Deciphering the fall and rise in the net capital share

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Abstract

In the postwar era, developed economies have experienced two substantial trends in the net capital share of aggregate income: a rise during the last several decades, which is well-known, and a fall of comparable magnitude that continued until the 1970s, which is less well-known. Overall, the net capital share has increased since 1948, but when disaggregated this increase comes entirely from the housing sector: the contribution to net capital income from all other sectors has been zero or slightly negative, as the fall and rise have offset each other. When decomposed into a return on fixed assets and a residual share of pure profits, the fall and rise of capital income outside the housing sector in the US owes mostly to the residual: it is not paralleled by fluctuations in the measured value of non-housing capital. This observation—combined with the theory of factor substitution, and simulation results from a multi-sector model—casts doubt on explanations of changes in the net capital share that rely on changes in the value of capital. There is greater support in the data for narratives that emphasize cyclical and trend variation in market power.

1 Introduction

How is aggregate income split between labor and capital? Ever since Ricardo (1821) pronounced it the “principal problem of Political Economy,” this question of distribution has puzzled and inspired economists. One of the key issues is empirical: what is the current split between factors, and how has it evolved over the years?

Views differ. In one popular interpretation, the division between labor and capital remains remarkably stable over time: Keynes (1939) called this “one of the most surprising,

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yet best-established, facts in the whole range of economic statistics,” and Kaldor (1957) immortalized it as one of the stylized facts of economic growth. In contrast, another tradition has emphasized variation in income shares: Solow (1958) was famously skeptical, disputing the labor share’s status as “one of the great constants of nature.” Recently, the latter view has experienced a resurgence, as the labor share has apparently trended downward. Elsby, Hobijn and Sahin (2013) carefully document this decline for the US, and Karabarbounis and Neiman (2014b) describe a broad, worldwide retreat of labor income.

At the same time, there has been an explosion of research into economic inequality, with a particular focus on the share of income accruing to the richest individuals. As Atkinson, Piketty and Saez (2011) explain, top income shares fell substantially in the first half of the twentieth century, owing mainly to a decline in capital income received by the richest. In the past thirty years, top incomes have mounted a recovery in English-speaking countries—principally as the result of rising concentration in labor income, but with increasing speculation that a resurgence in capital will also play a role. This case is made most forcefully by Piketty (2014), who hypothesizes that slower growth will lead to expansion in both the magnitude and concentration of capital relative to aggregate income.

There is an substantial overlap between these two research areas: the macroeconomic division of income between labor and capital influences the extent to which capital income, which tends to be highly concentrated, can contribute to inequality. Supposing, for instance, that Piketty (2014)’s projection of a rising ratio of capital to aggregate income is ultimately realized, a key question is whether the capital share of income will rise as well. If so, then the concentration of income may indeed increase; if not, then the rate of return on capital will fall substantially, and capital owners’ claim on aggregate output will be no larger than before.

This makes it important to understand the recent behavior of the labor and capital shares of income. Much of the discussion focuses on the apparent fall in the labor share—and rise in the capital share—over the last few decades. There are several leading narratives of this decline. Karabarbounis and Neiman (2014b) connect the global fall in the labor share to a coinciding fall in the relative price of investment goods: if capital and labor are substitutable enough, then cheaper capital implies that its share in production will rise. Piketty (2014) and Piketty and Zucman (2014), in contrast, argue that a rise in the quantity of capital from the increasing accumulation of savings has pushed up its share; although this hypothesis is distinct from that of Karabarbounis and Neiman (2014b), it also requires that capital and labor are sufficiently substitutable. Elsby et al. (2013) high-
light the role of offshoring, while other papers emphasize additional structural and institutional forces.\(^1\)

Relative to the existing literature, this paper makes several contributions. First, it argues that as a purely descriptive matter, the recent behavior of income shares is widely misunderstood: rather than experiencing a steady rise, the net capital share for large developed economies has followed a U-shaped trajectory in the postwar era, and its long-term expansion originates entirely in the housing sector. Although both gross and net income shares can be meaningful, net concepts are more relevant to the debate on inequality. Second, this paper observes that when net capital income outside the housing sector is disaggregated between the return on fixed assets and a residual share of pure profits, the U-shaped trajectory is driven mainly by the residual—calling the mechanisms in Karabarbounis and Neiman (2014b) and Piketty and Zucman (2014), which rely on changes in the aggregate value of capital, into question. Finally, it describes the theory of factor shares and the role of elasticities of substitution, evaluating various hypotheses by performing counterfactual simulations in a multisector model.

The paper starts by looking at the data, and progressively introduces more theory to interpret its salient features. Section 2 starts by describing the conceptual issues involved in decomposing income into labor and capital shares. It argues that when interpreted properly, both gross and net measures are worthwhile, but the net viewpoint—much rarer among recent entries in the literature—is more directly applicable to the discussion of distribution and inequality, because it reflects the resources that individuals are ultimately able to consume.\(^2\) It also discusses the imputations and choices that are subsequently used to measure income shares in a 1948–2010 panel of the G7 advanced economies.

This measurement reveals a striking discrepancy in the long-term behavior of gross and net shares, echoing the claims of Bridgman (2014); and it shows that the net capital share generally fell from the beginning of the sample through the mid-1970s, at which point the trends reversed. In the long run, there is a moderate increase in the aggregate net capital share, but this owes entirely to the housing sector—where, since ownership is less concentrated, the consequences for inequality may be less severe. In the shorter run, there is both the U-shaped pattern in capital income and a strong cyclical element.

Section 3 uses a simple theoretically-motivated decomposition to analyze the trends in net capital share further, restricting itself to the US to make use of more detailed data. It starts by disaggregating net capital income in the corporate sector into components that

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\(^1\)See, for instance, Azmat, Manning and Reenen (2012), who address the role of privatization, and Arpaia, Perez and Pichelmann (2009), who draw attention to capital-skill complementarity.

\(^2\)As Baker (2010) remarks, “you can’t eat depreciation”.
reflect the return on fixed assets—equipment, structures, and land—as well as a residual term reflecting “pure” profits. Although the decomposition between return on assets and this residual can be somewhat ambiguous, both crude and more sophisticated approaches indicate that the residual plays a central role. To some extent, this residual reflects cyclical variation in markups, but its long-term pattern—which parallels the U-shaped pattern in capital income as a whole—may hint at broader changes in market power and the importance of monopoly profits.

In contrast to the narratives in Karabarbounis and Neiman (2014b) and Piketty and Zucman (2014), changes in the net capital share do not appear to be driven by contemporaneous changes in the value of measured corporate capital relative to income. Meanwhile, extending the decomposition to all private sector capital income, I note some intriguing shifts in composition: for instance, there has been a long-term shift in net capital income from land in the non-housing sector to land in the housing sector.

Section 4 introduces more theory, to clarify and extend the analysis in section 3. I describe the behavior of factor income shares in the canonical one-sector model—especially the role of the elasticity of substitution—and the implications for various claims about the net capital share. Motivated by the limitations of a one-sector model, I construct a multisector model that reflects the disaggregation performed in section 3, and use this model (with a tentative calibration of elasticities) to perform counterfactual simulations that study the influence of key forces on the net capital share—including the observed changes in the relative price of various kinds of investment, and also changes in the real interest rate.

Section 5 concludes.

2 Evidence on factor income shares in developed countries

2.1 Conceptual issues

The notion of a “labor” or “capital” share is not monolithic; there are several ways to define and measure these concepts, and different choices can lead to strikingly different interpretations of the data.

Decomposing gross value added. In the national accounts, the gross value added of a sector at market prices—the value of its gross output, minus the intermediate inputs used
in production—can be divided into three components: labor income (which includes both wages and supplementary compensation), taxes on production, and gross capital income (usually called “gross operating surplus” in the national accounts). Since the second component, taxes on production, does not accrue to either labor or capital, when analyzing the distribution of income between factors it is often convenient to subtract this part, leaving us with gross value added at factor cost. This can be divided entirely into labor and gross capital shares, which sum to 1. Since I focus in this paper on the division of income between capital and labor, I will generally use this approach.

It is important to recognize that the split of value added between labor and capital is only the initial distribution. Labor income goes both to wages and to supplementary benefits, and a sizable share of wage income is subsequently paid to the government in taxes. Capital income is ultimately apportioned between many recipients, including the government (in the form of corporate and proprietor income taxes) and both debt and equity investors.

For instance, consider a sawmill. The gross value added at factor cost is the difference between its sales of lumber and the cost of logs, excluding taxes on production. Once all compensation of employees at the sawmill is subtracted, the remainder is its gross capital income. Some of this capital income will be paid to lenders in the form of interest, some will be paid to the government in taxes on profits, and the rest may be retained on the balance sheet of the sawmill or distributed as dividends to shareholders. Gross capital income is thus a very broad concept, encompassing funds that are ultimately paid out to many different recipients—it is unaffected, for instance, by the split in financing between debt and equity.\footnote{This invariance can be very useful in analyzing trends—for instance, when high inflation pushes up nominal interest rates, a large share of capital income is often paid to bondholders in the form of nominal interest. As Modigliani and Cohn (1979) memorably observed in the context of late-1970s inflation, this causes recorded profits to dramatically understate true profits, since they do not reflect the gain from real depreciation in nominal liabilities.}

\textbf{Gross versus net: concepts.} An alternative to gross value added is net value added, which subtracts depreciation. This can be divided into labor and net capital income; the latter is gross capital income minus depreciation. Whether a gross or net measure is more appropriate depends on the question being asked: the allocation of gross value added between labor and gross capital more directly reflects the structure of production, while the allocation of net value added between labor and net capital reflects the ultimate command

\footnote{This decomposition potentially applies at many levels of aggregation: for instance, the “sector” may be the entire domestic economy, in which case gross value added at market prices is called gross domestic product (GDP).}
over resources that accrues to labor versus capital.

For instance, in an industry where most of the output is produced by short-lived software, the gross capital share will be high, evincing the centrality of capital’s direct role in production. At the same time, the net capital share may be low, indicating that the returns from production ultimately go more to software engineers than capitalists—whose return from production is offset by a loss from capital that rapidly becomes obsolete.

Both measures are important: indeed, a rise in the gross capital share in a particular industry is particularly salient to an employee whose job has been replaced by software, and it may proxy for an underlying shift in distribution within aggregate labor income—for instance, from travel agents to software engineers. The massive reallocation of gross income in manufacturing from labor to capital, documented by Elsby et al. (2013), has certainly come as unwelcome news to manufacturing workers. But when considering the ultimate breakdown of income between labor and capital, particularly in the context of concern about inequality in the aggregate economy, the net measure is likely more relevant. This point is accepted by Piketty (2014), who uses net measures; the general rationale for excluding depreciation is pithily summarized by Baker (2010), who remarks that “you can’t eat depreciation.”

**Gross versus net shares: measurement and history.** Historically, the study of income shares has spanned both gross and net concepts: indeed, the famous quote by Keynes (1939) about the stability of labor’s share referred to data on net shares, as did Kaldor (1957)’s influential stylized fact.

