Estimating Wage Rigidity for the International Wage Flexibility Project

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Nearly all previous attempts to use cross national data to assess the causes and consequences of wage rigidity have relied on estimates derived from macro data.[1] Typically these studies have estimated Phillips curve relationships and interpreted differences in the sensitivity of inflation to unemployment as indicative of the extent of wage rigidity (the less sensitive the more rigid wages are). Such an approach assumes wage rigidity takes the form of slow adjustment of wages to economic fundamentals. However, several other sources of wage rigidity have been suggested. Downward nominal wage rigidity only slows adjustment under certain circumstances and is likely due to different causes than generalized slow adjustment. Minimum wages or national bargains are yet other kinds of rigidity, and menu costs one more. The presence of one or more of these other types of rigidity could complicate or invalidate studies attempting to measure slow adjustment using macro data.

The initial phase of the International Wage Flexibility Project suggests an alternative to studies that use macro data. The 12 country teams all have access to micro data on individual earnings. Examination of distributions of individual earnings changes for the 12 countries for a number of years suggests that it may be possible to identify the presence and importance of a number of different types of rigidity using micro data on individual wage changes. Wage change distributions differ considerably across countries and even across time within some countries. It is possible that if we can find ways to summarize and quantify these differences in the wage change distribution that we will be able to identify measures of the extent of different types of wage rigidity and to relate them to their causes and consequences in a cross-country-time-series analysis. Doing this may contribute significantly to our understanding of how wage rigidity affects economic outcomes and the institutional sources of rigidity.

However, before we can measure wage rigidity using wage change distributions we need an accurate measure of the distribution of wage changes. In nearly all the data sets available to the IWFP the observed measure of earnings is distorted either by reporting and recording error, by an absence of accurate data on time worked to compliment earnings data, or by a divergence between the concept of wages we wish to

[1] See Holden and Wulfsberg (2004) for a recent exception. Several other efforts related to this project have been circulated in recent years as well.

measure and what is available (for example average hourly earnings including overtime when we would want base wage). To deal with these problems we need a way to transform the observed wage change distribution into an estimate of the true distribution without errors.

Section I of this memo describes an estimator for a semi-non-parametric representation of the underlying wage change distribution and for the parameters of the error process, and an estimator for the variance covariance-matrix of the parameters and of rigidity measures based on them. We dub this estimator a mixed method-of-moments estimator or MMM. The likelihood function for the problem is computationally intractable, given available resources. There is certainly more information available than needed for a just identified method-of-moments estimator, but estimation of the model by generalized-method-of-moments would be impractical given the computing resources available to the IWFP teams. The MMM estimator solves the problem of computational complexity by concentrating out the estimates of the parameters of the true wage change distribution and the one time varying parameter of the error distribution. Given the other parameters of the model it is possible to solve for these moments exactly. We then use an iterative process to minimize a GLS distance measure for the remaining moments which are much fewer in number and a function of only three parameters.

With these new estimates of the true wage change distribution we turn to the task of estimating the extent of different types of rigidity. We have developed four different ways to measure the extent of rigidity using distributions of wage changes. What is needed is some way of knowing what the distribution of wages would look like in the absence of rigidity and then that can be compared to the actual distribution. Candidate methods for establishing the counter factual of what the distribution would look like in the absence of rigidity are: 1. assume a particular form for the distribution and estimate a model of the true wage change distribution based on it, 2. assume that the effects of rigidity are seen only below the median and that the distribution would be symmetric in the absence of rigidity, 3. assume that the notional wage change distribution is fixed over time except for a changing mean, 4. assume that in the absence of changes in the extent of rigidity certain aspects of the wage change distribution are constant (such as skew, kurtosis, smoothness). We tried all four, but settled on the first as providing the most reliable estimates.

The methods described in sections I and II will not work if the proxy for wages is annual earnings divided by annual hours. Unless the typically annual wage change takes place at the same time as the survey, a year’s income reflects two different wages. This causes two problems. First, it induces a positive correlation between the change in earnings in one year and in the next year which violates one of the important assumptions of the error correction model. Second, the change in income from one year to the next confounds two changes in wages. So, for example, someone would have to go two years without a wage change in order to have no change in annual income for two years (unless the wage change was synchronized with the period of observation for income). Section III describes methods for adapting both our error correction model and the method that estimates rigidity using an ideal distribution to the case where we only observe annual income.

Finally, an appendix presents a list of the notation used in the first three sections, definitions of variables and parameters and a list of the page in the text where each is defined.

I. MMM Estimator for True Distribution

Our first attempt to develop parametric rigidity estimates using ML was not completely successful. That estimator could identify the extent of measurement error only because we assumed that we knew the form of the distribution of the notional wage changes and the measurement error. However, the non-normality of the notional wage change distribution – and probably also the error distribution – made this identification suspect. In particular, the identification of the extent of measurement error in the ML estimator probably depends on conditions on high order moments that are very sensitive to treatment of observations in the tails of the distribution. It would be good if this could be avoided.

While it is not possible to estimate the model without making some use of distributional assumptions, it is possible to estimate the true wage change distribution semi-non-parametrically with relatively innocuous distributional assumptions if we can assume that the only source of auto-correlation in wage changes is measurement error. Such an assumption allows us to identify measurement error variance without distributional assumptions. Only relatively minor distributional assumptions are then necessary to separately identify the frequency of the error and the variance of the error when it is made.

The assumption that all auto correlation in changes in log wages is due to measurement error is suggested by the findings of Abowd and Card (1989) who show that the best characterization of the stochastic process generating individual wages in US panel data is an ARIMA(0,1,1) – a process which is MA1 in first differences. Measurement error with no serial correlation added to a random walk will generate this sort of process. Since all the covariance in wage changes is due to the MA1 process, the assumption that the measurement error is the only source of serial correlation in wage changes is tantamount to assuming that any observed wage change that goes away within a year was an error. This is probably a reasonable point of departure for attempts to estimate the true wage change distribution. Calculations based on Bound and Krueger's (1999) estimates of the extent of measurement error in US survey data suggest that error could be sufficient to explain all the negative correlation between wage changes observed in data sets such as the PSID.[2]

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[2] Their estimated ratio of noise to signal plus noise in wage changes for men is 85% of our estimate for the PSID for 1987-89 (Table 6 "Classical measurement error"). Bound and Krueger also find a significant positive correlation in the errors in adjacent periods and a negative correlation between errors and wage levels for men. Both correlations are inconsequential for women. The presence of a positive correlation in the errors would lead us to underestimate the true extent of wage variance. Their point estimates suggest an underestimate of about 20 percent. If errors in levels are also correlated with changes in wages our estimated true wage distributions likely have more variance than the true distribution. Also, if this correlation exists the total variance of wage changes would be less than the sum of the signal and noise variance. Bound and Krueger don't report the variance of log wage changes in their data, but under the assumption that wages are a random walk it is possible to estimate the variance as the variance of the signal minus the covariance of the signal in adjacent years. The ratio of the noise to this variance is 60% greater than our estimate of this value. Although this is much larger than our estimates, it is for total income which

Further, we have now analyzed data produced by Gottschalk (forthcoming). He uses a regression discontinuity model in a data set with multiple wage observations a year for each person to separate true wage changes from errors.[3] We have computed the auto-correlation of his estimated true wage changes and find a statistically insignificant value of .006. If the assumption that true wage changes are uncorrelated is incorrect it probably isn't off by much.

