply from normal capacity output display some persistence, so that $y_t - k_t$ depends partly on a distributed lag of prediction errors, as equation (1″) shows. If (1″) replaces (1) in the version of the model with rational expectations, both $y_t - k_t$ and the real rate of interest remain independent of the systematic part of the money supply. To see this, one has only to note that the systematic parts of current and lagged values of the money supply contribute nothing to the prediction errors that appear in (1″), nor do they influence the Us. Of course, the random parts of the money supply, ξ_m, will still influence $y - k$. Under (1″), the effects of ξ_m on $y - k$ will be distributed over time, but the two propositions about the systematic parts of policy variables remain unaltered.

In essence, two features of the model must hold to validate these propositions. First, expectations must be rational. Second, the model must possess "super-neutrality," by which I mean that proportionate changes in either the levels or the rates of change of all endogenous and exogenous variables

---

all of the properties of prediction errors that are used in the text to show the behavior of the model under rationality.

By invoking the expectations theory of the term structure of interest rates, yields on bonds with maturities greater than one period could be included in the model. It would be straightforward, for example, to enter an n-period rate in the aggregate demand schedule, modifying the price expectation term accordingly, while keeping a one-period rate in the portfolio balance curve.

denominated in dollars (prices, wages, and stocks of paper assets of fixed dollar value such as money and bonds) do not disturb an initial equilibrium. It should be noted that current and expected values of endogenous and exogenous nominal variables are among those changed proportionately in the experiment defining super-neutrality.

Appendix B demonstrates that key features of the results remain intact even when individuals have much less information and wisdom than I have imputed to them so far, so long as they have access to information at least about lagged prices and use it rationally in forecasting the price level. Appendix B also shows that dropping the assumption that bonds and equities are perfect substitutes does not change the essential character of the model.

Testing the Model

A "WRONG" TEST

The usual way of implementing Fisher's theory about interest and expected inflation has been to regress nominal interest rates on current and lagged values of the logarithm of the price level, interpreting the coefficients as estimates of the distributed lag by which the public seems to form its expectations about inflation. The implausibility of those distributed lags as devices for forming predictions of inflation has weakened the appeal of Fisher's doctrine. However, according to the version of the model with rational expectations described here, these regressions are not a valid test. In particular, there is no reason to expect that the distributed lags estimated in such regressions provide the basis for plausible, or in some sense optimal, forecasts of inflation. This is so even though the model predicts that the real rate of interest is independent of the money supply rule, a proposition that can be taken as capturing the essence of Fisher's theory.

To establish the inappropriate nature of the standard regressions, I use equation (15) to calculate the regression of the nominal interest rate on current and lagged prices:

$$(17)\quad \begin{aligned} E(r_t \mid p_t, p_{t-1}, \ldots) &= E[(p_{t+1} - p_t) \mid p_t, p_{t-1}, p_{t-2}, \ldots] \\ &+ c^{-1} E(U_t \mid p_t, p_{t-1}, \ldots) - \frac{d}{c} E(Z_t \mid p_t, p_{t-1}, \ldots) \\ &+ c^{-1} \gamma E\{[p_t - E(p_t \mid \theta_{t-1})] \mid p_t, p_{t-1}, \ldots\} \\ &- c^{-1} E(\epsilon_t \mid p_t, p_{t-1}, \ldots). \end{aligned}$$

Regressions of interest on current and lagged prices have been interpreted as yielding estimates of the regression $E[(p_{t+1} - p_t) \mid p_t, p_{t-1}, \ldots]$. In the model here, however, that interpretation is erroneous because of the presence of the second, third, fourth, and fifth terms in (17). The model predicts that the exogenous variables Z_t will be correlated with current and perhaps past values of the price level. The model also predicts that ϵ_t and U_t will be correlated with the current price level: a positive "pip" in ϵ_t increases both r_t and p_t, an effect that has nothing to do with the formation of expectations of inflation.[27] The presence of this effect pollutes the relationship between r and p from the point of view of extracting an estimate of $E[(p_{t+1} - p_t) \mid p_t, p_{t-1}, \ldots]$. The presence of the third and fourth terms similarly biases the regression of r on current and past ps taken as a device for recovering forecasts of inflation.

The biases pinpointed by equation (17) could easily be spectacularly large and could in principle give rise to the presence of a Gibson paradox in data generated by the model. Very long and implausible distributed lags of interest on inflation could be generated, since the model embodies sources of dependence between the interest rate and price *level* that are not accounted for by the presence of expected inflation. This fact implies that demonstrations of the "implausibility" of regressions of interest on inflation cannot refute the version of Fisher's theory embodied in the model.

A "PROPER" TEST

The straightforward approach to testing the model would be to subject the theory's centerpiece, the natural rate hypothesis, to an empirical test. However, as Lucas has forcefully pointed out, almost all such work has been wholly inadequate.[28] Basically, these improper tests[29] have all involved fitting a structure that can be rearranged to yield an expression for unemployment of the following form:

27. This is presumably the kind of effect that Tobin had in mind when he questioned Irving Fisher's explanation of the Gibson paradox. See his "Comment" in *Proceedings of a Symposium on Inflation: Its Causes, Consequences, and Control* (Wilton, Conn.: Kazanjian Economics Foundation, Inc., 1968), pp. 53–54.

28. Lucas, "Econometric Testing of the Natural Rate Hypothesis."

29. The test was described by both Robert Solow and James Tobin in their contributions to the *Proceedings of a Symposium on Inflation.* One of the best-known applications of the test is Gordon, "Recent Acceleration of Inflation."

$$Un_t = \hat{\beta}(p_t - \sum_{i=1}^{n} \hat{v}_i\, p_{t-i}) + \text{residual}_t, \tag{18}$$

where the unemployment rate Un_t can be regarded as an inverse index of $y_t - k_t$. In every case, $\hat{\beta}$ has been less than 0, indicating a short-run tradeoff between inflation and employment. The standard test of the natural rate hypothesis has been to determine whether, according to the estimates of equation (18), a once-and-for-all increase in the rate of inflation implies a permanent change in the unemployment rate.[30] But even if it doesn't, a once-and-for-all jump in some higher-order difference in the (log of the) price level will always imply a permanent change in the unemployment rate in the context of equation (18) with any fixed set of $\hat{v}_i$s. Thus, if the authorities can make the price level follow a path

$$p_t = \sum_{i=1}^{n} \hat{v}_i\, p_{t-i} + \phi,$$

they can, by increasing ϕ by $d\phi$, have a permanent, predictable effect on unemployment of $\hat{\beta}d\phi$. This conclusion, however, is incompatible with the natural rate hypothesis, which requires that certain, foreseen, once-and-for-all jumps in *any* order difference of the price level have no permanent effect on the unemployment rate. Put another way, the natural rate hypothesis requires that changing from one deterministic (and hence perfectly predictable) process for the price level to another will leave the unemployment rate unaltered. *No* values of the $\hat{v}_i$s of equation (18) are capable of representing that hypothesis, given the way the estimated $\hat{v}_i$s are manipulated in the test. The test, therefore, cannot possibly be fair.[31]

Lucas has described and implemented two proper tests of the rational expectations version of the natural rate hypothesis.[32] One involves testing a set of cross-equation restrictions implied by the hypothesis, the other, testing across countries for a relationship between the slope of a country's short-run inflation-unemployment tradeoff and the variance of its nominal aggregate demand implied by the hypothesis. Lucas is unable to reject the natural rate hypothesis on the basis of either of these tests.

Although, to my knowledge, Lucas' are the only proper tests of the

30. Usually, the weights are constrained to satisfy $\sum_{i=1}^{\infty} \hat{v}_i = 1$, so that a once-and-for-all jump in the log of the price level is *not* permitted to imply a permanent change in the unemployment rate.

31. Again, see Lucas, "Econometric Testing of the Natural Rate Hypothesis."

32. Lucas, "Some International Evidence on Output-Inflation Tradeoffs."

natural rate hypothesis implemented to date, there are other tests of the natural rate hypothesis. One exploits the implications under rationality of the hypothesis that aggregate supply is a function of the error in predicting the current price level on the basis of data available at some previous moment. Using the unemployment rate Un_t as an inverse index of $y_t - k_t$, the aggregate supply schedule (1′) can be written

$$(19) \qquad Un_t = \beta(p_t - Ep_t \mid \theta_{t-1}) + \sum_{i=1}^{q} \lambda_i \, Un_{t-i} + u_t, \qquad \beta < 0.$$

Here u_t is a random disturbance assumed to be normally distributed and to obey $E(u_t \mid \theta_{t-1}, u_{t-1}, u_{t-2}, \ldots) = E(u_t \mid u_{t-1}, u_{t-2}, \ldots)$. To take a special example that will illustrate the idea behind the test, suppose that u_t is not serially correlated and that all of the λ_is equal zero. Taking expectations in (19) conditional on any subset θ_{lt-1} of θ_{t-1} gives

$$E(Un_t \mid \theta_{lt-1}) = 0,$$

an implication that could be tested empirically by regressing Un_t on components of θ_{lt-1}. However, the presence of nonzero λ_is or serial correlation in u_t would destroy this implication, since then

$$E(Un_t \mid \theta_{lt-1}, Un_{t-1}, \ldots, Un_{t-q}) = \sum_{i=1}^{q} \lambda_i \, Un_{t-i} + E(u_t \mid \theta_{lt-1}) \neq 0.$$

The term $\sum_{i=1}^{q} \lambda_i \, Un_{t-i}$ obviously would not be zero; if u_t is serially correlated, then $E(u_t \mid \theta_{lt-1})$ also departs from zero to the extent that components of θ_{lt-1} proxy for lagged *u*s.