More recently, however, the vast majority of work on the topic—including Karabarbounis and Neiman (2014b)’s well-known documentation of the declining global labor share—has examined gross shares. To a large extent, this is because gross shares are easier to measure and interpret: as economists since Kalecki (1938) have observed, net income inherently involves a somewhat arbitrary computation of depreciation. High-quality data on gross shares is available for more countries, more years, and more levels of aggregation within each country.

Recently, debate has intensified about the empirical importance of this distinction. Bridgman (2014) argues that the inclusion of depreciation—and, to a lesser extent, taxes on production—in the denominator of the labor share has caused economists to greatly overstate the magnitude and novelty of the labor share’s decline. Augmenting their global dataset with information on depreciation, Karabarbounis and Neiman (2014a) argue to the contrary that gross and net labor shares have mainly moved together, and that moving from gross to net shares at most moderately attenuates the downward trend. In
my data analysis, I will focus on net shares, finding that the concerns in Bridgman (2014) are valid, especially in the years preceding the start of the Karabarbounis and Neiman (2014a) sample.

**Mixed income and other concerns.** The distinction between gross and net is not the only concern when computing income shares. Another crucial problem is how to allocate “mixed” income—income earned by the self-employed that is recorded in the national accounts as going to capital. The central difficulty is that this income includes both returns to labor and returns to the capital investments made by the self-employed, with no data available to disentangle the two. This was an essential question for early students of the labor share in the US: as Johnson (1954) and others pointed out, the dramatic rise in workers’ share of income in the first half of the twentieth century was in large part due to the shift from entrepreneurial income (often on farms) to formal labor income.

One solution is to disregard the entrepreneurial sector of the economy, limiting attention (for instance) to the labor share within the corporate sector. In any attempt to measure the labor share for the economy as a whole, however, some approach to dividing mixed income must be chosen—and this choice can matter a great deal. Indeed, Elsby et al. (2013) demonstrate that the “headline” measure provided by the BLS most likely exaggerates the decline in the US gross labor share, due to weaknesses in its approach to imputing labor income for the self-employed. This approach assumes that the self-employed receive the same average compensation per hour as all other workers—an imputation that, although popular and tractable, has some unlikely implications for the US data.

Alternative approaches to dividing mixed income, discussed by Gollin (2002), take several forms: they may do a more sophisticated estimation of labor income for the self-employed based on personal characteristics, or assume that the entrepreneurial sector has the same division between labor and capital as either some other sector or the economy as a whole. I follow Piketty and Zucman (2014) in adopting a form of the latter imputation, assuming that the non-corporate sector (excluding housing) has the same net capital share as the corporate sector.

Finally, another difficult point is the treatment of general government, as well as any other sectors whose output is valued in the national accounts “at cost”—meaning that gross value added is set equal to labor and depreciation costs—rather than by the market. Here, net capital income equals zero by construction; and regardless, it is unclear what net capital income would mean in the context of government.
2.2 Income shares in the G7

To better understand the recent evolution of factor shares, I turn to a panel with national accounts data from the G7—which consists of the US, Japan, Germany, France, the UK, Italy, and Canada, currently the seven largest advanced economies by nominal GDP. Most of the data for the panel is derived from the Piketty and Zucman (2014) database, which in turn is taken directly from each country’s national accounts publications.

Although this is a much narrower selection of countries than in the global panels of Karabarbounis and Neiman (2014a,b), it has several offsetting advantages. Most importantly, it covers a longer timespan: five countries have data starting in 1960 or earlier, and three countries have data starting in 1950 or earlier. By contrast, Karabarbounis and Neiman (2014a,b)’s dataset starts in 1975, and for many small and developing countries data only starts becoming available much later. Since the net labor share in most countries was close to its postwar peak in the mid-1970s, this offers an incomplete view of the overall trend. The dataset here also permits greater disaggregation, particularly along a dimension that will turn out to be crucial (housing versus the rest of the economy). By focusing on developed economies, it loses some generality but stays closer to the contemporary debate about inequality and income distribution, which has mostly dealt with the developed world.

Estimated average shares. To summarize the evolution over time of various income share measures $s_{i,t}$, I follow Karabarbounis and Neiman (2014a,b) by running panel regressions of the form

$$s_{i,t} = \phi_t + \alpha_t + \epsilon_{i,t}$$

for countries $i$ and years $t$. I then display the yearly fixed effects $\alpha_t$, normalizing them such that the fixed effect for the first year of the sample, $\alpha_{1948}$, equals the average share in the dataset in 1948. When countries have different trends in $s_{i,t}$, there will be an artifactual discontinuity in $\alpha_t$ when a country enters the sample, which in principle could deliver a misleading impression of the actual year-to-year changes in $s_{i,t}$. In practice, this does not seem to be much of an issue here, and alternative approaches—for instance, averaging the first differences $\Delta s_{i,t}$ across countries in the sample for each year $t$, then plotting the cumulative average first difference over time—deliver similar results.

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5 The full set of start dates is 1948 (France, UK, US), 1955 (Japan), 1960 (Canada), 1990 (Italy), and 1991 (Germany). Data for the France, the UK, and the US is available starting even earlier, but I focus on 1948 onward because that is when the necessary data starts becoming available for my subsequent, more detailed exercise for the US in section 3. This also keeps the focus on postwar dynamics, detached from the sizable dislocations associated with depression and wartime, and mostly postdates the transition from agricultural self-employment to formal employment that bedeviled older analysts like Johnson (1954).

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version 8.0 of the Penn World Table.\textsuperscript{7} (For convenience, I will refer to these normalized time fixed effects as yearly “averages”.)

Unlike in the usual presentation, I deal with the \textit{capital} share rather than its complement, the labor share. Of course, since I deal with value added at factor cost, the capital share is always one minus the labor share; I focus on the former because I will emphasize the composition of capital income.

**Overall capital shares: net and gross.** First I consider average capital shares for the private economy (excluding government, whose net capital share is zero by construction). As discussed in section 2.1, I deal with the problem of self-employment income by following Piketty (2014) and Piketty and Zucman (2014) in the assumption that the net capital share in non-corporate, non-housing sector equals the net capital share in the corporate sector.\textsuperscript{8}

Figures 1 and 2 report the average net and gross capital shares, respectively. As figure 1 demonstrates, the postwar behavior of the net capital share is characterized not so much by a secular rise as by a precipitous \textit{fall} in the 1970s, which preceded a steady rebound. In this light, it is clear why Karabarbounis and Neiman (2014a,b)—with a sample starting in 1975, the year in which the unweighted estimate for the net capital share hits its minimum—observe such a dramatic and pervasive rise in capital income relative to labor.

Although Piketty (2014) and others have documented an overall U-shaped trend in the capital share, the claims about timing are quite different: for instance, Piketty (2014) observes that capital’s aggregate valuation and share of income fell greatly in the first half of the twentieth century, during the depression and two world wars. The postwar period is characterized as a period of recovery from this decline. Yet figure 1 shows that if anything, the first half of the postwar era experienced a fall in the net capital share, and we are only today returning to levels achieved in the immediate aftermath of the war.

Set against figure 1, figure 2 reveals that there is a remarkable difference between the long-run behavior of net and gross shares, echoing the results of Bridgman (2014): since average depreciation as a share of gross value added has risen, the gross capital share displays much more of a long-term upward trend. Crucially, much of this disparity emerges before the 1970s, perhaps explaining why Karabarbounis and Neiman (2014a) do not detect such an important role for depreciation in their sample.

\textsuperscript{7}See Feenstra, Inklaar and Timmer (2013).

\textsuperscript{8}There are two exceptions: the Canadian national accounts already provide a decomposition of mixed income into labor and capital, which I use; and the Japanese national accounts do not fully break out the corporate sector, necessitating some additional imputations.
As argued in section 2.1, net shares are likely most relevant for discussions of distribution and inequality. Still, figure 1 paints a perhaps ambiguous picture of the net capital share: the recent rise might be in part just a recovery from the anomalously low levels of the 1970s, but the capital share is now reaching and even surpassing the heights previously achieved in the 1950s and 60s. To what extent, then, is the current high share of capital income a truly novel phenomenon? This question is best addressed by disaggregating further along an important dimension, distinguishing between capital income from housing and capital income from the rest of the economy.

Composition of the net capital share: the role of housing. Figure 3 subdivides the aggregate net capital share from figure 1 into two components: net capital income originating in the housing sector, and net capital income from all other sectors of the economy. It reveals that the aggregate net capital share originating in sectors other than housing has seen only a partial recovery since the 1970s; it remains below the levels of the 1950s, and slightly below or at par with the levels of the 1960s. In contrast, housing’s contribution to net capital income has expanded enormously, from roughly 3pp in 1950 to nearly 10pp today.

Housing’s central role in the long-term behavior of the aggregate net capital share has, to my knowledge, not been emphasized elsewhere. It demands careful scrutiny. Income from housing is unlike most other forms of capital income recorded in the national accounts: in countries where homeownership is dominant, most output in the housing sector is recorded as imputed rent paid by homeowners to themselves. It may not be a coincidence that Germany, which table 1 reveals to have by far the lowest housing component of net capital income, also has the lowest homeownership rate in the G7. Indeed, imputed rents from owner-occupied housing should arguably be treated as a form of mixed income akin to self-employment income: in part, they reflect labor by the homeowners themselves. Figure 3 may therefore exaggerate the level of true “capital” income originating in the housing sector.

Nevertheless, even if figure 3 exaggerates the level of capital income from the housing sector, this does not necessarily explain the vast increase in housing capital income—unless the bias is greater today than in the past. One contributor to the trend could be a rise in the rate of homeownership; but this has not been nearly dramatic enough to ac-

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9For Canada and Japan, the “housing” sector is actually the owner-occupied housing sector due to data limitations. Importantly, Canada and Japan do not drive the trend here: to the contrary, from 1960 (when Canada enters the sample) to 2010, the average contribution of housing to net capital income in Canada and Japan increases by 3pp, while in France, the UK, and the US it increases by 4.5pp.
count for a more than 3x increase in housing capital income.\textsuperscript{10} Another, distinct source of bias could be rent control: if the rents imputed for homeowners in the national accounts improperly reflect controlled rents in the tenant-occupied sector, then the ebb and flow of rent regulations will have an inflated impact on income in the housing sector as a whole.

These possible biases notwithstanding, the main thrust of figure 3 is that housing has a pivotal role in the modern story of income distribution.\textsuperscript{11} Since housing has relatively broad ownership, it does not conform to the traditional story of labor versus capital, nor can its growth be easily explained with many of the stories commonly proposed for the income split elsewhere in the economy—the bargaining power of labor, the growing role of technology, and so on.