A Description of the Estimator

The estimator works as follows. The underlying true change in log wage distribution is assumed to be a discrete. The log wage change variable can take one of 76 values from -.245 to .495 in steps of .01 or it can take the value zero. The histogram of observed wage changes for each pair of years in the data is calculated with the number of histogram cells in each year equal to the number of elements in the underlying discrete distribution. If wages are measured with error that is uncorrelated from year to year, the error will produce a negative covariance between wage changes in adjacent years. The magnitude of this covariance will depend on the frequency and the variance of the errors. We make an initial guess at the two parameters determining the frequency of errors and use them, and the auto-covariances, to compute the implied variance of the error in each period.

With an initial guess at the parameters of the error distribution, the estimates of the variance of the error distribution, and an assumption that errors, when made, are drawn from a two-sided Weibull distribution[4], it is possible to compute the fraction of observations associated with each element of the true (discrete) change-in-log-wage distribution that can be expected to be found in each cell of the observed histogram of log wage changes. These fractions can be arranged into a matrix with each row representing the fraction of the observations associated with each element of the discrete true change distribution that is expected to be found in each cell of the histogram of observed changes. One can then represent the expected number of observations in each cell of the histogram as the result of multiplying that matrix times the vector which contains the fraction of observations at each point in the discrete true change distribution. By inverting the matrix and multiplying it times the vector of the fraction of observations in each cell of the histogram one can estimate the fraction of observations generated by each element of the discrete true change-in-log-wage distribution.

Now we need some way to check our original guess about the frequency of errors and the curvature of the error distribution. When a large positive error is made it will at first cause a large increase in the observed wage. If errors are not correlated from one year to the next, the next year after a large increase due to an error there will likely be a large fall in the reported wage. Using the estimated true wage distribution and the parameters of the error distribution it is possible to compute the predicted number of such

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is probably measured less precisely than hourly wage (our measure in the PSID). The large differences between the error processes for men and women suggest that the model should be estimated separately by sex, but when we tried this in several IWFP data sets we found no practical or statistically significant differences between men and women.

[3] This method isn't available to us since our country teams have only annual wage observations.

[4] The Weibull distribution has support on interval $(0,\infty]$. The two-sided Weibull has support on the entire real number line excluding 0. The density is given by $d(x)=b^{-a}\,|x|^{a-1}\exp[-(|x|/b)^{a}]$.

changes from one year to the next at several points in the distribution (we call people who make such moves "switchers"). We use an iterative process to minimize a quadratic distance measure of the difference between the actual and predicted fraction of people "switching" from high to low, or low to high, as a function of the remaining parameters.

In theory we could have any number of such "switcher" moments however the cost of computing them is very high. As a compromise between the efficiency from the addition of more such moments and computational speed we chose to use two switcher moments, one centered around zero (with a switcher being someone who is on different sides of zero in the two periods) and one around 1% (with a switcher being someone who whose wage change is above 1% in one of the two periods and below it in the other). These values were chosen because they showed a fair amount of variation over the sample which was not as highly correlated as other values tried (for example zero and plus or minus 2% around zero).

Notation and Assumptions

[The appendix contains a list of all the variable names used in these notes, their meaning, and the point in the notes where they are defined.]

To begin, we will assume that the following process generates observed wages:

$$(1) \quad w_{it} = w_{it-1} + e_{it}$$

$$(2) \quad w^{o}_{it} = w_{it} + \eta'_{it} \quad where \quad \eta'_{it} = \begin{cases} 0 & if \;\; \mu_{it} > 0 \;\; or \;\; if \;\; \tau_i > 0 \\ \eta_{it} & otherwise \end{cases}$$

w_{it} is the natural log of the actual wage for person i at time t, and w^{o}_{it} is the natural log of the wage for that person at time t observed in the data. The random variable η_{it} will be assumed to be an i.i.d. two-sided Weibull with mean zero and parameters b_t and a, μ_{it} is assumed to be an i.i.d. random variable with a uniform distribution over the interval [-c, (1-c)], and τ_i is assumed to be an i.i.d. random variable with a uniform distribution over the interval [p-1, p][5]. We, will also assume that the e_{it}s are i.i.d. and drawn from a discrete distribution that can take one of K known values which we designate as the K length vector q. The probability that any e_{it} is equal to each of the K $q_j s$ is represented as the K vector m_t* and is *not* assumed to be known. Since the sum of the elements of m_t* equals one, only K-1 of them need be estimated. We denote the vector with the last element dropped m_t. We will denote the true wage change $d_{it}=w_{it}-w_{it-1}$ and observed wage change $d^{o}_{it}=w^{o}_{it}-w^{o}_{it-1}$ and the vector of such observations in time period t as d^{o}_{t}.

The data used to estimate the model will be log wages for T+1 sequential periods. We will denote the first period for which wage data are available period 0 and the last period T. Change in log wages will be computed between overlapping pairs of periods (ie. period 0 to period 1 and period 1 to period 2). Let N_t denote the number of

[5] This structure for the error – the two-sided Weibull with a fraction of people never making errors – was chosen to match the distribution of estimated errors in Gottschalk's data. His estimated errors had a distinctly peaked distribution and showed some auto-correlation in the probability of an error that was simply accounted for by having a group of people who didn't make errors.

individuals for whom data on wage changes are available from period t-1 to t, and $N_{t,t+1}$ will denote the number of people with wage change observations between periods t-1 and t and periods t and t+1. We will assume that the process generating missing data is random so that no bias is imparted by ignoring missing data in computing sample moments.

Define l and u as K vectors of upper and lower limits to categories of a histogram, chosen in advance, for the observed wage change data. Define the K-1 length column vector g_{it} to have zeros in all positions except j where $u_j > d^o_{it} \geq l_j$ for j =1 to K-1 (the sum of the g vectors across observations yields the frequency count of observations in the cells of a histogram). Define U_{tj} and L_{tj} as pairs of upper and lower limits for defining switchers (there will be (T-1)Q with Q being the number of pairs per year). Define h_{itj} as equal to 1 if $d^o_{it} > U_{tj}$ and $d^o_{it+1} < L_{tj}$ or if $d^o_{it} < L_{tj}$ and $d^o_{it+1} > U_{tj}$ and zero otherwise (a one indicating that the observation is a "switcher" in year t).