To illustrate how a feasible test could be carried out under these circumstances, suppose that u_t follows the first-order Markov process

$$u_t = \rho u_{t-1} + \xi_{ut}, \qquad |\rho| < 1,$$

where ξ_{ut} is a normally distributed, serially uncorrelated random variable. Then notice that (19) can be written as

$$(20) \quad Un_t = (\lambda_1 + \rho)\, Un_{t-1} + \sum_{i=2}^{q} (\lambda_i - \rho\lambda_{i-1})\, Un_{t-i} - \rho\lambda_q \, Un_{t-q-1}$$
$$+ \beta(p_t - Ep_t \mid \theta_{t-1}) - \beta\rho\,(p_{t-1} - Ep_{t-1} \mid \theta_{t-2}) + \xi_{ut}.$$

Taking expectations in (20) conditional on $Un_{t-1}, \ldots, Un_{t-q-1}$ and any subset θ_{lt-2} of θ_{t-2} yields

$$(21)\quad E(Un_t \mid Un_{t-1}, \ldots, Un_{t-q-1}, \theta_{lt-2}) = (\lambda_1 + \rho)\, Un_{t-1} + \sum_{i=2}^{q} (\lambda_i - \rho\lambda_{i-1})\, Un_{t-i} - \rho\lambda_q\, Un_{t-q-1} - \rho\beta E[(p_{t-1} - Ep_{t-1} \mid \theta_{t-2}) \mid Un_{t-1}].$$

Equation (21) holds because the prediction error $p_t - E\, p_t \mid \theta_{t-1}$ is independent of all components of θ_{t-1}, which include the regressors in (21), while the lagged prediction error is independent of $Un_{t-2}, \ldots, Un_{t-q-1}$ and θ_{lt-2}, but not of Un_{t-1}. According to (21), the regression of the unemployment rate against $Un_{t-1}, \ldots, Un_{t-q-1}$, and some components of θ_{t-2} ought, on the natural rate hypothesis, to have zero coefficients on components of θ_{t-2}. This implication can be tested empirically by calculating the regression indicated in (21). If $\rho = 0$, then (20) implies that

$$E(Un_t \mid Un_{t-1}, \ldots, Un_{t-q-1}, \theta_{t-1}) = E(Un_t \mid Un_{t-1}, \ldots, Un_{t-q-1}),$$

so that if the us in (19) are serially uncorrelated, components of θ_{t-1} ought not to obtain coefficients significantly different from zero when they are added to a regression of Un_t on enough lagged values of itself. On the other hand, if u_t is governed by an nth order autoregressive process

$$u_t = \sum_{i=1}^{n} \rho_i\, u_{t-i} + \xi_{ut},$$

where ξ_{ut} has the same properties imputed to it above, then it is readily shown that the natural rate hypothesis implies only that

$$E(Un_t \mid Un_{t-1}, Un_{t-2}, \ldots, Un_{t-n-q}, \theta_{lt-n-1}) = E(Un_t \mid Un_{t-1}, Un_{t-2}, \ldots, Un_{t-n-q}).$$

The higher the order of serial correlation in the us, the more periods components of θ_t must be lagged to warrant the implication that their coefficients are zero.

One can view the test from a slightly different perspective by considering the following very general mixed autoregressive, moving-average representation of the unemployment rate,

$$(22)\qquad Un_t = \sum_{i=1}^{q} \lambda_i\, Un_{t-i} + \sum_{i=0}^{f} \alpha_i\, \xi_{ut-i},$$

where the λ_is and α_is are parameters and where ξ_{ut} is again a serially uncorrelated, normally distributed random variable. The natural rate hypothesis can be viewed as permitting ξ_{ut} to be correlated with values of endogenous variables dated t and later, but as requiring ξ_{ut} to be uncorrelated with past endogenous and exogenous variables, so that

$$E(\xi_{ut} \mid \theta_{t-1}) = 0.$$

This means that the "innovation," or new random part of the unemployment rate, cannot be predicted from past values of any variables, and that it cannot be affected by movements in past values of government policy variables. This specification captures the heart of the natural unemployment rate hypothesis, and implies that there is no better way to predict subsequent rates of unemployment than fitting and extrapolating a mixed autoregressive, moving-average process in the unemployment rate itself. This suggests that the natural unemployment rate hypothesis can be tested against specific competing hypotheses by setting up statistical prediction "horse races." My proposed regression test is an alternative test, and exploits the notion that, if $E(\xi_{ut} \mid \theta_{t-1}) = 0$, then (22) implies that

$$E(Un_t \mid Un_{t-1}, \ldots, Un_{t-q}, \theta_{t-f-1}) = \sum_{i=1}^{q} \lambda_i \, Un_{t-i}.$$

To provide material for the test, regressions (1), shown below, are autogressions for the unemployment rate.

$$(1) \qquad \underset{(0.164)}{Un_t = 0.418} + \underset{(0.116)}{1.715} \, Un_{t-1} - \underset{(0.199)}{1.046} \, Un_{t-2} + \underset{(0.115)}{0.245} \, Un_{t-3}$$

$\bar{R}^2 = 0.9245$, standard error of estimate = 0.318, Durbin-Watson statistic = 1.984.
Period of fit: 1952:1–1970:4.

$$Un_t = \underset{(0.158)}{0.538} + \underset{(0.089)}{1.553} \, Un_{t-1} - \underset{(0.089)}{0.665} \, Un_{t-2},$$

$\bar{R}^2 = 0.9208$, standard error of estimate = 0.325, Durbin-Watson statistic = 1.616.
Period of fit: 1952:1–1970:4.

where Un is the unemployment rate for all civilian workers, seasonally adjusted, and t indicates time (data for regressions (1), and for regressions (2) and (3) below, unless stated otherwise, were obtained from the data bank for the Wharton Econometric Model). The numbers in parentheses here and in the following regressions are standard errors.

Regressions (2) and (3) include various components of θ_{t-1}, as well as lagged values of the unemployment rate. In regression (2), these components are the logarithm of the GNP deflator (p), seasonally adjusted, lagged one through four quarters, and the log of average hourly earnings in manufacturing corrected for overtime payments, not seasonally adjusted (w), lagged one through four quarters (from various issues of *Employment and Earnings*).

$$
\begin{aligned}
(2)\quad Un_t = & \underset{(1.806)}{-0.723} + \underset{(0.097)}{1.600}\, Un_{t-1} - \underset{(0.101)}{0.722}\, Un_{t-2} \\
& - \underset{(13.607)}{20.982}\, p_{t-1} + \underset{(20.223)}{15.805}\, p_{t-2} + \underset{(20.087)}{0.153}\, p_{t-3} + \underset{(14.002)}{2.574}\, p_{t-4} \\
& + \underset{(8.960)}{5.509}\, w_{t-1} + \underset{(10.125)}{3.152}\, w_{t-2} - \underset{(10.014)}{3.807}\, w_{t-3} - \underset{(8.327)}{3.080}\, w_{t-4}.
\end{aligned}
$$

$\bar{R}^2 = 0.917$, standard error of estimate = 0.333, Durbin-Watson statistic = 1.684. $F(8,65) = 0.594$.

The F-statistic pertinent for testing the null hypothesis that the coefficients on lagged p and lagged w are zero is 0.594, which implies that the null hypothesis cannot be rejected at the 95 percent confidence level. Accordingly, the natural unemployment rate hypothesis cannot be rejected on the basis of this regression. The adjusted standard error of the residuals in regression (2) (0.333) is actually larger than that obtained by excluding the ps and ws (0.325), reported in regressions (1).

Regression (3) implements the test by employing a much larger set of elements of θ_{t-1}. In addition to three lagged values of the unemployment rate, the regression includes values of the logarithm of the money supply (currency plus demand deposits), seasonally adjusted (m), the federal and state and local government deficit on the national income accounts basis (Def); and the logs of the GNP deflator, seasonally adjusted (p), of the implicit deflator for personal consumption expenditures (pc), of the average hourly wage rate in manufacturing, seasonally adjusted (wr), of government purchases of goods and services (g), of total federal and state and local government employment, seasonally adjusted (ng), and of GNP (y). Each of these arguments is included lagged one, two, and three periods.

$$
\begin{aligned}
(3)\quad Un_t = & \underset{(12.427)}{39.622} + \underset{(0.136)}{1.223}\, Un_{t-1} - \underset{(0.211)}{0.546}\, Un_{t-2} - \underset{(0.169)}{0.129}\, Un_{t-3} \\
& - \underset{(9.839)}{3.852}\, m_{t-1} - \underset{(15.926)}{11.835}\, m_{t-2} + \underset{(9.620)}{16.801}\, m_{t-3} + \underset{(0.016)}{0.023}\, Def_{t-1} \\
& - \underset{(0.020)}{0.006}\, Def_{t-2} + \underset{(0.018)}{0.020}\, Def_{t-3} + \underset{(21.702)}{26.268}\, p_{t-1} - \underset{(24.210)}{25.552}\, p_{t-2} \\
& + \underset{(19.511)}{27.416}\, p_{t-3} - \underset{(20.868)}{7.807}\, pc_{t-1} + \underset{(24.375)}{28.701}\, pc_{t-2} - \underset{(20.328)}{57.719}\, pc_{t-3}
\end{aligned}
$$

$$- \underset{(8.068)}{1.631}\, wr_{t-1} + \underset{(10.286)}{1.461}\, wr_{t-2} + \underset{(7.623)}{8.567}\, wr_{t-3} + \underset{(2.917)}{3.448}\, g_{t-1}$$

$$- \underset{(3.723)}{1.508}\, g_{t-2} + \underset{(2.662)}{3.812}\, g_{t-3} + \underset{(10.333)}{4.909}\, ng_{t-1} - \underset{(14.168)}{13.424}\, ng_{t-2}$$

$$+ \underset{(10.053)}{4.725}\, ng_{t-3} + \underset{(6.228)}{1.151}\, y_{t-1} - \underset{(7.913)}{8.560}\, y_{t-2} - \underset{(6.879)}{3.824}\, y_{t-3}.$$

$\bar{R}^2 = 0.9497$, standard error of estimate $= 0.259$, Durbin-Watson statistic $= 2.161$.
$F(24{,}48) = 2.503$.

For regression (3) the pertinent F-statistic for testing the null hypothesis that elements of θ_{t-1} (other than lagged rates of unemployment) have zero coefficients is 2.503. This statistic is distributed with 24,48 degrees of freedom and so is significant at the 99 percent confidence level. As a result, the null hypothesis must be rejected. The adjusted standard error of estimate falls from the 0.318 reported in (1) to 0.259 when the components of θ_{t-1} are added to the regression, indicating a modest but statistically significant gain in explanatory power. Consequently, this application of the test requires rejection of the version of the natural rate hypothesis that assumes rational expectations formed on the basis of at least the information contained in the particular set θ_{ut-1} used in the regression under discussion.