**Net capital share within the corporate sector.** For additional clarity, figure 4 plots the average net capital share within the corporate sector. Restricting attention to the corporate sector is a common way to deal with perceived conceptual and measurement difficulties elsewhere in the economy—including ambiguity in the labor/capital split of mixed income, as well as the crucial role of housing. Figure 4 echoes the behavior of the non-housing component in figure 3, with a sustained fall until the 1970s and a partial recovery in the decades since. Indeed, this resemblance is no coincidence: as discussed above, figure 3 imputes the net capital share in the non-housing, non-corporate sector to be the same as in the corporate sector, so that movement in all non-housing capital income is fundamentally driven by the corporate capital share visible in figure 4.\textsuperscript{12}

Although it does not show any decisive, long-term trend, figure 4 does clash with the Kaldor (1957) view of stable income shares, and it also contrasts with the relatively steady upward creep of housing capital income in figure 3. Fluctuations in the average corporate capital share have been rapid and macroeconomically significant—dropping from a high around 26% in 1950 to a trough around 18% in the 1970s and 80s, then rebounding to a peak of 24% in the 2000s. Indeed, the fall in the unweighted average share from 26.4% in 1950 to 17.7% in 1975, all else equal, contributed nearly half a percentage point annually

\textsuperscript{10}See, e.g., Andrews and Sanchez (2011) for some discussion of trends in homeownership.

\textsuperscript{11}One dissonant note is in Bonnet, Bono, Chapelle and Wasmer (2014), who argue that Piketty (2014)’s emphasis on the rising wealth-income ratio is misplaced, since most of the increase comes from housing values—whose spectacular rise, when adjusted for the rising price-to-rent ratio recorded by the OECD, mostly disappears. At face value, this claim appears very close to the claim that gross rents (both actual and imputed) on the housing stock have not increased—and this is inconsistent with the expansion of gross rents recorded in the national accounts, which is the key driver behind the expansion of net capital income visible in figure 3. The source of this apparent discrepancy is unclear and merits further investigation.

\textsuperscript{12}As described in footnote 8, different imputations are used for Canada and Japan. Furthermore, since separate data for the corporate sector is not available in Japan, figure 4 displays the overall capital share for Japan instead.
to growth in corporate labor compensation during that interval. In contrast, the rapid rise from 17.7% in 1975 to 23.6% in 1988 subtracted slightly more than half a percentage point of annual compensation growth.\textsuperscript{13}

Yet over the long term, the role of fluctuating corporate income shares is comparatively quite mild. For both the weighted and unweighted averages, the impact on annual compensation growth from the 1948-2010 change in net shares is roughly three-hundredths of a percentage point.\textsuperscript{14} The overall message is clear, and arguably consistent with the Kaldor (1957) perspective on long-run growth. Changes in the distribution of corporate income—even systematic ones spread across several countries—can have a marked effect on the short-to-medium-run growth of paychecks. The impact on long-run labor compensation, however, appears to be little more than a rounding error when set against trend growth.\textsuperscript{15}

There is also a pronounced cyclical pattern in figure 4. This is has long been recognized: Rotemberg and Woodford (1999) observe, for instance, that the labor share tends to rise late in expansions and fall late in recessions. The economic explanation for this pattern, however, is somewhat harder to discern. Conventional wisdom is that low unemployment puts upward pressure on real wages and hence the labor share, while high unemployment keeps real wage growth subdued. Rotemberg and Woodford (1999) emphasize, however, that this story implicitly involves variation in markups—they cite Mitchell (1941), who mentions "a problem still remains: Why cannot businessmen defend their profit margins against the threatened encroachment of costs by marking up their selling prices?"

The business cycle literature offers an abundance of proposed explanations for the cyclical pattern of markups, of which Rotemberg and Woodford (1999) is a good summary. One of the simplest is the standard sticky-price New Keynesian model, in which the labor share moves procyclically because firms cannot immediately adjust their prices to match fluctuations in wages; Sbordone (2005) even uses the labor share as a proxy for marginal costs in a New Keynesian Phillips curve specification, showing that the expected future value of the labor share plays a role in current inflation. This literature is

\textsuperscript{13}Explicitly, \((1 - .177)/(1 - .264))^{(1/25)} - 1 \approx .45\%\) and \((1 - .236)/(1 - .177))^{(1/13)} - 1 \approx -.57\%.

\textsuperscript{14}Explicitly, for unweighted: \((1 - .214)/(1 - .229))^{(1/62)} - 1 \approx .03\%.\) For weighted: \((1 - .231)/(1 - .245))^{(1/62)} - 1 \approx .03\%.

\textsuperscript{15}To be clear, the long-run impact in individual countries can be larger. Perhaps the most extreme example is Japan, which table 1 shows to have experienced a decline in the average non-housing share of aggregate capital income from 31% in the 1960s to 20% in the 2000s, implying an annualized contribution to wage growth of roughly three-tenths of a percentage point. But Table 2 does not suggest any long-run tendency for corporate capital shares in different countries to diverge from each other; the distinct paths across countries are therefore probably best interpreted as mean-reverting variations around an apparently trendless average.
informative for section 3, where I find that variation in markups appears to drive most of the variation in the capital versus labor share of corporate income.

3 Decomposing the capital share

3.1 Theory

In principle, aggregate capital income comes from several sources—the return on reproducible capital, rents on land, monopolists’ profits, and so on. One way to gain additional insight into the fluctuations identified in section 2 is to divide capital income between these components.

Let $K_1, \ldots, K_n$ be different types of capital, and let $Y = F(N, K_1, \ldots, K_n)$ be a constant-returns-to-scale production function that takes labor $N$ and capital $K_1, \ldots, K_n$ as inputs. Suppose that output $Y$ is sold at a price $P$ that represents a markup of $\mu \geq 1$ over the marginal cost of production, such that the share of “pure” profits—profits above and beyond the user cost of capital $K_1, \ldots, K_n$—is $\pi = 1 - \mu^{-1}$. I allow for the potentially time-varying markup $\mu$ in part because of the discussion of the corporate capital share in section 2.2, which notes a pronounced cyclical pattern that has been explained in the literature through markup variation.

Letting $W_N$ denote the wage paid to labor and $W_{K_1}, \ldots, W_{K_n}$ denote the user costs of capital, we have

\[
(1 - \pi)PY = W_N N + \sum_{i=1}^{n} W_{K_i} K_i \tag{1}
\]

Suppose further that we are in continuous time (suppressing time subscripts for convenience), and that the flow real cost of funds is $r$. Capital $K_i$ has real price $P_i$, with expected real growth rate $g_{P_i}$, as well as a flow depreciation rate of $\delta_i$. The user cost $W_{K_i}$ is then

\[
W_{K_i} = P_i (r + \delta_i - g_{P_i}) \tag{2}
\]

reflecting the real cost $P_i r$ of financing each unit of capital and the expected combined effect $P_i (\delta_i - g_{P_i})$ of depreciation and price growth on the value of capital held.

Combining (1) with (2), we see that we can divide net output into labor income $W_N N$ and net capital income; the latter can further be divided into a share $\pi PY$ of profits and a
component \((r - g_{P_i})P_iK_i\) corresponding to each type of capital \(i\).

\[
PY - \sum_{i=1}^{n} \delta_i P_iK_i = WN + \pi PY + \sum_{i=1}^{n} (r - g_{P_i})P_iK_i
\]

Suppose that in practice we have disaggregated capital into \(n\) types, and we want to divide observed net capital income into the components identified in (3). First, the nominal value \(P_iK_i\) of each type of capital is needed; this data is relatively easy to obtain. Second, expected price growth \(g_{P_i}\) is needed; this is very difficult to obtain in principle, since we rarely observe agents’ individual expectations of price growth, but it can be roughly approximated by assuming that \(g_{P_i}\) matches the trend rate of growth over some interval.

The most difficult parts of (3) are \(\pi\) and \(r\): with knowledge of one, we can infer the other, but neither is readily available in the data. In principle, \(r\) could be obtained from financial markets, perhaps as some function of bond and equity prices. But this is a notoriously hard problem: it is challenging to know how exactly the costs of borrowing or equity finance map onto the effective cost of funds faced by an enterprise. Moreover, due to capital adjustment costs, the relevant \(r\) for most businesses is probably much slower-moving than the rates seen in financial markets.

### 3.2 Implementing the theory: decomposing the net corporate capital share in the US, 1948–2013

I first attempt the disaggregation in (3) for the net capital share in the US corporate sector, at an annual frequency for the years 1948 through 2013.\(^{16}\) I disaggregate fixed capital into its three most important components: structures, equipment, and land (denoted by \(i = s, e, l\)), and I obtain the values \(P_iK_i\) for the corporate sector from the flow of funds.\(^{17}\) I assume that the expected price growth \(g_{P_i}\) of each form of capital is its actual average real price change from the end of 1947 to the end of 2013. I then try several approaches to resolving the difficulties identified at the end of section 3.1.

**Baseline approach: constant \(r\), average profit over time of zero.** My baseline assumption is that \(r\) is constant over the sample period, 1948–2013, and takes a value such that

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\(^{16}\)Ideally, this exercise would extend to all seven of the G7 countries covered in section 2, but the additional data required makes this difficult.

\(^{17}\)Since the flow of funds provides end-of-year values for capital, I average the adjacent end-of-year values to obtain the effective capital stock used in production during each year.
the average profit share \( \pi \) of corporate revenue over the sample is zero. This implies \( r \approx 11\% \).\(^{18}\) Taking this \( r \) as given, in each year I can subtract the sum \( \sum_i (r - g_P) P_i K_i \) in (3) from observed net capital income to obtain profits \( \pi PY \) as a residual.

Although this assumption on \( r \) is admittedly ad-hoc, it is simple and transparent, and it is well-suited for evaluating hypotheses that do not rely on variations in \( r \). Effectively, the assumption here is that in the long run, there are no pure profits in the corporate sector—on average, net capital income reflects a return on equipment, structure, or land. Figure 6 shows how the net capital income for the US corporate sector in figure 5 breaks down into the four components in (3). Though there are some fluctuations in each component’s contribution, both the U-shaped pattern and the cyclical fluctuations in the corporate capital share in figure 5 appear to be dominated by the movement of the profit share, \( \pi \).

In particular, there is little direct support for the hypotheses in either Piketty and Zucman (2014) or Karabarbounis and Neiman (2014b), both of which emphasize the role of changes in the value \( P_i K_i \) of capital as the proximate source of fluctuations in the capital share. Piketty and Zucman (2014)—as well as Piketty (2014)—highlight how the buildup of savings can cause the value of aggregate capital to increase, while the net return \( r \) stays relatively stable owing to a very high elasticity of substitution. Karabarbounis and Neiman (2014b) stress the role of increasing \( P_i K_i \) as \( P_i \) falls and \( K_i \) rises, given an elasticity of substitution above 1; they assume \( r \) is pinned down in the long run by households’ time preference. Yet in figure 5, where \( r \) is similarly assumed to be constant, the main force is \( \pi \), while the \( P_i K_i \) terms in (3) contribute variation that is relatively minor and not aligned with the large time-series movements in net share.