Assuming $b_0 = b_1$, *and* $b_{T-1} = b_T$[6] we can define a mixed method of moments estimator for a, p, c, the T-1 b_ts, and the (K-1)T m_ts as the values of those parameters that solve:

$$(3)\quad \begin{aligned} &0 = v_1 - E(v_1 \mid a,c,p,b_t \ \forall t = 1 \ to \ T-1 \quad and \quad m_t \ \forall t = 1 to \ T) \\ &\ and \\ &\min_{w.r.t.\,a,c,p} \quad (v_2 - E(v_2 \mid v_1,a,c,p))'\hat{\Lambda}_{v_2}(v_2 - E(v_2 \mid v_1,a,c,p\,)) \end{aligned}$$

where v_1, is the KT-1 column vector of sample moments that stacks the T vectors

$$(4)\quad v_{1t} = \begin{bmatrix} \sum_{i=1}^{N_t} g_{it} / N_t \\ \sum_{i=1}^{N_{t,t+1}} \dfrac{(d^o_{it} - \bar{d}_t)(d^o_{it+1} - \bar{d}_{t+1})}{N_{t,t+1} - 1} \end{bmatrix} \quad t = 1 \ to \ T-1 \qquad v_{1T} = \left[\sum_{i=1}^{N_T} g_{iT} / N_T \right],$$

v_2 is Q(T-1) and stacks the T-1 vectors

$$v_{2t} = \begin{bmatrix} \sum_{i=1}^{N_t} h_{it1} / N_{t,t+1} \\ \vdots \\ \sum_{i=1}^{N_t} h_{itQ} / N_{t,t+1} \end{bmatrix},$$

[6] We need a covariance of wage changes to identify b. We restrict the first and last pairs of *b* to be equal in order to allow us to use all the years of data on wage changes we have available. If we didn't do this we would have to restrict estimation of the true wage distribution to years t+1 through T-1 in order to have enough covariances to identify all the *b*s. When we tried this for the PSID results were essentially unchanged

$\bar{d}_t = \sum_{i=1}^{N_t} d^o_{it} / N_t$ and $\hat{\Lambda}_{v_2}$ is the sample covariance matrix of v_2.

Finally, we will denote $m^o_t = \sum_{i=1}^{N_t} \frac{g_{it}}{N_t}$.

This estimator will be consistent, but it will not be efficient. It is equivalent to GMM with infinite weight on the v_1 moments.

Deriving the Expectations

First note that

$$(5) \quad E(m^o_t) = (R_t - R_{t.K} 1'_{K-1}) m_t + R_{t.K}$$

where R_t is the (K-1)x(K-1) matrix with elements

$$(6) \quad \begin{aligned} R_{tij} &= (1-p)c^2\left(B_t\left(u_i - q_j\right) - B_t\left(l_i - q_j\right)\right) \\ &+ (1-p)c(1-c)\left[\left(W_t\left(u_i - q_j\right) - W_t\left(l_i - q_j\right)\right)\right. \\ &+ \left.\left(W_{t-1}\left(u_i - q_j\right) - W_{t-1}\left(l_i - q_j\right)\right)\right] \\ &+ [1-(1-p)(2c-c^2)]\, i(u_i \geq q_j > l_i), \end{aligned}$$

$R_{t.K}$ is the (K-1) vector, the elements of which are defined by (6) for i=1,K-1, and 1_{K-1} is a K-1 vector of ones. In (6)

$$B_t(x) = \int_{-\infty}^{\infty} \int_{-\infty}^{x-y} w(z \mid a, b_t) w(y \mid a, b_{t-1}) dz dy,$$

$$W_t(x) = \int_{-\infty}^{x} w(y \mid a, b_t) dy,$$

i(.) is an indicator function which returns the value 1 if the condition in parenthesis is true and zero otherwise, and w(x|a,b) denotes the two-sided Weibull density. In practice we will approximate these integrals (and several more below) using Gauss-Legendre quadrature (Judd 1998, p260).

Next, the covariance is

$$(7) \quad \begin{aligned} &Cov(d^o_{it}, d^o_{it+1}) = E([d^o_{it} - E(d^o_{it})][d^o_{it+1} - E(d^o_{it+1})]) = \\ &E([e_{it} - E(e_{it}) + \eta'_{it} - \eta'_{it-1}][e_{it+1} - E(e_{it+1}) + \eta'_{it+1} - \eta'_{it}]) = -E(\eta'^2_{it}) = \\ &-(1-p)c\,\sigma^2_t = -(1-p)c\, b^2_t \Gamma\left(1 + \frac{2}{a}\right) \end{aligned}$$

where Γ (x) is the gamma function. We will abbreviate $\Gamma_2 = \Gamma(1+2/a)$. Finally, the expected value of the fraction of switchers is given by

$$(8) \quad E\left(\sum_{i=1}^{N_{t,t+1}} h_{it} / N_{t,t+1}\right) = m_t^{*}{}' S_t m_{t+1}^{*}$$

where

$$\begin{aligned}
(9)\ S_{tij} = {} & (1-p)c^3\left[C_t\left(L_t - q_i, q_j - U_{t+1}\right) + C_t\left(q_i - U_t, L_{t+1} - q_j\right)\right] + \\
& (1-p)c^2(1-c)\left[D_2\left(L_t - q_i, q_j - U_{t+1} \mid a, b_t, b_{t-1}\right) + \right. \\
& \quad \left. D_1\left(L_{t+1} - q_j, q_i - U_t \mid a, b_t, b_{t-1}\right)\right] + \\
& (1-p)(1-c)c^2\left[D_2\left(L_{t+1} - q_j, q_i - U_t \mid a, b_t, b_{t+1}\right) + \right. \\
& \quad \left. D_1\left(L_t - q_i, q_j - U_{t+1} \mid a, b_t, b_{t+1}\right)\right] + \\
& (1-p)(1-c)^2 c\left[W_t\left(\min(L_t - q_i, q_j - U_{t+1})\right) + \right. \\
& \quad \left. W_t\left(\min(L_{t+1} - q_j, q_i - U_t)\right)\right] + \\
& (1-p)c^2(1-c)\left[W_{t-1}\left(L_t - q_i\right)W_{t+1}\left(q_j - U_{t+1}\right) + W_{t+1}\left(L_{t+1} - q_j\right)W_{t-1}\left(q_i - U_t\right)\right] + \\
& (1-p)(1-c)^2 c\left[i\left(L_t > q_i\right)W_{t+1}\left(q_j - U_{t+1}\right) + W_{t+1}\left(L_{t+1} - q_j\right)i\left(U_t < q_i\right)\right] + \\
& (1-p)c(1-c)^2\left[W_{t-1}\left(L_t - q_i\right)i\left(U_{t+1} < q_j\right) + i\left(L_{t+1} > q_j\right)W_{t-1}\left(q_i - U_t\right)\right] + \\
& [p + (1-p)(1-c)^3]\left[i\left(L_t > q_i\right)i\left(U_{t+1} < q_j\right) + i\left(L_{t+1} > q_j\right)i\left(U_t < q_i\right)\right],
\end{aligned}$$

where

$$C_t(x,y) = \int_{-\infty}^{\infty}\int_{\eta_t - x}^{\infty}\int_{\eta_t - y}^{\infty} w(\eta_{t-1} \mid a, b_{t-1}) w(\eta_t \mid a, b_t) w(\eta_{t+1} \mid a, b_{t+1}) d\eta_{t+1} d\eta_{t-1} d\eta_t,$$

$$D_1(x, y \mid a, b_1, b_2) = \int_{-\infty}^{x}\int_{-\infty}^{\eta_1 + y} w(\eta_2 \mid a, b_2) w(\eta_1 \mid a, b_1) d\eta_2 d\eta_1, \ and$$

$$D_2(x, y \mid a, b_1, b_2) = \int_{-\infty}^{y}\int_{\eta_1 - x}^{\infty} w(\eta_2 \mid a, b_2) w(\eta_1 \mid a, b_1) d\eta_2 d\eta_1.$$