Several reasons suggest caution in interpreting the verdict of this test. First, as shown above, the test assumes that the us in equation (19) are not serially correlated. If, in fact, they are, the test becomes biased in favor of rejecting the natural rate hypothesis. Second, the essence of the natural rate hypothesis could stand unrefuted even though tests using large subsets θ_{ut-1} find systematic effects of θ_{ut-1} on Un_t. This can occur if individuals form their expectations rationally on less information than is represented by θ_{ut-1}. In this regard, it is noteworthy that the natural rate hypothesis cannot be rejected on the basis of regression (2), which includes only lagged ws and ps as components of θ_{ut-1}. Third, the test could lead to rejection of the natural rate hypothesis if the us are correlated with components of θ_{ut-1}. This might occur, because, for example, current and lagged θs have a direct effect on unemployment that requires no movement in the price level, contrary to the hypothesis in (19). In this event, systematic changes in the price level could still leave the unemployment rate unaffected, so that policy makers confront no "cruel choice" between the average rate of inflation and the average unemployment rate.

Finally, it should be noted that the results of the test reported in (3) have not been shown to be of comfort to advocates of any particular alternatives to the natural rate hypothesis. That is, it has not been shown that an autoregression for unemployment yields ex ante predictions of unemployment inferior to those of a particular structural macroeconometric model that embodies a particular aggregate supply theory other than the natural rate hypothesis. A particular alternative aggregate supply hypothesis might well be able to predict unemployment better than an autoregressive moving-average process, but there is no way of knowing for sure until a horse race is held.[33]

ANOTHER TEST

An alternative test of the natural unemployment rate hypothesis can be carried out by directly estimating the parameters of a version of equation (19), where u_t is now assumed to be a serially uncorrelated random term satisfying $E(u_t \mid \theta_{t-1}) = 0$. Equation (19) embodies the null hypothesis to be tested, the natural unemployment rate hypothesis.[34] I propose to test it against the following alternative hypothesis:

(23)

$$Un_t = \sum_{i=1}^{q} \lambda_i \, Un_{t-i} + \beta(p_t - Ep_t \mid \theta_{t-1}) + \beta(1 - \alpha)(Ep_t \mid \theta_{t-1} - p_{t-1}) + u_t.$$

Equation (23) states that if $\alpha < 1$ ($\alpha > 1$), then increases in the systematic part of the rate of inflation decrease (increase) the unemployment rate, contrary to the natural rate hypothesis. On the natural rate hypothesis, $\alpha = 1$, which makes (23) equivalent to equation (19). My strategy is to

33. Charles Nelson found that the predictions of the unemployment rate from a version of the Fed-M.I.T. model were inferior to the predictions from an autoregression. This was so even though for my purposes Nelson's procedure is biased in favor of the Fed-M.I.T. model because he permits it to use the actual values of the exogenous variables at the same date for which unemployment is being forecast. See Charles R. Nelson, "The Prediction Performance of the FRB-MIT-PENN Model of the U.S. Economy," *American Economic Review*, Vol. 62 (December 1972), p. 914.

34. It is common to write the natural rate hypothesis in a way that, under rational expectations, would take the form

(a) $$Un_t = \sum_{i=1}^{q} \lambda_i Un_{t-1} + \beta[(p_t - p_{t-1}) - E(p_t - p_{t-1}) \mid \theta_{t-1}] + u_t,$$

so that the surprise increase in the rate of inflation is what boosts aggregate supply. But so long as p_{t-1} is one of the variables in θ_{t-1}, it is straightforward to show that

$$(p_t - p_{t-1}) - E[(p_t - p_{t-1}) \mid \theta_{t-1}] = p_t - Ep_t \mid \theta_{t-1}.$$

It follows, then, that (a) is equivalent with (19) in the text.

estimate (23) and to test the null hypothesis, $\alpha = 1$, against the alternative hypothesis, $\alpha \neq 1$.[35]

In conducting this test, two econometric problems must be overcome. First, macroeconomic theory implies that Un_t (or equivalently $y_t - k_t$) and p_t are simultaneously determined, implying that u_t and p_t may be correlated. For example, take a standard macroeconomic model in which aggregate demand, $(y - k)_d$, depends inversely on the current price level, while aggregate supply, $(y - k)_s$, responds directly to the current price level. With predetermined expectations, equation (1) or (19) is an example of such an aggregate supply schedule, while an aggregate demand schedule in the p, $y - k$ plane is derived by using the portfolio balance schedule (3) to eliminate the nominal interest rate from the IS curve (2). It is evident that an increase in u_t causes the aggregate supply schedule to shift upward in the p, $y - k$ plane, causing the price level to rise, $y - k$ to fall, and unemployment to rise. This leads to a positive correlation between u_t and p_t' provided, for example, that u is uncorrelated with the disturbances in the aggregate demand schedule in the p, $y - k$ plane. The correlation between u and p makes least squares estimation of (19) or (23) inappropriate. This problem can be overcome in the standard way, by using the technique of instrumental variables: replacing p_t in (23) by $\hat{p}_t$, the predicted value of p_t from a first-stage regression including a constant, Un_{t-1} through Un_{t-q}, and predetermined variables including lagged prices, lagged values of other variables thought to be endogenous to the system, and current and lagged values of exogenous variables.

The second econometric challenge is to produce an appropriate proxy for $Ep_t \mid \theta_{t-1}$. Here I am assuming that the regression $Ep_t \mid \theta_{t-1}$ is linear in θ_{t-1}, so that $Ep_t \mid \theta_{t-1}$ is in effect formed as if it were the prediction from a least squares regression of p_t on θ_{t-1}, and therefore

$$Ep_t \mid \theta_{t-1} = \delta\theta_{t-1}$$
$$p_t = \hat{\delta}\theta_{t-1} + e_t \equiv \tilde{p}_t + e_t,$$

35. The test here is related to Lucas' ("Econometric Testing of the Natural Rate Hypothesis"), which tests the restrictions across the reduced forms for the price level and for output that are implied by rational expectations in conjunction with an aggregate supply schedule like (1). For the test used here it is necessary neither to specify nor to estimate the full reduced forms for aggregate supply and the price level. The test requires that a list of some predetermined variables influencing the price level be available; but there is no necessity to have a complete list of the predetermined variables appearing in the reduced form for the price level.

where $\hat{\delta}$ is a vector of least squares parameter estimates conformable to θ_{t-1}, while e_t is a least squares residual vector that is orthogonal to θ_{t-1} by construction. I propose to use $\tilde{p}_t$ in place of $Ep_t \mid \theta_{t-1}$ in equation (23).[36]

I then substitute $\tilde{p}_t$ for $Ep_t \mid \theta_{t-1}$ in (23) to obtain

$$(24)\quad Un_t = \sum_{i=1}^{q} \lambda_i\, Un_{t-i} + \beta(p_t - \tilde{p}_t) + (1-\alpha)\,\beta(\tilde{p}_t - p_{t-1}) + u_t.$$

Since it is assumed that $E(u_t \mid \theta_{t-1}) = 0$, it follows that u_t is uncorrelated with $\tilde{p}_t$. Furthermore, by construction $(p_t - \tilde{p}_t)$ is orthogonal both to Un_{t-1} through Un_{t-q}, to p_{t-1}, and to $\tilde{p}_t$, by the orthogonality of least squares residuals to regressors. However, as I have argued above, simultaneity leads to a prediction that u_t and p_t, and hence u_t and $p_t - \tilde{p}_t$, are positively correlated. Under these circumstances, in which $p_t - \tilde{p}_t$ is correlated with the disturbance while the remaining regressors are orthogonal both to the disturbance and to $p_t - \tilde{p}_t$, it follows that least squares yields consistent estimates of the coefficients on all regressors except $p_t - \tilde{p}_t$.[37] Consequently, in (24), application of least squares yields consistent estimates of $(1-\alpha)\beta$ and the λ_is, but inconsistent estimates of β. On the hypothesis that $\beta \neq 0$, a consistent estimate of $(1-\alpha)\beta$ is really all that is required to test the natural rate hypothesis, $\alpha = 1$.

As mentioned above, the inconsistency in the estimates of β can be eliminated by replacing p_t by $\hat{p}_t$ in (24) to obtain

$$(24')\quad Un_t = \sum_{i=1}^{q} \lambda_i\, Un_{t-1} + \beta(\hat{p}_t - \tilde{p}_t) + (1-\alpha)\,\beta(\tilde{p}_t - p_{t-1}) + u_t + \beta f_t,$$

36. Suppose that instead of using θ_{t-1} to obtain $\tilde{p}_t$, $\tilde{p}_t^0$ is obtained from a regression of p_t on some subset θ_{1t-1} of θ_{t-1}, so that

$$p_t = \delta_0\theta_{1t-1} + e_{0t} = \tilde{p}_t^0 + e_{0t},$$

where $\hat{\delta}_0$ is a vector of least squares coefficients conformable to θ_{1t-1} and e_{0t} is a least squares residual. But individuals' expectations really equal the $\tilde{p}_t$ of the text. Then

$$\tilde{p}_t = \tilde{p}_t^0 + e_{0t} - e_t,$$

so that (24) can be written

$$(b)\quad Un_t = \sum_{i=1}^{q} \lambda_i Un_{t-1} + \beta(p_t - \tilde{p}_t^0) + (1-\alpha)\beta(\tilde{p}_t^0 - p_{t-1}) + u_t - \alpha\beta(e_{0t} - e_t).$$

So long as θ_{1t-1} includes the constant, Un_{t-1} through Un_{t-q}, and p_{t-1}, $e_{0t} - e_t$ is orthogonal to all arguments of (b) except $p_t - \tilde{p}_t^0$. It can readily be shown that using $\tilde{p}_t^0$ rather than $\tilde{p}_t$ leads to statistical inconsistency only in the estimate of β, and in particular that its use does not produce an inconsistent estimate of $(1-\alpha)\beta$, the parameter that must be estimated to test the natural rate hypothesis.

37. Theil's specification theorem is being invoked here. See Henri Theil, *Principles of Econometrics* (Wiley, 1971), pp. 548–49.

where f_t is a least squares residual in the first-stage regression used to form $\hat{p}_t$; f_t is orthogonal to $\hat{p}_t$, $\tilde{p}_t$, p_{t-1}, and the lagged *Un*s, so long as p_{t-1}, the lagged *Un*s, and all the "first-stage" variables used to obtain $\tilde{p}_t$ are used in the first stage to obtain $\hat{p}_t$. Since u_t and p_t are expected to be positively correlated, and since the *u*s, the *f*s, and $(p - \tilde{p})$s are orthogonal to the other regressors in (24′), estimating (24) rather than (24′) should produce an estimate of β that is biased upward in large samples.