Indeed, the contribution from equipment in figure 5 is if anything the inverse of the U-shaped pattern in the corporate net capital share in figure 5: it rises in the 1970s and 1980s, and then later trends downward. Since equipment is the component of fixed capital that has experienced a decline\(^{19}\) in real price, this is (at least superficially) hard to reconcile with a central role for falling investment prices in the dynamics of capital’s share. Without a structural model, of course, this exercise is not decisive: falling investment prices

\(^{18}\)Although this seems high for a real return, note that it is a pretax return: the return before taxes are applied either to corporate profits or distributions of interest or dividends. Interestingly, it is slightly lower than the constant return in figure 7 estimated using my alternative approach, which is roughly 12.8\%. As explained below, this return is higher because according to the flow of funds, the total market value of the corporate sector in the US has actually been lower than the book value, on average, in the postwar era—suggesting that pure profits are, if anything, negative, and that the assumption that pure profits are zero on average is not misattributing these profits to an exaggerated return \( r \) on capital.

\(^{19}\)The equipment investment deflator has relative to the GDP deflator at an annualized rate of 1.5\% during the sample period, as opposed to a 1.1\% average rise in the deflator for nonresidential structures.
might contribute to a rising capital share via some more indirect causal channel, and indeed Karabarbounis and Neiman (2014a) suggest one such possibility. I address these concerns by performing simulations with a multisector model in section 4.3, where I generally do not find a major role for such indirect mechanisms.

To some extent, the importance of \( \pi \) in figure 5 is unremarkable: since \( \pi \) is computed here as a residual, it can be expected to mirror the net capital share as long as the other terms in (3) do not provide much explanatory power. And this—the fact that changes in measured \( P_i K_i \) do not reliably accompany changes in the net capital share—is the most robust finding from the decomposition. If we enforced \( \pi = 0 \) in (3) for every year and allowed \( r \) to vary instead to satisfy the identity, the movements in \( \pi \) in figure 5 would instead be absorbed by movements in \( r \).

Taking the baseline decomposition at face value, however, the changes in \( \pi \) suggest some large-scale role for varying markups and market power. In part, this reflects the cyclical variation in markups already discussed in section 2.2, which macroeconomists have devoted great energy to explaining. The low levels of \( \pi \) in the 1970s and 1980s are arguably consistent with the emphasis in the New Keynesian literature on the relationship between markups and inflation: lower expected markups, all else equal, imply higher inflation via the New Keynesian Phillips curve. The long-term U-shape in \( \pi \) may also indicate some changes in market power or the scope for monopoly profits. To the extent that the rents giving rise to pure profits can sometimes be shifted to workers, fluctuations in \( \pi \) could in principle reflect changes in worker/firm bargaining power or the role of unions. In practice, however, the U-shape path for \( \pi \) is difficult to interpret along these lines: \( \pi \) is lowest in the 1980s, a decade that certainly was not known for union strength.

**Alternative approach: identify time path for \( r \) from market minus book value.** To cross-check the results from the baseline approach, I also consider a less ad-hoc and more technically involved way of specifying \( r \). The basic idea is that the difference between the

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20 Karabarbounis and Neiman (2014a) devise a model where two types of capital, high-depreciation (which can be interpreted as equipment) and low-depreciation (which can be interpreted as structures) combine to form a capital aggregate; the elasticity of substitution between these types of capital is less than 1, while the elasticity of substitution between the capital aggregate and labor is greater than 1. A decline in the price of equipment lowers the price of the capital aggregate, which induces substitution from labor to the capital aggregate; but since the elasticity of substitution between equipment and structures is less than 1, this also causes a decline in equipment relative to structures. With the right parameters, it is possible for a decline in the price of equipment to increase the net capital share while net capital income from equipment itself actually declines, owing to the particular pattern of substitutability between equipment, structures, and labor that is assumed. Although this model cannot directly address the patterns in figure 5 (where structures are not an important contributor), it does show how causality becomes more complex in a structural model.

21 See, for instance, Sbordone (2005).
market value of corporations and the value of their fixed assets should reflect the expected stream of future pure profits $\pi PY$ (up, possibly, to some stochastic pricing error). We can use this observation as a strategy to estimate the implied $r$: for instance, if market value is much higher than the value of the firm’s assets, the expected stream of pure profits $\pi PY$ is high, and $r$ in the future must be low enough that there are pure profits left over in (3) after the direct return from capital $\sum_i (r - g_i) P_i K_i$ is subtracted.\footnote{For simplicity, I will call the total value of the firm’s fixed assets its “book value”, even though this is not necessarily book value in the usual sense: I will define it to exclude financial assets—these are instead subtracted from the market value, which includes net financial liabilities—and it uses values from the flow of funds for real estate and equipment, which are updated to reflect changes in price.}

**Description of the method.** Appendix C provides the technical details, along with the specific theoretical assumptions in a continuous-time model that are needed to make the procedure valid. The core equation implied by the theory is (see (33) and (34)):

$$
E \left[ \frac{\phi(t)}{\text{output between } t-1 \text{ and } t} \times (OMV(t) - \text{discount} \times OMV(t+1)) \right] = E \left[ \frac{\phi(t)}{\text{output between } t-1 \text{ and } t} \times \text{pure profits between } t \text{ and } t+1 \right]
$$

(4)

where $OMV(t)$ denotes the difference between the market value and book value of corporations recorded at time $t$, and $\phi(t)$ is an arbitrary time-dependent function. Implicit in (4) is a (nonstochastic) time path $r(t)$ for the real interest rate, which is needed to calculate profits $\pi(t) P(t) Y(t)$ as a residual in (3) and to calculate the proper discount factors.

The interpretation of (4) is straightforward: it states that the expected difference between the present value of next year’s excess market value $OMV(t+1)$ and this year’s excess market value $OMV(t)$ reflects expected pure profits between $t$ and $t+1$. This relation continues to hold, in expectation, when both sides are normalized by the previous year’s recorded output, which I do to render values comparable across time; and it also holds when both sides are multiplied by any choice of the time-dependent function $\phi(t)$.

Technically speaking, equation (4) can be used as a moment condition to estimate $r(t)$. If we have $n$ functions $\{\phi_1(t), \ldots, \phi_n(t)\}$, we obtain $n$ distinct moment conditions (4), and can enforce these conditions in the sample to solve for an $n$-parameter functional form for $r(t)$. I choose $\phi_1(t) = 1$, $\phi_2(t) = t$, and $\phi_3(t) = t^2$, and estimate three specifications for $r(t)$: a constant value $r(t) = \bar{r}$, a linear trend $r(t) = a_0 + a_1 t$, and a quadratic trend $r(t) = a_0 + a_1 t + a_2 t^2$, using the moment conditions implied by $\{\phi_1(t)\}$, $\{\phi_1(t), \phi_2(t)\}$, and $\{\phi_1(t), \phi_2(t), \phi_3(t)\}$, respectively.
Effectively, I am solving for the constant $\bar{r}$ such that the expression

$$\frac{OMV(t) - \text{discount} \times OMV(t + 1) - \text{pure profits between } t \text{ and } t + 1}{\text{output between } t - 1 \text{ and } t}$$

equals zero on average throughout the sample; and I am also solving for the linear $r(t) = a_0 + a_1 t$ and the quadratic $r(t) = a_0 + a_1 t + a_2 t^2$ such that (5) does not have any linear or quadratic trends over time, respectively.

When calculating $OMV$, the difference between the market value of the corporate sector and the book value of its fixed capital, I interpret the “market value” to be the total value of all financial claims on a corporation—both its equity market capitalization and its net financial liabilities—in order to be consistent with the computation of capital income in the national accounts, which includes income that ultimately goes to both shareholders and bondholders.\footnote{Both market and book value are taken from the flow of funds.}

**Results.** Figure 7 shows the estimated constant, linear, and quadratic time trends for the corporate rate of return $r(t)$ following the procedure above. The most striking feature of these plots is the general downward trend in $r(t)$: according to this procedure, the required return on capital for the US corporate sector has fallen over the postwar era. This reflects the fact that the market value of corporations has grown relative to book value over this period, albeit unevenly, as can be seen in figure 9.

Another interesting feature of figure 7 is that estimated constant $\bar{r}$, at roughly 12.8%, is actually higher than the $r$ chosen in my benchmark decomposition to set the average share of pure profits to zero. This reflects the fact that according to the flow of funds, on average, the aggregate market value of corporations has actually been slightly below the book value during the sample period, as depicted in figure 9. This suggests that the assumption of zero average pure profits for the benchmark decomposition was not too far out of line: corporations, on average, have not been worth more than the underlying value of their assets.

Redoing the decomposition in figure 6, using the linear trend for $r(t)$ rather than a constant, produces figure 8. The impact of the change in $r(t)$ is unsurprising. Relative to figure 6, figure 8 initially attributes a larger share of returns to fixed capital, offset by substantial negative pure profits; over time, the return on fixed capital falls, and the role

\footnote{This causes some anomalies in the early postwar years, when the corporate sector was left with large cash balances and relatively little debt, making net liabilities negative while equity valuations were already quite low, and leading to an extremely low market relative to book value. To avoid undue influence from this period, I exclude data from prior to 1955 in the benchmark results displayed here; otherwise, there is an even more dramatic estimated downward trend in $r(t)$.}
of pure profits grows substantially. As in figure 6, pure profits are still responsible for most high-frequency fluctuations, and they play a central role in the U-shaped path for the overall corporate net capital share—but these movements come in addition to broad offsetting trends, in which pure profits have replaced income from fixed assets in (3).

It is difficult to say how literally these trends should be interpreted. Given the methodology for identifying $r(t)$, they are ultimately the consequence of the long-term rise in the ratio of market value to book value in the US corporate sector, as seen in figure 9; and this, in turn, may be the result of other, unmodeled changes in financial markets, not a rise in $\pi$. Nevertheless, figure 8 is certainly suggestive, and it casts additional doubt on claims that the recent rise in the corporate capital share comes from the return on fixed capital.

### 3.3 Extending the decomposition: the net capital share for the private economy

I now extend the decomposition in section 3.2 to the net capital share for the private domestic economy as a whole—excluding the non-housing government and NPISH (non-profit institutions serving households) sectors, which have zero net capital share by construction in the national accounts.

Due to the inherent difficulties in apportioning mixed income between labor and capital, as discussed in section 2.1, this requires some imputations. I will assume that both the rate of return $r$ and the pure profit share $\pi$ are the same in the non-housing, non-corporate sector and the corporate sector, and use the benchmark value $r \approx 11\%$ from the previous section. For the housing sector, I will assume that there is no pure profit, and allow $r$ to vary over time in (3) such that net housing capital income always equals $(r - g_P)P_s K_s + (r - g_P)L_2$, where $P_s K_s$ is the value of residential structures and $P_2 L_2$ is the value of residential land.

The results are displayed in figure 11, which decomposes the net capital share displayed in figure 10. Figure 11 is noisy, and for the most part it combines the lessons from sections 2.2 and 3.2: there is a strong, long-term upward trend in net capital income from housing, and the volatile capital share elsewhere in the economy is driven principally by pure profits.