Solving the Model

If we use a 76 element distribution to approximate the true wage change distribution, then we only need a couple of years of data before solving for the 2+TK parameters using standard non-linear estimation procedures becomes unmanageable. However, the form of the problem allows us to solve for the T(K-1) m_ts as functions of the other parameters. Similarly, we can solve for the T-1 b_ts as functions of the covariances and the other parameters. These can then be substituted out of the remaining 3 equations making a solution practical.

Specifically, we can substitute the estimate of *m*

$$(10) \quad m_t^e = (R_t^{-1} - R_{t.K} 1'_{K-1})^{-1}(m_t^o - R_{t.K})$$

for m_t in the remaining 2+T equations in (3) and solve for the *b*s as.

$$(11)\quad b_t = \sqrt{\left(\frac{-\hat{\sigma}^2_{d_t d_{t+1}}}{c(1-p)}\right) \Big/ \Gamma_2}$$

where

$$\hat{\sigma}^2_{d_t d_{t+1}} = \sum_{i=1}^{N_{t,t+1}} \frac{(d_{it} - \bar{d}_t)(d_{it+1} - \bar{d}_t)}{N_{t,t+1} - 1}.$$

This leaves only a, c and p to be found by minimizing the quadratic distance measure. Since the quadrature formula we use to compute the integrals in (6) and (9) causes discontinuities in the derivatives of the objective function we use a modified version of Powell's method, a non-derivative method, to minimize the distance measure. Since we encountered a great deal of difficulty with local minimums we started Powell's method from an extensive grid search on all three parameters.

Variance-Covariance Matrix for Parameters

To find the covariance matrix for the parameters we linearize and stack the solution to the first TK-1 moment conditions and the 3 first order conditions for the minimization in (3) to get TK+2 equations

$$(12)\quad Z(v - v^*) - M(\hat{\beta} - \beta^*) =$$

$$\left[\begin{array}{c:c} I_{TK-1} & \begin{matrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{matrix} \\ \hdashline \begin{matrix} 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \end{matrix} & \left.\dfrac{\partial E v_2}{\partial \beta_2}\right|_{\sim\beta_2}' \hat{\Lambda}^{-1}_{v_2} \end{array}\right] \left[\begin{array}{c} v_1 - v_1^* \\ \hdashline v_2 - v_2^* \end{array}\right] - \left[\begin{array}{c:c} \dfrac{\partial E v_1}{\partial \beta_1} & \dfrac{\partial E v_1}{\partial \beta_2} \\ \hdashline \left.\dfrac{\partial E v_2}{\partial \beta_2}\right|_{\sim\beta_2}' \hat{\Lambda}^{-1}_{v_2} \dfrac{\partial E v_2}{\partial \beta_1} & \left.\dfrac{\partial E v_2}{\partial \beta_2}\right|_{\sim\beta_2}' \hat{\Lambda}^{-1}_{v_2} \dfrac{\partial E v_2}{\partial \beta_2} \end{array}\right] \left[\begin{array}{c} \hat{\beta}_1 - \beta_1^* \\ \hdashline \hat{\beta}_2 - \beta_2^* \end{array}\right],$$

where

$$\beta_1 = \left[\begin{array}{c} m_{1,1} \\ \vdots \\ m_{T,K-1} \\ \hdashline b_1 \\ \vdots \\ b_{T-1} \end{array}\right], \quad \beta_2 = \begin{bmatrix} a \\ p \\ c \end{bmatrix},$$

and $\left.\dfrac{\partial E v_2}{\partial \beta_2}\right|_{\sim\beta_2}$ denotes the derivative of the second set of theoretical moments with respect to each element of β_2 holding constant only the other elements of β_2 with the elements of

β_1 concentrated out. Using equation (12) we can approximate the variance-covariance matrix of the parameters as

$$\text{(13)} \quad \hat{\Omega} = M^{-1} Z \hat{\Lambda} Z' M^{-1}{}'$$

were the matrix Λ can be estimated as

$$\hat{\Lambda}_{jk} = \sum_{i=1}^{N} \frac{(v_{ij} - \bar{v}_j)(v_{ik} - \bar{v}_k)}{(N_j - 1)(N_k - 1)}$$

N_i is the number of observations used to construct the *i*th moment and N is the total number of individuals in the sample. Missing observations are set equal to their mean so that any observation i with missing values for j or k will contribute nothing to the jkth covariance. The derivatives of the moments with respect to the parameters can be calculated numerically.

II. Estimating Rigidity

We have previously proposed the following taxonomy of sources of wage rigidity:

Nominal Rigidity
- Symmetric rigidity at zero due to menu costs
- Asymmetric or downward nominal rigidity or resistance to wage cuts

Real Rigidity
- Resistance to real wage cuts or downward real rigidity
- Insensitivity of real wages to economic conditions

Institutional Rigidity
- Statutory minimum wages, and
- Collectively bargained wages.

For both types of nominal rigidity and downward real rigidity we can develop specific estimates of the fraction of people subject to such rigidity.

Evidence for rigidity will be discerned by comparing the estimated true wage distribution to a counterfactual estimate of what the distribution would look like in the absence of the hypothesized rigidity. We have attempted to approach this problem of constructing a counter-factual in four ways: 1. assume an ideal type and estimate the parameters of the distribution from moments that are assumed not to be affected by rigidity (for example the 90^{th} and 75^{th} percentile of the distribution), 2. assume that the effects of rigidity are seen only below the median and that the distribution would be symmetric in the absence of rigidity, 3. assume that the notional wage change distribution is fixed over time except for a changing mean, 4. assume that in the absence of changes in the extent of rigidity certain aspects of the wage change distribution are constant (such as skew, kurtosis, smoothness). However, symmetry based measures could not be estimated for many years when reasonable measures of the expected rate of inflation put it above the median wage change for several countries. The method of assuming a constant wage change distribution (following Kahn 1997) did not work because in many countries there was insufficient variation in the median of the distribution to allow us to identify the extent of all three types of rigidity. Clear differences emerged between countries in several aspects of the shape of the distribution and these were clearly related to differences in wage setting institutions, but it proved difficult to relate these aspects of the distribution systematically to wage rigidity. Thus we settled on the first method.