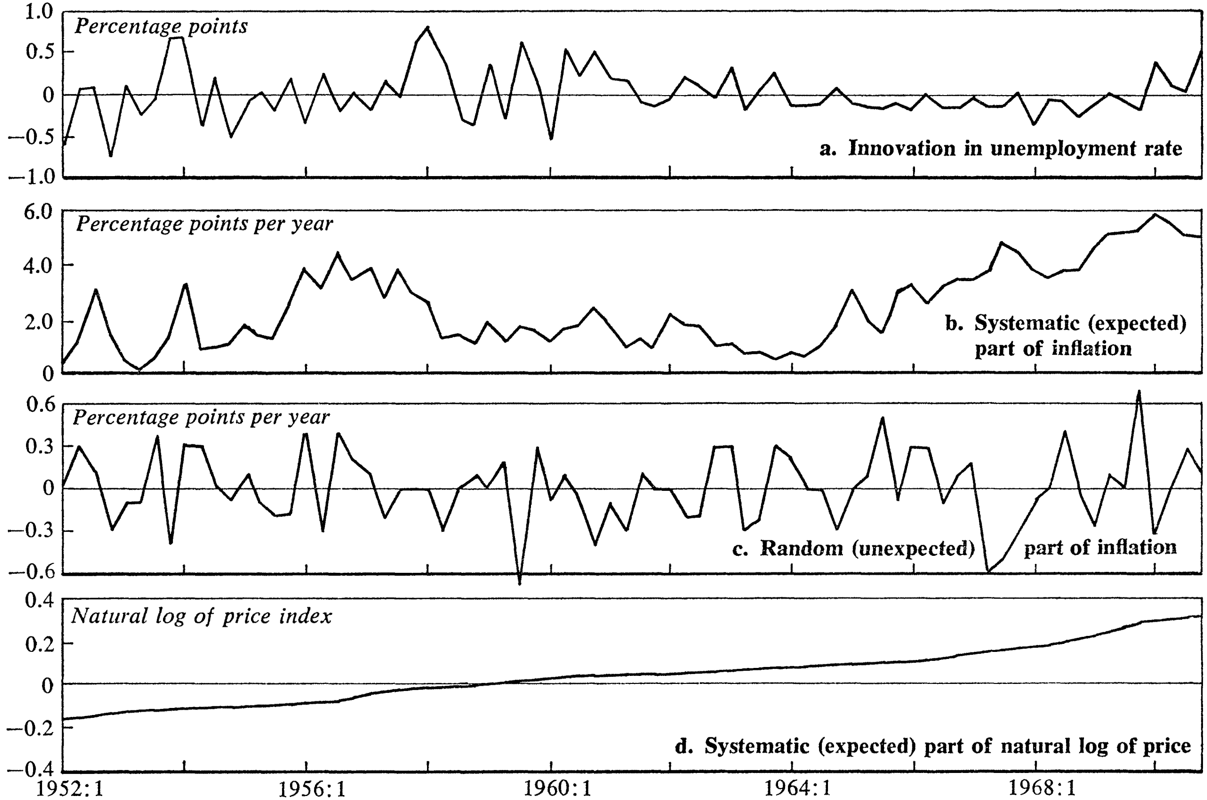
In summary, my strategy is to decompose the rate of inflation into two parts: a systematic part that is predictable from variables known in the past, and a random part that cannot be predicted from past data. The natural unemployment rate hypothesis permits the random part of the log of the price level (which equals the random part of the rate of inflation) to have an effect on the unemployment rate, but denies that the systematic part of the rate of inflation can affect unemployment. That hypothesis can be tested by regressing the unemployment rate against lagged values of itself and the random and systematic parts of the rate of inflation.

Table 1 reports the results of applying the test to quarterly data for the United States over the period 1952:1–1970:4. Two measures of the price level were used: the logarithm of the GNP deflator (p), and the log of a straight-time wage index in manufacturing (w).[38] Regressions (4.1) and (4.2) are estimates of equations (24) and (24′) for p, while regressions (5.1) and (5.2) are estimates of the same two equations using w as the index of the price level.

The data that form the raw material for these regressions are plotted in Figures 1 and 2. In each figure, panel (a) depicts the estimated innovation in the unemployment rate—that is, the residual in a regression of the unemployment rate against a constant and three lagged values of itself. The natural unemployment rate hypothesis permits this innovation to be inversely related to the random or unexpected part of the current price level, but denies that it is related to the systematic or expected part of the price level or rate of inflation. Panels (b) depict $\tilde{p} - p_{-1}$ and $\tilde{w} - w_{-1}$, respectively, and panels (c) report the unexpected parts, $\hat{p} - \tilde{p}$ and $\hat{w} - \tilde{w}$. Panels (d) report the paths of $\tilde{p}$ and $\tilde{w}$. What is claimed for these numbers

38. To form $\tilde{p}$ or $\tilde{w}$, p or w was regressed against a constant, time, and three lagged values each of p and w, as well as three lagged values each of *pc*, *g*, *Def*, *m*, *y*, *ng*, *Un*, and *wr*, where each of these variables is defined as in regression (3). To obtain $\hat{p}$ or $\hat{w}$, p or w was regressed against all of the variables just listed and also the current values of *g*, *ng*, *m*, and *Def*.

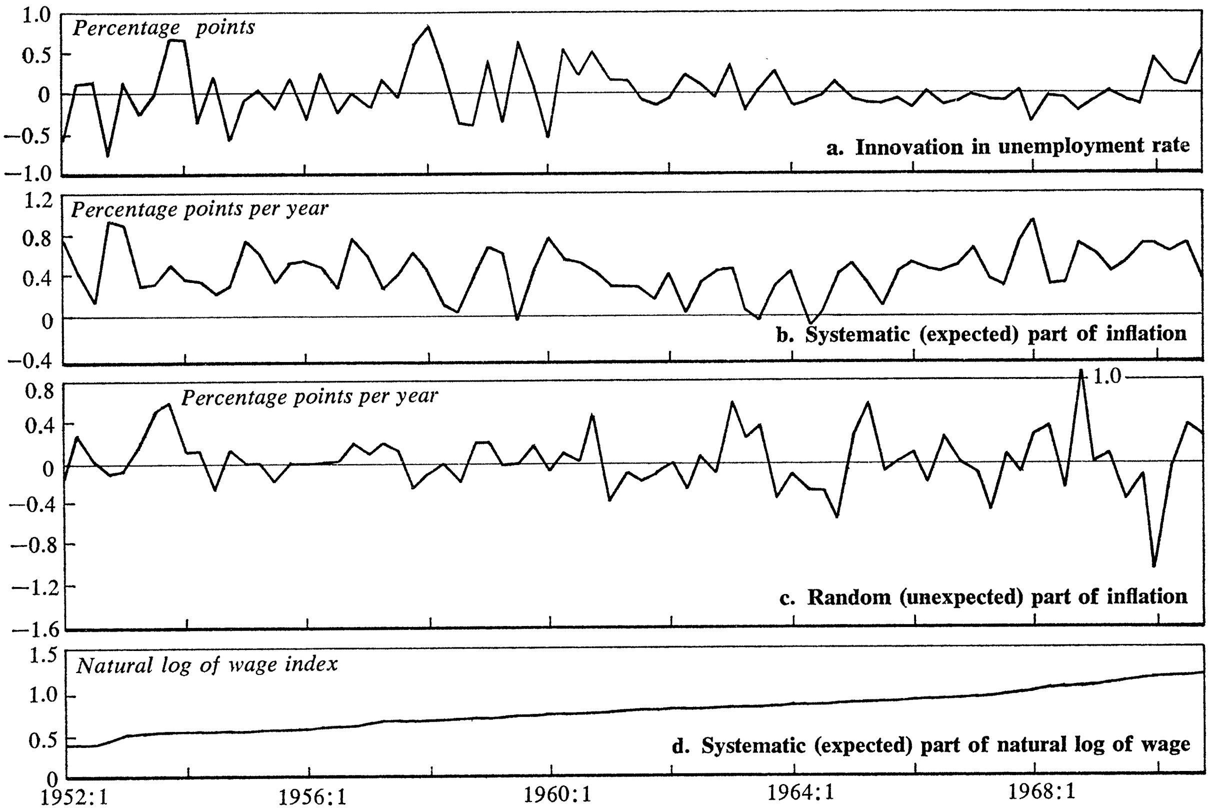
Figure 1. Innovation in the Unemployment Rate,[a] and Decomposition of Inflation into Systematic and Random Parts, Using the GNP Deflator to Measure Inflation



Source: Derived from the relevant official U.S. series from the data bank for the Wharton Econometric Model. Here and in Figure 2, equal vertical distances on various panels do not necessarily signify equal changes.

a. Innovation in the unemployment rate is the residual in a regression of the unemployment rate against a constant and three lagged values of itself.

Figure 2. Innovation in the Unemployment Rate,[a] and Decomposition of Inflation into Systematic and Random Parts, Using the Manufacturing Wage Index to Measure Inflation



Source: Same as Figure 1.
a. See Figure 1, note a.

Table 1. Regression Results for Alternative Tests of the Natural Unemployment Rate Hypothesis[a]

Variable and regression statistic	*Regression*			
	4.1	*4.2*	*5.1*	*5.2*
Variable				
Constant	0.2380	0.2380	0.694	0.694
	(2.172)		(0.216)	
Unemployment rate lagged one quarter, Un_{-1}	1.717	1.717	1.667	1.667
	(0.116)		(0.116)	
Unemployment rate lagged two quarters, Un_{-2}	−1.029	−1.029	−0.999	−0.999
	(0.199)		(0.196)	
Unemployment rate lagged three quarters, Un_{-3}	0.246	0.246	0.216	0.216
	(0.115)		(0.114)	
Random (unexpected) part of inflation, based on the GNP deflator, p, or the wage index, w				
$p - \tilde{p}$	−8.694	...	...	...
	(19.656)			
$\hat{p} - \tilde{p}$	...	−75.156	...	...
		(68.672)		
$w - \tilde{w}$	...	...	−11.884	...
			(10.053)	
$\hat{w} - \tilde{w}$	...	...	...	−50.500
				(52.109)
Systematic (expected) part of inflation				
$\tilde{p} - p_{-1}$	14.085	14.085	...	...
	(11.130)	(11.903)		
$\tilde{w} - w_{-1}$	...	...	−13.215	−13.215
			(6.925)	(7.634)
Regression statistic				
$\bar{R}^2$	0.924	...	0.928	...
Durbin-Watson statistic	1.980	...	1.952	...
$\hat{\alpha}$	2.62	1.19	0.11	0.74
t-statistic	1.27	1.19	−1.91	−1.73

Source: Derived from equations (24) and (24′), using relevant official U.S. series from the data bank of the Wharton Econometric Model.

a. The period of fit is 1952:1–1970:4. The dependent variable is the unemployment rate. Standard errors are in parentheses. The standard errors of coefficients for regressions (4.2) and (5.2) are asymptotic. For detailed definitions of symbols see discussion in text.

is that, on my assumptions, they represent appropriate decompositions of $\hat{w}$, $\hat{p}$, $\hat{w} - w_{-1}$, and $\hat{p} - p_{-1}$ for the purpose of estimating equation (24′).