There are, however, some additional insights in the figure 11 decomposition. For instance, the rise in net income for the housing sector has come both from residential struc-

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24Note that this imputation, which uses data on the value of fixed assets in the non-corporate sector, is different from the imputation in section 2.2, where this data was not available for the full sample and the net capital share of income—rather than the return $r$—in the non-housing, non-corporate sector was assumed to be the same as in the corporate sector.
tures and land, but figure 11 attributes a larger portion of the increase (and of the level) to structures.

This may come as a surprise, since one plausible hypothesis for the growth of net housing income is the rising scarcity of land. In part, the secondary role of residential land here comes from its more rapid price appreciation. Since I assume that the net rate of return including expected capital gains is equalized between residential structures and land, the net rate of return excluding expected capital gains—which is used in the decomposition, because income in the national accounts also excludes capital gains—is significantly lower for land. In a sense, then, the lesser role of land is due to the idiosyncrasies of national accounting; and an alternative definition of net capital income that included some form of expected capital gains would show a larger impact from land. (With this in mind, it is remarkable that housing plays such a large aggregate role in section 2.2 already: if the G7 national accounts data were modified to include capital gains, housing’s centrality would only increase.)

Another interesting feature of figure 11 is that there has been a sizable decline in the role of capital income from non-residential land over time, from nearly 5% of net private value added in the first half of the sample to an (erratic) average of roughly 2.5% today. In other words, there has been a shift in net capital income from non-residential to residential land—but the decline in the former has been larger than the growth in the latter, suggesting that the direct contribution of land to net capital income in the US has actually fallen.

4 Influences on the capital share

Thus far, section 2 has covered the direct, atheoretical evidence on the split between capital and labor income, while section 3 has introduced a simple, theory-motivated decomposition of net capital income. In this section, I will introduce additional theory, in order to think more rigorously about how different forces influence the evolution of the capital share.

4.1 Canonical theory

One-sector, one-good model. I start by taking a step back from the decomposition in section 3—with its multiple capital goods—to recount the simplest, traditional model of income shares.

Let $F(K, N)$ be a constant returns to scale production function, with capital $K$ and labor
As factor inputs, and positive but diminishing returns in each factor. Assume that this is a one-good model, where the relative price of capital and output is fixed at one. The **elasticity of substitution** \(\sigma\) between \(K\) and \(N\) is defined as

\[
\sigma \equiv \left( \frac{d(\log(F_K/F_N))}{d(\log(K/N))} \right)^{-1}
\]  

(6)

This gives us the (inverse) elasticity of the ratio \(F_K/F_N\) of marginal products to the ratio \(K/N\) of capital. Equivalently, \(\sigma\) tells us the extent to which a cost-minimizing producer’s relative demand for \(K/N\) will change if there is a change in the relative cost \(R/W\) of using capital and labor as inputs.

From the definition (6), one can show that \(\sigma\) also gives the inverse elasticity of \(F_K\) with respect to a change in the capital-output ratio \(K/F\):

\[
\sigma = -\left( \frac{d(\log(F_K))}{d(\log(K/F))} \right)^{-1}
\]  

(7)

which implies that the elasticity of the capital income share \(F_KK/F\) with respect to the capital-output ratio \(K/F\) is

\[
\frac{d(\log(F_KK/F))}{d(\log(K/F))} = 1 - \frac{1}{\sigma}
\]  

(8)

This indicates the critical importance of the threshold \(\sigma = 1\). If \(\sigma > 1\), the elasticity is positive, so that the capital income share will increase as \(K/F\) rises. Inversely, if \(\sigma < 1\), the capital income share will fall as \(K/F\) rises. In the important special case \(\sigma = 1\), diminishing returns exactly offset the increased quantity of capital, and the share remains constant.

Indeed, one of the original motivations behind **Cobb and Douglas (1928)**’s eponymous production function was the apparent constancy of capital and labor shares in the data; this is guaranteed by the Cobb-Douglas production function \(F(K,N) = K^\alpha N^{1-\alpha}\), which has a constant elasticity of substitution \(\sigma = 1\).

### Net versus gross.

Thus far, I have been ambiguous about whether the function \(F\) gives gross production, or production net of depreciation. In principle, either interpretation is legitimate—especially since this is a one-good model, where the relative price of capital and output is fixed at one, and losses from capital depreciation can reasonably be included as part of the production function.

If \(F\) is gross production, then \(1 - 1/\sigma\) is the elasticity of gross capital income with respect to the ratio of capital to gross output. If \(F\) is net production, then \(1 - 1/\sigma\) is the elas-
ticity of net capital income with respect to the ratio of capital to net output. As discussed in section 2.1, both measures are useful, but net concepts are probably more meaningful when studying income distribution.

It is important to recognize that $\sigma$ depends greatly on which measure is used—a subtlety that is often overlooked. Suppose $F(K, N)$ is the gross production function, with an elasticity of substitution of $\sigma$. Then the net production function is $\tilde{F}(K, N) = F(K, N) - \delta K$, and from (7) the elasticity of substitution for $\tilde{F}$ is

$$\tilde{\sigma} = \frac{d(\log(\tilde{F}/K))}{d(\log F_K)}$$

$$= \frac{d(F/K - \delta)/(F/K - \delta)}{d(F_K - \delta)/(F_K - \delta)}$$

$$= \frac{d(F/K)/(F/K) \cdot F_K - \delta}{d(F_K)/F_K \cdot F_K - \delta} \cdot \frac{F}{F_K - \delta K}$$

$$= \sigma \cdot \frac{F_K - \delta}{F_K} \cdot \frac{F}{F - \delta K} \cdot \frac{F}{F - \delta K}$$

Hence the elasticity of substitution $\tilde{\sigma}$ for the net production function (“net elasticity”) equals the elasticity of substitution $\sigma$ for the gross production function (“gross elasticity”) times

(A) the ratio $(F_K - \delta)/F_K$ of the net and gross returns from capital, and

(B) the ratio $F/(F - \delta K)$ of gross and net output.

The ratio in (A) is below 1, while the ratio in (B) is above 1. Critically, the product of these ratios is always less than 1, so that the net elasticity is always below the gross elasticity. $(F_K - \delta)/F_K$ is the ratio of net to gross capital income, while $(F - \delta K)/F$ is the ratio of net to gross total income. Since there is no distinction between net and gross for labor income—the other component of total income—the former ratio is smaller than the latter.

The intuition behind the dominant term, (A), is as follows. The net return on capital $\tilde{F}_K$ is less than the gross return $F_K$ by a constant—the depreciation rate $\delta$—meaning that a given change in $F_K$ translates into an equal absolute, and a larger relative, change in $\tilde{F}_K$. For instance, if $\delta = 5\%$, and $F_K$ declines from 10% to 8%, $\tilde{F}_K$ will decline from 5% to 3%. A 20% decline in the gross return becomes a 40% decline in the net return, and the ratio of the two is (A). As we increase capital relative to labor, the net marginal product of capital declines more rapidly than the gross—in short, capital is less substitutable for labor from a net perspective.\(^{25}\)

\(^{25}\)The other term, (B), adjusts for the fact that the net capital/output ratio is higher than the gross.
Empirical implications. Ever since Arrow, Chenery, Minhas and Solow (1961) first proposed the constant elasticity of substitution (CES) production function, researchers have attempted to estimate the key elasticity parameter $\sigma$. These studies have virtually always looked at the elasticity of substitution in the gross production function.

The literature is vast and its conclusions muddled, but one consistent theme has been the rarity of high elasticity estimates. Chirinko (2008) provides an excellent summary of the empirical literature, listing estimates from many different sources and empirical strategies. Of the 31 sources listed for the gross elasticity, fully 30 out of 31 show $\sigma < 2$. 29 out of 31 show $\sigma < 1.5$, and 26 out of 31 show $\sigma < 1$. The median is $\sigma = 0.52$, and Chirinko concludes that “the weight of the evidence suggests that $\sigma$ lies in the range between 0.40 and 0.60”. As per (9), the corresponding net elasticities $\tilde{\sigma}$ are even lower than these gross elasticities $\sigma$, suggesting that $\tilde{\sigma} \geq 1$ is very unlikely.

From (8), it follows that a rise in the capital-output ratio, holding the production function constant, most likely will cause a decline in the net share of capital income. This is inconsistent with the Piketty (2014) and Piketty and Zucman (2014) hypothesis that the accumulation of capital relative to aggregate income has led—and will lead going forward—to a rise in capital’s net share.

Two-good model. The canonical model presented above can be enriched slightly by allowing the price $P_K$ of capital relative to the output good to vary. This modification is central to the account in Karabarbounis and Neiman (2014b), who attribute the rise in the gross capital share to high capital demand induced by a fall in $P_K$.

To be more explicit, take the net required return $r$ on capital as given. Ignoring expected capital gains, demand for capital is pinned down by the condition

$$F_K(K, N) = P_K(r + \delta)$$

(10)

The elasticity of the gross capital/output ratio $K/F$ with respect to $P_K$ is then

$$\frac{\partial \log(K/F)}{\partial \log P_K} = \frac{d \log(K/F)}{d \log F_K} \cdot \frac{\partial \log F_K}{\partial \log P_K} = -\sigma$$

(11)

where $\partial \log F_K/\partial \log P_K = 1$ follows directly from (10), and $d \log(K/F)/d \log F_K = 26$.

For a few sources that list a range of elasticities, I take the midpoint. This has minimal effect on the distribution.
−σ follows from (7). The elasticity of the gross capital share with respect to $P_K$ becomes

$$\frac{\partial (\log (F_K K / F))}{\partial (\log P_K)} = \left( 1 + \frac{\partial (\log (K / F))}{\partial (\log F_K)} \right) \cdot \frac{\partial (\log F_K)}{\partial (\log P_K)} = 1 - \sigma$$

implying that a decline in the relative price $P_K$ of capital will increase the gross capital share if $\sigma > 1$.

Meanwhile, the elasticity of the net capital share with respect to $P_K$ is, after a somewhat more involved\(^27\) computation,

$$\frac{\partial (\log ((F_K - \delta P_K) K / (F - \delta P_K K)))}{\partial (\log P_K)} = (1 - \sigma) \cdot \frac{F}{F - \delta P_K K}$$

(13)

and $\sigma = 1$ is still the critical threshold: a decline in the relative price $P_K$ of capital increases both the net and gross capital shares if $\sigma > 1$. This consistency is a noteworthy contrast with the distinction (9) between gross and net elasticities of substitution, where a rise in the capital-output ratio could produce an increase in the gross capital share and a decrease in the net capital share. From an intuitive standpoint, this is unsurprising: since we are holding $r$ constant, the ratio $r / (r + \delta)$ of net to gross capital income is fixed, and the two move in the same direction in response to a change in $P_K$.