Estimates Based on an Ideal Distribution

Examination of both the estimated true distributions we have seen so far in developing the estimator, and an analysis of Gottschalk's estimates of true wages, suggests that wage changes have a distribution that is both more peaked and has fatter tails than the normal. The half of the distribution above the median appears to be well approximated by a Weibull distribution. The lower tail, in countries where real rigidity does not appear to be much of a problem, seems to be a mirror image of the upper tail for those parts that are above zero when the distribution is not affected by real rigidity. Thus it seems reasonable to model the notional wage change distribution as symmetric with the shape of a Weibull.

The Weibull is a three parameter distribution with support on the positive real numbers with cumulative distribution function $P(d<x) = 1-\exp(-([x-\mu]/a_t)^{c_t})$ where μ, a and c are the parameters. If it is assumed that the notional wage change density is symmetric around the mean it could be given by $f_t(x) = .5\, a_t^{1-c_t} c_t\, |(x-\mu)/a_t|^{c_t-1} \exp(-(|x-\mu|/a_t)^{c_t})$. The integral of this density (excluding the singularity at x=μ) yields the cumulative wage change distribution for the two-sided Weibull. Denote that cumulative distribution function for period t $F_t(x)$.

We assume that the true wage change is determined from the notional wage change by the following process. Person i's notional wage change in period t d^n_{it} will be a draw from the distribution for period t. From the notional wage change the notional real adjusted wage change will be computed as

$$(14) \qquad d^r_{it} = \begin{cases} d^n_{it} & if \quad \varepsilon^r_{it} > \rho_t \\ \max(d^n_{it}, \pi^e_{it}) & otherwise \end{cases}$$

where π^e_{it} is a normally distributed random variable with mean π^e_t and variance $\sigma_t^{\pi^e}$ (with cumulative distribution function in period t $\Phi_t(x)$) representing the expected rate of inflation determining this wage, ε^r_{it} is an i.i.d. random variable that is drawn from a uniform distribution on the unit interval, and ρ_t is the probability of an individual being subject to downward real wage rigidity if the notional wage is less than the expected rate of inflation. The true wage change is then determined as

$$(15) \quad d^T_{it} = \begin{cases} 0 & if\ (d^r_{it} \le 0\ and\ \varepsilon^n_{it} < n_t)\ or\ (-.01 \le d^r_{it} \le .01\ and\ \varepsilon^1_{it} < s_{1t}) \\ & or\ (-.02 \le d^r_{it} < -.01\ or\ .01 < d^r_{it} \le .02\ and\ \varepsilon^2_{it} < s_{2t}) \\ d^r_{it} & otherwise \end{cases}$$

where ε^n_{it}, ε^1_{it}, and ε^2_{it}, are all uniform distributed random variables with support on the unit interval, n_t is the probability of being subject to downward nominal rigidity if one's notional real adjusted wage is less than zero, and s_{1t} and s_{2t} are the probability of being subject to symmetric nominal rigidity if notional real adjusted wage changes are close enough to zero. Unless the true wage change is zero, it is then rounded to its nearest masspoint in the sequence {-.245, -.235, …, .485, .495}.

Given this model of wage changes we represent the expected mass at each point in m^c_t as

$$(16) \quad E(m^c_{jt}) = \begin{cases} j < -2 & (1-n_t)x_{jt} \\ j = -2 & (1-s_{2t})(1-n_t)x_{jt} \\ j = -1 & (1-s_{t1})(1-n_t)x_{jt} \\ j = 0 & s_{2t}x_{2t} + s_{1t}x_{1t} + (s_{1t} + n_t - n_t s_{1t})x_{-1t} \\ & + (s_{2t} + n_t - n_t s_{2t})x_{-2t} + n_t \sum_{k=-25}^{-3} x_{kt} \\ j = 1 & (1-s_{1t})x_{jt} \\ j = 2 & (1-s_{2t})x_{jt} \\ j > 2 & x_{jt} \end{cases}$$

where

$$(17) \quad x_{jt} = [1-\rho_t + \rho_t \Phi_t(l_j)][F_t(u_j) - F_t(l_j)] + \rho_t[\Phi_t(u_j) - \Phi_t(l_j)]F_t(u_j) \cdot$$

and u_j = (.01)(j+1) and lj = (.01)j for j<0 and u_j = (.01)j and lj = (.01)(j-1) for j>0. The x_{jt} represents the fraction of notional real adjusted wages that would fall in interval j of the histogram if there was no downward or symmetric nominal rigidity. This is the fraction of notional wage changes that would fall in the interval ($F_t(u_j)$-$F_t(l_j)$) times the fraction of people in that interval not affected by real rigidity. Those will include people whose ε^r_{it} is too large as well as those whose $\varepsilon^r_{it}<\rho_t$ but whose expected rate of inflation lies below the lower end of the interval. This is added to the fraction of people subject to real rigidity whose expected rate of inflation falls in the interval and whose notional wage change is in or below the interval.

This model can be estimated by GMM using the estimated m^cs as the observed moments and their covariance matrix as the weights. We estimate the model for each year for each data set used in the project. The program described below also allows structural break modeling to choose sub-periods over which to estimate the model holding rigidity parameters constant. In some years in some data sets where real rigidity is relatively unimportant the estimator tries to use the real rigidity regime to create a denser upper tail than that predicted by the symmetric Weibull. To avoid this we specify relatively narrow ranges for the expected inflation parameter based on actual inflation and simple predictions of inflation.

III. Annual Income Data

When instead of wage observations we observe annual income we have two additional problems. First, if all wage changes aren't exactly synchronized with the period over which income is observed the overlap of the effects of wage changes from one year to the next will cause positive auto-covariance in the observed wage changes which will prevent us from using the error correction method used for wages. This can be seen if we assume that each person gets at most one wage change a year and define income as

(18) $$y_{it} = x_i\, w_{it-1} + (1-x_i)\, w_{it}$$

where w_{it} is person i's wage at time t and x_i is the fraction of the year that passes before a wage change takes place. With this, if observed income is written as

(19) $$y^o_{it} = y_{it} + \eta'_{it}$$

the observed change in income from one year to the next will be

(20) $$d^{y0}_{it} = x_i\, e_{it-1} + (1-x_i)\, e_{it} + \eta'_{it} - \eta'_{it-1}$$

where e_{it} is the innovation in wages in period t and η'_{it} is the observation error in period t. As with wages the presence of the observation error will tend to induce negative autocorrelation in the income data, but the presence of the lagged wage innovation will tend to induce an offsetting positive autocorrelation as long as the wage change isn't synchronized with the wage change (x=0 or x=1).

Using income data produces another problem as well. The confounding of two wage change distributions in each income change distribution makes it impossible to use any of the measures of rigidity proposed in the last section. Here we propose solutions to both problems.