Regressions (4.1), (4.2), (5.1), and (5.2) test the natural rate hypothesis against the alternative hypothesis that the systematic part of the rate of inflation affects the unemployment rate. The coefficient on $\tilde{p}_t - p_{t-1}$ or $\tilde{w}_t - w_{t-1}$ estimates $\beta(1 - \alpha)$ and should equal zero on the natural rate hypothesis. The t-statistic reports the ratio of the coefficient on $\tilde{p}_t - p_{t-1}$ $(\tilde{w}_t - w_{t-1})$ to its standard error, and provides the basis for a statistical test of the null hypothesis. Regressions (5.1) and (5.2) come closest to supporting a rejection of the natural rate hypothesis. The t-statistic for regression (5.1) is -1.91 and is distributed according to the t-distribution with 70 degrees of freedom; its absolute value is thus slightly below the critical value of 1.99 for a two-tailed test at the 95 percent confidence level. However, for a one-tailed test, which is pertinent for testing the hypothesis $\alpha = 1$ against the alternative $\alpha < 1$, the critical value of the t-statistic is 1.66 at the 95 percent confidence level, so that the natural rate hypothesis can be rejected at that confidence level on a one-tailed test. The t-statistic for regression (5.2), being based on instrumental variable estimates, has only an asymptotic justification. Its absolute value is below the critical level for a normal variate of 1.96 for a two-tailed test at the 95 percent confidence level, but exceeds the critical value of 1.65 for a one-tailed test at this level. The regressions using w thus provide some evidence for rejecting the natural rate hypothesis, although not at an unusually high confidence level. On the other hand, the t-statistics for regressions (4.1) and (4.2) fail to support rejection of the hypothesis. The point estimates in regressions (5.1) and (5.2) indicate an inverse tradeoff ($\alpha < 1$) between unemployment and the expected change in w, a tradeoff consistent with a negatively sloped long-run Phillips curve. But the point estimates in regressions (4.1) and (4.2) indicate a direct tradeoff ($\alpha > 1$) between unemployment and the systematic part of inflation in the GNP deflator, and are thus not compatible with a negatively sloped long-run Phillips curve. Yet the short-run Phillips curve in regression (4.2) has the usual slope.

The coefficients on $\hat{p} - \tilde{p}$ and $\hat{w} - \tilde{w}$ in regressions (4.2) and (5.2) exceed in absolute value the coefficients on $p - \tilde{p}$ and $w - \tilde{w}$ in regressions (4.1) and (5.1), respectively. This is consistent with the argument that in large samples the least squares estimate of the coefficient on $p - \tilde{p}$ is biased upward due to simultaneous-equations bias.

The coefficients on all regressors except $p - \tilde{p}$ or $\hat{p} - \tilde{p}$ ($w - \tilde{w}$ or $\hat{w} - \tilde{w}$) are identical in the pairs of regressions (4.1) and (4.2) and (5.1) and (5.2). This is no accident but stems from the fact that by construction, $p - \tilde{p}$ and $\hat{p} - \tilde{p}$ ($w - \tilde{w}$ and $\hat{w} - \tilde{w}$) are each orthogonal to the remaining regressors, which are the same in these pairs of regressions. Consequently, the coefficients on those remaining regressors are the same whichever of these two "random" terms is included in the regression.

The magnitudes of the coefficients in regressions (4.2) and (5.2) support Lucas' notion that the surprise, or random, part of the rate of inflation has a much larger effect on the unemployment rate than does the systematic part. However, in each regression, the *t*-statistic for the coefficient on the surprise part of inflation indicates statistical insignificance. If anything, there seems to be less evidence for a stable relationship between unemployment and the surprise in inflation than between unemployment and expected inflation. The results suggest that it is difficult to isolate even a stable short-run tradeoff between inflation and unemployment in these data. Some evidence remains for an inverse tradeoff between the unemployment rate and the systematic part of the rate of inflation in the straight-time wage index, w, but it is not strong enough to reject the natural rate hypothesis at a very high confidence level. I imagine that that evidence would not be sufficiently compelling to persuade someone to abandon a strongly held prior belief in the natural rate hypothesis.

Conclusion

This paper has set out a macroeconomic model for which a version of Irving Fisher's theory about the relationship between interest rates and expected inflation is correct. The model turns out to be characterized by a number of properties that monetarists have attributed to the economy. Its structural equations themselves do not differ from those of the standard IS-LM-Phillips curve models used to rationalize Keynesian prescriptions for activist, countercyclical monetary and fiscal policies. In fact, the statics of the model with fixed or exogenous expectations about the price level are of the usual Keynesian variety.[39] Where the model does differ from standard implementations of the IS-LM-Phillips curve model is in the replacement of the usual assumption of fixed-weight, extrapolative or "adaptive" expectations by the assumption that expectations about future prices are rational and do not differ systematically from the predictions of the model. The result of this change in assumptions is to produce a model with the following implications:

39. There are models that, with exogenous expectations, display static properties that are very much more "monetarist" than those possessed by the model in this paper. An example is James Tobin's "A Dynamic Aggregative Model," *Journal of Political Economy*, Vol. 63 (February 1955), pp. 103–15.

1. The rate of output is independent of the systematic parts of both the money supply and fiscal policy variables.

2. The real rate of interest is independent of the systematic part of the money supply.

3. The monetary authority should not adopt a systematic policy of pegging the nominal interest rate at some fixed level over many periods. Such a policy would be very inflationary or deflationary, since strictly speaking, no equilibrium price level exists under it.

4. The distributed lag coefficients of money income behind money are variables, being dependent on, among other things, the money supply rule. Changes in the rule have the effect of altering the lag of money income behind money. More generally, the distributed lags in all of the reduced form equations change with changes in the rule governing any policy variable.

These four implications of the model are among the most prominent doctrines associated with the Chicago school. Furthermore, the model's assumption that expectations are rational and its stress on the distinction between the effects of random and systematic movements in the price level have long been important elements of macroeconomics at Chicago. For example, Milton Friedman has written:

> . . . it is argued that once it becomes widely recognized that prices are rising, the advantages [in terms of higher real output] . . . will disappear: escalator clauses or their economic equivalent will eliminate the stickiness of prices and wages and the greater stickiness of wages than of prices; strong unions will increase still further their wage demands to allow for price increases; and interest rates will rise to allow for the price rise. If the advantages are to be obtained, the rate of price rise will have to be accelerated and there is no stopping place short of runaway inflation. From this point of view, there may clearly be a major difference between the effects of a superficially similar price rise, according as it is an undesigned and largely unforeseen effect of such impersonal events as the discovery of gold, or a designed result of deliberative policy action by a public body.[40]

While the model described in this paper is consistent with a number of policy prescriptions associated with monetarism, or the Chicago school,

40. Milton Friedman, "The Supply of Money and Changes in Prices and Output," in *The Optimum Quantity of Money and Other Essays* (Aldine, 1969), p. 183; originally published in *The Relationship of Prices to Economic Stability and Growth*, Compendium of Papers Submitted by Panelists Appearing before the Joint Economic Committee, 85 Cong. 2 sess. (1958).

it does not embody the naive monetarism of textbooks, which requires either a vertical LM curve or a horizontal IS curve or a vertical short-run Phillips curve. On the contrary, the model requires only weak "sign" restrictions on the parameters of those three curves.

Given the empirical evidence of which I am aware, there is room for disagreement about the usefulness of the kind of model described in this paper. On the one hand, one test of the natural unemployment rate hypothesis above—which is the model's centerpiece—points to rejection of that hypothesis and seems to imply some scope for policy makers to influence the mean of the unemployment rate via a suitable policy rule. On the other hand, I am aware of no evidence that shows that any particular existing structural model embodying a specific alternative to the natural rate hypothesis can outperform it in predicting the course of the unemployment rate. Such evidence ought to be in hand before it is reasonable to believe that economists know enough to design policies that can affect the expected value of the unemployment rate.

APPENDIX A

Equilibrium of the Model with Rational Expectations

TO DETERMINE the equilibrium of the system formed by equations (1), (2), (3), and (8), I first solve (3) for r_t:

$$r_t = b^{-1} m_t - b^{-1} p_t - b^{-1} y_t - b^{-1} \eta_t.$$

Substituting the above equation and (8) into (2) yields

$$y_t = k_t + cb^{-1} m_t - cb^{-1} p_t - cb^{-1} y_t - cb^{-1} \eta_t - c \underset{t}{E} p_{t+1} + cp_t + \epsilon_t + dZ_t.$$

Solving this equation for $y_t - k_t$, and equating the result to the expression for $y_t - k_t$ derived by substituting (8) into (1), gives

$$\gamma p_t - \gamma \underset{t-1}{E} p_t + U_t = \frac{cb^{-1}}{1 + cb^{-1}} m_t + \frac{c(1 - b^{-1})}{(1 + cb^{-1})} p_t - \left(\frac{c}{1 + cb^{-1}}\right) \underset{t}{E} p_{t+1} - \frac{cb^{-1}}{1 + cb^{-1}} \eta_t + \left(\frac{1}{1 + cb^{-1}}\right) \epsilon_t - \left(\frac{cb^{-1}}{1 + cb^{-1}}\right) k_t + \frac{d}{1 + cb^{-1}} Z_t.$$