Karabarbounis and Neiman (2014a) stress this observation, which shows that their focus on the role of changes in $P_K$ can potentially account for simultaneous changes in both the gross and net capital shares, assuming that the gross elasticity of substitution $\sigma$ is greater than 1. In light of the existing micro estimates surveyed in Chirinko (2008), $\sigma > 1$ still appears unlikely, but certainly a gross elasticity $\sigma$ above 1 is much more plausible than a net elasticity $\tilde{\sigma}$ above 1, given the relationship between the two in (9).

### 4.2 A multisector model

The theory in section 4.1 enables a first-pass analysis of how the distribution of income is affected by various forces. It shows that accumulation of capital—all else equal—will likely result in a decline in the net capital share, since the net elasticity of substitution is almost certainly below one. This counters the central hypothesis of Piketty (2014). It also

\(^{27}\)For instance, one can write

$$\frac{\partial (\log ((F_K - \delta P_K) K / (F - \delta P_K K)))}{\partial (\log P_K)} = \frac{\partial (\log (F_K - \delta P_K))}{\partial (\log P_K)} + \frac{\partial (\log (K / F))}{\partial (\log F_K)} - \frac{\partial (\log (1 - \delta P_K K / F))}{\partial (\log P_K)}$$

$$= 1 - \sigma + \frac{\partial (\log (\delta P_K K / F))}{\partial (\log P_K)} \cdot \frac{\delta P_K K / F}{1 - \delta P_K K / F} = (1 - \sigma) + (1 - \sigma) \cdot \frac{\delta P_K K}{F - \delta P_K K} = (1 - \sigma) \cdot \frac{F}{F - \delta P_K K}$$
shows that a decline in the relative price $P_K$ of capital, holding the required return $r$ constant, will result in an increase in the net capital share if the gross elasticity of substitution is above 1—a claim that is still hard to reconcile with the bulk of empirical evidence, but for which Karabarbounis and Neiman (2014b) mount a spirited case.

Nevertheless, the one-sector model in section 4.1 is in many ways unsatisfactory as a model of the distribution between capital and labor. For instance, sections 2 and 3 demonstrated the decisive role of the housing sector in the long-term trajectory of the net capital share—but a one-sector model is by construction unable to account for a shift toward housing. Indeed, Piketty (2015) has recently voiced discomfort with the one-sector interpretation of the rising capital share, arguing that “the right model to think about rising capital-income ratios and capital shares in recent decades is a multi-sector model of capital accumulation.” In this section I will construct a tentative version of such a model.

**Designing a multisector framework.** Given the central role of housing in sections 2 and 3, it first seems important to distinguish between non-housing and housing output. If household preferences are homothetic in these two types of output, the household objective can be written as a monotonic transformation of a constant returns to scale aggregator $Z(Y_{nh}, Y_h)$ that takes non-housing and housing services as inputs. We can view $Z$ as the “top-level” production function for the economy.

For the non-housing sector, it will be useful to model the production process in a way that reflects the different types of capital studied in section 3 (equipment, structures, and land), so that the results from that disaggregation exercise can be used to inform the model. One natural approach is to assume that structures and land together provide “real estate” services that serve as an input to production, while labor and equipment together provide all other services.

Concretely, let $H(N, K_e)$ be a constant returns to scale aggregator combining labor $N$ and equipment $K_e$, and let $G_1(K_{s1}, L_1)$ be another constant returns to scale aggregator combining nonresidential structures $K_{s1}$ and land $L_1$. Finally, let $F$ be another constant returns to scale aggregator that combines $H$ and $G_1$, so that the consolidated production function for the non-housing sector takes the form

$$Y_{nh} = F(H(N, K_e), G_1(K_{s1}, L_1))$$

(14)

Following section 3, I assume that gross output in the non-housing sector is sold at some markup $\mu$ over marginal cost.
Similarly, suppose that residential structures $K_{s2}$ and land $L_2$ are combined by an aggregate $G_2(K_{s2}, L_2)$ to provide housing services, so that the production function for the housing sector takes the form

$$Y_h = G_2(K_{s2}, L_2) \quad (15)$$

Finally, as already mentioned, $Z$ combines $Y_{nh}$ and $Y_h$ into an aggregate that reflects household preferences:

$$Y = Z(Y_{nh}, Y_h) \quad (16)$$

This multisector economy captures the distinction between the non-housing and housing sectors, as well as all five forms of capital analyzed in section 3: equipment ($K_e$), nonresidential structures ($K_{s1}$), nonresidential land ($L_1$), residential structures ($K_{s2}$), and residential land ($L_2$).

The aggregate, nested structure of production in the economy is depicted in the tree below.

**Parametrizing the model.** For the sake of tractability, I will assume that the constant returns to scale aggregators ($Z, F, H, G_1, G_2$) take the constant elasticity of substitution form, potentially with factor-augmenting productivity that varies over time. Letting, for
instance, $Z^t$ denote the aggregator $Z$ at time $t$, I specify:

$$Z^t(Y^t_{nh}, Y^t_h) = \left( A^t_{nh} \cdot (Y^t_{nh})^{\sigma_{Z^{-1}}_{Z}} + A^t_h \cdot (Y^t_h)^{\sigma_{Z^{-1}}_{Z}} \right)^{\frac{1}{\sigma_Z}} \quad (17)$$

$$F^t(Y^t_H, Y^t_{G_1}) = \left( A^t_H \cdot (Y^t_H)^{\sigma_{F^{-1}}_{F}} + A^t_{G_1} \cdot (Y^t_{G_1})^{\sigma_{F^{-1}}_{F}} \right)^{\frac{1}{\sigma_F}} \quad (18)$$

$$G^t_1(K^t_{s1}, K^t_{l1}) = \left( A^t_{s1} \cdot (K^t_{s1})^{\sigma_{G_1^{-1}}_{G_1}} + A^t_{l1} \cdot (L^t_1)^{\sigma_{G_1^{-1}}_{G_1}} \right)^{\frac{1}{\sigma_{G_1}}} \quad (19)$$

$$G^t_2(K^t_{s2}, K^t_{l2}) = \left( A^t_{s2} \cdot (K^t_{s2})^{\sigma_{G_2^{-1}}_{G_2}} + A^t_{l2} \cdot (L^t_2)^{\sigma_{G_2^{-1}}_{G_2}} \right)^{\frac{1}{\sigma_{G_2}}} \quad (20)$$

$$H^t(N^t, K^t_e) = \left( A^t_n \cdot (N^t)^{\sigma_{H^{-1}}_{H}} + A^t_e \cdot (K^t_e)^{\sigma_{H^{-1}}_{H}} \right)^{\frac{1}{\sigma_H}} \quad (21)$$

I take final output from $Z$ to be the numeraire, and assume that the prices $P^t_{nh}, P^t_{s1},$ and $P^t_{s2}$ of reproducible capital in terms of this numeraire are exogenously fixed by technology, though potentially time-varying.\(^{28}\) The time-varying factor-augmenting shocks $A^t_i$ allow the model to fit the actual time series of value added for each sector perfectly; I take these time series, as well as the real interest rates for the non-housing and housing sectors, from the baseline decomposition exercise in 3, and back out the implied $A^t_i$.

Given these $A^t_i$, the CES functional forms (17)-(21) will allow for counterfactual simulation in section 4.3 of how the economy would have behaved given different reproducible capital prices $P^t_{nh}, P^t_{s1},$ and $P^t_{s2}$, different real interest rates, and so on.

**Elasticities of substitution.** To work with the multisector model, it is necessary to choose some values for the elasticities of substitution ($\sigma_Z, \sigma_F, \sigma_{G_1}, \sigma_{G_2}, \sigma_H$) in (17)-(21).\(^{29}\) These elasticities have been studied empirically to varying degrees:

- $\sigma_Z$ equals the elasticity of demand for housing services (as a share of total output) with respect to its price (relative to the aggregate price index for $Z$). Closely related elasticities of demand for housing have been studied in the literature, which has generally obtained relatively low values: for instance, Ermisch, Findlay and Gibb (1996) find values between 0.5 and 0.8 in a review of the literature, and themselves provide an estimate of 0.4. I tentatively set $\sigma_Z = 0.6$.

\(^{28}\)I will approximate the price of $Z$ in the data using the GDP deflator.

\(^{29}\)Note that these are all elasticities of substitution for gross production functions.
- $\sigma_F$, the elasticity of substitution between real estate and other services in the non-housing sector, does not map closely onto any empirically studied elasticity. In the absence of direct evidence, I will use $\sigma_F = 1$.

- $\sigma_{G_1}$ and $\sigma_{G_2}$ are the elasticities of substitution between structures and land in the non-housing and housing sectors, respectively. These elasticities play an important role in the urban economics literature, where substitutability between structures and land in the provision of real estate services is of great practical and theoretical interest. The more voluminous literature is for housing, $\sigma_{G_2}$, with a widely cited early entry by Muth (1971), who estimates $\sigma_{G_2} = 0.5$ using several approaches. More recently, Thorsnes (1997) surveys the literature and finds that recent estimates have generally been below 1, in the range $[0.5, 1]$; but he also argues that some of these estimates may be biased downward due to measurement error, and that the true elasticity may not be much below 1. This claim is seconded by Ahlfeldt and McMillen (2014). As a compromise, I set $\sigma_{G_2} = 0.8$. The literature for non-housing real estate, $\sigma_{G_1}$, is more scattered, with a range of elasticity estimates similar to that for housing—generally below one, but with concerns about bias from measurement error. Interpretation is complicated by the fact that non-housing real estate is much more heterogenous, spanning everything from high-rise office towers to farmland. Amid this uncertainty, I also choose $\sigma_{G_1} = 0.8$.

- $\sigma_H$ is the elasticity of substitution between equipment and labor. This is of great speculative interest—there are frequent discussions about the extent to which automation, for instance, can replace existing workers, and $\sigma_H$ governs the extent to which the decline in equipment prices documented by Karabarbounis and Neiman (2014b) will lead to substitution away from labor. In his survey, Chirinko (2008) reports a wide range of relevant estimates; the majority are still below one, but several are above one as well, and he suggests that the elasticity for equipment may be higher than the aggregate elasticity. Since $\sigma_H$ is such an important parameter, I will experiment with several different values: $\sigma_H = 0.5, 1, 1.5$.

As this discussion makes clear, estimating elasticities of substitution is difficult and imprecise; no single value for any $\sigma$ can be selected with confidence. At this stage, the goal is instead to explore the implications of the multisector model for plausible parameter values, and to see whether the results confirm or upturn the benchmark understanding from the one-sector model.
4.3 Counterfactual exercises

To better understand the forces driving the capital share in the US, I now use the multi-sector model from section 4.2 to perform counterfactual simulations. The idea is to study the impact of a given force influencing the capital share (for instance, the decline in real equipment investment prices) by simulating a counterfactual economy where that force is removed.

These are partial equilibrium simulations, holding real interest rates constant (or, in the case of scenario 3, varying real interest rates and holding all other features of the environment constant). I have not closed the model by making assumptions on household savings—although this lack of a general equilibrium side prevents the model from addressing all issues, it also avoids the need to specify household consumption/savings behavior, making the model and results easier to interpret.