Correcting the Income Change Histogram

The same general correction technique can be used, but another method must be found to estimate the parameters of the error distribution used to compute the correction matrix. For the wage change histogram correction we used the auto-covariance and counts of the number of switchers, but the positive auto-correlation of the income changes makes the computation of the expected number of switchers intractable. However, similar information can be recovered from the auto-covariance of higher order moments. If these are used we must also use higher order moments of the wage changes to identify some higher order moments of the error and wage change distribution. Thus we propose to use the following six moments, constructed for each year and pair of years for which we have data in order to identify the parameters of the error distribution. Those moments and their expected values are (dropping the subscripts and superscripts on d^{yo}_{it} except for those denoting the time period):

$$\mathrm{cov}(d_t, d_{t-1}) = \sum_{j=1}^{12} \omega_j x_j (1-x_j)\sigma^2_{e_{t-1}} - (1-p)c_{t-1}\sigma^2_{\eta_{t-1}} + O(1/N^2)$$

$$\mathrm{var}(d_t) = \sum_{j=1}^{12} \omega_j [x_j^2 \sigma^2_{e_{t-1}} + (1-x_j)^2 \sigma^2_{e_t}] + (1-p)c_t \sigma^2_{\eta_t} + (1-p)c_{t-1}\sigma^2_{\eta_{t-1}} + O(1/N^2)$$

$$\begin{aligned}
E((d_t - \bar{d}_t)^2(d_{t-1} - \bar{d}_{t-1})^2) = \sum_{j=1}^{12} \omega_j [& x_j^2(1-x_j)^2 \sigma^4_{e_{t-1}} + x_j \sigma^2_{e_{t-1}} \{x_j^3 \sigma^2_{e_{t-2}} + x_j(1-p)c_{t-1}\sigma^2_{\eta_{t-1}} \\
&+ x_j(1-p)c_{t-2}\sigma^2_{\eta_{t-2}} + 4(1-x_j)(1-p)c_{t-1}\sigma^2_{\eta_{t-1}}\} + (1-x_j)\sigma^2_{e_t} \\
&\{x_j^2(1-x_j)\sigma^2_{e_{t-2}} + (1-x_j)^3 \sigma^2_{e_{t-1}} + (1-x_j)(1-p)c_{t-1}\sigma^2_{\eta_{t-1}} \\
&+ (1-x_j)(1-p)c_{t-2}\sigma^2_{\eta_{t-2}}\} + (1-p)c_t \sigma^2_{\eta_t} \{x_j^2 \sigma^2_{e_{t-2}} + (1-x_j)^2 \sigma^2_{e_{t-1}} \\
&+ c_{t-1}\sigma^2_{\eta_{t-1}} + c_{t-2}\sigma^2_{\eta_{t-2}}\} + (1-p)c_{t-1}\sigma^2_{\eta_{t-1}} \{x_j^2 \sigma^2_{e_{t-2}} \\
&+ (1-x_j)^2 \sigma^2_{e_{t-1}} + c_{t-2}\sigma^2_{\eta_{t-2}}\} + (1-p)c_{t-1}\sigma^4_{\eta_{t-1}}] + O(1/N^2)
\end{aligned}$$

$$\begin{aligned}
(21)\ E((d_t - \bar{d}_t)^3(d_{t-1} - \bar{d}_{t-1})) = \sum_{j=1}^{12} \omega_j [& x_j^3(1-x_j)\sigma^4_{e_{t-1}} + x_j(1-x_j)\sigma^2_{e_{t-1}} 3\{(1-x_j)^2 \sigma^2_{e_t} + (1-p)c_t \sigma^2_{\eta_t} \\
&+ (1-p)c_{t-1}\sigma^2_{\eta_{t-1}}\} - (1-p)c_{t-1}\sigma^4_{\eta_{t-1}} - (1-p)c_{t-1}\sigma^2_{\eta_{t-1}} 3\{x_j^2 \sigma^2_{e_{t-1}} \\
&+ (1-x_j)^2 \sigma^2_{e_t} + c_t \sigma^2_{\eta_t}\}] + O(1/N^2)
\end{aligned}$$

$$\begin{aligned}
E((d_t - \bar{d}_t)(d_{t-1} - \bar{d}_{t-1})^3) = \sum_{j=1}^{12} \omega_j [& x_j(1-x_j)^3 \sigma^4_{e_{t-1}} + x_j(1-x_j)\sigma^2_{e_{t-1}} 3\{x_j^2 \sigma^2_{e_{t-2}} + (1-p)c_{t-2}\sigma^2_{\eta_{t-2}} \\
&+ (1-p)c_{t-1}\sigma^2_{\eta_{t-1}}\} - (1-p)c_{t-1}\sigma^4_{\eta_{t-1}} - (1-p)c_{t-1}\sigma^2_{\eta_{t-1}} 3\{x_j^2 \sigma^2_{e_{t-2}} \\
&+ (1-x_j)^2 \sigma^2_{e_{t-1}} + c_{t-2}\sigma^2_{\eta_{t-2}}\}] + O(1/N^2)
\end{aligned}$$

$$E(d_t - \overline{d}_t)^4 = \sum_{j=1}^{12} \omega_j [x_j^4 \sigma_{e_{t-1}}^4 + x_j^2 \sigma_{e_{t-1}}^2 6\{(1-x_j)^2 \sigma_{e_t}^2 + (1-p)c_t \sigma_{\eta_t}^2 + (1-p)c_{t-1}\sigma_{\eta_{t-1}}^2\}$$
$$+ (1-x_j)^4 \sigma_{e_t}^4 + (1-x_j)^2 \sigma_{e_t}^2 6\{(1-p)c_{t-1}\sigma_{\eta_{t-1}}^2 + (1-p)c_t \sigma_{\eta_t}^2\}]$$
$$+ (1-p)c_t \sigma_{\eta_t}^4 + 6(1-p)c_{t-1}\sigma_{\eta_{t-1}}^2 c_t \sigma_{\eta_t}^2 + (1-p)c_{t-1}\sigma_{\eta_{t-1}}^4$$
$$+ O(1/N^2)$$

where the ωs are fraction of the population with wage changes in each month and, assuming that wage changes take place only on the first of each month, $x_j = (j-1)/12$, where all observations are made on the first of the first month. As with the wage change model we assume that errors are made by a fraction (1-p) of the population with probability c_t in period t so that $\sigma^2_{\eta' t} = \sigma^2_{\eta t}(1-p)c_t$. Given our sample sizes we will ignore the terms of order $1/N^2$.

The parameters σ^2_{et}, σ^4_{et} (for t beyond the first period), $\sigma^2_{\eta t}$ and $\sigma^4_{\eta t}$ are themselves functions of other parameters we will estimate. We assume that the errors have a two-sided Weibull distribution so $\sigma^i_{\eta t}=b^i_{\eta t}\Gamma(1+i/a)$ where $b^2_{\eta t}$ and a are the parameters we will be estimating. The parameters σ^2_{et} and σ^4_{et} can be computed recursively given the parameters of the true income change distribution and the initial values of σ^2_{et} and σ^4_{et}

$$(22) \quad \sigma^i_{e_t} = \frac{\sum_{j=1}^{K} m^{e*}_{tj}\left(\frac{u_j - l_j}{2} - \sum_{k=1}^{K} m^{e*}_{tk}\frac{u_k - l_k}{2}\right)^i - \sum_{j=1}^{12}\omega_j x_j^i \sigma^i_{e_{t-1}}}{\sum_{j=1}^{12}\omega_j (1-x_j)^i}.$$

note that the m^{e*}_{t}s are themselves functions of $\sigma^2_{\eta t-1}$, $\sigma^2_{\eta t}$, c_{t-1}, c_t and p.