Solving this equation for p_t gives

$$\text{(A.1)} \quad p_t = B_0 \underset{t-1}{E} p_t + B_1 m_t + B_2 \underset{t}{E} p_{t+1} + B_3 \eta_t + B_4 \epsilon_t + B_5 U_t + B_6 k_t + B_7 Z_t,$$

where

$$B_0 = \left(\frac{\gamma}{\phi}\right)$$

$$B_1 = \frac{cb^{-1}}{1 + cb^{-1}}/\phi > 0$$

$$B_2 = -\frac{c}{1 + cb^{-1}}/\phi > 0$$

$$B_3 = -B_1 < 0$$

$$B_4 = \frac{1}{1 + cb^{-1}}/\phi > 0$$

$$B_5 = -\frac{1}{\phi} < 0$$

$$B_6 = B_3 < 0$$

$$B_7 = dB_4$$

$$\phi = \gamma - \frac{c(1 - b^{-1})}{1 + cb^{-1}} > 0.$$

To simplify the notation, I define the (5×1) vector

$$\mu_t = [B_3 \eta_t, B_4 \epsilon_t, B_5 U_t, B_6 k_t, B_7 Z_t]'.$$

Defining I as the 1×5 vector $[1, 1, 1, 1, 1]$, (A.1) can be written as

$$\text{(A.2)} \qquad p_t = B_0 \underset{t-1}{E} p_t + B_1 m_t + B_2 \underset{t}{E} p_{t+1} + I\mu_t.$$

To derive an expression for $\underset{t}{E} p_{t+1}$, shift equation (A.2) forward one period and take expectations conditional on information available at time t:

$$\text{(A.3)} \quad \underset{t}{E} p_{t+1} = \frac{B_1}{1 - B_0} \underset{t}{E} m_{t+1} + \frac{B_2}{1 - B_0} \underset{t}{E} p_{t+2} + \frac{1}{1 - B_0} I \underset{t}{E} \mu_{t+1}.$$

More generally, for any $j \geq 1$,

$$\text{(A.4)} \quad \underset{t}{E} p_{t+j} = \frac{B_1}{1 - B_0} \underset{t}{E} m_{t+j} + \frac{B_2}{1 - B_0} \underset{t}{E} p_{t+j+1} + \frac{1}{1 - B_0} I \underset{t}{E} \mu_{t+j}.$$

Repeatedly substituting (A.4) into (A.3) yields the following expression for $\underset{t}{E} p_{t+1}$:

$$\text{(A.5)}\quad \underset{t}{E}p_{t+1} = \frac{B_1}{1-B_0}\sum_{j=1}^{\infty}\delta^{j-1}\,\underset{t}{E}m_{t+j} + \frac{1}{1-B_0}\sum_{j=1}^{\infty}\delta^{j-1}\,I\,\underset{t}{E}\mu_{t+j},$$

where

$$\delta = \frac{B_2}{1-B_0} = \frac{\dfrac{-c}{1+cb^{-1}}/\phi}{1-\dfrac{\gamma}{\phi}}$$

or

$$0 < \delta = \frac{1}{1-b^{-1}} < 1.$$

Here I am imposing the terminal condition

$$\lim_{j\to\infty}\delta^{j-1}\,\underset{t}{E}p_{t+j} = 0,$$

which rules out speculative bubbles. Equation (A.5) states that under rationality, the currently held expectation of the price level for next period depends on current expectations about the whole future course of the money supply, as well as that of the vector μ, which includes as components U, ϵ, η, k, and Z. Notice that as long as $b < 0$, the parameter δ is between zero and unity, which permits the infinite sums in (12) to converge.

To make (A.5) operational, I must specify how the expectations of future m and μ are formed. I do this by positing that m and μ are governed by autoregressive processes known to the public, and that the public properly takes into account the nature of those processes in forecasting the variables. For example, the money supply is assumed to be governed by the known feedback rule

$$\text{(A.6)}\quad m_{t+1} = \sum_{i=0}^{\infty} w_i\, m_{t-i} + \sum_{i=0}^{\infty} v_i\, \mu_{t-i} + \xi_{mt+1},$$

where ξ_{mt} is a serially uncorrelated random term that is normally distributed with mean zero, while the w_is are fixed parameters and each v_i is a 1×5 vector of parameters, $i = 0, \ldots, \infty$. The random variable ξ_m denotes the component of the money supply that cannot be predicted on the basis of past ms or μs (the part perhaps attributable to discretion). The (5×1) vector μ is assumed to be governed by the autoregressive process

$$\text{(A.7)}\quad \mu_{t+1} = \sum_{i=0}^{\infty} X_i\, \mu_{t-i} + \psi_{t+1},$$

where each X_i is a diagonal (5×5) matrix of parameters and ψ_{t+1} is a

(5×1) vector of mutually uncorrelated, serially uncorrelated, normally distributed random variables with means zero.

Given (A.6) and (A.7), Wold's "chain rule of forecasting"[1] can be used to give the expected value of m_{t+j} for any j, conditional on information available at t. These forecasts have the form

$$\text{(A.8)} \qquad \underset{t}{E} m_{t+j} = \sum_{i=0}^{\infty} w_{ji}\, m_{t-i} + \sum_{i=0}^{\infty} X_{ji}\, \mu_{t-i},$$

where the w_{ji}s are known functions of the w_is of (A.6), and the (1×5) X_{ji}s are known functions of the v_is of (A.6) and the X_is of (A.7).

Using (A.8), the first term on the right side of equation (A.5) becomes

$$\frac{B_1}{1 - B_0} \sum_{j=1}^{\infty} \delta^{j-1} \left(\sum_{i=0}^{\infty} w_{ji}\, m_{t-i} + \sum_{i=0}^{\infty} X_{ji}\, \mu_{t-i} \right) = \sum_{i=0}^{\infty} W_i\, m_{t-i} + \sum_{i=0}^{\infty} V_i\, \mu_{t-i},$$

where

$$W_i = \frac{B_1}{1 - B_0} \sum_{j=1}^{\infty} \delta^{j-1}\, w_{ji},$$

$$V_i = \frac{B_1}{1 - B_0} \sum_{j=1}^{\infty} \delta^{j-1}\, X_{ji}, \qquad \text{a } (1 \times 5) \text{ matrix.}$$

Using this procedure, equation (A.5) can be rewritten to express $\underset{t}{E}\, p_{t+1}$ in terms of current and past values of m_t and μ_t:

$$\text{(A.9)} \qquad \underset{t}{E} p_{t+1} = \sum_{i=0}^{\infty} W_i\, m_{t-i} + \sum_{i=0}^{\infty} \tilde{V}_i\, \mu_{t-i} = W * m_t + \tilde{V} * \mu_t,$$

where

$$W * m_t \equiv \sum_{i=0}^{\infty} W_i\, m_{t-i}, \text{ and so on,}$$

and where each $\tilde{V}_i$ is a (1×5) matrix. Here the $\tilde{V}_i$s depend on the parameters of the monetary "rule" (A.6), the parameters of autoregressive processes that underlie (A.7), and the parameters of the economic structure, equations (1), (2), and (3). The W_is depend both on the model's structural parameters and on the parameters w_i of the monetary rule.

This expression for $\underset{t}{E} p_{t+1}$ can now be substituted into equation (A.2) to get the reduced form for the price level:

1. See Herman O. A. Wold (ed.), *Econometric Model Building: Essays on the Causal Chain Approach* (Amsterdam: North-Holland, 1964).

$$(A.10)\quad p_t = B_0\, W{*}m_{t-1} + B_0\, \tilde{V}{*}\mu_{t-1} + B_1\, m_t + I\, \mu_t + B_2\, W{*}m_t + B_2\, \tilde{V}{*}\mu_t.$$

Equation (A.10) is the reduced form equation that appears as equation (11) in the text, while equation (A.9) corresponds to the conditional expectation (13) in the text.

APPENDIX B

Modifications to the Model

A More Realistic Portfolio Sector

BUILDING IN a more realistic portfolio sector forces a modification of the proposition that the real rate of interest is independent of the systematic part of the money supply. Suppose the assumption that bonds and equities are viewed as perfect substitutes by owners of wealth were abandoned and replaced by separate demand schedules for equities and bonds of various maturities, perhaps assuming that the assets are gross substitutes.[1] It would then be standard to assume that the pertinent interest rate to enter in the aggregate demand schedule is the yield on equities.[2] The real yields on all of the paper assets would appear in each member of the set of equations describing the conditions for portfolio balance—that is, equations expressing equality of the stock demand for each paper asset with the quantity of each in existence. In such a system, it remains true that the real yield on equities that appears in the aggregate demand schedule is independent of the systematic part of the money supply. Systematic, predictable movements in the money supply are thus not able to influence the equity yield, which can be characterized as the "critical" yield from the point of view of affecting aggregate demand.[3] However, by conducting debt management

1. For example, see James Tobin, "A General Equilibrium Approach to Monetary Theory," *Journal of Money, Credit and Banking*, Vol. 1 (February 1969), pp. 15–29.

2. See ibid. Also see Franco Modigliani and Merton H. Miller, "The Cost of Capital, Corporation Finance and the Theory of Investment," *American Economic Review*, Vol. 48 (June 1958), pp. 261–97.

3. See Tobin, "General Equilibrium Approach."

or open market operations, the monetary authority can systematically influence both the relationships borne by the yields on other paper assets to the equity yield and the relationships among those other yields. In this way, debt management can have systematic effects on the yields of certain assets, whose strength depends on the extent to which wealth owners regard alternative paper assets as good substitutes for one another. In such a system, debt management operations might well permit the monetary authority to peg the nominal rate on, say, three-month Treasury bills. But that pegging would have no persistent effect on the critical yield on equities that governs aggregate demand.[4]

It might be useful to consider an additional change in the system that would further modify the second proposition, without, I believe, touching any of the policy implications of the model. Assume again the existence of various paper assets that are imperfect substitutes for one another; but abandon the notion that the real rate of return on one single asset, such as the yield on a certain class of equities, is the one crucial yield that belongs in the aggregate demand schedule, and instead, assume that aggregate demand depends on the real rates of return on all n assets, so that, instead of text equation (2), the aggregate demand schedule becomes

$$y_t - k_t = \sum_{i=1}^{n} c_i \, r_{it} + dZ_t + \epsilon_t, \tag{2'}$$

where $c_i < 0$ for all i, and r_{it} is the *real* rate of return on the ith paper asset. Define an index $\bar{r}_t$ of real yields as

$$\bar{r}_t \equiv \sum_{i=1}^{n} \frac{c_i}{\sum_{j=1}^{n} c_j} \, r_{it}.$$

Then notice that equation (2′) can be rewritten as

$$y_t - k_t = \left(\sum_{j=1}^{n} c_j\right) \bar{r}_t + dZ_t + \epsilon_t. \tag{2''}$$

4. The literature on the term structure of interest rates has in large part been devoted to attempting to detect evidence of imperfect substitutability among bonds of different maturities. For example, see Franco Modigliani and Richard Sutch, "Debt Management and the Term Structure of Interest Rates: An Empirical Analysis of Recent Experience," *Journal of Political Economy*, Vol. 75 (August 1967), Pt. 2, pp. 569–89, and David Meiselman, *The Term Structure of Interest Rates* (Prentice-Hall, 1962). Very little convincing evidence has been assembled that debt management has important effects on the yield curve.