Scenario 1: no change in equipment prices.  Figure 12 shows the relative price of equipment investment in the US, which has declined substantially over the last 50 years, particularly in the 1980s and 1990s. This decline in equipment prices is hypothesized by Karabarbounis and Neiman (2014b) and others to be a leading source of the rise in the net capital share.

Using the multisector model, I simulate the path that the net capital share would have taken conditional on the same paths of technology, relative capital prices, expected capital gains, markups, and real interest rates, with a single modification: the relative price of equipment investment is kept at its 1948 level for the entire 1948–2013 period, thereby eliminating the decline displayed in figure 12. I use the choice of elasticities in section 4.2, including all three possibilities $\sigma_H = 0.5, 1, 1.5$ for the elasticity of substitution between equipment and labor.

Figure 13 displays the results. It is no surprise that the net capital share changes in the wrong direction when $\sigma_H = 0.5$: in that case, removing the downward trend in equipment prices causes an increase in the value of equipment and its net capital income, leading to an even more spectacular rise in aggregate net capital income from 1980 to 2010 than in the actual time series.

Somewhat more interesting is the result for $\sigma_H = 1$, in which the path of net capital income is nearly unchanged in the counterfactual. This would be immediate in the one-sector model; there, (13) shows that when the gross elasticity is 1, the net capital share is invariant to changes in the relative price of capital. But this need no longer be true in the multisector model: although $\sigma_H = 1$ implies that the net shares of labor and equipment will move in tandem, it is possible for income shares to change in many other
Indeed, the simulation with $\sigma_H = 1$ does not precisely match the original net capital share—but the difference is so minor that in practice, the intuition from the one-sector model remains fully intact, which is an important result in its own right.

Finally, the case with a high elasticity $\sigma_H = 1.5$ is consonant with the Karabarbounis and Neiman (2014b) account: when the decline in equipment prices is removed, the upward trend in net capital income since the mid-70s disappears. If indeed $\sigma_H = 1.5$, this suggests that the observed rise in net capital income could have been caused by the decline in equipment prices. Figure 14, however, casts doubt on this explanation: the $\sigma_H = 1.5$ simulation achieves stable net income after 1975 only through a precipitous decline in the net capital share from equipment, offsetting the increase in the net capital share from all other sources (which is barely different in the counterfactual). In other words, if we believe the decline in equipment prices caused the rise in net capital share, we must accept a counterfactual in which capital income from equipment would otherwise have collapsed—and although this is hard to rule out, it does not seem like the most natural interpretation.

Scenario 2: no change in residential structures prices. This is analogous to the simulation in scenario 1, but inspired by the prominent role of housing in sections 2 and 3: rather than looking at equipment, I now simulate the path the net capital share would have taken if the relative price of investment in residential structures was kept at its 1948 level for the 1948–2013 period. Figure 15 shows the actual 1948–2013 trend, which has substantial appreciation in the relative price of investment, and figure 16 shows the counterfactual path of the net capital share when that trend is removed. The net capital share does show less of a long-term increase, but certainly it is not stable: there is still the overall U-shaped path and recent rise.

Figure 17 decomposes both the actual and counterfactual net capital share between housing and non-housing capital income. It reveals that aggregate net capital income is lower in the counterfactual solely because of a decline within housing, which is more capital-intensive than the rest of the economy; housing’s share, in turn, declines due to the low assumed elasticity $\sigma_Z = 0.6$ of substitution between housing and non-housing income, combined with the lower path of prices for residential investment.

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For instance, a higher equipment price makes the non-housing aggregate relatively more expensive, and since $\sigma_Z < 1$, this will lead to an increase in the share of the non-housing aggregate; this, in turn, has consequences for the net capital share.

This decomposition uses the methodology developed in section 3.

Since the elasticity of substitution between labor and equipment does not play a central role here, I show the results for only $\sigma_H = 1$. 
The figure 15 trend in residential investment prices, therefore, is a plausible contributor to the long-term rise in net capital income in the housing sector—but it does not explain the full increase in housing, much less the overall behavior of the net capital share.

Scenario 3: a different real interest rate. Finally, I evaluate the Piketty (2014) and Piketty and Zucman (2014) account of the capital share, which emphasizes the supply-side role of accumulated savings. In general equilibrium, increased savings influences capital income by pushing down the real interest rate; hence, to learn the sign of the effect of savings on the net capital share, it suffices to study the partial equilibrium effect of a change in the real interest rate.

For simplicity, I will look at the counterfactual effect of changes in the real interest rate starting from the 2013 data. Since the non-housing and housing sectors in the multisector model (following section 3) have different real interest rates, I assume that there is an equal additive shock to both rates. I report the results in table 3 for every combination of choice $\sigma_H = 0.5, 1, 1.5, \sigma_F = 0.5, 1, 1.5$, and $\sigma_Z = 0.5, 1, 1.5$ of the elasticities $\sigma_H, \sigma_F, \text{and } \sigma_Z$ in order to convey a sense of how the net capital share responds under a broad range of realistic parametric assumptions.

Results in table 3 are reported in elasticity form, namely as

$$\frac{\partial (\text{net capital share})}{\partial r} \bigg/ \left( \frac{\text{net capital share}}{r_{\text{ave}}} \right)$$

where the $r_{\text{ave}}$ rate scaling the denominator is the average real interest rate in the economy across the non-housing and housing sectors, weighted by value.

The outcome is unanimous: all simulated elasticities in table 3 are positive, meaning that in every case, a rise in the real interest rate causes an increase in the net capital share. As per the discussion above, this implies that in general equilibrium, increased savings will result in a decline, rather than an increase, in the net capital share, contradicting the mechanism proposed by Piketty (2014) and Piketty and Zucman (2014).

Some intuition for this outcome can be gleaned from the one-sector, one-good model in section 4.1, for which the conversion formula (9) shows how a given gross elasticity of substitution maps onto a smaller net elasticity of substitution. This net elasticity of substitution $\tilde{\sigma}$, in turn, governs the elasticity of the net capital share to a change in the real interest rate, which in the one-sector model is simply $1 - \tilde{\sigma}$: since net elasticities are smaller than gross elasticities, it is more likely that $1 - \tilde{\sigma}$ will be positive, as are the elasticities in table 3. Although the full intuition for the multisector model is much more
complicated, it appears to deliver the same basic outcome.

5 Conclusion

The story of the postwar net capital share is not a simple one. It has fallen and then recovered— with a large long-term increase in net capital income from housing, and a more volatile contribution from the rest of the economy. Outside of housing, it is difficult to explain the observed path of the net capital share via returns on the underlying assets. Instead, the decomposition in section 3 attributes most of the variation to pure profits, or markups.

Given the important role of housing, observers concerned about the distribution of income should keep an eye on housing costs— many urban economists, including Glaeser, Gyourko and Saks (2005) and Quigley and Raphael (2005), have documented how restrictions on land use and residential construction inflate the cost of housing.

Beyond housing, the results in this paper (if anything) tentatively suggest that concern about inequality should be shifted away from the split between capital and labor, and toward other aspects of distribution, such as the within-labor distribution of income. Although the net capital share has at times seen dramatic shifts both up and down, away from housing its long-term movement has been quite small, and there is not strong reason to suspect that this pattern will change going forward. Although there are several influential stories for why the capital share will experience a secular rise— most notably Piketty (2014) and Karabarbounis and Neiman (2014b)— sections 3 and 4 find these mechanisms difficult to reconcile with both theory and data.

Of course, the distribution between capital and labor will continue to be a salient issue: we surely have not seen the last of Ricardo (1821)’s principal problem of Political Economy. Time will tell whether it is also a principal problem for our own economy.

References


A Figures

![Graph of net capital share](image1)

Figure 1: Average net capital share of private domestic value added for G7 countries.

![Graph of gross capital share](image2)

Figure 2: Average gross capital share of private domestic value added for G7 countries.
Figure 3: Components of average net capital share of private domestic value added for G7 countries: housing (h) versus other (nh) sectors, weighted (w) and unweighted (uw).

Figure 4: Average net capital shares of corporate sector value added for G7 countries.
Figure 5: Net capital share of corporate sector value added in the US.

Figure 6: Decomposition of net capital share of corporate sector value added in the US: return on equipment, structures, land, and pure profits $\pi$. 


Figure 7: Estimated constant, linear, and quadratic time trends for the corporate rate of return $r(t)$.

Figure 8: Decomposition of net capital share of corporate sector value added in the US: return on equipment, structures, land, and pure profits $\pi$, using linear trend for $r(t)$. 
Figure 9: Ratio of total market value to the recorded value of equipment, structures, and land ("book value"), US corporate sector.
Figure 10: Net capital share of private value added in the US.

Figure 11: Decomposition of net capital share of private domestic value added in the US: return on equipment (eq), non-residential structures (st-nh), non-residential land (l-nh), pure profits $\pi$, residential structures (st-h), and residential land (l-h).
Figure 12: Real price of equipment investment in the US (relative to the GDP deflator).

Figure 13: Counterfactual paths for the net capital share of private value added in the US, assuming no change in the real price of equipment investment, versus the path of the actual share. Simulations for assumed elasticities of substitution between labor and equipment $\sigma_H = 0.5, 1, 1.5$. 
Figure 14: Decomposition of net capital share between net income on equipment (eq) and net income from all other sources (ne): actual series, and counterfactual simulation with no decline in equipment price ($\sigma_H = 1.5$).

Figure 15: Real price of residential structures investment in the US (relative to the GDP deflator).
Figure 16: Counterfactual paths for the net capital share of private value added in the US, assuming no change in the real price of residential structures investment, versus the path of the actual share.

Figure 17: Decomposition of net capital share between net capital income from housing (h) and net capital income from all other sources (nh): actual series, and counterfactual simulation with no rise in investment price for residential structures.
### Table 1: Decadal averages for the net capital share of private domestic value added, broken into housing and non-housing (“other”) components.

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### Table 2: Decadal averages for the net capital share of value added in the domestic corporate sector.

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<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Canada</strong></td>
<td>24.5%</td>
<td>26.1%</td>
<td>28.5%</td>
<td>24.3%</td>
<td>30.1%</td>
<td></td>
</tr>
</tbody>
</table>
Table 3: Elasticities of the simulated net capital share in 2013 with respect to the real interest rate, for various assumptions on $\sigma_H$, $\sigma_F$, and $\sigma_Z$.