The empirical counterparts to these theoretical moments are:

$$(23) \quad \begin{aligned}
\hat{E}((d_t - \overline{d}_t)(d_{t-1} - \overline{d}_{t-1})) &= \sum_{i=1}^{N_{t,t-1}} (d_{it} - \overline{d}_t)(d_{it-1} - \overline{d}_{t-1}) / N_{t,t-1} \\
\hat{E}((d_t - \overline{d}_t)^2) &= \sum_{i=1}^{N_t} (d_{it} - \overline{d}_t)^2_{it} / N_t \\
\hat{E}((d_t - \overline{d}_t)^2 (d_{t-1} - \overline{d}_{t-1})^2) &= \sum_{i=1}^{N_{t,t-1}} (d_{it} - \overline{d}_t)^2 (d_{it-1} - \overline{d}_{t-1})^2 / N_{t,t-1} \\
\hat{E}((d_t - \overline{d}_t)^3 (d_{t-1} - \overline{d}_{t-1})) &= \sum_{i=1}^{N_{t,t-1}} (d_{it} - \overline{d}_t)^3 (d_{it-1} - \overline{d}_{t-1}) / N_{t,t-1} \\
\hat{E}((d_t - \overline{d}_t)(d_{t-1} - \overline{d}_{t-1})^3) &= \sum_{i=1}^{N_{t,t-1}} (d_{it} - \overline{d}_t)(d_{it-1} - \overline{d}_{t-1})^3 / N_{t,t-1} \\
\hat{E}((d_t - \overline{d}_t)^4) &= \sum_{i=1}^{N_t} (d_{it} - \overline{d}_t)^4 / N_t
\end{aligned}.$$

The second and fourth moment can be computed for each year, and the other four moments can be computed for each pair of years giving 6T-4 moments to compute 2T+6 parameters. If there are inadequate numbers of moments, restrictions can be imposed on the parameters.

To estimate the parameters of the error distribution we minimize the distance measure

(24) $$(E^* - T^*)'\hat{V}^{-1}(E^* - T^*)$$

with respect to the 2T+6 parameters where E* is a 6T-4 vector of empirical moments, T* is the conforming vector of corresponding theoretical moments, and $\hat{V}$ is the empirical covariance matrix of the moments.

Estimating Rigidity

The fact that each income change reflects two wage changes, unless the period of observation is synchronized with the period of the wage change, makes it impossible to use any of the symmetry based estimates. However, the ideal distribution based estimator can be extended to deal with annual income data.

If we assume that the underlying wage change densities are the same for each month of the year for which an observation is made, that the densities are piecewise uniform in one percentage point change increments (except for a mass point at zero which will be treated in what follows as an infinitesimal interval), and if we continue to assume that wage changes are independent, then the expected fraction of observations of the true income change distribution in each cell of a histogram can be represented as

(25) $$m^y_t = \Psi \, vec(m_{t-1} \, m_t')$$

where vec is a function which turns a KxK matrix into a K^2x1 vector and Ψ is a KxK^2 matrix with elements

(26) $$\Psi_{i,K(l-1)+k} = \Psi_{i,l,k} = P[l_i < d^y_t < u_i \mid l_k < e_t < u_k, l_l < e_{t-1} < u_l]$$

where *l* and *u* are the upper and lower bounds for cells of a K length histogram. Under the assumption that e_t is piecewise uniform Ψ will depend only on the weights ω thus allowing the representation of m^y_t in (31).

For the case with 75 1 percentage point wide cells and a mass point at zero in position 26 of the histogram the elements of Ψ can be computed as

(27) $$\Psi_{i,l,k} = \sum_{j=1}^{12} \omega_j P[l_i < d^y_t < u_i \mid l_k < e_t < u_k, l_l < e_{t-1} < u_l, x_j]$$

where

(34) $P[l_i < d_t^y < u_i \mid l_k < e_t < u_k, l_l < e_{t-1} < u_l, x_j] =$

$$
\left\{
\begin{array}{ll}
\text{if } (u_i - x_j l_l)/(1-x_j) < l_k \text{ or } (l_i - x_j u_l)/(1-x_j) \ge u_k & 0 \\
\text{if } l_k \le (u_i - x_j l_l)/(1-x_j) < u_k \text{ and } (u_i - x_j u_l)/(1-x_j) < l_k & \frac{.5}{(u_k - l_k)(u_l - l_l)}\left(\frac{u_i - x_j l_l}{1-x_j} - l_k\right)\left(\frac{u_i - (1-x_j) l_k}{x_j} - l_l\right) \\
\text{if } l_k \le (u_i - x_j l_l)/(1-x_j) < u_k \text{ and } l_k \le (u_i - x_j u_l)/(1-x_j) < u_k & \frac{1}{u_k - l_k}\left(\frac{u_i - x_j(l_l + \frac{u_l - l_l}{2})}{1-x_j} - l_k\right) \\
\text{if } u_k \le (u_i - x_j l_l)/(1-x_j) \text{ and } (u_i - x_j u_l)/(1-x_j) < l_k & \frac{1}{u_l - l_l}\left(\frac{u_i - (1-x_j)(l_k + \frac{u_k - l_k}{2})}{x_j} - l_l\right) \\
\text{if } u_k \le (u_i - x_j l_l)/(1-x_j) \text{ and } l_k \le (u_i - x_j u_l)/(1-x_j) < u_k & 1 - \frac{.5}{(u_k - l_k)(u_l - l_l)}\left(u_k - \frac{u_i - x_j u_l}{1-x_j}\right)\left(u_l - \frac{u_i - (1-x_j) u_k}{x_j}\right) \\
\text{if } l_k \le (l_i - x_j l_l)/(1-x_j) < u_k \text{ and } (l_i - x_j u_l)/(1-x_j) < l_k & 1 - \frac{.5}{(u_k - l_k)(u_l - l_l)}\left(\frac{l_i - x_j l_l}{1-x_j} - l_k\right)\left(\frac{l_i - (1-x_j) l_k}{x_j} - l_l\right) \\
\text{if } l_k \le (l_i - x_j l_l)/(1-x_j) < u_k \text{ and } l_k \le (l_i - x_j u_l)/(1-x_j) < u_k & \frac{1}{u_k - l_k}\left(u_k - \frac{l_i - x_j(l_l + \frac{u_l - l_l}{2})}{1-x_j}\right) \\
\text{if } u_k \le (l_i - x_j l_l)/(1-x_j) \text{ and } (l_i - x_j u_l)/(1-x_j) < l_k & \frac{1}{u_l - l_l}\left(u_l - \frac{l_i - (1-x_j)(l_k + \frac{u_k - l_k}{2})}{x_j}\right) \\
\text{if } u_k \le (l_i - x_j l_l)/(1-x_j) \text{ and } l_k \le (l_i - x_j u_l)/(1-x_j) < u_k & \frac{.5}{(u_k - l_k)(u_l - l_l)}\left(u_k - \frac{l_i - x_j u_l}{1-x_j}\right)\left(u_l - \frac{l_i - (1-x_j) u_k}{x_j}\right) \\
\text{if } u_k \le (u_i - x_j u_l)/(1-x_j) \text{ and } (l_i - x_j l_l)/(1-x_j) < l_k & 1
\end{array}
\right.
$$

Using the Ψ matrix defined this way it is possible to compute the expected distribution of changes in income given the ideal distribution models of the wage change distribution allowing GMM estimation of the underlying parameters.