In the system that is formed by replacing (2) with (2′) and replacing (3) as before with a system of portfolio equilibrium conditions for a set of assets, it is easily shown that the real yield index $\bar{r}_t$ is independent of the systematic part of the money supply. Moreover, this yield index certainly qualifies as the "crucial" yield affecting aggregate demand, if anything does. In such a system, debt management policies are able systematically to affect the relationships among the real rates that are components of $\bar{r}_t$, but that is irrelevant from the point of view of affecting $\bar{r}_t$ and aggregate output.

The Model with "Partly Rational" Expectations

One criticism that has been made of the kind of model presented here is that it seems to require extraordinary amounts of wisdom and information on the part of those whose expectations are described by equation (13).[5] They are assumed to act as if they know the probability distribution (12) and then use it together with data on all of the conditioning variables to form their expectation about next period's price level. While assuming such a well-informed public may or may not strain credulity, the key aspects of the theory carry through even if the public is much less wise and knowledgeable.

First, the orthogonality of the public's prediction errors to the set of variables on which its expectations are based applies when the public gets its knowledge of the conditional expectation of p_{t+1} as if it were simply computing a linear least squares regression of the price level on lagged values of the conditioning variables for the historical data available. The well-known properties of least squares prediction errors—in particular, their orthogonality to the regressors in the sample period—will guarantee that the prediction error in the aggregate supply schedule is uncorrelated with past values of the conditioning variables. That in turn implies that $y - k$ will be independent of lagged values of those conditioning variables.

Now to indicate the minimal amount of information and wisdom that must be imputed to the public in order to preserve the key policy implications of the model, assume for the moment that in forming its forecast of the price level, the public has access to information only about lagged

5. For example, see James Tobin's discussion of Lucas' paper in Tobin's "The Wage-Price Mechanism: Overview of the Conference," in *Econometrics of Price Determination*, p. 13.

prices (and by implication lagged values of its own forecasts). The public again is assumed to put this information together in such a way as to extract the best (least squares) forecast of p_t, so that

$$ {}_tp^*_{t-1} = E(p_t \mid p_{t-1}, p_{t-2}, \ldots) \equiv E[p_t \mid \theta_{lt-1}], $$

where θ_{lt-1} includes only past values of p_t.[6] Taking note of the fact that the variables $p_{t-1} - {}_{t-1}p^*_{t2}, p_{t-2} - {}_{t-2}p^*_{t-3}, \ldots$ form a subset of the variables in θ_{lt-1} conditioning the above expectation, I calculate the regression of the current prediction error on past prediction errors,

$$ E[(p_t - {}_tp^*_{t-1}) \mid p_{t-1} - {}_{t-1}p^*_{t-2}, p_{t-2} - {}_{t-2}p^*_{t-3}, \ldots] = 0, $$

which establishes that the prediction errors are serially uncorrelated.

Now combine the above hypothesis about expectations formation with the modified aggregate supply hypothesis (1′):

$$ y_t - k_t = \gamma(p_t - Ep_t \mid \theta_{lt-1}) + \sum_{i=1}^{q} \lambda_i(y_{t-i} - k_{t-i}) + U_t. $$

Taking expectations conditional on θ_{t-1}, I obtain

$$ E[(y_t - k_t) \mid \theta_{t-1}] = \gamma(Ep_t \mid \theta_{t-1} - Ep_t \mid \theta_{lt-1}) + \sum_{i=1}^{q} \lambda_i(y_{t-i} - k_{t-i}) + E(U_t \mid U_{t-1}, U_{t-2}, \ldots). $$

In general, the term $Ep_t \mid \theta_{t-1} - Ep_t \mid \theta_{lt-1}$ will not be zero: one obtains a better prediction of p_t by taking into account the components of θ_{t-1} that are excluded from θ_{lt-1}. Consequently, on the hypothesis that the public's expectation is conditioned only on past prices, the forecast error $p_t - Ep_t \mid \theta_{lt-1}$ that appears in the aggregate supply schedule is not in general independent of the elements of θ_{t-1} that are excluded from θ_{lt-1}. In particular, the forecast error is generally correlated with past values of the money supply. This means that by choosing the money supply rule (9) appropriately, the monetary authority can systematically influence the forecast errors that appear in the aggregate supply schedule. The systematic part of the money supply then has effects on both the rate of output and the real rate of interest, so that neither of the two propositions about the neutrality of the systematic part of the money supply continues to hold.

6. Changing the assumption about ${}_{t+1}p^*_t$ in this way will itself change the form of the probability distribution (12) that governs p_{t+1}, as can be seen easily by pursuing the kind of calculations reported in Appendix A. The arguments of (12) would remain the same, however.

But the potential accomplishments of stabilization policy are still severely circumscribed. While the monetary authority can have a systematic effect on the prediction errors in the aggregate supply schedule, there exists no feedback policy that is capable of inducing serial correlation in those forecast errors. So long as the forecasts are conditioned on at least lagged prices, the errors will be serially uncorrelated. The monetary authority's ability systematically to affect the public's forecast errors then comes down to an ability to affect the variance of those errors without being able to affect their mean or serial correlation properties. It follows that there are no feedback rules for the money supply and fiscal policy variables that can be expected to produce "runs" of forecast errors that will in themselves be a source of persistent movements in output. Under the assumptions here, then, the monetary and the fiscal authorities still face no "cruel choice" between the average rate of inflation they shoot for and the expected value of the unemployment rate. But there remains a nontrivial problem in choosing stabilization policies, for different deterministic feedback rules deliver different variances for the public's errors in forecasting the price level, and thereby are associated with different variances for the unemployment rate.

Comments and Discussion

David Fand: Thomas Sargent has written a very stimulating paper. It covers complex analytical issues, and I had to read it a number of times; but each time I read it, I learned some more.

According to Sargent, the statistical models applying Fisher's theory have generally assumed adaptive expectations, and have estimated unrealistically long lags. Moreover, they require a number of special assumptions about the LM curve or the IS curve, which make the empirical results hard to take seriously.

He therefore offers a different kind of model, which introduces a particular kind of endogenous expectations—so-called rational expectations. In such a world, Fisher's theory would hold and so would the natural rate hypothesis. Sargent then tests the model by trying to reject the natural rate hypothesis; since at least some of his tests do not reject it, he infers that Fisher's theory cannot be dismissed.

I would like to raise a few questions about the paper, more in elaboration than in criticism. First, Sargent's model salvages Fisher's theory, but does not really permit a direct test of it. Perhaps it is possible to obtain an indirect test by using Sargent's derived measure of expected inflation. Presumably, that could be subtracted from the nominal rate of interest, and the resulting series on the real rate could be examined to see whether it is constant or stable or at least varies plausibly over time.

Second, I suspect that Sargent exaggerates the dispute between those who believe that the effect of expected inflation on nominal interest rates is immediate and those who think that it will show up fully but only ultimately. I don't think that is where the key difference lies. The more important issue is between those who think the inflation effect on rates occurs within, say, a year and those who think it takes five years or more; between those who think it is fully reflected and those who think it need not ever be fully reflected in nominal interest rates.

I also wonder how the model of rational expectations would apply in a variety of situations or different types of worlds. For example, is the public supposed to view a devaluation, or a shift from fixed to floating rates, as a change in structure? If so, doesn't that introduce more possibilities? As another example, how could this model be applied to the kind of world James Tobin described in his presidential address to the American Economic Association? I would also have welcomed an elaboration of the effects of introducing a real-balance effect—a matter that Sargent mentions only briefly in a footnote. Also, while Sargent's model has several monetarist implications, it contrasts with other monetarist models in one respect, since they offer a theory of nominal income rather than of real income. That may be an important distinction and it deserves some discussion.

My final comments concern Sargent's statistical tests. In his regressions, unemployment, as an independent variable, is used as a proxy for the deviation of actual output from trend output. With due respect for Okun's law, I would note that the short-run relationship between unemployment and output is not perfect, and it would be interesting to see whether using a direct measure of the output gap would alter the results of Sargent's tests. Also, Sargent sets up one test with an autoregression of order 3 in the unemployment rate. Then he uses the "innovation"—that is, the error in the autoregression—to try to explain inflation. The autoregression of order 3 seems arbitrary; it does not rest on the model which, so far as I can see, points only to an autoregression of order 1. Moreover, once an autoregression of order 3 is used, it will be very hard for the error to explain anything and very difficult for that term to show the statistical significance that in turn would refute the natural rate hypothesis.

Finally, even after Sargent's ingenious efforts, I am not convinced that Fisher's theory and the natural rate hypothesis are an inseparable package. I still think there is scope for a paper that tries to test Fisher's theory separately and directly, since I think more people would be inclined to accept the Fisher theory than the natural rate hypothesis.

Stephen Goldfeld: I, too, learned a lot from Sargent's paper. Essentially, he demonstrates two kinds of propositions. First, most previous conventional tests of the Fisher hypothesis have suffered from a number of specification problems, even in a world of adaptive expectations. Second, in a world of rational expectations, the Fisher theory can be reinterpreted so that it does hold and the natural rate hypothesis will also hold.

The paper contains a very able technical discussion of the econometric problems of testing such a model. Indeed, with its footnotes, it is almost encyclopedic in anticipating the technical comments one would make about the particular tests Sargent applies.

I would like to raise three kinds of questions. First, should one believe the assumptions underlying the model? And, if one modifies them, how robust are the theoretical conclusions? Second, are the empirical tests up to the high theoretical standards set for them, and is the interpretation of those tests acceptable? Third, what, if anything, does the paper tell us about short-run policy for the real world?

With respect to the assumptions underlying the model, I am puzzled by several features, although none of the issues seems terribly important. Why does productive capacity (k_t) appear in the demand equation? Why does the interest rate variable appear in linear form, when all other variables are logarithmic? Why is the income elasticity of the demand for money assumed to be precisely unity? Does the absence of a money supply mechanism that responds to interest rates alter the basic story? Similarly, would the introduction of a government budget constraint change the story? I don't know whether the presence or absence of these features makes a difference in this context, but in some cases it may.