<table>
<thead>
<tr>
<th>$\sigma_Z$</th>
<th>$\sigma_H$</th>
<th>$\sigma_F = 0.5$</th>
<th>$\sigma_F = 1$</th>
<th>$\sigma_F = 1.5$</th>
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<tr>
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<td>0.99</td>
<td>0.88</td>
<td>0.79</td>
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<tr>
<td>1.5</td>
<td>1.0</td>
<td>1.07</td>
<td>0.95</td>
<td>0.86</td>
</tr>
<tr>
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<td>1.5</td>
<td>0.99</td>
<td>0.88</td>
<td>0.79</td>
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<tr>
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<td>2.0</td>
<td>1.07</td>
<td>0.95</td>
<td>0.86</td>
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<tr>
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<td>1.07</td>
<td>0.95</td>
<td>0.86</td>
</tr>
<tr>
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<td>1.07</td>
<td>0.95</td>
<td>0.86</td>
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<td>0.95</td>
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<tr>
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</table>
C Description of alternative procedure in section 3.2 for estimating path of $r$.

I sketch here the procedure in section 3.2 for estimating the effective required return $r(t)$ on capital for the US corporate sector. I specify the model in continuous time, and use superscripts to denote the time $t$ for economy of notation. I am also more explicit here about how the underlying continuous time flows are aggregated into the measured flow for a given time period.

Assumed stochastic processes

I assume two stochastic processes beyond what is already visible in the data:

- $\pi^t$, which is a stationary, ergodic process for the share of gross output that goes to profits.

- $\zeta^t$, which reflects stochastic pricing error for the total market value of corporations, with mean 1 (where 1 corresponds to no error).

Core relations

First relation: profit share of flows. We know that all non-profit income will be allocated between depreciation, labor, and the various types of capital. This is a flow relation

$$(1 - \pi^t)Y^t = w^tL^t + \sum_i (\delta_i + r^t - g_{P_i}) P^t_i K^t_i$$

which can be rewritten as

$$\pi^t = 1 - \frac{w^tL^t}{Y^t} - \sum_i \frac{\delta_i P^t_i K^t_i}{Y^t} - \sum_i (r^t - g_{P_i}) \frac{P^t_i K^t_i}{Y^t}$$  \hspace{1cm} (23)$$

or, if we don’t want to divide by $Y^t$, as

$$\pi^t Y^t = Y^t - w^tL^t - \sum_i \delta_i P^t_i K^t_i - \sum_i (r^t - g_{P_i}) P^t_i K^t_i$$

Consolidating into an accumulated flow. Suppose that we write

$$\int_{t}^{t+\Delta t} \pi^s Y^s \, ds = \int_{t}^{t+\Delta t} (Y^s - w^sL^s - \sum_i \delta_i P^s_i K^s_i) \, ds - \sum_i \int_{t}^{t+\Delta t} (r^s - g_{P_i}) P^s_i K^s_i \, ds$$  \hspace{1cm} (24)$$
We can identify the first part as simply real net capital income during the period, while for the second term we must write
\[
\int_t^{t+\Delta t} (r^*-g_{P_i})P_i^sK_i^s \, ds \approx \left( (r^t+\Delta t - g_{P_i})P_i^{t+\Delta t}K_i^{t+\Delta t} + (r^t - g_{P_i})P_i^tK_i^t \right) \frac{\Delta t}{2} \quad (25)
\]

**Second relation: asset pricing.** The expected discounted value of the profit stream from time \( t \) onward is (in real terms)
\[
Y^t \cdot \int_0^\infty e^{(g_Y-\delta_\pi)s-\int_t^{t+s}r^u \, du} E_t[\pi^{t+s}] \, ds + \sum_i P_i^tK_i^t
\]
where \( \delta_\pi \) is the rate at which pure profits decay. (We can think of it as the rate at which a given company, for instance, on average loses the ability to make pure profits. There is no clear basis for picking \( \delta_\pi \), and I will choose \( \delta_\pi = .015 \), which implies a half-life of just below 50 years—within reason given the typical lifetimes of American corporations. Fortunately, the precise choice of \( \delta_\pi \) does not matter much for the results.)

This expected discounted value plus the value of capital itself is (again in real terms)
\[
Y^t \cdot \int_0^\infty e^{(g_Y-\delta_\pi)s-\int_t^{t+s}r^u \, du} E_t[\pi^{t+s}] \, ds + \sum_i P_i^tK_i^t
\]
I assume that the market value of the corporate sector equals this overall value times \( \zeta^t \), the multiplicative stochastic pricing error that has mean 1, follows a stationary, ergodic process, and is drawn independently of \( \pi_t \), \( \{P_i^t\} \), \( \{K_i^t\} \), and \( Y^t \):
\[
MV^t = \zeta^t \left( Y^t \cdot \int_0^\infty e^{(g_Y-\delta_\pi)s-\int_t^{t+s}r^u \, du} E_t[\pi^{t+s}] \, ds + \sum_i P_i^tK_i^t \right) \quad (26)
\]
Define \( OMOV^t \equiv MV^t - \sum_i P_i^tK_i^t \), and rewrite (26) as
\[
OMV^t = \zeta^t \left( Y^t \cdot \int_0^\infty e^{(g_Y-\delta_\pi)s-\int_t^{t+s}r^u \, du} E_t[\pi^{t+s}] \, ds \right) + (\zeta^t - 1) \sum_i P_i^tK_i^t \quad (27)
\]

**Second relation, part two: taking first differences.** Now use (27) to compute
\[
\frac{\phi(t)}{Y^{t-1,t}} \left( OMOV^t - e^{-\delta_\pi \Delta t - \int_t^{t+\Delta t}r^u \, du} \cdot OMOV^{t+\Delta t} \right) \quad (28)
\]
for some small \( \Delta t \), dividing by \( Y^{t-1,t} \) (to be defined later, but known at time \( t \)) and multiplying by any deterministic function \( \phi(t) \) of \( t \). Expanding the term inside the parentheses
in (28), we obtain
\[
OMV^t - e^{-\delta_t \Delta t - \int_t^{t+\Delta t} \pi^s du} OMV^{t+\Delta t} = \zeta^t \left( Y^t \int_0^{\Delta t} e^{(s - \delta_t) s - \int_s^{t+\Delta t} \pi^s du} \mathbb{E}_t[\pi^{t+s}] ds \right) - \zeta^{t+\Delta t} \left( e^{-\delta_t \Delta t} Y^{t+\Delta t} \int_0^\infty e^{(s - \delta_t) s - \int_s^{t+\Delta t} \pi^s du} \mathbb{E}_{t+\Delta t}[\pi^{t+s}] ds \right) + (\zeta^t - 1) Y^t \sum_i P_i^t K_i^t + e^{-\delta_t \Delta t - \int_t^{t+\Delta t} \pi^s du} (\zeta^{t+\Delta t} - 1) Y^{t+\Delta t} \sum_i P_i^{t+\Delta t} K_i^{t+\Delta t} \tag{29}
\]

Suppose now that we take the unconditional expectation of (28). Given the assumed independence of \( \zeta^t, Y^t, \) and \( \pi^t, \) (29) simplifies dramatically and we are left with
\[
\mathbb{E} \left[ \frac{\phi(t)}{Y_{t-1,t}} \left( OMV^t - e^{-\delta_t \Delta t - \int_t^{t+\Delta t} \pi^s du} \cdot OMV^{t+\Delta t} \right) \right] = \mathbb{E} \left[ \frac{\phi(t)}{Y_{t-1,t}} Y^t \int_0^{\Delta t} e^{(s - \delta_t) s - \int_s^{t+\Delta t} \pi^s du} \mathbb{E}_t[\pi^{t+s}] ds \right]
\]

We can further manipulate (30), using the law of iterated expectations to obtain
\[
\mathbb{E} \left[ \frac{\phi(t)}{Y_{t-1,t}} Y^t \int_0^{\Delta t} e^{(s - \delta_t) s - \int_s^{t+\Delta t} \pi^s du} \pi^{t+s} ds \right] = \mathbb{E} \left[ \frac{\phi(t)}{Y_{t-1,t}} \mathbb{E}_t \left[ \int_0^{\Delta t} (e^{s \pi^s} Y^t) e^{-\delta_t \Delta t - \int_s^{t+\Delta t} \pi^s du} \mathbb{E}_t[\pi^{t+s}] ds \right] \right]
\]
\[
= \mathbb{E} \left[ \frac{\phi(t)}{Y_{t-1,t}} \mathbb{E}_t \left[ \int_0^{\Delta t} e^{-\delta_t \Delta t - \int_s^{t+\Delta t} \pi^s du} \pi^{t+s} Y^{t+s} ds \right] \right]
\]
\[
= \mathbb{E} \left[ \frac{\phi(t)}{Y_{t-1,t}} \int_0^{\Delta t} e^{-\delta_t \Delta t - \int_s^{t+\Delta t} \pi^s du} \pi^{t+s} Y^{t+s} ds \right] \tag{31}
\]

Assuming that \( \Delta t \) is small enough and \( \pi^{t+s} Y^{t+s} \) is sufficiently close to being continuous, we can approximate the integral inside (31) by
\[
\int_0^{\Delta t} e^{-\delta_t \Delta t - \int_s^{t+\Delta t} \pi^s du} \pi^{t+s} Y^{t+s} ds \approx \frac{1}{2} e^{-\delta_t \Delta t - (r^t + r^{t+\Delta t})/2} \int_0^{\Delta t} \pi^{t+s} Y^{t+s} ds \tag{32}
\]

**Full estimation strategy.** We have shown that the unconditional expectation of (28), which we can approximate by
\[
\mathbb{E} \left[ \frac{\phi(t)}{Y_{t-1,t}} \left( OMV^t - e^{-\delta_t \Delta t - (r^t + r^{t+\Delta t})/2} OMV^{t+\Delta t} \right) \right] \tag{33}
\]
has unconditional expectation approximately equal to
\[
\mathbb{E} \left[ \frac{\phi(t)}{Y_{t-1,t}} \cdot \frac{1 + e^{-\delta_t \Delta t - (r^t + r^{t+\Delta t})/2}}{2} \cdot \int_0^{\Delta t} \pi^{t+s} Y^{t+s} ds \right] \tag{34}
\]
where according to (24) and (25), we can obtain the approximate flow of pure profits \( \int_0^{\Delta t} \pi^{t+s}Y^{t+s} \, ds \) in (34) as

\[
\int_0^{\Delta t} \pi^{t+s}Y^{t+s} \, ds \\
\approx \int_{t}^{t+\Delta t} \left( Y^s - w^sL^s - \sum_i \delta_i P_i^s K_i^s \right) ds - \sum_i \left( (r^{t+\Delta t} - gP_i) P_i^{t+\Delta t} K_i^{t+\Delta t} + (r^t - gP_i) P_i^t K_i^t \right) \frac{\Delta t}{2}
\]

where the first term is just the recorded net return on capital in the period \([t, t + \Delta t]\) as measured in the national accounts, while the second term can be derived from the nominal quantities \(P_iK_i\) of each type of capital.

The moments (33) and (34) are equal, and we can set the corresponding sample moments equal to each other. Generally I will look at an annual frequency, such that \(\Delta t = 1\). Given a functional form for \(r^t\) with \(n\) free parameters to be pinned down, we can choose \(n\) functions for \(\phi(t)\) to give us \(n\) sample moment conditions that determine those parameters.