To construct the Ψ matrix requires knowledge of the distribution of wage changes over the months of the year. Different IWFP teams used different methods to obtain this information. The Swiss conducted a small labor force survey. Several countries had data on the timing of wage changes in contracts, and others used employer surveys.

Appendix: Notation

Duplication of variable and parameter names has been avoided in the first two sections, but some duplication was unavoidable in the section on rigidity estimates. The page number in parenthesis after each definition tells where in the text the symbol is defined and first used.

a	parameter of the 2 sided Weibull distribution that determines curvature (p4 footnote)
a_t	scaling parameter for the Weibull density used to model the true change in log wage distribution (p11)
b_t	parameter of the 2 sided Weibull distribution that scales the argument (p4 footnote)
$B_t(x)$	the integral of the product of two two-sided Weibull distributions (p7)
c	parameter of the error distribution that is equal to the probability that someone who is prone to errors makes one (p5)
c_t	the curvature parameter of the true change in log wage distribution for one of the rigidity estimators (p11)
$C_t()$	function yielding the integral of three two-sided Weibull densities (p8)
d_{it}	the change in person *i*s log wage between t and t-1 (d_t is the N_t vector of the d_{it}s) (p6)
d^o_{it}	the change in person *i*s observed log wage between t and t-1 (d_t^o is the N_t vector of the d_{it}^o s) (p5)
$D_j()$	function yielding the integral of two two-sided Weibull densities (p8)
e_{it}	log wage change for person *i* from period t-1 to period t (p5)
E*	the spike based estimator of downward nominal wage rigidity (p16)
E()	expectation operator (p7)
$f_t(x)$	the density for the true change in log wage distribution in period t (p11)
$F_t(x)$	cumulative 2-sided Weibul distribution function for true wage changes in period t (p11)
g_{it}	a K-1 length column vector that is zero except in the position corresponding to person *i*s period t wage change (where it equals 1) (p6)
h_{itj}	a variable equal to 1 if person i is defined as a switcher by criteria j in period t (zero otherwise) (p6)
i	index number normally differentiating individuals (p5)
j	index number normally differentiating elements of a vector (p5)

K	the number of discrete mass points in the true wage change distribution (p5)
l	a K vector of lower bounds for wage change categories (p6)
L_t	the number of constraints imposed on the estimated wage change distribution (p8)
L_{tj}	The lower limit for the *j*th definition of switchers in period t (p6)
m_t	a K-1 vector giving the probability that any e_{it} is equal to each of the first K-1 *q*s (p5)
m_t*	a K vector that has m_t as the first K-1 elements and one minus the sum of the other elements as the last element (p5)
m^c_t	the K-1 vector of constrained estimates of *m* (p12)
m^e_t	the K-1 vector of estimated *m*s (p9)
m^o_t	the K-1 vector of observed *m*s (p7)
M	a matrix of coefficients in a linear expansion of the estimator's normal equations (p9)
N_t	the number of observed wage changes between period t and t-1 (p5)
N_j	the number of observations used to construct the *j*th moment (p10)
$N_{t,t+1}$	the number of individuals with observed wage changes in both period t and t+1 (p6)
n_t	parameter for the fraction of observations affected by DNWR and DRWR in period t (p12)
O	a K vector used to transform *m** into *m* (p14)
p	parameter of the error distribution that determines the probability that someone is not prone to reporting errors (p5)
q_j	location of the *j*th element of the true wage distribution (p5)
Q	the number of ways in which switchers are measured (p6) also the set of cells around the expected rate of inflation in the Kahn estimator (p17)
R_{tij}	an element of the KxK matrix R_t whose *ij*th element is the probability that an true wage change of q_j will be observed in the *i*th range of the empirical frequency distribution in period t (when subscripted only with *t* the matrix is K-1xK-1 leaving out the last row and column, when subscripted *t.K* it is a K-1 vector containing the last column) (p7)
S_{tij}	an element of the matrix S_t which is used to compute the expected number of switchers in period t (p8)
s_{1t}	parameter for the fraction unaffected by menu costs within one percent of zero (p12)
s_{2t}	parameter for the fraction of wage changes unaffected by menu costs within two percent of zero but more one percent (p12)

t index number for time period (p5)

T the number of periods for which wage changes are measured (one minus the number of periods for which wages are measured (p5)

u a K vector of upper bounds for wage change categories (p6)

U_{tj} The upper limit for the *j*th definition of switchers in period t (p6)

v_1 a TK-1 vector of the moments (when a t subscript is present it is the subvector containing only those parameters for year t) (p6)

v_2 a vector with the elements a, p, and α (p6)

w_{it} log of person *i*s true wage in period t (p5)

w^o_{it} log of person *i*s observed wage in period t (p5)

w() two-sided Weibull density function (p7)

$W_t(x)$ integral of the two-sided Weibull from minus infinity to x (p7)

Z a matrix of coefficients in the linear expansion of the normal equations for the estimator (p9)

β a vector that stacks all the parameters of the model (subscripts 1 and 2 denote particular subsets of the parameters) (p9)

$\Phi_\sigma(x)$ cumulative normal distribution with variance sigma (p11)

$\Gamma(x)$ the gamma function (the integral from zero to one with respect to t of the function ln(1/t)^(x-1) (Γ_2 is used to designate $\Gamma(1+2/a)$) (p8)

η_{it} measurement error in the log wage of person i in period t if there is measurement error (p5)

η'_{it} actual measurement error in log wage in period t (p5)

Λ the covariance matrix of the empirical moments (p6)

μ the mean of the true change in log wage distribution (p11)

μ_{it} uniform random variable on $[-c_t,(1-c_t)]$ which determines whether a person prone to errors makes one in period t (p5)

π^e_t parameter for the expected rate of inflation in period t (p11)

ρ_t parameter for the fraction of observations affected by real rigidity (p11)

σ_t variance of η (the error when an error is made) (p7)

τ_i uniform random variable on [-p,(1-p)] which determines whether a person is prone to errors in reporting wages (p5)

Ω covariance matrix of the model parameters (p10)