The heart of Sargent's paper is the model based on rational expectations, in which the public knows essentially everything about the models and monetary policy follows a money supply rule. From that model Sargent derives two principal theoretical conclusions: (1) the unemployment rate is uninfluenced by the money supply rule, and (2) the expected real rate of interest is similarly uninfluenced by the money supply rule. How robust are these propositions to the various assumptions?

As Sargent recognizes, the real interest rate, or Fisher proposition, depends critically on the absence of a real-balance effect. That is an assumption about which tastes differ among economists, and its flavor is nonmonetarist in an otherwise monetarist type of model. More seriously, it is very strongly restrictive to assume that the public really knows the model and its parameters. Sargent sees a way out of this tight spot, however. Under some assumptions, the public need merely estimate relevant things by making an unbiased forecast. The expected values can be validly replaced by their least squares forecasts. But that validity depends on the linearity of the model, and would not apply in a model that had nonlinearities. So, softening the assumption of complete knowledge really requires adding the assumption of linearity. Thus, a number of aspects and assumptions of the model are crucial, even on a theoretical level, to the conclusions that the model generates. And one may not find all those assumptions congenial.

As to the empirical tests, basically they consist of regressions of the unemployment rate on its own lagged values and on a number of other variables; under the maintained natural rate hypothesis, the set of other variables should have no effect on the unemployment rate. The first two tests differ in the choice of variables included in the set that is not supposed to influence the unemployment rate. One test—using just wages and prices—tends to confirm the natural rate story. A second, related, test, with a longer list of possible explanatory variables, tends to reject the natural rate hypothesis. Instrumental variables are used to produce another pair of tests; the third test—using expected and unexpected price movements—is consistent with the hypothesis, while the fourth, using wages, rejects it.

Sargent seems to play down the implications of his second test, by stressing that the equation that seems to reject the natural rate hypothesis provides no comfort to any alternative theory of aggregate supply. According to his argument, any alternative supply theory would impose restrictions on the equation he actually estimates; therefore, the fit would deteriorate and hence the alternative theory would not necessarily do as well in terms of explanatory power as the equation in the paper. All of that is true. But it has to be interpreted with several caveats. An alternative theory would impose restrictions, but it might have fewer variables and hence leave a larger number of degrees of freedom. The loss of explanatory power would not necessarily mean an alternative theory would lose out. On the other hand and even more important, an alternative theory might introduce new variables and thus might make the whole game different.

On the tests themselves, I share David Fand's concern about the number of lags included. Basically, the number of lags should reflect the assumed specification of the supply side of the economy. No strong justification is—or probably can be—offered for any particular number of lags. I am not sure how that affects the tests. For example, Fand conjectures that it would become easier to reject the natural rate hypothesis if no lags or only one lag were included in the unemployment rate. I do not know what the right answer is, but I would like to know the sensitivity of that test to the number of unemployment lags in the equation.

On a related matter, Sargent discusses carefully what happens in the presence of autocorrelation. The appropriate form of the test is slightly different in that case. He actually raises the specter of autocorrelation in questioning the validity of the test that refutes the natural rate hypothesis. It seems appropriate to examine the effects on the results of estimating an allowance for autocorrelation and of interacting autocorrelation and lags. Finally, the tests presented in the paper are designed as large sample tests and yet they must be conducted with a fairly small number of time series observations.

So, on balance, it seems to me that the tests are not quite as careful or as widely encompassing in their examination of the sensitivity of the results as the standards for testing set forth in the theoretical discussion of the paper. There seems to be a lot of room for further testing along those lines, within the spirit of raising the standards.

The third set of questions in my mind relates to the policy implications. There are some obvious long-run implications if the natural rate and Fisher hypotheses are right. But I have difficulty assessing the short-run implications. A variety of questions arises. For example, how much room is there, even in this kind of world, for policy strategies to minimize variance? And how does that answer depend on various parameters of the model? What kind of short-run dynamics does this model generate? How are rational expectations supposed to be formed in this world if the government imposes an incomes policy (as well as a change in exchange rates or rules, such as Fand mentioned)?

In general, I would have been happier if some of the questions about the short-run implications had been drawn more finely. Nevertheless, I found Sargent's paper quite interesting and provocative.

General Discussion

James Tobin probed more deeply into the particular assumptions required to produce the conclusions of Sargent's theoretical model. He focused on a model of a simple world in which money and capital were the only assets and in which expectations were rational and money was neutral in the sense that doubling the money supply would merely double the price level and leave all real variables unchanged. Nonetheless, if that economy was observed under two situations—with the growth rate of money zero in the first and 10 percent in the second—the real rate of interest would not be the same in the two. If in the second situation the price level rose by 10 percent a year and everyone expected that, then the real return on money would be minus 10 percent, and the resulting diminution in the demand for real cash balances would encourage capital formation and lower the real rate of interest. After some discussion, Franco Modigliani, Sargent, and Tobin agreed that the Tobin example was not "super-neutral"—that is, the growth rate of the money supply (although not its level) would affect real magnitudes. In a sense, Tobin's example revealed to the group that super-neutrality is a highly restrictive assumption.

The nature of the aggregate supply relationship in Sargent's model was subjected to several critical comments. Modigliani emphasized the importance of Sargent's assumption that output differs from its trend value only because suppliers are "fooled" about prices. That assumption assures that a model will have a "natural" rate of unemployment—that its long-run Phillips curve will be vertical. Robert J. Gordon was very skeptical that output fluctuations in the real world could be linked to errors in price predictions. For seven consecutive years, between 1958 and 1964, output was below trend (the unemployment rate was above anyone's estimate of the natural rate). Gordon thought it highly unlikely that prices consistently ran below expected prices during this period, especially since the actual price level was creeping up very steadily. Nor could he believe that prices were lower than expected in the 1970 recession, when no economist had predicted price increases as large as those that actually occurred. Responding to Gordon, William Poole noted that Sargent's model permitted output adjustments to be spread out over long periods of time, and that the lags could produce cyclical patterns and sustained periods of underutilization even when price anticipations were fully realized.

Gordon also felt that the downward inflexibility of the overall price level made a rational expectations model inappropriate in explaining anticipated movements. While that model has passed a number of statistical tests in the context of commodity and security markets, upward and downward movements of prices are essentially symmetrical in those markets, whereas they are not symmetrical for aggregate price behavior. Richard Freeman questioned Sargent's justification for the aggregate supply schedule: that suppliers sell more as a result of higher prices because they get information more rapidly about the prices of goods they sell than about the aggregate price level. Presumably the relevant comparison would be with items they buy rather than with any overall index. Freeman saw no reason to believe that people or firms take more time to perceive changes in buying prices as opposed to selling prices.

The hypothesis that expectations are formed rationally was also discussed at some length. Thomas Juster stressed the cost of forming good expectations. Different types of economic agents have different incentives that determine how much effort and expense to put into the formulation of forecasts. To Juster, it seemed plausible that a small class of actors on the economic scene would behave in accordance with the rational expectations model, but he doubted that this class would include most business firms or virtually any households. Arthur Okun suggested that the issue might be whether enough participants had the incentive to arbitrage to make rationality dominate in markets. Gordon noted that even the forecasts of experts do not seem to be unbiased and serially uncorrelated in the way that Sargent's model requires. For example, economists consistently underpredicted prices for years after the Second World War; similarly, businessmen's errors in anticipations of their own investments display a clear cyclical pattern.

Poole felt, however, that the rational expectations model was a major improvement over the highly subjective initial Keynesian formulation of expectations and the subsequent model of adaptive expectations that assumes that views of the future are formed purely by naive inspection of the past. He urged Sargent to take explicit account of the accumulation of knowledge. Rational expectations mean that the public makes use of all the information and knowledge available at any one moment. But the amounts change over time. People could not have predicted economic variables the way they now do back at the turn of the century, any more than the Wright brothers could have flown to the moon. Accordingly, Poole raised the possibility that the policy makers may have more rapid access to knowledge and fuller information about some aspects of the economy than would private economic agents. Hence, under some circumstances, they may be able to take actions that benefit the nation.

Modigliani agreed with Fand's conjecture that the Fisher proposition might hold in a world that did not conform to the hypothesis of a natural unemployment rate. He pointed out that the FRB-MIT-Penn (FMP) econometric model maintains the Fisher proposition in the long run, while it contains a long-run Phillips curve that is not vertical. He emphasized that some tradeoff between inflation and unemployment could persist in the long run even in a world of rational expectations, reflecting the existence of vacancies and other features that depart from pure market clearing. Modigliani also suggested that, if it takes a very long time for the Phillips curve to become vertical, then the natural rate hypothesis may be just an intellectual curiosity. Sargent took issue with that view; he insisted that, whether it was right or wrong, the long-run natural rate thesis has immediate relevance because it says something important about the impact of systematic and predictable changes on the economic system.

A number of participants focused on Sargent's empirical work. Modigliani wondered whether Sargent's tests were capable of distinguishing between a Phillips curve that was truly vertical and one that became very steep after a moderate period. Gordon suspected that the particular wage index used by Sargent might cause problems since it was not corrected for interindustry shifts and hence tended to drop more during recessions than would a more accurately weighted wage measure. Poole found it interesting to compare Sargent's empirical findings with those in a 1972 article by Charles Nelson.[1] Nelson had evaluated the predictive performance of the FMP model against that of a simple autoregressive scheme, and found that the autoregressive model was better for estimating unemployment but inferior in tracking prices. Those findings accord with the implications of Sargent's model that structural variables should help to improve predictions of prices but not of unemployment.

Sargent responded to Fand's question about why his measure of the expected inflation rate should not be subtracted from observed short-term nominal rates of interest to derive an estimate of the real rate of interest over time. Sargent argued that this procedure would be valid only if the expected inflation rate had been derived from a complete econometric model. Moreover, Sargent doubted that such measures of real rates of interest could answer any relevant questions about the world that would not be illuminated by the market yields on equities or other real assets. But Modigliani and Alan Greenspan cautioned that observations of the stock market do not provide a meaningful estimate of the expected rate of return on equities. Greenspan felt that an interest rate series corrected for expected inflation would provide valuable information to the economic analyst.

1. Charles R. Nelson, "The Prediction Performance of the FRB-MIT-Penn Model of the U.S. Economy," *American Economic Review*, Vol. 62 (December 1972), pp. 902–17.